Study Guide
Exam FM: Financial Mathematics
Society of Actuaries (SOA)

Alec James van Rassel

## Table des matières

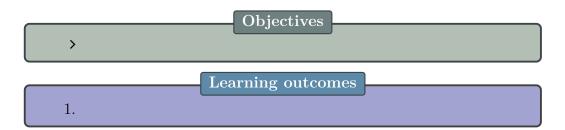
	Information	5
	Autres ressources	6
	Notes sur les vidéos YouTube	6
1	Time Value of Money	8
	Information	8
	Résumés des chapitres	10
	1a. Basic Concepts	10
	1b. Why Do We Need a Force of Interest?	10
	1c. Defining the Force of Interest	10
	1d. Finding the Fund in Terms of the Force of Interest	10
	1e. The Simplest Case : A Constant Force of Interest .	10
	1f. Power Series	10
	1g. The Variable Force of Interest Trap	10
	1h. Equivalent Rates	10
	2a. Equations of Value, Time Value of Money, and	
	Time Diagrams	10
	2b. Unknown Time and Unknown Interest Rate	11
	Notes sur les vidéos YouTube	11
2	Topic: Annuities / cash flows with non-contingent payments	12
	Information	12
	Résumés des chapitres	14
	3a. The Geometric Series Trap	14
	3b. Annuity-Immediate and Annuity-Due	14
	3c. The Great Confusion: Annuity-Immediate and Annuit	y-
	Due	14
	3d. Deferred Annuities	14
	3e. A Short-Cut Method for Annuities with "Block"	
	Payments	14
	3f. Perpetuities	14
	3g. The $a_{\overline{2n}}/a_{\overline{n}}$ Trick (and Variations)	14
	3h. What If the Rate Is Unknown?	15

	3i. What If the Rate Varies?	15
	4a. Annuities with "Off-Payments" Part I	15
	4b. Annuities with "Off-Payments" Part II	15
	4c. Avoiding the $m^{\text{thly}}$ Annuity Trap	15
	4d. Continuous Annuities	15
	4e. "Double-Dots Cancel" (and so do "upper $m$ 's")	15
	4f. A Short Note on Remembering Annuity Formulas .	15
	4g. The $s_{\overline{n} }$ Trap When Interest Variess	15
	4h. Payments in Arithmetic Progression	16
	4i. Remembering Increasing Annuity Formulas	16
	4j. Payments in Geometric Progression	16
	4k. The Amazing Expanding Money Machine (Or Conti-	
	nouss Varying Annuities)	16
	4l. A Short-Cut Method for the Palindromic Annuity .	16
	4m. The $0\%$ Test : A Quick Check of Symbolic Answers	16
	Notes sur les vidéos YouTube	16
3	Topic : Loans	17
•	Information	
	Résumés des chapitres	
	XX. Title-of-ASM-chapter	18
	Notes sur les vidéos YouTube	
4	Topic: Bonds	19
•	Information	
	Résumés des chapitres	
	XX. Title-of-ASM-chapter	
	Notes sur les vidéos YouTube	
5	Topic: General Cash Flows and Portfolios	21
•	Information	21
		23
	XX. Title-of-ASM-chapter	
	Notes sur les vidéos YouTube	
6	Topic: Immunization	<b>2</b> 4
_	Information	24
	Résumés des chapitres	25

	10h. Redington Immunization	25
	10i. Full Immunization	25
	10j. A Note on Rebalancing	25
	10k. Immunization by Exact Matching ("Dedication").	25
	Notes sur les vidéos YouTube	25
7	Topic : Interest Rate Swaps	26
	Information	26
	Résumés des chapitres	27
	11b. What is an Interest Rate Swap?	27
	Notes sur les vidéos YouTube	27
8	Topic : Determinants of Interest Rates	28
	Information	28
	Résumés des chapitres	30
	9a. What is Interest?	30
	9b. Quotation Bases for Interest Rates	30
	9c. Components of the Interest Rate: No Inflation or	
	Default Risk	30
	9d. Components of the Interest Rate: no Inflation but	
	with Default Risk	30
	9e. Components of the Interest Rate: Known Inflation	30
	9f. Components of the Interest Rate: Uncertain Infla-	
	tiono	30
	9g. Savings and Lending Interest Rates	30
	9h. Government and Corporate Bonds	30
	9i. The Role of Central Banks	30
	Notes sur les vidéos YouTube	31

