Factorial Treatment Stuctures

Definition 1. Consider an experiment involving n factors where factor A_1 has a_1 possibilities, factor A_2 has a_2 possibilities, …, and factor A_n has a_n possibilities. Such an experiment is called a $a_1 \times a_2 \times \ldots \times a_n$ factorial experiment.

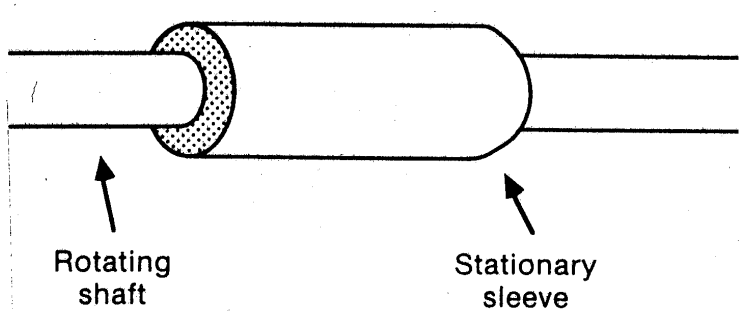
2^n Factorial Treatment Stuctures

Definition 2. Consider an experiment involving n factors where each factor has two possibilities. Such an experiment is called a $2\times 2 \times \ldots \times 2 = 2^n$ experiment.

Examples.

2×3	2 factors
2^2	2 factors
2^4	4 factors
$2^3 \times 3^2$	5 factors
$2 \times 3 \text{ x } 5$	3 factors
$2^3 \times 3 \times 5$	5 factors

Factor	Levels
Shaft alloy	Steel, aluminum
Sleeve metal	Porous, nonporous
Lubricant Type	Lub 1, Lub 2, Lub 3, Lub 4



Factors for torque experiment.

Table 1 Factor-Level Combinations for Torque Study

Combination Number	Shaft Alloy	Sleeve Metal	Lubricant Type
1	Steel	Porous	Lub 1
2	Steel	Porous	Lub 2
3	Steel	Porous	Lub 3
4	Steel	Porous	Lub 4
5	Steel	Nonporous	Lub 1
6	Steel	Nonporous	Lub 2
7	Steel	Nonporous	Lub 3
8	Steel	Nonporous	Lub 4
9	Aluminum	Porous	Lub 1
10	Aluminum	Porous	Lub 2
11	Aluminum	Porous	Lub 3
12	Aluminum	Porous	Lub 4
13	Aluminum	Nonporous	Lub 1
14	Aluminum	Nonporous	Lub 2
15	Aluminum	Nonporous	Lub 3
16	Aluminum	Nonporous	Lub 4

Table 2 Randomized Test Sequence for Torque Study

Run Number	Combination Number	Shaft Alloy	Sleeve Metal	Lubricant Type
1	8	Steel	Nonporous	Lub 4
2	13	Aluminum	Nonporous	Lub 1
3	4	Steel	Porous	Lub 4
4	7	Steel	Nonporous	Lub 3
5	5	Steel	Nonporous	Lub 1
6	1	Steel	Porous	Lub 1
7	11	Aluminum	Porous	Lub 3
8	15	Aluminum	Nonporous	Lub 3
9	9	Aluminum	Porous	Lub 1
10	3	Steel	Porous	Lub 3
11	12	Aluminum	Porous	Lub4
12	10	Aluminum	Porous	Lub 2
13	6	Steel	Nonporous	Lub 2
14	14	Aluminum	Nonporous	Lub 2
15	16	Aluminum	Nonporous	Lub 4
16	2	Steel	Porous	Lub 2

Table 3 Randomized Test Sequence for Torque Study Including Replicate Tests

Run Number	Combination Number	Shaft Alloy	Sleeve Metal	Lubricant Type
1	4	Steel	Porous	Lub 4
2	2	Steel	Porous	Lub 2
3	7	Steel	Nonporous	Lub 3
4	16	Aluminum	Nonporous	Lub 4
5	10	Aluminum	Porous	Lub 2
6	4	Steel	Porous	Lub 4
7	11	Aluminum	Porous	Lub 3
8	12	Aluminum	Porous	Lub 4
9	8	Steel	Nonporous	Lub 4
10	16	Aluminum	Nonporous	Lub 4
11	7	Steel	Nonporous	Lub 3
12	8	Steel	Nonporous	Lub 4
13	12	Aluminum	Porous	Lub 4
14	3	Steel	Porous	Lub 3
15	5	Steel	Nonporous	Lub 1
16	11	Aluminum	Porous	Lub 3
17	5	Steel	Nonporous	Lub 1
18	1	Steel	Porous	Lub 1
19	10	Aluminum	Porous	Lub 2
20	9	Aluminum	Porous	Lub 1
21	1	Steel	Porous	Lub 1
22	9	Aluminum	Porous	Lub 1
23	15	Aluminum	Nonporous	Lub 3
24	15	Aluminum	Nonporous	Lub 3
25	6	Steel	Nonporous	Lub 2
26	13	Aluminum	Nonporous	Lub 1
27	13	Aluminum	Nonporous	Lub 1
28	6	Steel	Nonporous	Lub 2
29	3	Steel	Porous	Lub 3
30	14	Aluminum	Nonporous	Lub 2
31	2	Steel	Porous	Lub 2
32	14	Aluminum	Nonporous	Lub 2

