ALEC CHENG PRE-SESSION MATH TASKS PAGE 1

1.1
$$\frac{z^{17}}{z^3 \cdot z^5} = \frac{z^{17}}{z^8} = \boxed{z^9}$$

1.2
$$6^2 \cdot 6^x = 6^6$$
$$6^{2+x} = 6^6$$
$$2+x = 6$$
$$\boxed{x=4}$$

1.3
$$x \cdot y = 5$$
$$x^3 \cdot y^3 = (x \cdot y)^3$$
$$= 5^3$$
$$= \boxed{125}$$

1.4
$$\frac{\sqrt{2^{10}}}{\sqrt{4^3}} = \frac{2^5}{\sqrt{2^6}} = \frac{2^5}{2^3} = 2^2 = \boxed{4}$$

1.5
a) TRUE
b) TRUE
c) FALSE
d) TRUE

1.6
$$\frac{2x-5}{2} \geq 4$$
$$2x \geq 13$$
$$\boxed{x \geq \frac{13}{2}}$$

2.1
$$(x_1, y_1) = (0, 32)$$
$$(x_2, y_2) = (100, 212)$$
$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$
$$y - 32 = \frac{212-32}{100-0}(x-0)$$
$$y = \frac{9}{5}x + 32$$
$$\Rightarrow x = y$$
$$x = \frac{9}{5}x + 32$$
$$-32 = \frac{4}{5}x$$
$$\boxed{x = -40^\circ C = -40^\circ F}$$

2.2
$$f(y) = 24 = 5x + 4$$
$$20 = 5x$$
$$\boxed{x = 4}$$

2.3
$$10^{x^2-2x+2} = 100 = 10^2$$
$$x^2 - 2x + 2 = 2$$
$$x^2 - 2x = 0$$
$$x(x-2) = 0$$
$$\boxed{x = 0 \quad x = 2}$$

2.4
$$(1.03)^x \geq 2$$
$$\ln(1.03)^x \geq \ln 2$$
$$x \ln 1.03 \geq \ln 2$$
$$x \geq \frac{\ln 2}{\ln 1.03}$$
$$\boxed{x = 24}$$

2.5
$$\ln(1/e)$$
$$\ln 1 - \ln e$$
$$0 - 1$$
$$\boxed{-1}$$

3.1
$$\sum_{i=0}^{\infty}\left(\frac{1}{8}^i + 0.5^i\right)$$
$$\sum_{i=0}^{\infty}\frac{1}{8}^i + \sum_{i=0}^{\infty} 0.5^i$$
$$\frac{1}{1-\frac{1}{8}} + \frac{1}{1-0.5}$$
$$\frac{8}{7} + 2 = \boxed{\frac{22}{7}}$$

