SGN – Assignment #2

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Disclaimer: The story plot contained in the following three exercises is entirely fictional.

Exercise 1: Uncertainty propagation

The Prototype Research Instruments and Space Mission Technology Advancement (PRISMA) is a technology in-orbit test-bed mission for demonstrating Formation Flying (FF) and rendezvous technologies, as well as flight testing of new sensors and actuator equipment. It was launched on June 15, 2010, and it involves two satellites: Mango (Satellite 1, ID 36599), the chaser, and Tango (Satellite 2, ID 36827), the target.

You have been provided with an estimate of the states of Satellites 1 and 2 at the separation epoch t_{sep} = `2010-08-12T05:27:39.114` (UTC) in terms of mean and covariance, as reported in Table 1. Assume Keplerian motion can be used to model the spacecraft dynamics.

1. Propagate the initial mean and covariance for both satellites within a time grid going from t_{sep} to $t_{sep} + N\,T_1$, with a step equal to T_1, where T_1 is the orbital period of satellite 1 and $N = 10$, using both a Linearized Approach (LinCov) and the Unscented Transform (UT). We suggest to use $\alpha = 0.1$ and $\beta = 2$ for tuning the UT in this case.

2. Considering that the two satellites are in close formation, you have to guarantee a sufficient accuracy about the knowledge of their state over time to monitor potential risky situations. For this reason, at each revolution, you shall compute:

    - the norm of the relative position (Δr), and
    - the sum of the two covariances associated to the position elements of the states of the two satellites (P_{sum})

    The critical conditions which triggers a collision warning is defined by the following relationship:
    $$\Delta r < 3\sqrt{\max(\lambda_i(P_{\text{sum}}))}$$
    where $\lambda_i(P_{\text{sum}})$ are the eigenvalues of P_{sum}. Identify the revolution N_c at which this condition occurs and elaborate on the results and the differences between the two approaches (UT and LinCov).

3. Perform the same uncertainty propagation process on the same time grid using a Monte Carlo (MC) simulation *. Compute the sample mean and sample covariance and compare them with the estimates obtained at Point 1). Provide the plots of:

    - the time evolution for all three approaches (MC, LinCov, and UT) of $3\sqrt{\max(\lambda_i(P_{r,\text{i}}))}$ and $3\sqrt{\max(\lambda_i(P_{v,\text{i}}))}$, where $i = 1, 2$ is the satellite number and P_r and P_v are the 3x3 position and velocity covariance submatrices.
    - the propagated samples of the MC simulation, together with the mean and covariance obtained with all methods, projected on the orbital plane.

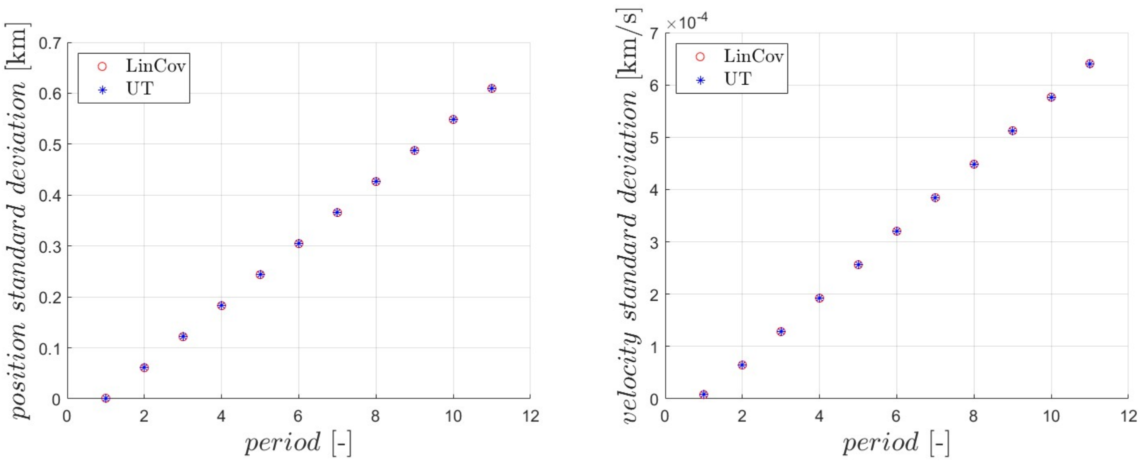
    Compare the results and discuss on the validity of the linear and Guassian assumption for uncertainty propagation.

*Use at least 100 samples drawn from the initial covariance

Table 1: Estimate of Satellite 1 and Satellite 2 states at t_0 provided in ECI J2000.

