

A 2^3 Experiment

Consider a 2^3 experiment with factors A , B , & C .

A	B	C	Expected Response
-	-	-	(1)
+	-	-	a
-	+	-	b
+	+	-	ab
-	-	+	c
+	-	+	ac
-	+	+	bc
+	+	+	abc

Effects

$$A = [- (1) + a - b + ab - c + ac - bc + abc]/4$$

Effects

$$A = [- (1) + a - b + ab - c + ac - bc + abc]/4$$

$$B = [- (1) - a + b + ab - c - ac + bc + abc]/4$$

Effects

$$A = [- (1) + a - b + ab - c + ac - bc + abc]/4$$

$$B = [- (1) - a + b + ab - c - ac + bc + abc]/4$$

$$A*B = [- b + ab - bc + abc]$$

$$- [- (1) + a - c + ac]/4$$

$$= [(1) - a - b + ab + c - ac - bc + abc]/4$$

Effects

$$A = [- (1) + a - b + ab - c + ac - bc + abc]/4$$

$$B = [- (1) - a + b + ab - c - ac + bc + abc]/4$$

$$A*B = [(1) - a - b + ab + c - ac - bc + abc]/4$$

$$C = [- (1) - a - b - ab + c + ac + bc + abc]/4$$

Effects

$$A = [- (1) + a - b + ab - c + ac - bc + abc]/4$$

$$B = [- (1) - a + b + ab - c - ac + bc + abc]/4$$

$$A*B = [(1) - a - b + ab + c - ac - bc + abc]/4$$

$$C = [- (1) - a - b - ab + c + ac + bc + abc]/4$$

$$A*C = [(1) - a + b - ab - c + ac - bc + abc]/4$$

$$B*C = [(1) + a - b - ab - c - ac + bc + abc]/4$$

$$A*B*C = [(1) - a - b + ab - c + ac + bc - abc]/4$$

Neat Tricks (Symbolism)

$$A \sim (a-1)(b+1)(c+1)/4$$

Neat Tricks (Symbolism)

$$A \sim (a-1)(b+1)(c+1)/4$$

$$B \sim (a+1)(b-1)(c+1)/4$$

Neat Tricks (Symbolism)

$$A \sim (a - 1)(b + 1)(c + 1)/4$$

$$B \sim (a + 1)(b - 1)(c + 1)/4$$

$$A*B \sim (a - 1)(b - 1)(c + 1)/4$$

Neat Tricks (Symbolism)

$$A \sim (a - 1)(b + 1)(c + 1)/4$$

$$B \sim (a + 1)(b - 1)(c + 1)/4$$

$$A*B \sim (a - 1)(b - 1)(c + 1)/4$$

$$C \sim (a + 1)(b + 1)(c - 1)/4$$

$$A*C \sim (a - 1)(b + 1)(c - 1)/4$$

$$B*C \sim (a + 1)(b - 1)(c - 1)/4$$

$$A*B*C \sim (a - 1)(b - 1)(c - 1)/4$$

				Expected Response	EFFECTS						
A	B	C			A	B	A*B	C	A*C	B*C	A*B*C
-	-	-		(1)	-						
+	-	-		a	+						
-	+	-		b	-						
+	+	-		ab	+						
-	-	+		c	-						
+	-	+		ac	+						
-	+	+		bc	-						
+	+	+		abc	+						

				Expected Response	EFFECTS						
A	B	C			A	B	A*B	C	A*C	B*C	A*B*C
-	-	-		(1)	-	-					
+	-	-		a	+	-					
-	+	-		b	-	+					
+	+	-		ab	+	+					
-	-	+		c	-	-					
+	-	+		ac	+	-					
-	+	+		bc	-	+					
+	+	+		abc	+	+					

					EFFECTS						
<i>A</i>	<i>B</i>	<i>C</i>	Expected Response		<i>A</i>	<i>B</i>	<i>A*B</i>	<i>C</i>	<i>A*C</i>	<i>B*C</i>	<i>A*B*C</i>
-	-	-	(1)		-	-	+				
+	-	-	<i>a</i>		+	-	-				
-	+	-	<i>b</i>		-	+	-				
+	+	-	<i>ab</i>		+	+	+				
-	-	+	<i>c</i>		-	-	+				
+	-	+	<i>ac</i>		+	-	-				
-	+	+	<i>bc</i>		-	+	-				
+	+	+	<i>abc</i>		+	+	+				

					EFFECTS						
<i>A</i>	<i>B</i>	<i>C</i>	Expected Response		<i>A</i>	<i>B</i>	<i>A*B</i>	<i>C</i>	<i>A*C</i>	<i>B*C</i>	<i>A*B*C</i>
-	-	-	(1)		-	-	+	-			
+	-	-	<i>a</i>		+	-	-	-			
-	+	-	<i>b</i>		-	+	-	-			
+	+	-	<i>ab</i>		+	+	+	-			
-	-	+	<i>c</i>		-	-	+	+			
+	-	+	<i>ac</i>		+	-	-	+			
-	+	+	<i>bc</i>		-	+	-	+			
+	+	+	<i>abc</i>		+	+	+	+			

