Chapter 8 – Blocking 2^n Experiments

To illustrate the idea of blocking in 2^n experiments, we will consider a 2^3 experiment first, although one cannot recommend blocking in a single replicate in experiments of this size.

Chapter 8 – Blocking a 2^3 Experiment

Consider the contrast that measures the 3-way interaction.

$$A*B*C = (a - 1)(b - 1)(c - 1)$$

$$= abc - ab - ac - bc + a + b + c - (1)$$

Suppose that we put the four treatment combinations in the above contrast that are preceded by a + sign in block 1 and those that are preceded by a – sign in block 2.

Table 8.1 A 2^3 Experiment in Two Blocks of Size 4

Block 1 Trt Comb	Block 2 Trt Comb
a	(1)
b	*ab*
c	*ac*
abc	*bc*

Table 8.1a A 2^3 Experiment in Two Blocks of Size 4

Block 1 Trt Comb			Block 2 Trt Comb		
A	*B*	*C*	*A*	*B*	*C*
1	0	0	0	0	0
0	1	0	1	1	0
0	0	1	1	0	1
1	1	1	0	1	1

Note that Block 1 contains all of the treatment combinations for which the sum of the three factor levels is odd and Block 2 contains all of the treatment combinations for which the sum of the three factor levels is even.

Next, consider the Effects.

						Effects				
Block	***A***	***B***	***C***	***A***	***B***	***A*B***	***C***	***A*C***	***B*C***	***A*B*C***
2	0	0	0	-	-	+	-	+	+	-
1	1	0	0	+	-	-	-	-	+	+
1	0	1	0	-	+	-	-	+	-	+
2	1	1	0	+	+	+	-	-	-	-
1	0	0	1	-	-	+	+	-	-	+
2	1	0	1	+	-	-	+	+	-	-
2	0	1	1	-	+	-	+	-	+	-
1	1	1	1	+	+	+	+	+	+	+

						Effects				
Block	*A*	*B*	*C*	*A*	*B*	*A*B*	*C*	*A*C*	*B*C*	*A*B*C*
2	0	0	0	-	-	+	-	+	+	-
1	1	0	0	+	-	-	-	-	+	+
1	0	1	0	-	+	-	-	+	-	+
2	1	1	0	+	+	+	-	-	-	-
1	0	0	1	-	-	+	+	-	-	+
2	1	0	1	+	-	-	+	+	-	-
2	0	1	1	-	+	-	+	-	+	-
1	1	1	1	+	+	+	+	+	+	+

						Effects				
Block	*A*	*B*	*C*	*A*	*B*	*A*B*	*C*	*A*C*	*B*C*	*A*B*C*
2	0	0	0	-	-	+	-	+	+	-
1	1	0	0	+	-	-	-	-	+	+
1	0	1	0	-	+	-	-	+	-	+
2	1	1	0	+	+	+	-	-	-	-
1	0	0	1	-	-	+	+	-	-	+
2	1	0	1	+	-	-	+	+	-	-
2	0	1	1	-	+	-	+	-	+	-
1	1	1	1	+	+	+	+	+	+	+

						Effects				
Block	*A*	*B*	*C*	*A*	*B*	*A*B*	*C*	*A*C*	*B*C*	*A*B*C*
2	0	0	0	-	-	+	-	+	+	-
1	1	0	0	+	-	-	-	-	+	+
1	0	1	0	-	+	-	-	+	-	+
2	1	1	0	+	+	+	-	-	-	-
1	0	0	1	-	-	+	+	-	-	+
2	1	0	1	+	-	-	+	+	-	-
2	0	1	1	-	+	-	+	-	+	-
1	1	1	1	+	+	+	+	+	+	+

