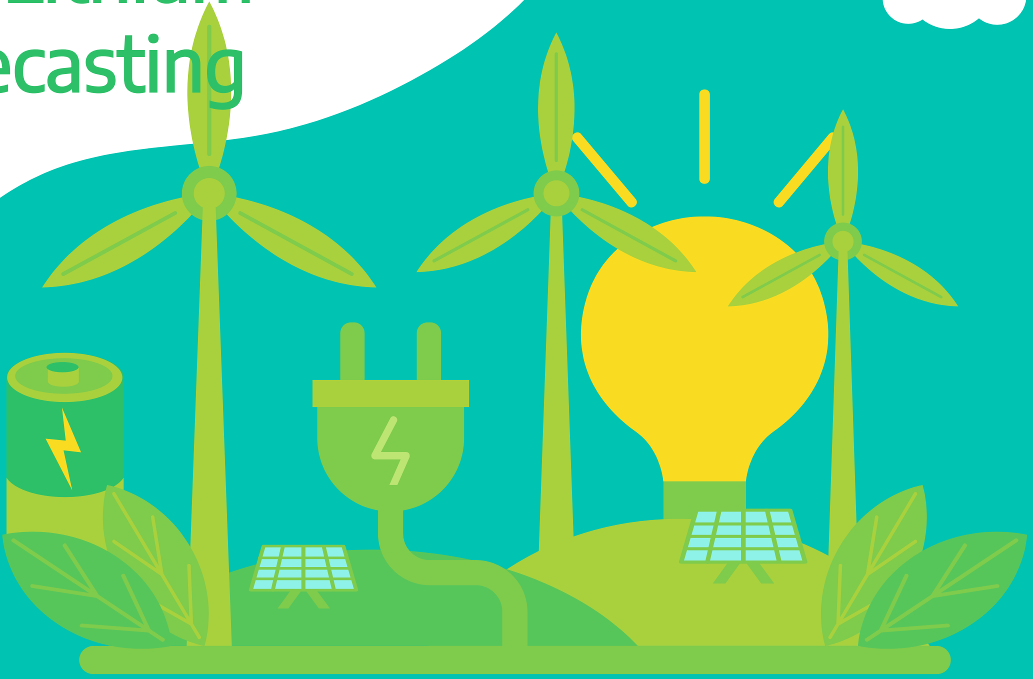
Powering the Future in a Sustainable way: Lithium Analysis and Forecasting

José Chacón
Alejandra Cruces
Mario Tapia



Program

01
Lithium demand factors
Key aspects of the increasing demand of lithium worldwide

02
Lithium time series
Comparison of lithium exports in Chile and Australia

03
Explainability analysis
Bass model, GBM, Competition model

04
Forecast
Exponential smoothing, KNN regression, ARIMA, GAM

05
Conclusions
Summary of the main findings

06
Future of lithium
Expectations in the Lithium market for the future

01

Lithium demand factors

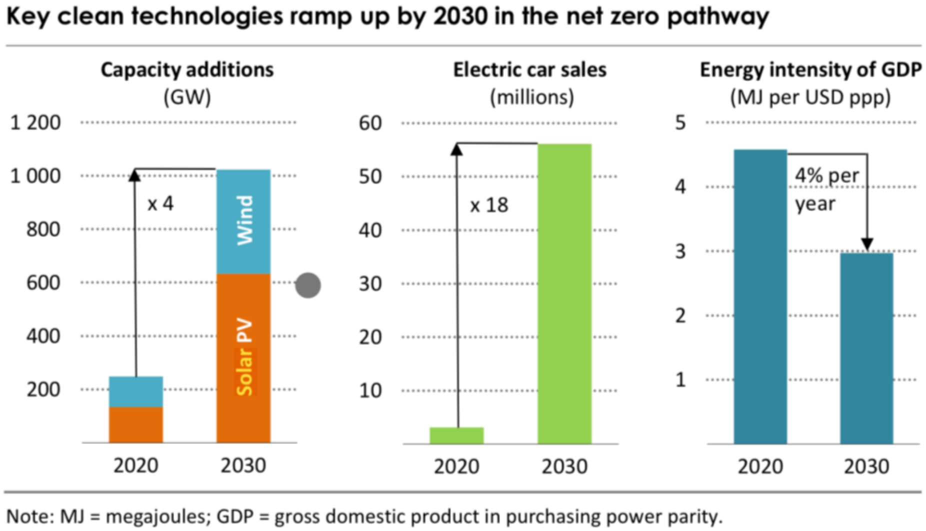
Key aspects for understanding the current demand for lithium

Net Zero by 2050

- To achieve global net-zero emissions by 2050 the conversion to electric transport is a cornerstone[1]

Source: [1]IEA (A Roadmap for the Global Energy Sector)

A clean technology expansion by 2030 is needed



Source: IEA (A Roadmap for the Global Energy Sector)

Sales of electric vehicles surge as fast-charging sites double across Australia in a year

EVs made up just 2% of new car sales in May 2022, but now 8.3% of new car sales in 2023 are battery powered

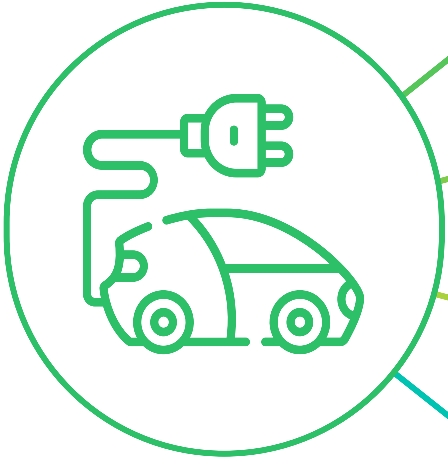
Source: [The Guardian]

Sales of electric vehicles surge as fast-charging sites double across Australia in a year

EVs made up just 2% of new car sales in May 2022, but now 8.3% of new car sales in 2023 are battery powered

Source: The Guardian

Lithium: From the nature to an electric vehicle



01 Lithium extraction

02 Conversion to Lithium Carbonate

03 Production of **Li-ion batteries**

04 EV battery pack assembly

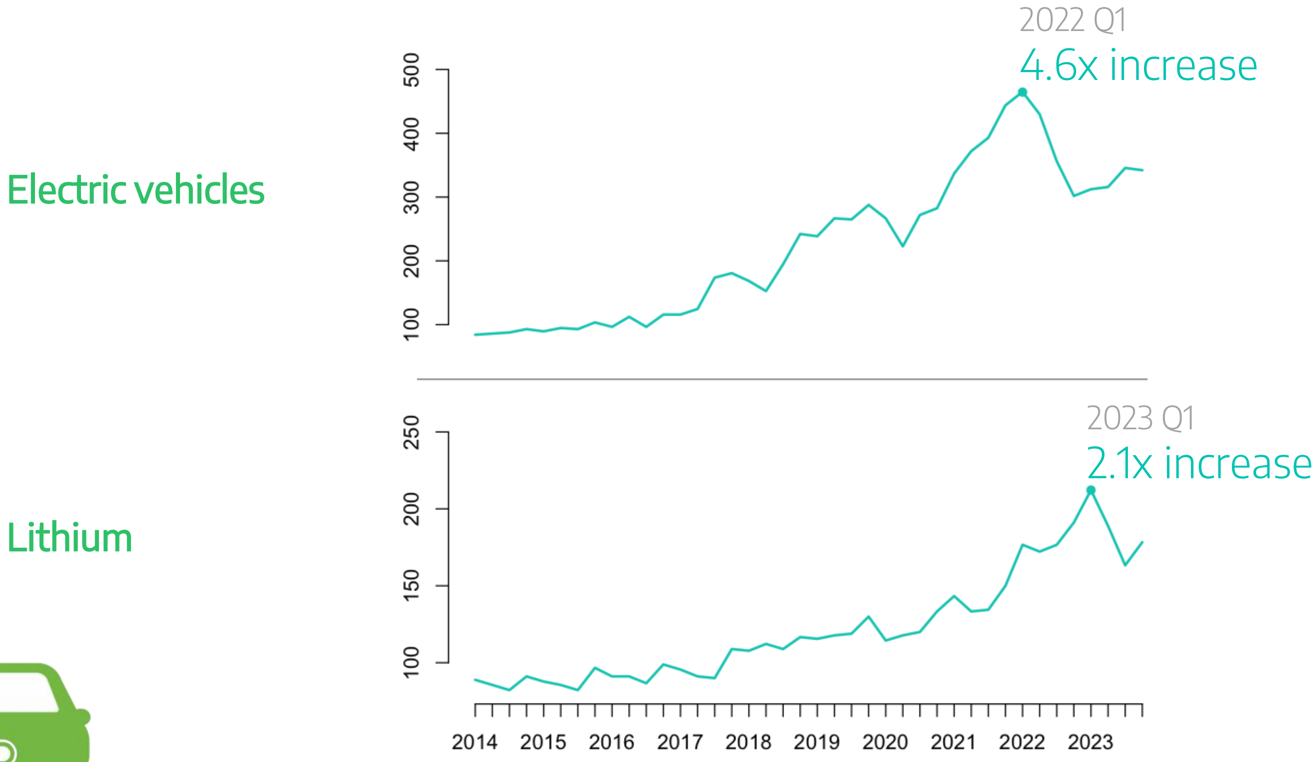
What is Lithium?

