Guide d'étude
Examen MAS-I: Modern Actuarial Statistics I
Casualty Actuarial Society (CAS)

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Table des matières

Préliminaire

Information

Objectives

> Set forth, usually in broad terms, what the candidate should be able to do in actual practice ;

> The objectives include methodologies that may be impossible to perform on an exam that the candidate is expected to be able to explain conceptually ;

> *For example* : The Hat Matrix couldn't be calculated, but conceptual questions about it could be asked ;

Learning outcomes

1. It's important to identify some of the key terms, concepts, and methods associated with each of the learning objectives ;

2. They aren't an exhaustive list of the material being tested, but rather illustrate the scope of each learning objective ;

Information additionnelle

> The learning objectives define the behaviours and the knowledge statements illustrate more fully their intended scope ;

> Learning objectives should not be seen as independent units but as building blocks for our understanding ;

> The ranges are just guidelines ;

> The overall section weights should be seen as having more significance than the individual section weights ;

> Tables include :
> - values for the illustrative life tables ;
> - Standard normal distribution ;
> - Abridged inventories of discrete and continuous probability distributions ;
> - Chi-square distribution ;
> - t-distribution ;

– F-distribution ;

› **There is a guessing adjustment** ;

Sujets à l'étude

1 Probability models (Stochastic Processes and Survival Models) (20% à 35%)

Information

Description

Notes du descriptif principal :
- › Stochastic processes
- › Survival models
  - – Covered in depth as part of probability modeling in generic terms ;
- › Markov Chains
  - – Provide the means to model how an entity can move through different states ;
- › Simplified version of life contingencies
  - – Life contingencies problems can be viewed as discounted cash flow problems which include thee effect of probability of payment ;
  - – Covered through a study note which link the generic survival model concepts to a subset of life actuarial concepts ;
  - – This study note illustrates how to calculate annuities or single premium insurance amounts ;

- -

Notes de la sous-section :
- › Résoudre des problèmes de processus aléatoires ;
- › Identifier les probabilités et distributions associées avec ces processus ;
  - – Particulièrement, être capable d'utiliser un processus de Poisson dans ces applications ;

> Les modèles de survie sont une rallonge aux modèles de probabilité de processus stochastiques ;
>> – En lieu, on estime la vie futur d'une entité avec quelques suppositions sur la distribution de la vraisemblance de survie ;
> Chaines de Markov utiles pour modéliser la mobilité entre états dans un processus et souligner les modèles Bayésien MCMC sous-jacent ;
> La simulation est incluse puisqu'elle peut s'avérer essentielle pour arriver à une solution de problème complexe ;

Learning objectives

1. Understand and apply the properties of Poisson processes ;
   > For increments in the homogeneous case ;
   > For interval times in the homogeneous case ;
   > For increments in the non-homogeneous case ;
   > Resulting from special types of events in the Poisson process ;
   > Resulting from sums of independent Poisson processes ;

   ### Knowledge Statements

   a. Poisson process ;
   b. Non-homogeneous Poisson process ;
   c. Memoryless property of Exponential and Poisson ;
   d. Relationship between Exponential and Gamma ;
   e. Relationship between Exponential and Poisson ;

2. For any Poisson process and the inter-arrival and waiting distributions associated with the Poisson process, calculate :
   > Expected values ;
   > Variances ;

> Probabilities;

Knowledge Statements

a. Probability calculations for Poisson process

b. Conditional distribution of arrival times;

c. Splitting grouped Poisson rate to subsets of population using probability distribution;

d. Conditional distribution of events by category within a group within a certain time period;

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3. For a compound Poisson process, calculate moments associated with the value of the process at a given time;

Knowledge Statements

a. Compound Poisson process mean and variance;

b. Normal approximation and hypothesis testing;

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4. Apply the Poisson process concepts to calculate the hazard function and related survival model concepts;

> Relationship between hazard rate, probability density function and cumulative distribution function;

> Effect of memoryless nature of Poisson distribution on survival time estimation;

Knowledge Statements

a. Failure time random variables;

b. Cumulative distribution functions;

c. Survival functions;

d. Probability density functions;

e. Hazard functions and relationship to Exponential distribution;

f. Relationships between failure time random variables in the functions above;

g. Greedy algorithms;

5. Given the joint distribution of more than one source of failure in a system (or life) and using Poisson Process assumptions:

> Calculate probabilities and moments associated with functions of these random variables' variances;

> Understand differences between a series system (joint life) and parallel system (last survivor) when calculating expected time to failure or probability of failure by a certain time;