						Effects				
Block	*A*	*B*	*C*	*A*	*B*	*A*B*	*C*	*A*C*	*B*C*	*A*B*C*
2	0	0	0	-	-	+	-	+	+	-
1	1	0	0	+	-	-	-	-	+	+
1	0	1	0	-	+	-	-	+	-	+
2	1	1	0	+	+	+	-	-	-	-
1	0	0	1	-	-	+	+	-	-	+
2	1	0	1	+	-	-	+	+	-	-
2	0	1	1	-	+	-	+	-	+	-
1	1	1	1	+	+	+	+	+	+	+

						Effects				
Block	*A*	*B*	*C*	*A*	*B*	*A*B*	*C*	*A*C*	*B*C*	*A*B*C*
2	0	0	0	-	-	+	-	+	+	-
1	1	0	0	+	-	-	-	-	+	+
1	0	1	0	-	+	-	-	+	-	+
2	1	1	0	+	+	+	-	-	-	-
1	0	0	1	-	-	+	+	-	-	+
2	1	0	1	+	-	-	+	+	-	-
2	0	1	1	-	+	-	+	-	+	-
1	1	1	1	+	+	+	+	+	+	+

						Effects				
Block	*A*	*B*	*C*	*A*	*B*	*A*B*	*C*	*A*C*	*B*C*	*A*B*C*
2	0	0	0	-	-	+	-	+	+	-
1	1	0	0	+	-	-	-	-	+	+
1	0	1	0	-	+	-	-	+	-	+
2	1	1	0	+	+	+	-	-	-	-
1	0	0	1	-	-	+	+	-	-	+
2	1	0	1	+	-	-	+	+	-	-
2	0	1	1	-	+	-	+	-	+	-
1	1	1	1	+	+	+	+	+	+	+

						Effects				
Block	*A*	*B*	*C*	*A*	*B*	*A*B*	*C*	*A*C*	*B*C*	*A*B*C*
2	0	0	0	-	-	+	-	+	+	-
1	1	0	0	+	-	-	-	-	+	+
1	0	1	0	-	+	-	-	+	-	+
2	1	1	0	+	+	+	-	-	-	-
1	0	0	1	-	-	+	+	-	-	+
2	1	0	1	+	-	-	+	+	-	-
2	0	1	1	-	+	-	+	-	+	-
1	1	1	1	+	+	+	+	+	+	+

From the previous slides, we can see that the *A, B, A*B, C, A*C, B*C* can be estimated and that these estimates do not depend on block effects.

However, the *A*B*C* interaction contrast is identical to the contrast that compares the two blocks. We say that the *A*B*C* interaction is confounded with blocks.

The *A, B, A*B, C, A*C, B*C* effects are not confounded with blocks.

Definition: An effect is defined to be *confounded* with another effect if the contrast which defines the effect actually estimates some linear combination of the effect of interest and one or more other effects.

Performing a 2^n Experiment in two blocks of size 2^{n-1}

Blocking 2^n factorial experiments into two blocks of size 2^{n-1} is a straightforward generalization of the results presented for the 2^3 experiment.

