Generalized Interactions

Remark: If one knows the effects that are used to assign treatment combinations into blocks, then one can find the additional effects that are also confounded with blocks by finding the *generalized interactions* between the defining effects.


Generalized Interactions

Definition: A *generalized interaction* of two or more effects can be found by combining all of the letters that appear in the effects, and canceling out those that occur an even number of times.


Generalized Interactions

Example: The generalized interaction between *A*B*C*D* and *A*B* is (*A*B*C*D*) (*A*B*) = *C*D* since the factors *A* and *B* each occur twice, and hence they cancel out.

Suppose one assigns treatment combinations to blocks using *A*B*C*D* and *A*B*C* as defining effects. The generalized interaction between these two defining effect is:

$$(A*B*C*D)(A*B*C) = D.$$

Hence, confounding the four factor interaction and a three factor interaction with blocks would also confound a main effect with blocks. In nonreplicated experiments, one would never want to confound a main effect with blocks.

Example: Consider designing a 2^6 experiment with factors *A, B, C, D, E, F* into $8 = 2^3$ blocks of size $8 = 2^{6-3}$.

This will require three defining effects. Suppose we decide to confound the *A*B*C*D* , *A*B*E*F*, and *A*C*E* with blocks.

If the levels *A, B, C, D, E, F* are denoted by 0's and 1's. Then the blocks are defined by the evenness and/or oddness of $A + B + C + D$,

$A + B + E + F$, and $A + C + E$.

Let $L_1 = A + B + C + D$

$L_2 = A + B + E + F$, and

$L_3 = A + C + E$.

	Block 1	Block 2	Block 3	Block 4	Block 5	Block 6	Block 7	Block 8
L_1	Even	Even	Even	Even	Odd	Odd	Odd	Odd
L_2	Even	Even	Odd	Odd	Even	Even	Odd	Odd
L_3	Even	Odd	Even	Odd	Even	Odd	Even	Odd

Question: What other effects are confounded with blocks?