Chemical Element

Abundant Element

Crucial for Batteries

Google Trends Interest over time

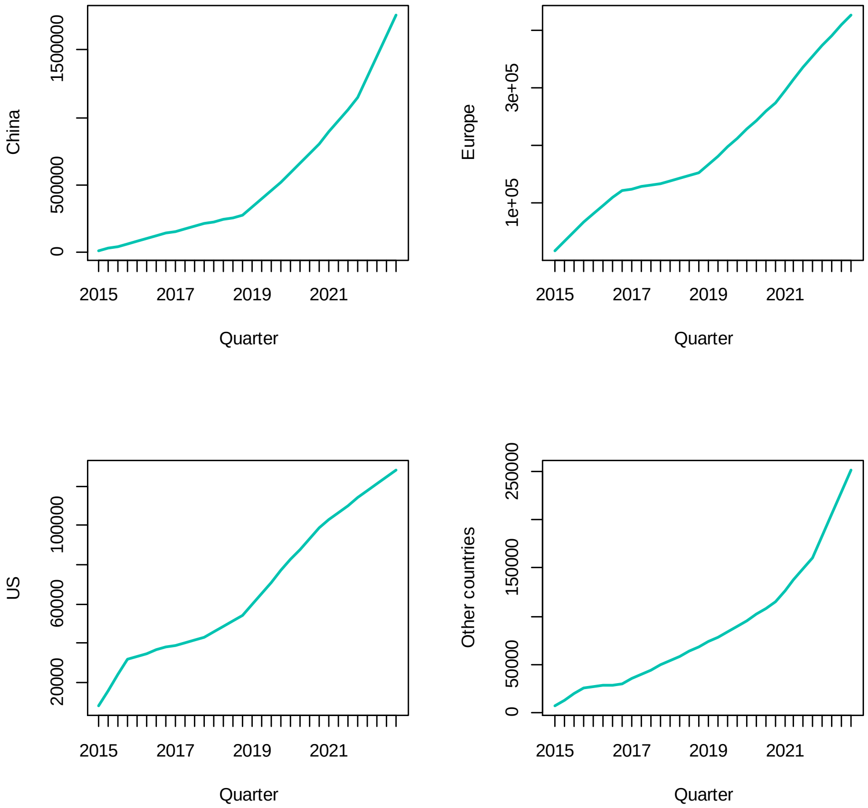


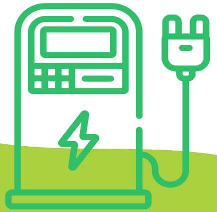
Source: Google Trends

Electric chargers Number of fast and slow chargers

- Increasing trend in electric chargers

- China stands out as the country with the greatest number of fast[1] and slow[2] electric chargers available

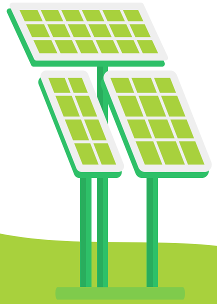




Source: [1]IEA (fast chargers), [2]IEA (slow chargers)

Solar investment Billion USD

- It is related to energy storage
- Can also be seen as an indicator of interest in renewable energies



Source: Statista

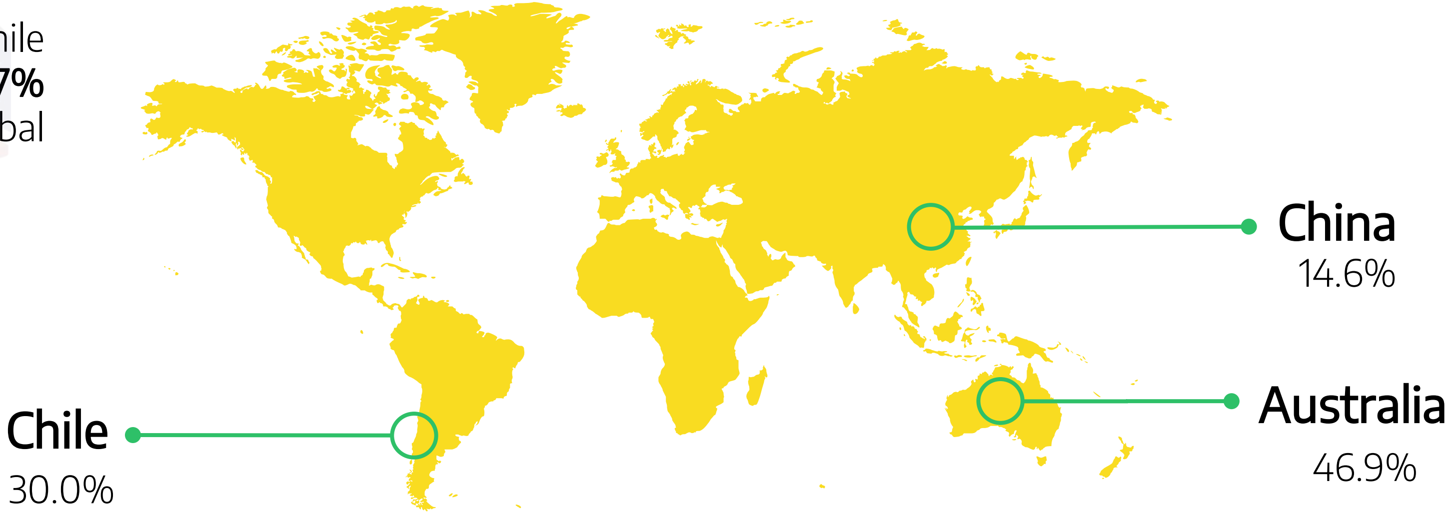
02

Lithium Time Series

About the main time series

Lithium production Worldwide map (2022)

Australia and Chile account for the **77%** of the global production



Source: United States Geological Survey

Lithium production Australia and Chile

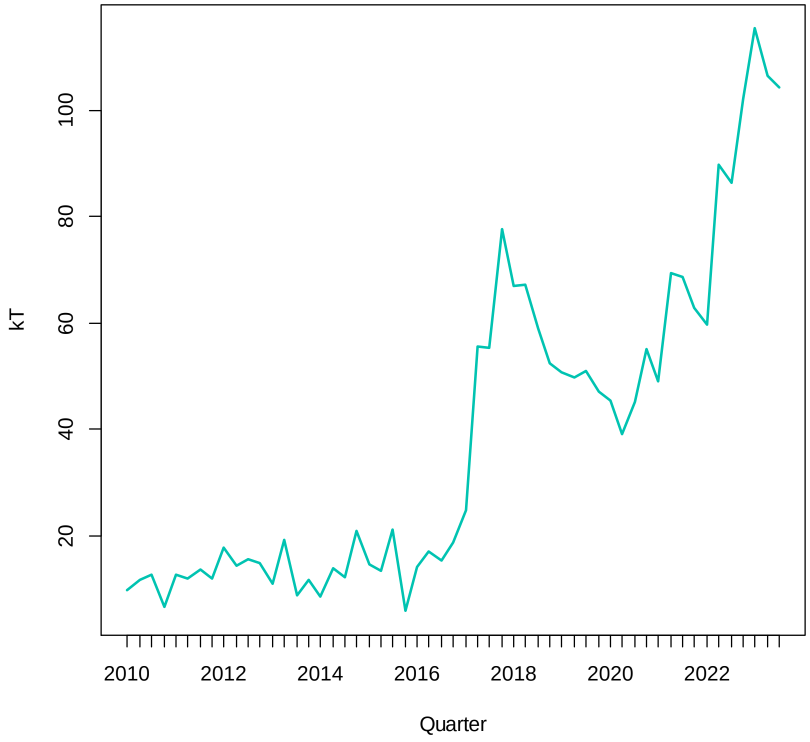
- Australia extracts lithium from hard rock mines
- Chile extracts the mineral from brines
- Common unit of measure: Lithium Carbonate Equivalent (LCE)
- We will focus on the exports of each country

Pilbara Minerals' Pilgangoora lithium tantalum mine, Australia

Brine pools and processing areas at SQM's lithium mine on the Atacama salt flat, Chile

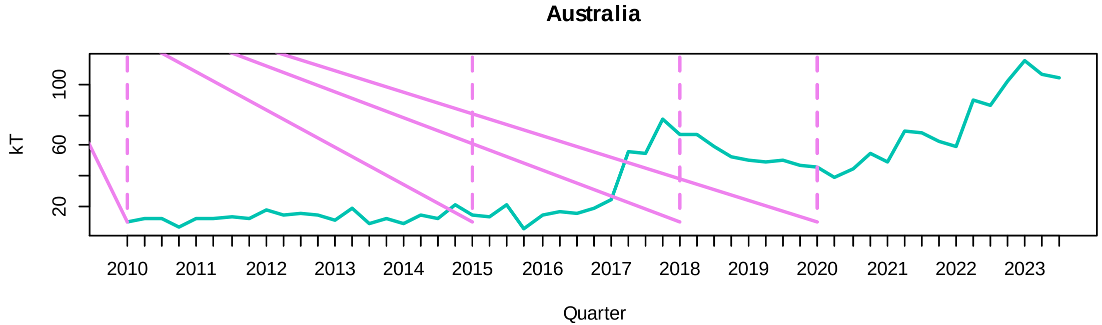
Lithium exports Australia

- Significant rise from 2016 to 2018
- Decline from 2018 to 2020
- Recovery from 2020 onwards
- Most of Australia's exports go to China



Source: Australian Government, Department of Industry, Science and Resources

Lithium Australia events



2010 → **2015** → **2018** → **2020**

2010: China began granting subsidies to EV buyers[1]

2015: China's government announced that it would phase out subsidies progressively from 2016 and by the end of 2020[2]

2018: Excess of supply, slower demand growth for EV, criteria to qualify for subsidies became more stringent in China[3]

2020: China's government extended the subsidies by two years to the end of 2022, due to the pandemic and the economic downturn[4]

Sources:

[1] China's National Development and Reform Commission

[2] International Council on Clean Transportation

[3] Reuters news agency

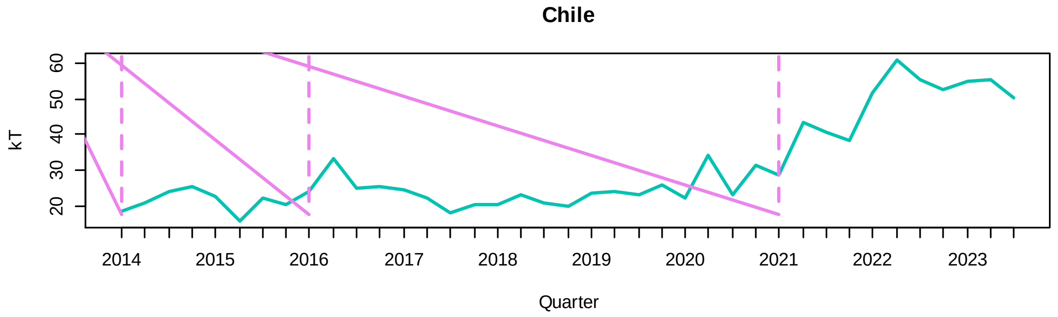
[4] Ministry of Finance of the People's Republic of China

Lithium exports Chile

- Chile started a National Lithium Commission in 2014
- Rise from 2015 to mid-2016
- Decline from mid-2016 to 208
- Significant rise from 2021 onwards, most of Chile's exports go to China, Japan and South Korea

Source: National Customs Service of Chile

Lithium Chile events



2014

Decree: National Lithium Commission in 2014

2016

Chile experienced a decrease in its lithium production and market share[1]

2021

In the first five months of 2021, Chile increased significantly the exportation of lithium carbonates (China, Japan, South Korea)[2]. Most of Chile's exports go to China[3]

Sources:

[1] Mining.com

[2] Reuters news agency

[3] World Integrated Trade Solution

Lithium exports Australia and Chile

- Australia has a larger experience in Lithium extraction and exploration
- Both countries have experienced an increase in their exportations since 2020.
- Mainly explained by the growing interest in this product

Lithium Carbonate Price per kilotonne (CNY)

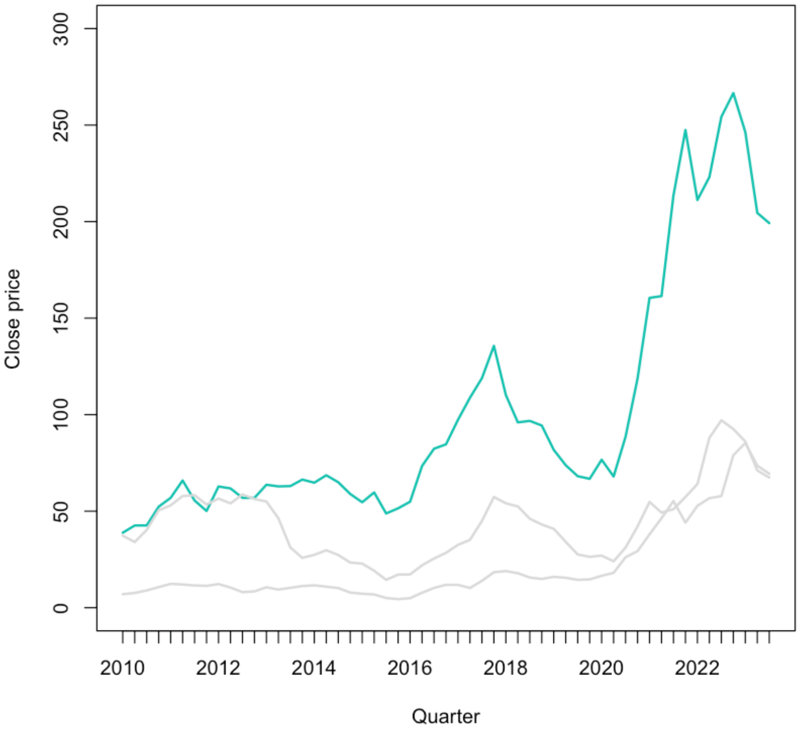
- China spot price
- Exponential increase from 2020
- Slowing demand causing the price to fall.

