MTRE 2610 Intermediate Programming for Mechatronics

Laboratory - Complex number class

Introduction

The goal of this laboratory is to create a surface plot in Excel with data produced using complex arithmetic in C++.

Mapping equally spaced points to index positions

The array

```
double y[5] = { -2,3,8,13,18 };
```

has five elements equally spaced between -2 and 18, inclusive. Consider initializing the array in a loop rather than manually where the minimum y_{min} and maximum y_{max} (which are -2 and 18, respectively, in the example), as well as the number of elements n (5 in the example), could be different.

```
const int n = 5;
double yMin = -2, yMax = 18, y[n];
for (int x = 0; x < n; i++)
    y[x] = ??? ;
```

The one missing piece is an equation for `y` which depends on `x`. For the points to be equally spaced, a linear function

$$y(x) = mx + b$$

is required. This function is constrained by the desired minimum and maximum values according to

$$y(0) = m \cdot 0 + b = y_{\min} \text{ and } y(n-1) = m \cdot (n-1) + b = y_{\max}$$

Note the n-1 results from the n^{th} value appearing at index n-1 since indices begin at 0 rather than 1. The first of these equations generates $b = y_{min}$ which combined with the second equation simplifies to

$$m = \frac{y_{\max} - y_{\min}}{n-1}$$

The question marks in the sample code can be replaced with the linear equation with the desired values of y_{min}, y_{max} and n. In the case of the example,

$$y(x) = mx + b = \left(\frac{y_{\max} - y_{\min}}{n-1}\right)x + y_{\min} = \frac{18-(-2)}{5-1}x - 2 = \frac{20}{4}x - 2 = 5x - 2$$

Note in this case that y increases by m = 5 for every increment in x, i.e. each of the numbers in the series -2, 3, 8, 13, 18 represent an increase of 5.

Complex arithmetic

Consider two complex numbers

$$z_1 = a + bi \text{ and } z_2 = c + di$$

Collecting real and imaginary components when adding results in

$$z_1 + z_2 = (a+bi)+(c+di) = (a+c)+(bi+di) = (a+c)+(b+d)i$$

Subtraction is analogously performed as

$$z_1 - z_2 = (a+bi)-(c+di) = (a-c)+(bi-di) = (a-c)+(b-d)i$$

Multiplication is more complicated

$$z_1 z_2 = (a+bi)(c+di) = ac + adi + bci + bdi^2 = ac + (ad+bc)i + bd(-1) = (ac-bd)+(ad+bc)i$$

where the first-outer-inner-last technique is applied to distribute multiplication over addition. Division

$$\frac{z_1}{z_2} = z_1 \cdot \frac{1}{z_2} = z_1 \cdot z_3$$

can be accomplished by first computing the reciprocal of the divisor and then performing multiplication. The reciprocal of a complex number

$$z_3 = \frac{1}{z_2} = \frac{1}{c+di}\cdot\frac{c-di}{c-di} = \frac{c-di}{c^2 - cdi + cdi - b^2 i^2} = \frac{c-di}{c^2 - d^2(-1)} = \frac{c-di}{c^2+d^2} = \left(\frac{c}{c^2+d^2}\right) - \left(\frac{d}{c^2+d^2}\right)i$$

can be found by employing the complex conjugate. The magnitude of a complex number

$$|z_1| = \sqrt{a^2 + b^2}$$

is its distance from the center of the complex plane.

Complex class

Create a class to instantiate objects representing complex numbers using the class definition

```
class Complex {
        double  real, imag;
public:
        Complex          (                    );
        Complex          (double  r, double i);
        void    setVals  (double  r, double i);
        double  getReal  (                    ); // Return real      part
        double  getImag  (                    ); // Return imaginary part
        double  getMag   (                    ); // Return magnitude
        Complex operator+(Complex c           ); // Add      two complex numbers
        Complex operator-(Complex c           ); // Subtract two complex numbers
        Complex operator*(Complex c           ); // Multiply two complex numbers
        Complex operator/(Complex c           ); // Divide   two complex numbers
};

ostream& operator<<(ostream& out, Complex z); // Write z to the output stream out
Complex operator/(double x, Complex z);     // Return reciprocal of z multiplied by scalar x
```

