Radiation Physics

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Contents

1 Basic definitions

	definition	units
Spectral radiance / Spectral specific intensity	I_ν	$\left[\frac{\mathrm{J}}{\mathrm{s{\cdot}m^2{\cdot}sr{\cdot}Hz}}\right]$
0^{th} moment	$J_\nu = \dfrac{1}{4\pi}\displaystyle\int_{4\pi} I_\nu \, d\Omega$	$\left[\frac{\mathrm{J}}{\mathrm{s{\cdot}m^2{\cdot}Hz}}\right]$
1^{st} moment	$\mathbf{H}_\nu = \dfrac{1}{4\pi}\displaystyle\int_{4\pi} I_\nu \boldsymbol{\Omega} \, d\Omega = \dfrac{\mathbf{F}_\nu}{4\pi}$	$\left[\frac{\mathrm{J}}{\mathrm{s{\cdot}m^2{\cdot}Hz}}\right]$
2^{nd} moment	$\mathbf{K}_\nu = \dfrac{1}{4\pi}\displaystyle\int_{4\pi} I_\nu \boldsymbol{\Omega}\boldsymbol{\Omega} \, d\Omega = \dfrac{c}{4\pi}\mathbf{P}_\nu$	$\left[\frac{\mathrm{J}}{\mathrm{s{\cdot}m^2{\cdot}Hz}}\right]$
Spectral radiant energy density	$E_\nu = \dfrac{1}{c}\displaystyle\int_{4\pi} I_\nu \, d\Omega = \dfrac{4\pi}{c}J_\nu$	$\left[\frac{\mathrm{J}}{\mathrm{m^3{\cdot}Hz}}\right]$
One-sided spectral radiant energy flux	$S_\nu^{\mathbf{A}} = \displaystyle\int_{\boldsymbol{\Omega}\cdot\mathbf{A}>0} I_\nu \boldsymbol{\Omega}\cdot\mathbf{A} \, d\Omega$	$\left[\frac{\mathrm{J}}{\mathrm{s{\cdot}m^2{\cdot}Hz}}\right]$

Table 1: Radiation quantities. In the above $\mathbf{F}_\nu$ is the radiation flux and $\mathbf{P}_\nu$ the radiation pressure tensor.

Consider an infinitesimal amount of energy dE_ν which is the energy at location $\mathbf{x}$ and time t with frequencies in the infinitesimal range $d\nu$ about the frequency ν and flowing in the direction

	total	spectral
Radiance / Specific intensity / 0^{th} moment	$I = J = \frac{1}{\pi}\sigma T^4$	$I_\nu = J_\nu = \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/kT) - 1}$
Radiant energy density	$E = \frac{4}{c}\sigma T^4$	$E_\nu = \frac{8\pi h\nu^3}{c^3}\frac{1}{\exp(h\nu/kT) - 1}$
One-sided radiant energy flux	$S^{\hat{\mathbf{z}}} = \sigma T^4$	$S^{\hat{\mathbf{z}}}_\nu = \frac{2\pi h\nu^3}{c^2}\frac{1}{\exp(h\nu/kT) - 1}$

Table 2: Radiation quantities for a blackbody spectrum

of the solid angle $d\mathbf{\Omega}$ about the unit vector $\mathbf{\Omega}$ and passing through an infinitesimal area $d\mathbf{A}$ with unit normal $\mathbf{A}$. We express this energy in terms of a distribution $I_\nu = I_\nu(\mathbf{x}, t, \nu, \mathbf{\Omega})$ as follows

$$dE_\nu = I_\nu dt d\nu d\Omega dA(\mathbf{\Omega} \cdot \mathbf{A}). \tag{1}$$

I_ν is referred to as the spectral radiance, or spectral specific intensity.

Any quantity dependent on ν can be integrated over all frequencies to obtain a total value. For example, for the spectral radiance/spectral specific intensity, we have

$$I = \int_0^\infty I_\nu \, d\nu. \tag{2}$$

In the above, $I = I(\mathbf{x}, t, \mathbf{\Omega})$ is the radiance, or specific intensity.

Various additional radiation quantities can be defined in terms of I_ν, as shown in table 1.

2 Blackbody radiation

For blackbody radiation we have

$$I_\nu = \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/k_B T) - 1}. \tag{3}$$

Consider the identity

$$\int_0^\infty \frac{x^3}{\exp(yx) - 1}\, dx = \frac{1}{15}\left(\frac{\pi}{y}\right)^4. \tag{4}$$

Using the above to integrate over all frequencies, we get

$$I = \frac{2h}{c^2}\frac{1}{15}\left(\frac{\pi k_B T}{h}\right)^4. \tag{5}$$

Defining the Stefan-Boltzmann constant as

$$\sigma = \frac{2\pi^5 k_B^4}{15c^2h^3} = 5.67037 \times 10^{-8}\left[\frac{\mathrm{W}}{\mathrm{m}^2\mathrm{K}^4}\right], \tag{6}$$

we have

$$I = \frac{1}{\pi}\sigma T^4. \tag{7}$$

For blackbody radiation I_ν is isotropic, that is, it is independent of the direction $\mathbf{\Omega}$. Thus $J_\nu = I_\nu$ and $J = I$. This then leads to $E_\nu = (4\pi/c)I_\nu$ and $E = (4\pi/c)I$.