Source: Investing

Stock Prices Lithium Companies

Albemarle Corporation

- US based company
- World’s largest lithium producer
- Presence in Australia and Chile

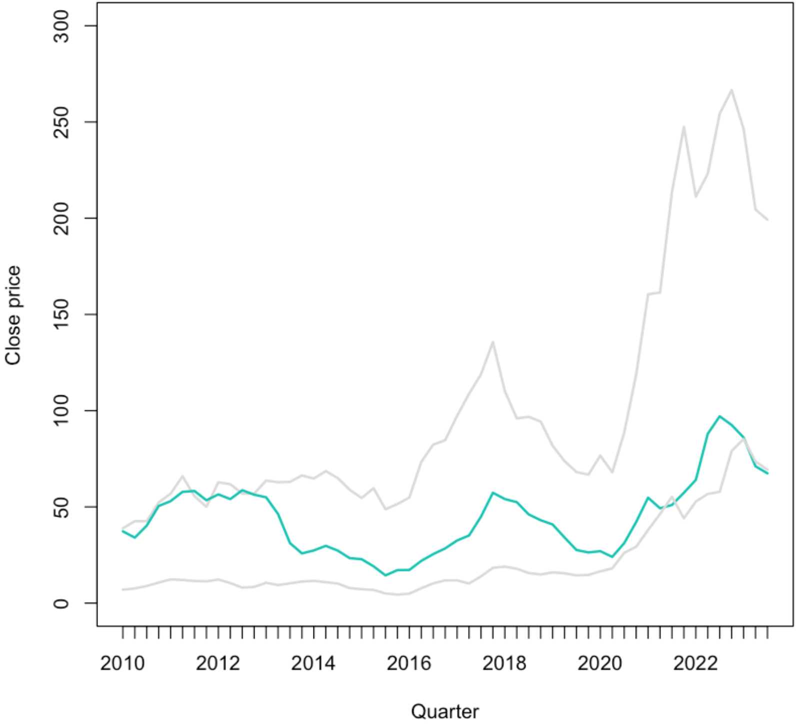


Source: Yahoo! Finance

Stock Prices Lithium Companies

Sociedad Química y Minera de Chile S.A. (SQM)

- Chilean company
- World's second-largest lithium producer

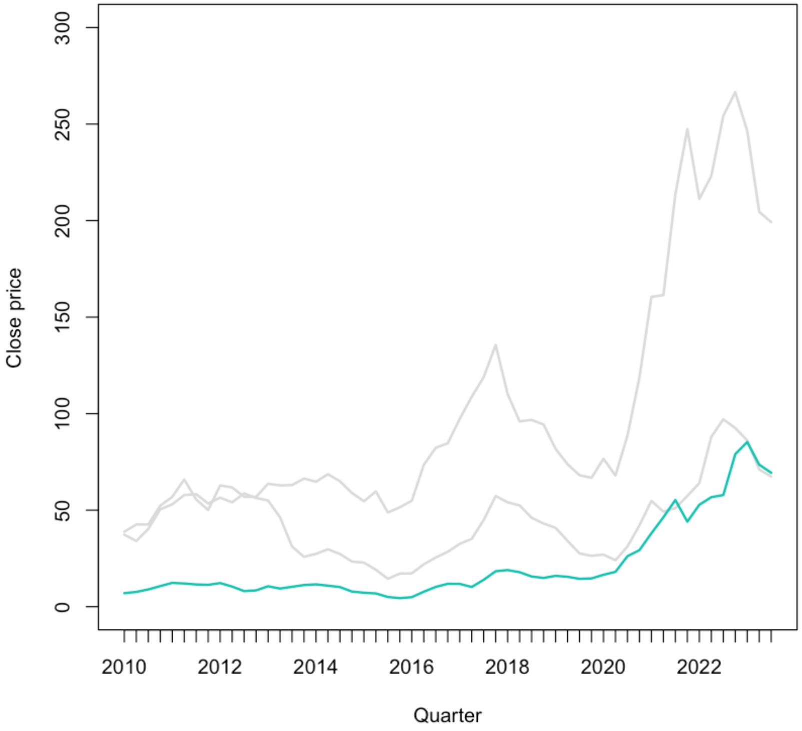


Source: Yahoo! Finance

Stock Prices Lithium Companies

Mineral Resources Limited

- Australian company
- Operates hard rock lithium mines in Western Australia



Source: Yahoo! Finance

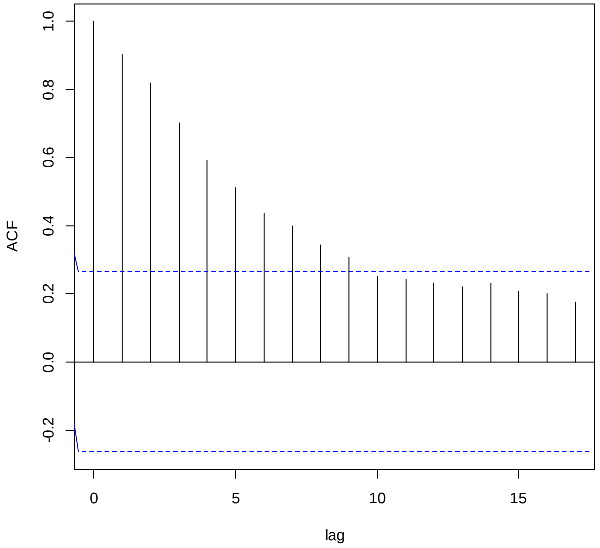
03

Explainability Analysis

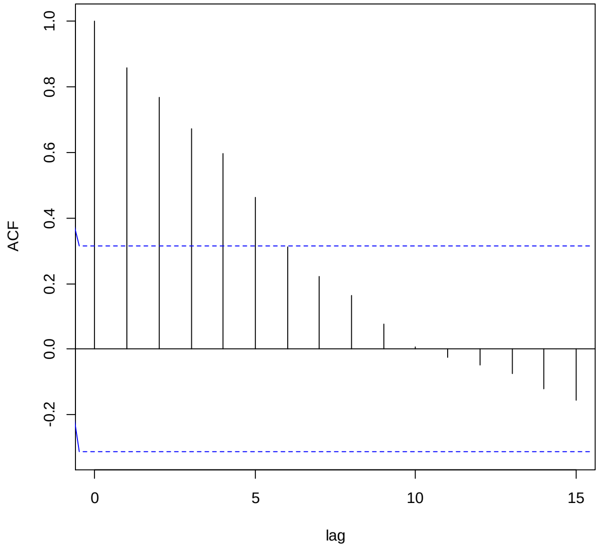
Bass model, GBM, Competition model

Lithium time series Australia and Chile

Australia



Chile

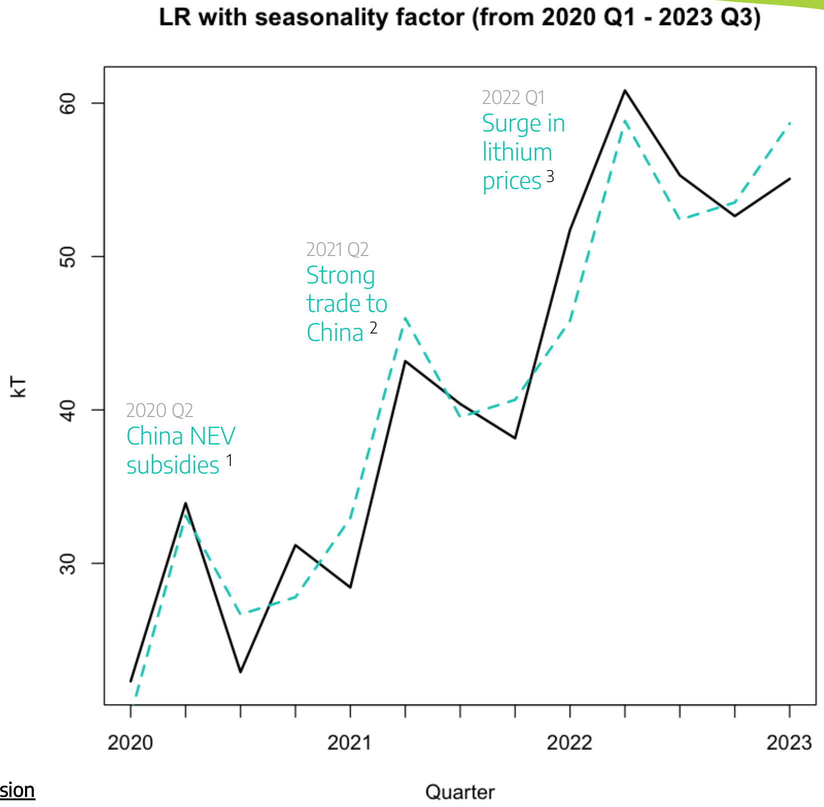


- Both cases: Trend is significant and Seasonality not significant (Linear Regression + season + trend) but...

