10_2Q3

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30 novembre 2017

10.2Q.3

a)

i)

$$
\begin{aligned}
F_{X_1,X_2}(X_1,X_2) &= P\big(X_1 \le x_1, X_2 \le x_2\big) \\
&= P\big(F_{X_1}^{-1}(U_1) \le x_1, F_{X_2}^{-1}(U_2) \le x_2\big) \\
&= P\big(U_1 \le F_{X_1}(x_1), U_2 \le F_{X_2}(x_2)\big) \\
&= C_{\alpha}^{Gumbel}\big(F_{X_1}(x_1), F_{X_2}(x_2)\big)
\end{aligned}
$$

ii)

```r
alpha <- 5
densityCopule <- function(u1, u2) {
exp(-((-log(u1) ^ alpha) + (-log(u2) ^ alpha)) ^ (1 / alpha))
}
FXX <- function(x1, x2) {
densityCopule(pexp(x1, 1 / 100), plnorm(x2, log(100) - 0.32, 0.8))
}
FXX(100, 100)
```

```
## [1] 0.601826
```

```r
FXX(200, 100)
```

```
## [1] 0.6551548
```

```r
FXX(100, 300)
```

```
## [1] 0.6321203
```

b)

$$
\begin{aligned}
F_{X_1,X_2}(X_1,X_2) &= P\big(X_1 \le x_1, X_2 \le x_2\big) \\
&= P\big(F_{X_1}^{-1}(U_1) \le x_1, F_{X_2}^{-1}(1-U_2) \le x_2\big) \\
&= P\big(U_1 \le F_{X_1}(x_1), 1-U_2 \le F_{X_2}(x_2)\big) \\
&= P\big(U_1 \le F_{X_1}(x_1), U_2 > 1-F_{X_2}(x_2)\big) \\
&= P\big(U_1 \le F_{X_1}(x_1), U_2 > \overline{F}_{X_2}(x_2)\big) \\
&= 1-\overline{F}_{X_2}(x_2) - C_{\alpha}^{Gumbel}\big(F_{X_1}(x_1), \overline{F}_{X_2}(x_2)\big)
\end{aligned}
$$

ii)

```r
FXX <- function(x1, x2) {
     1 - pexp(x1, 1 / 100, lower.tail = FALSE) - densityCopule(pexp(x1, 1 / 100),
     1 - plnorm(x2, log(100) - 0.32, 0.8))
}
FXX(100, 100)
```

```
## [1] 0.28862
```

```r
FXX(200, 100)
```

```
## [1] 0.5200899
```

```r
FXX(100, 300)
```

```
## [1] 0.5940295
```

c)

$$
\begin{aligned}
F_{X_1,X_2}(X_1, X_2) &= P\big(X_1 \le x_1, X_2 \le x_2\big) \\
&= P\big(F_{X_1}^{-1}(1-U_1) \le x_1, F_{X_2}^{-1}(U_2) \le x_2\big) \\
&= P\big(1-U_1 \le F_{X_1}(x_1), U_2 \le F_{X_2}(x_2)\big) \\
&= P\big(U_1 > 1 - F_{X_1}(x_1), U_2 \le F_{X_2}(x_2)\big) \\
&= P\big(U_1 > \overline{F}_{X_1}(x_1), U_2 \le F_{X_2}(x_2)\big) \\
&= 1 - \overline{F}_{X_1}(x_1) - C_\alpha^{Gumbel}\big(\overline{F}_{X_1}(x_1), F_{X_2}(x_2)\big)
\end{aligned}
$$

ii)

```r
FXX <- function(x1, x2) {
     1 - (1 - plnorm(x2, log(100) - 0.32, 0.8)) - densityCopule(pexp(x1, 1 /
     100, lower.tail = FALSE),
     plnorm(x2, log(100) - 0.32, 0.8))
}
FXX(100, 100)
```

```
## [1] 0.288526
```

```r
FXX(200, 100)
```

```
## [1] 0.5201092
```

```r
FXX(100, 300)
```

```
## [1] 0.5940282
```

d)

$$
\begin{aligned}
F_{X_1,X_2}(X_1,X_2) &= P\big(X_1 \le x_1, X_2 \le x_2\big) \\
&= P\big(F_{X_1}^{-1}(1-U_1) \le x_1, F_{X_2}^{-1}(U_2) \le x_2\big) \\
&= P\big(1-U_1 \le F_{X_1}(x_1), 1-U_2 \le F_{X_2}(x_2)\big) \\
&= P\big(U_1 > 1-F_{X_1}(x_1), U_2 > 1-F_{X_2}(x_2)\big) \\
&= P\big(U_1 > \overline{F}_{X_1}(x_1), U_2 > \overline{F}_{X_2}(x_2)\big) \\
&= 1 - \overline{F}_{X_1}(x_1)\overline{F}_{X_2}(x_2) + C_\alpha^{Gumbel}\big(\overline{F}_{X_1}(x_1), \overline{F}_{X_2}(x_2)\big)
\end{aligned}
$$

ii)

```
FXX <- function(x1, x2) {
     1 - (1 - plnorm(x2, log(100) - 0.32, 0.8)) - pexp(x1, 1 / 100, lower.tail = FALSE) +
     densityCopule(pexp(x1, 1 /
     100, lower.tail = FALSE),
     plnorm(x2, log(100) - 0.32, 0.8))
}
FXX(100, 100)
```

```
## [1] 0.6544381
```

```
FXX(200, 100)
```

```
## [1] 0.655399
```

```
FXX(100, 300)
```

```
## [1] 0.9619076
```