> Understand the effect of multiple sources of failure (multiple decrement) on expected system time to failure (expected lifetime);

a. Joint distribution of failure times;

b. Probabilities and moments;

c. Time until failure of the system (life);

d. Time until failure of the system (life) from a specific cause;

e. Time until failure of the system (life) for parallel or series systems with multiple components;

f. Paths that lead to parallel or series system failures for systems with multiple components;

g. Relationship between failure time and minimal path and minimal cut sets;

h. Bridge system and defining path to failure;

i. Random graphs and defining path to failure;

j. Effect of multiple sources of failure (multiple decrements) on failure time calculations (competing risk);

k. Non-uniform probability of component failure (multiple decrement);

l. Method of inclusion and exclusion as applied to failure time estimates;

m. Expected system lifetime as function of component lifetime and properties of expected lifetime estimates;

n. Linkage between reliability function for a system and future expected lifetime;

6. For discrete Markov Chains under both homogeneous and non-homogenous states:

> Definition of a Markov Chain;

> Chapman-Kolmogorov Equations for n-step transition calculations;

> Accessible states;

> Ergodic Markov Chains and limiting probabilities;

Knowledge Statements

a. Random Walk;

b. Classification of states and classes of states (absorbing, accessible, transition, irreducible, and recurrent);

c. Transition step probabilities;

d. Stationary probabilities;

e. Recurrent vs. transient states;

f. Gamblers ruin problem;

g. Branching processes;

h. Homogeneous transition probabilities;

i. Memoryless property of Markov Chains;

j. Limiting probabilities;

7. Solve Life Contingency problems using a life table in a spreadsheet as the combined result of discount, probability of payment and amount of payment vectors. Understand the linkage between the life table and the corresponding probability models;

> Calculate annuities for discrete time;

> Calculate life insurance single net premiums (or P & C pure premiums) for discrete time;

> Solve for net level premiums (**not** including fractional lives);

a. Discounted cash flow;
b. Relationship between annuity values and insurance premiums;
c. Life table linkage to probability models;
d. Equivalence property;

8. The candidate should be familiar with basic computer simulation methods.

> Understand the basic framework of Monte Carlo Simulation;

> Understand the mechanics of generating uniform random numbers;

> Generate random numbers from a variety of distributions using the inversion method;

> Be able to explain when and how to use the Acceptance-Rejection method;

a. Random Number Generation;
b. Uniform Random Numbers;
c. Inversion Method;
d. Acceptance-Rejection Method;

Related lessons ASM

1. Probability Review
2. Parametric Distributions
3. Mixtures

4. Markov Chains : Chapman-Kolmogorov Equations

Vidéos YouTube

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Résumés des chapitres

Probability Review

> ### Chapter 1: Probability Review
>
> Introduction to Mathematical Statistics 1 - 3, 5
>
> ❯

> ### Chapter 2: Parametric Distributions
>
> Introduction to Mathematical Statistics 2.2, 2.7 Nonlife Actuarial Models—Theory Methods and Evaluation 2.2
>
> ❯

> ### Chapter 3: Mixtures
>
> Introduction to Mathematical Statistics 3.7 Nonlife Actuarial Models—Theory Methods and Evaluation 2.3.2
>
> ❯

Stochastic Processes

> ### Chapter 4: Markov Chains : Chapman-Kolmogorov Equations
>
> Ross 4.1 - 4.2, 4.5.1 - 4.5.2
>
> ❯

> ### Chapter 5: Markov Chains : Classification of States
>
> Ross 4.3
>
> ❯

Chapter 6: Discrete Markov Chains : Long-Run Proportions and Limiting Probabilities

Ross 4.4

>

Chapter 7: Markov Chains : Time in Transient States

Ross 4.6

>

Chapter 8: Markov Chains : Branching Processes

Ross 4.7

>

Chapter 9: Markov Chains : Time Reversible

Ross 4.8

>

Chapter 10: Exponential Distribution

Ross 5.2

>

Chapter 11: The Poisson Process : Probabilities of Events

Ross 5.3.1 - 5.3.2 Daniel Poisson Study Note 1.1, 1.4.1

>

Chapter 12: The Poisson Process : Time To Next Event

Ross 5.2, 5.3.3 Daniel Poisson Study Note 1.1.1

>

Chapter 13: The Poisson Process : Thinning, or Couting Special Types of Events

Ross 5.3.4 Daniel Poisson Study Note 1.3.1, 1.4.3

>

Chapter 14: The Poisson Process : Other Characteristics

Ross 5.2.3, 5.3.4, 5.3.5

>

Chapter 15: The Poisson Process : Sums and Mixtures

Ross 5.4.3 Daniel Poisson Study Note 1.3.2, 1.3.3

>

Chapter 16: Compound Poisson Processes

Ross 5.4.2 Daniel Poisson Study Note 1.2, 1.4.2

>

Chapter 17: Reliability : Structure Functions

Ross 9.1 - 9.2

>

Chapter 18: Reliability : Probabilities

Ross 9.3 - 9.4

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Chapter 19: Reliability : Time to Failure

