Example: A 2<sup>6</sup> experiment was conducted to improve the quality of a frozen food product. The factors that were thought to influence product quality were mix temperature, mixing speed, freezing temperature, air velocity, end product temperature, and package type. The beginning levels for each factor are given on the next slide.





# Table 8.6 Definition of the Treatment Structure in a 2<sup>6</sup> Factorial Experiment to Improve the Quality of a Frozen Food Product.

		Level	
Variable	Low		High
MT: mixing time	3 min		6 min
MS: mixing speed	75 rpm		150 rpm
FT: freezing temperature	-40 degrees F		-20 degrees F
AV: air velocity	100 fpm		300 fmp
ET: end product temperature	0 degrees F		10 degrees F
PT: package type	Rectangular		Round





Since the raw materials used in making the frozen food product change on a daily basis, it was decided that we should block on days. Also, only 8 runs could be completed on a given day. So we need to create 8 blocks of size 8.





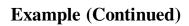
We will used the defining effects A\*B\*C\*D, A\*B\*E\*F, and A\*C\*E, and then randomly assign the treatment factors to the letters A, B, C, D, E, & F. Then we will also randomly assign the eight blocks to the 8 days. Also the run order will be randomized on each day.





**Table 8.7 Random Assignment of Factors to Symbols and Days to Blocks** 

Factor	Symbol	Day	Block
MT	В	1	3
MS	D	2	8
FT	С	3	6
AV	F	4	1
ET	A	5	7
PT	E	6	2
		7	5
		8	4



Obs	DAY	RUN	ЕТ	MT	FT	MS	PT	AV	QUALITY
1	1	1	1	0	1	0	0	0	4.5
2	1	4	0	1	0	1	0	0	9.2
3	1	7	1	1	0	0	1	0	6.9
4	1	3	0	0	1	1	1	0	5.6
5	1	8	0	0	0	0	0	1	6.2
6	1	2	1	1	1	1	0	1	5.8
7	1	6	0	1	1	0	1	1	6.3
8	1	5	1	0	0	1	1	1	7.2
9	2	6	1	0	0	0	0	0	6.3
10	2	3	0	1	1	1	0	0	6.0
11	2	2	1	1	1	0	1	0	5.6
12	2	1	0	0	0	1	1	0	7.9





#### PROC ANOVA;

TITLE 'AN ANOVA FOR THE DATA IN TABLE 8.8';
CLASS DAY MT MS FT AV ET PT;

MODEL QUALITY = DAY MT MS FT AV ET PT

MT\*MS MT\*FT MT\*AV MT\*ET MT\*PT MS\*FT MS\*AV MS\*ET

MS\*PT FT\*AV FT\*ET FT\*PT AV\*ET AV\*PT ET\*PT

MT\*MS\*FT MT\*MS\*AV MT\*MS\*ET MT\*FT\*ET MT\*FT\*PT

MT\*AV\*ET MT\*AV\*PT MT\*ET\*PT MS\*FT\*AV MS\*FT\*ET

MS\*FT\*PT MS\*AV\*PT MS\*ET\*PT FT\*AV\*ET FT\*AV\*PT

AV\*ET\*PT;





Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	44	148.1993750	3.3681676	8.41	<.0001
Error	19	7.6104688	0.4005510		
<b>Corrected Total</b>	63	155.8098438			