3.2 $\lim_{x \to 3} \frac{x-3}{2} = \frac{3-3}{2} = \boxed{0}$

3.3 $f(x) = x^2 - 4$

$f'(x) = 2x \qquad (-1,-3)$

$f'(-1) = 2(-1) = \boxed{-2}$

3.4 $\frac{d}{dx} \frac{x^2+3}{x+2} = \frac{f(x)}{g(x)}$

$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

$= \frac{2x(x+2) - (x^2+3)}{(x+2)^2}$

$= \frac{2x^2 + 4x - x^2 - 3}{x^2 + 4x + 4}$

$= \boxed{\frac{(x^2+4x-3)}{(x^2+4x+4)}}$

$=$

3.5 $f(x) = 4x^3 + 4$

$f'(x) = 12x^2$

$f''(x) = \boxed{24x}$

3.6 No. It is undefined at $x = 0$

3.7 $f(x) = 3x^3 - 9x$

$f'(x) = 9x^2 - 9$

$9x^2 - 9 = 0$

$x^2 - 1 = 0$

$(x-1)(x+1) = 0$

$x = -1, 1$

$P_1(-1, 6) \qquad P_2(1, -6)$

$f''(x) = 18x$

$f''(-1) = -18 \qquad f''(1) = 18$

P_1 : local maxima ~~loca~~

P_2 : local minima

3.7 cont $f''(x) = 18x = 0$

$x = 0$

$P_3\ (0,0)$: inflection point

$(-\infty, 0)$ locally concave

$(0, \infty)$ locally convex //

3.8 $f(x,y) = x^2 y^3$

$f(2,3) = 2^2\, 3^3$

$= 4 \cdot 27$

$= \boxed{108}$

3.9 $f(x,y) = \ln(x-y)$

Domain of ln is $(0, \infty)$

$x - y > 0$

$\therefore \forall x, y$ ST $x > y$ //

3.10 $\frac{\partial^2}{\partial x^2} x^5 + xy^3$

$\frac{\partial}{\partial x} 5x^4 + y^3$

$\boxed{20x}$

3.11 $f(x,y) = \sqrt{xy} - 0.5x - 0.5y$

$f'_x = \frac{\sqrt{y}}{2\sqrt{x}} - 0.5 \qquad f''_x = \frac{-\sqrt{y}}{4\sqrt{x^3}}$

$f'_y = \frac{\sqrt{x}}{2\sqrt{y}} - 0.5 \qquad f''_y = \frac{-\sqrt{x}}{4\sqrt{y^3}}$

$f'_{xy} = \frac{1}{4\sqrt{xy}}$

$\frac{\sqrt{y}}{2\sqrt{x}} - 0.5 = 0 \qquad \frac{\sqrt{x}}{2\sqrt{y}} - 0.5 = 0$

$\sqrt{y} = \sqrt{x} \qquad \sqrt{x} = \sqrt{y}$

$y = x \qquad x = y$

3.12 max x^2y^2 ST $x+y=5$

$$\mathcal{L} = x^2y^2 - \lambda(x+y-5)$$

$$\frac{\partial \mathcal{L}}{\partial x} = 2xy^2 - \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial y} = 2x^2y - \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = -x-y+5 = 0$$

$$x = 5-y$$

$$2xy^2 - 2x^2y = 0$$

$$2(5-y)y^2 - 2(5-y)^2y = 0$$

$$10y^2 - 2y^3 - 50y + 10y^2 - 2y^3 = 0$$

$$2y^3 - 15y^2 + 25y = 0$$

$$y(2y^2 - 15y + 25) = 0$$

$$y(2y-5)(y-5) = 0$$

$$y = 0, \tfrac{5}{2}, 5$$

$$x = 5, \tfrac{5}{2}, 0$$

When x or $y = 0$; $x^2y^2 = 0$

$$(x,y) = \left(\tfrac{5}{2}, \tfrac{5}{2}\right)$$

$$x^2y^2 = \left(\tfrac{5}{2}\right)^2\left(\tfrac{5}{2}\right)^2 = \frac{25}{4}\cdot\frac{25}{4} = \boxed{\frac{625}{16}}$$

4.1 $\begin{bmatrix} 2 & 3 \\ 4 & 1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 4 & 1 \\ 2 & 1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 8 & 11 & 8 \\ 6 & 17 & 6 \\ 5 & 6 & 5 \end{bmatrix} //$$

4.2 $\begin{bmatrix} 1 & 4 & 1 \\ 2 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 4 & 1 \\ 1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 19 & 9 \\ 10 & 11 \end{bmatrix} //$$

4.3 $\begin{bmatrix} 3.3 & 5.1 & 4.7 \\ 2 & 6.1 & 1.23 \\ 4 & 5.74 & 0 \end{bmatrix}^T$

$$= \begin{bmatrix} 3.3 & 2 & 4 \\ 5.1 & 6.1 & 5.74 \\ 4.7 & 1.23 & 0 \end{bmatrix} //$$

4.4 $\det\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = (2)(5) - (3)(4)$

$$= 10 - 12$$

$$= \boxed{-2}$$

5.1 {HHHH, HHHT, HHTH, HHTT,
HTHH, HTHT, HTTH, HTTT,
THHH, THHT, THTH, THTT,
TTHH, TTHT, TTTH, TTTT} //

5.2 P(A): drug user
P(B): positive drug test

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)}$$

$$= \frac{(0.99)(0.01)}{(0.99)(0.01) + (0.005)(0.99)}$$

$$= \boxed{\frac{2}{3}}$$

5.3 Expected sum of 1 die

$$S = \frac{1+2+3+4+5+6}{6} = 3.5$$

$\therefore$ Expected sum of 2 is 7 //