Ross 9.5 - 9.6

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Life Contingencies

> ### Chapter 20: Survival Models
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> Struppeck 1, 2, 6, 7
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> >

> ### Chapter 21: Contingent Payments
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> Struppeck 3, 4, 5, 6
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> >

Simulation

> ### Chapter 22: Simulation—Inverse Transformation Method
>
> Ross 11.1, 11.2.1
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> >

> ### Chapter 23: Simulation—Applications
>
> Ross 11.1, 11.2.1
>
> >

> ### Chapter 24: Simulation—Rejection Method
>
> Ross 11.2.2
>
> >

Notes sur les vidéos YouTube

> ### StatQuest: A Gentle Introduction to Machine Learning
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2   Statistics (15% à 30%)

Information

> ## Description
>
> Notes du descriptif principal :
>
> › Topics which would commonly be covered in a 2-semester Probability & Statistics sequence ;

> ## Learning objectives
>
> 1. Perform point estimation of statistical parameters using Maximum likelihood estimation (MLE). Apply criteria to estimates such as :
>
> | | |
> |---|---|
> | › Consistency ; | › Efficiency ; |
> | › Unbiasedness ; | › Minimum variance ; |
> | › Sufficiency ; | › MSE ; |

Knowledge Statements

a. Equations for MLE of mean, variance from a sample ;

b. Estimation of mean and variance based on samples ;

c. General equations for MLE of parameters ;

d. Recognition of consistency property of estimators and alternative measures of consistency ;

e. Application of criteria for measurement when estimating parameters through minimisation of variance, MSE ;

f. Definition of statistical bias and recognition of estimators that are unbiased or biased ;

g. Application of Rao-Cramer Lower Bound and Efficiency ;

h. Relationship between Sufficiency and Minimum Variance ;

i. Develop and estimate a sufficient statistic for a distribution ;

j. Factorization Criterion for sufficiency ;

k. Application of Rao-Cramer Lower Bound and Fisher Information ;

l. Application of MVUE for the exponential class of distributions ;

m. Linkage between Score Function, Fisher Information and maximum likelihood ;

n. Method of Moments ;

o. Percentile Matching ;

p. Kernel Density Estimation ;

q. Maximum Likelihood with Censoring and Truncation ;

Calculate parameter estimates using methods other than maximum likelihood ;

2. Test statistical hypotheses including Type I and Type II errors using :

> Neyman-Pearson theorem ;

Apply Neyman-Pearson theorem to construct likelihood ratio equation ;

> Likelihood ratio tests ;

> First principles ;

Use critical values from a sampling distribution to test means and variances ;

Knowledge Statements

a. Presentation of fundamental inequalities based on general assumptions and normal assumptions ;

b. Definition of Type I and Type II errors ;

c. Significance levels ;

d. One-sided versus two-sided tests ;

e. Estimation of sample sizes under normality to control for Type I and Type II errors ;

f. Determination of critical regions ;

g. Definition and measurement of likelihood ratio tests ;

h. Determining parameters and testing using tabular values (from a table) ;

i. Recognizing when to apply likelihood ratio tests versus chi-square or other goodness of fit tests ;

j. Apply paired t-test to two samples ;

k. Test for difference in variance under Normal distribution between two samples through the application of F-test ;

l. Test of significance of means from two samples under Normal distribution assumptions in both large and small sample cases ;

m. Test for significance of difference in proportions between two samples under the Binomial distribution assumption in both large and small sample cases ;

n. Application of contingency tables to test independence between effects ;

o. Asymptotic relationship between likelihood ratio tests and the Chi-Square distribution ;

p. Application of Neyman-Pearson theorem to Uniformly Most Powerful hypothesis tests ;

q. Equivalence between critical regions and confidence intervals ;

r. Kolmogorov-Smirnov test ;

3. For the Exponential, Gamma, Weibull, Pareto, Lognormal, Beta, and mixtures thereof :

    > Identify the applciations to Insurance claim modeling in which each distribution is used and reasons why ;
    > Transformation of distributions ;

    Knowledge Statements

    a. Frequency, severity and aggregate loss ;
    b. Common continuous distributions for modeling claim severity ;
    c. Mixing distributions ;
    d. Tail properties of claim severity ;
    e. Effects of coverage modifications including, for example : limits, deductibles, loss elimination ratios and effects of inflation ;

