

# Determining Statistically Significant Effects

**CASE 2:** There are no independent replications of any of the treatment combinations.

CASE 2 (A):  $n$  is large

CASE 2 (B):  $n$  is small



# *n* is large

Many researchers have observed that four-way, five-way, and higher order interaction are rarely significant, and hence, they most likely correspond to contrasts that are measuring experimental error rather than effects of the sampled treatment combinations.



# ***n* is large**

When  $n$  is large, say  $n > 5$ , the sums of squares for 4-way and higher order interactions are often pooled together and used to estimate the experimental error variance.



**Example:** Consider a  $2^6$  experiment.

Effect Type	Number
Main	6
2-Way Interactions	15
3-Way Interactions	20
4-Way Interaction	15
5-Way Interactions	6
6-Way Interaction	1
Total	63



By pooling the four-way and higher order interaction sums of squares into error in a  $2^6$  experiment, we can get an approximate estimate of the experimental error variance that has

$$15 + 6 + 1 = 22 \text{ degrees of freedom.}$$



## ***n* is small**

When  $n$  is small, say  $n < 6$ , then half-normal plots can be used to identify the statistically significant effects. Then after the statistically significant effects are identified, another half-normal plot can be created to help get an estimate of the experimental error variance.



**Remark:** Many authors suggest pooling 3-way and higher order interactions into error. While I have only occasionally observed statistically significant 4-way and higher order interactions during my career as a statistician, I have seen lots of statistically significant 3-way interactions.

Consequently, I would rather use half-normal plots to help estimate the experimental error than depend on being able to use 3-way interaction effects to estimate experimental error.



**Table 7.4 Definition of the Treatment Structure in a  $2^5$  Factorial Experiment to Study Cake Quality**

Variable	Level	
	Low	High
<i>W</i> : amount of water	40	120
<i>M</i> : mixing time	3	6
<i>T</i> : temperature	300	500
<i>C</i> : cooking oil	6	12
<i>P</i> : mixer type	$P_1$	$P_2$





Obs	W	M	T	C	P	QUALITY
1	0	0	0	0	0	4.8
2	1	0	0	0	0	3.9
3	0	1	0	0	0	5.0
4	1	1	0	0	0	2.2
5	0	0	1	0	0	3.9
6	1	0	1	0	0	4.2
7	0	1	1	0	0	3.0
8	1	1	1	0	0	2.2
9	0	0	0	1	0	5.7
10	1	0	0	1	0	2.2
11	0	1	0	1	0	8.4
12	1	1	0	1	0	8.3
13	0	0	1	1	0	5.3
14	1	0	1	1	0	2.3
15	0	1	1	1	0	8.6
16	1	1	1	1	0	8.9
17	0	0	0	0	1	4.2
18	1	0	0	0	1	5.0
19	0	1	0	0	1	5.8
20	1	1	0	0	1	5.2
21	0	0	1	0	1	4.6
22	1	0	1	0	1	4.1
23	0	1	1	0	1	5.4
24	1	1	1	0	1	5.2
25	0	0	0	1	1	2.9
26	1	0	0	1	1	3.0
27	0	1	0	1	1	6.7
28	1	1	0	1	1	6.6
29	0	0	1	1	1	5.0
30	1	0	1	1	1	2.7
31	0	1	1	1	1	7.0
32	1	1	1	1	1	7.1



Obs	W	M	T	C	P	QUALITY
1	0	0	0	0	0	4.8
2	1	0	0	0	0	3.9
3	0	1	0	0	0	5.0
4	1	1	0	0	0	2.2
5	0	0	1	0	0	3.9
6	1	0	1	0	0	4.2
7	0	1	1	0	0	3.0
8	1	1	1	0	0	2.2
9	0	0	0	1	0	5.7
10	1	0	0	1	0	2.2
11	0	1	0	1	0	8.4
12	1	1	0	1	0	8.3
13	0	0	1	1	0	5.3
14	1	0	1	1	0	2.3
15	0	1	1	1	0	8.6
16	1	1	1	1	0	8.9



<b>17</b>	0	0	0	0	1	4.2
<b>18</b>	1	0	0	0	1	5.0
<b>19</b>	0	1	0	0	1	5.8
<b>20</b>	1	1	0	0	1	5.2
<b>21</b>	0	0	1	0	1	4.6
<b>22</b>	1	0	1	0	1	4.1
<b>23</b>	0	1	1	0	1	5.4
<b>24</b>	1	1	1	0	1	5.2
<b>25</b>	0	0	0	1	1	2.9
<b>26</b>	1	0	0	1	1	3.0
<b>27</b>	0	1	0	1	1	6.7
<b>28</b>	1	1	0	1	1	6.6
<b>29</b>	0	0	1	1	1	5.0
<b>30</b>	1	0	1	1	1	2.7
<b>31</b>	0	1	1	1	1	7.0
<b>32</b>	1	1	1	1	1	7.1



```
PROC ANOVA;
```

```
CLASSES W M T C P;
```

```
MODEL QUALITY = W M W*M T W*T  
M*T W*M*T C W*C M*C W*M*C T*C  
W*T*C M*T*C P W*P M*P W*M*P T*P  
W*T*P M*T*P C*P W*C*P M*C*P  
T*C*P;
```

```
RUN;
```



Class Level Information		
Class	Levels	Values
W	2	0 1
M	2	0 1
T	2	0 1
C	2	0 1
P	2	0 1

Number of observations

32



Source	D F	Sum of Squares	Mean Square	F Value	Pr > F
Model	25	116.9262	4.6770500	12.63	0.0023
Error	6	2.22250	0.3704167		
Corrected Total	31	119.1487			

R-Square	Coeff Var	Root MSE	QUALITY Mean
0.981347	12.21819	0.608619	4.981250



Source	D F	Sum of Squares	Mean Square	F Value	Pr > F
Model	25	116.9262	4.6770500	12.63	0.0023
Error	6	2.22250	0.3704167		
Corrected Total	31	119.1487			