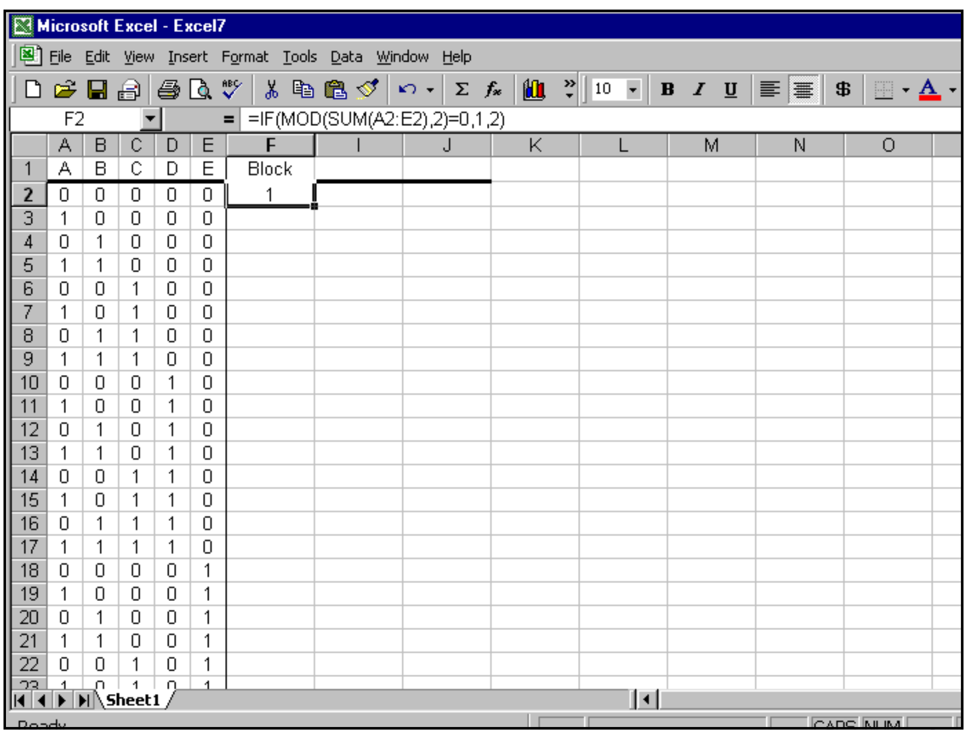
Performing a 2^n Experiment in two blocks of size 2^{n-1}

The effect that should be confounded with blocks is the *n*-way interaction effect. This can be accomplished by writing out the contrast for the *n*-way interaction, and then placing the treatment combinations that have positive signs into one block and those treatment combinations with negative signs would go into the other block.

Equivalently, if we denote the factor levels by zeros and ones, the treatment combinations for which the sum of the factor levels is even would be placed in one block and those for which the sum of the factor levels is odd would be placed in the second block. This confounds the *n*-way interaction with blocks, and it will be the only effect that is confounded with blocks.

Using Excel to Assign Treatment Combinations into Blocks

Next, we consider using Excel to assign treatment combinations into blocks.



Microsoft Excel - Excel7

F2 =IF(MOD(SUM(A2:E2),2)=0,1,2)

	A	B	C	D	E	F
1	A	B	C	D	E	Block
2	0	0	0	0	0	1
3	1	0	0	0	0	
4	0	1	0	0	0	
5	1	1	0	0	0	
6	0	0	1	0	0	
7	1	0	1	0	0	
8	0	1	1	0	0	
9	1	1	1	0	0	
10	0	0	0	1	0	
11	1	0	0	1	0	
12	0	1	0	1	0	
13	1	1	0	1	0	
14	0	0	1	1	0	
15	1	0	1	1	0	
16	0	1	1	1	0	
17	1	1	1	1	0	
18	0	0	0	0	1	
19	1	0	0	0	1	
20	0	1	0	0	1	
21	1	1	0	0	1	
22	0	0	1	0	1	
23	1	0	1	0	1	

Sheet1

Trt Comb	A	B	C	D	E	Block
1	0	0	0	0	0	1
2	1	0	0	0	0	2
3	0	1	0	0	0	2
4	1	1	0	0	0	1
5	0	0	1	0	0	2
6	1	0	1	0	0	1
7	0	1	1	0	0	1
8	1	1	1	0	0	2
9	0	0	0	1	0	2
10	1	0	0	1	0	1
11	0	1	0	1	0	1
12	1	1	0	1	0	2
13	0	0	1	1	0	1
14	1	0	1	1	0	2
15	0	1	1	1	0	2
16	1	1	1	1	0	1
17	0	0	0	0	1	2
18	1	0	0	0	1	1
19	0	1	0	0	1	1
20	1	1	0	0	1	2
21	0	0	1	0	1	1
22	1	0	1	0	1	2
23	0	1	1	0	1	2
24	1	1	1	0	1	1
25	0	0	0	1	1	1
26	1	0	0	1	1	2
27	0	1	0	1	1	2
28	1	1	0	1	1	1
29	0	0	1	1	1	2
30	1	0	1	1	1	1
31	0	1	1	1	1	1

