Alec Portelli

Module 2 Homework

(1)

$$R_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$ switch R_2 & R_3 $\to R_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ $R_1 = R_1 \cdot -1$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

The rank of R_1 is $\underline{3}$ because in row echelon form it has 3 non zero rows

$$R_2 = \begin{bmatrix} 0 & 2 & 1 \\ 2 & 6 & 2 \\ 3 & 9 & 3 \end{bmatrix}$$ $R_3 = R_3/3$ $\begin{bmatrix} 0 & 2 & 1 \\ 2 & 6 & 2 \\ 1 & 3 & 1 \end{bmatrix}$ $R_2 = R_2/2 \to \begin{bmatrix} 0 & 2 & 1 \\ 1 & 3 & 2 \\ 1 & 3 & 1 \end{bmatrix}$ $R_2 = R_2 + R_3 \cdot -1$ $\to$

$\begin{bmatrix} 0 & 2 & 1 \\ 0 & 0 & 1 \\ 1 & 3 & 1 \end{bmatrix}$ $R_1 = R_1/2$ $\begin{bmatrix} 0 & 1 & 1/2 \\ 0 & 0 & 1 \\ 1 & 3 & 1 \end{bmatrix}$ Switch R_2 and R_1 $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1/2 \\ 1 & 3 & 1 \end{bmatrix}$ Switch R_1 and R_3

$\begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 1/2 \\ 0 & 0 & 1 \end{bmatrix}$ The rank of R_2 is $\underline{3}$ because in row echelon form it has 3 non zero rows

$$R_3 = \begin{bmatrix} 5 & 2.5 & 7.5 \\ 2.5 & 1 & 3 \end{bmatrix}$$ $R_3 = R_2 + -2R_3$ $\begin{bmatrix} 5 & 2.5 & 7.5 \\ 0 & .5 & 1.5 \end{bmatrix}$ $R_3 = R_3 \cdot 2 = \begin{bmatrix} 5 & 2.5 & 7.5 \\ 0 & 1 & 3 \end{bmatrix}$

The rank of R_3 is $\underline{2}$ because it has 2 non zero rows

(2)

Based on the properties of R listed during the lecture, we can rewrite the matrix as:

$i_0 i_1 = \cos(\theta) = \cos(0) = 1$ $\quad j_0 k_1 = -\sin(\theta)$

$i_0 j_1 = 0$ $\quad k_0 j_1 = \sin(\theta)$

$j_0 j_1 = \cos(\theta)$ $\quad k_0 k_1 = \cos(\theta)$

Therefore the matrix can be rewritten as: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$ for $R_{x,\theta}$

For $R_{y,\theta}$, it is a similar process

$i_0 i_1 = \cos\theta$ $\quad j_0 k_1 = \sin(\theta)$

$i_0 j_1 = 0$ $\quad j_0 j_1 = 1$

$i_0 k_1 = \sin(\theta)$ $\quad k_0 k_1 = -\sin(\theta)$

$k_0 k_1 = \cos(\theta)$

$$R_{y,\theta} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

(3)

R_x needs to be orthogonal and have a determinant of $\underline{1}$

Let's take a look at R_1!

$$R_1 = \begin{bmatrix} .5 & -.6124 & .6124 \\ 0.86 & 0.3536 & -.3536 \\ 0 & .7071 & .7071 \end{bmatrix} \quad R_1^T = \begin{bmatrix} 0.5 & .866 & 0 \\ -.6124 & .3536 & 0.7071 \\ 0.6124 & -.3536 & 0.7071 \end{bmatrix}$$

$$R_1^{-1} = \begin{bmatrix} 1/2 & .866 & 0 \\ -.6123 & .3535 & .7071 \\ 0.6123 & -.3535 & .7071 \end{bmatrix}, \quad R^T = R^{-1},$$ so we now need to check the $\det(R_1)$

$\det(R') = .9998 \approx 1$, so $\underline{\text{YES}}$ rotational matrix

R_2!

$$R_2 = \begin{bmatrix} 0.5 & -0.6124 & 0.6124 \\ 0 & 0.7071 & 0.7071 \\ 0.866 & 0.3536 & -.3536 \end{bmatrix} \quad R_2^T = \begin{bmatrix} 0.5 & 0 & 0.866 \\ -0.6 & 0.7071 & 0.3536 \\ 0.6 & 0.7071 & -0.3536 \end{bmatrix}$$

$$R_2^{-1} = \begin{bmatrix} 0.5 & 0 & .866 \\ -0.61 & .707 & .3535 \\ 0.61 & 0.707 & -.3535 \end{bmatrix} \quad R^T = R^{-1},$$ so we check determinant

$\det(R_2) = -1$, so $\underline{\text{NOT}}$ Rotational

R_3!

$$R_3 = \begin{bmatrix} 0.5 & -.6124 & .6124 \\ 0 & 0.7071 & 0.7071 \\ 0.866 & 0.3536 & 0.3536 \end{bmatrix} \quad R_3^T = \begin{bmatrix} 0.5 & 0 & 0.866 \\ -0.6214 & 0.7071 & 0.3536 \\ 0.6214 & 0.7071 & 0.3536 \end{bmatrix}$$

$$R_3^{-1} = \begin{bmatrix} 0 & -0.577 & 1.1547 \\ -0.81 & 0.47 & 0.47 \\ 0.81 & 0.14 & -0.47 \end{bmatrix} \quad R_3^T \neq R_3^{-1},$$ so $\underline{\text{NOT}}$ rotational