Alexander Caines
MATH 301
1-30-2021

HMWK 1

1. (a) Since the calculated t-statistic, 4.81, is less than the calculated critical value of 7.96, the null hypothesis—that the concentration in the sampled region exceeds the stated background value—is rejected. The result surprises me as the concentration, 45.31 is much greater than the background, 20. However, because the sample size of the population is only 3, an inaccurate result from the above test can be expected.

Work

```
[1] n = 3; x̄ = 45.31; ste=5.26; μ0 = 20
[2] t = (x̄ − μ0)/(ste)
[3] 1-qt(0.01, 2)
```

2. (a) Despite the fact that the distribution is not normal (the proposed mean is nowhere near the median of the range provided), because there are sufficiently many entries in the population, the central limit theorem allows us to test the hypothesis about the value of the population mean consumption.

   (b) Upon running a t-test, under the following hypothesis

$$H_0 : \mu < 200, H_a : \mu > 200$$

did not fall under the critical region. So the null is rejected, implying that the mean consumption was not at most 200mg.

Work

```
[1] n = 47; x̄ = 215; s = 235; μ0 = 200
[2] t = (x̄ − μ0)/(s/√n) # test statistic
[3] qt(0.01, 46, lower.tail=FALSE) # t critical value
```

3. No, the data does not suggest that the condition has not been met as the average is within the critical region.

Work

```
[1] data <- read.csv('HW1prob3.csv')[,1]
[2] n = length(data)
[3] x̄ = mean(data)
[4] μ0 = 9.75
[5] std = sd(data)
[6] qt(0.1, n, lower.tail=FALSE)
```

4. (a) The relevant test statistic in this case would be testing for proportion. Consider the following hypothesis:

$$H_0 : \hat{p} = p_0$$

$$H_a : p_0 < \frac{1}{75} \text{ or } -\frac{1}{75} < p_0$$

Since the pvalue is greater than the significance level (0.05), it cannot be concluded that the incidence rate of the defect among prisoners differs from the presumed rate for the entire male population. A Type 2 Error could have been made when arriving at the conclusion.

Work

[1] $\hat{p} = 16/800$; $p_0 = 1/75$; n = 800
[2] z = $(\hat{p} - p_0)$/($\sqrt{p_0 \cdot (1-p_0)/n}$)
[3] qnorm(0.975) # compare to z score
[4] pvalue=2*(1-pnorm(1.75))# greater than significance level 0.05

(b) The pvalue calculated came out to be 0.074, which is greater than 0.05. So, it would have been rejected at a significance level of 0.2.

Work

```
[1] 2*(1-pnorm(1.74)) > 0.05 # Returns TRUE
```

5. Since the calculated zscore was 0.74 and is less than the calculated critical value (1.65), we fail to reject the null under an upper tail test at significance level 0.05. So, it cannot be concluded that more than 10% of the population has abstained from alcohol use.

Work

[1] $\hat{p} = 51/462$; $p_0 = 0.1$; n=462
[2] z = $(\hat{p} - p_0)$/($\sqrt{p_0 \cdot (1-p_0)/n}$)
[3] qnorm(0.95)