Trt Comb	A	B	C	D	E	Block
1	0	0	0	0	0	1
2	1	0	0	0	0	2
3	0	1	0	0	0	2
4	1	1	0	0	0	1
5	0	0	1	0	0	2
6	1	0	1	0	0	1
7	0	1	1	0	0	1
8	1	1	1	0	0	2
9	0	0	0	1	0	2
10	1	0	0	1	0	1
11	0	1	0	1	0	1
12	1	1	0	1	0	2
13	0	0	1	1	0	1
14	1	0	1	1	0	2
15	0	1	1	1	0	2

A	B	C	D	E	Block
0	0	0	0	0	1
1	1	0	0	0	1
1	0	1	0	0	1
0	1	1	0	0	1
1	0	0	1	0	1
0	1	0	1	0	1
0	0	1	1	0	1
1	1	1	1	0	1
1	0	0	0	1	1
0	1	0	0	1	1
0	0	1	0	1	1
1	1	1	0	1	1
0	0	0	1	1	1
1	1	0	1	1	1
1	0	1	1	1	1
0	1	1	1	1	1

A	B	C	D	E	Block
1	0	0	0	0	2
0	1	0	0	0	2
0	0	1	0	0	2
1	1	1	0	0	2
0	0	0	1	0	2
1	1	0	1	0	2
1	0	1	1	0	2
0	1	1	1	0	2
0	0	0	0	1	2
1	1	0	0	1	2
1	0	1	0	1	2
0	1	1	0	1	2
1	0	0	1	1	2
0	1	0	1	1	2
0	0	1	1	1	2
1	1	1	1	1	2

Microsoft Excel - Excel7

File Edit View Insert Format Tools Data Window Help

G2 = =RAND()

	A	B	C	D	E	F	G
1	A	B	C	D	E	Block	Random
2	0	0	0	0	0	1	0.809279
3	1	0	0	0	0	2	0.139151
4	0	1	0	0	0	2	0.808803
5	1	1	0	0	0	1	0.46194
6	0	0	1	0	0	2	0.922871
7	1	0	1	0	0	1	0.72917
8	0	1	1	0	0	1	0.391871
9	1	1	1	0	0	2	0.773714
10	0	0	0	1	0	2	0.859285
11	1	0	0	1	0	1	0.249804
12	0	1	0	1	0	1	0.534668
13	1	1	0	1	0	2	0.190659
14	0	0	1	1	0	1	0.670319
15	1	0	1	1	0	2	0.413719
16	0	1	1	1	0	2	0.350129
17	1	1	1	1	0	1	0.828004
18	0	0	0	0	1	2	0.795732
19	1	0	0	0	1	1	0.095013
20	0	1	0	0	1	1	0.337784
21	1	1	0	0	1	2	0.584129
22	0	0	1	0	1	1	0.945763
23	1	0	1	0	1	2	[illegible]

Microsoft Excel - Excel7

File Edit View Insert Format Tools Data Window Help

H2 = =RANK(G2,$G$2:$G$17,0)

	A	B	C	D	E	F	G	H
1	A	B	C	D	E	Block	Random	Run
2	0	0	0	0	0	1	0.8092789	4
3	1	0	0	0	0	2	0.1391505	16
4	0	1	0	0	0	2	0.8088026	5
5	1	1	0	0	0	1	0.4619399	10
6	0	0	1	0	0	2	0.9228712	1
7	1	0	1	0	0	1	0.7291702	7
8	0	1	1	0	0	1	0.3918707	12
9	1	1	1	0	0	2	0.7737143	6
10	0	0	0	1	0	2	0.8592851	2
11	1	0	0	1	0	1	0.2498038	14
12	0	1	0	1	0	1	0.5346685	9
13	1	1	0	1	0	2	0.1906587	15
14	0	0	1	1	0	1	0.6703189	8
15	1	0	1	1	0	2	0.4137195	11
16	0	1	1	1	0	2	0.350129	13
17	1	1	1	1	0	1	0.8280042	3
18	0	0	0	0	1	2	0.795732	2
19	1	0	0	0	1	1	0.0950134	13
20	0	1	0	0	1	1	0.3377836	11
21	1	1	0	0	1	2	0.5841294	7
22	0	0	1	0	1	1	0.9457633	1
23	1	0	1	0	1	2	0.5530434	8

