Study Guide

Exam FM: Financial Mathematics
Society of Actuaries (SOA)

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Preliminary

Information

Objectives

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Learning outcomes

1.

Autres ressources

Liens

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Vidéos YouTube

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Notes sur les vidéos YouTube

Subjects of study

1 Time Value of Money (10%-15%)

Information

Objective

The Candidate will understand and be able to perform calculations relating to present value, current value, and accumulated value.

Learning outcomes

The candidate will be able to :

a) Define and recognize the *definitions* of the following terms :

- Interest rate (rate of interest) ;
- Simple interest ;
- Compound interest ;
- Accumulation function ;
- Future value ;
- Current value ;
- Present value ;
- Net present value ;
- Discount factor ;
- Discount rate (rate of discount) ;
- Convertible m-thly (...?) ;
- Nominal rate ;
- Effective rate ;
- Inflation ;
- Real rate of interest ;
- Force of interest ;
- Equation of value.

b) Given any 3 of :

- Interest rate ;
- Period of time ;
- Present value ;
- Current value ;
- Future value,

calculate the remaining item using *simple* or *compound* interest ;

Solve time value of money equations involving variable force of interest ;

c) Given any 1 of :
   - Effective interest rate ;
   - Nominal interest rate convertible m-thly ;
   - Force of interst,

   calculate any of the other items ;
d) Write the equation of value given a set of cash flows and interest rate.

Related lessons ASM

Section 1 : Interest rates and Discount Rates

- 1a. Basic Concepts
- 1b. Why Do We Need a Force of Interest ?
- 1c. Defining the Force of Interest
- 1d. Finding the Fund in Terms of the Force of Interest
- 1e. The Simplest Case : A Constant Force of Interest
- 1f. Power Series
- 1g. The Variable Force of Interest Trap
- 1h. Equivalent Rates

Section 2 : Practical Applications

- 2a. Equations of Value, Time Value of Money, and Time Diagrams
- 2b. Unknown Time and Unknown Interest Rate

Vidéos YouTube

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Résumés des chapitres

1a. Basic Concepts

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Effective rate of interest

$a(t)$ **Accumulation function** defined as the Accumulated Value (AV) of the fund at time t of an initial investment of \$1.00 at time 0.

- $a(0) \equiv 1$.
- Generally **continuous** and **increasing**.

$a(t) - a(t-1)$ ***Amount* of growth** in the t^{th} year.

- a.k.a. the interest earned

$\frac{a(t)-a(t-1)}{a(t-1)}$ ***Rate* of growth** in the t^{th} year.

- a.k.a. effective rate of interest denoted i_t.

$A(t)$ **Amount function** defined as the Accumulated Value (AV) of the fund at time t of an initial investment of \$$k$ at time 0.

- $A(t) = ka(t)$.

i_t **Effective rate of interest** defined as the rate of growth based on the amount in the fund at the ***beginning*** of the year.

- $i_t = \frac{A(t)-A(t-1)}{A(t-1)}$.
- We deduce $A(t) = (1 + i_t)A(t-1)$.

Effective Rate of Discount

d_t **Effective rate of discount** defined as the rate of growth based on the amount in the fund at the ***end*** of the year.

- $d_t = \frac{A(t)-A(t-1)}{A(t)}$.
- Although we could get by without it, it's useful to determine the amount to pay today for a specified amount in the future.

Discounting Finding the price we'd be willing to pay for the promise to receive a future amount.

- a.k.a. finding the present value which is why $i = \frac{d}{1-d}$.
- $v = (1-d) = \frac{1}{1+i}$.
- $d = \frac{i}{1+i}$.

Nominal Rates of Interest

$i^{(m)}$ **Nominal** annual rate of of interest **compounded** m **times a year**.

$\frac{i^{(m)}}{m}$ **Effective** rate of of interest **for an** $m^{\textbf{th}}$ **of a year**.

- Thus $(1+i) = \left(1+\frac{i^{(m)}}{m}\right)^m$.

1b. Why Do We Need a Force of Interest ?

- An effective rate of interest only gives information about the starting and ending values, but give no information about in between.
- Thus, the force of interest can give information at any given time about the rate of growth.
- The image of the four different fund's growth curves with the same starting and ending values is a perfect visualization.

1c. Defining the Force of Interest

- The derivative is divided by the amount function to obtain a rate of growth proportional to the amount invested.
- Two funds can have the same rate of change but different amounts originally invested.
- If one fund's growing with a smaller amount of money, then it's rate of change is actually less than the other.