Laboratory exercise procedure

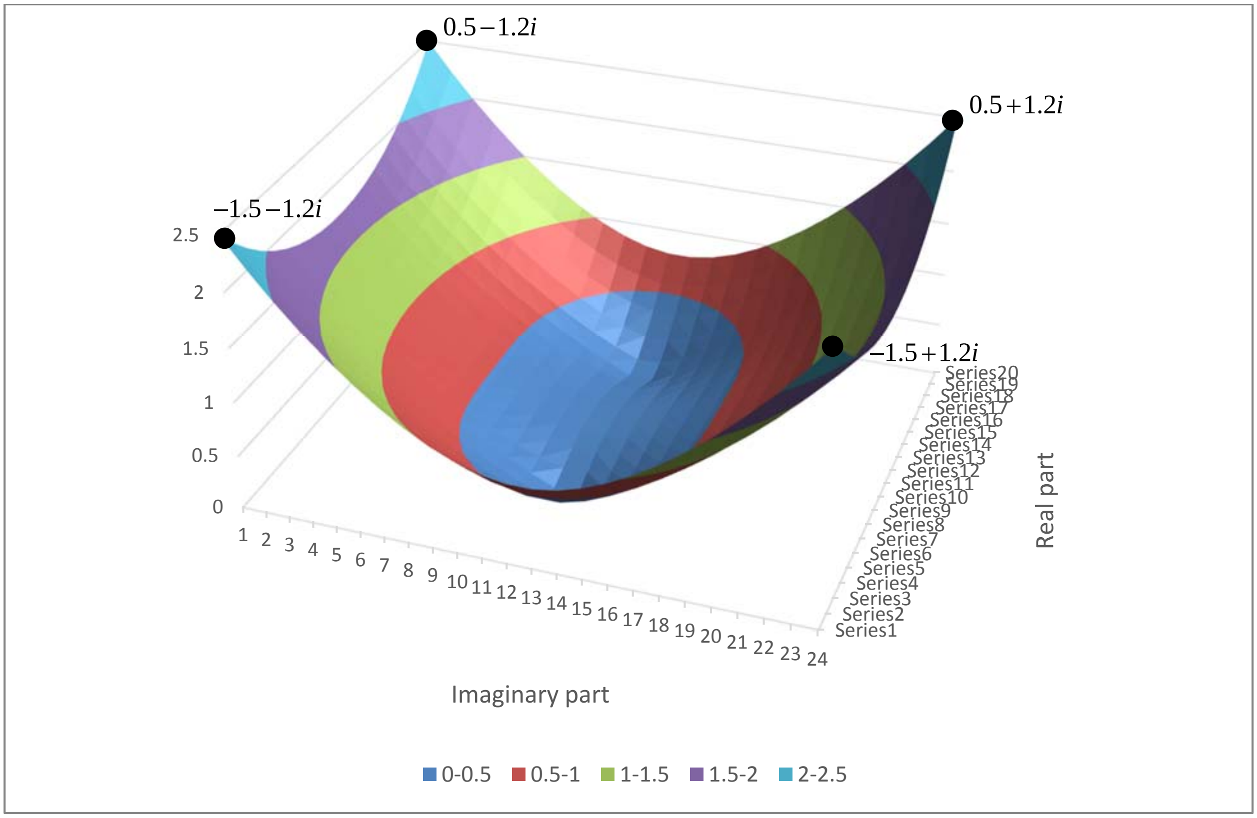
Complete the complex number class and confirm it by displaying the result

$$z_1 z_2 + \frac{z_1}{z_2 - z_1} = -14.1834 - 0.760117i \text{ for } z_1 = 1.4 + 2.1i \text{ and } z_2 = -3.2 + 4.6i \tag{1}$$

using `cout` and the insertion operator `<<`.

Create an array of 20 instances of the `Complex` class all with the same -1.2 imaginary part but with real parts varying between -1.5 and 0.5. Use the index-to-value mapping presented earlier to vary the real part for index values from 0 to 19. For each complex number, compute $z^2 + z$ and write its magnitude to a text file with the 20 magnitudes on a single row separated by spaces with a single `endl` at the end.

Take the program created in the previous paragraph, and place it inside a loop iterating 24 times. Have the imaginary part of z depend on this loop with values between -1.2 and 1.2. This requires a 2-dimensional array, or matrix, of complex instances who size is 24×20. Again, write the magnitudes of each $z^2 + z$ to a text file where each row in the file corresponds to complex numbers with the same imaginary part. Load the text file into Excel by selecting spaces as delimiters. Select all the data and display as a surface plot, similar to the figure below.



Grading rubric

1. 50 points: Completed `Complex` class demonstrates correct calculation of Equation (1)
2. 25 points: Single-row text file created with magnitudes of complex numbers with -1.2 imaginary part
3. 25 points: Surface plot from complex numbers ranging between -1.5 to 0.5 real and -1.2 to 1.2 imaginary