Remark: A fractional factorial experiment may be appropriate when

- **The number of treatment combinations is very large**
- **The number of levels of each factor is equal to 2**

1

Remark: A fractional factorial experiment may also be appropriate when

- **The experimenter has some "a priori" reason for believing that a number of the higher-order interactions are zero or relatively small as compared to main effects and two-way interaction effects**
- **The experiment is exploratory in nature and follow-up experiments will be performed**

2

Remark: A fractional factorial design of resolution *R* contains complete factorials (possibly replicated) in every set of *R* - 1 factors.

3

Suppose an experimenter believes that among all of the factors that might affect a process, at most *R* – 1 will have detectable effects, then if he/she uses a design of resolution *R*, and his supposition is justified, he/she will have a complete factorial design in the factors that affect the process.

4

The preceding slide gives another justification for fractional factorial designs that does not depend on higher order interactions being zero or near zero.

5

Example: Consider a 2^{6-2} fractional factorial experiment. A resolution IV design is given to the right.

The defining effects are ABCD and CDEF. Their GI is ABEF.

A	B	C	D	E	F
0	0	0	0	0	0
1	1	0	0	0	0
0	0	1	1	0	0
0	0	0	0	1	1
1	1	1	1	1	1
0	0	1	1	1	1
1	1	0	0	1	1
1	1	1	1	0	0
1	0	1	0	1	0
1	0	1	0	0	1
1	0	0	1	1	0
1	0	0	1	0	1
0	1	1	0	1	0
0	1	1	0	0	1
0	1	0	1	1	0
0	1	0	1	0	1

6

A	B	C	D	E	F
0	0	0	0	0	0
1	1	0	0	0	0
0	0	1	1	0	0
0	0	0	0	1	1
1	1	1	1	1	1
0	0	1	1	1	1
1	1	0	0	1	1
1	1	1	1	0	0
1	0	1	0	1	0
1	0	1	0	0	1
1	0	0	1	1	0
1	0	0	1	0	1
0	1	1	0	1	0
0	1	1	0	0	1
0	1	0	1	1	0
0	1	0	1	0	1

7

A	B	C	D	E	F
0	0	0	0	0	0
1	1	0	0	0	0
0	0	1	1	0	0
0	0	0	0	1	1
1	1	1	1	1	1
0	0	1	1	1	1
1	1	0	0	1	1
1	1	1	1	0	0
1	0	1	0	1	0
1	0	1	0	0	1
1	0	0	1	1	0
1	0	0	1	0	1
0	1	1	0	1	0
0	1	1	0	0	1
0	1	0	1	1	0
0	1	0	1	0	1

8

A	B	C	D	E	F
0	0	0	0	0	0
1	1	0	0	0	0
0	0	1	1	0	0
0	0	0	0	1	1
1	1	1	1	1	1
0	0	1	1	1	1
1	1	0	0	1	1
1	1	1	1	0	0
1	0	1	0	1	0
1	0	1	0	0	1
1	0	0	1	1	0
1	0	0	1	0	1
0	1	1	0	1	0
0	1	1	0	0	1
0	1	0	1	1	0
0	1	0	1	0	1

9

Remark: Suppose an experimenter wishes to investigate five factors, each at two levels, and is contemplating a 2^5 design involving 32 runs. It is almost always better to run a half-fraction of 16 runs first, and analyze the results. If necessary, a second fraction can be run later to complete a full 2^5 design. Frequently, however, the first half-fraction by itself will allow one to proceed to a new stage of experimental iteration, which may involve, for example, new variables and/or different levels of the old variables.

10

If one does what is suggested on the previous slide, then

- **The experimenter should randomize within each fraction**
- **If the experimenter runs both halves, then each half is considered a block, and the effect used to generate the halves will confounded with blocks**

11

- **No information will be lost except for the interaction effect that is confounded with blocks**
- **A design ran as two randomized fractions can give greater precision than the whole design run in a random order because the effects due to blocks is eliminated from all other comparisons.**

12

Some Fractions of 2^n Designs

The design generators are in a Word file on the course website.

Number of Factors	Number of Test Runs	Fraction	Resolution	Defining Equations
5	8	1/ 4	*III*	*I=ABD* *I=ACE*
6	8	1/8	*III*	*I=ABD* *I=ACE* *I=BCF*
	16	1 /4	*IV*	*I=ABCE* *I=BCDF*
7	8	1/16	*III*	*I=ABD* *I=ACE* *I=BCF* *I=ABCG*
	16	1/8	*IV*	*I=ABCE* *I=BCDF* *I=ABDG*
	32	1 /4	*IV*	*I=ABCDF* *I=ABDEG*
8	16	1/16	*IV*	*I=BCDE* *I=ACDF* *I=ABCG* *I=ABDH*
	32	1/8	*IV*	*I=ABCF* *I=ABDG* *I=BCDEH*
	64	1 /4	*V*	*I=ABCDG* *I=ABEFH*

13

Number of Factors	Number of Test Runs	Fraction	Resolution	Defining Equations
5	8	1/ 4	*III*	*I=ABD* *I=ACE*
6	8	1/8	*III*	*I=ABD* *I=ACE* *I=BCF*
	16	1 /4	*IV*	*I=ABCE* *I=BCDF*

