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MATH-310
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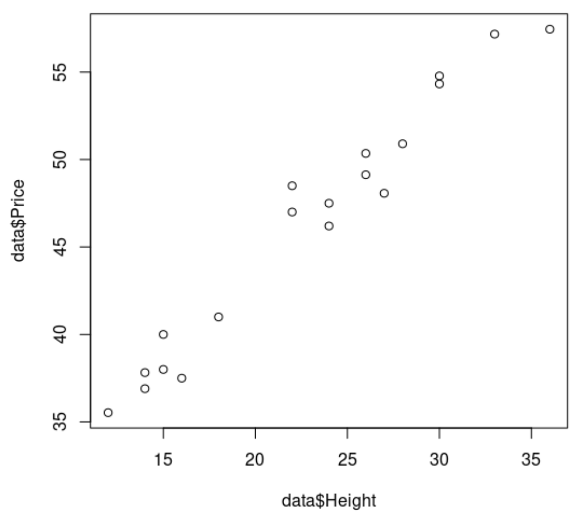
EXAM 2

1. The following are the results of running an ANOVA on the three factors (temperature, fabric denier, and air pressure) at each of their respective three levels.

2. (a)
   (b) The scatter plot indicates a positive, linear relatonship between height and price.



   (c) The equation of least squares is $y = 0.98715x + 23.77215$.
   (d) At a height of 27, the corresponding price is \$50.43. Given the summary below, the corresponding residual is the result of the least squares (50.4252) plus the median residual (-0.8796), which is 49.55.
   (e) In referring to the above figure, one will see that the adjusted R-squared is 0.96. So, 96% of the variation in sales price can be attributed to the approximate linear relationship between truss height and price.

3. (a) Below is the result of the following line of code, given the appropriate datset, *data*:

$$model \leftarrow lm(NO3\ CO, data = data);\ summary(model)$$

   (b) The equation produced by the linear regression in R was

$$\hat{y} = 13.63x + 21.97$$

   So, the prediction for $x = 400$ is 5473.97. Given that the mean of CO is 211.89, the following work shows that the prediction interval is $(-139.5264, 563)$.

$$\hat{y} = mean(data\$CO);\ t = 0.627;\ error = 21.734;\ r^2 = deviance(model)$$

```
Call:
lm(formula = Price ~ Height, data = data)

Residuals:
     Min       1Q   Median       3Q      Max 
-2.35522 -0.63584 -0.08796  0.92263  3.01053 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 23.77215    1.11347   21.35 1.03e-13 ***
Height       0.98715    0.04684   21.07 1.27e-13 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.416 on 17 degrees of freedom
Multiple R-squared:  0.9631,	Adjusted R-squared:  0.961 
F-statistic: 444.1 on 1 and 17 DF,  p-value: 1.271e-13
```

Figure 1: Summary of linear model.

```
Call:
lm(formula = NO3 ~ CO, data = data)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.5398 -0.7906  0.2801  0.9139  2.5317 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.220414   0.991656  -0.222     0.83    
CO           0.043620   0.003429  12.719  4.3e-06 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 2.024 on 7 degrees of freedom
Multiple R-squared:  0.9585,	Adjusted R-squared:  0.9526 
F-statistic: 161.8 on 1 and 7 DF,  p-value: 4.298e-06
```

Figure 2: Model of NO3 and CO data.

(c) Without the largest value of CO, the fitted equation is $\hat{y} = 15.21x + 21.70$, yielding 6105.7 as the prediction: a number far outside the previous interval. So, the largest CO value has a substantial affect on the model.

4. Below I show that $SSE = S_{yy} - \hat{\beta}_1 S_{xy}$.

$$\begin{aligned}
SSE &= \sum y_i^2 - \hat{\beta}_0 \sum y_i - \hat{\beta}_1 \sum x_i y_i \\
&= \sum y_i^2 - (\bar{y} - \hat{\beta}_1 \bar{x}) \sum y_i - \hat{\beta}_1 \sum x_i y_i \\
&= \sum y_i^2 - \bar{y} \sum y_i + \hat{\beta}_1 \bar{x} \sum y_i - \hat{\beta}_1 \sum x_i y_i \\
&= \sum y_i^2 - \frac{1}{n} \sum y_i \sum y_i + \hat{\beta}_1 (\sum \bar{x} y_i - \sum x_i y_i) \\
&= \sum y_i^2 - \frac{1}{n} (\sum y_i)^2 - \hat{\beta}_1 ((\sum x_i y_i) - \sum \bar{x} y_i) \\
&= S_{yy} - \hat{\beta}_1 ((\sum x_i y_i) - \sum \bar{x} y_i) \\
&= S_{yy} - \hat{\beta}_1 S_{xy}
\end{aligned}$$