				EFFECTS							
A	B	C	Expected Response	A	B	A*B	C	A*C	B*C	A*B*C	
-	-	-	(1)	-	-	+	-	+			
+	-	-	a	+	-	-	-	-			
-	+	-	b	-	+	-	-	+			
+	+	-	ab	+	+	+	-	-			
-	-	+	c	-	-	+	+	-			
+	-	+	ac	+	-	-	+	+			
-	+	+	bc	-	+	-	+	-			
+	+	+	abc	+	+	+	+	+			

				EFFECTS							
A	B	C	Expected Response	A	B	A*B	C	A*C	B*C	A*B*C	
-	-	-	(1)	-	-	+	-	+	+		
+	-	-	a	+	-	-	-	-	+		
-	+	-	b	-	+	-	-	+	-		
+	+	-	ab	+	+	+	-	-	-		
-	-	+	c	-	-	+	+	-	-		
+	-	+	ac	+	-	-	+	+	-		
-	+	+	bc	-	+	-	+	-	+		
+	+	+	abc	+	+	+	+	+	+		

				EFFECTS							
A	B	C	Expected Response	A	B	A*B	C	A*C	B*C	A*B*C	
-	-	-	(1)	-	-	+	-	+	+	-	
+	-	-	a	+	-	-	-	-	+	+	
-	+	-	b	-	+	-	-	+	-	+	
+	+	-	ab	+	+	+	-	-	-	-	
-	-	+	c	-	-	+	+	-	-	+	
+	-	+	ac	+	-	-	+	+	-	-	
-	+	+	bc	-	+	-	+	-	+	-	
+	+	+	abc	+	+	+	+	+	+	+	

				EFFECTS							
A	B	C	Expected Response	A	B	A*B	C	A*C	B*C	A*B*C	
-	-	-	(1)	-	-	+	-	+	+	-	
+	-	-	a	+	-	-	-	-	+	+	
-	+	-	b	-	+	-	-	+	-	+	
+	+	-	ab	+	+	+	-	-	-	-	
-	-	+	c	-	-	+	+	-	-	+	
+	-	+	ac	+	-	-	+	+	-	-	
-	+	+	bc	-	+	-	+	-	+	-	
+	+	+	abc	+	+	+	+	+	+	+	

<u>Example</u>			
			Observed
Temp	Conc	Catalyst	Response
160	20	C1	60
180	20	C1	72
160	40	C1	54
180	40	C1	68
160	20	C2	52
180	20	C2	83
160	40	C2	45
180	40	C2	80

<u>Example</u>											
Temp	Conc	CAT	Observed Response	Total	EFFECTS						
					A	B	A*B	C	A*C	B*C	A*B*C
160	20	C1	60	+	-	-	+	-	+	+	-
180	20	C1	72	+	+	-	-	-	-	+	+
160	40	C1	54	+	-	+	-	-	+	-	+
180	40	C1	68	+	+	+	+	-	-	-	-
160	20	C2	52	+	-	-	+	+	-	-	+
180	20	C2	83	+	+	-	-	+	+	-	-
160	40	C2	45	+	-	+	-	+	-	+	-
180	40	C2	80	+	+	+	+	+	+	+	+
Value				514							

Example											
Temp	Conc	CAT	Observed Response	Total	EFFECTS						
					<i>A</i>	<i>B</i>	<i>A*B</i>	<i>C</i>	<i>A*C</i>	<i>B*C</i>	<i>A*B*C</i>
160	20	C1	60	+	-	-	+	-	+	+	-
180	20	C1	72	+	+	-	-	-	-	+	+
160	40	C1	54	+	-	+	-	-	+	-	+
180	40	C1	68	+	+	+	+	-	-	-	-
160	20	C2	52	+	-	-	+	+	-	-	+
180	20	C2	83	+	+	-	-	+	+	-	-
160	40	C2	45	+	-	+	-	+	-	+	-
180	40	C2	80	+	+	+	+	+	+	+	+
Value				514	92	-20	6	6	40	0	2