For the one-sided spectral radiant energy flux, we make reference to the diagram for spherical coordinates in fig. 1. Let's assume $\mathbf{A} = \mathbf{z}$ without loss of generality. Then, we have

$$S_\nu^{\hat{\mathbf{z}}} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I_\nu \cos\theta \, d\Omega = I_\nu \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \cos\theta \sin\theta \, d\theta d\phi = \pi I_\nu. \tag{8}$$

Similarly as before, integrating over all frequencies leads to $S^{\hat{\mathbf{z}}} = \pi I$.

The above relationships and others are shown in table 2.

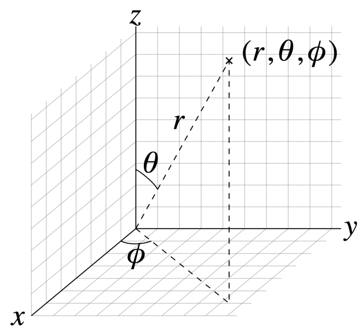

Figure 1: Spherical coordinates from Wikipedia.

3 Radiation hydrodynamics

3.1 Transport

3.2 Diffusion

Conservation equations

$$\frac{\partial \rho Y_\alpha}{\partial t} + \frac{\partial \rho Y_\alpha u_i}{\partial x_i} = -\frac{\partial J_{\alpha,i}}{\partial x_i} + w_\alpha$$

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = \frac{\partial}{\partial x_j} \sum_\alpha \eta_\alpha \sigma_{\alpha,ij} - \frac{\partial}{\partial x_i} \sum_g \frac{E_g}{3} + \rho f_i$$

$$\frac{\partial \rho Y_\alpha e_\alpha}{\partial t} + \frac{\partial \rho Y_\alpha e_\alpha u_j}{\partial x_j} = \eta_\alpha \sigma_{\alpha,ij} \frac{\partial u_i}{\partial x_j} - \bar{p}\alpha_\alpha - c\eta_\alpha \sum_g \sigma_{p,g,\alpha} \left[B_g(T_\alpha) - E_g\right] - \frac{\partial q_{\alpha,j}}{\partial x_j}.$$

$$\frac{\partial E_g}{\partial t} + \frac{\partial E_g u_j}{\partial x_j} + \frac{\partial F_{g,i}}{\partial x_j} = \sum_\alpha c\eta_\alpha \sigma_{p,g,\alpha} \left[B_g(T_\alpha) - E_g\right] - \frac{E_g}{3} \frac{\partial u_j}{\partial x_j}$$

Transport models

$$t_{\alpha,ij} = 2\mu_\alpha S^*_{ij}$$

$$q_{\alpha,i} = -\kappa_\alpha \frac{\partial T_\alpha}{\partial x_i} + \sum_\alpha h_\alpha J_{\alpha,i}$$

$$J_{\alpha,i} = -\rho \left(D_\alpha \frac{\partial Y_\alpha}{\partial x_i} - Y_\alpha \sum_\beta D_\beta \frac{\partial Y_\beta}{\partial x_i} \right)$$

Transport coefficients

$$\mu_\alpha = \mu_{0,\alpha}\left(\frac{T_\alpha}{T_{\alpha,0}}\right)^n$$

$$\kappa_\alpha = \frac{\mu_\alpha C_{p,\alpha}}{Pr_\alpha}$$

$$D_\alpha = \frac{\mu_\alpha}{\rho Sc_\alpha}$$

Equations of state

$$p_\alpha = \phi_\alpha(\rho_\alpha, T_\alpha),$$

$$e_\alpha = \psi_\alpha(\rho_\alpha, T_\alpha).$$

Multi-material models

$$\frac{\partial \eta_\alpha}{\partial t} + u_j \frac{\partial \eta_\alpha}{\partial x_j} = \alpha_\alpha. \tag{9}$$

$$\alpha_\alpha = f(\dots)$$

$$\bar{p} = g(\dots)$$

Additional relations

$$\sigma_{\alpha,ij} = -p_\alpha \delta_{ij} + t_{\alpha,ij}$$

$$S^*_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{1}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}$$

$$h_\alpha = e_\alpha + \frac{p}{\rho_\alpha}$$

$$1 = \sum_\alpha Y_\alpha$$

$$\rho_\alpha = \rho \frac{Y_\alpha}{\eta_\alpha}$$