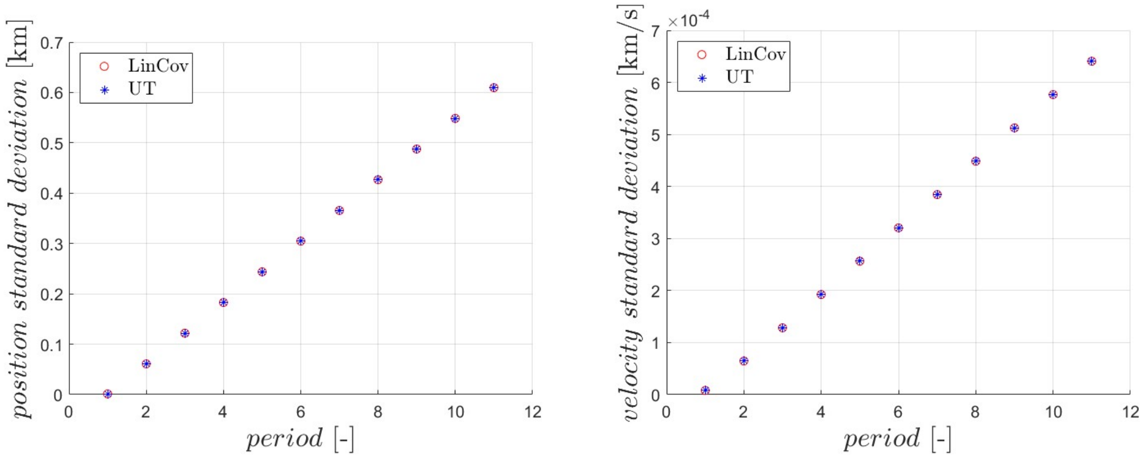
Parameter	Value
Ref. epoch t_{sep} [UTC]	2010-08-12T05:27:39.114
Mean state $\hat{\boldsymbol{x}}_{0,\text{sat1}}$ [km, km/s]	$\hat{\boldsymbol{r}}_{0,\text{sat1}} =$ [4622.232026629, 5399.3369588058, -0.0212138165769957] $\hat{\boldsymbol{v}}_{0,\text{sat1}} =$ [0.812221125483763, -0.721512914578826, 7.42665302729053]
Mean state $\hat{\boldsymbol{x}}_{0,\text{sat2}}$ [km, km/s]	$\hat{\boldsymbol{r}}_{0,\text{sat2}} =$ [4621.69343340281, 5399.26386352847, -3.09039248714313] $\hat{\boldsymbol{v}}_{0,\text{sat2}} =$ [0.813960847513811, -0.719449862738607, 7.42706066911294]
Covariance P_0 [km^2, km^2/s, km^2/s^2]	$\begin{bmatrix} +5.6e-7 & +3.5e-7 & -7.1e-8 & 0 & 0 & 0 \\ +3.5e-7 & +9.7e-7 & +7.6e-8 & 0 & 0 & 0 \\ -7.1e-8 & +7.6e-8 & +8.1e-8 & 0 & 0 & 0 \\ 0 & 0 & 0 & +2.8e-11 & 0 & 0 \\ 0 & 0 & 0 & 0 & +2.7e-11 & 0 \\ 0 & 0 & 0 & 0 & 0 & +9.6e-12 \end{bmatrix}$

Both $LinCov$ and UT are implemented on a single period time (related to Mango, $T \approx 6004$ s), so at every iteration both the initial mean state and covariance are updated from the previous results, starting from the given ones.
The trend of the position and velocity standard deviations for both satellites is shown below:



(a) Position σ (b) Velocity σ

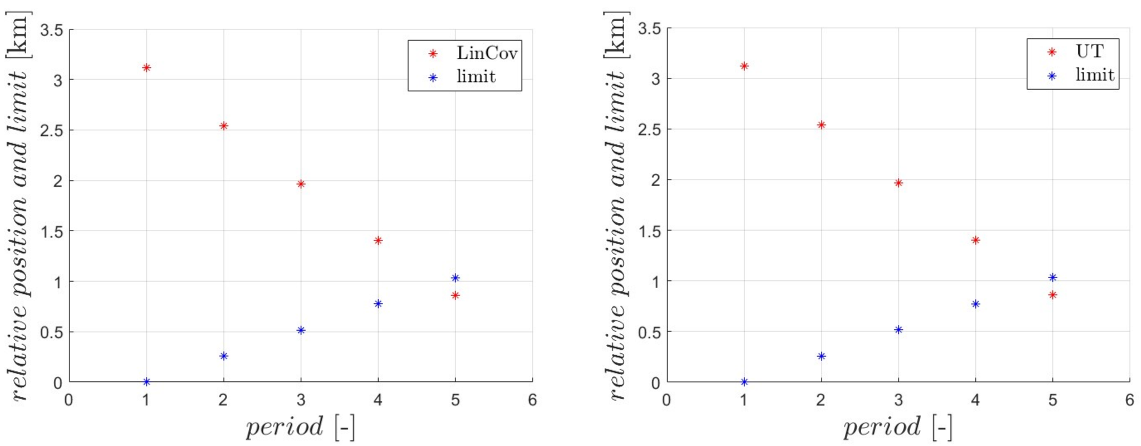
Figure 1: Position and velocity standard deviation for satellite 1.



(a) Position σ **(b)** Velocity σ

Figure 2: Position and velocity standard deviation for satellite 2.

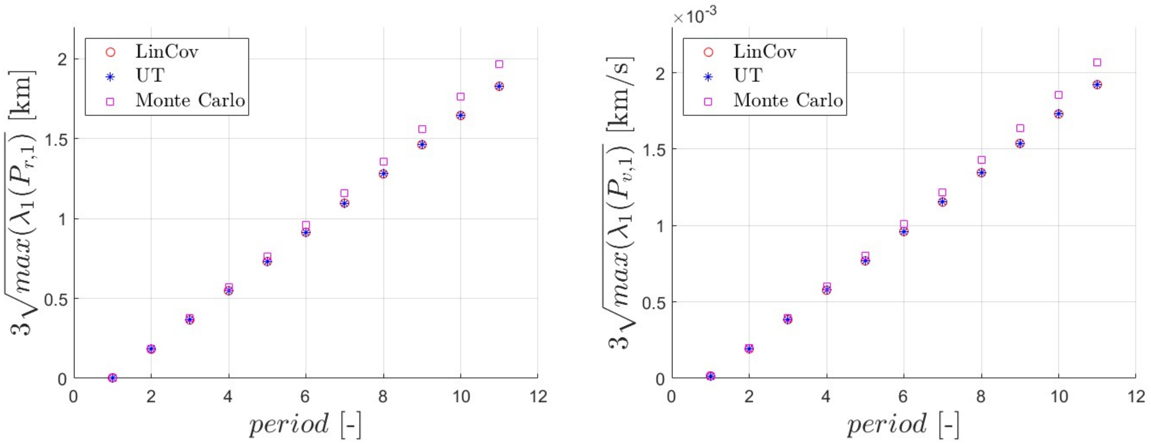
As expected, every standard deviation trend grows over time, as the uncertainty on position and velocity accumulates at every revolution, and both methods deliver almost coincident results: this is caused as the dynamics are not highly nonlinear and the propagation time T is not so long.
Considering this behavior, after a certain time the uncertainty on the position of both satellites has grown so much that the collision warning is issued: this is triggered when the relative position between Mango and Tango is lower than a value directly proportional to the largest eigenvalue of the combined covariance matrix, which corresponding eigenvector indicates the direction of the largest variance of the data. Being the two methods almost overlapping, this happens after 4 revolution for both.



