¼ Replicates

Suppose we consider a ¼ replicate of a 2^n factorial experiment. This will require two defining effects. How many aliases will each effect have?

¼ Replicates

Suppose we consider a ¼ replicate of a 2^n factorial experiment. This will require two defining effects. How many aliases will each effect have?

Answer: In a ¼ replicate, each effect will have three aliases.

Remark: The aliases of any effect are given by the generalized interactions of the effect and the defining effects as well as with the generalized interactions of the defining effects.

Consider taking a ¼ replicate of a 2^5 experiment with factors *A,B,C,D, & E.*

What should we use as defining effects?

How about using *ABCDE* and *ABC* ?

What are the alias sets?

Note that the generalized interaction of *ABCDE* and *ABC* is *DE*.

Therefore, the aliases of *A* are *BCDE*, *BC*, & *ADE*.

Alias sets:

Defining Effects			
	I=ABCDE	*I=ABC*	*DE*
Effect	Alias 1	Alias 2	Alias 3
A	*BCDE*	*BC*	*ADE*
B	*ACDE*	*AC*	*BDE*
AB	*CDE*	*C*	*ABDE*
ABC	*DE*	*TOTAL*	*ABCDE*
D	*ABCE*	*ABCD*	*E*
AD	*BCE*	*BCD*	*AE*
BD	*ACE*	*ACD*	*BE*
ABD	*CE*	*CD*	*ABE*

Alias sets:

Defining Effects			
	I=ABCDE	I=ABC	DE
Effect	Alias 1	Alias 2	Alias 3
A	BCDE	BC	ADE
B	ACDE	AC	BDE
AB	CDE	C	ABDE
ABC	DE	TOTAL	ABCDE
D	ABCE	ABCD	E
AD	BCE	BCD	AE
BD	ACE	ACD	BE
ABD	CE	CD	ABE

Remark: The previous design would not be a good one since the main effect D has the main effect E as an alias.

Question: Can we do better?

Consider using ABD and ACE as defining effects. The generalized interaction between these two effects is $BCDE$. Find the alias sets.

Defining Effects			
	I=ABD	*I=ACE*	*BCDE*
Effect	Alias 1	Alias 2	Alias 3
A	*BD*	*CE*	*ABCDE*
B	*AD*	*ABCE*	*CDE*
C	*ABCD*	*AE*	*BDE*
D	*AB*	*ACDE*	*BCE*
E	*ABDE*	*AC*	*BCD*
BC	*ACD*	*ABE*	*DE*
BE	*ADE*	*ABC*	*CD*
TOTAL	*ABD*	*ACE*	*BCDE*

In the previous design, no main effect is aliased with any other main effects. However, each main effect is aliased with one or more two-way interaction effects.

Question: Can we find a ¼ replicate of a 2^5 such that no main effect is aliased with a two-way interaction?

Answer: There are 5 main effects and 10 two-way interaction effects in a 2^5 experiment. A ¼ replicate would only have 8 runs. So we can estimate a maximum of eight effects including the Total Effect. So it is not possible to find a ¼ replicate of a 2^5 experiment without aliasing at least one main effect with a two-way interaction effect .

A ¼ Replicate of a 2^5

```
DATA ONE;
DO A = 0 TO 1;  DO B = 0 TO 1;
  DO C = 0 TO 1;    DO D = 0 TO 1;
    DO E = 0 TO 1;
     IF MOD(A+B+D,2)= 1 AND
MOD(A+C+E,2)  = 0 THEN OUTPUT;
    END; END; END; END; END;
    RUN;
```

```
ODS RTF FILE = 'C:\TEMP\TEMP8.RTF';
PROC PRINT;
 TITLE 'A 1/4 REPLICATE OF A 2^5
EXPERIMENTAL DESIGN';
 RUN;
ODS RTF CLOSE;
```

A ¼ Replicate of a 2^5 Factorial Design.

Obs	A	B	C	D	E
1	0	0	0	1	0
2	0	0	1	1	1
3	0	1	0	0	0
4	0	1	1	0	1
5	1	0	0	0	1
6	1	0	1	0	0
7	1	1	0	1	1
8	1	1	1	1	0

An alternative way to find the treatment combinations using the defining effects:

$I = ABD$ **and**

$I = ACE$

is to note that these are equivalent to

$D = AB$ **and** $E = AC$.

