Guide d'étude

Examen MAS-I: Modern Actuarial Statistics I

Casualty Actuarial Society (CAS)

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Table des matières

Préliminaire

Information

Objectives

- Set forth, usually in broad terms, what the candidate should be able to do in actual practice ;
- The objectives include methodologies that may be impossible to perform on an exam that the candidate is expected to be able to explain conceptually ;
- *For example* : The Hat Matrix couldn't be calculated, but conceptual questions about it could be asked ;

Learning outcomes

1. It's important to identify some of the key terms, concepts, and methods associated with each of the learning objectives ;
2. They aren't an exhaustive list of the material being tested, but rather illustrate the scope of each learning objective ;

Information additionnelle

- The learning objectives define the behaviours and the knowledge statements illustrate more fully their intended scope ;
- Learning objectives should not be seen as independent units but as building blocks for our understanding ;
- The ranges are just guidelines ;
- The overall section weights should be seen as having more significance than the individual section weights ;
- Tables include :
  - values for the illustrative life tables ;
  - Standard normal distribution ;
  - Abridiged inventories of discrete and continuous probability distributions ;
  - Chi-square distribution ;
  - t-distribution ;

- F-distribution ;

› **There is a guessing adjustement** ;

Sujets à l'étude

1 Probability models (Stochastic Processes and Survival Models) (20% à 35%)

Information

Description

Notes du descriptif principal :

- Stochastic processes
- Survival models
  - Covered in depth as part of probability modeling in generic terms ;
- Markov Chains
  - Provide the means to model how an entity can move through different states ;
- Simplified version of life contingencies
  - Life contingencies problems can be viewed as discounted cash flow problems which include thee effect of probability of payment ;
  - Covered through a study note which link the generic survival model concepts to a subset of life actuarial concepts ;
  - This study note illustrates how to calculate annuities or single premium insurance amounts ;

Notes de la sous-section :

- Résoudre des problèmes de processus aléatoires ;
- Identifier les probabilités et distributions associées avec ces processus ;
  - Particulièrement, être capable d'utiliser un processus de Poisson dans ces applications ;

- Les modèles de survie sont une rallonge aux modèles de probabilité de processus stochastiques ;
  - En lieu, on estime la vie futur d'une entité avec quelques suppositions sur la distribution de la vraisemblance de survie ;
- Chaines de Markov utiles pour modéliser la mobilité entre états dans un processus et souligner les modèles Bayésien MCMC sous-jacent ;
- La simulation est incluse puisqu'elle peut s'avérer essentielle pour arriver à une solution de problème complexe ;

Learning objectives

1. Understand and apply the properties of Poisson processes ;
   - For increments in the homogeneous case ;
   - For interval times in the homogeneous case ;
   - For increments in the non-homogenous case ;
   - Resulting from special types of events in the Poisson process ;
   - Resulting from sums of independant Poisson processes ;
2. For any Poisson process and the inter-arrival and waiting distributions associated with the Poisson process, calculate :
   - Expected values ;
   - Variances ;
   - Probabilities ;
3. For a compound Poisson process, calculate moments associated with the value of the process at a given time ;
4. Apply the Poisson process concepts to calculate the hazard function and related survival model concepts ;

- Relationship between hazard rate, probability density function and cumulative distribution function;
- Effect of memoryless nature of Poisson distribution on survival time estimation;

5. Given the joint distribution of more than one source of failure in a system (or life) and using Poisson Process assumptions :
   - Calculate probabilities and moments associated with functions of these random variables' variances;
   - Understand differences between a series system (joint life) and parallel system (last survivor) when calculating expected time to failure or probability of failure by a certain time;
   - Understand the effect of multiple sources of failure (multiple decrement) on expected system time to failure (expected lifetime);

6. For discrete Markov Chains under both homogeneous and non-homogenous states :
   - Definition of a Markov Chain;
   - Chapman-Kolmogorov Equations for n-step transition calculations;
   - Accessible states;
   - Ergodic Markov Chains and limiting probabilities;