Advantages of 2^n Experiments

- Great for exploratory purposes
- Allow researchers to consider large numbers of factors in a single experiment
- Much more efficient to study all factors that might influence a process simultaneously than it is to study each factor one-at-a-time
- Allow researchers to identify the factors that are likely to have the biggest impact on a process

Disadvantages of 2^n Experiments

- Usually require some follow-up experiments in order to fine-tune the resulting models
- May require more test runs than one can afford to do

2^n Experiments (Notation)

Consider a 2^3 experiment that involves the factors *A*, *B*, & *C*.

Treatment Design

A	*B*	*C*
Low	Low	Low
High	Low	Low
Low	High	Low
High	High	Low
Low	Low	High
High	Low	High
Low	High	High
High	High	High

2^n Experiments (Notation)

Treatment Design

A	*B*	*C*	**Expected Response**
Low	Low	Low	μ_{000}
High	Low	Low	μ_{100}
Low	High	Low	μ_{010}
High	High	Low	μ_{110}
Low	Low	High	μ_{001}
High	Low	High	μ_{101}
Low	High	High	μ_{011}
High	High	High	μ_{111}

2^n Experiments (Notation)

Treatment Design

A	**B**	**C**	**Expected Response (1)**	**Expected Response (2)**
Low	Low	Low	μ_{000}	(1)
High	Low	Low	μ_{100}	*a*
Low	High	Low	μ_{010}	*b*
High	High	Low	μ_{110}	*ab*
Low	Low	High	μ_{001}	*c*
High	Low	High	μ_{101}	*ac*
Low	High	High	μ_{011}	*bc*
High	High	High	μ_{111}	*abc*

2^n Experiments (Other Forms)

Treatment Design

A	*B*	*C*
-	-	-
+	-	-
-	+	-
+	+	-
-	-	+
+	-	+
-	+	+
+	+	+

2^n Experiments (Other Forms)

Treatment Design

A	*B*	*C*
-1	-1	-1
+1	-1	-1
-1	+1	-1
+1	+1	-1
-1	-1	+1
+1	-1	+1
-1	+1	+1
+1	+1	+1

2^n Experiments (Other Forms)

Treatment Design

A	***B***	***C***
0	0	0
1	0	0
0	1	0
1	1	0
0	0	1
1	0	1
0	1	1
1	1	1

Factorial Effects

Consider a 2^2 factorial experiment with factors *A* and *B*.

A	*B*	Expected response
0	0	$(1) = \mu_{00}$
1	0	$a = \mu_{10}$
0	1	$b = \mu_{01}$
1	1	$ab = \mu_{11}$

A *B*	Expected response
0 0	(1) = μ_{00}
1 0	a = μ_{10}
0 1	b = μ_{01}
1 1	ab = μ_{11}

The Main Effect of *A* is defined by:

$$A = [\ -(1) + a - b + ab\ ]/2$$

and/or

$$A = \left[-\mu_{00} + \mu_{10} - \mu_{01} + \mu_{11}\right]/2$$

The Main Effect of *B* is defined by:

$$B = [\ -(1) - a + b + ab\ ]/2$$

and/or

$$B = \left[-\mu_{00} - \mu_{10} + \mu_{01} + \mu_{11}\right]/2$$

The Interaction between A and B is defined by:

$$A*B = [\ (1) - a - b + ab\ ]/2$$

and/or

$$A*B = [\mu_{00} - \mu_{10} - \mu_{01} + \mu_{11}]/2$$

A Numerical Example

Treatment Design					
		Observed			
A	*B*	Response			
0	0	70			
1	0	73			
0	1	58			
1	1	62			

A Numerical Example

Treatment Design					
		Observed	*A*	*B*	*A*B*
A	*B*	Response	Contrast	Contrast	Contrast
0	0	70	-	-	+
1	0	73	+	-	-
0	1	58	-	+	-
1	1	62	+	+	+

A Numerical Example

Treatment Design					
		Observed	*A*	*B*	*A*B*
A	*B*	Response	Contrast	Contrast	Contrast
0	0	70	-	-	+
1	0	73	+	-	-
0	1	58	-	+	-
1	1	62	+	+	+
	Contrast	Value	7	-23	1

A Numerical Example

Treatment Design					
		Observed	*A*	*B*	*A*B*
A	*B*	Response	Contrast	Contrast	Contrast
0	0	70	-	-	+
1	0	73	+	-	-
0	1	58	-	+	-
1	1	62	+	+	+
	Contrast	Value	7	-23	1
		Effect	3.5	-11.5	0.5

A Second Example

Example 2.