So we can write down the eight treatment combinations for A, B, **and** C **using pluses and minuses, and then take**

$D = AB$ **and** $E = AC$.

A 1/4 Rep of a 2^5 Design		
A	*B*	*C*
-1	-1	-1
1	-1	-1
-1	1	-1
1	1	-1
-1	-1	1
1	-1	1
-1	1	1
1	1	1

A 1/4 Replicate of a 2^5 Design				
A	*B*	*C*	*D*	
-1	-1	-1	1	
1	-1	-1	-1	
-1	1	-1	-1	
1	1	-1	1	
-1	-1	1	1	
1	-1	1	-1	
-1	1	1	-1	
1	1	1	1	

A 1/4 Replicate of a 2^5 Design				
A	***B***	***C***	***D***	***E***
-1	-1	-1	1	1
1	-1	-1	-1	-1
-1	1	-1	-1	1
1	1	-1	1	-1
-1	-1	1	1	-1
1	-1	1	-1	1
-1	1	1	-1	-1
1	1	1	1	1

A 1/4 Replicate of a 2^5 Design				
A	***B***	***C***	***D***	***E***
-1	-1	-1	1	1
1	-1	-1	-1	-1
-1	1	-1	-1	1
1	1	-1	1	-1
-1	-1	1	1	-1
1	-1	1	-1	1
-1	1	1	-1	-1
1	1	1	1	1

A 2^{6-2} Design

Note: The notation 2^{6-2} is used to represent a ¼ replicate of a 2^6 design. Likewise, 2^{n-k} is used to represent a $1/2^k$ replicate of a 2^n experiment.

A 2^{n-k} Fractional Design

Consider a 2^{n-k} fractional factorial design.

How many runs are required?

How many defining effects are required?

How many aliases will each effect have?

How many effects will be in each alias set?

A 2^{n-k} Fractional Design

Consider a 2^{n-k} fractional factorial design.

How many runs are required? 2^{n-k}

How many defining effects are required? k

How many aliases will each effect have? $2^k - 1$

How many effects will be in each alias set? 2^k

Design Resolution

Definition: A fraction of a 2^n design is defined to be of resolution R if all s level effects are not aliased with any $R - s - 1$ level effects or lower level effects.

Design Resolution

Example: Suppose a fractional design is of resolution IV. This requires that all 1 level effects be unaliased with 4-1-1 = 2 level or lower order effects, and that all 2 level effects be unaliased with 4-2-1=1 level or lower order effects. That is, no main effects can be aliased with 2-factor interactions nor can they be aliased with other main effects.

Design Resolution

Example: A design of resolution III may have main effects confounded with two-way interactions, but can have no main effect confounded with another main effect.

Design Resolution

Remark: A resolution V design cannot have any main effect confounded with a 3-way or lower order interaction effect and no two-way interaction effect can be confounded with another two-way interaction effect.

Design Resolution

Remark: For a 2^{n-k} fractional factorial design, its resolution is equal to the minimum number of factors in any of its defining effects and their generalized interactions with one another.

Recall our ¼ rep of a 2^5 with ABCDE and *ABC* as defining effects.

Note that the generalized interaction of *ABCDE* and *ABC* is *DE*.

Therefore, this design would be a Resolution II design.

Remark: The previous design was not a good one since at least one main effect will be aliased with another main effect. Recall that *D* and *E* were aliases in this design.

Question: Can we do better?

Question: Can we do better?

Consider again using *ABD* and *ACE* as defining effects. The generalized interaction between these two effects is *BCDE*. So this design will be of Resolution III.

Here no main effects will be aliased with any other main effect, but some main effects will be aliased with two factor interactions.

Blocking a Fractional Factorial Design

Remark: A fractional factorial design can also be blocked by using additional defining effects other than those used to create the fraction.

Example: Create a ½ replicate of a 2^5 factorial design in two blocks of size 8.