Preliminary

## Information



## Autres ressources



Subjects of study

#### Time Value of Money (10%-15%)1

#### Information

#### Objective

The Candidate will understand and be able to perform calculations relating to present value, current value, and accumulated value.

#### Learning outcomes

The candidate will be able to:

- a) Define and recognize the definitions of the following terms:
  - > Interest rate (rate of inter- > Discount rate (rate of disest);
    - count);
  - > Simple interest;
- $\rightarrow$  Convertible *m*-thly (...?);
- > Compound interest;
- > Nominal rate;
- > Accumulation function;
- > Effective rate;

> Future value;

> Inflation;

> Current value;

- > Real rate of interest;
- > Present value;
- > Force of interest;
- > Net present value; > Discount factor;
- > Equation of value.

- b) Given any 3 of:

> Interest rate:

- > Present value: > Future value,
- > Period of time; > Current value;

calculate the remaining item using *simple* or *compound* interest; Solve time value of money equations involving variable force of interest;

- c) Given any 1 of:
  - > Effective interest rate;
  - > Nominal interest rate convertible *m*-thly;
  - > Force of interst, calculate any of the other items;
- d) Write the equation of value given a set of cash flows and interest rate.

#### Related lessons ASM

Section 1 : Interest rates and Discount Rates

- > 1a. Basic Concepts
- > 1b. Why Do We Need a Force of Interest?
- > 1c. Defining the Force of Interest
- > 1d. Finding the Fund in Terms of the Force of Interest
- > 1e. The Simplest Case : A Constant Force of Interest
- > 1f. Power Series
- > 1g. The Variable Force of Interest Trap
- > 1h. Equivalent Rates

Section 2: Practical Applications

- > 2a. Equations of Value, Time Value of Money, and Time Diagrams
- > 2b. Unknown Time and Unknown Interest Rate

#### Vidéos YouTube

>

## Résumés des chapitres

1a. Basic Concepts
>
1b. Why Do We Need a Force of Interest?
>
1c. Defining the Force of Interest
>
1d. Finding the Fund in Terms of the Force of Interest
>
1e. The Simplest Case: A Constant Force of Interest
>
1f. Power Series
>
1g. The Variable Force of Interest Trap
>
1h. Equivalent Rates
>
2a. Equations of Value, Time Value of Money, and Time Diagrams
>

#### 2b. Unknown Time and Unknown Interest Rate

>

## 2 Topic: Annuities / cash flows with non-contingent payments (exam weight)

#### Information

#### Objective

The Candidate will be able to calculate present value, current value, and accumulated value for sequences of non-contingent payments.

#### Learning outcomes

The candidate will be able to:

- a) Define and recognize the *definitions* of the following terms :
  - > Annuity-immediate;
  - > Annuity-due;
  - > Perpetuity;
  - > Payable *m*-thly or continously;
  - > Level payment annuity;
- > Arithmetic increasing/decreasing annuity;
- > Geometric increasing/decreasing annuity;
- > Term of annuity;
- b) For each of the following types of annuity / cash flows, given sufficient information of :
  - > Immediate or due;
- > Interest rate;

- > Present value;
- > Payment amount;
- > Futur value;> Current value;

> Term of annuity,

calculate any remaining item.

The types are:

> Level annuity, finite term;

- > Level perpetuity;
- > Non-level annuities / cash flows;
  - Arithmetic progression, finite term and perpetuity;
  - Geometric progression, finite term and perpetuity;
  - Other non-level annuities / cash flows.