R-Square	Coeff Var	Root MSE	QUALITY Mean
0.981347	12.21819	0.608619	4.981250

$$\hat{\sigma} = 0.609$$



Source	DF	Anova SS	Mean Square	F Value	Pr > F
W	1	5.44500000	5.44500000	14.70	0.0086
M	1	31.60125000	31.60125000	85.31	<.0001
W*M	1	0.72000000	0.72000000	1.94	0.2127
T	1	0.00500000	0.00500000	0.01	0.9113
W*T	1	0.03125000	0.03125000	0.08	0.7812
M*T	1	0.04500000	0.04500000	0.12	0.7393
W*M*T	1	0.78125000	0.78125000	2.11	0.1966
C	1	15.12500000	15.12500000	40.83	0.0007
W*C	1	0.45125000	0.45125000	1.22	0.3120
M*C	1	34.44500000	34.44500000	92.99	<.0001
W*M*C	1	5.28125000	5.28125000	14.26	0.0092
T*C	1	1.36125000	1.36125000	3.67	0.1037
W*T*C	1	0.40500000	0.40500000	1.09	0.3360
M*T*C	1	0.06125000	0.06125000	0.17	0.6984
P	1	0.08000000	0.08000000	0.22	0.6585
W*P	1	1.90125000	1.90125000	5.13	0.0641
M*P	1	0.32000000	0.32000000	0.86	0.3885
W*M*P	1	0.21125000	0.21125000	0.57	0.4787
T*P	1	0.45125000	0.45125000	1.22	0.3120
W*T*P	1	1.62000000	1.62000000	4.37	0.0815
M*T*P	1	0.01125000	0.01125000	0.03	0.8674
C*P	1	11.28125000	11.28125000	30.46	0.0015
W*C*P	1	0.00500000	0.00500000	0.01	0.9113
M*C*P	1	5.28125000	5.28125000	14.26	0.0092
T*C*P	1	0.00500000	0.00500000	0.01	0.9113





Source	DF	Anova SS	Mean Square	F Value	Pr > F
<b>W</b>	<b>1</b>	<b>5.44500000</b>	<b>5.44500000</b>	<b>14.70</b>	<b>0.0086</b>
<b>M</b>	<b>1</b>	<b>31.60125000</b>	<b>31.60125000</b>	<b>85.31</b>	<b>&lt;.0001</b>
<b>W*M</b>	1	0.72000000	0.72000000	1.94	0.2127
<b>T</b>	1	0.00500000	0.00500000	0.01	0.9113
<b>W*T</b>	1	0.03125000	0.03125000	0.08	0.7812
<b>M*T</b>	1	0.04500000	0.04500000	0.12	0.7393
<b>W*M*T</b>	1	0.78125000	0.78125000	2.11	0.1966
<b>C</b>	<b>1</b>	<b>15.12500000</b>	<b>15.12500000</b>	<b>40.83</b>	<b>0.0007</b>
<b>W*C</b>	1	0.45125000	0.45125000	1.22	0.3120
<b>M*C</b>	<b>1</b>	<b>34.44500000</b>	<b>34.44500000</b>	<b>92.99</b>	<b>&lt;.0001</b>
<b>W*M*C</b>	<b>1</b>	<b>5.28125000</b>	<b>5.28125000</b>	<b>14.26</b>	<b>0.0092</b>
<b>T*C</b>	1	1.36125000	1.36125000	3.67	0.1037
<b>W*T*C</b>	1	0.40500000	0.40500000	1.09	0.3360
<b>M*T*C</b>	1	0.06125000	0.06125000	0.17	0.6984



<b>P</b>	1	0.08000000	0.08000000	0.22	0.6585
<b>W*P</b>	1	1.90125000	1.90125000	5.13	0.0641
<b>M*P</b>	1	0.32000000	0.32000000	0.86	0.3885
<b>W*M*P</b>	1	0.21125000	0.21125000	0.57	0.4787
<b>T*P</b>	1	0.45125000	0.45125000	1.22	0.3120
<b>W*T*P</b>	1	1.62000000	1.62000000	4.37	0.0815
<b>M*T*P</b>	1	0.01125000	0.01125000	0.03	0.8674
<b>C*P</b>	<b>1</b>	<b>11.28125000</b>	<b>11.28125000</b>	<b>30.46</b>	<b>0.0015</b>
<b>W*C*P</b>	1	0.00500000	0.00500000	0.01	0.9113
<b>M*C*P</b>	<b>1</b>	<b>5.28125000</b>	<b>5.28125000</b>	<b>14.26</b>	<b>0.0092</b>
<b>T*C*P</b>	1	0.00500000	0.00500000	0.01	0.9113



# Significant Effects

The statistically significant effects are:

$W, M, C, M*C, W*M*C, C*P, M*C*P$



# Half-normal Plot Method

To obtain the standardized effects for utilizing a half-normal plot to identify the statistically significant effects, we fit a full 5-way factorial model to the data.



```
PROC ANOVA;
```

```
  TITLE2 'ANALYSIS USING A HALF NORMAL PLOT';
```

```
  CLASSES W M T C P;
```

```
  MODEL QUALITY = W|M|T|C|P;
```

```
  ODS OUTPUT MODELANOVA=EFFECTS;
```

```
RUN;
```



Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	31	119.1487500	3.8435081	.	.
Error	0	0.0000000	.		
Corrected Total	31	119.1487500			