Example											
Temp	Conc	CAT	Observed Response	Total	EFFECTS						
					<i>A</i>	<i>B</i>	<i>A*B</i>	<i>C</i>	<i>A*C</i>	<i>B*C</i>	<i>A*B*C</i>
160	20	C1	60	+	-	-	+	-	+	+	-
180	20	C1	72	+	+	-	-	-	-	+	+
160	40	C1	54	+	-	+	-	-	+	-	+
180	40	C1	68	+	+	+	+	-	-	-	-
160	20	C2	52	+	-	-	+	+	-	-	+
180	20	C2	83	+	+	-	-	+	+	-	-
160	40	C2	45	+	-	+	-	+	-	+	-
180	40	C2	80	+	+	+	+	+	+	+	+
Value				514	92	-20	6	6	40	0	2
Effect				64.25	23	-5	1.5	1.5	10	0	0.5

$$\text{Standardized Effect} = \text{Value}/\sqrt{8} = \text{Value}/2.8284$$

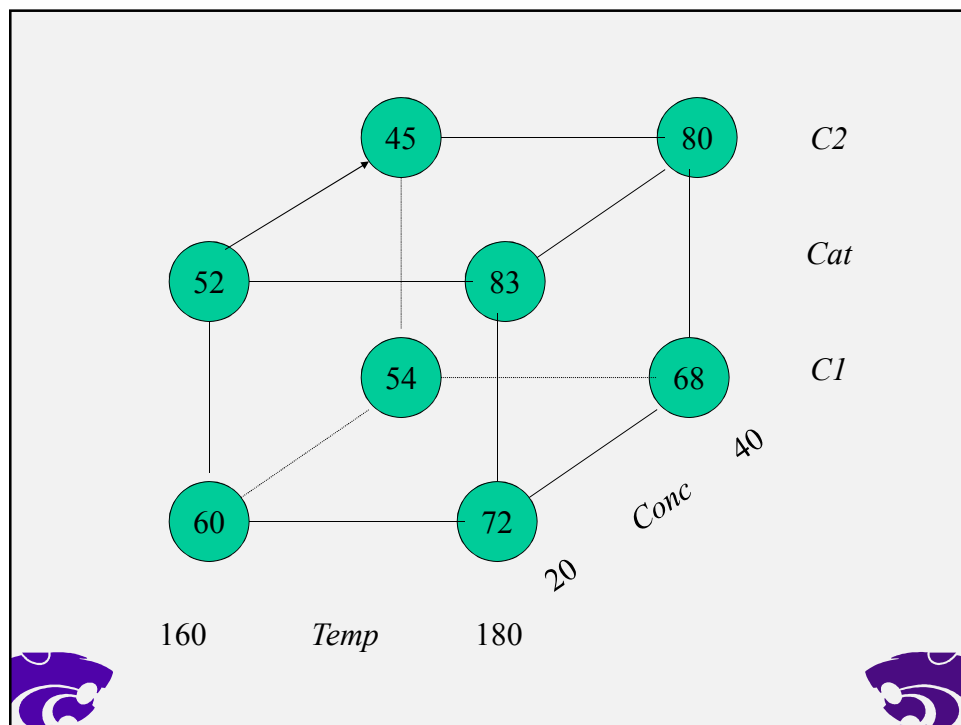
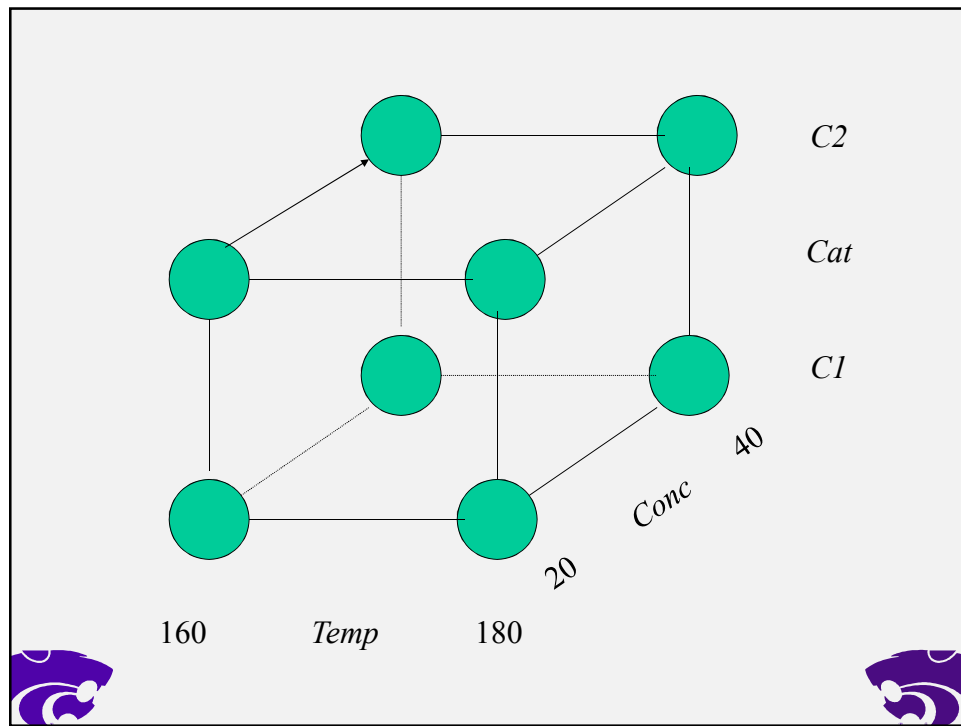
Example

