Hyperbolic Functions

The hyperbolic sine, cosine, tangent, secant, cotangent and cosecant functions sinh, cosh, tanh, sech, coth and cosech are defined by

$$\sinh x = \frac{e^x - e^{-x}}{2},$$

$$\cosh x = \frac{e^x + e^{-x}}{2},$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1},$$

$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}},$$

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}} = \frac{e^{2x} + 1}{e^{2x} - 1} \qquad (x \neq 0),$$

$$\operatorname{cosech} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}} \qquad (x \neq 0).$$

Properties of the Hyperbolic Functions

(1) $\sinh 0 = 0, \quad \sinh(-x) = -\sinh x \quad$ for all x.

(2) $\cosh 0 = 1, \quad \cosh(-x) = \cosh x \quad$ for all x.

(3) $\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y \quad$ for all x, y.

(4) $\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y \quad$ for all x, y.

(5) $\sinh 2x = 2 \sinh x \cosh x \quad$ for all x.

(6) $\cosh 2x = \cosh^2 x + \sinh^2 x \quad$ for all x.

(7) $\cosh^2 x - \sinh^2 x = 1 \quad$ for all x.

(8) $\dfrac{d}{dx}(\sinh x) = \cosh x, \quad \dfrac{d}{dx}(\cosh x) = \sinh x,$

$\dfrac{d}{dx}(\tanh x) = \operatorname{sech}^2 x, \quad \dfrac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x.$

(9) $\cosh x \geqslant 1 \quad$ and $\quad \cosh x > \sinh x \quad$ for all x.

(10) $\tanh x \to 1 \quad$ as $x \to \infty$.

$\tanh x \to -1 \quad$ as $x \to -\infty$.

(11) Graphs

Proofs

(1) and (2) are clear.

(3)

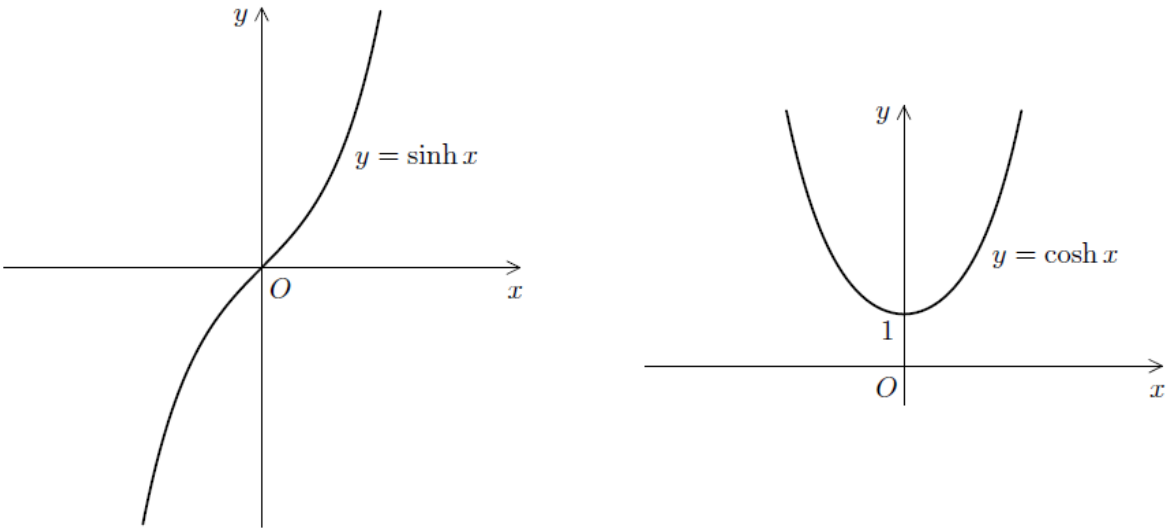

Figure 6:

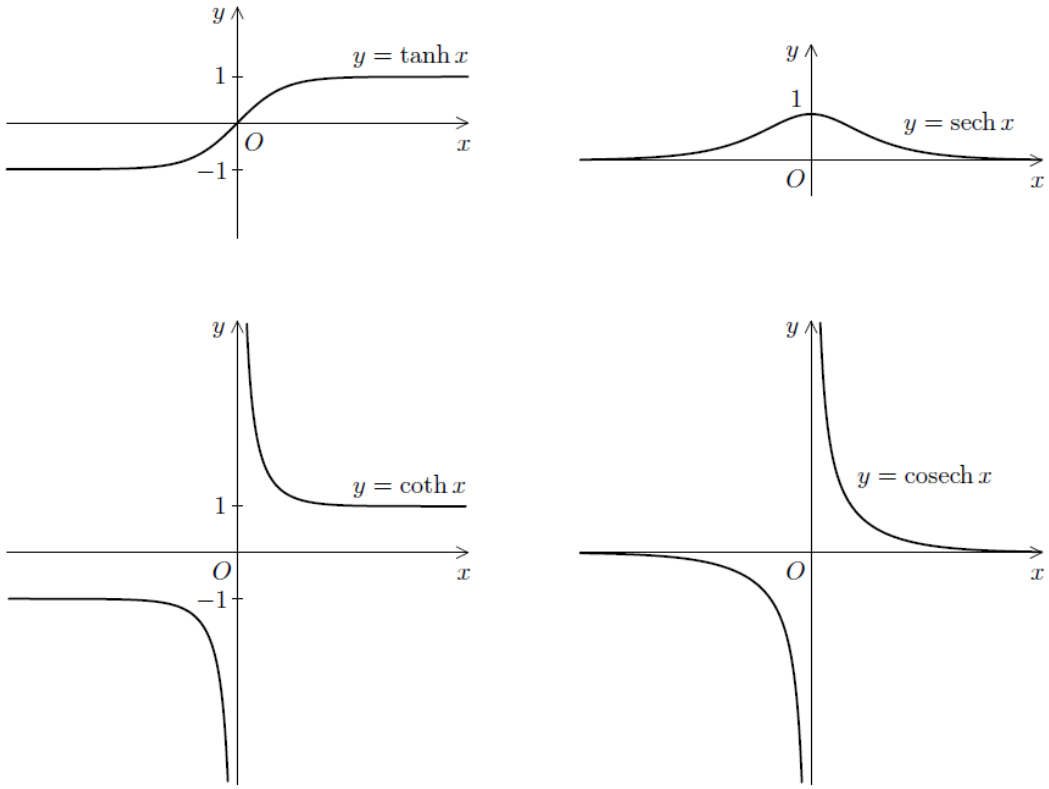

Figure 7: