Adrian Lechuga Diaz
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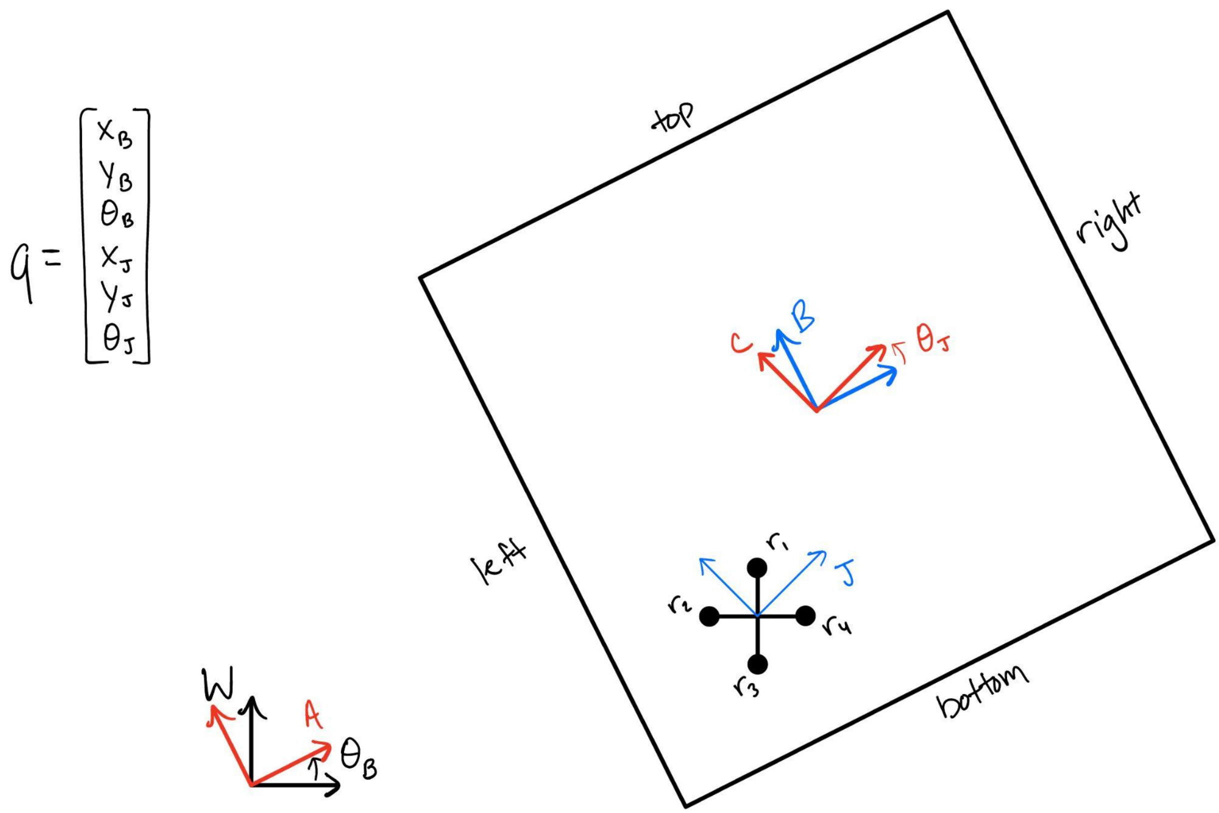
ME 314 Final Project

1. Summary

For the final project, I chose to do the default project of animating a jack-in-a-box. This jack-in-a-box animation is a 2D model, where a jack is placed inside a box and shaken around by an external force. This makes for a system where there are 16 different possible impacts. To note, this model assumes that the impacts don't occur simultaneously and gravity points downwards.

2. System

A drawing of the system I am modeling is shown below. It includes all the frames I am using with labels. Below that, all the rigid body transformations between the frames are shown as well.