4. Calculate Order Statistics of a sample for a given distribution ;

    Knowledge Statements

    a. General form for distribution of nth largest element of a set ;
    b. Application to a given distributional form ;

Related lessons ASM

25. Estimator Quality
26. Kernel Density Estimation
27. Method of Moments
28. Percentile Matching
29. Maximum Likelihood Estimators
30. Maximum Likelihood Estimators—Special Techniques

31. Variance of Maximum Likelihood Estimator

32. Sufficient Statistics

33. Hypothesis Testing

34. Confidence Intervals and Sample Size

35. Confidence Intervals for Means

36. Kolmogorov-Smirnov Tests

37. Chi Square Tests

38. Confidence Intervals for Variances

39. Uniformly Most Powerful Critical Regions

40. Likelihood Ratio Tests

41. Q.-Q. Plots

Vidéos YouTube

>

Likely Questions

> Question where we calculate the sample variance with the STAT function of the calculator ;
  − MAS-I F19, # 15 ;

Résumés des chapitres

Chapter 25: Estimator Quality

Introduction to Mathematical 4.1.3, 5.1, 7.1

>

Chapter 26: Kernel Density Estimation

Nonlife Actuarial Models : Theory Methods and Evaluation 11.1

>

Chapter 27: Method of Moments

Nonlife Actuarial Models : Theory Methods and Evaluation 12.1.1

>

Chapter 28: Percentile Matching

Nonlife Actuarial Models : Theory Methods and Evaluation 11.1.1, 12.1.2

>

Chapter 29: Maximum Likelihood Estimators

Introduction to Mathematical Statistics 4.1, 6.1 Nonlife Actuarial Models : Theory Methods and Evaluation 10.2, 12.3

>

Chapter 30: Maximum Likelihood Estimators—Special Techniques

Introduction to Mathematical Statistics 4.1, 6.1 Nonlife Actuarial Models : Theory Methods and Evaluation 10.2, 12.3

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Chapter 31: Variance of Maximum Likelihood Estimator

Introduction to Mathematical Statistics 6.2, 6.5

>

Chapter 32: Sufficient Statistics

Introduction to Mathematical Statistics 7

>

Chapter 33: Hypothesis Testing

Introduction to Mathematical Statistics 4.5, 4.6

>

Chapter 34: Confidence Intervals and Sample Size

Introduction to Mathematical Statistics 4.5, 4.6

>

Chapter 35: Confidence Intervals for Means

Introduction to Mathematical Statistics 4.2

>

Chapter 36: Kolmogorov-Smirnov Tests

Nonlife Actuarial Models : Theory Methods and Evaluation 13.2.1

>

Chapter 37: Chi Square Tests

Introduction to Mathematical 4.7 Nonlife Actuarial Models : Theory Methods and Evaluation 13.2.3

>

Chapter 38: Confidence Intervals for Variances

Introduction to Mathematical Statistics 8.3

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Chapter 39: Uniformly Most Powerful Critical Regions

Introduction to Mathematical Statistics 8.1 - 8.2

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Chapter 40: Likelihood Ratio Tests

Introduction to Mathematical Statistics 8.3

>

Chapter 41: Q.-Q. Plots

Introduction to Mathematical Statistics 4.4 Larsen Study Note

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Notes sur les vidéos YouTube

StatQuest: A Gentle Introduction to Machine Learning

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3 Extended Linear Models (30% à 50%)

Information

from list below :

- OLS ;
- GLM ;
- ANOVA ;
- GAM ;
- Local Regression ;

- Lasso ;
- Ridge Regression ;
- Partial Least Squares ;
- PCA regression ;

Knowledge Statements

a. Understand the relationship between mean and variance by model family member for the exponential distribution ;

b. Understand how to select the appropriate distribution function for the dependent variable and the implication for the appropriate model form ;

c. Link functions (Identity, Log, Logit, Power, Inverse) ;

d. Characteristics of Exponential Family (Binomial, Normal, Exponential, Gamma, Poisson, Inverse Gaussian, Negative Binomial, and Tweedie) ;

e. Canonical Forms of link function and effect of non-canonical link function on bias ;

f. Penalized Regression as implemented using the Lasso or Ridge Regression ;

g. Understand concept of models within models for Generalized Additive Models (GAM) ;

h. Understand dimension reduction using Partial Least Squares (PLS) or Principal Components Regression (PCR) ;

