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[L]Master Thesis[C][R]Alessandro Bellotta [L]USI - Università della Svizzera Italiana, MSc in Finance[C][R] fancy plain 0.4pt 0.4pt

- Università della Svizzera Italiana, Lugano
Facoltà di Scienze Economiche
MSc in Finance
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Alessandro Bellotta Supervised by: Prof. Patrick Gagliardini, PhD Second Reader: Prof. Paul Schneider, PhD February 16, 2017February 2017
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1 Abstract

The aim of this thesis is to develop a broad analysis on option pricing models such as Black-Scholes model, Heston stochastic volatility model and jump-diffusion model. In addition we want also to analyse a non-ordinary theory that is called Fractal Market Hypothesis in which we also price options with, the so-called, Fractal Brownian Motion.
What we want to show is a computational approach to this field: we start from theoretical and mathematical formulations and then we apply IT models in order to implement the theory; the choosen software is R.
The aim is not only to plug-in numbers in already-built function but also to develop step-by-step the theory and then to create algorithms to achieve the task (pricing) that are calibrated on real-world data and that are able to adapt when another input is chosen.
What we are calling pricing, is, for the sake of preciseness, a computation of future expected values; starting from this concept, an investor could implement its own strategy.

2 Introduction

This paragraph is aimed to introduce the following work.
What we want to achieve is both an academic and practical analysis and, in addition, a not-so-conventional approach to a fundamental financial field.
Our objective is to deliver a broad analysis about option pricing and to analyse something that is not mainstream and widespread.
First we want to present models from the theory and from a mathematical perspective. In order to achieve this goal, we will deliver a broad but concise analysis of models: model formulation, useful mathematics, properties and derivation.
In a second step, we explain what was done in order to implement the theory with a computational approach: here, we explain formulae, methodologies, calibration and results that we get.
In the third step we provide code chunks that are useful in order to understand how the analysis evolved.
How is this thesis built?
First of all we define an option: to be more precise, in this thesis we deal with European Call Option.
Then, we start with models: the Black-Scholes model, the Heston model, the jump-diffusion model.
In the end we analyse the Fractal Market Hypothesis as an alternative to the Efficient Market Hypothesis: in order to do this we first define "fractal" with some examples, then we perform a "Rescaled Range Analysis" that aims to find the famous Hurst Exponent (H) and, in the end, we price a call option by applying fractal Brownian motion.
In addition, we deliver an analysis in a Monte Carlo framework and an appendix with mathematics useful for this work.

3 Options: Basic Concepts

3.1 What is an Option?

A derivative asset can be defined as a financial instrument whose value depends on the value of other underlying variables. The variables underlying derivatives can belong to different asset classes such as traded securities, interest rates, volatility and so on.
There exist two big classes of options: call options and put options.
From this dichotomy, we can analyse whether the position is long or short: it is long when the investor has bought the option; it is short when the investor has written the option. A call option gives the holder the right to buy an asset by a certain date (maturity date, T) for a certain price (strike price, K). The payoff at maturity is:

equation call 0.2cm payoff $= \max(\mathrm{S}_T - K, 0)$