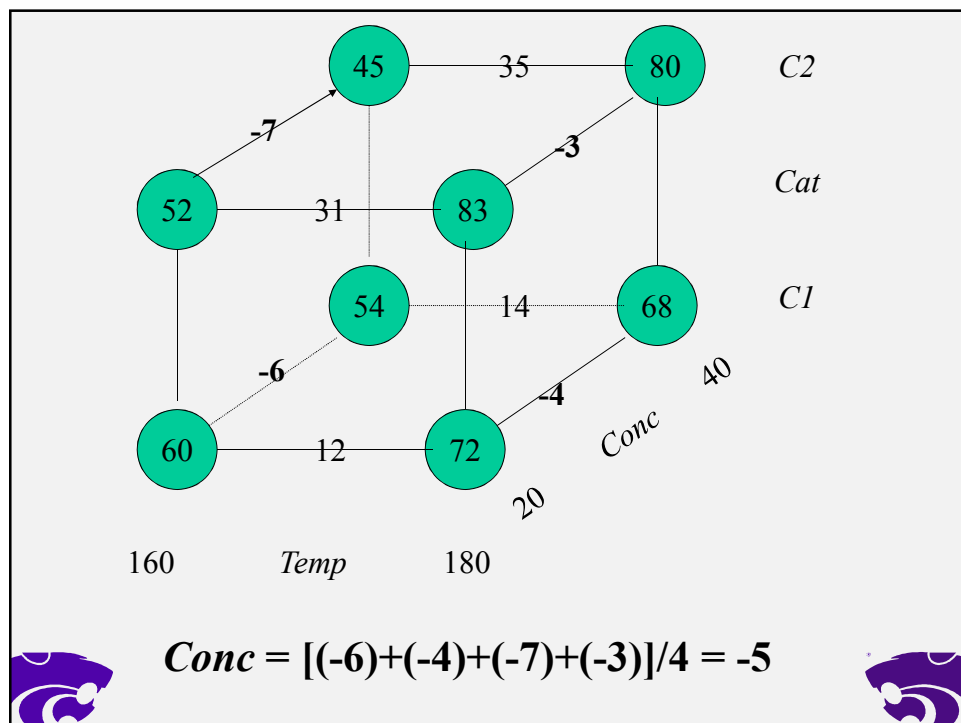
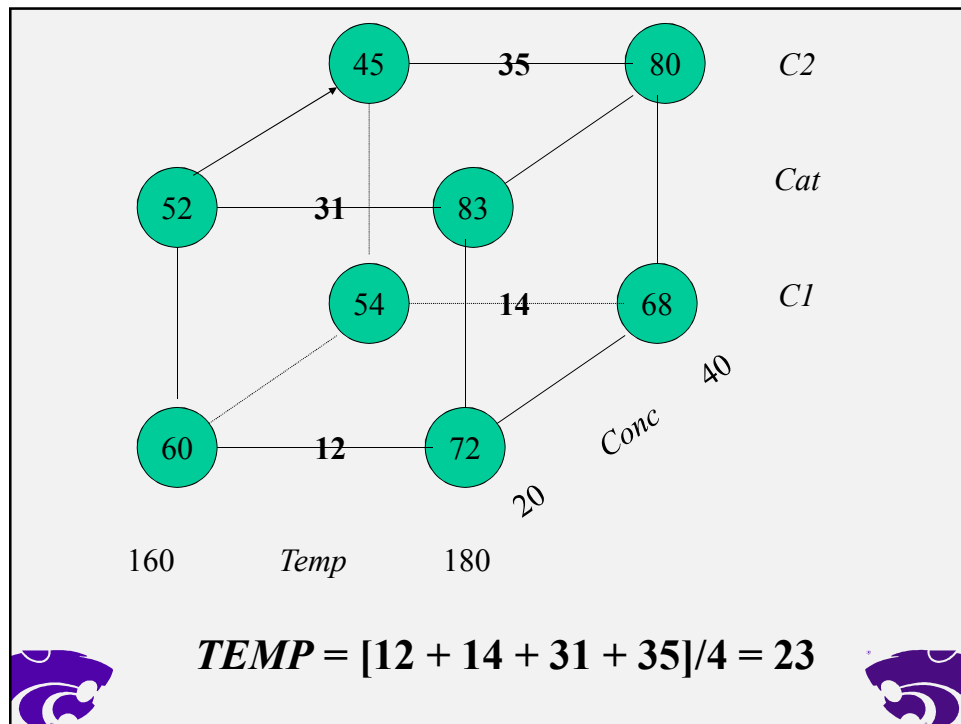
Temp	Conc	CAT	Observed Response	Total	EFFECTS						
					A	B	A*B	C	A*C	B*C	A*B*C
160	20	C1	60	+	-	-	+	-	+	+	-
180	20	C1	72	+	+	-	-	-	-	+	+
160	40	C1	54	+	-	+	-	-	+	-	+
180	40	C1	68	+	+	+	+	-	-	-	-
160	20	C2	52	+	-	-	+	+	-	-	+
180	20	C2	83	+	+	-	-	+	+	-	-
160	40	C2	45	+	-	+	-	+	-	+	-
180	40	C2	80	+	+	+	+	+	+	+	+
Value				514	92	-20	6	6	40	0	2
Effect				64.25	23	-5	1.5	1.5	10	0	0.5
Standardized Effect					32.5269	-7.0711	2.12132	2.12132	14.1421	0	0.70711