(a) LinCov case **(b)** UT case

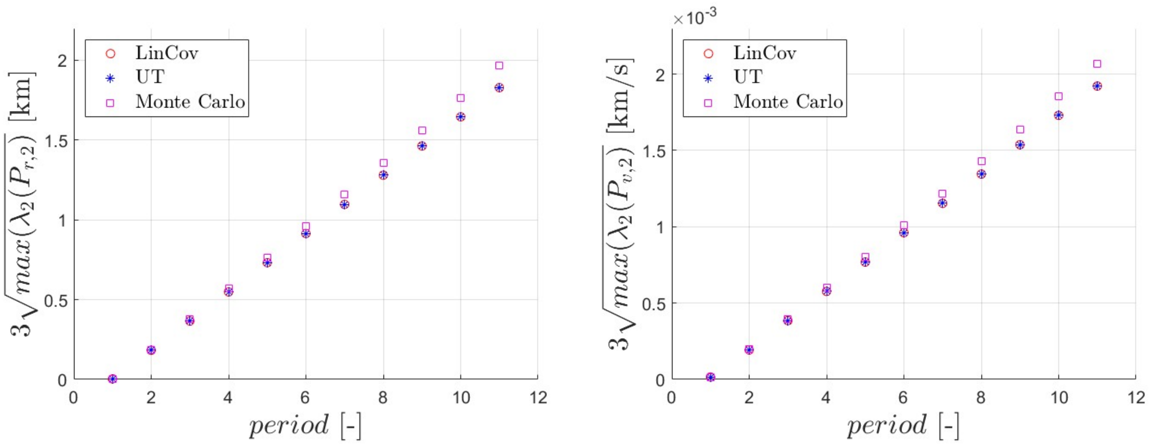
Figure 3: Relative position and collision warning.

The Monte Carlo method is performed generating 500 points from the given mean state and covariance matrix under the assumption of multivariate normal distribution. While LinCov and UT are always almost overlapped, the trend for Monte Carlo of $3\sqrt{(max(\lambda_i(P_i)))}$ slightly diverges over time. Selecting a higher number of points, the accuracy of the method grows and it trails the evolution of the other methods for longer times.



(a) Position **(b)** Velocity

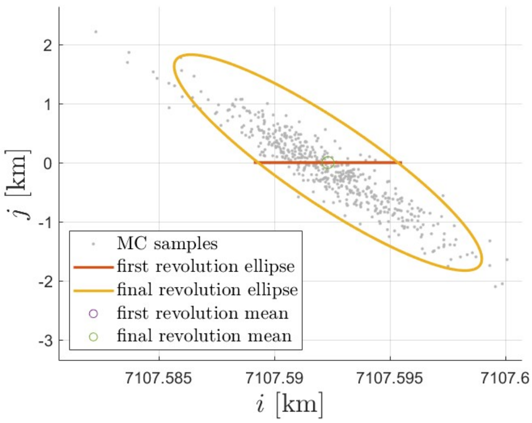
Figure 4: Position and velocity $3\sqrt{max(\lambda_1(P_1))}$ for satellite 1.



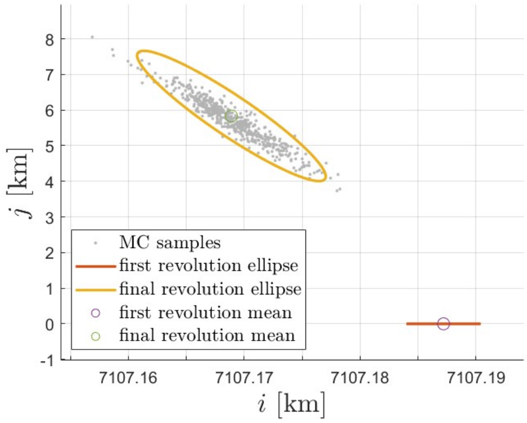
(a) Position **(b)** Velocity

Figure 5: Position and velocity $3\sqrt{max(\lambda_2(P_2))}$ for satellite 2.

The error ellipses and mean values for both satellites with every method, together with the propagated samples of the Monte Carlo analysis rotated on the orbital plane are also hereby shown. Differences between the ellipses related to the first and last revolutions are visible: As expected for Mango, the last one is far more spread due to an accumulation of uncertainty over time, but the mean states are almost coincident. On the other hand there is a more remarked difference between the ellipses and means for Tango, and that is caused by the fact that the propagations have been carried out still using as reference period the one of Mango, that slightly differs from that of the second satellite.

