The following is a derivation of the equation for a two dimensional density distribution of a Bose-Einstein Condensate (BEC) and thermal cloud.

We begin by recognizing that our density distribution is simply the sum of the density distribution of the thermal cloud, n_{ex}, and the density distribution of the BEC, n_0. Thus:

$$n\left(x,y\right)=n_0\left(x,y\right)+n_{ex}\left(x,y\right) \tag{1}$$

Let's derive an equation for the BEC's density distribution first. Pethick and Smith Eq. 2.32 gives us that:

$$n_0\left(x,y\right)=N_0\left|\psi_0\left(x,y\right)\right|^2 \tag{2}$$

where N_0 is the number of particles in the BEC.

Furthermore, based on Pethick and Smith Eq. 2.33's three dimensional formula, we can describe the ground-state wave function of a two dimensional anisotropic harmonic oscillator with the following equation:

$$\psi_0\left(x,y\right)=\frac{1}{\sqrt{\pi a_x a_y}}e^{\frac{-x^2}{2a_x^2}}e^{\frac{-y^2}{2a_y^2}} \tag{3}$$

where a_i represents the width of the wave function in the direction i given by the square root of Pethick and Smith Eq. 2.34:

$$a_i=\sqrt{\frac{\hbar}{m\omega_i}} \tag{4}$$

Squaring this wavefunction gives us:

$$\left|\psi_0\left(x,y\right)\right|^2=\frac{1}{\pi a_x a_y}e^{\frac{-x^2}{a_x^2}}e^{\frac{-y^2}{a_y^2}} \tag{5}$$

Pethick and Smith Eq. 2.29 tells us that the number of particles in a BEC for a two dimensional harmonic oscillator potential is given by:

$$N_0=\left\{\begin{array}{ll} N\left[1-\left(\frac{T}{T_c}\right)^2\right], & \text{if } T<T_c \\ 0, & \text{if } T\geq T_c \end{array}\right\} \tag{6}$$

where N is the total number of particles in the system and T_c is the transition temperature, which Pethick and Smith define as "the highest temperature at which the macroscopic occupation of the lowest-energy state appears." When T is greater than T_c, we don't expect to see any particles in the condensate.

Now, substituting Eq. 5 into Eq. 2, we find that:

$$n_0\left(x,y\right) = N_0 \frac{1}{\pi a_x a_y} e^{\frac{-x^2}{a_x^2}} e^{\frac{-y^2}{a_y^2}} \tag{7}$$

Before continuing further, we want to make all of our quantities dimensionless for easier analysis. Let's establish these dimensionless quantities:

$$\tilde{kT} \equiv \frac{kT}{\hbar\omega} \tag{8}$$

$$\tilde{kT_c} \equiv \frac{kT_c}{\hbar\omega} \tag{9}$$

$$\tilde{x} \equiv \frac{x}{a_x} \tag{10}$$

$$\tilde{y} \equiv \frac{y}{a_y} \tag{11}$$

As a small aside, using Pethick and Smith Eq. 2.19, we can define the unitless transition temperature energy, $\tilde{kT_c}$, in a two dimensional system as:

$$\tilde{kT_c} = \left(\frac{3N}{\pi^2}\right)^{\frac{1}{2}} \tag{12}$$

Let's substitute these dimensionless quantities into Eq. 6 and Eq. 7:

$$N_0 = \left\{ \begin{array}{ll} N\left[1-\left(\frac{\tilde{kT}}{\tilde{kT_c}}\right)^2\right], & \text{if } T < T_c \\ 0, & \text{if } T \geq T_c \end{array} \right\} \tag{13}$$

$$n_0\left(x,y\right) = N_0 \frac{1}{\pi a_x a_y} e^{-\tilde{x}^2} e^{-\tilde{y}^2} \tag{14}$$