Linear Regression Window from 2020 onwards

Chile

- Trend is significant
- Q2 is slightly significant, compared to Q1
- $R^2 = 0.94$



Sources:

1 China's National Development and Reform Commission

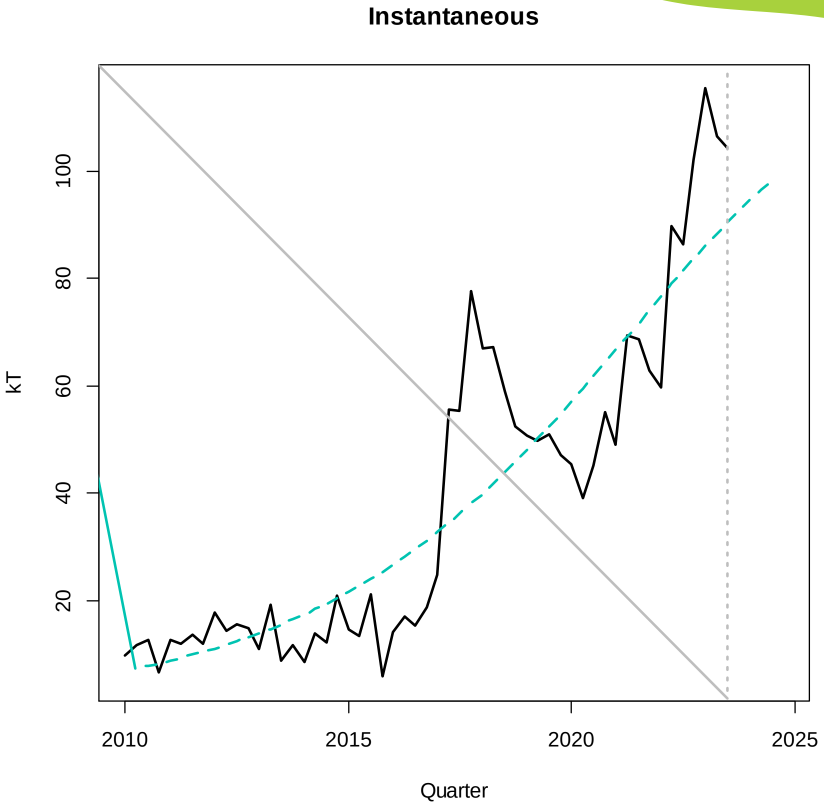
2 S & P

3 S & P

Bass model Australia

- Forecast for one year
- Slightly pessimistic
- Innovation and imitation are significant

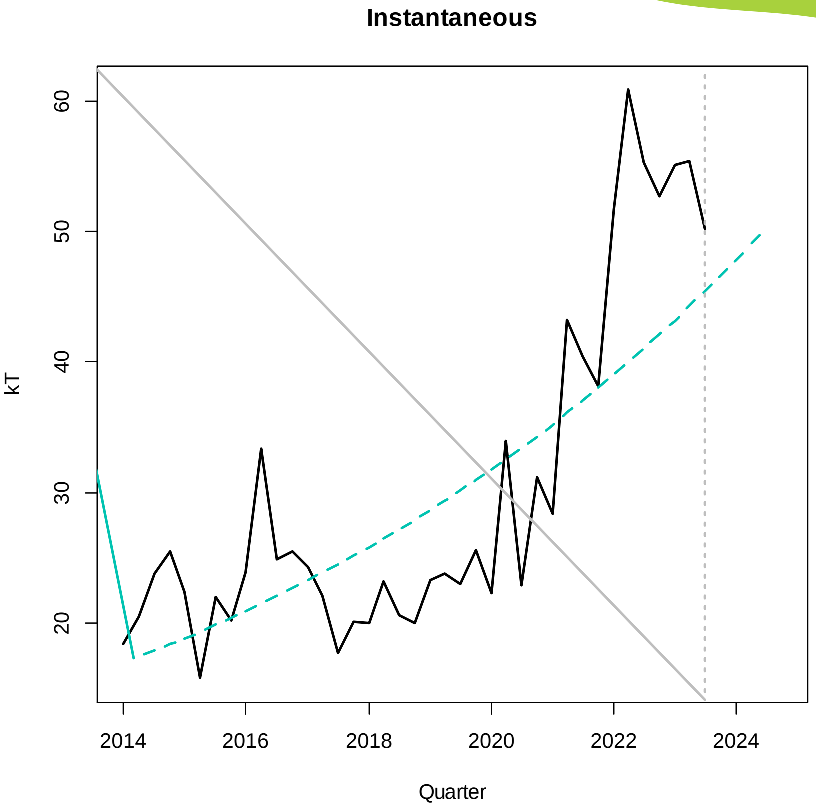
Market Potential	Innovation	Imitation	R^2
6.99e+03**	9.7e-04***	5.9e-02***	0.99



Bass model Chile

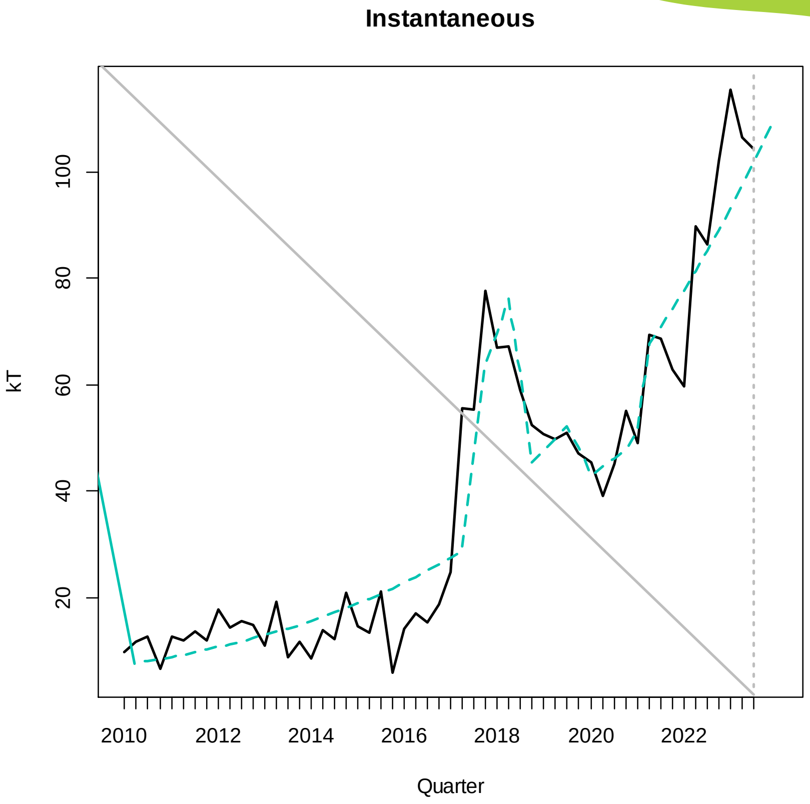
- No significant parameters
- Forecast for one year
- No indication that the peak has been reached

Market Potential	Innovation	Imitation	R^2
5.67+04	2.9-04	2.66-02	0.99



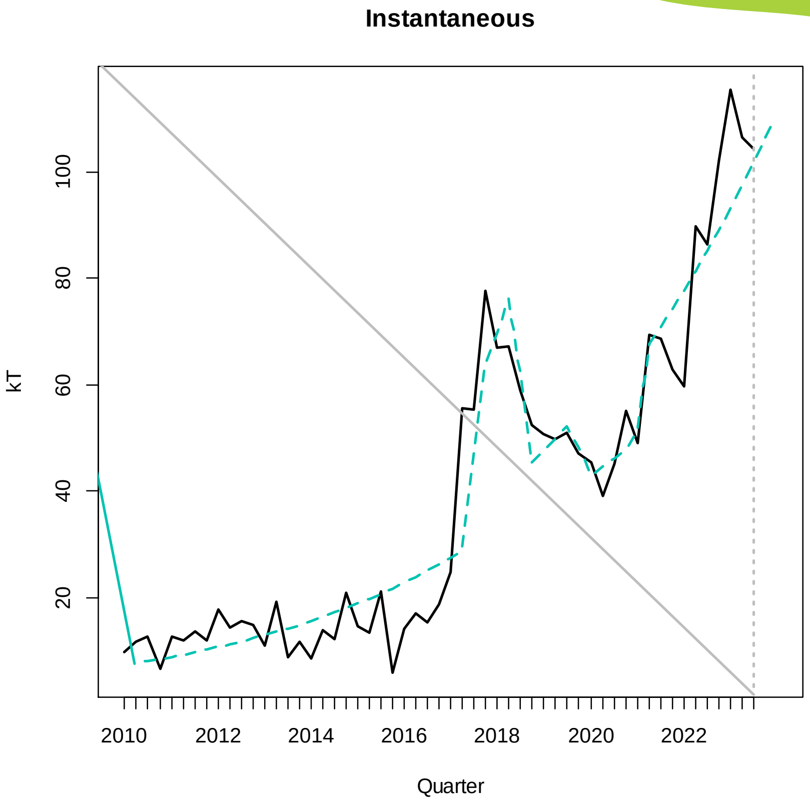
GBM Australia

- Double Rectangular shock between **2017.Q2 – 2018.Q2**, and **2019.Q4 – 2021.Q1**
- These periods could be approximately explained with the excess of supply of 2018, and the extension of the subsidies in 2020



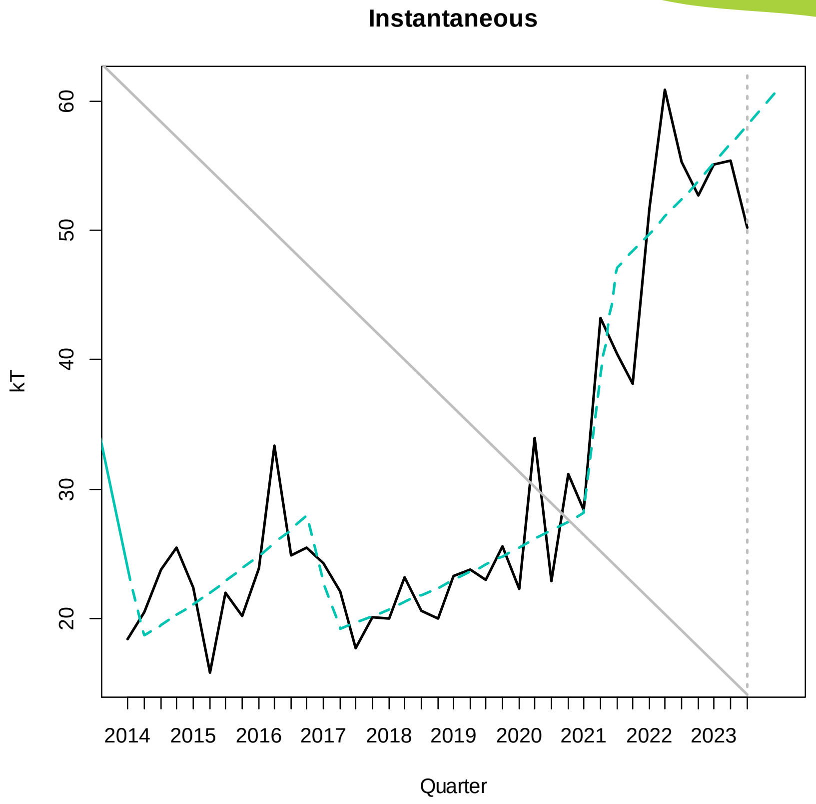
GBM Australia

- $R^2 = 0.998889$
- Significant auto-correlated residuals (better model found by using Double Exponential shock)
- Expected out-of-sample behaviour (four quarters)