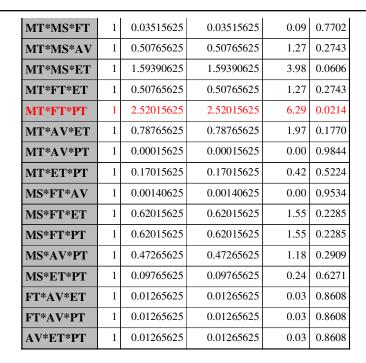


		_				
Source	DI	F	Anova SS	Mean Square	F Value	Pr > F
DAY			41.44859375	5.92122768		<.0001
MT		1	19.03140625	19.03140625	47.51	<.0001
MS		1	8.77640625	8.77640625	21.91	0.0002
FT		1	52.74390625	52.74390625	131.68	<.0001
AV		1	3.01890625	3.01890625	7.54	0.0129
ET		1	1.65765625	1.65765625	4.14	0.0561
PT		1	0.83265625	0.83265625	2.08	0.1656
MT*MS		1	1.78890625	1.78890625	4.47	0.0480
MT*FT		1	0.17015625	0.17015625	0.42	0.5224
MT*AV		1	0.04515625	0.04515625	0.11	0.7407
MT*ET		1	0.11390625	0.11390625	0.28	0.6000
MT*PT		1	0.28890625	0.28890625	0.72	0.4063
MS*FT		1	0.00015625	0.00015625	0.00	0.9844
MS*AV		1	0.09765625	0.09765625	0.24	0.6271
MS*ET		1	0.13140625	0.13140625	0.33	0.5735
MS*PT		1	0.43890625	0.43890625	1.10	0.3083
FT*AV		1	5.34765625	5.34765625	13.35	0.0017
FT*ET		1	2.36390625	2.36390625	5.90	0.0252
FT*PT		1	0.00140625	0.00140625	0.00	0.9534
AV*ET		1	0.28890625	0.28890625	0.72	0.4063
AV*PT		1	0.28890625	0.28890625	0.72	0.4063
ET*PT		1	1.35140625	1.35140625	3.37	0.0819
MT*MS*	FT	1	0.03515625	0.03515625	0.09	0.7702
MT*MS*	AV	1	0.50765625	0.50765625	1.27	0.2743
MT*MS*	ET	1	1.59390625	1.59390625	3.98	0.0606
MT*FT*	ET	1	0.50765625	0.50765625	1.27	0.2743
MT*FT*	PT	1	2.52015625	2.52015625	6.29	0.0214
MT*AV*	ET	1	0.78765625	0.78765625	1.97	0.1770
MT*AV*	PT	1	0.00015625	0.00015625	0.00	0.9844
MT*ET*	PT	1	0.17015625	0.17015625	0.42	0.5224
MT*ET* MS*FT*	V	1	0.00140625	0.00140625	0.00	0.9534
MS*FT*	T	1	0.62015625	0.62015625	1.55	0.2285

		I	I	ı	1 1	
DAY	7	41.44859375	5.92122768	14.78	<.0001	
MT	1	19.03140625	19.03140625	47.51	<.0001	
MS	1	8.77640625	8.77640625	21.91	0.0002	
FT	1	52.74390625	52.74390625	131.68	<.0001	
AV	1	3.01890625	3.01890625	7.54	0.0129	
ET	1	1.65765625	1.65765625	4.14	0.0561	
PT	1	0.83265625	0.83265625	2.08	0.1656	0

MT*MS	1	1.78890625	1.78890625	4.47	0.0480
MT*FT	1	0.17015625	0.17015625	0.42	0.5224
MT*AV	1	0.04515625	0.04515625	0.11	0.7407
MT*ET	1	0.11390625	0.11390625	0.28	0.6000
MT*PT	1	0.28890625	0.28890625	0.72	0.4063
MS*FT	1	0.00015625	0.00015625	0.00	0.9844
MS*AV	1	0.09765625	0.09765625	0.24	0.6271
MS*ET	1	0.13140625	0.13140625	0.33	0.5735
MS*PT	1	0.43890625	0.43890625	1.10	0.3083
FT*AV	1	5.34765625	5.34765625	13.35	0.0017
FT*ET	1	2.36390625	2.36390625	5.90	0.0252
FT*PT	1	0.00140625	0.00140625	0.00	0.9534
AV*ET	1	0.28890625	0.28890625	0.72	0.4063
AV*PT	1	0.28890625	0.28890625	0.72	0.4063
ЕТ*РТ	1	1.35140625	1.35140625	3.37	0.0819









Level of	Level of	Level of		QUA	LITY
MT	FT	PT	N	Mean	Std Dev
0	0	0	8	5.43750000	1.33195613
0	0	1	8	6.18750000	1.08290284
0	1	0	8	4.11250000	0.94783588
0	1	1	8	4.08750000	1.54127730
1	0	0	8	7.16250000	1.22467197
1	0	1	8	6.85000000	0.81940745
1	1	0	8	4.83750000	1.27832200
1	1	1	8	5.33750000	1.19754213



$$LSD_{0.05} = 2\sqrt{\frac{2\hat{\sigma}^2}{8}} = 2 \cdot \frac{\hat{\sigma}}{2} = \hat{\sigma} = 0.633$$