7. Solve Life Contingency problems using a life table in a spreadsheet as the combined result of discount, probability of payment and amount of payment vectors. Understand the linkage between the life table and the corresponding probability models;
   - Calculate annuities for discrete time;
   - Calculate life insurance single net premiums (or P & C pure premiums) for discrete time;
   - Solve for net level premiums (**not** including fractional lives);

8. The candidate should be familiar with basic computer simulation methods.
   - Understand the basic framework of Monte Carlo Simulation ;
   - Understand the mechanics of generating uniform random numbers ;
   - Generate random numbers from a variety of distributions using the inversion method ;
   - Be able to explain when and how to use the Acceptance-Rejection method ;

Related lessons ASM

1. Probability Review
2. Parametric Distributions
3. Mixtures
4. Markov Chains : Chapman-Kolmogorov Equations
5. Markov Chains : Classification of States
6. Discrete Markov Chains : Long-Run Proportions and Limiting Probabilities
7. Markov Chains : Time in Transient States
8. Markov Chains : Branching Processes
9. Markov Chains : Time Reversible
10. Exponential Distribution
11. The Poisson Process : Probabilities of Events
12. The Poisson Process : Time To Next Event
13. The Poisson Process : Thinning, or Couting Special Types of Events
14. The Poisson Process : Other Characteristics
15. The Poisson Process : Sums and Mixtures

16. Compound Poisson Processes
17. Reliability : Structure Functions
18. Reliability : Probabilities
19. Reliability : Time to Failure

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20. Survival Models
21. Contingent Payments

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22. Simulation—Inverse Transformation Method
23. Simulation—Applications
24. Simulation—Rejection Method

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Résumés des chapitres

Probability Review

Chapter 1: Probability Review

Introduction to Mathematical Statistics 1 - 3, 5

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Chapter 2: Parametric Distributions

Introduction to Mathematical Statistics 2.2, 2.7 Nonlife Actuarial Models—Theory Methods and Evaluation 2.2

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Chapter 3: Mixtures

Introduction to Mathematical Statistics 3.7 Nonlife Actuarial Models—Theory Methods and Evaluation 2.3.2

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Stochastic Processes

Chapter 4: Markov Chains : Chapman-Kolmogorov Equations

Ross 4.1 - 4.2, 4.5.1 - 4.5.2

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Chapter 5: Markov Chains : Classification of States

Ross 4.3

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Chapter 6: Discrete Markov Chains : Long-Run Proportions and Limiting Probabilities

Ross 4.4

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Chapter 7: Markov Chains : Time in Transient States

Ross 4.6

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Chapter 8: Markov Chains : Branching Processes

Ross 4.7

›

Chapter 9: Markov Chains : Time Reversible

Ross 4.8

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Chapter 10: Exponential Distribution

Ross 5.2

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Chapter 11: The Poisson Process : Probabilities of Events

Ross 5.3.1 - 5.3.2 Daniel Poisson Study Note 1.1, 1.4.1

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Chapter 12: The Poisson Process : Time To Next Event

Ross 5.2, 5.3.3 Daniel Poisson Study Note 1.1.1

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Chapter 13: The Poisson Process : Thinning, or Couting Special Types of Events

Ross 5.3.4 Daniel Poisson Study Note 1.3.1, 1.4.3

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Chapter 14: The Poisson Process : Other Characteristics

Ross 5.2.3, 5.3.4, 5.3.5

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Chapter 15: The Poisson Process : Sums and Mixtures

Ross 5.4.3 Daniel Poisson Study Note 1.3.2, 1.3.3

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Chapter 16: Compound Poisson Processes

Ross 5.4.2 Daniel Poisson Study Note 1.2, 1.4.2

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Chapter 17: Reliability : Structure Functions