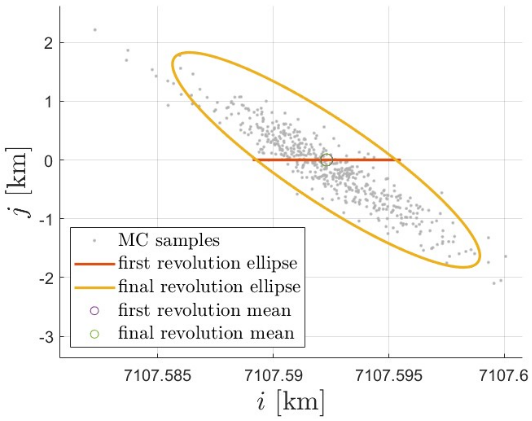


(a) Mango

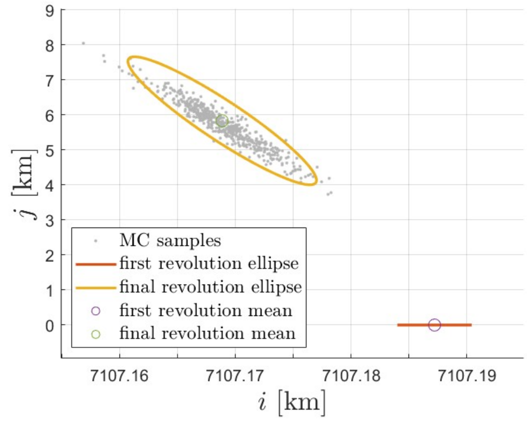


(b) Tango

Figure 6: Error ellipse using LinCov.

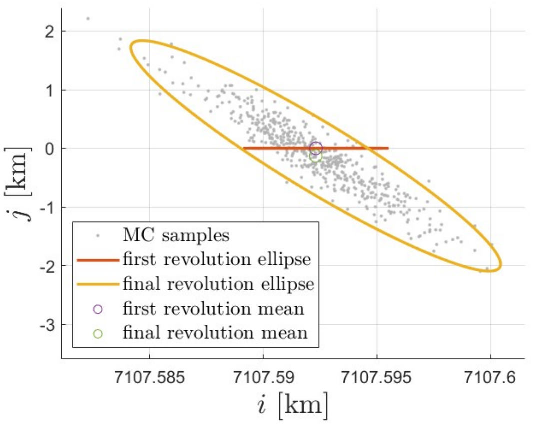


(a) Mango

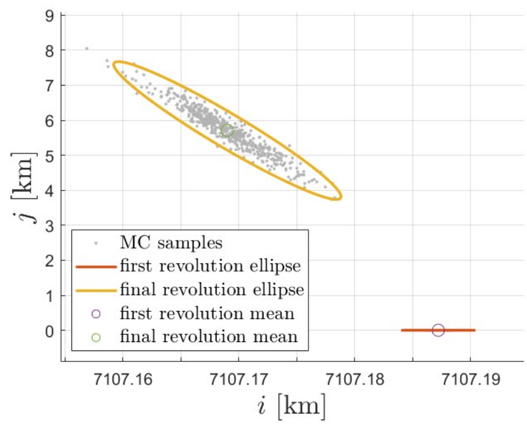


(b) Tango

Figure 7: Error ellipse using UT.



(a) Mango



(b) Tango

Figure 8: Error ellipse using Monte Carlo.

Exercise 2: Batch filters

You have been asked to track Mango to improve the accuracy of its state estimate. To this aim, you shall schedule the observations from the two ground stations reported in Table 2.

1. *Compute visibility windows.* By using the mean state reported in Table 1 and by assuming Keplerian motion, predict the trajectory of the satellite over a uniform time grid (with a time step of 60 seconds) and compute all the visibility time windows from the available stations in the time interval from t_0 = `2010-08-12T05:30:00.000` (UTC) to t_f = `2010-08-12T11:00:00.000` (UTC). Plot the resulting predicted Azimuth and Elevation profiles in the visibility windows.

2. *Simulate measurements.* The Two-Line Elements (TLE) set of Mango are reported in Table 3 (and in WeBeep as 36599.3le). Use SGP4 and the provided TLEs to simulate the measurements acquired by the sensor network in Table 2 by:

   (a) Computing the spacecraft position over the visibility windows identified in Point 1 and deriving the associated expected measurements.

   (b) Simulating the measurements by adding a random error to the expected measurements (assume a Gaussian model to generate the random error, with noise provided in Table 2). Discard any measurements (i.e., after applying the noise) that does not fulfill the visibility condition for the considered station.

3. *Solve the navigation problem.* Using the measurements simulated at the previous point:

   (a) Find the least squares (minimum variance) solution to the navigation problem without a priori information using
      - the epoch t_0 as reference epoch;
      - the reference state as the state derived from the TLE set in Table 3 at the reference epoch;
      - the simulated measurements obtained for the KOROU ground station only;
      - pure Keplerian motion to model the spacecraft dynamics.

   (b) Repeat step 3a by using all simulated measurements from both ground stations.

   (c) Repeat step 3b by using J2-perturbed motion to model the spacecraft dynamics.

4. Provide the obtained navigation solutions and elaborate on the results, comparing the different solutions.

5. Select the best combination of dynamical model and ground stations and perform the orbit determination for the other satellite.

Table 2: Sensor network to track Mango and Tango: list of stations, including their features.

Station name	KOUROU	SVALBARD
Coordinates	LAT = 5.25144° LON = -52.80466° ALT = -14.67 m	LAT = 78.229772° LON = 15.407786° ALT = 458 m
Type	Radar (monostatic)	Radar (monostatic)
Provided measurements	Az, El [deg] Range (one-way) [km]	Az, El [deg] Range (one-way) [km]
Measurements noise (diagonal noise matrix R)	$\sigma_{Az,El} = 100$ mdeg $\sigma_{range} = 0.01$ km	$\sigma_{Az,El} = 125$ mdeg $\sigma_{range} = 0.01$ km
Minimum elevation	10 deg	5 deg

Table 3: TLE of Mango.

```
1␣36599U␣10028B␣␣␣10224.22752732␣-.00000576␣␣00000-0␣-16475-3␣0␣␣9998
2␣36599␣098.2803␣049.5758␣0043871␣021.7908␣338.5082␣14.40871350␣␣8293
```