$(A*B*C*D)\,(A*B*E*F) = C*D*E*F$

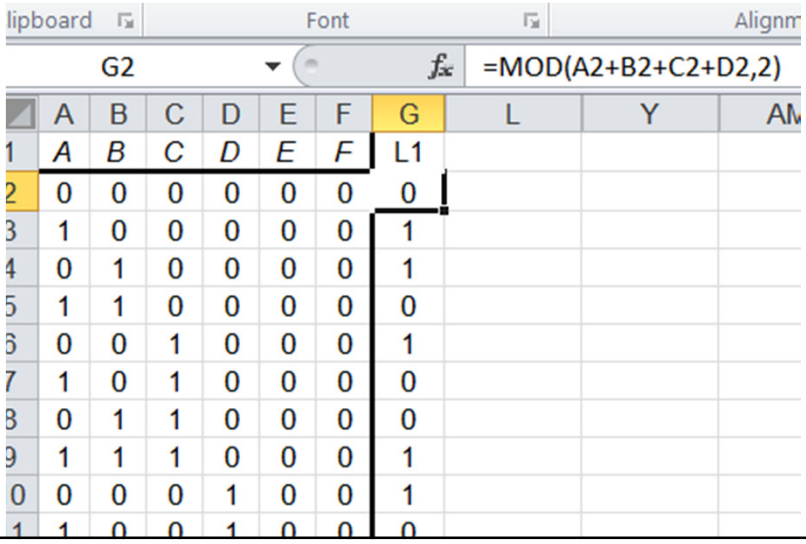
$(A*B*C*D)(A*C*E) = B*D*E$

$(A*B*E*F)(A*C*E) = B*C*F$, and

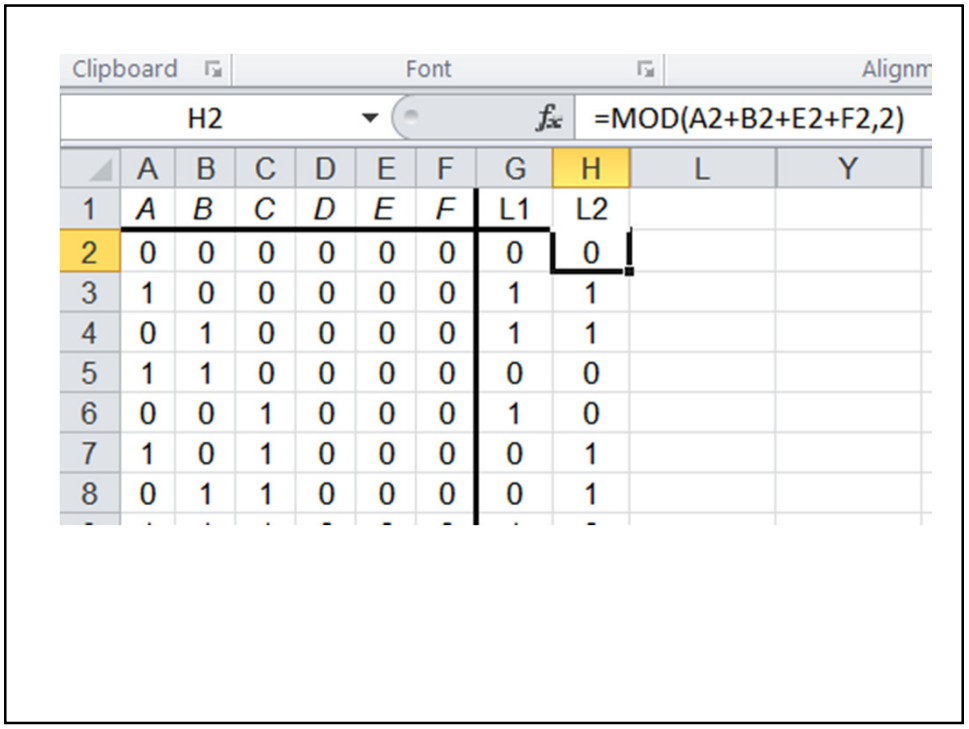
$(A*B*C*D)\,(A*B*E*F)(A*C*E) = A*D*F$

are also confounded with blocks.

