

# 10\_2Q4

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## 10.2Q.4

a)

i)

$$\begin{aligned}F_{X_1, X_2}(X_1, X_2) &= P(X_1 \leq x_1, X_2 \leq x_2) \\&= P(F_{X_1}^{-1}(U_1) \leq x_1, F_{X_2}^{-1}(U_2) \leq x_2) \\&= P(U_1 \leq F_{X_1}(x_1), U_2 \leq F_{X_2}(x_2)) \\&= C_{\alpha}^{EFGM}(F_{X_1}(x_1), F_{X_2}(x_2))\end{aligned}$$

ii)

```
alpha <- 1
densityCopule <- function(u1, u2) {
  u1 * u2 + alpha * u1 * u2 * (1 - u1) * (1 - u2)
}
FXX <- function(x1, x2) {
  densityCopule(pexp(x1, 1 / 100), plnorm(x2, log(100) - 0.32, 0.8))
}
FXX(100, 100)
```

```
## [1] 0.4668243
```

```
FXX(200, 100)
```

```
## [1] 0.5931482
```

```
FXX(100, 300)
```

```
## [1] 0.6165623
```

b)

$$\begin{aligned}F_{X_1, X_2}(X_1, X_2) &= P(X_1 \leq x_1, X_2 \leq x_2) \\&= P(F_{X_1}^{-1}(U_1) \leq x_1, F_{X_2}^{-1}(1 - U_2) \leq x_2) \\&= P(U_1 \leq F_{X_1}(x_1), 1 - U_2 \leq F_{X_2}(x_2)) \\&= P(U_1 \leq F_{X_1}(x_1), U_2 > 1 - F_{X_2}(x_2)) \\&= P(U_1 \leq F_{X_1}(x_1), U_2 > \overline{F}_{X_2}(x_2)) \\&= 1 - \overline{F}_{X_2}(x_2) - C_{\alpha}^{EFGM}(F_{X_1}(x_1), \overline{F}_{X_2}(x_2))\end{aligned}$$

ii)

```
FXX <- function(x1, x2) {  
  1 - pexp(x1, 1 / 100, lower.tail = FALSE) - densityCopule(pexp(x1, 1 / 100),  
    1 - plnorm(x2, log(100) - 0.32, 0.8))  
}  
FXX(100, 100)  
  
## [1] 0.3617868  
FXX(200, 100)  
  
## [1] 0.5402919  
FXX(100, 300)  
  
## [1] 0.5995209
```

c)

$$\begin{aligned} F_{X_1, X_2}(X_1, X_2) &= P(X_1 \leq x_1, X_2 \leq x_2) \\ &= P(F_{X_1}^{-1}(1 - U_1) \leq x_1, F_{X_2}^{-1}(U_2) \leq x_2) \\ &= P(1 - U_1 \leq F_{X_1}(x_1), U_2 \leq F_{X_2}(x_2)) \\ &= P(U_1 > 1 - F_{X_1}(x_1), U_2 \leq F_{X_2}(x_2)) \\ &= P(U_1 > \bar{F}_{X_1}(x_1), U_2 \leq F_{X_2}(x_2)) \\ &= 1 - \bar{F}_{X_1}(x_1) - C_{\alpha}^{EFGM}(\bar{F}_{X_1}(x_1), F_{X_2}(x_2)) \end{aligned}$$

ii)

```
FXX <- function(x1, x2) {  
  1 - (1 - plnorm(x2, log(100) - 0.32, 0.8)) - densityCopule(pexp(x1, 1 /  
    100, lower.tail = FALSE),  
    plnorm(x2, log(100) - 0.32, 0.8))  
}  
FXX(100, 100)  
  
## [1] 0.3617868  
FXX(200, 100)  
  
## [1] 0.5402919  
FXX(100, 300)  
  
## [1] 0.5995209
```

d)

$$\begin{aligned}
F_{X_1, X_2}(X_1, X_2) &= P(X_1 \leq x_1, X_2 \leq x_2) \\
&= P(F_{X_1}^{-1}(1 - U_1) \leq x_1, F_{X_2}^{-1}(U_2) \leq x_2) \\
&= P(1 - U_1 \leq F_{X_1}(x_1), 1 - U_2 \leq F_{X_2}(x_2)) \\
&= P(U_1 > 1 - F_{X_1}(x_1), U_2 > 1 - F_{X_2}(x_2)) \\
&= P(U_1 > \bar{F}_{X_1}(x_1), U_2 > \bar{F}_{X_2}(x_2)) \\
&= 1 - \bar{F}_{X_1}(x_1)\bar{F}_{X_2}(x_2) + C_{\alpha}^{EFGM}(\bar{F}_{X_1}(x_1), \bar{F}_{X_2}(x_2))
\end{aligned}$$

ii)

```

FXX <- function(x1, x2) {
  1 - (1 - plnorm(x2, log(100) - 0.32, 0.8)) - pexp(x1, 1 / 100, lower.tail = FALSE) +
  densityCopule(pexp(x1, 1 /
    100, lower.tail = FALSE),
    plnorm(x2, log(100) - 0.32, 0.8))
}
FXX(100, 100)

## [1] 0.5811772
FXX(200, 100)

## [1] 0.6352163
FXX(100, 300)

## [1] 0.956415

```