TRAP If given the derivative of the accumulation function, $a'(t)$, use the property that the fund at the beginning is 1, $a(0) = 1$, to define the $+C$ when integrating for $a(t)$.

1d. Finding the Fund in Terms of the Force of Interest

- If we want to find the accumulation, or amount, function from the force of interest we inverse the equation.
- To do so, recall that $\frac{\partial}{\partial x}\ln(f(x)) = \frac{f'(x)}{f(x)}$.
- Also $\int_0^t \frac{\partial}{\partial r}\ln(a(r))dr = \ln(a(r))\Big|_0^t = \ln(a(t))$.

Force of interest

Force of interest the rate of growth at a point in time.

- a.k.a. finding the present value which is why $i = \frac{d}{1-d}$.
- $v = (1-d) = \frac{1}{1+i}$.
- $d = \frac{i}{1+i}$.

δ_t The **Force of interest** at time t.

- $\delta_t = \frac{A'(t)}{A(t)}$.
- $a(t) = \mathrm{e}^{\int_0^t \delta_r dr}$

1e. The Simplest Case : A Constant Force of Interest

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Simple Force of interest

δ The constant force of interest.

- a.k.a. the nominal rate of interest compounded continuously.
- $\delta = \lim_{m\to\infty} i^{(m)} = i^{(\infty)} = \ln(1+i)$.
- $a(t) = \mathrm{e}^{\int_0^t \delta dr} = \mathrm{e}^{\delta t}$.

1f. Power Series

- Not really on past exams, section is « just in case ».

1g. The Variable Force of Interest Trap

- When we want the accumulated value of an amount not invested at the beginning, we integrate the force of interest over the respective integral.
- Alternatively, we can take the ratio of the accumulation function at both times.

Variable Force of interest

$$\begin{aligned} FV &= \mathrm{e}^{\int_{t_1}^{t_2} \delta_r dr} \\ &= \frac{a(t_2)}{a(t_1)} \end{aligned}$$

1h. Equivalent Rates

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2a. Equations of Value, Time Value of Money, and Time Diagrams

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2b. Unknown Time and Unknown Interest Rate

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Notes sur les vidéos YouTube

2 Topic : Annuities / cash flows with non-contingent payments (exam weight)

Information

Objective

The Candidate will be able to calculate present value, current value, and accumulated value for sequences of non-contingent payments.

Learning outcomes

The candidate will be able to :

a) Define and recognize the *definitions* of the following terms :

- Annuity-immediate ;
- Annuity-due ;
- Perpetuity ;
- Payable m-thly or continously ;
- Level payment annuity ;
- Arithmetic increasing/decreasing annuity ;
- Geometric increasing/decreasing annuity ;
- Term of annuity ;

b) For each of the following types of annuity / cash flows, given sufficient information of :

- Immediate or due ;
- Present value ;
- Futur value ;
- Current value ;
- Interest rate ;
- Payment amount ;
- Term of annuity,

calculate any remaining item.

The types are :

- Level annuity, finite term ;

- Level perpetuity ;
- Non-level annuities / cash flows ;
  - Arithmetic progression, finite term and perpetuity ;
  - Geometric progression, finite term and perpetuity ;
  - Other non-level annuities / cash flows.

Related lessons ASM

Section 3 : Annuities

- 3a. The Geometric Series Trap
- 3b. Annuity-Immediate and Annuity-Due
- 3c. The Great Confusion : Annuity-Immediate and Annuity-Due
- 3d. Deferred Annuities
- 3e. A Short-Cut Method for Annuities with "Block" Payments
- 3f. Perpetuities
- 3g. The $a_{\overline{2n}|}/a_{\overline{n}|}$ Trick (and Variations)
- 3h. What If the Rate Is Unknown ?
- 3i. What If the Rate Varies ?

Section 4 : Complex Annuities

- 4a. Annuities with "Off-Payments" Part I
- 4b. Annuities with "Off-Payments" Part II
- 4c. Avoiding the m^{thly} Annuity Trap
- 4d. Continuous Annuities
- 4e."Double-Dots Cancel" (and so do "upper m's")
- 4f. A Short Note on Remembering Annuity Formulas
- 4g. The $s_{\overline{n}|}$ Trap When Interest Variess
- 4h. Payments in Arithmetic Progression
- 4i. Remembering Increasing Annuity Formulas
- 4j. Payments in Geometric Progression
- 4k. The Amazing Expanding Money Machine (Or Continouss Varying Annuities)

- 4l. A Short-Cut Method for the Palindromic Annuity
- 4m. The 0% Test : A Quick Check of Symbolic Answers

Vidéos YouTube

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Résumés des chapitres

3a. The Geometric Series Trap

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3b. Annuity-Immediate and Annuity-Due

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3c. The Great Confusion : Annuity-Immediate and Annuity-Due

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3d. Deferred Annuities

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3e. A Short-Cut Method for Annuities with "Block" Payments

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3f. Perpetuities

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3g. The $a_{\overline{2n}|}/a_{\overline{n}|}$ Trick (and Variations)

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3h. What If the Rate Is Unknown?

