Radiation

Alejandro Campos

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1 Basic definitions

	definition	units
Spectral radiance / Spectral specific intensity	I_ν	$\left[\frac{\mathrm{J}}{\mathrm{s{\cdot}m^2{\cdot}sr{\cdot}Hz}}\right]$
0^{th} moment	$J_\nu = \frac{1}{4\pi}\int_{4\pi} I_\nu \, d\Omega$	$\left[\frac{\mathrm{J}}{\mathrm{s{\cdot}m^2{\cdot}Hz}}\right]$
1^{st} moment	$\mathbf{H}_\nu = \frac{1}{4\pi}\int_{4\pi} I_\nu \boldsymbol{\Omega} \, d\Omega = \frac{\mathbf{F}_\nu}{4\pi}$	$\left[\frac{\mathrm{J}}{\mathrm{s{\cdot}m^2{\cdot}Hz}}\right]$
2^{nd} moment	$\mathbf{K}_\nu = \frac{1}{4\pi}\int_{4\pi} I_\nu \boldsymbol{\Omega}\boldsymbol{\Omega} \, d\Omega = \frac{c}{4\pi}\mathbf{P}_\nu$	$\left[\frac{\mathrm{J}}{\mathrm{s{\cdot}m^2{\cdot}Hz}}\right]$
Spectral radiant energy density	$E_\nu = \frac{1}{c}\int_{4\pi} I_\nu \, d\Omega = \frac{4\pi}{c} J_\nu$	$\left[\frac{\mathrm{J}}{\mathrm{m^3{\cdot}Hz}}\right]$
One-sided spectral radiant energy flux	$S_\nu^{\mathbf{A}} = \int_{\boldsymbol{\Omega}\cdot\mathbf{A}>0} I_\nu \boldsymbol{\Omega}\cdot\mathbf{A} \, d\Omega$	$\left[\frac{\mathrm{J}}{\mathrm{s{\cdot}m^2{\cdot}Hz}}\right]$

Table 1: Radiation quantities. In the above $\mathbf{F}_\nu$ is the radiation flux and $\mathbf{P}_\nu$ the radiation pressure tensor.

Consider an infinitesimal amount of energy dE_ν which is the energy at location $\mathbf{x}$ and time t with frequencies in the infinitesimal range $d\nu$ about the frequency ν and flowing in the direction of the solid angle $d\boldsymbol{\Omega}$ about the unit vector $\boldsymbol{\Omega}$ and passing through an infinitesimal area $d\mathbf{A}$ with unit normal $\mathbf{A}$. We express this energy in terms of a distribution $I_\nu = I_\nu(\mathbf{x}, t, \nu, \boldsymbol{\Omega})$ as follows

$$dE_\nu = I_\nu dt d\nu d\Omega dA(\boldsymbol{\Omega}\cdot\mathbf{A}). \tag{1}$$

	total	spectral
Radiance / Specific intensity / 0^{th} moment	$I = J = \frac{1}{\pi}\sigma T^4$	$I_\nu = J_\nu = \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/kT) - 1}$
Radiant energy density	$E = \frac{4}{c}\sigma T^4$	$E_\nu = \frac{8\pi h\nu^3}{c^3}\frac{1}{\exp(h\nu/kT) - 1}$
One-sided radiant energy flux	$S^{\hat{\mathbf{z}}} = \sigma T^4$	$S^{\hat{\mathbf{z}}}_\nu = \frac{2\pi h\nu^3}{c^2}\frac{1}{\exp(h\nu/kT) - 1}$

Table 2: Radiation quantities for a blackbody spectrum

I_ν is referred to as the spectral radiance, or spectral specific intensity.

Any quantity dependent on ν can be integrated over all frequencies to obtain a total value. For example, for the spectral radiance/spectral specific intensity, we have

$$I = \int_0^\infty I_\nu \, d\nu. \tag{2}$$

In the above, $I = I(\mathbf{x}, t, \mathbf{\Omega})$ is the radiance, or specific intensity.

Various additional radiation quantities can be defined in terms of I_ν, as shown in table 1.

2 Blackbody radiation

For blackbody radiation we have

$$I_\nu = \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/k_BT) - 1}. \tag{3}$$

Consider the identity

$$\int_0^\infty \frac{x^3}{\exp(yx) - 1}\, dx = \frac{1}{15}\left(\frac{\pi}{y}\right)^4. \tag{4}$$

Using the above to integrate over all frequencies, we get

$$I = \frac{2h}{c^2}\frac{1}{15}\left(\frac{\pi k_B T}{h}\right)^4. \tag{5}$$

Defining the Stefan-Boltzmann constant as

$$\sigma = \frac{2\pi^5 k_B^4}{15c^2h^3} = 5.67037 \times 10^{-8}\left[\frac{\mathrm{W}}{\mathrm{m}^2\mathrm{K}^4}\right], \tag{6}$$

we have

$$I = \frac{1}{\pi}\sigma T^4. \tag{7}$$

For blackbody radiation I_ν is isotropic, that is, it is independent of the direction $\mathbf{\Omega}$. Thus $J_\nu = I_\nu$ and $J = I$. This then leads to $E_\nu = (4\pi/c)I_\nu$ and $E = (4\pi/c)I$.

For the one-sided spectral radiant energy flux, we make reference to the diagram for spherical coordinates in fig. 1. Let's assume $\mathbf{A} = \mathbf{z}$ without loss of generality. Then, we have

$$S^{\hat{\mathbf{z}}}_\nu = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} I_\nu \cos\theta \, d\Omega = I_\nu \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} \cos\theta \sin\theta \, d\theta d\phi = \pi I_\nu. \tag{8}$$

Similarly as before, integrating over all frequencies leads to $S^{\hat{\mathbf{z}}} = \pi I$.

The above relationships and others are shown in table 2.

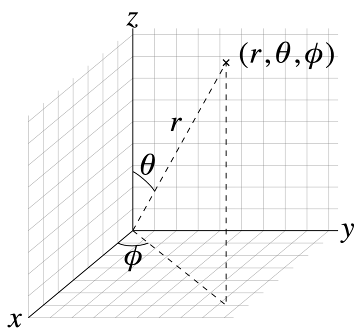

Figure 1: Spherical coordinates from Wikipedia.