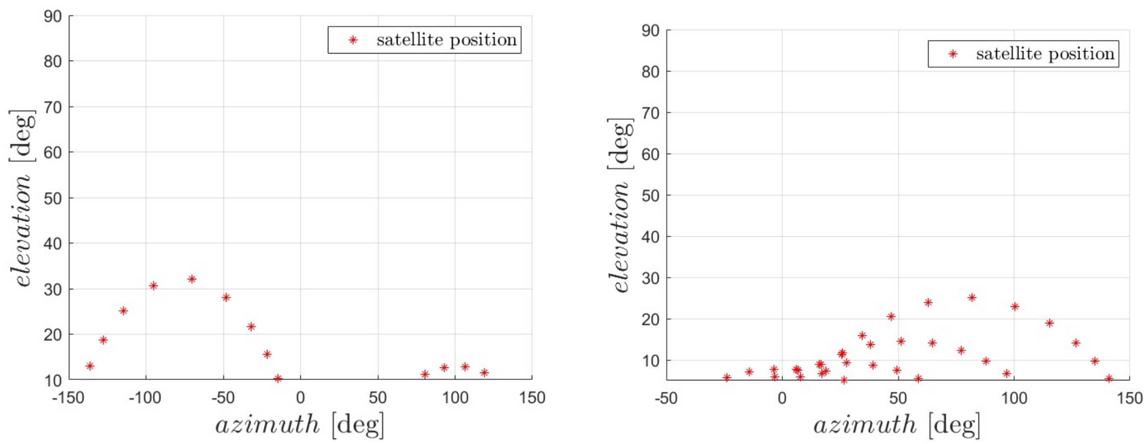
Table 4: TLE of Tango.

```
1␣36827U␣10028F␣␣␣10224.22753605␣␣.00278492␣␣00000-0␣␣82287-1␣0␣␣9996
2␣36827␣098.2797␣049.5751␣0044602␣022.4408␣337.8871␣14.40890217␣␣␣␣55
```

Starting from the mean state of Mango reported in Table 2 (referred to the separation time t_{sep}), the initial state at time t_0 is retrieved by propagation. At this point, considering Keplerian motion, the position of the spacecraft in the ECI reference frame in the given time window is found; once the relative position between Mango and the considered station is calculated and transposed to the Topocentric frame of the station, azimuth and elevation are easily retrieved. The visibility windows of the stations correspond to those time intervals in which the satellite's elevation is above the minimum elevation of the considered station.

Table 5: Visibility windows for Kourou and Svalbard station.

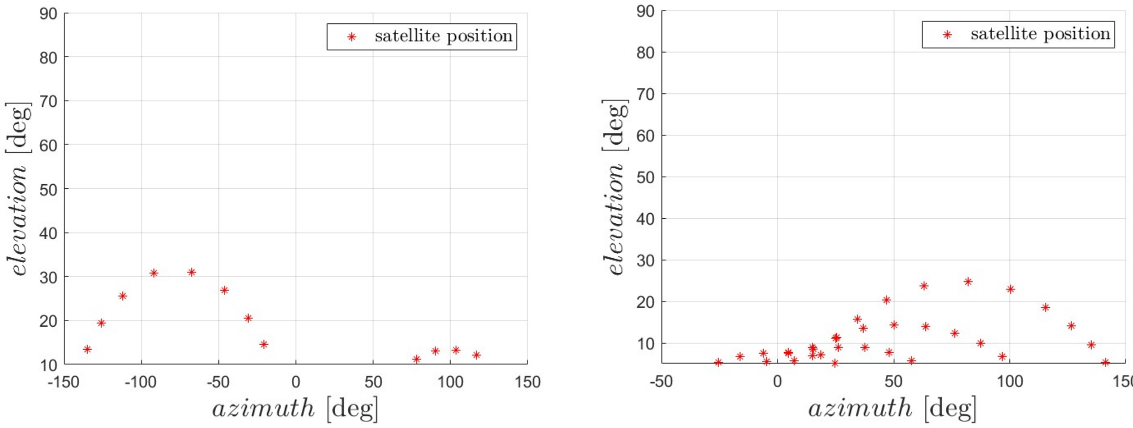
Station name	Start time (UTC)	End time (UTC)
Svalbard	2010-08-12-05:44:00.0	2010-08-12-05:54:00.0
Svalbard	2010-08-12-07:26:00.0	2010-08-12-07:34:00.0
Kourou	2010-08-12-08:46:00.0	2010-08-12-08:54:00.0
Svalbard	2010-08-12-09:08:00.0	2010-08-12-09:14:00.0
Kourou	2010-08-12-10:27:00.0	2010-08-12-10:30:00.0
Svalbard	2010-08-12-10:50:00.0	2010-08-12-10:55:00.0



(a) Predicted Mango position from Kourou **(b)** Predicted Mango position from Svalbard

Figure 9: Predicted azimuth and elevation of the satellite in the visibility windows.

Starting from the provided TLE set of Mango, the satellites' state is analytically propagated with $SGP4$ in the visibility time window and then converted to azimuth, elevation and range as explained earlier. These measurements are however ideal and not affected by the uncertainty of the acquisition process: noise is simulated as a random Gaussian process with zero mean and given standard deviation.



(a) Mango position acquired by Kourou **(b)** Mango position acquired by Svalbard

Figure 10: Acquired azimuth and elevation of the satellite in the visibility windows.

The navigation problem is then solved exploiting a Batch filter without a priori information: the local solver is run in a *MultiStart* environment to locate different local minima and then select the global one. Three different cases are presented:

a) The measurements from the Kourou station only are available and the predicted measurements are retrieved from a pure Keplerian motion model.

Table 6: Solution of the navigation problem for Mango with Kourou station only.