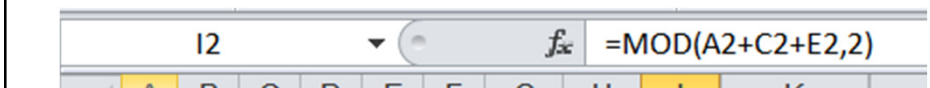
Using Excel



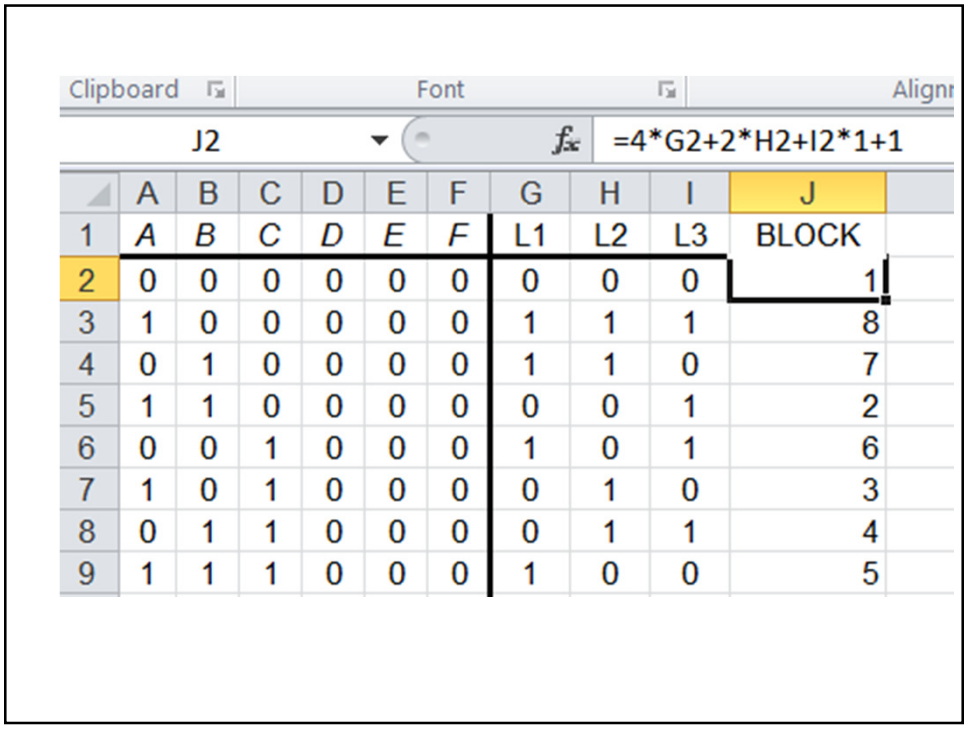
	A	B	C	D	E	F	G	L	Y	AM
1	*A*	*B*	*C*	*D*	*E*	*F*	L1			
2	0	0	0	0	0	0	0			
3	1	0	0	0	0	0	1			
4	0	1	0	0	0	0	1			
5	1	1	0	0	0	0	0			
6	0	0	1	0	0	0	1			
7	1	0	1	0	0	0	0			
8	0	1	1	0	0	0	0			
9	1	1	1	0	0	0	1			
10	0	0	0	1	0	0	1			
11	1	0	0	1	0	0	0			



	A	B	C	D	E	F	G	H	L	Y
1	*A*	*B*	*C*	*D*	*E*	*F*	L1	L2		
2	0	0	0	0	0	0	0	0		
3	1	0	0	0	0	0	1	1		
4	0	1	0	0	0	0	1	1		
5	1	1	0	0	0	0	0	0		
6	0	0	1	0	0	0	1	0		
7	1	0	1	0	0	0	0	1		
8	0	1	1	0	0	0	0	1		

