10_2Q1

David Beauchemin

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10.2Q.1

a)

i)

$$\begin{aligned} F_{X_1,X_2}(X_1,X_2) &= P\big(X_1 \leq x_1, X_2 \leq x_2\big) \\ &= P\big(F_{X_1}^{-1}(U_1) \leq x_1, F_{X_2}^{-1}(U_2) \leq x_2\big) \\ &= P\big(U_1 \leq F_{X_1}(x_1), U_2 \leq F_{X_2}(x_2)\big) \\ &= C_{\alpha}^{Frank}\big(F_{X_1}(x_1), F_{X_2}(x_2)\big) \end{aligned}$$

ii)

```
alpha <- 5
densityCopule <- function(u1, u2) {
-1 / alpha * log(1 + ((exp(-alpha * u1) - 1) * (exp(-alpha * u2) - 1) /
(exp(-alpha) - 1)))
}

FXX <- function(x1, x2) {
densityCopule(pexp(x1, 1 / 100), plnorm(x2, log(100) - 0.32, 0.8))
}

#Évaluation
FXX(100, 100)
```

```
## [1] 0.5254238
```

```
FXX(200, 100)
```

```
## [1] 0.6244787
```

```
FXX(100, 300)
```

```
## [1] 0.6257929
```

b)

$$\begin{aligned} F_{X_1,X_2}(X_1,X_2) &= P\big(X_1 \le x_1, X_2 \le x_2\big) \\ &= P\big(F_{X_1}^{-1}(U_1) \le x_1, F_{X_2}^{-1}(1-U_2) \le x_2\big) \\ &= P\big(U_1 \le F_{X_1}(x_1), 1-U_2 \le F_{X_2}(x_2)\big) \\ &= P\big(U_1 \le F_{X_1}(x_1), U_2 > 1-F_{X_2}(x_2)\big) \\ &= P\big(U_1 \le F_{X_1}(x_1), U_2 > \overline{F}_{X_2}(x_2)\big) \\ &= 1-\overline{F}_{X_2}(x_2) - C_\alpha^{Frank}\big(F_{X_1}(x_1), \overline{F}_{X_2}(x_2)\big) \end{aligned}$$

ii)

```
FXX <- function(x1, x2) {
    1 - pexp(x1, 1 / 100, lower.tail = FALSE) - densityCopule(pexp(x1, 1 / 100),
    1 - plnorm(x2, log(100) - 0.32, 0.8))
}
FXX(100, 100)
```

```
## [1] 0.3181181
```

```
FXX(200, 100)
```

```
## [1] 0.5260351
```

```
FXX(100, 300)
```

```
## [1] 0.5955291
```

c)

$$\begin{aligned} F_{X_1,X_2}(X_1,X_2) &= P\big(X_1 \le x_1, X_2 \le x_2\big) \\ &= P\big(F_{X_1}^{-1}(1-U_1) \le x_1, F_{X_2}^{-1}(U_2) \le x_2\big) \\ &= P\big(1-U_1 \le F_{X_1}(x_1), U_2 \le F_{X_2}(x_2)\big) \\ &= P\big(U_1 > 1-F_{X_1}(x_1), U_2 \le F_{X_2}(x_2)\big) \\ &= P\big(U_1 > \overline{F}_{X_1}(x_1), U_2 \le F_{X_2}(x_2)\big) \\ &= 1-\overline{F}_{X_1}(x_1) - C_\alpha^{Frank}\big(\overline{F}_{X_1}(x_1), F_{X_2}(x_2)\big) \end{aligned}$$

ii)

```
FXX <- function(x1, x2) {
    1 - (1 - plnorm(x2, log(100) - 0.32, 0.8)) - densityCopule(pexp(x1, 1 /
    100, lower.tail = FALSE),
    plnorm(x2, log(100) - 0.32, 0.8))
}

FXX(100, 100)
```

```
## [1] 0.3181181
```

```
FXX(200, 100)
```

```
## [1] 0.5260351
```

```
FXX(100, 300)
```

```
## [1] 0.5955291
```

d)

$$
\begin{aligned}
F_{X_1,X_2}(X_1,X_2) &= P\big(X_1 \leq x_1, X_2 \leq x_2\big) \\
&= P\big(F_{X_1}^{-1}(1-U_1) \leq x_1, F_{X_2}^{-1}(U_2) \leq x_2\big) \\
&= P\big(1-U_1 \leq F_{X_1}(x_1), 1-U_2 \leq F_{X_2}(x_2)\big) \\
&= P\big(U_1 > 1-F_{X_1}(x_1), U_2 > 1-F_{X_2}(x_2)\big) \\
&= P\big(U_1 > \overline{F}_{X_1}(x_1), U_2 > \overline{F}_{X_2}(x_2)\big) \\
&= 1-\overline{F}_{X_1}(x_1)\overline{F}_{X_2}(x_2) + C_\alpha^{Frank}\big(\overline{F}_{X_1}(x_1), \overline{F}_{X_2}(x_2)\big)
\end{aligned}
$$

ii)

```
FXX <- function(x1, x2) {
    1 - (1 - plnorm(x2, log(100) - 0.32, 0.8)) - pexp(x1, 1 / 100, lower.tail = FALSE) +
        densityCopule(pexp(x1, 1 /
    100, lower.tail = FALSE),
    plnorm(x2, log(100) - 0.32, 0.8))
}
FXX(100, 100)
```

```
## [1] 0.6248459
```

```
FXX(200, 100)
```

```
## [1] 0.6494731
```

```
FXX(100, 300)
```

```
## [1] 0.9604067
```