Source	DF	Anova SS	Mean Square	F Value	Pr > F
W	1	5.44500000	5.44500000	.	.
M	1	31.60125000	31.60125000	.	.
W*M	1	0.72000000	0.72000000	.	.
T	1	0.00500000	0.00500000	.	.
W*T	1	0.03125000	0.03125000	.	.
M*T	1	0.04500000	0.04500000	.	.
W*M*T	1	0.78125000	0.78125000	.	.
C	1	15.12500000	15.12500000	.	.
W*C	1	0.45125000	0.45125000	.	.
M*C	1	34.44500000	34.44500000	.	.
W*M*C	1	5.28125000	5.28125000	.	.
T*C	1	1.36125000	1.36125000	.	.
W*T*C	1	0.40500000	0.40500000	.	.
M*T*C	1	0.06125000	0.06125000	.	.
W*M*T*C	1	0.00000000	0.00000000	.	.
P	1	0.08000000	0.08000000	.	.
W*P	1	1.90125000	1.90125000	.	.
M*P	1	0.32000000	0.32000000	.	.
W*M*P	1	0.21125000	0.21125000	.	.
T*P	1	0.45125000	0.45125000	.	.
W*T*P	1	1.62000000	1.62000000	.	.
M*T*P	1	0.01125000	0.01125000	.	.
W*M*T*P	1	0.40500000	0.40500000	.	.
C*P	1	11.28125000	11.28125000	.	.
W*C*P	1	0.00500000	0.00500000	.	.



```
PROC PRINT DATA=EFFECTS;  
  VAR SOURCE MS;  
RUN;
```





Obs	Source	MS
1	W	5.44500000
2	M	31.60125000
3	W*M	0.72000000
4	T	0.00500000
5	W*T	0.03125000
6	M*T	0.04500000
7	W*M*T	0.78125000
8	C	15.12500000
9	W*C	0.45125000
10	M*C	34.44500000
11	W*M*C	5.28125000
12	T*C	1.36125000
13	W*T*C	0.40500000
14	M*T*C	0.06125000
15	W*M*T*C	0.00000000



<b>16</b>	P	0.08000000
<b>17</b>	W*P	1.90125000
<b>18</b>	M*P	0.32000000
<b>19</b>	W*M*P	0.21125000
<b>20</b>	T*P	0.45125000
<b>21</b>	W*T*P	1.62000000
<b>22</b>	M*T*P	0.01125000
<b>23</b>	W*M*T*P	0.40500000
<b>24</b>	C*P	11.28125000
<b>25</b>	W*C*P	0.00500000
<b>26</b>	M*C*P	5.28125000
<b>27</b>	W*M*C*P	1.28000000
<b>28</b>	T*C*P	0.00500000
<b>29</b>	W*T*C*P	0.03125000
<b>30</b>	M*T*C*P	0.40500000
<b>31</b>	W*M*T*C*P	0.10125000



```
DATA EFFECTS2;
```

```
SET EFFECTS;
```

```
STDEFF = SQRT(MS) ;
```

```
KEEP SOURCE MS STDEFF;
```

```
RUN;
```



```
PROC RANK OUT=RANKS;
```

```
RANKS R;
```

```
VAR STDEFF;
```

```
RUN;
```



```
DATA PLOTDATA;
```

```
SET RANKS;
```

```
RSTAR = (R - .5) / 31;
```

```
P = (RSTAR + 1) / 2;
```

```
V = PROBIT(P) ;
```

```
RUN;
```



```
PROC SORT; BY R;
```

```
PROC PRINT;
```

```
    TITLE3 'PRINTOUT OF RANKED  
ABSOLUTE VALUES IN RANK  
ORDER' ;
```

```
RUN;
```



15	W*T*C	0.70000000	0.80000000	15.0	0.40000000	0.70000000	0.621300
16	M*T*C*P	0.40500000	0.63640	16.0	0.50000	0.75000	0.67449
17	W*C	0.45125000	0.67175	17.0	0.53226	0.76613	0.72616
18	T*P	0.45125000	0.67175	18.0	0.56452	0.78226	0.77984
19	W*M	0.72000000	0.84853	19.0	0.59677	0.79839	0.83587
20	W*M*T	0.78125000	0.88388	20.0	0.62903	0.81452	0.89466
21	W*M*C*P	1.28000000	1.13137	21.0	0.66129	0.83065	0.95672
22	T*C	1.36125000	1.16673	22.0	0.69355	0.84677	1.02270
23	W*T*P	1.62000000	1.27279	23.0	0.72581	0.86290	1.09346
24	W*P	1.90125000	1.37886	24.0	0.75806	0.87903	1.17016
25	M*C*P	5.28125000	2.29810	25.0	0.79032	0.89516	1.25445
26	W*M*C	5.28125000	2.29810	26.0	0.82258	0.91129	1.34874
27	W	5.44500000	2.33345	27.0	0.85484	0.92742	1.45684
28	C*P	11.28125000	3.35876	28.0	0.88710	0.94355	1.58528
29	C	15.12500000	3.88909	29.0	0.91935	0.95968	1.74695
30	M	31.60125000	5.62150	30.0	0.95161	0.97581	1.97395
31	M*C	34.44500000	5.86899	31.0	0.98387	0.99194	2.40598



```
PROC GPLOT;
```

```
TITLE 'A HALF-NORMAL PLOT OF THE  
EFFECTS IN CAKE QUALITY STUDY';
```

```
SYMBOL V=DOT I=NONE;
```