Suppose we use the *ABCDE* interaction to create the ½ replicate, and then suppose we create the blocks by placing the treatment combinations in the ½ replicate into blocks according to whether the sum of the levels of *A, B, & C* is odd or even.

A	B	C	D	E
0	0	0	0	1
1	0	0	0	0
0	1	0	0	0
1	1	0	0	1
0	0	1	0	0
1	0	1	0	1
0	1	1	0	1
1	1	1	0	0
0	0	0	1	0
1	0	0	1	1
0	1	0	1	1
1	1	0	1	0
0	0	1	1	1
1	0	1	1	0
0	1	1	1	0
1	1	1	1	1

A	B	C	D	E	BLOCK
0	0	0	0	1	1
1	0	0	0	0	2
0	1	0	0	0	2
1	1	0	0	1	1
0	0	1	0	0	2
1	0	1	0	1	1
0	1	1	0	1	1
1	1	1	0	0	2
0	0	0	1	0	1
1	0	0	1	1	2
0	1	0	1	1	2
1	1	0	1	0	1
0	0	1	1	1	2
1	0	1	1	0	1
0	1	1	1	0	1
1	1	1	1	1	2

A	B	C	D	E	BLOCK
0	0	0	0	1	1
1	1	0	0	1	1
1	0	1	0	1	1
0	1	1	0	1	1
0	0	0	1	0	1
1	1	0	1	0	1
1	0	1	1	0	1
0	1	1	1	0	1

A	B	C	D	E	BLOCK
1	0	0	0	0	2
0	1	0	0	0	2
0	0	1	0	0	2
1	1	1	0	0	2
1	0	0	1	1	2
0	1	0	1	1	2
0	0	1	1	1	2
1	1	1	1	1	2

The alias pairs are:

A +/- BCDE   *AB +/- CDE*   *BD +/- ACE*

B +/- ACDE   *AC +/- BDE*   *BE +/- ACD*

C +/- ABDE   *AD +/- BCE*   *CD +/- ABE*

D +/- ABCE   *AE +/- BCD*   *CE +/- ABD*

E +/- ABCD   *BC+/- ADE*   *DE +/- ABC*

The alias pairs are:

A +/- BCDE	*AB +/- CDE*	*BD +/- ACE*
B +/- ACDE	*AC +/- BDE*	*BE +/- ACD*
C +/- ABDE	*AD +/- BCE*	*CD +/- ABE*
D +/- ABCE	*AE +/- BCD*	*CE +/- ABD*
E +/- ABCD	*BC+/- ADE*	*DE +/- ABC*

Note that the last of these, *DE +/- ABC* is confounded with blocks.

Example: A 2^{6-2} experiment was performed and the data are in Table 9.8 of the text. The treatment combinations were selected by using the *ABCD* and *ABEF* interaction effects as defining effects. The generalized interaction of the two defining effects is *CDEF*. Therefore this is a resolution IV design. The treatment combinations that were selected were those where $A+B+C+D$ = Even and $A+B+E+F$ = Even where the low and high values of the factor levels are denoted by 0's and 1's.

