Contributeurs

MAS-II: Modern Actuarial Statistics II

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Référence (manuels, YouTube, notes de cours) En ordre alphabétique :
Contributeurs

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Context

The ***limitation fluctuation credibility*** approach, or ***classical credibility*** approach, calculates an updated prediction (U) of the **loss measure** as a weighted (Z) average of recent claim experience (D) and a rate (M) specified in the manual. Thus, we calculate the *premium* paid by the *risk group* as $\boxed{U = ZD + (1-Z)M}$.

Notation

M Predicted loss based on the "***manual***".

D Observe**d** losses based on the recent experience of the risk group.

Z Weight assigned to the recent experience D called the ***credibility factor*** with $\boxed{Z \in [0,1]}$.

U **U**pdated prediction of the premium.

Terminology

Risk group block of insurance policies, covered for a period of time upon payment of a *premium*.

Claim frequency The number of claims denoted N.

Claim severity The amount of the i^{th} claim denoted X_i.

Aggregate loss The total loss denoted S where $S = X_1 + X_2 + \ldots + X_N$.

Pure premium The pure premium denoted P where $P = S/E$ with E denoting the number of exposure units.

Exam tips

Typical questions about this involve being given 3 of M, D, Z, and U then finding the missing one.

Context

With $\min\{D, M\} \leq U \leq \max\{D, M\}$, we can see that the credibility factor determines the relative importance of the claim experience of the risk group D relative to the manual rate M.

If $Z = 1$, we obtain *Full Credibility* where the predicted premium depends only on the data ($U = D$). It follows that with $Z < 1$, we obtain *Partial Credibility* as the weighted average of both D and M.

Full Credibility

Contexte

The classical credibility approach determines the ***minimum*** *data size* required for the experience data (D) to be given ***full credibility***. The minimum data size, or ***standard for full credibility***, depends on the **loss measure**.

Claim Frequency

The claim frequency random variable N has mean μ_N and variance σ_N^2.
If we assume $N \approx \mathcal{N}(\mu_N, \sigma_N^2)$, then the probability of observing claim frequency **within** k **of the mean** is $\boxed{\Pr(\mu_N - k\mu_N \leq N \leq \mu_N + k\mu_N) = 2\Phi\left(\frac{k\mu_N}{\sigma_N}\right) - 1}$.

We often assume that the claim frequency $\boxed{N \sim \text{Pois}(\lambda_N)}$ and then apply the normal approximation to find the standard for full credibility for claim frequency $\boxed{\lambda_F}$. First, we impose that the probability of the claim being with k of the mean must be at least $1-\alpha$. Then, we rewrite $\boxed{\frac{k\mu_N}{\sigma_N} = k\sqrt{\lambda_N}}$ and set $\boxed{\lambda_N \geq \left(\frac{z_{1-\alpha/2}}{k}\right)^2}$ where $\boxed{\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2}$.

Claim Severity

We assume that the loss amounts $X_1, X_2, \ldots, X_N$ are independent and identically distributed random variables with mean μ_X and variance σ_X^2. Full credibility is attributed to $\boxed{D = \bar{X}}$ if $\boxed{2\Phi\left(\frac{k\mu_X}{\sigma_N/\sqrt{N}}\right) - 1 \geq 1 - \alpha}$.

Similarly to claim frequency, we apply the normal approximation with $\boxed{\bar{X} \approx \mathcal{N}\left(\mu_X, \sigma_X^2/N\right)}$. Then, we find $\boxed{N \geq \left(\frac{z_{1-\alpha/2}}{k}\right)^2 \cdot \left(\frac{\sigma_X}{\mu_X}\right)^2 = \lambda_F CV_X^2}$ where the ***standard for full credibility for claim severity*** is $\lambda_F CV_X^2$.

Aggregate Loss

For the aggregate loss $S = X_1 + X_2 + \ldots + X_N$, we have $\boxed{\mu_S = \mu_N \mu_X}$ and $\boxed{\sigma_S^2 = \mu_N \sigma_X^2 + \mu_X^2 \sigma_N^2}$.

With the same normality assumptions for the Poisson distributed N, we find $\boxed{\lambda_N \geq \left(\frac{z_{1-\alpha/2}}{k}\right)^2 \cdot \left(\frac{\mu_X^2 + \sigma_X^2}{\mu_X^2}\right) = \lambda_F(1 + CV_X^2)}$ where the ***standard for full credibility for claim severity*** is $\lambda_F(1 + CV_X^2)$.

Note The conditions are the same for the ***Pure Premium*** as for the aggregate loss.

Partial Credibility

The ***credibility factor*** for :

Claim Frequency is $\boxed{Z = \sqrt{\frac{\lambda_N}{\lambda_F}}}$.

Claim Severity is $\boxed{Z = \sqrt{\frac{N}{\lambda_F CV_X^2}}}$.

Aggregate Loss and Pure Premium is $\boxed{Z = \sqrt{\frac{\lambda_N}{\lambda_F(1+CV_X^2)}}}$

Bühlmann Credibility

Basic framework

Variance components

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Bayesian Credibility

Basic framework

Premium

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Linear Mixed Models

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Bayesian Analysis and Markov Chain Monte Carlo

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Principal Components Analysis (PCA)

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