Ross 9.1 - 9.2

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Chapter 18: Reliability : Probabilities

Ross 9.3 - 9.4

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Chapter 19: Reliability : Time to Failure

Ross 9.5 - 9.6

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Life Contingencies

Chapter 20: Survival Models

Struppeck 1, 2, 6, 7

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Chapter 21: Contingent Payments

Struppeck 3, 4, 5, 6

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Simulation

Chapter 22: Simulation—Inverse Transformation Method

Ross 11.1, 11.2.1

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Chapter 23: Simulation—Applications

Ross 11.1, 11.2.1

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Chapter 24: Simulation—Rejection Method

Ross 11.2.2

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2 Statistics (15% à 30%)

Information

Description

Notes du descriptif principal :

- Topics which would commonly be covered in a 2-semester Probability & Statistics sequence ;

Learning objectives

1. Perform point estimation of statistical parameters using Maximum likelihood estimation (MLE). Apply criteria to estimates such as :
   - Consistency ;
   - Efficiency ;
   - Unbiasedness ;
   - Minimum variance ;
   - Sufficiency ;
   - MSE ;

   Calculate parameter estimates using methods other than maximum likelihood ;
2. Test statistical hypotheses including Type I and Type II errors using :
   - Neyman-Pearson theorem ;

     Apply Neyman-Pearson theorem to construct likelihood ratio equation ;
   - Likelihood ratio tests ;
   - First principles ;

     Use critical values from a sampling distribution to test means and variances ;

3. For the Exponential, Gamma, Weibull, Pareto, Lognormal, Beta, and mixtures thereof :
   - Identify the applciations to Insurance claim modeling in which each distribution is used and reasons why ;
   - Transformation of distributions ;
4. Calculate Order Statistics of a sample for a given distribution ;

Knowledge Statements

a. Equations for MLE of mean, variance from a sample ;
b. Estimation of mean and variance based on samples ;
c. General equations for MLE of parameters ;
d. Recognition of consistency property of estimators and alternative measures of consistency ;
e. Application of criteria for measurement when estimating parameters through minimisation of variance, MSE ;
f. Definition of statistical bias and recognition of estimators that are unbiased or biased ;
g. Application of Rao-Cramer Lower Bound and Efficiency ;
h. Relationship between Sufficiency and Minimum Variance ;
i. Develop and estimate a sufficient statistic for a distribution ;
j. Factorization Criterion for sufficiency ;
k. Application of Rao-Cramer Lower Bound and Fisher Information ;
l. Application of MVUE for the exponential class of distributions ;
m. Linkage between Score Function, Fisher Information and maximum likelihood ;
n. Method of Moments ;
o. Percentile Matching ;
p. Kernel Density Estimation ;
q. Maximum Likelihood with Censoring and Truncation ;

a. Presentation of fundamental inequalities based on general assumptions and normal assumptions;
b. Definition of Type I and Type II errors;
c. Significance levels;
d. One-sided versus two-sided tests;
e. Estimation of sample sizes under normality to control for Type I and Type II errors;
f. Determination of critical regions;
g. Definition and measurement of likelihood ratio tests;
h. Determining parameters and testing using tabular values (from a table);
i. Recognizing when to apply likelihood ratio tests versus chi-square or other goodness of fit tests;
j. Apply paired t-test to two samples;
k. Test for difference in variance under Normal distribution between two samples through the application of F-test;
l. Test of significance of means from two samples under Normal distribution assumptions in both large and small sample cases;
m. Test for significance of difference in proportions between two samples under the Binomial distribution assumption in both large and small sample cases;
n. Application of contingency tables to test independence between effects;
o. Asymptotic relationship between likelihood ratio tests and the Chi-Square distribution;
p. Application of Neyman-Pearson theorem to Uniformly Most Powerful hypothesis tests;
q. Equivalence between critical regions and confidence intervals;
r. Kolmogorov-Smirnov test;

a. Frequency, severity and aggregate loss ;
b. Common continuous distributions for modeling claim severity ;
c. Mixing distributions ;
d. Tail properties of claim severity ;
e. Effects of coverage modifications including, for example : limits, deductibles, loss elimination ratios and effects of inflation ;