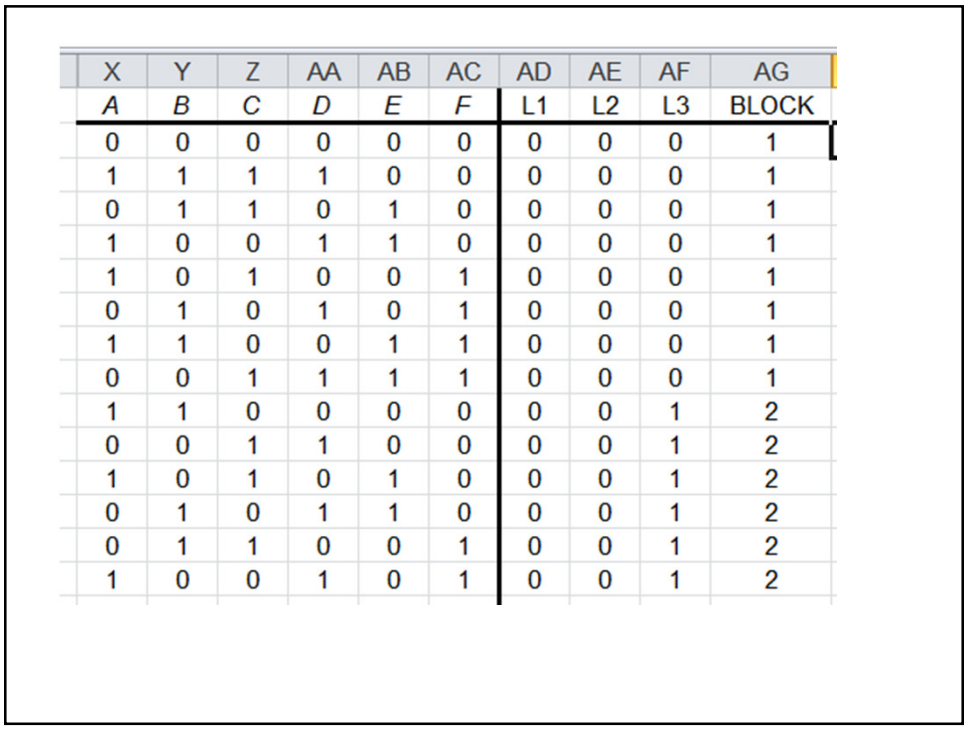


	A	B	C	D	E	F	G	H	I	K
1	*A*	*B*	*C*	*D*	*E*	*F*	L1	L2	L3	
2	0	0	0	0	0	0	0	0	0	
3	1	0	0	0	0	0	1	1	1	
4	0	1	0	0	0	0	1	1	0	
5	1	1	0	0	0	0	0	0	1	
6	0	0	1	0	0	0	1	0	1	
7	1	0	1	0	0	0	0	1	0	
8	0	1	1	0	0	0	0	1	1	


Clipboard Font Align

J2 =4*G2+2*H2+I2*1+1

	A	B	C	D	E	F	G	H	I	J
1	*A*	*B*	*C*	*D*	*E*	*F*	L1	L2	L3	BLOCK
2	0	0	0	0	0	0	0	0	0	1
3	1	0	0	0	0	0	1	1	1	8
4	0	1	0	0	0	0	1	1	0	7
5	1	1	0	0	0	0	0	0	1	2
6	0	0	1	0	0	0	1	0	1	6
7	1	0	1	0	0	0	0	1	0	3
8	0	1	1	0	0	0	0	1	1	4
9	1	1	1	0	0	0	1	0	0	5



X	Y	Z	AA	AB	AC	AD	AE	AF	AG
A	*B*	*C*	*D*	*E*	*F*	L1	L2	L3	BLOCK
0	0	0	0	0	0	0	0	0	1
1	1	1	1	0	0	0	0	0	1
0	1	1	0	1	0	0	0	0	1
1	0	0	1	1	0	0	0	0	1
1	0	1	0	0	1	0	0	0	1
0	1	0	1	0	1	0	0	0	1
1	1	0	0	1	1	0	0	0	1
0	0	1	1	1	1	0	0	0	1
1	1	0	0	0	0	0	0	1	2
0	0	1	1	0	0	0	0	1	2
1	0	1	0	1	0	0	0	1	2
0	1	0	1	1	0	0	0	1	2
0	1	1	0	0	1	0	0	1	2
1	0	0	1	0	1	0	0	1	2

Using Excel

A	B	C	D	E	F	BLOCK
0	0	0	0	0	0	1
1	1	1	1	0	0	1
0	1	1	0	1	0	1
1	0	0	1	1	0	1
1	0	1	0	0	1	1
0	1	0	1	0	1	1
1	1	0	0	1	1	1
0	0	1	1	1	1	1
1	1	0	0	0	0	2
0	0	1	1	0	0	2
1	0	1	0	1	0	2
0	1	0	1	1	0	2
0	1	1	0	0	1	2
1	0	0	1	0	1	2
0	0	0	0	1	1	2
1	1	1	1	1	1	2
1	0	1	0	0	0	3
0	1	0	1	0	0	3
1	1	0	0	1	0	3
0	0	1	1	1	0	3
0	0	0	0	0	1	3
1	1	1	1	0	1	3
0	1	1	0	1	1	3

Using SAS

```
DATA EXPDSGN;
 DO F=0 TO 1;
  DO E = 0 TO 1;
   DO D = 0 TO 1;
    DO C = 0 TO 1;
     DO B = 0 TO 1;
      DO A = 0 TO 1;
```