Conclusion: We want MT high, FT low, and PT does not matter





Level of	Level of		QUAL	ITY
MT	MS	N	Mean	Std Dev
0	0	16	4.41875000 c	1.28073872
0	1	16	5.49375000 b	1.53643256
1	0	16	5.84375000 ab	1.51171812
1	1	16	6.25000000 a	1.45876660

$$LSD_{0.05} = 2\sqrt{\frac{2\hat{\sigma}^2}{16}} = 2 \cdot \frac{\hat{\sigma}}{\sqrt{8}} = \frac{(2)(0.633)}{2.8284} = 0.448$$





Conclusion: We want MT high. Although the two high combinations are not significantly different at the 5% level, we might also want to take MS high.





Level of Level of			QUALITY						
FT	AV	N	Mean	Std Dev					
0	0	16	6.48125000 a	1.45749500					
0	1	16	6.33750000 a	1.09048919					
1	0	16	4.08750000 c	1.14302231					
1	1	16	5.10000000 b	1.30128142					

$$LSD_{0.05} = 2\sqrt{\frac{2\hat{\sigma}^2}{16}} = 2 \cdot \frac{\hat{\sigma}}{\sqrt{8}} = \frac{(2)(0.633)}{2.8284} = 0.448$$





Level of Level of			QUALITY						
FT	AV	N	Mean	Std Dev					
0	0	16	6.48125000 a	1.45749500					
0	1	16	6.33750000 a	1.09048919					
1	0	16	4.08750000 c	1.14302231					
1	1	16	5.10000000 b	1.30128142					

$$LSD_{0.05} = 2\sqrt{\frac{2\hat{\sigma}^2}{16}} = 2 \cdot \frac{\hat{\sigma}}{\sqrt{8}} = \frac{(2)(0.633)}{2.8284} = 0.448$$

**Conclusion:** We want FT low, AV does not matter.



Level of	Level of Level of		QUALITY			
FT	ET	N	Mean	Std Dev		
0	0	16	6.76250000 a	1.2701049		
0	1	16	6.05625000 b	1.2022028		
1	0	16	4.56250000 c	1.2622070		
1	1	16	4.62500000 c	1.3969013		

$$LSD_{0.05} = 2\sqrt{\frac{2\hat{\sigma}^2}{16}} = 2 \cdot \frac{\hat{\sigma}}{\sqrt{8}} = \frac{(2)(0.633)}{2.8284} = 0.448$$





Level of	Level of		QUALITY			
FT	ET	N	Mean	Std Dev		
0	0	16	6.76250000 a	1.2701049		
0	1	16	6.05625000 b	1.2022028		
1	0	16	4.56250000 c	1.2622070		
1	1	16	4.62500000 c	1.3969013		

$$LSD_{0.05} = 2\sqrt{\frac{2\hat{\sigma}^2}{16}} = 2 \cdot \frac{\hat{\sigma}}{\sqrt{8}} = \frac{(2)(0.633)}{2.8284} = 0.448$$

Conclusion: We want FT low and ET low.

Conclusion: We want MT high, FT

low, and PT does not matter

Conclusion: We want MT high and MS

high.

Conclusion: We want FT low, AV

does not matter.

Conclusion: We want FT low and ET low.

Overall Conclusion: We want FT low, MT high, MS high, and ET low. AV and PT do not matter.





Remark: The text also analyzes this data using a half-normal plot. See the text for more information.

You can now work Assignments 4 and 5.





## Partially Confounded Designs

Consider performing a 2<sup>3</sup> experiment in blocks of size 4. Since this is a relatively small experiment, we can consider performing more than 8 runs. Let us suppose we are going to do three replicates of the 8 treatment combinations in a total of 24 runs. If we used blocks of size 4, then we would have six blocks of size 4.





## Partially Confounded Designs

Three pairs of blocks could be obtained by confounding the three-factor interaction, A\*B\*C, in each pair of blocks. However, such a set of blocks would provide no information about the three factor interaction since the three factor interaction would be confounded with each pair of blocks.