14

Number of Factors	Number of Test Runs	Fraction	Resolution	Defining Equations
7	**8**	**1/16**	***III***	***I=ABD*** ***I=ACE*** ***I=BCF*** ***I=ABCG***
	16	**1/8**	***IV***	***I=ABCE*** ***I=BCDF*** ***I=ABDG***
	32	**1 /4**	***IV***	***I=ABCDF*** ***I=ABDEG***

15

Number of Factors	Number of Test Runs	Fraction	Resolution	Defining Equations
8	**16**	**1/16**	***IV***	***I=BCDE*** ***I=ACDF*** ***I=ABCG*** ***I=ABDH***
	32	**1/8**	***IV***	***I=ABCF*** ***I=ABDG*** ***I=BCDEH***
	64	**1 /4**	***V***	***I=ABCDG*** ***I=ABEFH***

16

Number of Factors	Number of Test Runs	Fraction	Resolution	Defining Equations
9	**16**	**1/32**	***III***	***I=ABCE*** ***I=BCDF*** ***I=ACDG*** ***I=ABDH*** ***I=ABCDJ***
	32	**1/16**	***IV***	***I=BCDEF*** ***I=ACDEG*** ***I=ABDEH*** ***I=ABCEJ***

17

Number of Factors	**Number of Test Runs**	**Fraction**	**Resolution**	**Defining Equations**
9	**64**	**1/8**	***IV***	***I=ABCDG*** ***I=ACEFH*** ***I=CDEFJ***
	128	**1 /4**	***VI***	***I=ACDFGH*** ***I=BCEFGJ***

18

Number of Factors	Number of Test Runs	Fraction	Resolution	Defining Equations
10	16	1/64	*III*	*I=ABCE* *I=BCDF* *I=ACDG* *I=ABDH* *I=ABCDJ* *I=ABK*
	32	1/32	*IV*	*I=ABCDF* *I=ABCEG* *I=ABDEH* *I=ACDEJ* *I=BCDEK*
	64	1/16	*IV*	*I=BCDFG* *I=ACDFH* *I=ABDEJ* *I=ABCEK*
	128	1/8	*V*	*I=ABCGH* *I=BCDEJ* *I=ACDFK*

19

Number of Factors	Number of Test Runs	Fraction	Resolution	Defining Equations
11	16	1/128	*III*	*I=ABCE* *I=BCDF* *I=ACDG* *I=ABDH* *I=ABCDJ* *I=ABK* *I=ACL*
	32	1/64	*IV*	*I=ABCF* *I=BCDG* *I=CDEH* *I=ACDJ* *I=ADEK* *I=BDEL*
	64	1/32	*IV*	*I=CDEG* *I=ABCDH* *I=ABFJ* *I=BDEFK* *I=ADEFL*
	128	1/16	*V*	*I=ABCGH* *I=BCDEJ* *I=ACDFK* *I=ABCDEFGL*

20

There is also a SAS procedure that can be used to generate fractions of 2^n experiments. The procedure is called

SAS-FACTEX.

The following examples are in ST722_9_5.SAS on the course web site.

21

Using SAS-FACTEX to get a 1/8 Replicate of a 2^7 Design.

```
PROC FACTEX;
 FACTORS A B C D E F G;
 MODEL RES=4;
  SIZE FRACTION=8;
   OUTPUT OUT=DESIGN;
   EXAMINE ALIASING
CONFOUNDING;
 RUN;
```

22

Using SAS-FACTEX to get a 1/8 Replicate of a 2^7 Design.

Factor Confounding Rules
E = B*C*D
F = A*C*D
G = A*B*D

23

Using SAS-FACTEX to get a 1/8 Replicate of a 2^7 Design.

Aliasing Structure
A
B
C
D
E
F
G
A*B = D*G = E*F
A*C = D*F = E*G
A*D = B*G = C*F
A*E = B*F = C*G
A*F = B*E = C*D
A*G = B*D = C*E
B*C = D*E = F*G

24

```
PROC PRINT DATA=DESIGN;
RUN;
```

25

Using SAS-FACTEX to get a 1/8 Replicate of a 2^7 Design.

Obs	A	B	C	D	E	F	G
1	-1	-1	-1	-1	-1	-1	-1
2	-1	-1	-1	1	1	1	1
3	-1	-1	1	-1	1	1	-1
4	-1	-1	1	1	-1	-1	1
5	-1	1	-1	-1	1	-1	1
6	-1	1	-1	1	-1	1	-1
7	-1	1	1	-1	-1	1	1
8	-1	1	1	1	1	-1	-1
9	1	-1	-1	-1	-1	1	1
10	1	-1	-1	1	1	-1	-1
11	1	-1	1	-1	1	-1	1
12	1	-1	1	1	-1	1	-1
13	1	1	-1	-1	1	1	-1
14	1	1	-1	1	-1	-1	1
15	1	1	1	-1	-1	-1	-1
16	1	1	1	1	1	1	1

26

Using SAS-FACTEX to get a 1/8 Replicate of a 2^7 Design ran in Two Blocks of size 8.

```
PROC FACTEX;
 FACTORS A B C D E F G;
 MODEL RES=4; SIZE FRACTION=8;
   BLOCKS NBLOCKS=2;
   OUTPUT OUT=DESIGN;
   EXAMINE ALIASING CONFOUNDING;
 RUN;
```

27

Using SAS-FACTEX to get a 1/8 Replicate of a 2^7 Design ran in Two Blocks of size 8.