2. Evaluate models developed using Extended Linear Model approach ;

Knowledge Statements

a. Raw or studentized Residuals;

b. R^2;

c. Cook's Distance and outliers;

d. Influential points;

e. Leverage;

f. AIC and BIC penalized log likelihood measures;

g. Standardized / Studentized Residuals;

h. Deviance, Deviance Residuals and relationship to likelihood;

i. Pearson Residuals vs. Deviance residuals;

j. Scatter, QQ and Box Plots;

k. Type III Sequential Chi-Square test;

l. T-test and Wald test for significance of regression coefficients;

m. Prediction intervals for response variable;

n. MSE and standard error;

o. Calculation and validity of F test to compare 2 models (under OLS);

p. Cross-Validation;

q. Test vs. Train Error;

r. Bootstrapping to test model validity;

s. Prediction vs. Forecast Error;

t. Overfitting;

u. Bias-Variance Tradeoff;

v. Evaluate collinearity using VIF;

w. Evaluate appropriateness of underlying assumptions including homoscedasticity and autocorrelation of residuals;

3. Understand the algorithms behind the numerical solutions for the different forms of the Extended Linear Model family to enable interpretation of output from the statistical software employed in modeling and to make appropriate modeling choices when selecting modeling options;

Knowledge Statements

   a. Maximum Likelihood and OLS;

   b. Fisher Scoring (iterative weighted least squares as implemented using the Information and Score functions from section B. 1);

   c. Quasi-Likelihood and relationship to maximum likelihood;

   d. Collinearity (Aliasing) and model stability;

   e. Hat matrix $\boldsymbol{H}$;

   f. Design matrix $\boldsymbol{X}$;

   g. Fitting adjoining, overlapping observations in groups for Local Regression;

   h. Supervised vs. Unsupervised learning methods;

   i. Modeling functions within functions for GAMs;

   j. Penalty function in Penalized regression models (Lasso and Ridge Regression);

   k. PLS supervised learning vs. PCR unsupervised learning;

4. Understand and be able to select the appropriate model structure for an Extended Linear Model given the behavior of the data set to be modeled;

Knowledge Statements

a. Predictor variables;

b. Response variables;

c. Regression through the origin;

d. Transformation of variables;

e. Categorical vs. continuous explanatory variables;

f. Interaction terms;

g. Significance and model comparison statistics;

h. Residuals and model parameter selection;

i. Piecewise linear and Smoothing Splines;

j. Smoothing parameter for splines;

k. Basis Functions;

l. Knot Selection for Splines;

m. Weighting function for local regression;

n. Selection of functions within functions for GAMs;

o. Selection of appropriate tuning factor for Lasso or Ridge Regression;

p. Select either Lasso or Ridge Regression depending on desired effect from penalized regression;

q. Curse of High Dimensionality;

r. Forward or backward or best subset selection;

Related lessons ASM

Vidéos YouTube

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Résumés des chapitres

Chapter 42: Introduction to Extended Linear Models

Introduction to Mathematical Statistics 4.4 Introduction to Generalized Linear Models 2 Introduction to Statistical Learning with R 2 (except 2.2.3)

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Chapter 43: How a Generalized Linear Model Works

Introduction to Generalized Linear Models 3.9 Introduction to Statistical Learning with R 3.3.1 - 3.3.2 Larsen Study Note

>

Chapter 44: How a Generalized Linear Model Works : Categorical Response

Introduction to Generalized Linear Models 7, 8 Introduction to Statistical Learning with R 4

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Chapter 45: Generalized Linear Model : Estimating Parameters

Introduction to Generalized Linear Models 4 Introduction to Statistical Learning with R 3.1.1, 3.2.1 Larsen Study Note

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Chapter 46: Generalized Linear Model : Measures of Fit

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Chapter 47: Normal Linear Model : Standard Error, R-squared, and t-statistic

Introduction to Generalized Linear Models 6.1 - 6.3 Introduction to Statistical Learning with R 3.1.2 - 3.1.3, 3.2.2 - 3.2.3

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Chapter 48: Normal Linear Model : F and VIF

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Chapter 49: Normal Linear Model : Validation

Introduction to Generalized Linear Models 6.2 Introduction to Statistical Learning with R 3.3.3

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Chapter 50: Normal Linear Model : Predictions

>

Chapter 51: ANOVA

Introduction to Generalized Linear Models 6.4 - 6.5, 9.3 - 9.7 Introduction to Mathematical Statistics 9.1 - 9.5

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Chapter 52: Generalized Linear Model : Measures of Fit II

Introduction to Generalized Linear Models 7, 8, 9

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Chapter 53: Resampling Methods

Introduction to Statistical Learning with R 5

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Chapter 54: Normal Linear Model : Subset Selection

Introduction to Statistical Learning with R 6.1

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Chapter 55: Normal Linear Model : Shrinkage and Dimension Reduction

Introduction to Statistical Learning with R 6.2 - 6.4

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Chapter 56: Extensions to the Linear Model