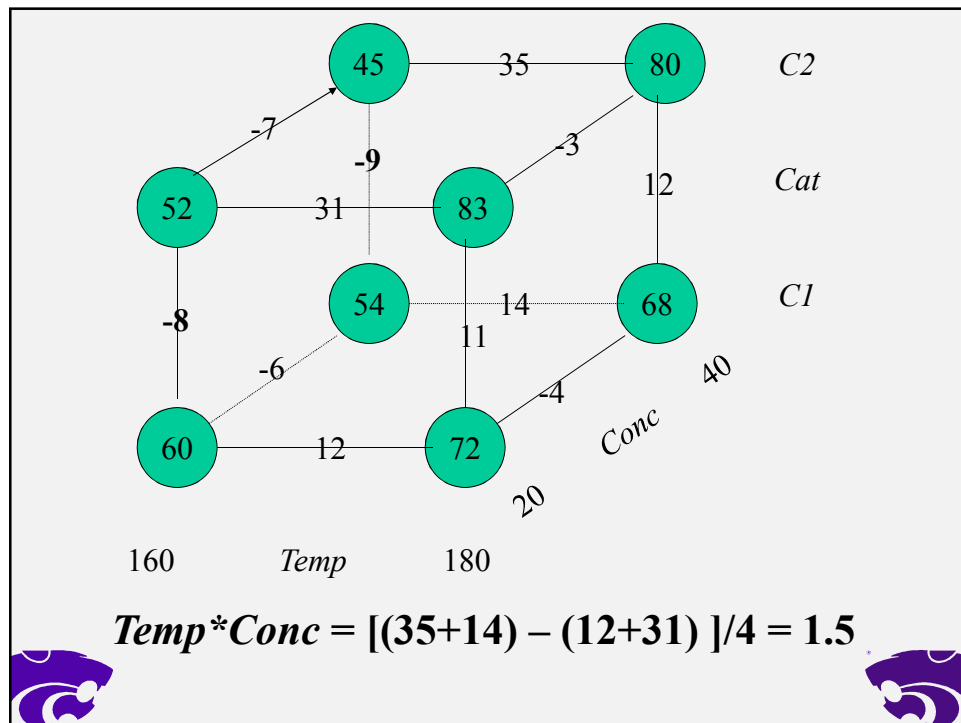
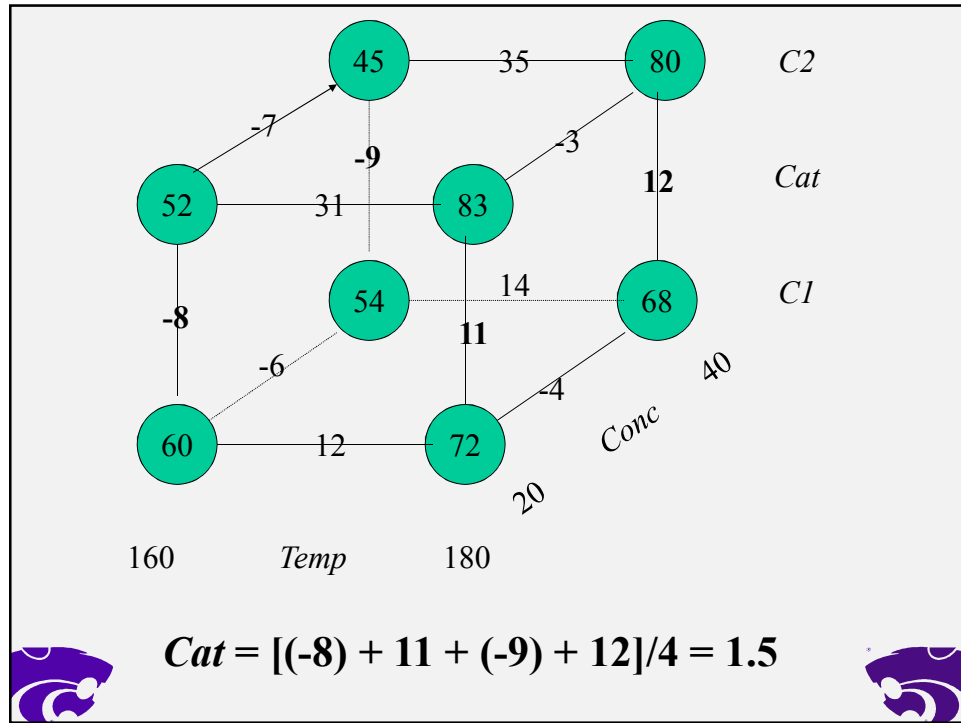
$$\text{Standardized Effect} = \text{Value}/\sqrt{8} = \text{Value}/2.8284$$