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3i. What If the Rate Varies?

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4a. Annuities with "Off-Payments" Part I

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4b. Annuities with "Off-Payments" Part II

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4c. Avoiding the m^{thly} Annuity Trap

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4d. Continuous Annuities

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4e."Double-Dots Cancel" (and so do "upper m's")

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4f. A Short Note on Remembering Annuity Formulas

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4g. The $s_{\overline{n}|}$ Trap When Interest Variess

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4h. Payments in Arithmetic Progression

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4i. Remembering Increasing Annuity Formulas

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4j. Payments in Geometric Progression

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4k. The Amazing Expanding Money Machine (Or Continouss Varying Annuities)

›

4l. A Short-Cut Method for the Palindromic Annuity

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4m. The 0% Test : A Quick Check of Symbolic Answers

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Notes sur les vidéos YouTube

3 Topic : Loans (10%-20%)

Information

Objective

The Candidate will understand key concepts concerning loans and how to perform related calculations.

Learning outcomes

The candidate will be able to :

a) Define and recognize the *definitions* of the following terms :

- Principal ;
- Interest ;
- Term of loan ;
- Outstanding balance ;
- Final payment ;
  - Drop payment ;
  - Baloon payment.
- Amortization.

b) Calculate :

- The missing item given any 4 of :
  - Term of loan ;
  - Interest rate ;
  - Payment amount ;
  - Payment period ;
  - Principal.
- The outstanding balance at any point in time ;
- The amount of interest and principal repayment in a given payment ;
- Similar calculations to the above when refinancing is involved.

Related lessons ASM

Section 6 : Loans

- XX. Title-of-ASM-chapter

Vidéos YouTube

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Résumés des chapitres

XX. Title-of-ASM-chapter

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Notes sur les vidéos YouTube

4 Topic : Bonds (10%-20%)

Information

Objective

The Candidate will understand key concepts concerning bonds, and how to perform related calculations.

Learning outcomes

The candidate will be able to :

a) Define and recognize the *definitions* of the following terms :

- Price ;
- Book value ;
- Amortization of premium ;
- Accumulation of discount ;
- Redemption value ;
- Par value / Face value ;
- Yield rate ;
- Coupon ;
- Coupon rate ;
- Term of bond ;
- Callable / Non-callable.

b) Given sufficient partial information about the items listed below, calculate any of the remaining items :

- Price, book value, amortization of premium, accumulation of discount ;
- Redemption value, face value ;
- Yield rate ;
- Coupon, coupon rate ;
- Term of bond, point in time that a bond has a given book value, amortization of premium, or accumulation of discount.

Related lessons ASM

Section 7 : Bonds

- XX. Title-of-ASM-chapter

Vidéos YouTube

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Résumés des chapitres

XX. Title-of-ASM-chapter

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5 Topic : General Cash Flows and Portfolios (15%-20%)

Information

Objective

The Candidate will understand key concepts concerning yield curves, rates of return, and measures of duration and convexity, and how to perform related calculations.

Learning outcomes

The candidate will be able to :

a) Define and recognize the *definitions* of the following terms :

- Yield rate / rate of return ;
- Dollar-weighted rate of return ;
- Time-weighted rate of return ;
- Current value ;
- Duration (Macaulay and modified) ;
- Convexity (Macaulay and modified) ;
- Portfolio ;
- Spot rate ;
- Forward rate ;
- Yield Curve ;
- Stock price ;
- Stock dividend ;

b) Calculate :

- The dollar-weighted and time-weighted rate of return ;
- The duration and convexity of a set of cash flows ;
- Either Macaulay or modified duration given the other ;
- The approximate change in present value due to a change in interest rate,

  - Using 1st-order linear approximation based on modified duration ;
  - Using 1st-order approximation based on Macaulay duration.
- The price of a stock using the dividend discount model ;
- The present value of a set of cash flows, using a yield curve developed from forward and spot rates.

Related lessons ASM

Section 5 :

- 5a
- 5b
- 5c
- 5d
- 5e
- 5f

Section 8 :

- 8a
- 8b
- 8c

Section 10 :

- **??**
- **??**
- **??**
- **??**
- **??**
- **??**
- **??**

Section 11 :

- **??**

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Résumés des chapitres

5a.