#### Related lessons ASM

#### Section 3: Annuities

- > 3a. The Geometric Series Trap
- > 3b. Annuity-Immediate and Annuity-Due
- > 3c. The Great Confusion : Annuity-Immediate and Annuity-Due
- > 3d. Deferred Annuities
- > 3e. A Short-Cut Method for Annuities with "Block" Payments
- > 3f. Perpetuities
- > 3g. The  $a_{\overline{2n}}/a_{\overline{n}}$  Trick (and Variations)
- > 3h. What If the Rate Is Unknown?
- > 3i. What If the Rate Varies?

#### Section 4 : Complex Annuities

- > 4a. Annuities with "Off-Payments" Part I
- > 4b. Annuities with "Off-Payments" Part II
- > 4c. Avoiding the  $m^{\text{thly}}$  Annuity Trap
- > 4d. Continuous Annuities
- $\rightarrow$  4e. "Double-Dots Cancel" (and so do "upper m's")
- > 4f. A Short Note on Remembering Annuity Formulas
- $\rightarrow$  4g. The  $s_{\overline{n}|}$  Trap When Interest Variess
- > 4h. Payments in Arithmetic Progression
- > 4i. Remembering Increasing Annuity Formulas
- > 4j. Payments in Geometric Progression
- > 4k. The Amazing Expanding Money Machine (Or Continouss Varying Annuities)

4l. A Short-Cut Method for the Palindromic Annuity
4m. The 0% Test: A Quick Check of Symbolic Answers

Vidéos YouTube

>

#### Résumés des chapitres

3a. The Geometric Series Trap

>

3b. Annuity-Immediate and Annuity-Due

>

 $3c.\ The\ Great\ Confusion: Annuity-Immediate and Annuity-Due$ 

>

3d. Deferred Annuities

>

3e. A Short-Cut Method for Annuities with "Block" Payments

>

3f. Perpetuities

>

3g. The  $a_{\overline{2n}}/a_{\overline{n}}$  Trick (and Variations)

>

3h. What If the Rate Is Unknown?
>
3i. What If the Rate Varies?
>
4a. Annuities with "Off-Payments" Part I
>
4b. Annuities with "Off-Payments" Part II
· ·
<b>&gt;</b>
A A 11 (1 thly A 1) Th
4c. Avoiding the $m^{\text{thly}}$ Annuity Trap
>
4d. Continuous Annuities
>
4e. "Double-Dots Cancel" (and so do "upper $m$ 's")
>
4f. A Short Note on Remembering Annuity Formulas
>
4g. The $s_{\overline{n} }$ Trap When Interest Variess
>

```
4h. Payments in Arithmetic Progression

4i. Remembering Increasing Annuity Formulas

4j. Payments in Geometric Progression

4k. The Amazing Expanding Money Machine (Or Continouss Varying Annuities)

4l. A Short-Cut Method for the Palindromic Annuity

4m. The 0% Test: A Quick Check of Symbolic Answers

>
```

## 3 Topic : Loans (10%-20%)

#### Information

#### Objective

The Candidate will understand key concepts concerning loans and how to perform related calculations.

### Learning outcomes

The candidate will be able to:

- a) Define and recognize the *definitions* of the following terms :
  - > Principal;

> Final payment;

> Interest;

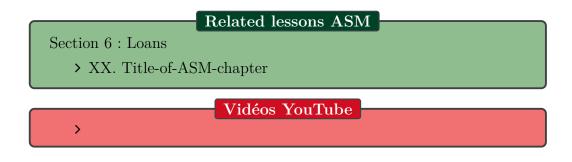
- Drop payment;

> Term of loan;

- Baloon payment.
- > Outstanding balance;
- > Amortization.

- b) Calculate:
  - > The missing item given any 4 of:
    - Term of loan;
- Payment period;

- Interest rate;
- Payment amount;
- Principal.
- > The outstanding balance at any point in time;
- > The amount of interest and principal repayment in a given payment;
- > Similar calculations to the above when refinancing is involved.