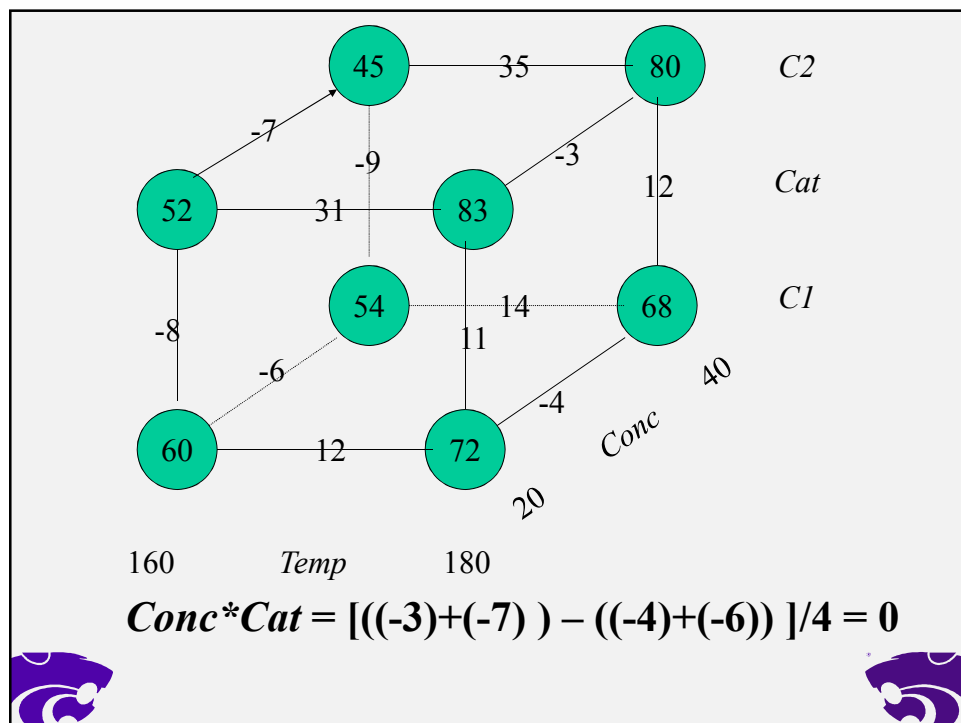
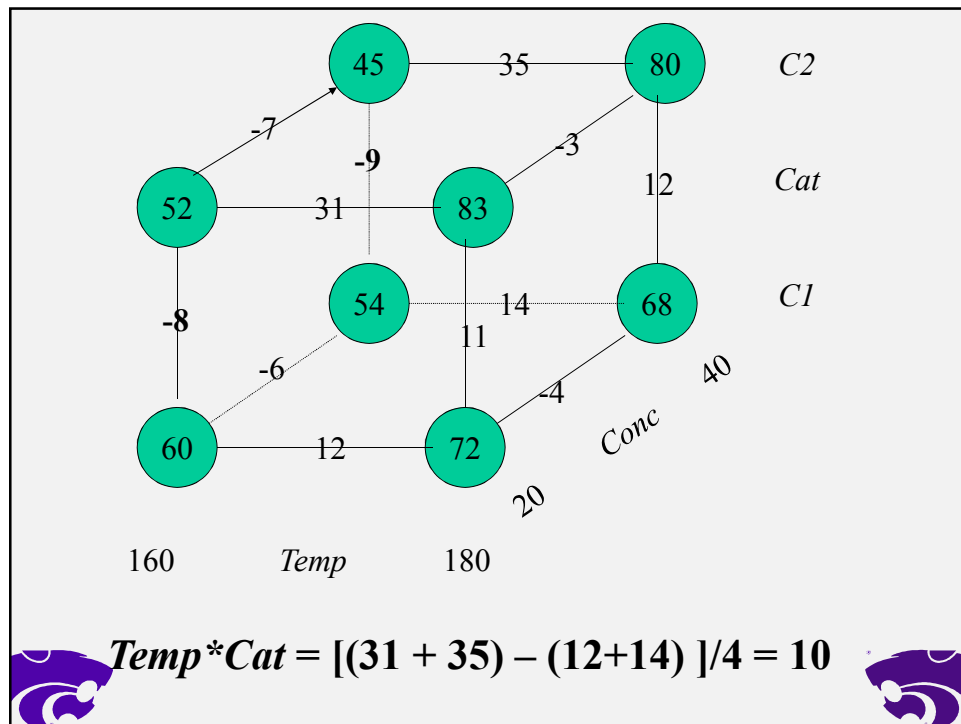
Example