Factor Confounding Rules
E = B*C*D
F = A*C*D
G = A*B*D

28

Using SAS-FACTEX to get a 1/8 Replicate of a 2^7 Design ran in Two Blocks of size 8.

Block Pseudo-factor Confounding Rules
[B1] = A*B*C*D

29

Using SAS-FACTEX to get a 1/8 Replicate of a 2^7 Design ran in Two Blocks of size 8.

Aliasing Structure
A
B
C
D
E
F
G
A*B = D*G = E*F
A*C = D*F = E*G
A*D = B*G = C*F
[B] = A*E = B*F = C*G
A*F = B*E = C*D
A*G = B*D = C*E
B*C = D*E = F*G

30

Using SAS-FACTEX to get a 1/8 Replicate of a 2^7 Design ran in Two Blocks of size 8.

Obs	BLOCK	A	B	C	D	E	F	G
1	1	-1	-1	-1	1	1	1	1
2	1	-1	-1	1	-1	1	1	-1
3	1	-1	1	-1	-1	1	-1	1
4	1	-1	1	1	1	1	-1	-1
5	1	1	-1	-1	-1	-1	1	1
6	1	1	-1	1	1	-1	1	-1
7	1	1	1	-1	1	-1	-1	1
8	1	1	1	1	-1	-1	-1	-1
9	2	-1	-1	-1	-1	-1	-1	-1
10	2	-1	-1	1	1	-1	-1	1
11	2	-1	1	-1	1	-1	1	-1
12	2	-1	1	1	-1	-1	1	1
13	2	1	-1	-1	1	1	-1	-1
14	2	1	-1	1	-1	1	-1	1
15	2	1	1	-1	-1	1	1	-1
16	2	1	1	1	1	1	1	1

31

Using SAS-FACTEX to get a 1/8 Replicate of a 2^9 Experiment that has Resolution V.

```
PROC FACTEX;
FACTORS A B C D E F G H J;
MODEL RES=5; SIZE FRACTION=8;
  OUTPUT OUT=DESIGN;
  EXAMINE ALIASING CONFOUNDING;
RUN;
```

32

SAS LOG

```
PROC FACTEX;
FACTORS A B C D E F G H J;
MODEL RES=5;
 SIZE FRACTION=8;
  OUTPUT OUT=DESIGN;
  EXAMINE ALIASING CONFOUNDING;
RUN;

ERROR: No such design exists.
ERROR: No design to output.
NOTE: The data set WORK.DESIGN has 0 observations
and 0 variables.
```

33

Using SAS-FACTEX to get a 1/16 Fraction of a 2^9 in four Blocks of size 8.

```
PROC FACTEX;
FACTORS A B C D E F G H J;
MODEL RES=4; SIZE FRACTION=16;
 BLOCKS NBLOCKS=4;
  OUTPUT OUT=DESIGN;
  EXAMINE ALIASING CONFOUNDING;
RUN;
```

34

Using SAS-FACTEX to get a 1/16 Fraction of a 2^9 in four Blocks of size 8.

```
PROC FACTEX;
FACTORS A B C D E F G H J;
MODEL RES=4; SIZE FRACTION=16;
 BLOCKS NBLOCKS=4;
  OUTPUT OUT=DESIGN;
  EXAMINE ALIASING CONFOUNDING;
RUN;
```

35

Using SAS-FACTEX to get a 1/16 Fraction of a 2^9 in four Blocks of size 8.

Factor Confounding Rules
F = A*B*C*D*E
G = C*D*E
H = B*D*E
J = A*D*E

36

Using SAS-FACTEX to get a 1/16 Fraction of a 2^9 in four Blocks of size 8.

Block Pseudo-factor Confounding Rules
[B1] = B*C*D*E
[B2] = A*C*D*E

37

Using SAS-FACTEX to get a 1/16 Fraction of a 2^9 in four Blocks of size 8.

Aliasing Structure
A
B
C
D
E
F
G
H
J
[B] = A*B = F*G = H*J
A*C = F*H = G*J
A*D = E*J
A*E = D*J
[B] = A*F = B*G = C*H
[B] = A*G = B*F = C*J
A*H = B*J = C*F
A*J = B*H = C*G = D*E
B*C = F*J = G*H
B*D = E*H
B*E = D*H
C*D = E*G
C*E = D*G
D*F
E*F

38

```
PROC PRINT DATA=DESIGN;
 RUN;
```

39

Obs	BLOCK	A	B	C	D	E	F	G	H	J
1	1	-1	-1	-1	-1	1	1	1	1	1
2	1	-1	-1	-1	1	-1	1	1	1	1
3	1	-1	-1	1	-1	-1	1	1	-1	-1
4	1	-1	-1	1	1	1	1	1	-1	-1
5	1	1	1	-1	-1	-1	-1	-1	1	1
6	1	1	1	-1	1	1	-1	-1	1	1
7	1	1	1	1	-1	1	-1	-1	-1	-1
8	1	1	1	1	1	-1	-1	-1	-1	-1