As can be seen in the drawing, I defined the configuration variables to be:

$$q = [x_B, y_B, \theta_B, x_J, y_J, \theta_J]$$

The frames are as follows (in order):

W. This is the world frame, and is that I use to reference all the frames to.

A. Frame A is defined to be a counter-clockwise rotation by the angle θ_B .

B. Frame B is defined to be a translation by x_C and y_C .

C. Frame C is defined to be a counter-clockwise rotation by the angle θ_J .

J. Frame J is defined to be a translation by x_J and y_J .

3. Methods

To start, I first defined the frames listed above by relating them back to the world frame in the form of SE(3) transformations. They are the following:

$$g_{WA} = se3(0, 0, \theta_B)$$
$$g_{AB} = se3(x_B, y_B, 0)$$
$$g_{BC} = se3(0, 0, \theta_J)$$
$$g_{CJ} = se3(x_J, y_J, 0)$$

$$g_{WB} = g_{WA} \cdot g_{AB}$$
$$g_{WJ} = g_{WB} \cdot g_{BC} \cdot g_{CJ}$$

$$g_{BJ} = g_{WB}^{-1} \cdot g_{WJ}$$

To note, the final SE(3) transformation was to be able to project the endpoints of the jack onto Frame B, which is the frame where the box walls will be defined for impact.

Next, I calculated the body velocity, kinetic energy, and potential energy for both the jack and the box, where m_B is the mass of the box, m_J is the mass of the jack, g is gravity, and the I's are the inertia tensors for each:

$$V_{WB}^{b} = (g_{WB}^{-1} \cdot \dot{g}_{WB})^{\vee}$$
$$V_{WJ}^{b} = (g_{WJ}^{-1} \cdot \dot{g}_{WJ})^{\vee}$$
$$KE_B = \frac{1}{2} \cdot (V_{WB}^{b})^{T} \cdot I_B \cdot V_{WB}^{b}$$
$$KE_J = \frac{1}{2} \cdot (V_{WJ}^{b})^{T} \cdot I_J \cdot V_{WJ}^{b}$$
$$V_B = m_B * g * h_B$$
$$V_J = m_J * g * h_J$$

This allowed me to then calculate the Lagrangian (L = KE - V). Because both bodies are unconstrained in gravity, and to simulate the shaking of a jack within a box, the following external forces were applied

$$F_x = 600 * cos(5t)$$

$$F_y = 1.3 * m_B * g + 3$$

Once I had all of these, I was able to solve for the Euler-Lagrange equations for each of the configuration variables. Finally, the next step was to set up and solve the impact equations, which was the biggest challenge of this project. As stated above, this system has 16 different impact conditions. To begin this step, used the transformation g_{BJ} to define all the coordinates of the box and jack in the same frame, which made it much easier to define all the impact conditions. They all looked very similar, as it was just the x and y coordinates of each vertex of the jack, and either adding or subtracting half the width/length of the box, so that phi_i is equal to 0 when impact occurs. After this, I made a few functions and loops such as a new impact_update, impact_condition, and simulate for this system, similar to a previous homework, as well as a modified animation function. When the code is run, the impact conditions get checked at every t, and if one of them is true, the new state vector is added.

4. **Discussion**

In the end, I believe my code simulated correctly, with a runtime of less than 3 and a half minutes. The animation, which is attached to this submission as a .mp4 file, looks physically possible. If I had a jack and a thin glass box, I feel like it would look similar to this animation (outside of the 3D aspect and impacts in the z-axis). When the jack hits a wall of the box, it immediately bounces in the opposite direction at an accurate angle that makes sense. The jack never goes through the wall, which is very important in making this animation correct. The jack rotates more than the box, which is expected behavior as the jack is a lot smaller. However, the box is also clearly affected whenever impact occurs (although much less than the jack is), which is expected as the jack has a mass that is one-fifth that of the box. The gravity aspect is also noticed, as when the jack has an upward trajectory, it clearly is accelerated downwards, which is not surprising to see and happens in the real world.