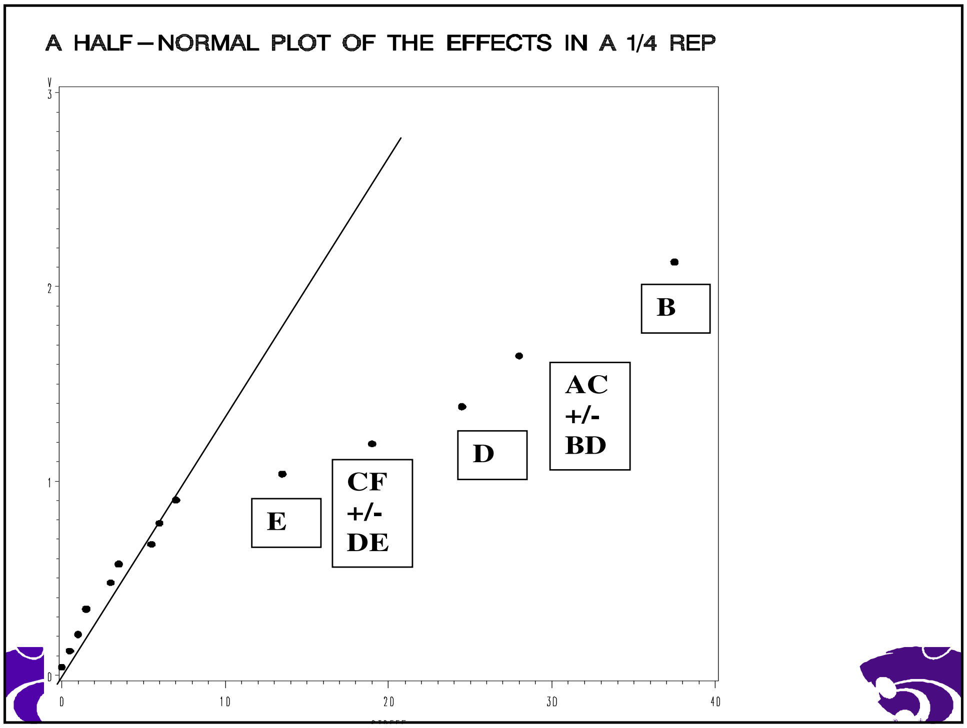
Alias Sets			
(1)	*ABCD*	*ABEF*	*CDEF*
A	*BCD*	*BEF*	*ACDEF*
B	*ACD*	*AEF*	*BCDEF*
C	*ABD*	*ABCEF*	*DEF*
D	*ABC*	*ABDEF*	*CEF*
E	*ABCDE*	*ABF*	*CDF*
F	*ABCDF*	*ABE*	*CDE*
AB	*CD*	*EF*	*ABCDEF*
AC	*BD*	*BCEF*	*ADEF*
AD	*BC*	*BDEF*	*ACEF*
AE	*BCDE*	*BF*	*ACDF*
AF	*BCDF*	*BE*	*ACDE*
CE	*ABDE*	*ABCF*	*DF*
CF	*ABDF*	*ABCE*	*DE*
ACE	*BDE*	*BCF*	*ADF*
ACF	*BDF*	*BCE*	*ADE*

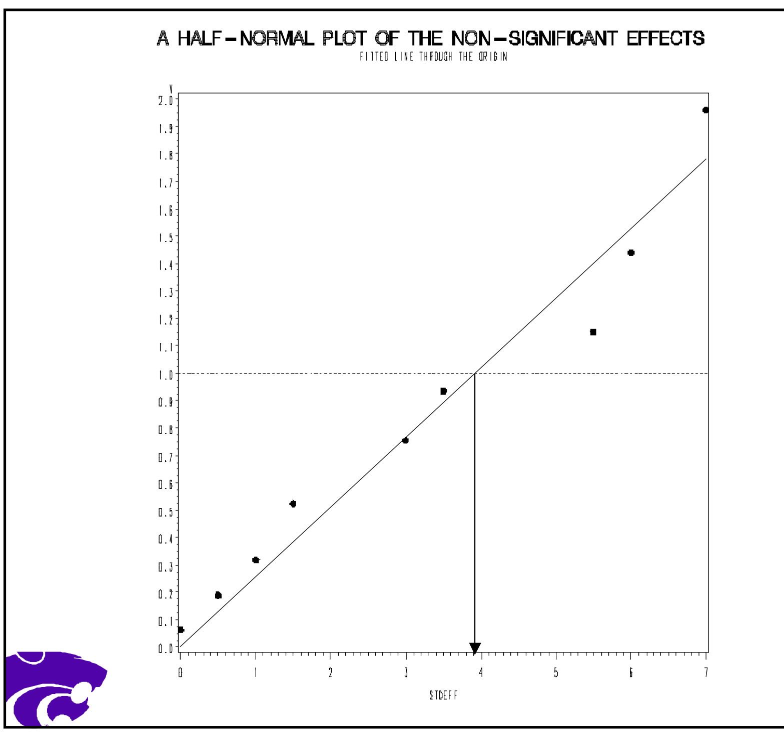
Table 9.8 Data from a ¼ rep of a 2^6 factorial experiment.