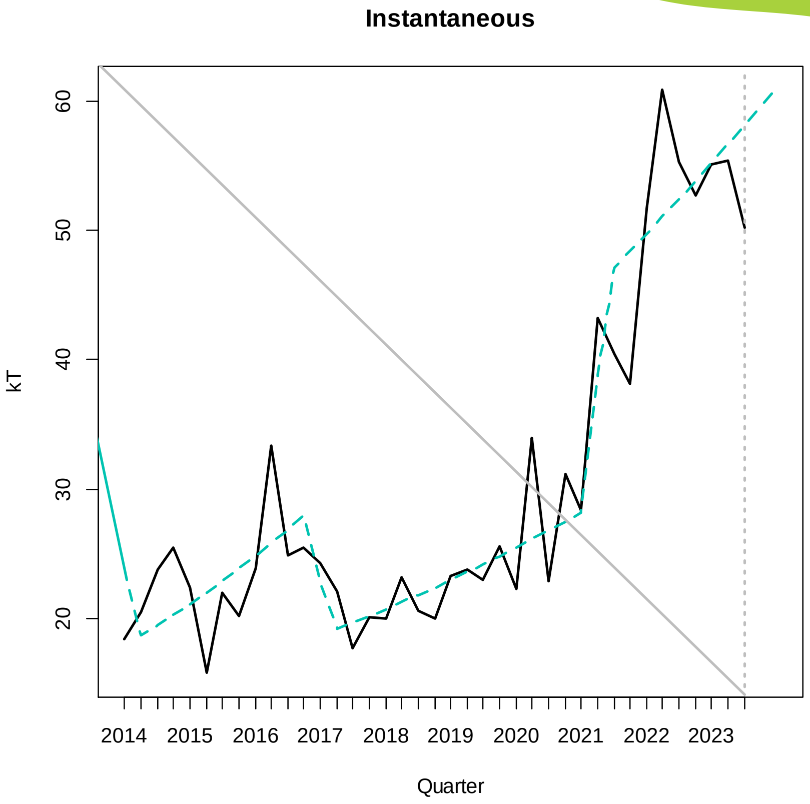
GBM Chile

- Mixed shock at **2016.Q4** (rectangular), and **2021.Q1** (exponential)
- These periods could be approximately explained with the decrease in lithium production and market share of 2016, and the increase in exportations to China, Japan, and South Korea from 2021 onwards



GBM Chile

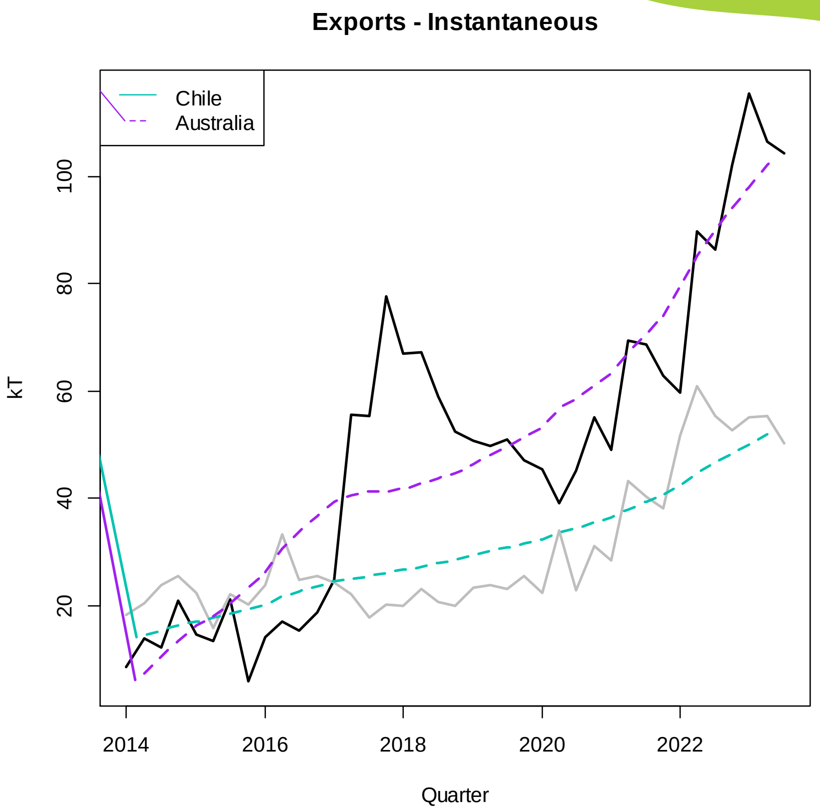
- Significant auto correlated residuals
- Expected out-of-sample behaviour (four quarters)



Competition model Australia vs. Chile

- Both series since 2014.Q1
- Chile collaborates with Australia, but Australia competes with Chile
- Both series show a better adjustment than Bass model

q1c	q2-gamma	R^2
1.47e-01**	-4.2e-03	0.83



04

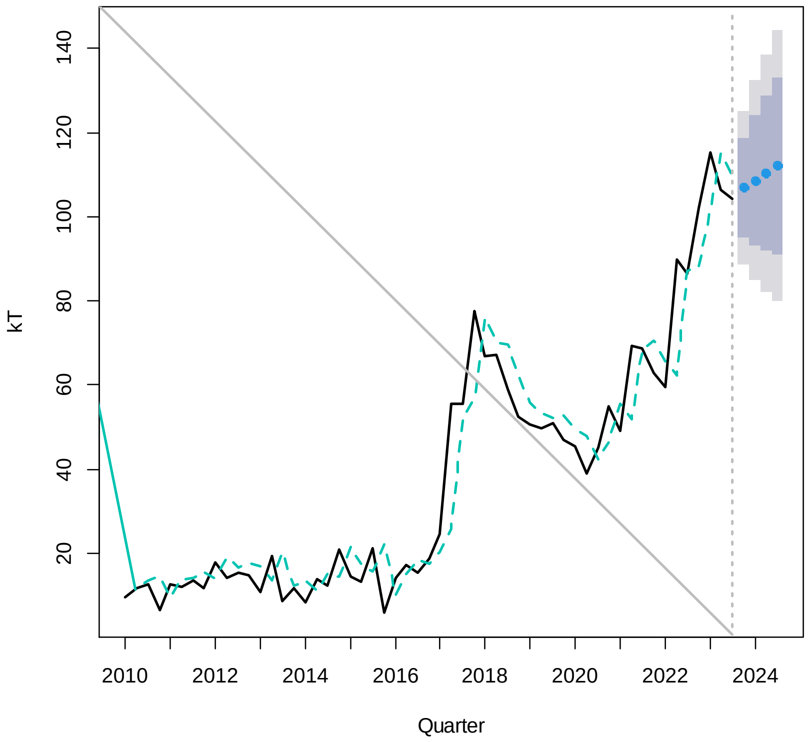
Forecast

Holt’s exponential smoothing, KNN regression, ARIMA

Holt's exponential smoothing Australia

Comments:

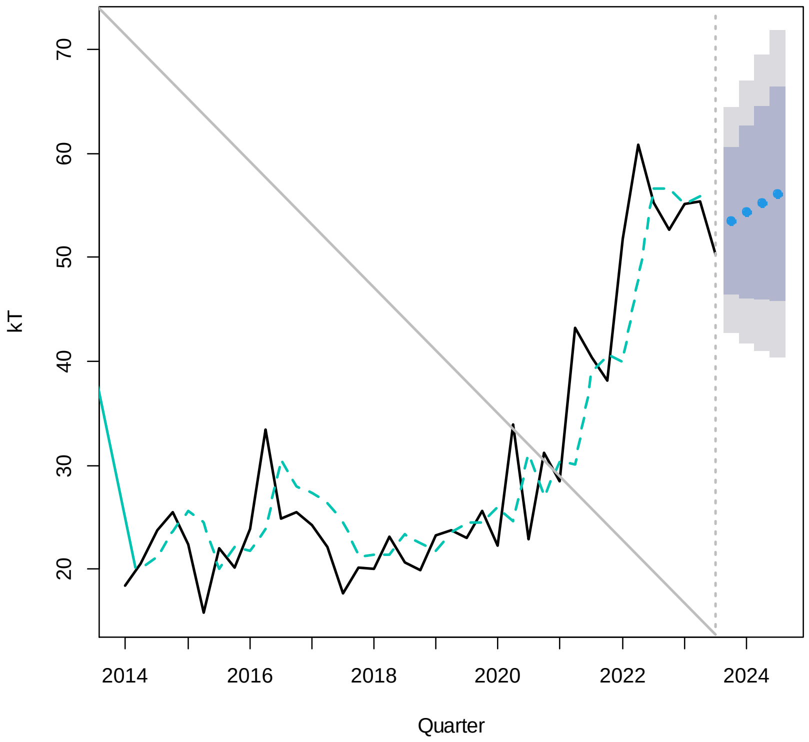
- MAPE 13.64
- Smoothing parameter chosen automatically (0.61)
- A time shift is observed
- Holt-Winters does not offer a better solution to this



Holt’s exponential smoothing Chile

Comments:

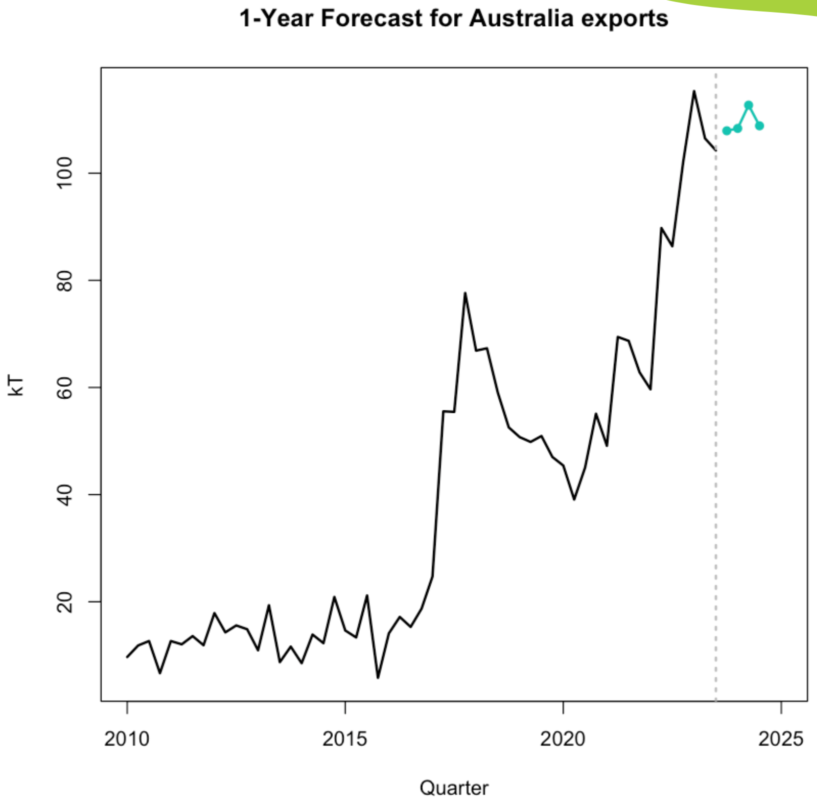
- MAPE 26.68
- Smoothing parameter chosen automatically (0.84)
- A time shift is observed
- Holt-Winters does not offer a better solution to this