## Partially Confounded Designs

As an alternative, suppose we confound the A\*B interaction in the first pair of blocks, the A\*C interaction in a second pair of blocks, and the B\*C interaction in the third pair of blocks.





#### **First Block Pair 1** (*A\*B* Confounded)

Block 1	Block 2
ABC	ABC
100	$0\ 0\ 0$
010	110
101	001
011	111

# Second Block Pair (A\*C

**Confounded**)

Block 3	Block 4
ABC	ABC
000	100
101	001
010	110
111	011

## Third Block Pair (B\*C Confounded)

Block 5	Block 6
ABC	ABC
000	$0\ 1\ 0$
011	$0\ 0\ 1$
100	110
111	101

# Partially Confounded Designs

Pair 1 (A*B Confounded)		Pair 2 (A*C Co	onfounded)	Pair 3 (B*C Confounded)		
Block 1	Block 2	Block 3	Block 4	Block 5	Block 6	
ABC	ABC	ABC	ABC	ABC	ABC	
100	000	000	100	000	010	
010	110	101	001	011	0 0 1	
101	0 0 1	010	110	100	110	
011	111	111	011	111	101	





# Partially Confounded Designs

Source	DF
BLKS	5
Α	1
В	1
A*B	1'
С	1
A*C	1'
B*C	1'
A*B*C	1
ERROR	11

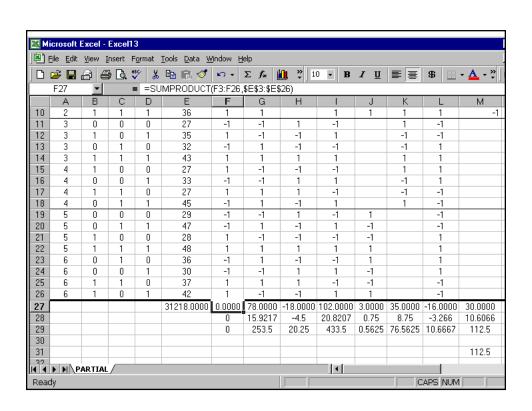


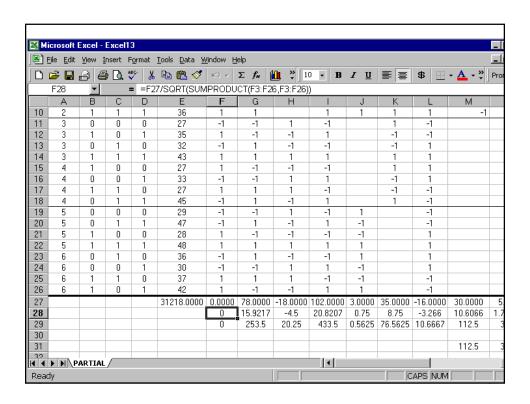


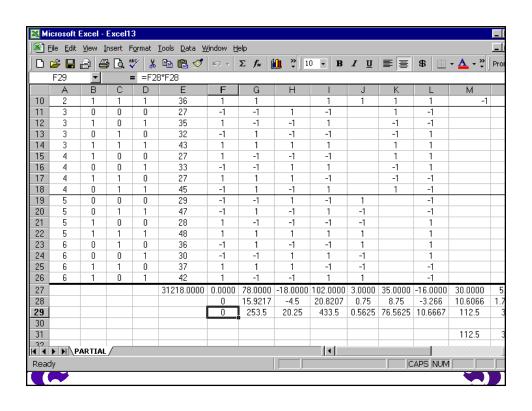
	DI OOK	_	_	_	VIELD	
	BLOCK	Α	В	С	YIELD	
	1	1	0	0	37	
	1	0	1	0	35	
	1	1	0	1	36	
	1	0	1	1	49	
	2	0	0	0	30	
	2	1	1	0	29	
	2	0	0	1	32	
	2	1	1	1	36	
	3	0	0	0	27	
	3	1	0	1	35	
	3	0	1	0	32	
	3	1	1	1	43	
	4	1	0	0	27	
	4	0	0	1	33	
	4	1	1	0	27	
	4	0	1	1	45	
	5	0	0	0	29	
	5	0	1	1	47	
	5	1	0	0	28	
	5	1	1	1	48	
	6	0	1	0	36	
	6	0	0	1	30	
	6	1	1	0	37	
C	6	1	0	1	42	5