## Résumés des chapitres



## 4 Topic : Bonds (10%-20%)

#### Information

#### Objective

The Candidate will understand key concepts concerning bonds, and how to perform related calculations.

#### Learning outcomes

The candidate will be able to:

- a) Define and recognize the *definitions* of the following terms :
  - > Price;

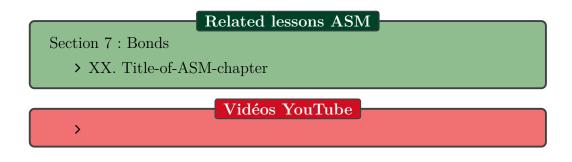
> Yield rate;

> Book value;

- > Coupon;
- > Amortization of premium;
- > Coupon rate;
- > Accumulation of discount;

> Par value / Face value;

- > Term of bond;
- > Redemption value;
- > Callable / Non-callable.
- b) Given sufficient partial information about the items listed below, calculate any of the remaining items :
  - > Price, book value, amortization of premium, accumulation of discount;
  - > Redemption value, face value;
  - > Yield rate;
  - > Coupon, coupon rate;
  - > Term of bond, point in time that a bond has a given book value, amortization of premium, or accumulation of discount.



## Résumés des chapitres



# 5 Topic : General Cash Flows and Portfolios (15%-20%)

#### Information

#### Objective

The Candidate will understand key concepts concerning yield curves, rates of return, and measures of duration and convexity, and how to perform related calculations.

#### Learning outcomes

The candidate will be able to:

- a) Define and recognize the *definitions* of the following terms :
  - > Yield rate / rate of return; > Convexity (Macaulay and modified);
  - Dollar-weighted rate of return;
     Portfolio;
  - > Time-weighted rate of re- > Spot rate;
  - turn; > Forward rate;
  - > Current value; > Yield Curve;
  - Duration (Macaulay and modified);
     Stock price;
     Stock dividend;
- b) Calculate:
  - $\succ$  The dollar-weighted and time-weighted rate of return ;
  - > The duration and convexity of a set of cash flows;
  - > Either Macaulay or modified duration given the other;
  - > The approximate change in present value due to a change in interest rate,

- Using 1st-order linear approximation based on modified duration;
- Using 1st-order approximation based on Macaulay duration.
- > The price of a stock using the dividend discount model;
- > The present value of a set of cash flows, using a yield curve developed from forward and spot rates.

	Related lessons ASM	
Section 5:		
> ??		
> ??		
> ??		
> ??		
> ??		
> ??		
Section 8:		
> ??		
> ??		
> ??		
Section 10:		
> ??		
> ??		
> ??		
> ??		
> ??		
> ??		
> ??		
Section 11:		
> ??		

## Vidéos YouTube

>

## Résumés des chapitres

5a.			
>			
5b.			
>			
5c.			
>			
5d.			
>			
5e.			
>			
5f.			
>			
8a.			
>			
01			
8b.			
>			

8c. >

## 6 Topic: Immunization (10%-15%)

#### Information

#### Objective

The Candidate will understand key concepts concerning cash flow matching and immunization, and how to perform related calculations.

#### Learning outcomes

The candidate will be able to:

- a) Define and recognize the *definitions* of the following terms :
  - > Cash flow matching;
  - > Immunization (including full immunization);
  - > Redington immunization.
- b) Construct an investment portfolio to:
  - > Redington immunize a set of liability cash flows;
  - > Fully immunize a set of liability cash flows;
  - > Exactly match a set of liability cash flows.