KNN Regression Australia

Comments:

- KNN adapted to time series using lagged values of the dependent variable[1].
- k = 2
- Recursive strategy for forecast
- Exports to be tightly balanced according to forecast

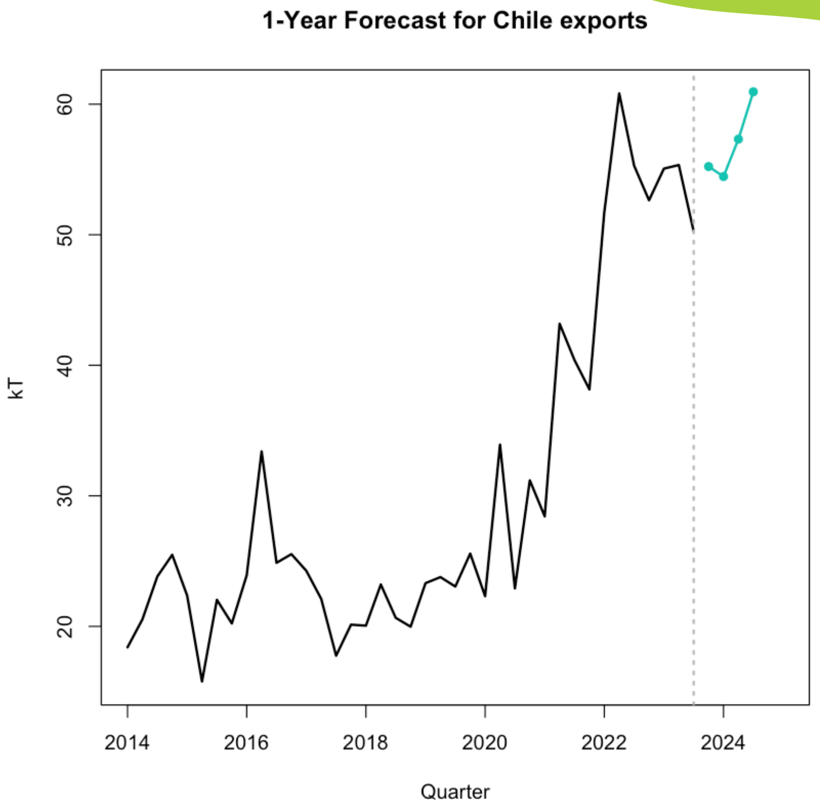


Source: CRAN

KNN Regression Chile

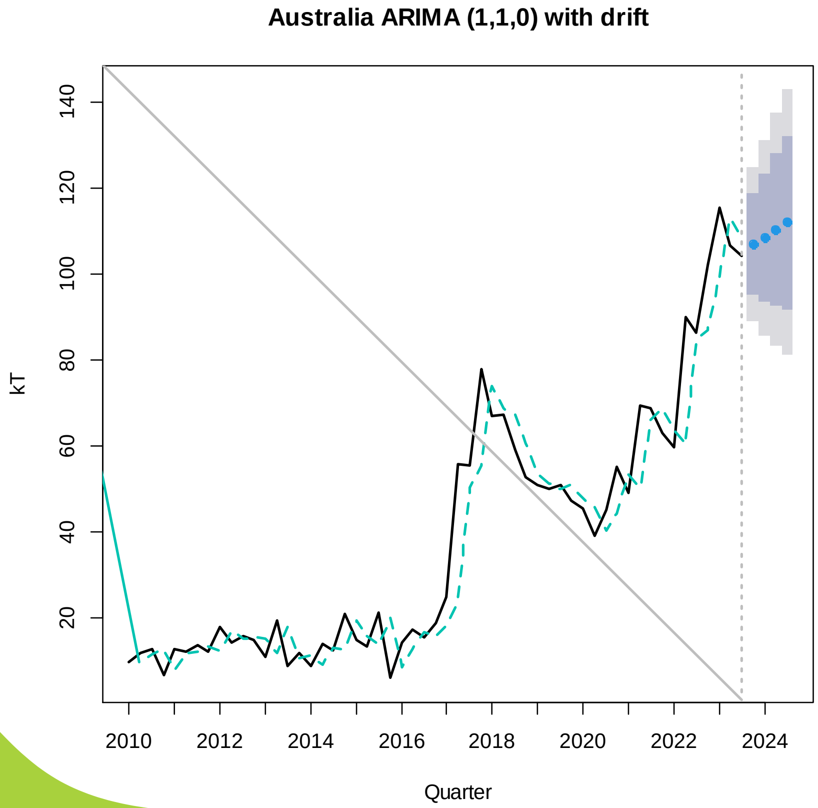
Comments:

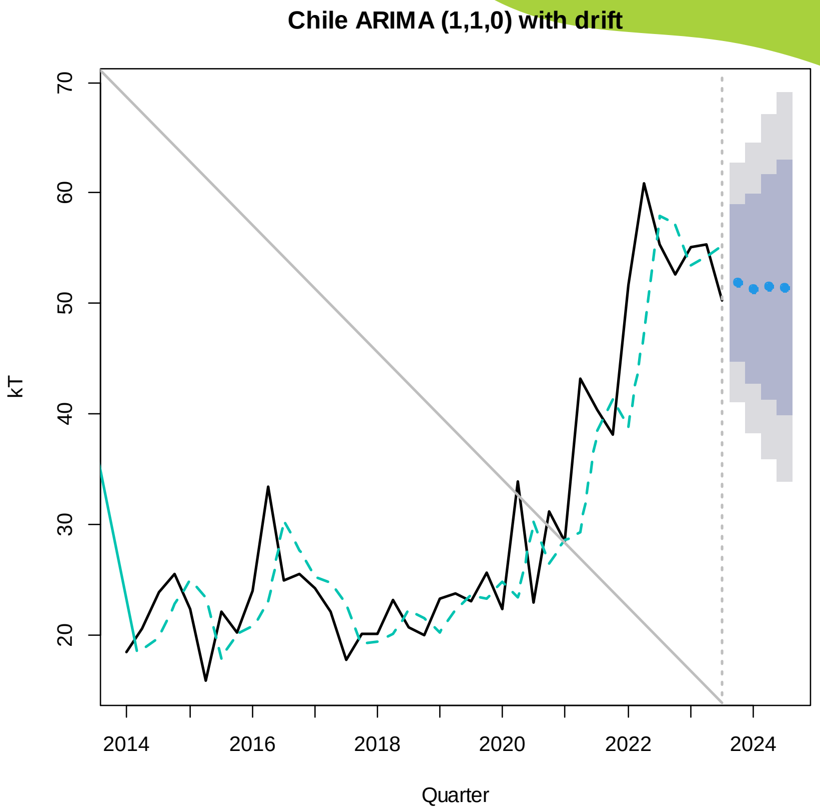
- KNN adapted to time series using lagged values of the dependent variable[1].
- k = 2
- Recursive strategy for forecast
- Optimistic forecast



Source: CRAN

ARIMA Australia and Chile





ARMAX Explanatory Variables

Economic

- GDP
- GDP per capita (working population)
- Yearly variations

Energy related

- Electric Vehicles Stock (China, Europe, USA, Total)
- Fast and Slow chargers (China, Europe, USA, Total)
- Solar Investment

Google Trends

- E-cars (AUS, CHL, World)
- Lithium (AUS, CHL, World)
- Lithium Batteries (AUS, CHL, World)

Stock Market

- Albemarle
- Mineral Resources
- SQM

ARMAX Explanatory Variables

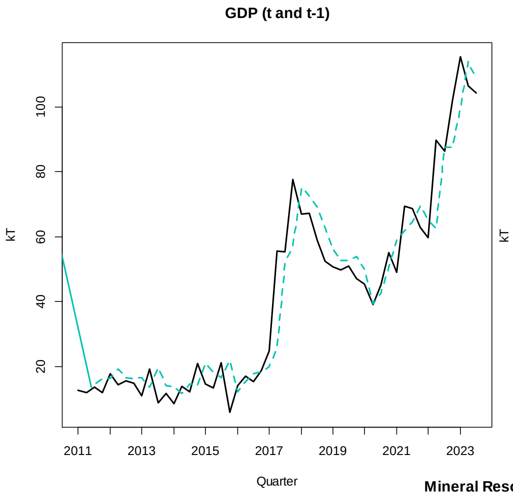
01	Different lags	t = 0, t = t-1, and both
02	Choose Best Adjustment	Based on AIC and model adjustment
03	Forecasting Explanatory variables	Exponential Smoothing or Holt

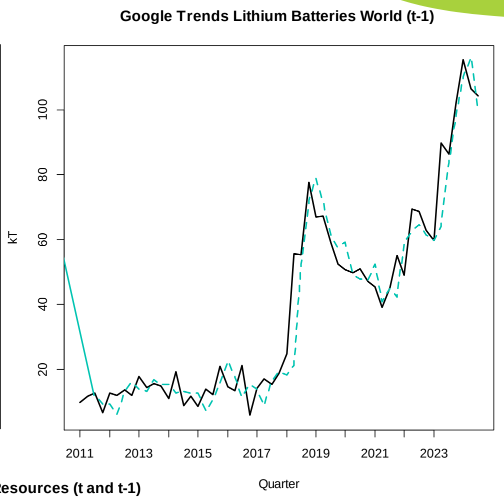
ARMAX Australia

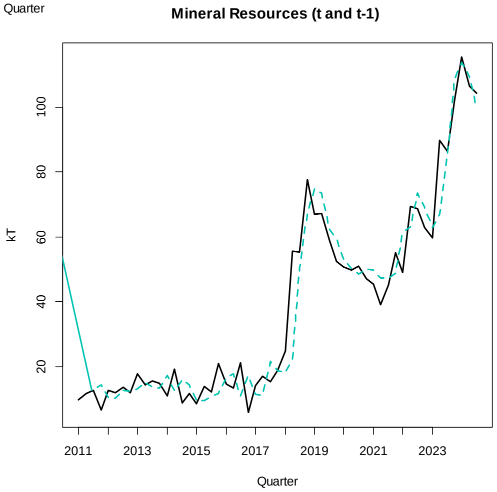
Comments:

- Similar adjustments. Differences in certain periods.