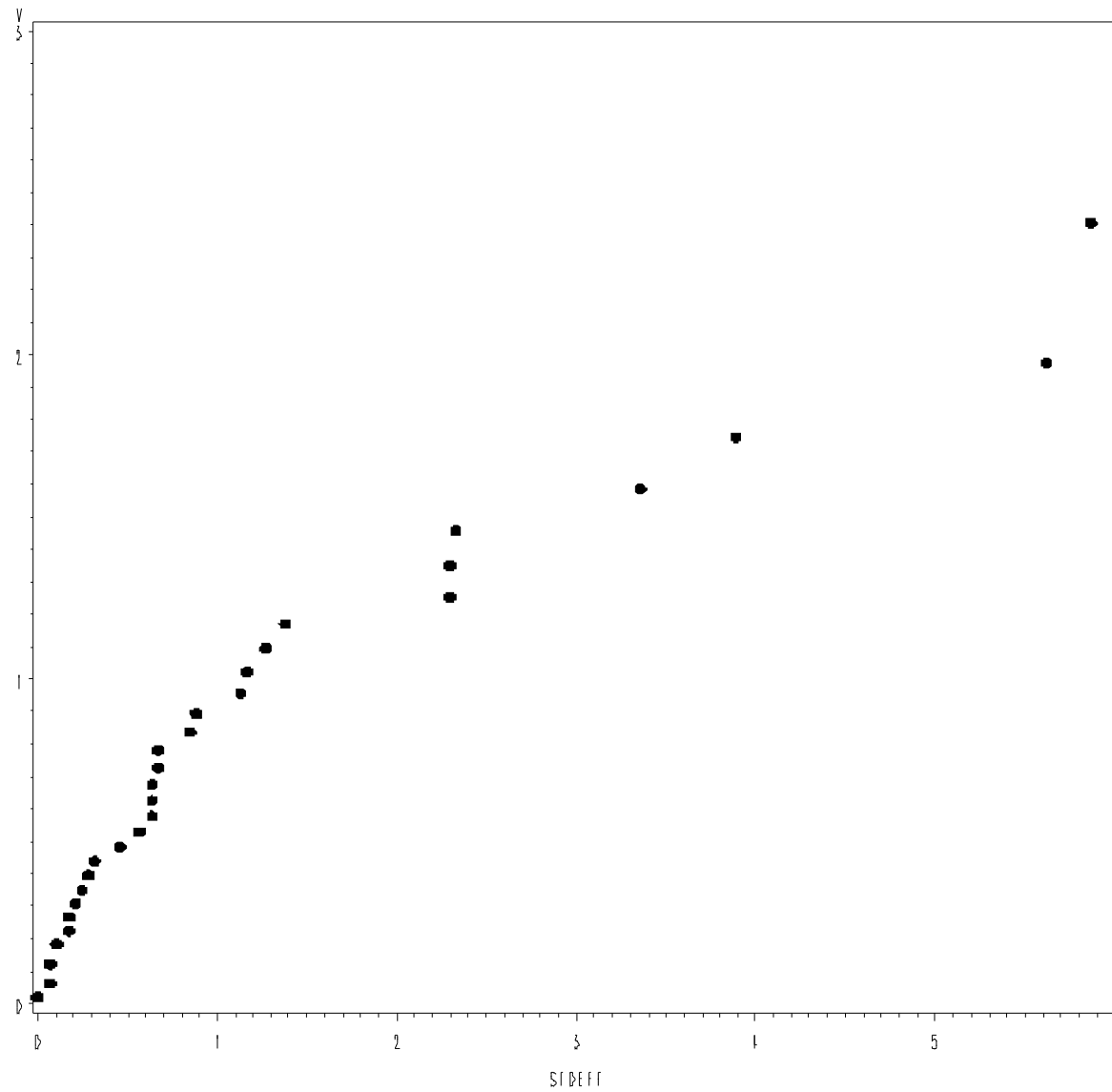
```
PLOT V*STDEFF;
```

```
RUN;
```

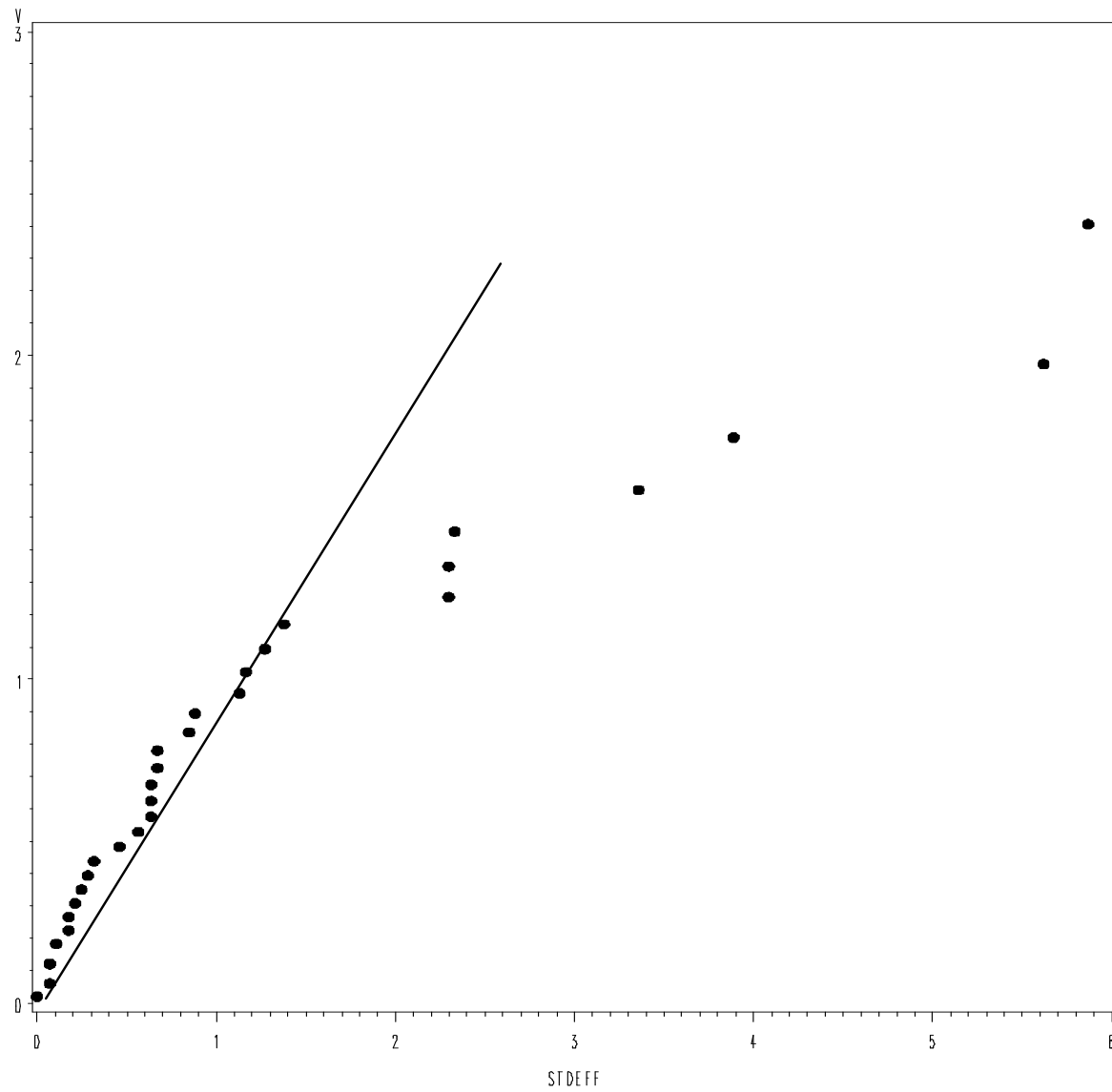




# A HALF-NORMAL PLOT OF THE EFFECTS IN CAKE QUALITY STUDY



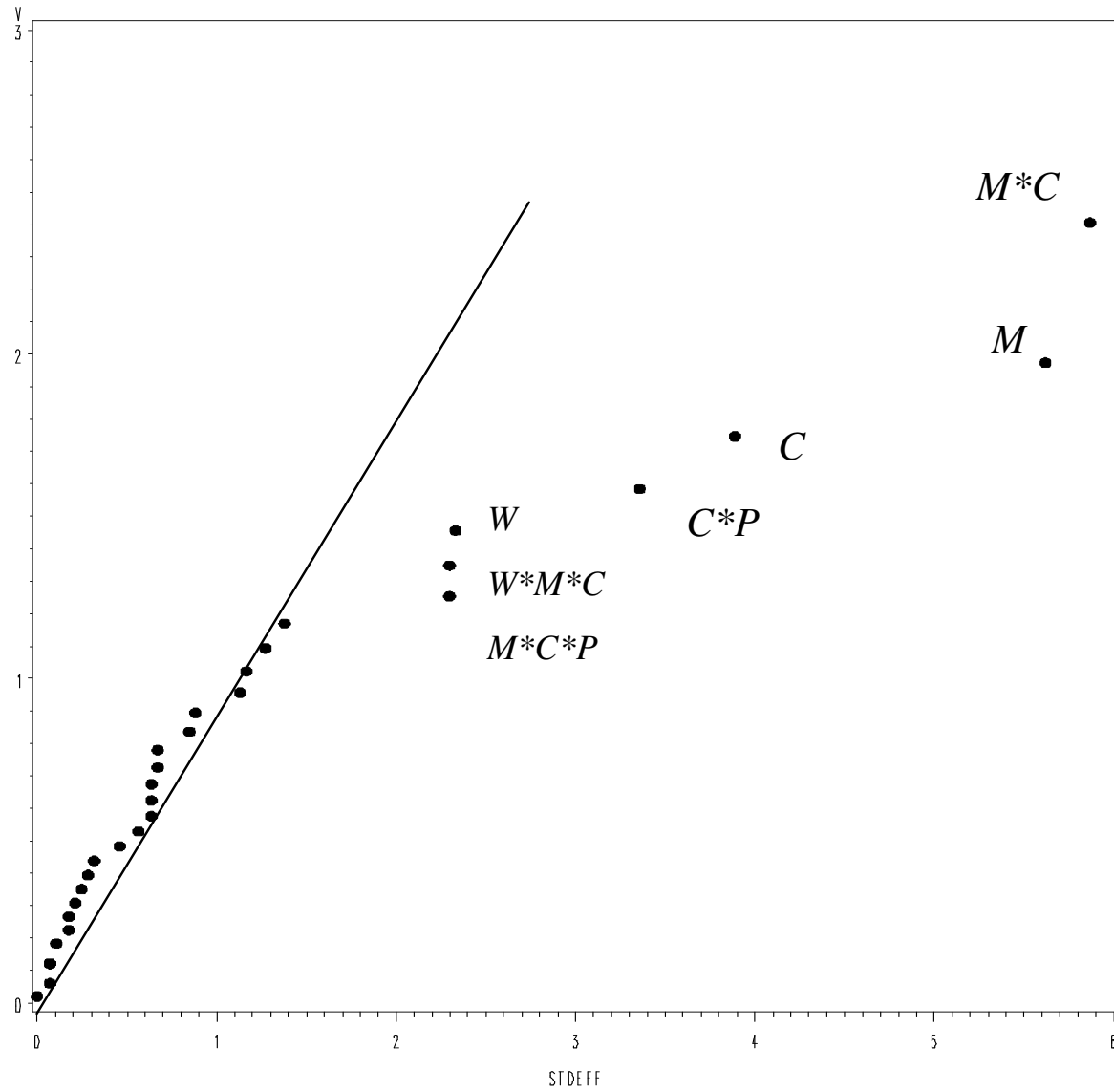
# A HALF-NORMAL PLOT OF THE EFFECTS IN CAKE QUALITY STUDY



<b>15</b>	W*M*T*P	0.40500000	0.63640	15.0	0.46774	0.73387	0.62456
<b>16</b>	M*T*C*P	0.40500000	0.63640	16.0	0.50000	0.75000	0.67449
<b>17</b>	W*C	0.45125000	0.67175	17.0	0.53226	0.76613	0.72616
<b>18</b>	T*P	0.45125000	0.67175	18.0	0.56452	0.78226	0.77984
<b>19</b>	W*M	0.72000000	0.84853	19.0	0.59677	0.79839	0.83587
<b>20</b>	W*M*T	0.78125000	0.88388	20.0	0.62903	0.81452	0.89466
<b>21</b>	W*M*C*P	1.28000000	1.13137	21.0	0.66129	0.83065	0.95672
<b>22</b>	T*C	1.36125000	1.16673	22.0	0.69355	0.84677	1.02270
<b>23</b>	W*T*P	1.62000000	1.27279	23.0	0.72581	0.86290	1.09346
<b>24</b>	W*P	1.90125000	1.37886	24.0	0.75806	0.87903	1.17016
<b>25</b>	M*C*P	5.28125000	2.29810	25.0	0.79032	0.89516	1.25445
<b>26</b>	W*M*C	5.28125000	2.29810	26.0	0.82258	0.91129	1.34874
<b>27</b>	W	5.44500000	2.33345	27.0	0.85484	0.92742	1.45684
<b>28</b>	C*P	11.28125000	3.35876	28.0	0.88710	0.94355	1.58528
<b>29</b>	C	15.12500000	3.88909	29.0	0.91935	0.95968	1.74695
<b>30</b>	M	31.60125000	5.62150	30.0	0.95161	0.97581	1.97395
<b>31</b>	M*C	34.44500000	5.86899	31.0	0.98387	0.99194	2.40598