Sheet1

Ready CAPS NUM

A	B	C	D	E	Block	Run
1	0	0	0	1	1	1
1	0	0	1	0	1	2
1	1	1	0	1	1	3
0	1	1	0	0	1	4
0	1	1	1	1	1	5
0	0	1	0	1	1	6
0	0	1	1	0	1	7
1	0	1	0	0	1	8
0	0	0	1	1	1	9
1	1	0	1	1	1	10
1	1	1	1	0	1	11
0	1	0	0	1	1	12
0	0	0	0	0	1	13
1	0	1	1	1	1	14
1	1	0	0	0	1	15
0	1	0	1	0	1	16

A	B	C	D	E	Block	Run
1	1	1	0	0	2	1
1	0	1	0	1	2	2
0	0	0	1	0	2	3
0	0	1	1	1	2	4
0	0	1	0	0	2	5
0	0	0	0	1	2	6
0	1	1	0	1	2	7
0	1	0	1	1	2	8
0	1	0	0	0	2	9
1	0	0	1	1	2	10
1	1	0	1	0	2	11
1	1	1	1	1	2	12
1	0	1	1	0	2	13
1	0	0	0	0	2	14
1	1	0	0	1	2	15
0	1	1	1	0	2	16

Using SAS to Assign Treatment Combinations to Blocks

Next, we show how to assign treatment combinations to blocks using SAS. We consider again a 2^5 experiment.

```
DATA EXPDSGN;
 DO E = 0 TO 1;
  DO D = 0 TO 1;
   DO C = 0 TO 1;
    DO B = 0 TO 1;
     DO A = 0 TO 1;
```

```
    IF MOD(A+B+C+D+E,2) = 0
THEN BLOCK = 1; ELSE BLOCK=2;
  RAN=UNIFORM(448091);
  OUTPUT;
  END; END; END; END; END;
  RUN;
```

```
PROC SORT; BY BLOCK;
PROC RANK; BY BLOCK;
 VAR RAN;
 RANKS RUNORDER;
 RUN;
PROC SORT; BY BLOCK RUNORDER;
 RUN;
```

```
ODS RTF FILE = 'C:\TEMP\TEMP5.RTF';
PROC PRINT;
 VAR BLOCK RUNORDER A B C D E;
RUN;
ODS RTF CLOSE;
```

Obs	BLOCK	RUNORDER	A	B	C	D	E
1	1	1	0	0	0	0	0
2	1	2	0	1	1	0	0
3	1	3	1	1	0	1	1
4	1	4	1	1	0	0	0
5	1	5	1	1	1	0	1
6	1	6	1	0	0	0	1
7	1	7	0	1	1	1	1
8	1	8	0	1	0	1	0
9	1	9	1	0	1	1	1
10	1	10	0	0	1	1	0
11	1	11	1	0	1	0	0
12	1	12	0	1	0	0	1
13	1	13	0	0	1	0	1
14	1	14	1	0	0	1	0
15	1	15	0	0	0	1	1
16	1	16	1	1	1	1	0

Obs	BLOCK	RUNORDER	A	B	C	D	E
17	2	1	1	0	0	0	0
18	2	2	1	0	0	1	1
19	2	3	0	1	1	0	1
20	2	4	0	0	0	1	0
21	2	5	1	1	1	0	0
22	2	6	1	1	1	1	1
23	2	7	0	0	1	1	1
24	2	8	0	1	0	1	1
25	2	9	1	1	0	1	0
26	2	10	0	1	0	0	0
27	2	11	1	0	1	0	1
28	2	12	0	1	1	1	0
29	2	13	1	0	1	1	0
30	2	14	0	0	1	0	0
31	2	15	0	0	0	0	1
32	2	16	1	1	0	0	1

Remark: If one were to assign the treatment combinations into blocks in some haphazard or random manner, it could be disastrous as many of the effects of primary interest could be confounded with blocks.