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a. General form for distribution of n^{th} largest element of a set ;
b. Application to a given distributional form ;

Related lessons ASM

25. Estimator Quality
26. Kernel Density Estimation
27. Method of Moments
28. Percentile Matching
29. Maximum Likelihood Estimators
30. Maximum Likelihood Estimators—Special Techniques
31. Variance of Maximum Likelihood Estimator
32. Sufficient Statistics
33. Hypothesis Testing
34. Confidence Intervals and Sample Size
35. Confidence Intervals for Means
36. Kolmogorov-Smirnov Tests
37. Chi Square Tests
38. Confidence Intervals for Variances
39. Uniformly Most Powerful Critical Regions
40. Likelihood Ratio Tests
41. Q.-Q. Plots

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Likely Questions

› Question where we calculate the sample variance with the STAT function of the calculator ;
  – MAS-I F19, # 15 ;

Résumés des chapitres

Chapter 25: Estimator Quality

Introduction to Mathematical 4.1.3, 5.1, 7.1

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Chapter 26: Kernel Density Estimation

Nonlife Actuarial Models : Theory Methods and Evaluation 11.1

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Chapter 27: Method of Moments

Nonlife Actuarial Models : Theory Methods and Evaluation 12.1.1

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Chapter 28: Percentile Matching

Nonlife Actuarial Models : Theory Methods and Evaluation 11.1.1, 12.1.2

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Chapter 29: Maximum Likelihood Estimators

Introduction to Mathematical Statistics 4.1, 6.1 Nonlife Actuarial Models : Theory Methods and Evaluation 10.2, 12.3

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Chapter 30: Maximum Likelihood Estimators—Special Techniques

Introduction to Mathematical Statistics 4.1, 6.1 Nonlife Actuarial Models : Theory Methods and Evaluation 10.2, 12.3

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Chapter 31: Variance of Maximum Likelihood Estimator

Introduction to Mathematical Statistics 6.2, 6.5

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Chapter 32: Sufficient Statistics

Introduction to Mathematical Statistics 7

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Chapter 33: Hypothesis Testing

Introduction to Mathematical Statistics 4.5, 4.6

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Chapter 34: Confidence Intervals and Sample Size

Introduction to Mathematical Statistics 4.5, 4.6

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Chapter 35: Confidence Intervals for Means

Introduction to Mathematical Statistics 4.2

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Chapter 36: Kolmogorov-Smirnov Tests

Nonlife Actuarial Models : Theory Methods and Evaluation 13.2.1

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Chapter 37: Chi Square Tests

Introduction to Mathematical 4.7 Nonlife Actuarial Models : Theory Methods and Evaluation 13.2.3

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Chapter 38: Confidence Intervals for Variances

Introduction to Mathematical Statistics 8.3

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Chapter 39: Uniformly Most Powerful Critical Regions

Introduction to Mathematical Statistics 8.1 - 8.2

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Chapter 40: Likelihood Ratio Tests

Introduction to Mathematical Statistics 8.3

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Chapter 41: Q.-Q. Plots

Introduction to Mathematical Statistics 4.4 Larsen Study Note

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3 Extended Linear Models (30% à 50%)

Information

Description

Notes du descriptif principal :

- Include GLMs which are commonly used to construct classification plans ;
- OLS model is covered as one member of the exponential family ;
- `R` is useful to better visualise and conceptualise the material ;

Notes de la sous-section :