Variable	Time
Mineral Resources	t, t-1
Lithium Bateries World Trends	t-1
GDP per capita Australia	t, t-1





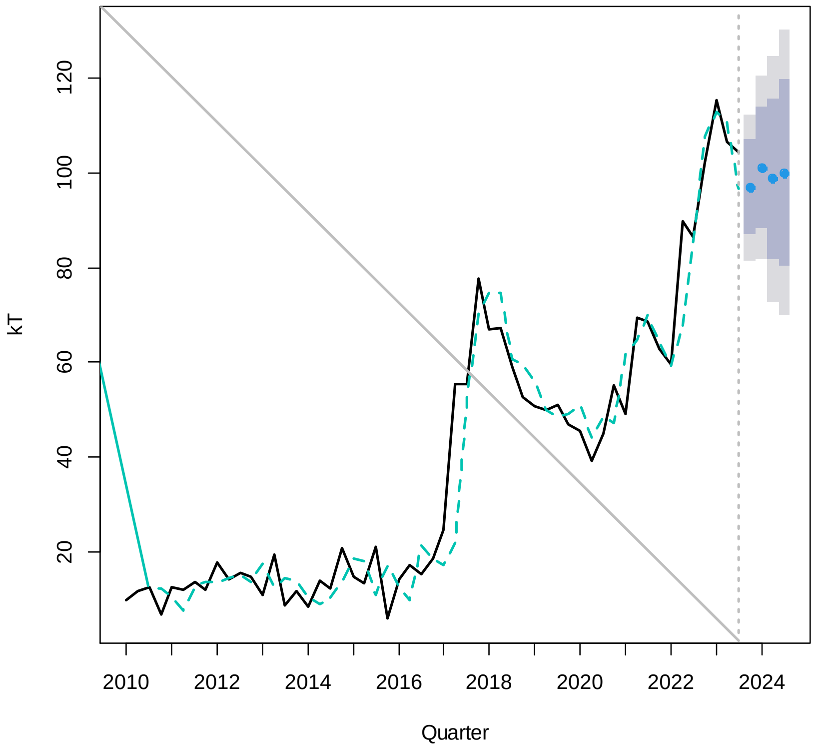


ARMAX Australia

Comments:

- ARIMA(2,1,0) with errors
- Slowly increasing forecasting

Variable	Time	Forecasting
Mineral Resources	t, t-1	Exponential Smoothing
Lithium Bateries World Trends	t-1	Exponential Smoothing

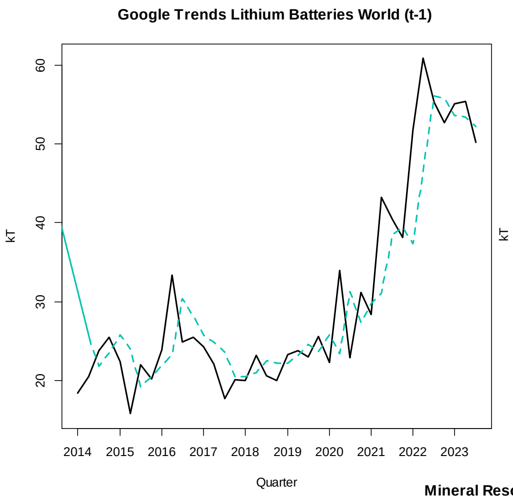


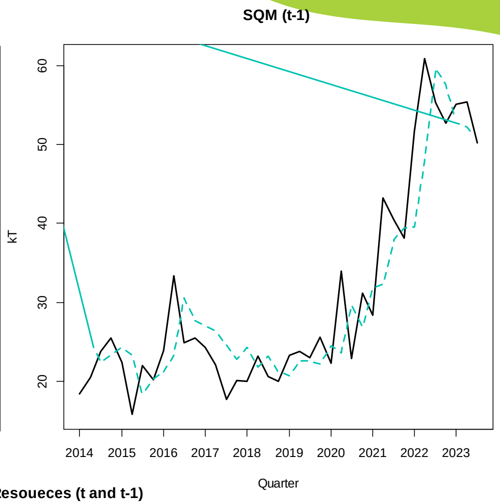
ARMAX Chile

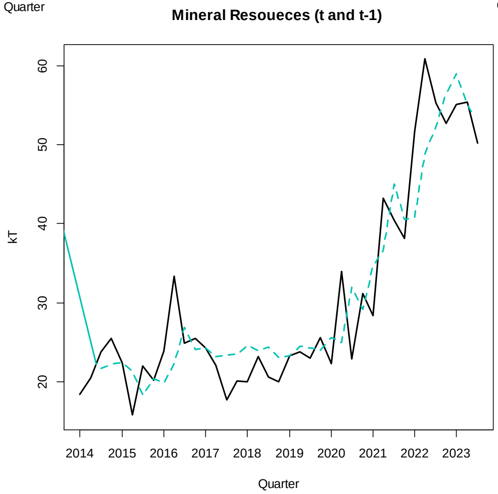
Comments:

- Differences in the adjustment, by period and peaks

Variable	Time
Mineral Resources	t, t-1
Lithium Bateries World Trends	t-1
SQM stock	t-1





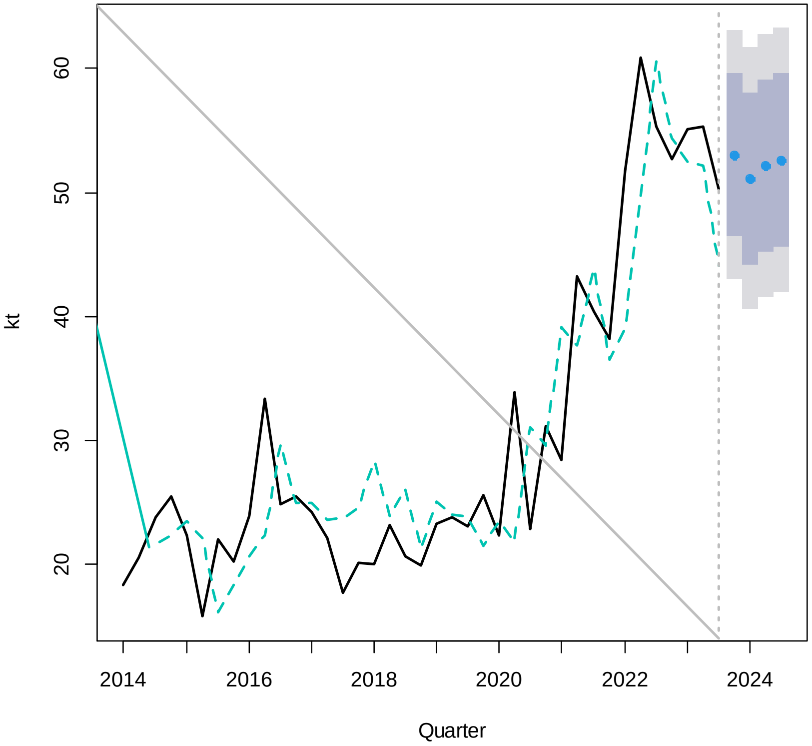


ARMAX Chile

Comments:

- ARIMA(1,1,0) with errors
- Moderate forecasting behaviour

Variable	Time	Forecasting
SQM stock	t-1	Exponential Smoothing
Lithium Bateries World Trends	t-1	Exponential Smoothing
Mineral Resources	t, t-1	Exponential Smoothing



05

Conclusions

Summary of the main findings

Product Growth Australia and Chile

Comments:

- The Bass Model and the Generalized Bass Model help to understand the general growth of Lithium up to now
- The market potential of Lithium has not been reached
- The Generalized Bass Model shows how the shocks impact the dynamics of the lithium export

Forecasting Australia

Model	MAPE
Holt's exponential smoothing	**13.64**
ARIMA(1,1,0)	26.25
ARMAX GDP	24.77
ARMAX Lithium Batteries World Trends	20.84
ARMAX Mineral Resources	22.48
ARMAX(2,1,0) with errors	21.87

Forecasting Chile

Model	MAPE
Holt's exponential smoothing	26.68
ARIMA(1,0,0)	13.17
ARMAX SQM	13.29
ARMAX Lithium Batteries World Trends	14.31
ARMAX Mineral Resources	12.77
ARMAX(1,1,0) with errors	**12.48**

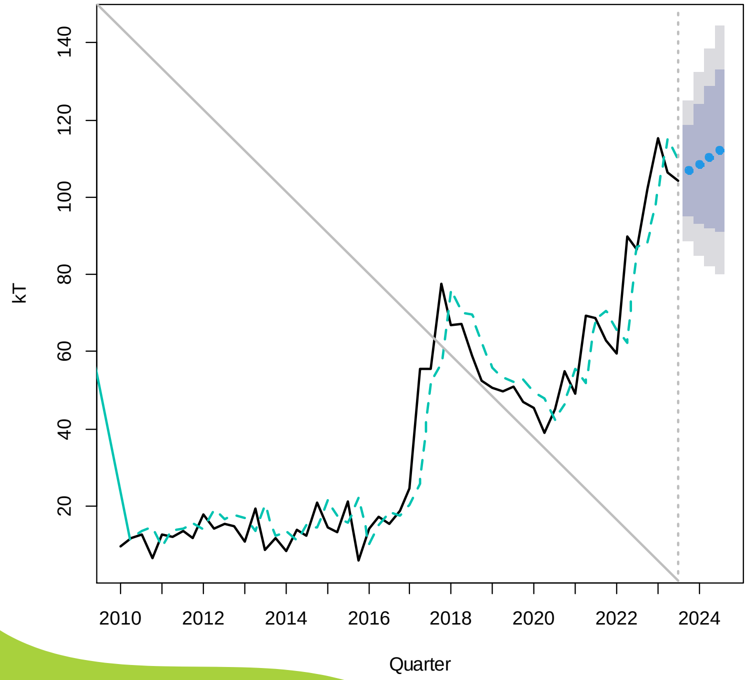
Forecasting Australia and Chile

Comments:

- In the case of Australia, Holt's Exponential Smoothing is the model with the best MAPE
- In the case of Chile, ARMAX with explanatory variables has the lowest MAPE
- The ARMAX model with multiple variables seems to have a better adjustment. However, forecasting will depend on the forecasting of explanatory variables
- It is expected that forecasting has a moderate or increasing behaviour
- Mineral Resources stock seems to be an important variable for **<u>both countries</u>**
- In general, **<u>stock prices</u>** of main companies and **<u>trends of Google</u>** can help forecast Lithium demands