Obs	A	B	C	D	E	F	Y
1	0	0	0	0	0	0	41
2	1	1	0	0	0	0	41
3	0	0	1	1	0	0	46
4	0	0	0	0	1	1	36
5	1	1	1	1	1	1	62
6	0	0	1	1	1	1	29
7	1	1	0	0	1	1	45
8	1	1	1	1	0	0	78
9	1	0	1	0	1	0	35
10	1	0	1	0	0	1	36
11	1	0	0	1	1	0	25
12	1	0	0	1	0	1	41
13	0	1	1	0	1	0	47
14	0	1	1	0	0	1	34
15	0	1	0	1	1	0	58
16	0	1	0	1	0	1	74

```
PROC ANOVA;
 TITLE2 'ANALYSIS OF A 1/4
REPLICATE OF A 2^6';
 CLASSES A B C D E F;
 MODEL Y = A B C D E F
    A*B A*C A*D A*E A*F C*E C*F
    A*C*E A*C*F;
 ODS OUTPUT MODELANOVA=EFFECTS;
      RUN;
```

Obs	Source	MS	STDEFF	R	RSTAR	P	V
1	A*D	0.000000	0.0	1.0	0.03333	0.51667	0.04179
2	A	0.250000	0.5	2.0	0.10000	0.55000	0.12566
3	A*E	1.000000	1.0	3.0	0.16667	0.58333	0.21043
4	C	2.250000	1.5	4.5	0.26667	0.63333	0.34069
5	A*C*F	2.250000	1.5	4.5	0.26667	0.63333	0.34069
6	C*E	9.000000	3.0	6.0	0.36667	0.68333	0.47704
7	F	12.250000	3.5	7.0	0.43333	0.71667	0.57297
8	A*C*E	30.250000	5.5	8.0	0.50000	0.75000	0.67449
9	A*F	36.000000	6.0	9.0	0.56667	0.78333	0.78350
10	A*B	49.000000	7.0	10.0	0.63333	0.81667	0.90273
11	E	182.250000	13.5	11.0	0.70000	0.85000	1.03643
12	C*F	361.000000	19.0	12.0	0.76667	0.88333	1.19182
13	D	600.250000	24.5	13.0	0.83333	0.91667	1.38299
14	A*C	784.000000	28.0	14.0	0.90000	0.95000	1.64485
15	B	1406.25000	37.5	15.0	0.96667	0.98333	2.12805


A HALF-NORMAL PLOT OF THE EFFECTS IN A 1/4 REP
E
CF +/- DE
D
AC +/- BD
B
0
10
20
30
40
0
1
2
3


A HALF-NORMAL PLOT OF THE NON-SIGNIFICANT EFFECTS
FITTED LINE THROUGH THE ORIGIN
STDEFF

$\hat{\sigma} = 3.9$

Remark: The significant main effects are *B, D,* and *E*. The significant two-way interactions are: *AC* +/- *BD* and

CF +/- *DE*. Since the *A, C,* and *F* main effects were not significant, it is likely that the significance of the two-way interactions are due to *DE* and *BD* being significant. So we will look at the *DE* and *BD* means.

Obs	B	D	_TYPE_	_FREQ_	N	YBAR
1	0	0	0	4	4	37.00
2	0	1	0	4	4	35.25
3	1	0	0	4	4	41.75
4	1	1	0	4	4	68.00

$$LSD_{0.05} = 2\hat{\sigma}/\sqrt{4} = 3.9$$

Obs	D	E	_TYPE_	_FREQ_	N	YBAR
1	0	0	0	4	4	38.00
2	0	1	0	4	4	40.75
3	1	0	0	4	4	59.75
4	1	1	0	4	4	43.50

Remark: To maximize *y*, we want *B* high, *D* high, and *E* low. *A, C, & F* do not matter.