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5b.

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5c.

-

5d.

-

5e.

-

5f.

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8a.

-

8b.

-

8c.

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Notes sur les vidéos YouTube

6 Topic : Immunization (10%-15%)

Information

Objective

The Candidate will understand key concepts concerning cash flow matching and immunization, and how to perform related calculations.

Learning outcomes

The candidate will be able to :

a) Define and recognize the *definitions* of the following terms :
   - Cash flow matching ;
   - Immunization (including full immunization) ;
   - Redington immunization.

b) Construct an investment portfolio to :
   - Redington immunize a set of liability cash flows ;
   - Fully immunize a set of liability cash flows ;
   - Exactly match a set of liability cash flows.

Related lessons ASM

Section 10 : Duration, Convexity, and Immunization

- 10h. Redington Immunization
- 10i. Full Immunization
- 10j. A Note on Rebalancing
- 10k. Immunization by Exact Matching ("Dedication")

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Résumés des chapitres

10h. Redington Immunization

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10i. Full Immunization

›

10j. A Note on Rebalancing

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10k. Immunization by Exact Matching ("Dedication")

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Notes sur les vidéos YouTube

7 Topic : Interest Rate Swaps (0-10%)

Information

Objective

The Candidate will understand key concepts concerning interest rate swaps, and how to perform related calculations.

Learning outcomes

The candidate will be able to :

a) Define and recognize the *definitions* of the following terms :

- Swap rate ;
- Swap term (tenor) ;
- Notional amount ;
- Market value of a swap ;
- Settlement dates ;
- Settlement period ;
- Counterparties ;
- Deferred swap ;
- Amortizing swap ;
- Accreting swap ;
- Interest rate swap net payments.

b) Given sufficient information, calculate :

- The market value ;
- Notional amount ;
- Spot rates or swap rate,
- deferred or otherwise ;
- with either constant or varying notional amount.

of an interest rate swap

Related lessons ASM

Section 11 : Interest Rate Swaps

- 11b. What is an Interest Rate Swap ?

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Résumés des chapitres

11b. What is an Interest Rate Swap ?

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8 Topic : Determinants of Interest Rates (0-10%)

Information

Objective

The Candidate will understand key concepts concerning the determinants of interest rates, the components of interest, and how to perform related calculations.

Learning outcomes

The candidate will be able to :

a) Define and recognize the *definitions* of the following terms :

- Real risk-free rate ;
- Inflation rate ;
- Default risk premium ;
- Liquidity premium ;
- Maturity risk premium.

b) Explain how the components of interest rates apply in various contexts, such as :

- Commercial loans ;
- Mortgages ;
- Credit cards ;
- Bonds ;
- Government securities.

c) Explain the **roles** of the Federal Reserve and the FOMC in carrying out *fiscal* policy and *monetary* policy and the **tools** used thereby including :

- Targeting the federal funds rate ;
- Setting reserve requirements ;

- Setting the discount rate.

d) Explain the theories of why interest rates differ by term, including :
   - Liquidity preference (opportunity cost) ;
   - Expectations ;
   - Preferred habitat ;
   - Market segmentation.

e) Explain how interest rates differ from one country to another (e.g., U.S. vs. Canada) ;

f) In the context of loans with and without inflation protection :
   - **Identify** the *real* interest and the *nominal* interest rate ;
   - **Calculate** the effect of changes in inflation on loans with inflation protection.

Related lessons ASM

Section 9 : Determinants of Interest Rates

- 9a. What is Interest ?
- 9b. Quotation Bases for Interest Rates
- 9c. Components of the Interest Rate : No Inflation or Default Risk
- 9d. Components of the Interest Rate : no Inflation but with Default Risk
- 9e. Components of the Interest Rate : Known Inflation
- 9f. Components of the Interest Rate : Uncertain Inflationo
- 9g. Savings and Lending Interest Rates
- 9h. Government and Corporate Bonds
- 9i. The Role of Central Banks

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Résumés des chapitres

9a. What is Interest ?

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9b. Quotation Bases for Interest Rates

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9c. Components of the Interest Rate : No Inflation or Default Risk

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9d. Components of the Interest Rate : no Inflation but with Default Risk

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9e. Components of the Interest Rate : Known Inflation

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9f. Components of the Interest Rate : Uncertain Inflationo

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9g. Savings and Lending Interest Rates

›

9h. Government and Corporate Bonds

›

9i. The Role of Central Banks

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