Introduction to Statistical Learning with R 7

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Notes sur les vidéos YouTube

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4 Time Series with Constant Variance (10% à 20%)

Information

Description

Notes du descriptif principal :

> Covers an introduction to modeling activity, such as financial results or stock prices, over time ;

> The model used is the Auto Regressive Integrated Moving Average (ARIMA) where activity in a given period may be linked to activity in subsequent time periods ;

> The connection between adjacent time periods violates one of the assumptions behind the Extended Linear Model techniques ;

> The ARIMA appproach incorporates that linkage as an aid for it's predictions ;

> Also covers the application of regression models to time series analysis ;

- -

Notes de la sous-section :

> Section covers basic applications of the ARIMA time series model ;

Learning objectives

1. Use time series to model trends ;
   > Estimation, data analysis, and forecasting ;
   > Forecast errors and confidence intervals ;

(a) Mean-reverting time series;

(b) Elimination of trends using differencing;

(c) Relationship between seasonality and autocorrelation;

2. Model relationships of current and past values of a statistic / metric;

> Estimation, data analysis, and forecasting;
> Forecast errors and confidence intervals;

(a) Calculation and use of lag k autocorrelation statistic and cross correlation statistics in determining model structure;

(b) Stationary series;

(c) AR(1) models;

(d) ARIMA models—AR(p), MA(q), ARMA(p, q), ARIMA(p, d, q), ARMA vs ARIMA;

(e) Invertible time series and relationship between AR and MA models;

(f) Converting between AR and MA models;

(g) Interpretation of auto-correlation function as aid to model selection (AR vs. MA and number of lags to include in model);

(h) Relationship between time series input and item modeled for AR vs. MA;

3. Understand forecasts produced by ARIMA;

Knowledge Statements

(a) Forecast using ARIMA models;

(b) **One** step ahead *prediction* vs **many** step ahead *projection*;

(c) Change in variance in prediction by AR vs MA model;

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4. Time Series with Regression;

Knowledge Statements

(a) Deterministic vs. Stochastic Trend;

(b) Serial correlation in regression error results;

(c) Correction in regression via Generalized Least Squares;

(d) Transformation of data using natural logarithms for regression modelling;

(e) Forecast error correction under natural logarithms transformation;

Related lessons ASM

57. Time Series : Trend and Seasonality

58. Time Series : Correlation

59. Time Series : White Noise and Random Walks

60. Time Series : Autoregressive Models

61. Time Series : Regression

62. Time Series : Moving Average Models

63. Time Series : ARMA Models

64. Time Series : ARIMA and SARIMA Models

Résumés des chapitres

57. Time Series : Trend and Seasonality

Introduction to Time Series with `R` 1

1.1 : Time Series Data—Purpose ;

- Understand the past and predict the future ;
- Usage by Singapore Airlines to increase their fleet based on future goals (expansion) and past trends (growing popularity) ;
- Usage by gas suppliers in the UK who must purchase the supply the day before it's demanded from offshore fields ;

  The price thus varies according to the temperature, the time of year, etc. and even the wind temperature.

1.2 : Time Series Data—Time Series ;

- Idea that variables are measured sequentially in time everywhere ;

  Banks record interest and exchange rates on a daily basis ;

  Governments measure GDP on a yearly basis ;

- Time series model, a.k.a. **discrete-time stochastic process**, is a **sequence** of *random variables* defined at **fixed sampling intervals** ;
- Main **features** of time series are **trends** and **seasonal variations** ;
- Another important feature is that most observations which are close together in time tend to be correlated *(serially dependent)* ;
- This can be applied to making predictions and test whether, for example, fluctuations in sales are due to an underlying change or just regular variation ;

- – Sampling intervals can differ in their relation to data.
  - ∗ The data could have been aggregated over the period (for example, the number of foreign tourists arriving per day);
  - ∗ The data could have been sampled over the period (for example, the daily close price of Apple stock).

1.3 : Time Series Data—`R language`;

1.4 : Time Series Data—Plots, trends, and seasonal variation;

- – **Trend** : systemic change in a time series which doesn't appear to be periodic;

  The simplest model for trends is a linear (de)increase and is often an adequate approximation;

- – **Seasonal variation** : Repeating pattern within each year;

  For example, restaurant bookings varying according to the weekday;

- – **Cycles** : do not correspond to some **fixed** natural period

  For example, the economic cycle;