- OLS treated as *one* type of model that may be used when the dependant variable follows the Normal distribution and the observations are (iid) with a constant variance ;
- All models assume data is (iid) from the exponentional family ;
- Assume linear relationship between dependant and independant variables ;
- Assume variance is a function of the mean ;
- **VIF** formula found on p. 102 of James et al. and p. 101 of Dobson is used and not hte one on p. 101 of James and et al. ;

$$\text{VIF}(b_j) = \frac{1}{1 - R^2_{(j)}}$$

- Questions may contain parameter tables and plots (of the type shown in texts) with which we should familiarise ourselves ;

Learning objectives

1. Understand the assumptions behind different forms of the Extended Linear Model and be able to select the appropriate model

from list below :

- OLS ;
- GLM ;
- ANOVA ;
- GAM ;
- Local Regression ;
- Lasso ;
- Ridge Regression ;
- Partial Least Squares ;
- PCA regression ;

2. Evaluate models developed using Extended Linear Model approach ;

3. Understand the algorithms behind the numerical solutions for the different forms of the Extended Linear Model family to enable interpretation of output from the statistical software employed in modeling and to make appropriate modeling choices when selecting modeling options ;

4. Understand and be able to select the appropriate model structure for an Extended Linear Model given the behavior of the data set to be modeled ;

5.

Related lessons ASM

42. Introduction to Extended Linear Models
43. How a Generalized Linear Model Works
44. How a Generalized Linear Model Works : Categorical Response
45. Generalized Linear Model : Estimating Parameters
46. Generalized Linear Model : Measures of Fit
47. Normal Linear Model : Standard Error, R-squared, and t-statistic
48. Normal Linear Model : F and VIF
49. Normal Linear Model : Validation

50. Normal Linear Model : Predictions
51. ANOVA
52. Generalized Linear Model : Measures of Fit II
53. Resampling Methods
54. Normal Linear Model : Subset Selection
55. Normal Linear Model : Shrinkage and Dimension Reduction
56. Extensions to the Linear Model

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Résumés des chapitres

Chapter 42: Introduction to Extended Linear Models

Introduction to Mathematical Statistics 4.4 Introduction to Generalized Linear Models 2 Introduction to Statistical Learning with R 2 (except 2.2.3)

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Chapter 43: How a Generalized Linear Model Works

Introduction to Generalized Linear Models 3.9 Introduction to Statistical Learning with R 3.3.1 - 3.3.2 Larsen Study Note

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Chapter 44: How a Generalized Linear Model Works : Categorical Response

Introduction to Generalized Linear Models 7, 8 Introduction to Statistical Learning with R 4

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Chapter 45: Generalized Linear Model : Estimating Parameters

Introduction to Generalized Linear Models 4 Introduction to Statistical Learning with R 3.1.1, 3.2.1 Larsen Study Note

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Chapter 46: Generalized Linear Model : Measures of Fit

Introduction to Generalized Linear Models 5

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Chapter 47: Normal Linear Model : Standard Error, R-squared, and t-statistic

Introduction to Generalized Linear Models 6.1 - 6.3 Introduction to Statistical Learning with R 3.1.2 - 3.1.3, 3.2.2 - 3.2.3

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Chapter 48: Normal Linear Model : F and VIF

Introduction to Generalized Linear Models 6.2 Introduction to Statistical Learning with R 3

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Chapter 49: Normal Linear Model : Validation

Introduction to Generalized Linear Models 6.2 Introduction to Statistical Learning with R 3.3.3

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Chapter 50: Normal Linear Model : Predictions

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Chapter 51: ANOVA

Introduction to Generalized Linear Models 6.4 - 6.5, 9.3 - 9.7 Introduction to Mathematical Statistics 9.1 - 9.5

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Chapter 52: Generalized Linear Model : Measures of Fit II

Introduction to Generalized Linear Models 7, 8, 9

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Chapter 53: Resampling Methods

Introduction to Statistical Learning with R 5

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Chapter 54: Normal Linear Model : Subset Selection