# A HALF-NORMAL PLOT OF THE EFFECTS IN CAKE QUALITY STUDY



```
DATA ERROR;  
  
    SET EFFECTS2;  
  
    IF MS > 2 THEN DELETE;  
  
RUN;  
  
PROC RANK OUT=RANKS2;  
  
RANKS R;  
  
VAR STDEFF;  
  
RUN;
```



```
DATA PLOTDATA2;
```

```
SET RANKS2;
```

```
RSTAR = (R - .5) / 24;
```

```
P = (RSTAR + 1) / 2;
```

```
V = PROBIT(P) ;
```

```
RUN;
```



```
PROC GPLOT;
```

```
    TITLE2 'FITTED LINE THROUGH THE  
    ORIGIN';
```

```
    SYMBOL V=DOT I=RL0;
```

```
    PLOT V*STDEFF/VREF=1 LVREF=2
```

```
    REGEQN;
```

```
    RUN;
```

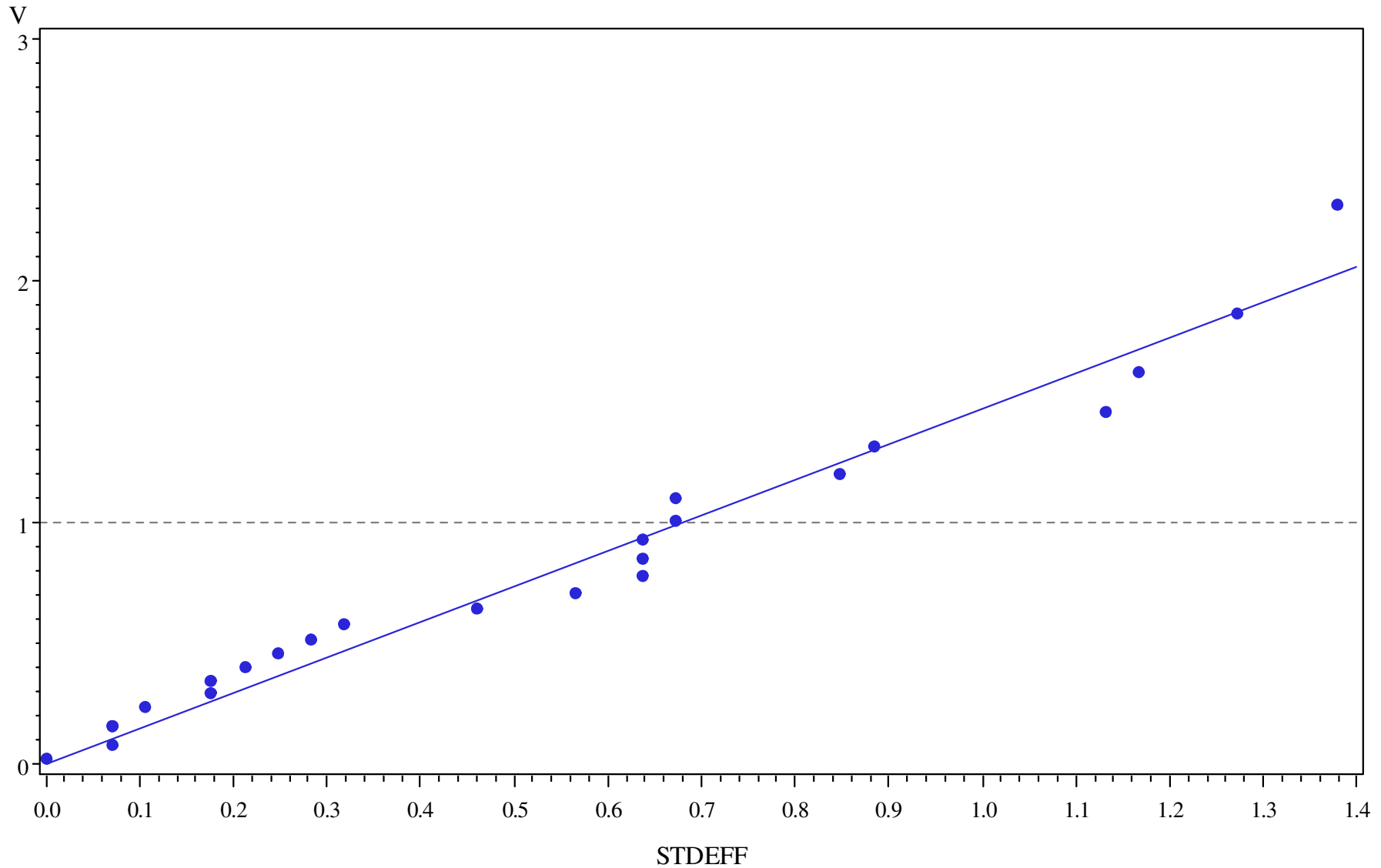
```
    ODS RTF CLOSE;
```

```
    QUIT;
```