	Treatment Combination			
Effect	(1)	*a*	*b*	*ab*
Total	+	+	+	+
A	-	+	-	+
B	-	-	+	+
*A*B*	+	-	-	+
y	50	62	44	58

A Second Example

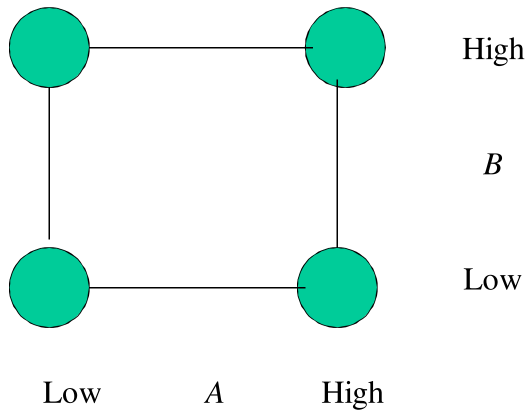
Example 2.

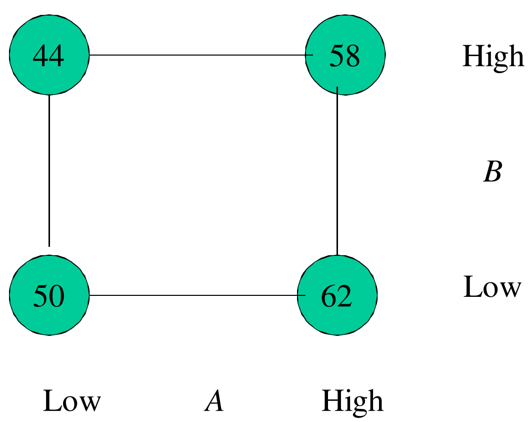
		Treatment Combination				Contrast
	Effect	(1)	*a*	*b*	*ab*	Value
	Total	+	+	+	+	214
	A	-	+	-	+	26
	B	-	-	+	+	-10
	*A*B*	+	-	-	+	2
	y	50	62	44	58	

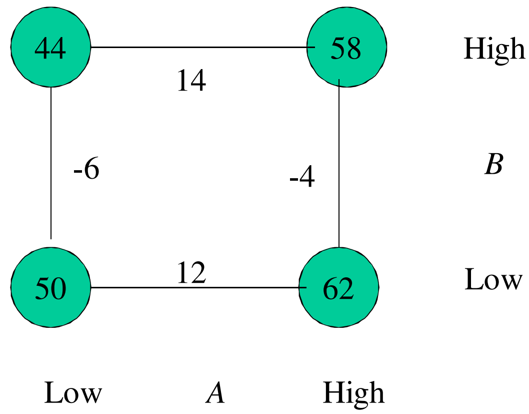
A Second Example

Example 2.

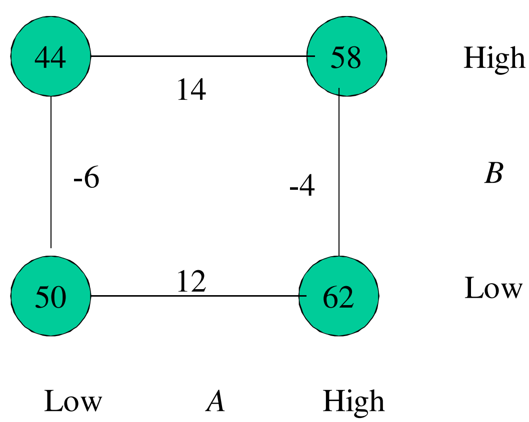
		Treatment Combination				Contrast	Estimated
	Effect	(1)	*a*	*b*	*ab*	Value	Effect
	Total	+	+	+	+	214	53.5
	A	-	+	-	+	26	13
	B	-	-	+	+	-10	-5
	*A*B*	+	-	-	+	2	1
	y	50	62	44	58		


High
B
Low
Low
A
High


44
58
High
B
50
62
Low
Low
A
High

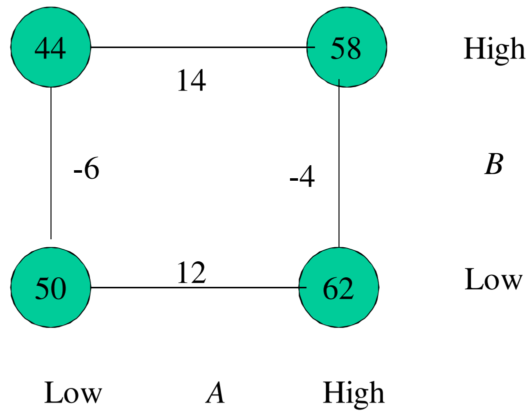


A = [12 + 14]/2 = 13



A = [12 + 14]/2 = 13

B = [(-6) + (-4)]/2 = -5



$A = [12 + 14]/2 = 13$

$B = [(-6) + (-4)]/2 = -5$

$A*B = [14-12]/2 = [(-4) - (-6)]/2 = 1$

You can now do Assignment 1, please e-mail it to me when you are finished.