Using SAS

```
L1 = MOD(A+B+C+D,2);
L2 = MOD(A+B+E+F,2);
L3 = MOD(A+C+E,2);

BLOCK = 4*L1+2*L2+L3+1;
OUTPUT;
END; END; END; END; END; END;
```


```
DATA; SET;
 RANDOM = UNIFORM(48091);
PROC SORT; BY BLOCK;
PROC RANK; BY BLOCK;
 VAR RANDOM;
  RANKS RUNORDER;
  RUN;
```


```
PROC SORT; BY BLOCK RUNORDER;
ODS RTF
FILE='C:\TEMP\TEMP5.RTF';
PROC PRINT; BY BLOCK;
 VAR RUNORDER A B C D E F;
 RUN;
ODS RTF CLOSE;
```

See ST722_8_2.sas for the SAS program.

Using SAS

BLOCK=1

Obs	RUNORDER	A	B	C	D	E	F
1	1	0	0	0	0	0	0
2	2	0	1	0	1	0	1
3	3	1	1	0	0	1	1
4	4	1	0	0	1	1	0
5	5	1	1	1	1	0	0
6	6	0	1	1	0	1	0
7	7	1	0	1	0	0	1
8	8	0	0	1	1	1	1

Using SAS

BLOCK=2

Obs	RUNORDER	A	B	C	D	E	F
9	1	1	1	0	1	1	1
10	2	0	0	1	0	1	1
11	3	0	1	0	0	0	1
12	4	1	0	1	1	0	1
13	5	1	0	0	0	1	0
14	6	1	1	1	0	0	0
15	7	0	1	1	1	1	0
16	8	0	0	0	1	0	0

Using SAS

BLOCK=8

Obs	RUNORDER	A	B	C	D	E	F
57	1	0	1	0	0	1	1
58	2	0	0	0	1	1	0
59	3	0	1	1	1	0	0
60	4	1	0	0	0	0	0
61	5	0	0	1	0	0	1
62	6	1	0	1	1	1	1
63	7	1	1	0	1	0	1
64	8	1	1	1	0	1	0

Question: The preceding 2^6 example had the three-way interactions *A*C*E, B*D*E, B*C*F,* and *A*D*F* confounded with blocks. Are there three defining effects that we could have used so that the only effects confounded with blocks are 4-way and higher order interactions?


First consider using two 5-way interaction effects for the first two defining effects. WLOG, take *A*B*C*D*E* and *A*B*C*D*F*. The GI of these two effects is *E*F*, a two-way interaction contrast. So we can't have two 5-way effects in our defining effects.

How about using a 4-way and a 5-way for the first two defining effects?


WLOG, consider consider *A*B*C*D*E* and *F*A*B*C*. The GI is *D*E*F,* a 3-way interaction effect.

Therefore, we will need to be able to find three 4-way interaction effects whose GIs are also 4-way interaction effects.


WOLG, consider using *A*B*C*D*, *A*B*E*F* for the first two effects. The GI between these two effects is *A*B*E*F*. Is there another 4-way effect whose GIs with the preceding 4-way effects are also 4-way effects? There are 15 4-way interaction effects, and one can try using each of the remaining 12 4-way effects. You will find that none work. Therefore, 3-way interactions will also need to be confounded with blocks when blocking a 2^6 experiment into blocks of size 8.


Statistical Analyses

As before both the half-normal plotting technique and the ANOVA method where one pools the high-order interactions that are not confounded with blocks into an estimate for experimental error can be used. Which is most appropriate will depend on the number of high-order interactions that can be pooled into an estimate of experimental error.

Question: When one is using the half-normal plot method, should one include the effects confounded with blocks in the half-normal plot?

I am not aware of any recommendations for or against this. I usually include them, but I would have no strong arguments against taking them out prior to doing the half-normal plot if an analyst would like to do so.