40

Obs	BLOCK	A	B	C	D	E	F	G	H	J
9	2	-1	1	-1	-1	1	-1	1	-1	1
10	2	-1	1	-1	1	-1	-1	1	-1	1
11	2	-1	1	1	-1	-1	-1	1	1	-1
12	2	-1	1	1	1	1	-1	1	1	-1
13	2	1	-1	-1	-1	-1	1	-1	-1	1
14	2	1	-1	-1	1	1	1	-1	-1	1
15	2	1	-1	1	-1	1	1	-1	1	-1
16	2	1	-1	1	1	-1	1	-1	1	-1

41

Obs	BLOCK	A	B	C	D	E	F	G	H	J
17	3	-1	1	-1	-1	-1	1	-1	1	-1
18	3	-1	1	-1	1	1	1	-1	1	-1
19	3	-1	1	1	-1	1	1	-1	-1	1
20	3	-1	1	1	1	-1	1	-1	-1	1
21	3	1	-1	-1	-1	1	-1	1	1	-1
22	3	1	-1	-1	1	-1	-1	1	1	-1
23	3	1	-1	1	-1	-1	-1	1	-1	1
24	3	1	-1	1	1	1	-1	1	-1	1

42

Obs	BLOCK	A	B	C	D	E	F	G	H	J
25	4	-1	-1	-1	-1	-1	-1	-1	-1	-1
26	4	-1	-1	-1	1	1	-1	-1	-1	-1
27	4	-1	-1	1	-1	1	-1	-1	1	1
28	4	-1	-1	1	1	-1	-1	-1	1	1
29	4	1	1	-1	-1	1	1	1	-1	-1
30	4	1	1	-1	1	-1	1	1	-1	-1
31	4	1	1	1	-1	-1	1	1	1	1
32	4	1	1	1	1	1	1	1	1	1

43

If you have not yet completed Assignment 6, you should do so now.

44

Selecting Additional Runs to Resolve Ambiguities (Tricks)

Suppose we run a Resolution III design, and find that a main effect factor that has one or more two-factor aliases appears to be statistically significant. If further runs were to be made, it would be good if they could be chosen so that the main effect factor no longer has the two-factor interaction effects as aliases.

45

Selecting Additional Runs to Resolve Ambiguities

This can be accomplished by adding a second fraction where all of the levels of the significant factor are switched.

46

Selecting Additional Runs to Resolve Ambiguities

Example: Suppose we perform 2^{6-3} design of resolution III where the treatment combinations are selected by using the defining effects *I=ABD, I=ACE,* and *I=BCF*. The generalized interactions of these defining effects are: *BCDE, ACDF, ABEF,* and *DEF.* So the *A* main effect has the two-factor effects *BD* and *CE* as aliases.

47

Suppose we use +'s and –'s to identify the factor levels in our 1/8 replicate of a 2^6 factorial experiment. And WOLG suppose the 1/8 replicate we selected satisfies:

***ABD*=+, *ACE*=+, and *BCF*=+ .**

If we ran the fraction where the signs of factor *A* were all changed, then this fraction contains the treatment combinations where *ABD* = - , *ACE* = -, and *BCF* =+ .

48

Note that in the first fraction *BD* = *CE,* and in the second fraction *BD* = *CE.* Equivalently, we could say that the defining effects in the two fractions together are:

$$I = BCDE \text{ and } I = BCF.$$

49

Since the defining effects in the two fractions together are:

$$I = BCDE \text{ and } I = BCF.$$

The generalized interaction of the two effects is: *DEF.*

Consequently, *A* is not aliased with any two factor interactions.

50

ABD **is still aliased with *ACE,* and these two effects along with their other two aliases, namely, *ACDF* and *ABEF* are confounded with blocks.**

51

Remark: One can also add a second fraction that will de-alias all main effects from two-factor interactions by switching signs in all of the columns of the design.

52

To illustrate, consider the 1/8 fraction of a 2^6 given by:

$$ABD = +,\ ACE = +, \text{ and } BCF = + .$$

Suppose we run a second fraction where the signs of all factors are switched. This implies that the second fraction can be generated by

$$ABD = - ,\ ACE = -, \text{ and } BCF = - .$$

53

That is,

$ABD = +$, $ACE = +$, and $BCF = +$ and

$ABD = -$, $ACE = -$, and $BCF = -$.

Now look at the generalized interactions of the defining effects. We see that:

54

We see that:

$(ABD)(ACE) = BCDE = +$ in both fractions,

$(ABD)(BCF) = ACDF = +$ in both fractions,

$(ACE)(BCF) = ABEF = +$ in both fractions.

$(ABD)(ACE)(BCF) = DEF = +$ in fraction 1 and $= -$ in fraction 2.

55

Therefore, the ¼ fraction of the 2^6 is defined by

$BCDE = +$ and $ACDF = +$ and the blocks are defined by DEF.

The generalized interaction of the two defining effects is $ABEF$.

Therefore, no main effect is confounded with a two-factor interaction.

The aliases of DEF are BCF, ACE, & ABD and these are confounded with blocks.

56

Resolution III Designs

Resolution III designs can also be easily constructed when the number of factors is equal to $2^k – 1$ for some *k*. To do so we can write out all of the effect contrasts for a full 2^k design for *k* factors. When finished, then one can randomly assign one of the factors to each of the $2^k – 1$ columns using +1's to represent the high levels of each factor, and –1's to represent the low levels.

57

Example: Find a Resolution III Design for 7 Factors. Note that $7 = 2^3 - 1$

A	*B*	*C*
-	-	-
+	-	-
-	+	-
+	+	-
-	-	+
+	-	+
-	+	+
+	+	+

58

Example: Find a Resolution III Design for 7 Factors.

