

Radiation Physics

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Chapter 1

Introduction

1.1 Definitions

Consider an infinitesimal amount of energy dE which is the energy at location \mathbf{x} and time t with frequencies in the infinitesimal range $d\nu$ about the frequency ν and flowing in the direction of the solid angle $d\Omega$ about the vector Ω and passing through an infinitesimal area dA with unit normal \mathbf{A} . We express this energy in terms of a distribution $I_\nu = I_\nu(\mathbf{x}, t, \nu, \Omega)$ as follows

$$dE = I_\nu(\Omega \cdot \mathbf{A}) dt d\nu d\Omega dA. \quad (1.1)$$

I_ν is referred to as the spectral radiance, or spectral specific intensity. Various additional radiation quantities can be defined in terms of I_ν , as shown in table 1.1.

Any quantity dependent on ν can be integrated over all frequencies to obtain a total value. For example, for the spectral radiance/spectral specific intensity, we have

$$I = \int_0^\infty I_\nu d\nu. \quad (1.2)$$

In the above, $I = I(\mathbf{x}, t, \Omega)$ is the radiance, or specific intensity.

	spectral	
	definition	units
Spectral radiance / Spectral specific intensity	I_ν	$\left[\frac{\text{J}}{\text{s} \cdot \text{m}^2 \cdot \text{sr} \cdot \text{Hz}} \right]$
0 th moment	$J_\nu = \frac{1}{4\pi} \int_{4\pi} I_\nu d\Omega$	$\left[\frac{\text{J}}{\text{s} \cdot \text{m}^2 \cdot \text{Hz}} \right]$
1 st moment	$\mathbf{H}_\nu = \frac{1}{4\pi} \int_{4\pi} I_\nu \boldsymbol{\Omega} d\Omega$ $= \frac{\mathbf{F}_\nu}{4\pi}$	$\left[\frac{\text{J}}{\text{s} \cdot \text{m}^2 \cdot \text{Hz}} \right]$
2 nd moment	$\mathbf{K}_\nu = \frac{1}{4\pi} \int_{4\pi} I_\nu \boldsymbol{\Omega} \boldsymbol{\Omega} d\Omega$ $= \frac{c}{4\pi} \mathbf{P}_\nu$	$\left[\frac{\text{J}}{\text{s} \cdot \text{m}^2 \cdot \text{Hz}} \right]$
Spectral radiant energy density	$E_\nu = \frac{1}{c} \int_{4\pi} I_\nu d\Omega$ $= \frac{4\pi}{c} J_\nu$	$\left[\frac{\text{J}}{\text{m}^3 \cdot \text{Hz}} \right]$
One-sided spectral radiant energy flux	$S_\nu^{\mathbf{A}} = \int_{\boldsymbol{\Omega} \cdot \mathbf{A} > 0} I_\nu (\boldsymbol{\Omega} \cdot \mathbf{A}) d\Omega$	$\left[\frac{\text{J}}{\text{s} \cdot \text{m}^2 \cdot \text{Hz}} \right]$

Table 1.1: Radiation quantities. In the above \mathbf{F}_ν is the radiation flux and \mathbf{P}_ν the radiation pressure tensor.

Chapter 2

Thermal radiation

For blackbody radiation we have

$$I_\nu = \frac{2h\nu^3}{c^2} \frac{1}{\exp(h\nu/kT) - 1}. \quad (2.1)$$

Consider the identity

$$\int_0^\infty \frac{x^3}{\exp(yx) - 1} dx = \frac{1}{15} \left(\frac{\pi}{y} \right)^4. \quad (2.2)$$

Using the above to integrate over all frequencies, we get

$$I = \frac{2h}{c^2} \frac{1}{15} \left(\frac{\pi kT}{h} \right)^4. \quad (2.3)$$

Defining the Stefan-Boltzmann constant as

$$\sigma = \frac{2\pi^5 k^4}{15c^2 h^3}, \quad (2.4)$$

we have

$$I = \frac{1}{\pi} \sigma T^4. \quad (2.5)$$

In this case I_ν is isotropic, that is, it is independent of the direction $\mathbf{\Omega}$. Thus $J_\nu = I_\nu$ and therefore $E_\nu = (4\pi/c)I_\nu$. Integrating over all frequencies leads to $E = (4\pi/c)I$.

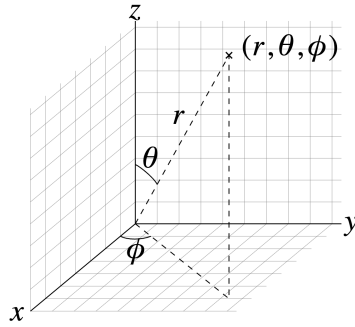


Figure 2.1: Spherical coordinates from Wikipedia.

For the one-sided spectral radiant energy flux, we make reference to the diagram for spherical coordinates in fig. 2.1. Let's assume $\mathbf{A} = \mathbf{z}$ without loss of generality. Then, we have

$$S_\nu^{\hat{\mathbf{z}}} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I_\nu \cos \theta d\Omega = I_\nu \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \cos \theta \sin \theta d\theta d\phi = \pi I_\nu. \quad (2.6)$$

	total	spectral
Radiance / Specific intensity	$I = \frac{1}{\pi}\sigma T^4$	$I_\nu = \frac{2h\nu^3}{c^2} \frac{1}{\exp(h\nu/kT) - 1}$
Radiant energy density	$E = \frac{4}{c}\sigma T^4$	$E_\nu = \frac{8\pi h\nu^3}{c^3} \frac{1}{\exp(h\nu/kT) - 1}$
One-sided radiant energy flux	$S^{\hat{\mathbf{z}}} = \sigma T^4$	$S_\nu^{\hat{\mathbf{z}}} = \frac{2\pi h\nu^3}{c^2} \frac{1}{\exp(h\nu/kT) - 1}$

Table 2.1: Radiation quantities for a blackbody spectrum

Similarly as before, integrating over all frequencies leads to $S^{\hat{\mathbf{z}}} = \pi I$. These and the other relations derived above are shown in table 2.1.