Introduction to Statistical Learning with R 6.1

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Chapter 55: Normal Linear Model : Shrinkage and Dimension Reduction

Introduction to Statistical Learning with R 6.2 - 6.4

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Chapter 56: Extensions to the Linear Model

Introduction to Statistical Learning with R 7

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4 Time Series with Constant Variance (10% à 20%)

Information

Description

Notes du descriptif principal :

- Covers an introduction to modeling activity, such as financial results or stock prices, over time ;
- The model used is the Auto Regressive Integrated Moving Average (ARIMA) where activity in a given period may be linked to activity in subsequent time periods ;
- The connection between adjacent time periods violates one of the assumptions behind the Extended Linear Model techniques ;
- The ARIMA appproach incorporates that linkage as an aid for it's predictions ;
- Also covers the application of regression models to time series analysis ;

Notes de la sous-section :

- Section covers basic applications of the ARIMA time series model ;

Learning objectives

1. Use time series to model trends ;
   - Estimation, data analysis, and forecasting ;
   - Forecast errors and confidence intervals ;
2. Model relationships of current and past values of a statistic / metric ;

- Estimation, data analysis, and forecasting ;
- Forecast errors and confidence intervals ;

3. Understand forecasts produced by ARIMA ;

4. Time Series with Regression ;

Related lessons ASM

57. Time Series : Trend and Seasonality
58. Time Series : Correlation
59. Time Series : White Noise and Random Walks
60. Time Series : Autoregressive Models
61. Time Series : Regression
62. Time Series : Moving Average Models
63. Time Series : ARMA Models
64. Time Series : ARIMA and SARIMA Models

Vidéos YouTube

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Résumés des chapitres

57. Time Series : Trend and Seasonality

Introduction to Time Series with R 1

1 : Time Series Data

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58. Time Series : Correlation

Introduction to Time Series with R 2 - 3.1

2 : Correlation

3.1 : Forecasting Strategies—Purpose

1. Second order properties of a time series ;
2. Relationships of different time series ;

59. Time Series : White Noise and Random Walks

Introduction to Time Series with R 4.1 - 4.4

4.1 : Basic Stochastic Models—Purpose

4.2 : Basic Stochastic Models—White Noise

4.3 : Basic Stochastic Models—Random Walks

4.4 : Basic Stochastic Models—Fitted models and diagnostic plots

1. White noise ;
2. Random walks ;

60. Time Series : Autoregressive Models

Introduction to Time Series with R 4.5 - 4.8

4.5 : Basic Stochastic Models—AR models

4.6 : Basic Stochastic Models—Fitted models

4.7 : Basic Stochastic Models—Summary of R commands

4.8 : Basic Stochastic Models—Exercices

Correlograms and partial correlograms ;

Stationnarity ;

Forecasting with $AP(p)$ series ;

61. Time Series : Regression

Introduction to Time Series with R 5

5 : Regression

1. Correcting for autocorrelation ;
2. Seasonality ;
3. Logarithmic transformations ;
4. Error correction factors ;

62. Time Series : Moving Average Models

Introduction to Time Series with R 6.1 - 6.4

6.1 : Stationary Models—Purpose

6.2 : Stationary Models—Strictly stationary series

6.3 : Stationary Models—MA models

6.4 : Stationary Models—Fitted MA models

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63. Time Series : ARMA Models

Introduction to Time Series with R 6.5 - 6.8

6.5 : Stationary Models—Mixed models : The ARMA process

6.6 : Stationary Models—ARMA models : Empirical Analysis

6.7 : Stationary Models—Summary of R commands

6.8 : Stationary Models—Exercices

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64. Time Series : ARIMA and SARIMA Models

Introduction to Time Series with R 7.1 - 7.3

7.1 : Non-stationary Models—Purpose

7.2 : Non-stationary Models—Non-Seasonal ARIMA models

7.3 : Non-stationary Models—Seasonal ARIMA models

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