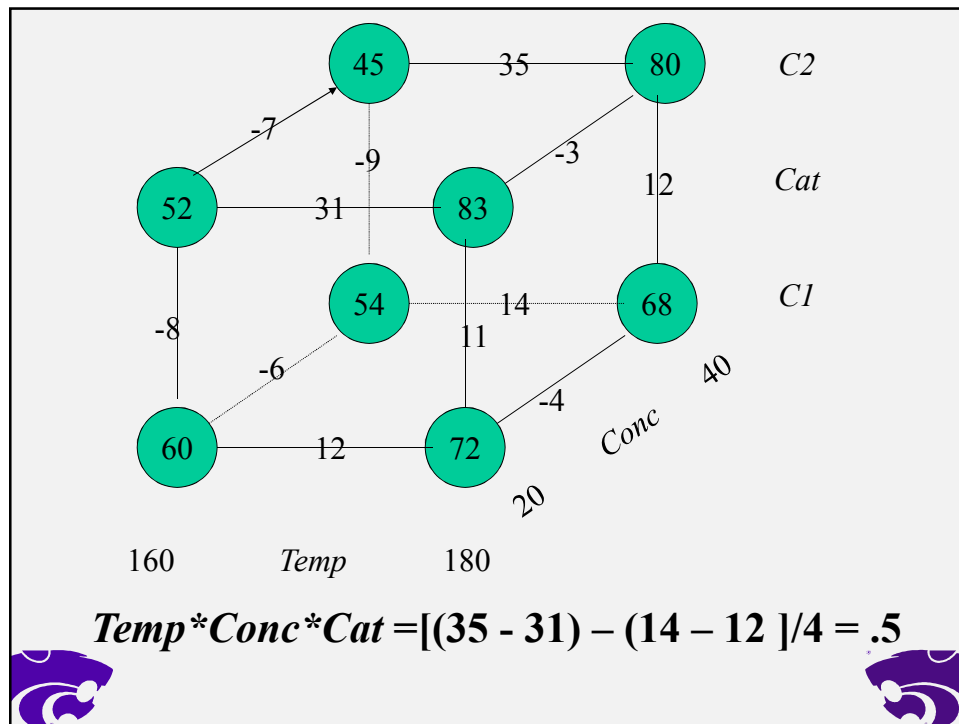
Temp	Conc	CAT	Observed Response	Total	EFFECTS						
					A	B	A*B	C	A*C	B*C	A*B*C
160	20	C1	60	+	-	-	+	-	+	+	-
180	20	C1	72	+	+	-	-	-	-	+	+
160	40	C1	54	+	-	+	-	-	+	-	+
180	40	C1	68	+	+	+	+	-	-	-	-
160	20	C2	52	+	-	-	+	+	-	-	+
180	20	C2	83	+	+	-	-	+	+	-	-
160	40	C2	45	+	-	+	-	+	-	+	-
180	40	C2	80	+	+	+	+	+	+	+	+
Value				514	92	-20	6	6	40	0	2
Standardized Effect					32.5269	-7.0711	2.12132	2.12132	14.1421	0	0.70711











Yates' Method

Yates provided a neat scheme for finding the effects in any 2^n experiment.

The method requires n steps and that one first lists the treatment combinations in our standard order. Then one adds successive pairs, and then subtracts successive pairs in each step.

<u>Example</u>				
			Observed	
Temp	Conc	CAT	Response	Step 1
160	20	C1	60	
180	20	C1	72	
160	40	C1	54	
180	40	C1	68	
160	20	C2	52	
180	20	C2	83	
160	40	C2	45	
180	40	C2	80	

<u>Example</u>				
			Observed	
Temp	Conc	CAT	Response	Step 1
160	20	C1	60	132
180	20	C1	72	
160	40	C1	54	
180	40	C1	68	
160	20	C2	52	
180	20	C2	83	
160	40	C2	45	
180	40	C2	80	

<u>Example</u>				
			Observed	
Temp	Conc	CAT	Response	Step 1
160	20	C1	60	132
180	20	C1	72	122
160	40	C1	54	135
180	40	C1	68	125
160	20	C2	52	
180	20	C2	83	
160	40	C2	45	
180	40	C2	80	

<u>Example</u>				
			Observed	
Temp	Conc	CAT	Response	Step 1
160	20	C1	60	132
180	20	C1	72	122
160	40	C1	54	135
180	40	C1	68	125
160	20	C2	52	12
180	20	C2	83	
160	40	C2	45	
180	40	C2	80	

Example

			Observed	
Temp	Conc	CAT	Response	Step 1
160	20	C1	60	132
180	20	C1	72	122
160	40	C1	54	135
180	40	C1	68	125
160	20	C2	52	12
180	20	C2	83	14
160	40	C2	45	31
180	40	C2	80	35