	Value
Initial state [km,km/s]	$\hat{\mathbf{r}}_0$ = [4662.3870 5259.9189 994.0435] $\hat{\mathbf{v}}_0$ = [0.0953 -1.4558 7.3686]
Displacement from initial reference state [km, km/s]	$\delta\mathbf{r}_0$ = [-22.9416 21.3358 -48.3921] $\delta\mathbf{v}_0 = [1.358 \times 10^{-2}\ 1.006 \times 10^{-1}\ 2.405 \times 10^{-2}\ ]$
Residual norm [km]	$\|\|res\|\| = 2516.4982$
Position standard deviation [km]	$\sigma_r = 52.3665$
Velocity standard deviation [km/s]	$\sigma_r = 0.3061$

b) The measurements from both Kourou and Svalbard stations are available and the predicted measurements are still retrieved from a pure Keplerian motion model.

Table 7: Solution of the navigation problem for Mango with Kourou and Svalbard stations.

	Value
Initial state [km, km/s]	$\hat{\mathbf{r}}_0$ = [4666.3089 5248.9042 1039.5268] $\hat{\mathbf{v}}_0$ = [0.0921 -1.5478 7.3503]
Displacement from initial reference state [km, km/s]	$\delta\mathbf{r}_0$ = [-19.020 10.321 -2.909] $\delta\mathbf{v}_0 = [1.045 \times 10^{-2}\ 8.647 \times 10^{-3}\ 5.733 \times 10^{-3}\ ]$
Residual norm [km]	$\|\|res\|\| = 659.5204$
Position standard deviation [km]	$\sigma_r = 0.4678$
Velocity standard deviation [km/s]	$\sigma_r = 4.665 \times 10^{-4}$

c) The measurements from both Kourou and Svalbard stations are available and the predicted measurements are retrieved from a J2-perturbed motion model.

Table 8: Solution of the navigation problem for Mango with Kourou and Svalbard stations and J2 perturbation.

	Value
Initial state [km, km/s]	$\hat{\mathbf{r}}_0$ = [4685.4108 5238.5839 1042.5619] $\hat{\mathbf{v}}_0$ = [0.0816 -1.5566 7.3445]
Displacement from initial reference state [km, km/s]	$\delta\mathbf{r}_0 = [8.205 \times 10^{-2}\ 8.146 \times 10^{-4}\ 1.262 \times 10^{-1}]$ $\delta\mathbf{v}_0 = [-1.250 \times 10^{-4}\ -1.351 \times 10^{-4}\ -1.016 \times 10^{-4}\ ]$
Residual norm [km]	$\|\|res\|\| = 11.5314$
Position standard deviation [km]	$\sigma_r = 1.1624 \times 10^{-2}$
Velocity standard deviation [km/s]	$\sigma_r = 1.2894 \times 10^{-5}$

As expected, results get more and more precise as we add information to the model (first a second station and then a refinement of the motion model): the solution approaches the initial reference state (drawn from the SGP4 propagation) and gets less and less spread, as both position and velocity standard deviations decrease.
The navigation problem is finally solved for the Tango satellite considering the measurements acquired by both stations and the perturbed motion to model the dynamics:

Table 9: Solution of the navigation problem for Tango with Kourou and Svalbard stations and J2 perturbation.

	Value
Initial state [km, km/s]	$\hat{\mathbf{r}}_0$ = [4684.9421 5238.7786 1038.7735] $\hat{\mathbf{v}}_0$ = [0.0838 -1.5542 7.3456]
Displacement from initial reference state [km, km/s]	$\delta\mathbf{r}_0 = [-8.783 \times 10^{-2}\ -1.109 \times 10^{-2}\ -6.836 \times 10^{-1}]$ $\delta\mathbf{v}_0 = [4.119 \times 10^{-4}\ 3.350 \times 10^{-4}\ 1.588 \times 10^{-4}\ ]$
Residual norm [km]	$\|\|res\|\| = 131.4093$
Position standard deviation [km]	$\sigma_r = 1.2613 \times 10^{-1}$
Velocity standard deviation [km/s]	$\sigma_r = 1.6095 \times 10^{-4}$

Exercise 3: Sequential filters

According to the Formation Flying In Orbit Ranging Demonstration experiment (FFIORD), PRISMA's primary objectives include testing and validation of GNC hardware, software, and algorithms for autonomous formation flying, proximity operations, and final approach and recede operations. The cornerstone of FFIORD is a Formation Flying Radio Frequency (FFRF) metrology subsystem designed for future outer space formation flying missions.

FFRF subsystem is in charge of the relative positioning of 2 to 4 satellites flying in formation. Each spacecraft produces relative position, velocity and line-of-sight (LOS) of all its companions.

You have been asked to track Mango to improve the accuracy of the estimate of its absolute state and then, according to the objectives of the PRISMA mission, validate the autonomous formation flying navigation operations by estimating the relative state between Mango and Tango by exploiting the relative measurements acquired by the FFRF subsystem. The Two-Line Elements (TLE) set of Mango and Tango satellites are reported in Tables 3 and 4 (and in WeBeep as 36599.3le, and 36827.3le).

The relative motion between the two satellites can be modelled through the linear, Clohessy-Wiltshire (CW) equations†

$$\begin{aligned}\ddot{x} &= 3n^2x + 2n\dot{y}\\ \ddot{y} &= -2n\dot{x}\\ \ddot{z} &= -n^2z\end{aligned} \tag{1}$$

where x, y,and z are the relative position components expressed in the LVLH frame, whereas n is the mean motion of Mango, which is assumed to be constant and equal to:

$$n = \sqrt{\frac{GM}{R^3}} \tag{2}$$

where R is the position of Mango at t_0.