#### Related lessons ASM

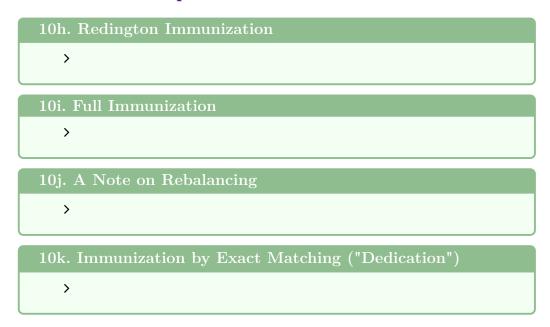
Section 10: Duration, Convexity, and Immunization

- > 10h. Redington Immunization
- > 10i. Full Immunization
- > 10j. A Note on Rebalancing
- > 10k. Immunization by Exact Matching ("Dedication")

#### Vidéos YouTube

>

## Résumés des chapitres



#### Topic: Interest Rate Swaps (0-10%) 7

#### Information

#### Objective

The Candidate will understand key concepts concerning interest rate swaps, and how to perform related calculations.

#### Learning outcomes

The candidate will be able to:

- a) Define and recognize the definitions of the following terms:
  - > Swap rate;

- > Counterparties;
- > Swap term (tenor);
- > Deferred swap;
- > Notional amount;
- > Amortizing swap;
- > Market value of a swap;
- > Accreting swap;
- > Settlement dates;
- > Settlement period;
- > Interest rate swap net pay-
- b) Given sufficient information, calculate:
  - > The market value;
- > deferred or otherwise;
- > Notional amount;
- > with either constant or varying notional amount.
- > Spot rates or swap rate,
- of an interest rate swap

#### Related lessons ASM

Section 11: Interest Rate Swaps

> 11b. What is an Interest Rate Swap?

## Vidéos YouTube

## Résumés des chapitres

11b. What is an Interest Rate Swap?

## 8 Topic : Determinants of Interest Rates (0-10%)

#### Information

#### Objective

The Candidate will understand key concepts concerning the determinants of interest rates, the components of interest, and how to perform related calculations.

#### Learning outcomes

The candidate will be able to:

- a) Define and recognize the definitions of the following terms :
  - > Real risk-free rate;
- > Liquidity premium;

- > Inflation rate;
- > Default risk premium;
- > Maturity risk premium.
- b) Explain how the components of interest rates apply in various contexts, such as:
  - > Commercial loans:
  - > Mortgages;
  - > Credit cards;
  - > Bonds;
  - > Government securities.
- c) Explain the **roles** of the Federal Reserve and the FOMC in carrying out *fiscal* policy and *monetary* policy and the **tools** used thereby including:
  - > Targeting the federal funds rate;
  - > Setting reserve requirements;

- > Setting the discount rate.
- d) Explain the theories of why interest rates differ by term, including :
  - > Liquidity preference (opportunity cost);
  - > Expectations;
  - > Preferred habitat;
  - > Market segmentation.
- e) Explain how interest rates differ from one country to another (e.g., U.S. vs. Canada);
- f) In the context of loans with and without inflation protection:
  - > **Identify** the *real* interest and the *nominal* interest rate;
  - > Calculate the effect of changes in inflation on loans with inflation protection.

#### Related lessons ASM

Section 9 : Determinants of Interest Rates

- > 9a. What is Interest?
- > 9b. Quotation Bases for Interest Rates
- > 9c. Components of the Interest Rate : No Inflation or Default Risk
- > 9d. Components of the Interest Rate : no Inflation but with Default Risk
- > 9e. Components of the Interest Rate: Known Inflation
- > 9f. Components of the Interest Rate: Uncertain Inflationo
- > 9g. Savings and Lending Interest Rates
- > 9h. Government and Corporate Bonds
- > 9i. The Role of Central Banks

#### Vidéos YouTube

>

## Résumés des chapitres

9a. What is Interest?
>
9b. Quotation Bases for Interest Rates
>
9c. Components of the Interest Rate : No Inflation or Default Risk
>
9d. Components of the Interest Rate : no Inflation but with Default Risk
>
9e. Components of the Interest Rate: Known Inflation
>
9f. Components of the Interest Rate: Uncertain Inflationo
>
9g. Savings and Lending Interest Rates
>
9h. Government and Corporate Bonds
>
9i. The Role of Central Banks
>