A	B	C	A*B	A*C	B*C	A*B*C
-	-	-	+	+	+	-
+	-	-	-	-	+	+
-	+	-	-	+	-	+
+	+	-	+	-	-	-
-	-	+	+	-	-	+
+	-	+	-	+	-	-
-	+	+	-	-	+	-
+	+	+	+	+	+	+

59

Resolution III Design for 7 factors

F	B	A	D	G	C	E
-	-	-	+	+	+	-
+	-	-	-	-	+	+
-	+	-	-	+	-	+
+	+	-	+	-	-	-
-	-	+	+	-	-	+
+	-	+	-	+	-	-
-	+	+	-	-	+	-
+	+	+	+	+	+	+

60

Remark: Resolution III designs formed in this manner for experiments with $2^k - 1$ factors are called *saturated* designs as all of the available effect contrasts of 2^k variables is associated with a factor.

61

Remark: Saturated fractional designs with N runs (rows) have the following property: If we take any two columns, then corresponding to the $N/2$ plus signs in the first column, there will be $N/4$ plus signs in the second column and $N/4$ minus signs in the second column. Similarly, for all of the $N/2$ minus signs in the first column. Provided that all interactions are negligible, designs with this property allow unbiased estimation of all main effects among $N - 1$ factors assigned to the columns.

62

The designs described above require that $N = 2^k$ for some k.

Plackett and Burman obtained arrangements with the above property whenever N is a multiple of 4. For example, their design for $k = 11$ factors in $N = 12$ runs is shown on the next slide.

63

A Plackett-Burman Design for 11 Factors

H	F	J	A	E	B	G	L	D	K	C
+	-	+	-	-	-	+	+	+	-	+
+	+	-	+	-	-	-	+	+	+	-
-	+	+	-	+	-	-	-	+	+	+
+	-	+	+	-	+	-	-	-	+	+
+	+	-	+	+	-	+	-	-	-	+
+	+	+	-	+	+	-	+	-	-	-
-	+	+	+	-	+	+	-	+	-	-
-	-	+	+	+	-	+	+	-	+	-
-	-	-	+	+	+	-	+	+	-	+
+	-	-	-	+	+	+	-	+	+	-
-	+	-	-	-	+	+	+	-	+	+
-	-	-	-	-	-	-	-	-	-	-

64

Plackett & Burman developed design generators that can be used to create Resolution III designs. Their designs require 12, 16, 20, 24 or 32 test runs. It is recommended that the design used should have at least 6 more test runs than there are factors in the experiment. Their design generators are given in a Word file on the course website.

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Plackett and Burman Design Generators

Factor Number	Number of Test Runs: 12	16	20	24	32	Factor Number
1	+	+	+	+	-	1
2	+	-	+	+	-	2
3	-	-	-	+	-	3
4	+	-	-	+	-	4
5	+	+	+	+	+	5
6	+	-	+	-	-	6
7	-	-	+	+	+	7
8	-	+	+	-	-	8
9	-	+	-	+	+	9
10	+	-	+	+	+	10
11	-	+	-	-	+	11
12		-	+	-	-	12
13		+	-	+	+	13
14		+	-	+	+	14
15		+	-	-	-	15
16			-	-	-	16
17			+	+	-	17
17			+	-	+	17
19			-	+	+	19
20				-	+	20
21				-	+	21
22				-	+	22
23				-	-	23
24					-	24
25					+	25
26					+	26
27					-	27
28					+	28
29					-	29
30					-	30
31					+	31

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Factor Number	12	16	20	24	32	Factor Number
1	+	+	+	+	-	1
2	+	-	+	+	-	2
3	-	-	-	+	-	3
4	+	-	-	+	-	4
5	+	+	+	+	+	5
6	+	-	+	-	-	6
7	-	-	+	+	+	7
8	-	+	+	-	-	8
9	-	+	-	+	+	9
10	+	-	+	+	+	10
11	-	+	-	-	+	11
12		-	+	-	-	12
13		+	-	+	+	13
14		+	-	+	+	14
15		+	-	-	-	15
16			-	-	-	16
17			+	+	-	17
17			+	-	+	17
19			-	+	+	19
20				-	+	20

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Create a Plackett-Burman Design for 10 factors in 16 runs.

Step 1: Select the design generator for a 16 run design. This is used for the first column of the design.

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Create a Plackett-Burman Design for 10 factors in 16 runs.

Step 2: Create a second column by using the same design generator but beginning in the 2nd row with the last element moved to the top of the column.

Step 3: Create a third column by using the same design generator but beginning in the 3rd row and with the last two elements moved to the top of the column.

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Create a Plackett-Burman Design for 10 factors in 16 runs.

Steps 4-15: Similar to Steps 3 and 4 where elements are shifted by one in each column.

Step 16: A row of minuses is added to the last row

Final Step: The ten factors are randomly assigned to 10 of the 15 columns in the design table created.