# A HALF-NORMAL PLOT OF THE EFFECTS IN CAKE QUALITY STUDY

## FITTED LINE THROUGH THE ORIGIN

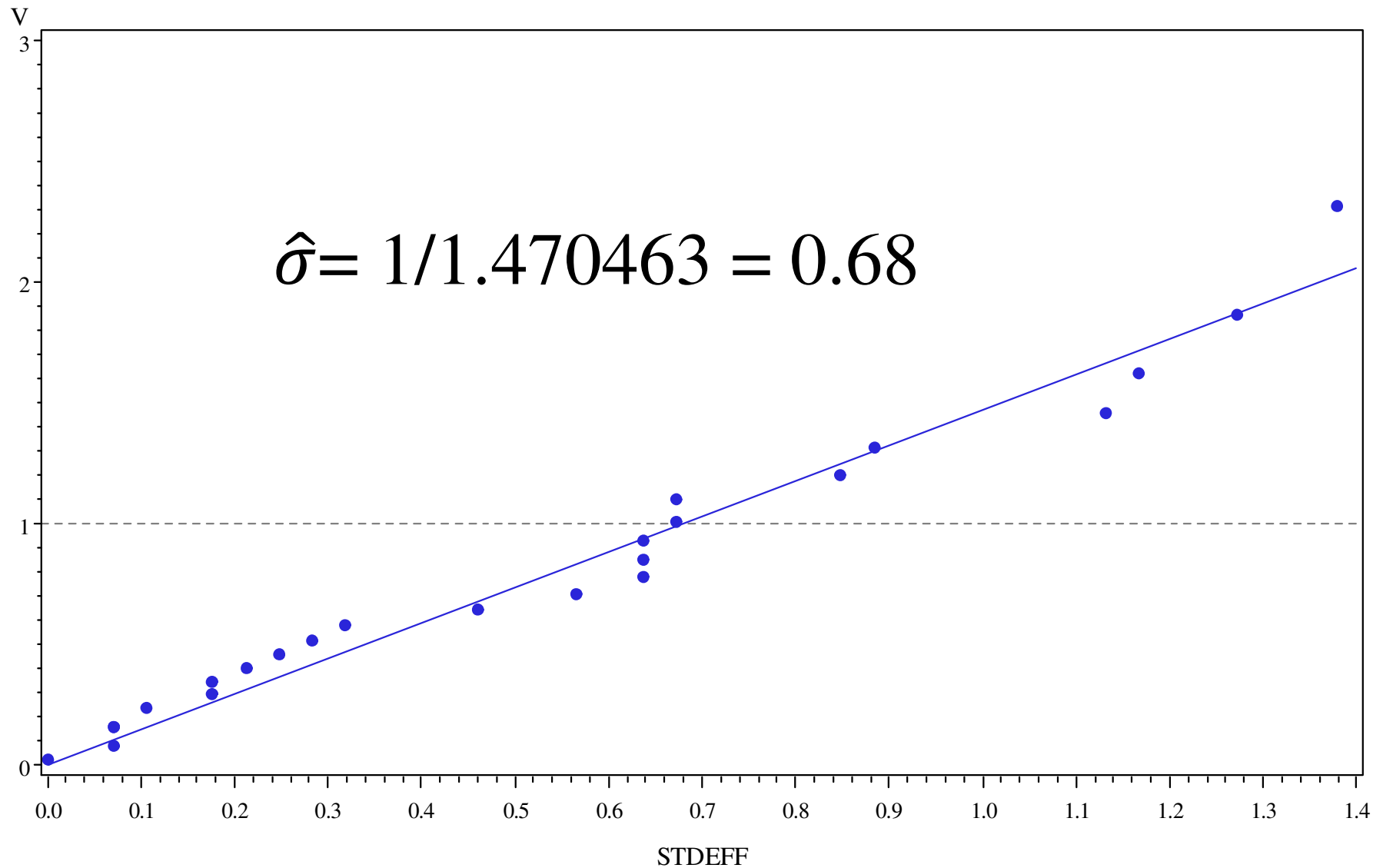


Regression Equation:  
 $V = 0 + 1.470463 \cdot \text{STDEFF}$



# A HALF-NORMAL PLOT OF THE EFFECTS IN CAKE QUALITY STUDY

## FITTED LINE THROUGH THE ORIGIN



**NOTE:** In the Cake Quality study, temperature did not occur in any of the significant effects. So it should not matter whether temperature is set at 300 or 500.

**NOTE:** The only qualitative factor, mixer type, was involved in the significant interactions,  $C * P$  and  $M * C * P$ . Therefore, the optimum choices for the other factors,  $W$ ,  $M$ , and  $C$ , will likely be different for each mixer type. Consequently, we should look at the data for each mixer separately.



# Mixer Type: P<sub>1</sub>

Mixer P1			
	Low Temp	Hi Temp	Total
(1)	4.8	3.9	8.7
w	3.9	4.2	8.1
m	5	3	8
wm	2.2	2.2	4.4
c	5.7	5.3	11
wc	2.2	2.3	4.5
mc	8.4	8.6	17
wmc	9.3	8.9	17.2



# Mixer Type: P<sub>1</sub>

	Total	Step 1	Step 2	Step 3	Std Eff
(1)	8.7	16.8	29.2	78.9	
<i>w</i>	8.1	12.4	49.7	-10.5	-2.625
<i>m</i>	8	15.5	-4.2	14.3	3.575
<i>wm</i>	4.4	34.2	-6.3	3.7	0.925
<i>c</i>	11	-0.6	-4.4	20.5	5.125
<i>wc</i>	4.5	-3.6	18.7	-2.1	-0.525
<i>mc</i>	17	-6.5	-3	23.1	5.775
<i>wmc</i>	17.2	0.2	6.7	9.7	2.425

$$LSD_{0.05} \doteq 2 \cdot \hat{\sigma} = 2 \cdot (0.68) = 1.36$$



# Mixer Type: $P_1$

The  $W*M*C$  interaction effect is significant for Mixer Type  $P_1$ , therefore we will look at the  $W*M*C$  treatment combination means.



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$$Var(\text{diff in two means}) = \sigma^2 \left( \frac{1}{2} + \frac{1}{2} \right) = \sigma^2$$

$$LSD_{0.05} \doteq 2 \cdot (0.68) = 1.36$$



# Mixer Type: $P_1$

	Means
(1)	4.35
$w$	4.05
$m$	4
$wm$	2.2
$c$	5.5
$wc$	2.25
$mc$	8.5
$wmc$	8.6

$$LSD_{0.05} \doteq 2 \cdot (0.68) = 1.36$$



**Table 7.4 Definition of the Treatment Structure in a  $2^5$  Factorial Experiment to Study Cake Quality**

Variable	Level	
	Low	High
<i>W</i> : amount of water	40	120
<i>M</i> : mixing time	3	6
<i>T</i> : temperature	300	500
<i>C</i> : cooking oil	6	12
<i>P</i> : mixer type	$P_1$	$P_2$





# Mixer Type: $P_1$

Note that the treatment combinations  $w_m$  and  $w_{mc}$  are not significantly different from each other, but are significantly higher than each of the other means.