The unit vectors of the LVLH reference frame are defined as follows:

$$\hat{\boldsymbol{i}} = \frac{\boldsymbol{r}}{r}, \quad \hat{\boldsymbol{j}} = \hat{\boldsymbol{k}} \times \hat{\boldsymbol{i}}, \quad \hat{\boldsymbol{k}} = \frac{\boldsymbol{h}}{h} = \frac{\boldsymbol{r} \times \boldsymbol{v}}{\|\boldsymbol{r} \times \boldsymbol{v}\|} \tag{3}$$

To perform the requested tasks you should:

1. *Estimate Mango absolute state.* You are asked to develop a sequential filter to narrow down the uncertainty on the knowledge of Mango absolute state vector. To this aim, you shall schedule the observations from the SVALBARD ground station‡ reported in Table 2, and then proceed with the state estimation procedure by following these steps:

   (a) By using the mean state reported in Table 1 and by assuming Keplerian motion, predict the trajectory of the satellite over a uniform time grid (with a time step of 5 seconds) and compute the first visibility time window from the SVALBARD station in the time interval from t_0 = `2010-08-12T05:30:00.000` (UTC) to t_f = `2010-08-12T06:30:00.000` (UTC).

   (b) Use SGP4 and the provided TLE to simulate the measurements acquired by the SVALBARD station for the Mango satellite only. For doing it, compute the spacecraft position over the visibility window using a time-step of 5 seconds, and derive the associated expected measurements. Finally, simulate the measurements by adding a random error (assume a Gaussian model to generate the random error, with noise provided in Table 2).

†Notice that the system is linear, therefore it has an analytic solution of the state transition matrix $\boldsymbol{\Phi}$

‡Note that these are the same ones computed in Exercise 2

(c) Using an Unscented Kalman Filter (UKF), provide an estimate of the spacecraft state (in terms of mean and covariance) by sequentially processing the acquired measurements in chronological order. Plot the time evolution of the error estimate together with the 3σ of the estimated covariance for both position and velocity.

2. *Estimate the relative state.* To validate the formation flying operations, you are also asked to develop a sequential filter to narrow down the uncertainty on the knowledge of the relative state vector. To this aim, you can exploit the relative azimuth, elevation, and range measurements obtained by the FFRF subsystem, whose features are reported in Table 10, and then proceed with the state estimation procedure by following these steps:

   (a) Use SGP4 and the provided TLEs to propagate the states of both satellites at epoch t_0 in order to compute the relative state in LVLH frame at that specific epoch.

   (b) Use the relative state as initial condition to integrate the CW equations over the time grid defined in Point 1a. Finally, simulate the relative measurements acquired by the Mango satellite through its FFRF subsystem by adding a random error to the expected measurements. Assume a Gaussian model to generate the random error, with noise provided in Table 10.

   (c) Consider a time interval of 20 minutes starting from the first epoch after the visibility window (always with a time step of 5 seconds). Use an UKF to provide an estimate of the spacecraft relative state in the LVLH reference frame (in terms of mean and covariance) by sequentially processing the measurements acquired during those time instants in chronological order. Plot the time evolution of the error estimate together with the 3σ of the estimated covariance for both relative position and velocity.

3. *Reconstruct Tango absolute covariance.* Starting from the knowledge of the estimated covariance of the absolute state of Mango, computed in Point 1, and the estimated covariance of the relative state in the LVLH frame, you are asked to provide an estimate of the covariance of the absolute state of Tango. You can perform this operation as follows:

   (a) Pick the estimated covariance of the absolute state of Mango at the last epoch of the visibility window, and propagate it within the time grid defined in Point 2c.

   (b) Rotate the estimated covariance of the relative state from the LVLH reference frame to the ECI one within the same time grid.

   (c) Sum the two to obtain an estimate of the covariance of the absolute state of Tango. Plot the time evolution of the 3σ for both position and velocity and elaborate on the results.

Table 10: Parameters of FFRF.

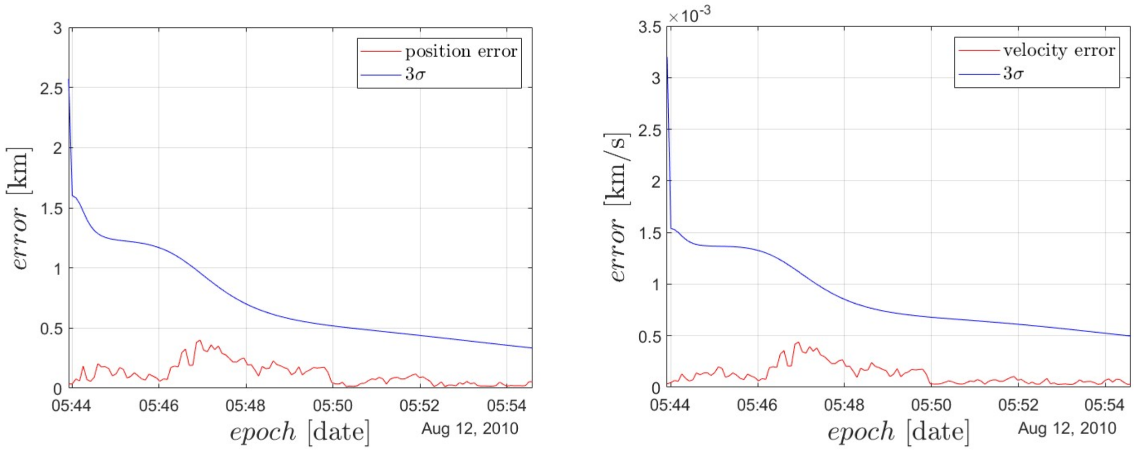
Parameter	Value
Measurements noise (diagonal noise matrix R)	$\sigma_{Az,El} = 1$ deg $\sigma_{range} = 1$ cm

Given the state of Mango at the separation time t_{sep}, it is firstly propagated until t_0. Then exploiting this new initial condition, the set of states in the given time window is provided: the propagations are performed with a J2-perturbed Keplerian model. Finally, the states are converted to azimuth, elevation and range as seen from the Svalbard station, and the visibility window is computed.

Table 11: Visibility windows for Svalbard station.