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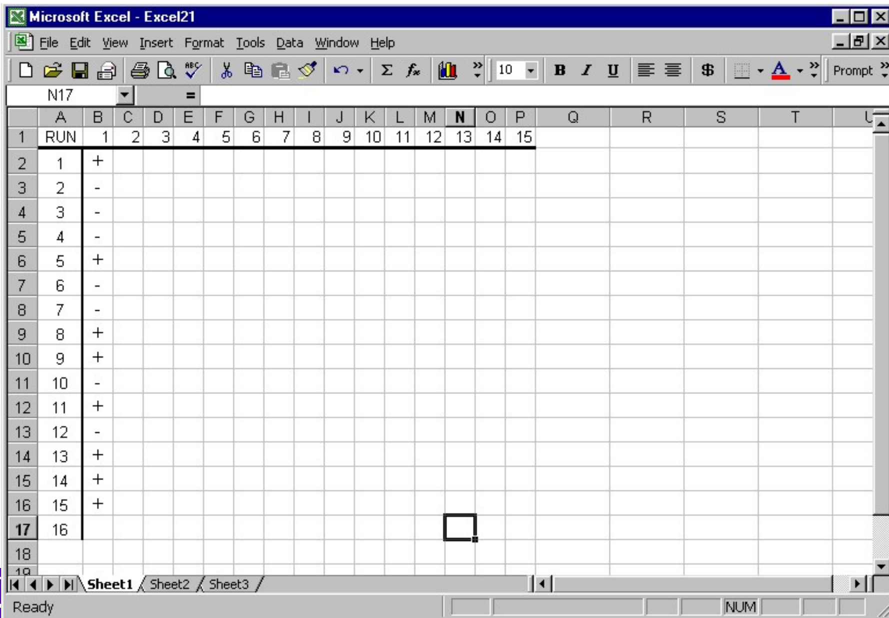
Step 1.



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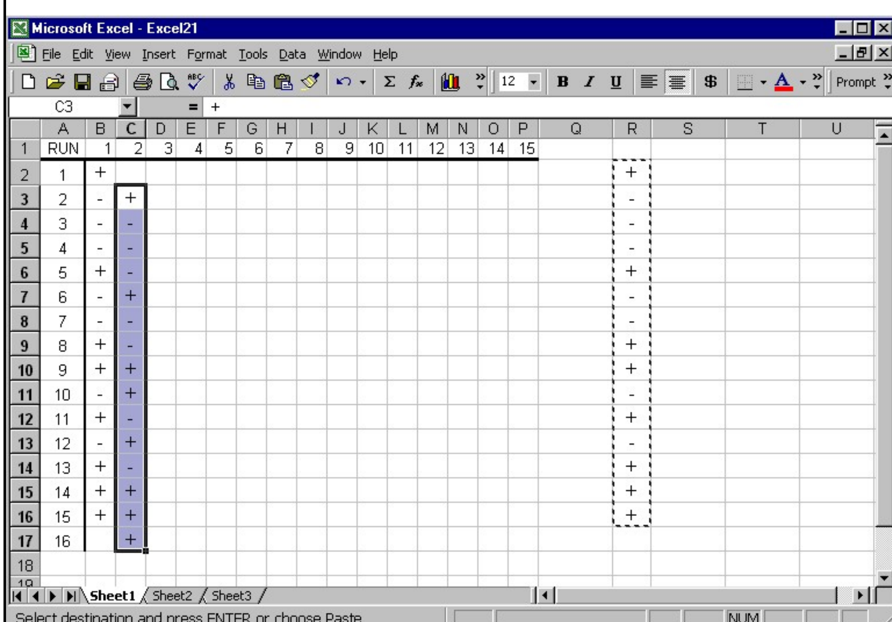
Step 2.



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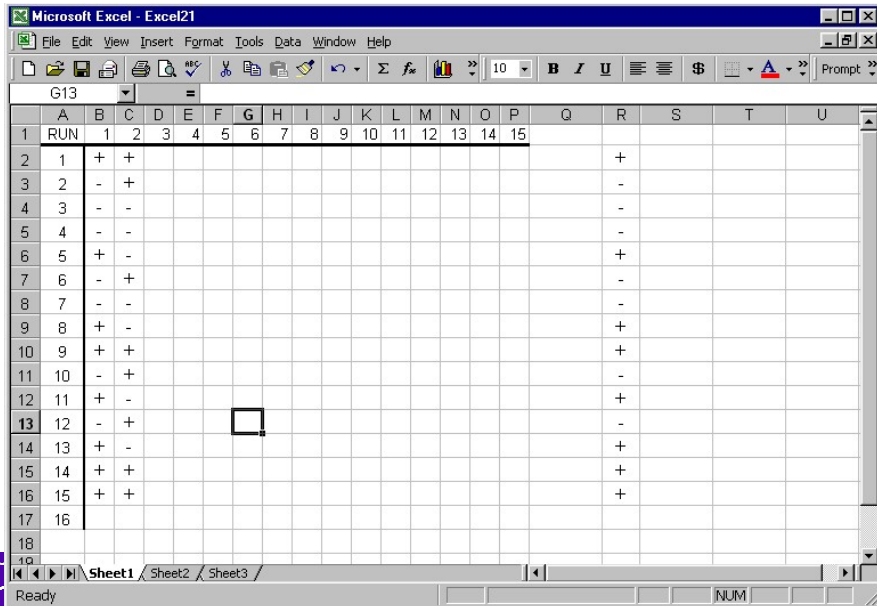
Step 2.