In this case, very little reliable information could be available from the data collected, and those resources used to collect the data would be wasted.

Analyzing 2^n experiments which have been conducted in two blocks of size 2^{n-1} is no more difficult than analyzing 2^n experiments where blocking has not been used.

Both the ANOVA method and the half-normal plot method can be used with some relatively minor alterations.

ANOVA METHOD

This method can be used as long as there are a sufficient number of high order interaction effects other than the *n*-way interaction that can be pooled into error.

If one is using SAS-ANOVA, the model statement should include an effect for blocks, and should not include the other effects that will be pooled into error.

As an example, suppose the Cake Quality experiment had used a blocked design where the 5-factor interaction was confounded with blocks.

The SAS commands that one can use are given on the next slide.

PROC ANOVA;

CLASSES W M T C P BLOCK;

MODEL QUALITY=BLOCK W M W*M T W*T M*T W*M*T C W*C M*C W*M*C T*C W*T*C M*T*C P W*P M*P W*M*P T*P W*T*P M*T*P

C*P W*C*P M*C*P T*C*P;

In this ANOVA analysis there would be only 5 degrees of freedom for error, all coming from the five pooled 4-way interaction effects.

Half-normal Plot Method

When using the half-normal plotting method to identify the statistically significant effects, there are no adjustments that need to be made. One just needs to remember that the n-way interaction is confounded with blocks, and if this effect is statistically significant, then it is most likely due to the block differences rather than the n-way interaction effect.

You can now begin to work on Assignment # 3.

Blocking 2^n Experiments into More than Two Blocks

It is possible to assign the treatment combinations of a 2^n experiment into more blocks of smaller sizes.

The advantage of having smaller block sizes is that the experimental units within a smaller block will generally be more homogeneous than those in larger blocks. Thus the smaller block sizes should provide smaller estimates of the experimental error variance which, in turn, will provide more accurate information about the effects of the treatment combinations under study.

Blocking 2^n Experiments into More than Two Blocks

The main disadvantage is additional effects must also be confounded with blocks.

This may not be a major disadvantage, if only high-order interaction effects are confounded with blocks.

Blocking 2^n Experiments into More than Two Blocks

If one is going to wisely conduct a 2^n experiment in 2^k blocks of size 2^{n-k}, then there will be 2^k-1 effects confounded with blocks.

If one conducts such an experiment unwisely, then almost all of the effects might be confounded with blocks.

In general, in order to conduct a 2^n experiment in 2^k blocks, one must be able to identify *k* effects to confound with blocks.

The *k* effects that are confounded with blocks will give rise to an additional

$2^k - k - 1$ effects that will also be confounded with blocks.

Example: Consider conducting a 2^4 experiment in 4 blocks of size 4. Suppose we decide to confound the *A*B*C*D* and the *A*B* interaction effects with blocks.

The contrast of the treatment combinations that define these two effects are:

$$A*B*C*D = abcd - abc - abd - acd - bcd + ab + ac + ad + bc + bd + cd - a - b - c - d + (1)$$

and

$$\mathrm{A*B} = abcd + abc + abd - acd - bcd + ab - ac - ad - bc - bd + cd - a - b + c + d + (1)$$

$$A*B*C*D = abcd - abc - abd - acd - bcd + ab + ac + ad + bc + bd + cd - a - b - c - d + (1)$$

$$\mathrm{A*B} = abcd + abc + abd - acd - bcd + ab - ac - ad - bc - bd + cd - a - b + c + d + (1)$$

Block 1 takes the treatment combinations that have positive signs in both effects, Block 2 contains those that are positive in the first effect and negative in the second, Block 3 contains those that are negative in the first effect and positive in the second, and Block 4 contains those are negative in both effects

