

Radiation Transport

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1 Basic definitions

	definition	units
Spectral radiance / Spectral specific intensity	I_ν	$\left[\frac{\text{J}}{\text{s} \cdot \text{m}^2 \cdot \text{sr} \cdot \text{Hz}} \right]$
0 th moment	$J_\nu = \frac{1}{4\pi} \int_{4\pi} I_\nu d\Omega$	$\left[\frac{\text{J}}{\text{s} \cdot \text{m}^2 \cdot \text{Hz}} \right]$
1 st moment	$\mathbf{H}_\nu = \frac{1}{4\pi} \int_{4\pi} I_\nu \boldsymbol{\Omega} d\Omega$ $= \frac{\mathbf{F}_\nu}{4\pi}$	$\left[\frac{\text{J}}{\text{s} \cdot \text{m}^2 \cdot \text{Hz}} \right]$
2 nd moment	$\mathbf{K}_\nu = \frac{1}{4\pi} \int_{4\pi} I_\nu \boldsymbol{\Omega} \boldsymbol{\Omega} d\Omega$ $= \frac{c}{4\pi} \mathbf{P}_\nu$	$\left[\frac{\text{J}}{\text{s} \cdot \text{m}^2 \cdot \text{Hz}} \right]$
Spectral radiant energy density	$E_\nu = \frac{1}{c} \int_{4\pi} I_\nu d\Omega$ $= \frac{4\pi}{c} J_\nu$	$\left[\frac{\text{J}}{\text{m}^3 \cdot \text{Hz}} \right]$
One-sided spectral radiant energy flux	$S_\nu^{\mathbf{A}} = \int_{\boldsymbol{\Omega} \cdot \mathbf{A} > 0} I_\nu \boldsymbol{\Omega} \cdot \mathbf{A} d\Omega$	$\left[\frac{\text{J}}{\text{s} \cdot \text{m}^2 \cdot \text{Hz}} \right]$

Table 1: Radiation quantities. In the above \mathbf{F}_ν is the radiation flux and \mathbf{P}_ν the radiation pressure tensor.

Consider an infinitesimal amount of energy dE_ν which is the energy at location \mathbf{x} and time t with frequencies in the infinitesimal range $d\nu$ about the frequency ν and flowing in the direction of the solid angle $d\boldsymbol{\Omega}$ about the unit vector $\boldsymbol{\Omega}$ and passing through an infinitesimal area $d\mathbf{A}$ with unit normal \mathbf{A} . We express this energy in terms of a distribution $I_\nu = I_\nu(\mathbf{x}, t, \nu, \boldsymbol{\Omega})$ as follows

$$dE_\nu = I_\nu dt d\nu d\Omega dA (\boldsymbol{\Omega} \cdot \mathbf{A}). \quad (1)$$

	total	spectral
Radiance / Specific intensity / 0 th moment	$I = J = \frac{1}{\pi}\sigma T^4$	$I_\nu = J_\nu = \frac{2h\nu^3}{c^2} \frac{1}{\exp(h\nu/kT) - 1}$
Radiant energy density	$E = \frac{4}{c}\sigma T^4$	$E_\nu = \frac{8\pi h\nu^3}{c^3} \frac{1}{\exp(h\nu/kT) - 1}$
One-sided radiant energy flux	$S^{\hat{\mathbf{z}}} = \sigma T^4$	$S_\nu^{\hat{\mathbf{z}}} = \frac{2\pi h\nu^3}{c^2} \frac{1}{\exp(h\nu/kT) - 1}$

Table 2: Radiation quantities for a blackbody spectrum

I_ν is referred to as the spectral radiance, or spectral specific intensity.

Any quantity dependent on ν can be integrated over all frequencies to obtain a total value. For example, for the spectral radiance/spectral specific intensity, we have

$$I = \int_0^\infty I_\nu d\nu. \quad (2)$$

In the above, $I = I(\mathbf{x}, t, \mathbf{\Omega})$ is the radiance, or specific intensity.

Various additional radiation quantities can be defined in terms of I_ν , as shown in table 1.

2 Blackbody radiation

For blackbody radiation we have

$$I_\nu = \frac{2h\nu^3}{c^2} \frac{1}{\exp(h\nu/k_B T) - 1}. \quad (3)$$

Consider the identity

$$\int_0^\infty \frac{x^3}{\exp(yx) - 1} dx = \frac{1}{15} \left(\frac{\pi}{y} \right)^4. \quad (4)$$

Using the above to integrate over all frequencies, we get

$$I = \frac{2h}{c^2} \frac{1}{15} \left(\frac{\pi k_B T}{h} \right)^4. \quad (5)$$

Defining the Stefan-Boltzmann constant as

$$\sigma = \frac{2\pi^5 k_B^4}{15c^2 h^3} = 5.67037 \times 10^{-8} \left[\frac{\text{W}}{\text{m}^2 \text{K}^4} \right], \quad (6)$$

we have

$$I = \frac{1}{\pi} \sigma T^4. \quad (7)$$

For blackbody radiation I_ν is isotropic, that is, it is independent of the direction $\mathbf{\Omega}$. Thus $J_\nu = I_\nu$ and $J = I$. This then leads to $E_\nu = (4\pi/c)I_\nu$ and $E = (4\pi/c)I$.

For the one-sided spectral radiant energy flux, we make reference to the diagram for spherical coordinates in fig. 1. Let's assume $\mathbf{A} = \mathbf{z}$ without loss of generality. Then, we have

$$S_\nu^{\hat{\mathbf{z}}} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I_\nu \cos \theta d\Omega = I_\nu \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \cos \theta \sin \theta d\theta d\phi = \pi I_\nu. \quad (8)$$

Similarly as before, integrating over all frequencies leads to $S^{\hat{\mathbf{z}}} = \pi I$.

The above relationships and others are shown in table 2.

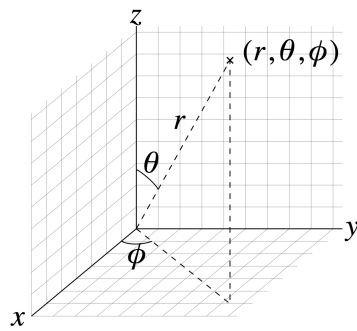


Figure 1: Spherical coordinates from Wikipedia.