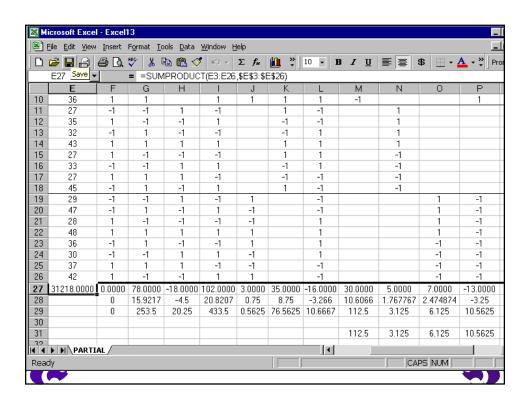
BLOCK	Α	В	С	YIELD	Α	В	A*B	С	A*C	B*C	A*B*C
1	1	0	0	37	1	-1		-1	-1	1	1
1	0	1	0	35	-1	1		-1	1	-1	1
1	1	0	1	36	1	-1		1	1	-1	-1
1	0	1	1	49	-1	1		1	-1	1	-1
2	0	0	0	30	-1	-1		-1	1	1	-1
2	1	1	0	29	1	1		-1	-1	-1	-1
2	0	0	1	32	-1	-1		1	-1	-1	1
2	1	1	1	36	1	1		1	1	1	1
3	0	0	0	27	-1	-1	1	-1		1	-1
3	1	0	1	35	1	-1	-1	1		-1	-1
3	0	1	0	32	-1	1	-1	-1		-1	1
3	1	1	1	43	1	1	1	1		1	1
4	1	0	0	27	1	-1	-1	-1		1	1
4	0	0	1	33	-1	-1	1	1		-1	1
4	1	1	0	27	1	1	1	-1		-1	-1
4	0	1	1	45	-1	1	-1	1		1	-1
5	0	0	0	29	-1	-1	1	-1	1		-1
5	0	1	1	47	-1	1	-1	1	-1		-1
5	1	0	0	28	1	-1	-1	-1	-1		1
5	1	1	1	48	1	1	1	1	1		1
6	0	1	0	36	-1	1	-1	-1	1		1
6	0	0	1	30	-1	-1	1	1	-1		1
6	1	1	0	37	1	1	1	-1	-1		-1
6	1	0	1	42	1	-1	-1	1	1		-1

					BLK	BLK	BLK	BLK	BLK	TOTAL
BLOCK	Α	В	С	YIELD	1 VS 2	3 VS 4	5 VS 6	1+2 VS 5+6		
1	1	0	0	37	1			1	1	1
1	0	1	0	35	1			1	1	1
1	1	0	1	36	1			1	1	1
1	0	1	1	49	1			1	1	1
2	0	0	0	30	-1			1	1	1
2	1	1	0	29	-1			1	1	1
2	0	0	1	32	-1			1	1	1
2	1	1	1	36	-1			1	1	1
3	0	0	0	27		1			-2	1
3	1	0	1	35		1			-2	1
3	0	1	0	32		1			-2	1
3	1	1	1	43		1			-2	1
4	1	0	0	27		-1			-2	1
4	0	0	1	33		-1			-2	1
4	1	1	0	27		-1			-2	1
4	0	1	1	45		-1			-2	1
5	0	0	0	29			1	-1	1	1
5	0	1	1	47			1	-1	1	1
5	1	0	0	28			1	-1	1	1
5	1	1	1	48			1	-1	1	1
6	0	1	0	36			-1	-1	1	1
6	0	0	1	30			-1	-1	1	1
6	1	1	0	37			-1	-1	1	1
6	1	0	1	42			-1	-1	1	1









Source	DF	SS	MS	F	
TOTAL	24	31218			
MEAN	1	30104.2			
BLKS	5	170.833	34.167	2.540	0.092
Α	1	0	0.000	0.000	1.000
В	1	253.5	253.500	18.847	0.001
A*B	1	20.25	20.250	1.505	0.245
С	1	433.5	433.500	32.229	0.000
A*C	1	0.5625	0.563	0.042	0.842
B*C	1	76.5625	76.563	5.692	0.036
A*B*C	1	10.6667	10.667	0.793	0.392
ERROR	11	148	13.451		

See Excel Example – Excel13.xls on website for more details.