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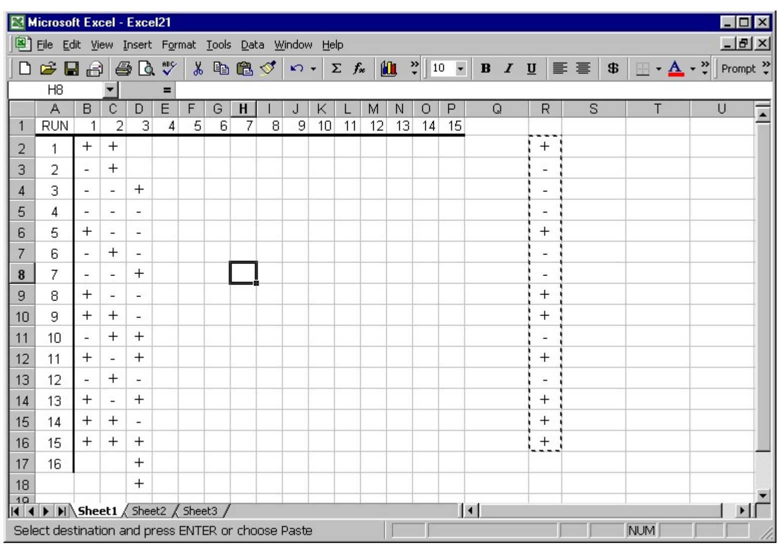
Step 3.



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Step 3.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1	RUN	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15		
2	1	+	+	+														+
3	2	-	+	+														-
4	3	-	-	+														-
5	4	-	-	-														-
6	5	+	-	-														+
7	6	-	+	-														-
8	7	-	-	+														-
9	8	+	-	-														+
10	9	+	+	-														+
11	10	-	+	+														-
12	11	+	-	+														+
13	12	-	+	-														-
14	13	+	-	+														+
15	14	+	+	-														+
16	15	+	+	+														+
17	16																	

Steps 4-15.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1	RUN	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15		
2	1	+	+	+														+
3	2	-	+	+														-
4	3	-	-	+														-
5	4	-	-	-	+													-
6	5	+	-	-	-	+												+
7	6	-	+	-	-	-	+											-
8	7	-	-	+	-	-	-	+										-
9	8	+	-	-	+	-	-	-	+									+
10	9	+	+	-	-	+	-	-	-	+								+
11	10	-	+	+	-	-	+	-	-	-	+							-
12	11	+	-	+	+	-	-	+	-	-	-	+						+
13	12	-	+	-	+	+	-	-	+	-	-	-	+					-
14	13	+	-	+	-	+	+	-	-	+	-	-	-	+				+
15	14	+	+	-	+	-	+	+	-	-	+	-	-	-	+			+
16	15	+	+	+	-	+	-	+	+	-	-	+	-	-	-	+		+
17	16				+	-	+	-	+	+	-	-	+	-	-	-		
18					+	+	-	+	-	+	+	-	-	+	-	-		

Steps 4-15.

Microsoft Excel - Excel21

RUN	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	+	+	+	+	-	+	-	+	+	-	-	+	-	-	-
2	-	+	+	+	+	-	+	-	+	+	-	-	+	-	-
3	-	-	+	+	+	+	-	+	-	+	+	-	-	+	-
4	-	-	-	+	+	+	+	-	+	-	+	+	-	-	+
5	+	-	-	-	+	+	+	+	-	+	-	+	+	-	-
6	-	+	-	-	-	+	+	+	+	-	+	-	+	+	-
7	-	-	+	-	-	-	+	+	+	+	-	+	-	+	+
8	+	-	-	+	-	-	-	+	+	+	+	-	+	-	+
9	+	+	-	-	+	-	-	-	+	+	+	+	-	+	-
10	-	+	+	-	-	+	-	-	-	+	+	+	+	-	+
11	+	-	+	+	-	-	+	-	-	-	+	+	+	+	-
12	-	+	-	+	+	-	-	+	-	-	-	+	+	+	+
13	+	-	+	-	+	+	-	-	+	-	-	-	+	+	+
14	+	+	-	+	-	+	+	-	-	+	-	-	-	+	+
15	+	+	+	-	+	-	+	+	-	-	+	-	-	-	+
16															

Ready

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Step 16.

Microsoft Excel - Excel21

RUN	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	+	+	+	+	-	+	-	+	+	-	-	+	-	-	-
2	-	+	+	+	+	-	+	-	+	+	-	-	+	-	-
3	-	-	+	+	+	+	-	+	-	+	+	-	-	+	-
4	-	-	-	+	+	+	+	-	+	-	+	+	-	-	+
5	+	-	-	-	+	+	+	+	-	+	-	+	+	-	-
6	-	+	-	-	-	+	+	+	+	-	+	-	+	+	-
7	-	-	+	-	-	-	+	+	+	+	-	+	-	+	+
8	+	-	-	+	-	-	-	+	+	+	+	-	+	-	+
9	+	+	-	-	+	-	-	-	+	+	+	+	-	+	-
10	-	+	+	-	-	+	-	-	-	+	+	+	+	-	+
11	+	-	+	+	-	-	+	-	-	-	+	+	+	+	-
12	-	+	-	+	+	-	-	+	-	-	-	+	+	+	+
13	+	-	+	-	+	+	-	-	+	-	-	-	+	+	+
14	+	+	-	+	-	+	+	-	-	+	-	-	-	+	+
15	+	+	+	-	+	-	+	+	-	-	+	-	-	-	+
16	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Select destination and press ENTER or choose Paste