Station name	Start time (UTC)	End time (UTC)
Svalbard	`2010-08-12-05:43:55.0`	`2010-08-12-05:54:35.0`

Once the visibility window is determined, exploiting $SGP4$, the states of the satellite in that period are retrieved and the expected measurements are then simulated considering a random Gaussian error on the acquisition process.
The spacecraft's state can then be estimated setting an Unscented Kalman Filter: measurements in the visibility windows are exploited to sequentially update the mean and covariance. Initial conditions on $\hat{\mathbf{x}}$ and P to start filtering are obtained propagating the separation time mean and covariance matrix with the Unscented Transform until the first epoch of the visibility window.



(a) Mango position error and $3\sigma_r$

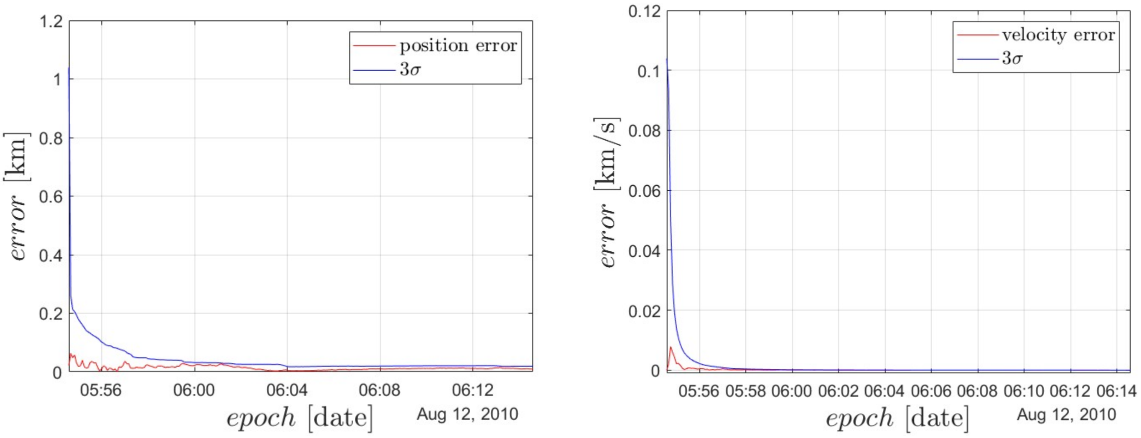
(b) Mango velocity error and $3\sigma_v$

Figure 11: Evolution of the error on Mango along with 3σ.

Then the states of both Mango and Tango are propagated until t_0, where the relative position is computed and then rotated to the LVLH rotating reference frame centered at the chaser satellite. Transformations from the inertial frame to the rotating frame for position and velocity are given by:

$$\begin{cases} \mathbf{r}_{LVLH} = [\mathbf{R}]\mathbf{r}_{ECI} \\ \dot{\mathbf{r}}_{LVLH} = [\dot{\mathbf{R}}]\mathbf{r}_{ECI} + [\mathbf{R}]\dot{\mathbf{r}}_{ECI} \end{cases} \tag{4}$$

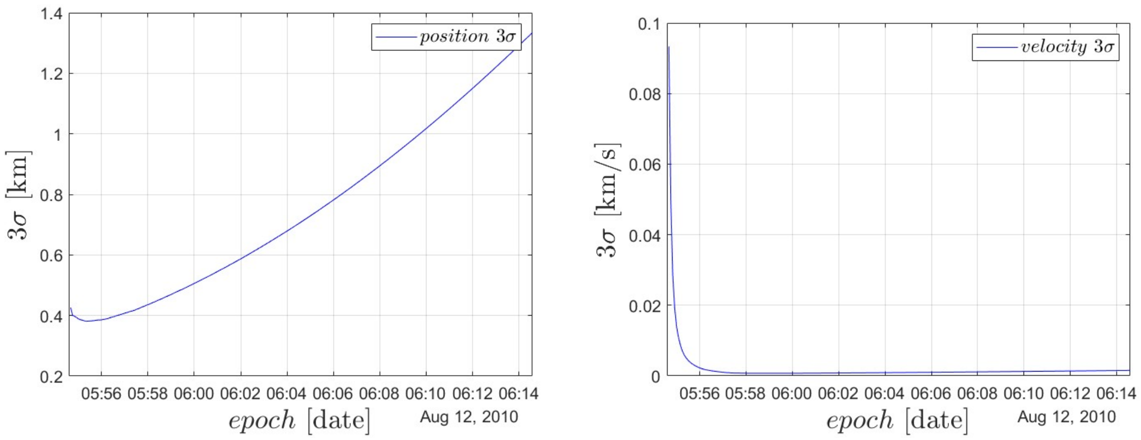
where $[\mathbf{R}]$ is the 3×3 rotation matrix to switch from ECI to LVLH reference frames and $[\dot{\mathbf{R}}]$ is its derivative.
The relative position is then propagated using the Clohessy-Wiltshire equations and then converted into azimuth, elevation and range measurements as acquired from the FFRF on board Mango (a random error to simulate the process is accounted). Considering the 20 minutes time interval after the visibility window, an Unscented Kalman Filter is exploited to provide an estimate of the spacecraft relative state by sequentially processing the measurements acquired during those time instants in chronological order.



(a) Relative position error and $3\sigma_r$ **(b)** Relative velocity error and $3\sigma_v$

Figure 12: Evolution of the relative error along with 3σ.

The covariance of the state of Mango at the last epoch obtained before with the UKF is then propagated in the latest time grid (duration of 20 minutes after the antenna visibility window); the relative state is brought back to the ECI frame and the two are summed to obtain an estimate on the covariance of Tango. Trends for the position and velocity 3σ are detailed: position increases while the velocity rapidly approaches zero.



(a) Position 3σ **(b)** Velocity 3σ

Figure 13: Tango position and velocity 3σ.