Multiplying Eq. 14 by $a_x a_y$, we find a dimensionless quantity, $\tilde{n_0}$, for a two dimensional BEC density distribution:

$$\tilde{n_0}(x,y) = N_0\frac{1}{\pi}e^{-\tilde{x}^2}e^{-\tilde{y}^2} \tag{15}$$

where:

$$\tilde{n_0}(x,y) = a_x a_y n_0(x,y) \tag{16}$$

Now, let's derive an equation for the density distribution of the thermal cloud. Pethick and Smith 2.39 gives us:

$$n_{ex}(x,y) = \frac{N_{ex}}{\pi R_x R_y}e^{\frac{-x^2}{R_x^2}}e^{\frac{-y^2}{R_y^2}} \tag{17}$$

where R_i is given by Pethick and Smith 2.40:

$$R_i^2 = \frac{2kT}{m\omega_i^2} \tag{18}$$

and i represents direction.

After multiplying by $\frac{\hbar\omega}{\hbar\omega}$ and rearranging, Eq. 18 can be rewritten as:

$$R_i^2 = \frac{2kT}{\hbar\omega_i}\frac{\hbar}{m\omega_i} \tag{19}$$

This can be further simplified using Eq. 4 and Eq. 8:

$$R_i^2 = 2k\tilde{T}a_i^2 \tag{20}$$

Finally, substituting Eq. 10 and Eq. 11 into this equation and rearranging, we get:

$$\frac{x^2}{R_x^2} = \frac{\tilde{x}^2}{2k\tilde{T}} \tag{21}$$

$$\frac{y^2}{R_y^2} = \frac{\tilde{y}^2}{2k\tilde{T}} \tag{22}$$

Now, let's substitute Eq. 21 and Eq. 22 into Eq. 17:

$$n_{ex}(x,y) = \frac{N_{ex}\tilde{x}\tilde{y}}{2\pi k\tilde{T}xy}e^{\frac{-\tilde{x}^2}{2k\tilde{T}}}e^{\frac{-\tilde{y}^2}{2k\tilde{T}}} \tag{23}$$

Pethick and Smith Eq. 2.27 gives us an equation for the number of particles in the thermal cloud:

$$N_{ex} = \left\{ \begin{array}{ll} N\left(\frac{T}{T_c}\right)^2, & \text{if } T < T_c \\ N, & \text{if } T \geq T_c \end{array} \right\} \tag{24}$$

where we can easily replace the temperatures with unitless quantities, giving us:

$$N_{ex} = \left\{ \begin{array}{ll} N\left(\frac{\tilde{kT}}{\tilde{kT_c}}\right)^2, & \text{if } T < T_c \\ N, & \text{if } T \geq T_c \end{array} \right\} \tag{25}$$

Multiplying Eq. 23 by $a_x a_y$, we find a dimensionless quantity, $\tilde{n_{ex}}$, for a two dimensional thermal cloud density distribution:

$$\tilde{n_{ex}}\left(x, y\right) = N_{ex}\frac{1}{2\pi\tilde{kT}}e^{\frac{-\tilde{x}^2}{2\tilde{kT}}}e^{\frac{-\tilde{y}^2}{2\tilde{kT}}} \tag{26}$$

where:

$$\tilde{n_{ex}}\left(x, y\right) = a_x a_y n_{ex}\left(x, y\right) \tag{27}$$

Finally, we can substitute Eq. 15 and Eq. 26 into Eq. 1 to find a dimensionless quantity, $\tilde{n}$, for a two dimensional density distribution including a BEC and thermal cloud:

$$\tilde{n}\left(x, y\right) = N_0\frac{1}{\pi}e^{-\tilde{x}^2}e^{-\tilde{y}^2} + N_{ex}\frac{1}{2\pi\tilde{kT}}e^{\frac{-\tilde{x}^2}{2\tilde{kT}}}e^{\frac{-\tilde{y}^2}{2\tilde{kT}}} \tag{28}$$

where:

$$\tilde{n}\left(x, y\right) = a_x a_y n\left(x, y\right) \tag{29}$$