Yates Method Adjusted – Defining effects *ABCD & ABEF*

A	B	C	E
0	0	0	0
1	0	0	0
0	1	0	0
1	1	0	0
0	0	1	0
1	0	1	0
0	1	1	0
1	1	1	0
0	0	0	1
1	0	0	1
0	1	0	1
1	1	0	1
0	0	1	1
1	0	1	1
0	1	1	1
1	1	1	1

A	B	C	E	D
0	0	0	0	0
1	0	0	0	1
0	1	0	0	1
1	1	0	0	0
0	0	1	0	1
1	0	1	0	0
0	1	1	0	0
1	1	1	0	1
0	0	0	1	0
1	0	0	1	1
0	1	0	1	1
1	1	0	1	0
0	0	1	1	1
1	0	1	1	0
0	1	1	1	0
1	1	1	1	1

A	B	C	E	D	F	
0	0	0	0	0	0	
1	0	0	0	1	1	
0	1	0	0	1	1	
1	1	0	0	0	0	
0	0	1	0	1	0	
1	0	1	0	0	1	
0	1	1	0	0	1	
1	1	1	0	1	0	
0	0	0	1	0	1	
1	0	0	1	1	0	
0	1	0	1	1	0	
1	1	0	1	0	1	
0	0	1	1	1	1	
1	0	1	1	0	0	
0	1	1	1	0	0	
1	1	1	1	1	1	

A	B	C	E	D	F	Y
0	0	0	0	0	0	41
1	0	0	0	1	1	41
0	1	0	0	1	1	74
1	1	0	0	0	0	41
0	0	1	0	1	0	46
1	0	1	0	0	1	36
0	1	1	0	0	1	34
1	1	1	0	1	0	78
0	0	0	1	0	1	36
1	0	0	1	1	0	25
0	1	0	1	1	0	58
1	1	0	1	0	1	45
0	0	1	1	1	1	29
1	0	1	1	0	0	35
0	1	1	1	0	0	47
1	1	1	1	1	1	62

Yates' Analysis

A	B	C	E	D	F	Y	Step 1	Step 2	Step3	Step4	Effects
0	0	0	0	0	0	41	82	197	391	728	-
1	0	0	0	1	1	41	115	194	337	-2	A
0	1	0	0	1	1	74	82	164	1	150	B
1	1	0	0	0	0	41	112	173	-3	28	AB
0	0	1	0	1	0	46	61	-33	63	6	C
1	0	1	0	0	1	36	103	34	87	112	AC
0	1	1	0	0	1	34	64	-24	21	0	BC
1	1	1	0	1	0	78	109	21	7	98	ABC
0	0	0	1	0	1	36	0	33	-3	-54	E
1	0	0	1	1	0	25	-33	30	9	-4	AE
0	1	0	1	1	0	58	-10	42	67	24	BE
1	1	0	1	0	1	45	44	45	45	-14	ABE
0	0	1	1	1	1	29	-11	-33	-3	12	CE
1	0	1	1	0	0	35	-13	54	3	-22	ACE
0	1	1	1	0	0	47	6	-2	87	6	BCE
1	1	1	1	1	1	62	15	9	11	-76	ABCE

Yates' Analysis

A	B	C	E	D	F	Y	Step 1	Step 2	Step3	Step4	Effects
0	0	0	0	0	0	41	82	197	391	728	-
1	0	0	0	1	1	41	115	194	337	-2	A
0	1	0	0	1	1	74	82	164	1	150	B
1	1	0	0	0	0	41	112	173	-3	28	AB =/- CD =/- EF
0	0	1	0	1	0	46	61	-33	63	6	C
1	0	1	0	0	1	36	103	34	87	112	AC=/-BD
0	1	1	0	0	1	34	64	-24	21	0	BC =/- AD
1	1	1	0	1	0	78	109	21	7	98	ABC +/- D
0	0	0	1	0	1	36	0	33	-3	-54	E
1	0	0	1	1	0	25	-33	30	9	-4	AE +/- BF
0	1	0	1	1	0	58	-10	42	67	24	BE +/- AF
1	1	0	1	0	1	45	44	45	45	-14	ABE +/- F
0	0	1	1	1	1	29	-11	-33	-3	12	CE +/- DF
1	0	1	1	0	0	35	-13	54	3	-22	ACE
0	1	1	1	0	0	47	6	-2	87	6	BCE
1	1	1	1	1	1	62	15	9	11	-76	ABCE +/- CF +/- DE

Still need to standardize the effects