  No cycles are apparent in the image below :
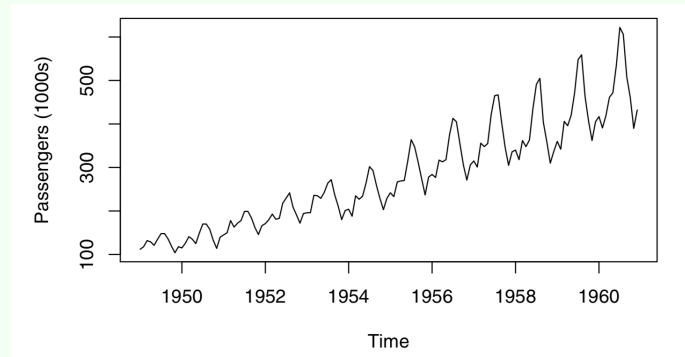
- – The trend has many possible explanations (post-WWII prosperity, cheaper flights, etc.) but if there are non then it would be a **stochastic trend**;

  In this case, regression's not appropriate;

- – Forecasting uses extrapolation supposing trends continue at a slow pace;

– Therefore, linear extrapolation is a reasonable approximation for a **few steps** ahead ;

  There's no empirical way to verify this, so we must explain the trend to justify extrapolation ;

– Forecasts beyond a year are thus better described as ***scenarios*** ;

– **Seasonal effects** can be *removed* by *aggregating to an annual basis* and evaluating the resulting time series ;

– Example of Chocolate/Beer/Electricity in Australia for *Multiple Time Series* ;

– Because autocorrelated series will often have the same trends and seasonal variations, they're often removed before comparison ;

– These trends will often be *deterministic*

  Australian population increase leads to an increase in electricity usage ;

– In contrast, finance will often have *stochastic* trends ;

– Sometimes these can be modeled with a **random walk** ;
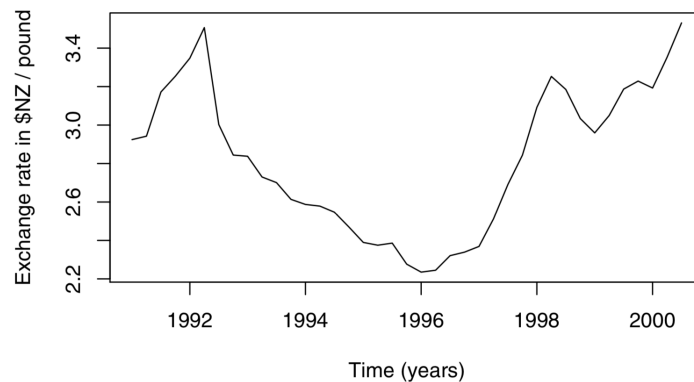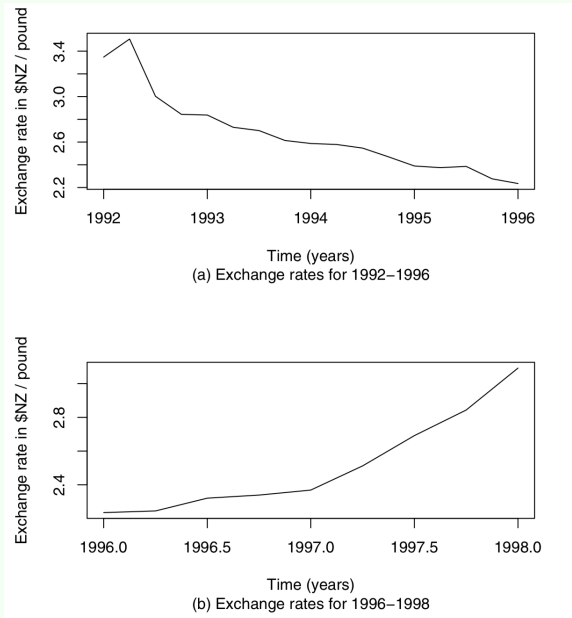
– Idea of there being two distinct trends

Fig. 1.9. Quarterly exchange rates for the period 1991–2000.

– Thereby, split the series in 2 (`window` in R);

Time (years)
(a) Exchange rates for 1992–1996

Time (years)
(b) Exchange rates for 1996–1998

– This also highlights the importance of not extrapolating—without additional information we don't know whether the trend will continue;

– Idea that 2 unrelated time series will be correlated if they both contain a trend thus we can't attribute global warming to fossil fuel increase without a physical explanation;

– As per scientists, we judge appropriate to attribute a **causal relationship** and to expect mean global temperature to continue rising if greenhouse gas emissions aren't reduced;

1.5 : Time Series Data—Decomposition of series;

– Time series of length n : $\{x_t \; : \; t \; = \; 1,\ldots,n\} = \{x_1, x_2, \ldots, x_n\}$;

It is a sequence of random variables x_t sampled at n discrete times $1, 2, \ldots, n$;