Therefore, for high cake quality with mixer  $P_1$ , mixing time should be near 6 and cooking oil should be near 12. When mixing time is near 6 and cooking oil is near 12, it does not matter whether water is near 40 or 120.



## Mixer Type $P_2$

Mixer P2					
	Total	Step 1	Step 2	Step 3	Std Eff
(1)	8.8	17.9	39.5	80.5	
w	9.1	21.6	41	-2.7	-0.675
m	11.2	13.6	-0.5	17.5	4.375
wm	10.4	27.4	-2.2	1.1	0.275
c	7.9	0.3	3.7	1.5	0.375
wc	5.7	-0.8	13.8	-1.7	-0.425
mc	13.7	-2.2	-1.1	10.1	2.525
wmc	13.7	0	2.2	3.3	0.825

$$LSD_{0.05} \doteq 2 \cdot \hat{\sigma} = 2 \cdot (0.68) = 1.36$$



# Mixer Type $P_2$

The significant effects for mixer type  $P_2$  are  $M$  and  $M*C$ . So we look at the  $M*C$  treatment combination means for this mixer type.



# Mixer Type $P_2$

Mixer P2					
	(1)	$t$	$w$	$tw$	Total
(1)	4.2	4.6	5	4.1	17.9
$m$	5.8	5.4	5.2	5.2	21.6
$c$	2.9	5	3	2.7	13.6
$mc$	6.7	7	6.6	7.1	27.4



## Mixer Type $P_2$

Mixer P2				
	Total	Step 1	Step 2	Std Eff
(1)	17.9	39.5	80.5	
$m$	21.6	41	17.5	4.375
$c$	13.6	3.7	1.5	0.375
$mc$	27.4	13.8	10.1	2.525

$$LSD_{0.05} \doteq 2 \cdot \hat{\sigma} = 2 \cdot (0.68) = 1.36$$



## Mixer Type $P_2$

trt comb	means
(1)	4.475
<i>m</i>	5.4
<i>c</i>	3.4
<i>mc</i>	6.85



## Mixer Type $P_2$

$$Var(\text{diff in two means}) = \sigma^2 \left( \frac{1}{4} + \frac{1}{4} \right) = \frac{\sigma^2}{2}$$

$$LSD_{0.05} \doteq \frac{2 \cdot (0.68)}{\sqrt{2}} = 0.962$$



# Mixer Type $P_2$

trt comb	means
(1)	4.475
$m$	5.4
$c$	3.4
$mc$	6.85

$$LSD_{0.05} \doteq \frac{2 \cdot (0.68)}{\sqrt{2}} = 0.962$$





We see that the treatment combination *mc* is significantly higher than each of the other treatment combination means. Thus when using Mixer Type  $P_2$ , mix time should be near 6 and cooking oil should be near 12. The amount of water used and the temperature used does not matter.



Mixer P1

trt comb	Means
(1)	4.35
<i>w</i>	4.05
<i>m</i>	4
<i>wm</i>	2.2
<i>c</i>	5.5
<i>wc</i>	2.25
<i>mc</i>	8.5
<i>wmc</i>	8.6

Mixer P2

trt comb	means
(1)	4.475
<i>m</i>	5.4
<i>c</i>	3.4
<i>mc</i>	6.85

Mixer P1

trt comb	Means
(1)	4.35
<i>w</i>	4.05
<i>m</i>	4
<i>wm</i>	2.2
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<i>mc</i>	8.5
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Mixer P2

trt comb	means
(1)	4.475
<i>m</i>	5.4
<i>c</i>	3.4
<i>mc</i>	6.85

How do *mc* and *wmc* for P1  
compare to  
*mc* for P2?

Mixer P1

trt comb	Means
(1)	4.35
<i>w</i>	4.05
<i>m</i>	4
<i>wm</i>	2.2
<i>c</i>	5.5
<i>wc</i>	2.25
<i>mc</i>	8.5
<i>wmc</i>	8.6

Mixer P2

trt comb	means
(1)	4.475
<i>m</i>	5.4
<i>c</i>	3.4
<i>mc</i>	6.85

How do *mc* and *wmc* for P1  
compare to  
*mc* for P2?

Recall that each of the mixer P1 means are the mean of two observations, and that each of the P2 means are the average of four observations. Therefore, the difference between a P1 mean and a P2 mean will have a standard error equal to

$$\hat{\sigma}\sqrt{\frac{1}{2} + \frac{1}{4}} = \hat{\sigma}\sqrt{\frac{3}{4}}$$

and the 5% LSD value  
would be approximately

$$\text{LSD} = 2\hat{\sigma}\sqrt{\frac{3}{4}} = 2 \cdot (0.68)(0.866) = 1.178$$

Mixer P1

trt comb	Means
(1)	4.35
<i>w</i>	4.05
<i>m</i>	4
<i>wm</i>	2.2
<i>c</i>	5.5
<i>wc</i>	2.25
<i>mc</i>	8.5
<i>wmc</i>	8.6

Mixer P2

trt comb	means
(1)	4.475
<i>m</i>	5.4
<i>c</i>	3.4
<i>mc</i>	6.85

LSD = 1.178

Mixer P1

trt comb	Means
(1)	4.35
<i>w</i>	4.05
<i>m</i>	4
<i>wm</i>	2.2
<i>c</i>	5.5
<i>wc</i>	2.25
<i>mc</i>	8.5
<i>wmc</i>	8.6

Mixer P2

trt comb	means
(1)	4.475
<i>m</i>	5.4
<i>c</i>	3.4
<i>mc</i>	6.85

$$\text{LSD} = 1.178$$

Thus, we want to use Mixer P1 with mix time near 6 and cooking oil near 12. The amount of water and temperature don't matter.



You can now do  
Assignment 2.