			Observed		
Temp	Conc	CAT	Response	Step 1	Step 2
160	20	C1	60	132	254
180	20	C1	72	122	
160	40	C1	54	135	
180	40	C1	68	125	
160	20	C2	52	12	
180	20	C2	83	14	
160	40	C2	45	31	
180	40	C2	80	35	

			Observed		
Temp	Conc	CAT	Response	Step 1	Step 2
160	20	C1	60	132	254
180	20	C1	72	122	260
160	40	C1	54	135	26
180	40	C1	68	125	66
160	20	C2	52	12	
180	20	C2	83	14	
160	40	C2	45	31	
180	40	C2	80	35	

			Observed		
Temp	Conc	CAT	Response	Step 1	Step 2
160	20	C1	60	132	254
180	20	C1	72	122	260
160	40	C1	54	135	26
180	40	C1	68	125	66
160	20	C2	52	12	-10
180	20	C2	83	14	-10
160	40	C2	45	31	2
180	40	C2	80	35	4

			Observed			
Temp	Conc	CAT	Response	Step 1	Step 2	Step 3
160	20	C1	60	132	254	514
180	20	C1	72	122	260	92
160	40	C1	54	135	26	-20
180	40	C1	68	125	66	6
160	20	C2	52	12	-10	
180	20	C2	83	14	-10	
160	40	C2	45	31	2	
180	40	C2	80	35	4	

			Observed			
Temp	Conc	CAT	Response	Step 1	Step 2	Step 3
160	20	C1	60	132	254	514
180	20	C1	72	122	260	92
160	40	C1	54	135	26	-20
180	40	C1	68	125	66	6
160	20	C2	52	12	-10	6
180	20	C2	83	14	-10	40
160	40	C2	45	31	2	0
180	40	C2	80	35	4	2

			Observed				
Temp	Conc	CAT	Response	Step 1	Step 2	Step 3	Effect
160	20	C1	60	132	254	514	64.25
180	20	C1	72	122	260	92	23
160	40	C1	54	135	26	-20	-5
180	40	C1	68	125	66	6	1.5
160	20	C2	52	12	-10	6	1.5
180	20	C2	83	14	-10	40	10
160	40	C2	45	31	2	0	0
180	40	C2	80	35	4	2	0.5

			Observed				Standardized
Temp	Conc	CAT	Response	Step 1	Step 2	Step 3	Effect
160	20	C1	60	132	254	514	
180	20	C1	72	122	260	92	32.527
160	40	C1	54	135	26	-20	-7.071
180	40	C1	68	125	66	6	2.121
160	20	C2	52	12	-10	6	2.121
180	20	C2	83	14	-10	40	14.142
160	40	C2	45	31	2	0	0.000
180	40	C2	80	35	4	2	0.707

A	B	C	Observed				Standardized	
Temp	Conc	CAT	Response	Step 1	Step 2	Step 3	Effect	EFFECT
160	20	C1	60	132	254	514		<i>Total</i>
180	20	C1	72	122	260	92	32.527	<i>A</i>
160	40	C1	54	135	26	-20	-7.071	<i>B</i>
180	40	C1	68	125	66	6	2.121	<i>A*B</i>
160	20	C2	52	12	-10	6	2.121	<i>C</i>
180	20	C2	83	14	-10	40	14.142	<i>A*C</i>
160	40	C2	45	31	2	0	0.000	<i>B*C</i>
180	40	C2	80	35	4	2	0.707	<i>A*B*C</i>

Justification of Yates' Method				
Trt				
Combination	Step 1	Step 2	Step 3	EFFECT
<i>(1)</i>	<i>(1)+a</i>	<i>(1)+a+b+ab</i>	<i>(1)+a+b+ab+c+ac+bc+abc</i>	<i>Total</i>
<i>a</i>	<i>b+ab</i>	<i>c+ac+bc+abc</i>	<i>a-(1)+ab-b+ac-c+abc-bc</i>	<i>A</i>
<i>b</i>	<i>c+ac</i>	<i>a-(1)+ab-b</i>	<i>b+ab-(1)-a+bc+abc-c-ac</i>	<i>B</i>
<i>ab</i>	<i>bc+abc</i>	<i>ac-c+abc-bc</i>	<i>ab-b-a+(1)+abc-bc-ac+c</i>	<i>A*B</i>
<i>c</i>	<i>a-(1)</i>	<i>b+ab-(1)-a</i>	<i>(1)+a+b+ab-c-ac-bc-abc</i>	<i>C</i>
<i>ac</i>	<i>ab-b</i>	<i>bc+abc-c-ac</i>	<i>a-(1)+ab-b-ac+c-abc+bc</i>	<i>A*C</i>
<i>bc</i>	<i>ac-c</i>	<i>ab-b-a+(1)</i>	<i>b+ab-(1)-a-bc-abc+c+ac</i>	<i>B*C</i>
<i>abc</i>	<i>abc-bc</i>	<i>abc-bc-ac+c</i>	<i>ab-b-a+(1)-abc+bc+ac-c</i>	<i>A*B*C</i>