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RUN	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	+	+	+	+	-	+	-	+	+	-	-	+	-	-	-
2	-	+	+	+	+	-	+	-	+	+	-	-	+	-	-
3	-	-	+	+	+	+	-	+	-	+	+	-	-	+	-
4	-	-	-	+	+	+	+	-	+	-	+	+	-	-	+
5	+	-	-	-	+	+	+	+	-	+	-	+	+	-	-
6	-	+	-	-	-	+	+	+	+	-	+	-	+	+	-
7	-	-	+	-	-	-	+	+	+	+	-	+	-	+	+
8	+	-	-	+	-	-	-	+	+	+	+	-	+	-	+
9	+	+	-	-	+	-	-	-	+	+	+	+	-	+	-
10	-	+	+	-	-	+	-	-	-	+	+	+	+	-	+
11	+	-	+	+	-	-	+	-	-	-	+	+	+	+	-
12	-	+	-	+	+	-	-	+	-	-	-	+	+	+	+
13	+	-	+	-	+	+	-	-	+	-	-	-	+	+	+
14	+	+	-	+	-	+	+	-	-	+	-	-	-	+	+
15	+	+	+	-	+	-	+	+	-	-	+	-	-	-	+
16	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

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RUN	J	C	3	A	5	G	7	B	H	10	D	K	F	14	E
1	+	+	+	+	-	+	-	+	+	-	-	+	-	-	-
2	-	+	+	+	+	-	+	-	+	+	-	-	+	-	-
3	-	-	+	+	+	+	-	+	-	+	+	-	-	+	-
4	-	-	-	+	+	+	+	-	+	-	+	+	-	-	+
5	+	-	-	-	+	+	+	+	-	+	-	+	+	-	-
6	-	+	-	-	-	+	+	+	+	-	+	-	+	+	-
7	-	-	+	-	-	-	+	+	+	+	-	+	-	+	+
8	+	-	-	+	-	-	-	+	+	+	+	-	+	-	+
9	+	+	-	-	+	-	-	-	+	+	+	+	-	+	-
10	-	+	+	-	-	+	-	-	-	+	+	+	+	-	+
11	+	-	+	+	-	-	+	-	-	-	+	+	+	+	-
12	-	+	-	+	+	-	-	+	-	-	-	+	+	+	+
13	+	-	+	-	+	+	-	-	+	-	-	-	+	+	+
14	+	+	-	+	-	+	+	-	-	+	-	-	-	+	+
15	+	+	+	-	+	-	+	+	-	-	+	-	-	-	+
16	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

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RUN	J	C	A	G	B	H	D	K	F	E
1	+	+	+	+	+	+	-	+	-	-
2	-	+	+	-	-	+	-	-	+	-
3	-	-	+	+	+	-	+	-	-	-
4	-	-	+	+	-	+	+	+	-	+
5	+	-	-	+	+	-	-	+	+	-
6	-	+	-	+	+	+	+	-	+	-
7	-	-	-	-	+	+	-	+	-	+
8	+	-	+	-	+	+	+	-	+	+
9	+	+	-	-	-	+	+	+	-	-
10	-	+	-	+	-	-	+	+	+	+
11	+	-	+	-	-	-	+	+	+	-
12	-	+	+	-	+	-	-	+	+	+
13	+	-	-	+	-	+	-	-	+	+
14	+	+	+	+	-	-	-	-	-	+
15	+	+	-	-	+	-	+	-	-	+
16	-	-	-	-	-	-	-	-	-	-

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A 16 run Plackett-Burman Design for 10 factors.

RUN	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*J*	*K*
1	+	+	+	-	-	-	+	+	+	+
2	+	-	+	-	-	+	-	+	-	-
3	+	+	-	+	-	-	+	-	-	-
4	+	-	-	+	+	-	+	+	-	+
5	-	+	-	-	-	+	+	-	+	+
6	-	+	+	+	-	+	+	+	-	-
7	-	+	-	-	+	-	-	+	-	+
8	+	+	-	+	+	+	-	+	+	-
9	-	-	+	+	-	-	-	+	+	+
10	-	-	+	+	+	+	+	-	-	+
11	+	-	-	+	-	+	-	-	+	+
12	+	+	+	-	+	+	-	-	-	+
13	-	-	-	-	+	+	+	+	+	-
14	+	-	+	-	+	-	+	-	+	-
15	-	+	+	+	+	-	-	-	+	-
16	-	-	-	-	-	-	-	-	-	-

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Remark: Resolution III designs are often called *Screening Designs.* They are often utilized with the hope that one can find a subset of a large number of factors that are going to affect a process the most. They try to use as few a number of runs that it is possible to use. Additional experiments will need to be conducted using the factors identified to be among those that affect the process the most.

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Statistical Analysis of a Resolution III design

The easiest way to analyze a Resolution III design is to simply run an ANOVA that consists of the main effects only. The so-called error from such an analysis contains information about many of the interactions between the main effect factors, and consequently, the resulting F-tests may not be very powerful.

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Statistical Analysis of a Resolution III design

One can also perform a half-normal plot of the square roots of the main effect mean squares with the hopes that the half-normal plot will identify the most important factors.

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You can now do Assignments 7 and 8.

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