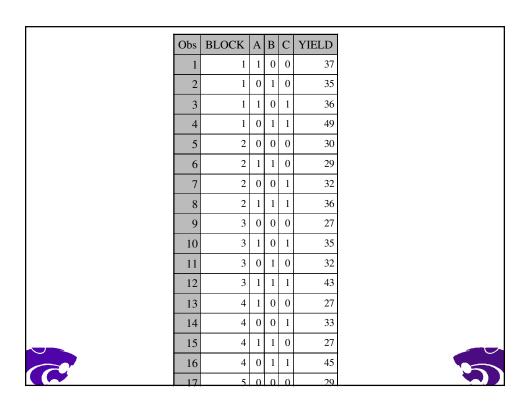
```
PROC GLM;

TITLE 'A CORRECT ANALYSIS USING SAS-GLM';

CLASSES BLOCK A B C;

MODEL YIELD = BLOCK A|B|C;

RUN;
```



Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	12	965.875000	80.489583	5.98	0.0029
Error	11	147.958333	13.450758		
<b>Corrected Total</b>	23	1113.833333			





				F	
Source	DF	Type I SS	Mean Square	Value	Pr > F
BLOCK	5	170.8333333	34.1666667	2.54	0.0918
A	1	0.0000000	0.0000000	0.00	1.0000
В	1	253.5000000	253.5000000	18.85	0.0012
A*B	1	20.2500000	20.2500000	1.51	0.2454
C	1	433.5000000	433.5000000	32.23	0.0001
A*C	1	0.5625000	0.5625000	0.04	0.8417
B*C	1	76.5625000	76.5625000	5.69	0.0361
A*B*C	1	10.6666667	10.6666667	0.79	0.3923





#### PROC ANOVA;

TITLE 'AN INCORRECT ANALYSIS USING SAS-ANOVA';

CLASSES BLOCK A B C;

MODEL YIELD = BLOCK A|B|C;

RUN;





#### **An Incorrect Analysis**

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	12	1040.666667	86.722222	13.04	<.0001
Error	11	73.166667	6.651515		
<b>Corrected Total</b>	23	1113.833333			

The portions shown in red are wrong!





#### **An Incorrect Analysis**

				F	
Source	DF	Anova SS	Mean Square	Value	Pr > F
BLOCK	5	170.8333333	34.1666667	5.14	0.0113
A	1	0.0000000	0.0000000	0.00	1.0000
В	1	253.5000000	253.5000000	38.11	<.0001
A*B	1	96.0000000	96.0000000	14.43	0.0029
C	1	433.5000000	433.5000000	65.17	<.0001
A*C	1	2.6666667	2.6666667	0.40	0.5396
В*С	1	73.5000000	73.5000000	11.05	0.0068
A*B*C	1	10.6666667	10.6666667	1.60	0.2315



The portions shown in red are wrong!



				F	
Source	DF	Type I SS	Mean Square	Value	Pr > F
BLOCK	5	170.8333333	34.1666667	2.54	0.0918
A	1	0.0000000	0.0000000	0.00	1.0000
В	1	253.5000000	253.5000000	18.85	0.0012
A*B	1	20.2500000	20.2500000	1.51	0.2454
C	1	433.5000000	433.5000000	32.23	0.0001
A*C	1	0.5625000	0.5625000	0.04	0.8417
В*С	1	76.5625000	76.5625000	5.69	0.0361
A*B*C	1	10.6666667	10.6666667	0.79	0.3923





### **B\*C** Means

Level of Level of			YIELD		
В	C	N	Mean	Std Dev	
0	0	6	29.66667	3.77712413	
0	1	6	34.66667	4.17931414	
1	0	6	32.66667	4.03319559	
1	1	6	44.66667	4.76095229	

$$LSD = t_{0.025,11} \cdot \hat{\sigma} \sqrt{\frac{1}{6} + \frac{1}{6}} = (2.201) \sqrt{\frac{13.45}{3}} = 4.66$$