$A*B*C*D = abcd - abc - abd - acd - bcd + ab + ac + ad + bc + bd + cd - a - b - c - d + (1)$

$A*B = abcd + abc + abd - acd - bcd + ab - ac - ad - bc - bd + cd - a - b + c + d + (1)$

$A*B*C*D = abcd - abc - abd - acd - bcd + ab + ac + ad + bc + bd + cd - a - b - c - d + (1)$

$A*B = abcd + abc + abd - acd - bcd + ab - ac - ad - bc - bd + cd - a - b + c + d + (1)$

$A*B*C*D = abcd - abc - abd - acd - bcd + ab + ac + ad + bc + bd + cd - a - b - c - d + (1)$

$A*B = abcd + abc + abd - acd - bcd + ab$

$- ac - ad - bc - bd + cd - a - b + c + d + (1)$

$A*B*C*D = abcd - abc - abd - acd - bcd + ab + ac + ad + bc + bd + cd - a - b - c - d + (1)$

$A*B = abcd + abc + abd - acd - bcd + ab$

$- ac - ad - bc - bd + cd - a - b + c + d + (1)$

Block 1	Block 2	Block 3	Block 4
+ +	+ -	- +	- -
abcd	ac	abc	acd
ab	ad	abd	bcd
cd	bc	c	a
(1)	bd	d	b

Equivalently, **the treatment combinations that go into each block are given below.**

Block 1	Block 2	Block 3	Block 4
A B C D	*A B C D*	*A B C D*	*A B C D*
0 0 0 0	1 0 1 0	1 1 1 0	1 0 1 1
1 1 0 0	1 0 0 1	1 1 0 1	0 1 1 1
0 0 1 1	0 1 1 0	0 0 1 0	1 0 0 0
1 1 1 1	0 1 0 1	0 0 0 1	0 1 0 0

Block 1	Block 2	Block 3	Block 4
A B C D	*A B C D*	*A B C D*	*A B C D*
0 0 0 0	1 0 1 0	1 1 1 0	1 0 1 1
1 1 0 0	1 0 0 1	1 1 0 1	0 1 1 1
0 0 1 1	0 1 1 0	0 0 1 0	1 0 0 0
1 1 1 1	0 1 0 1	0 0 0 1	0 1 0 0

Note that in the above table, Block 1 contains the treatment combinations where $A+B+C+D$ is even and $A+B$ is also even; Block 2 contains the treatment combinations where $A+B+C+D$ is even and $A+B$ is odd;

Block 1	Block 2	Block 3	Block 4
A B C D	*A B C D*	*A B C D*	*A B C D*
0 0 0 0	1 0 1 0	1 1 1 0	1 0 1 1
1 1 0 0	1 0 0 1	1 1 0 1	0 1 1 1
0 0 1 1	0 1 1 0	0 0 1 0	1 0 0 0
1 1 1 1	0 1 0 1	0 0 0 1	0 1 0 0

Note that in the above table, Block 3 contains the treatment combinations where $A+B+C+D$ is odd and $A+B$ is even; and Block 4 contains the treatment combinations where $A+B+C+D$ is odd and $A+B$ is also odd.

Note that the oddness and/or evenness of $A+B+C+D$ and $A+B$ implies the oddness and/or evenness of $C+D$.

Thus, in addition to $A*B*C*D$ and $A*B$ being confounded with blocks, it would seem that $C*D$ is also confounded with blocks. This is indeed true as shown on the next slide.

***C*D* = *abcd* – *abc* – *abd* + *acd* + *bcd* + *ab* – *ac* – *ad* – *bc* – *bd* + *cd* + *a* + *b* – *c* – *d* + (1)**

All of the treatment combinations that have positive signs in the above contrast fall into Blocks 1 and 4, and all of those with negative signs fall into Blocks 2 and 3. Therefore the *C*D* interaction contrast also contrasts the difference between Blocks 1 & 4 and Blocks 2 & 3, and hence *C*D* is confounded with blocks.