– Forecast at time t for a future value $t + k$: $\hat{x}_{t+k|t}$;

k is the **lead time**;

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- If the seasonal effect tends to increase as the trend increases, a multiplicative model may be more appropriate ;

- If there's a multiplicative factor modelled with the random variable (for example we have really large numbers) then an additive decomposition model for $\log(x_t)$ may be more appropriate ;

- Moving average centered around x_t is one of the simplest ways to estimate a trend ;

  Because the average time is $t = 6.5$ and we have integer values, we have one half of both x_{t-6} and x_{t+6} ;

- We can estimate the seasonal effect of each month $\bar{s}_t$ by averaging $\hat{s}_t$ per month ;

- We can take the mean of the averages and substract it from the time series and obtain the **seasonally adjusted series** ;

- **loess** : locally weighted regression technique ;

  « *local* » as it uses a « small » number of points « around » it ;

  Thereby, this reduces the impact of outliers ;

Notation :

time series : $\{x_t : t = 1, \ldots, n\} = \{x_1, x_2, \ldots, x_n\}$
forecast : $\hat{x}_{t+k|t}$

Models

additive decomposition model : $x_t = m_t + s_t + z_t$
multiplicative model : $x_t = m_t \cdot s_t + z_t$
additive decomposition model for $\log(x_t)$: $x_t = m_t + s_t + z_t$
forecast when $z_t \sim \mathcal{N}(0, \sigma^2)$: $\hat{x}_t = e^{m_t + s_t} e^{\frac{1}{2}\sigma^2}$

où

x_t observed series ; $\qquad\qquad$ s_t seasonal effect ;

m_t trend ; $\qquad\qquad\qquad$ z_t error term.

z_t is generally a sequence of random variables with a mean of zero ;
Moving average of a monthly series

$$\hat{m}_t = \frac{\frac{1}{2}x_{t-6} + x_{t-5} + \cdots + x_t + \cdots + x_{t+5} + x_{t+6}}{12}$$

Monthly additive effect :

$$\hat{s}_t = x_t - \hat{m}_t$$

Adjustment such that $\bar{s} = 0$.
Monthly multiplicative effect :

$$\hat{s}_t = \frac{x_t}{\hat{m}_t}$$

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Adjustment such that $\bar{s} = 1$.
$\bar{s} = \frac{\sum_{t=1}^{t=m} \hat{s}_t}{m}$. Seasonally adjusted data : $x_t - \hat{s}_t$.

58. Time Series : Correlation

Introduction to Time Series with `R` 2 - 3.1

  2 : Correlation

3.1 : Forecasting Strategies—Purpose

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1. Second order properties of a time series ;
2. Relationships of different time series ;

59. Time Series : White Noise and Random Walks

Introduction to Time Series with `R` 4.1 - 4.4

4.1 : Basic Stochastic Models—Purpose

4.2 : Basic Stochastic Models—White Noise

4.3 : Basic Stochastic Models—Random Walks

4.4 : Basic Stochastic Models—Fitted models and diagnostic plots

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1. White noise ;
2. Random walks ;

60. Time Series : Autoregressive Models

Introduction to Time Series with `R` 4.5 - 4.8

4.5 : Basic Stochastic Models—AR models

4.6 : Basic Stochastic Models—Fitted models

4.7 : Basic Stochastic Models—Summary of `R` commands

4.8 : Basic Stochastic Models—Exercices

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Correlograms and partial correlograms ;

Stationnarity ;

Forecasting with $AP(p)$ series ;

61. Time Series : Regression

Introduction to Time Series with `R` 5

   5 : Regression

1. Correcting for autocorrelation ;
2. Seasonality ;
3. Logarithmic transformations ;
4. Error correction factors ;

62. Time Series : Moving Average Models

Introduction to Time Series with `R` 6.1 - 6.4

6.1 : Stationary Models—Purpose

6.2 : Stationary Models—Strictly stationary series

6.3 : Stationary Models—MA models

6.4 : Stationary Models—Fitted MA models

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63. Time Series : ARMA Models

Introduction to Time Series with `R` 6.5 - 6.8

6.5 : Stationary Models—Mixed models : The ARMA process

6.6 : Stationary Models—ARMA models : Empirical Analysis

6.7 : Stationary Models—Summary of `R` commands

6.8 : Stationary Models—Exercices

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64. Time Series : ARIMA and SARIMA Models

Introduction to Time Series with `R` 7.1 - 7.3

7.1 : Non-stationary Models—Purpose

7.2 : Non-stationary Models—Non-Seasonal ARIMA models

7.3 : Non-stationary Models—Seasonal ARIMA models

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Notes sur les vidéos YouTube

StatQuest: A Gentle Introduction to Machine Learning

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