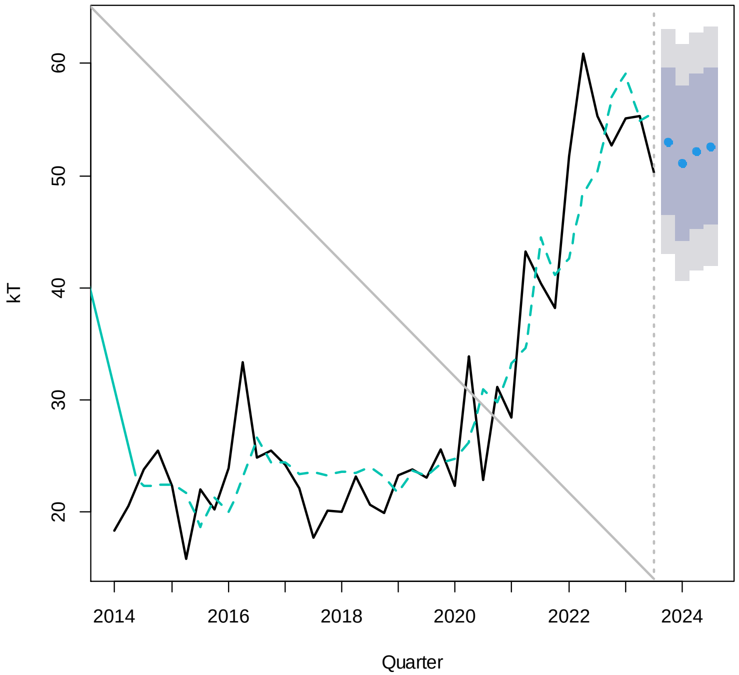
Forecasting Australia and Chile

Holt's Exponential Smoothing Australia



ARIMA(1,0,0) with errors Chile

SQM stock, Lithium trends, Mineral Resources stock



06

Future of Lithium

Expectations in the Lithium market for the future

Future of lithium

- Companies Albemarle and Tianqi, as well as the Australian Government through grants, are still investing in Western Australia for lithium extraction[1,2]
- Australia is facing competition from the "lithium triangle" of Chile, Bolivia, and Argentina[3]
- Chile's state-owned copper mining company Codelco reached a deal with miner SQM to take a majority stake in a new partnership for future lithium projects in the country until 2060[4]

Albemarle's lithium refinery plant, Australia

Latin America's Lithium Triangle

Source: [1]ABC News Australia, [2]Australian Resources and Investment Mining Journal, [3]The Guardian, [4]Financial Times

Padova, another player in the clean technology expansion race

Powering the Future in a Sustainable way: Lithium Analysis and Forecasting

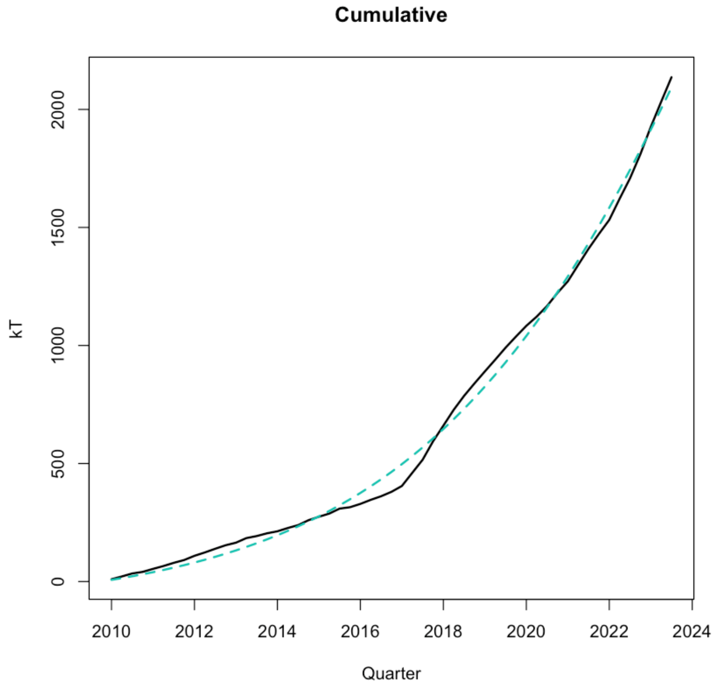
José Chacón
Alejandra Cruces
Mario Tapia

Annex

Models details

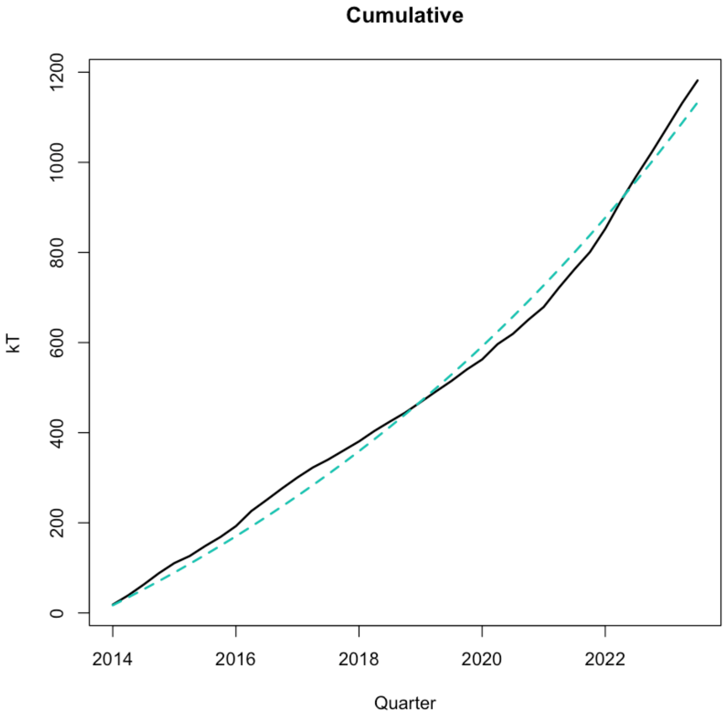
Bass model Australia

```
Residuals:
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-92.441  -20.592   11.698    3.162   30.712   64.192

Coefficients:
       Estimate     Std.Error         Lower         Upper  p-value
m 6.996733e+03 2.023331e+03 3.031077e+03 1.096239e+04 1.09e-03  **
p 9.742995e-04 2.199984e-04 5.431106e-04 1.405488e-03 4.90e-05 ***
q 5.918484e-02 4.335764e-03 5.068690e-02 6.768278e-02 7.99e-19 ***
---
 Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

 Residual standard error  40.09025  on  52  degrees of freedom
 Multiple R-squared:   0.998009  Residual sum of squares:  83575.88
```



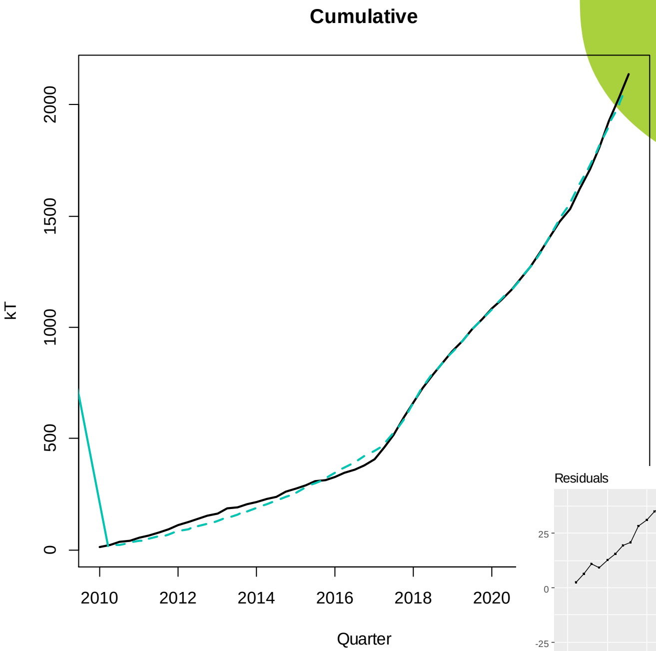
Bass model Chile

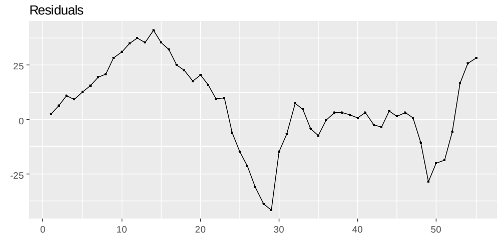
```
Residuals:
    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-48.005 -16.801  11.857   5.834  24.918  48.728

Coefficients:
       Estimate    Std.Error         Lower         Upper p-value
m 5.672974e+04 1.308179e+06 -2.507253e+06 2.620713e+06   0.966
p 2.953937e-04 6.789610e-03 -1.301200e-02 1.360278e-02   0.966
q 2.662497e-02 1.669377e-02 -6.094216e-03 5.934416e-02   0.119
---
 Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

 Residual standard error  29.49248  on  36  degrees of freedom
 Multiple R-squared:   0.997473  Residual sum of squares:  31313.04
```

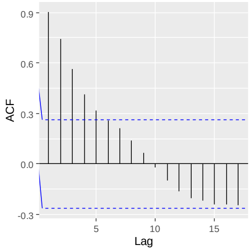


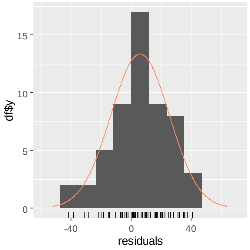
GBM Australia

```
Call: ( Generalized Bass model with 2  Rectangular  shock )

  GBM(series = lit.aus.exp.quarter.kton, shock = "rett", nshock = 2,
    prelimestimates = c(5742.86, 0.001144887, 0.06183144, 29,
        40, 0.1, 46, 48, -0.1), oos = 4)

Residuals:
    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-41.558  -4.966   3.872   5.829  19.845  41.011

Coefficients:
           Estimate    Std.Error          Lower          Upper  p-value
m      8.416800e+04 4.715245e+05 -8.400030e+05  1.008339e+06 8.59e-01
p      8.566532e-05 4.920639e-04 -8.787622e-04  1.050093e-03 8.63e-01
q      4.724516e-02 6.224386e-03  3.504559e-02  5.944473e-02 1.20e-09 ***
a1     3.040689e+01 8.713057e-01  2.869916e+01  3.211462e+01 9.55e-35 ***
b1     3.448762e+01 6.650405e-01  3.318417e+01  3.579108e+01 1.84e-42 ***
c1     9.210240e-01 2.880427e-01  3.564706e-01  1.485577e+00 2.51e-03  **
a2     3.952839e+01 1.733262e+00  3.613126e+01  4.292553e+01 1.03e-26 ***
b2     4.486250e+01 1.652551e+00  4.162356e+01  4.810144e+01 5.86e-30 ***
c2    -2.382478e-01 1.196473e-01 -4.727523e-01 -3.743350e-03 5.24e-02   .
---
 Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

 Residual standard error  21.97029  on  46  degrees of freedom
 Multiple R-squared:   0.998889  Residual sum of squares:  22203.92


             Ljung-Box test

    data:  Residuals
    Q* = 125.01, df = 10, p-value < 2.2e-16

    Model df: 0.   Total lags used: 10
```



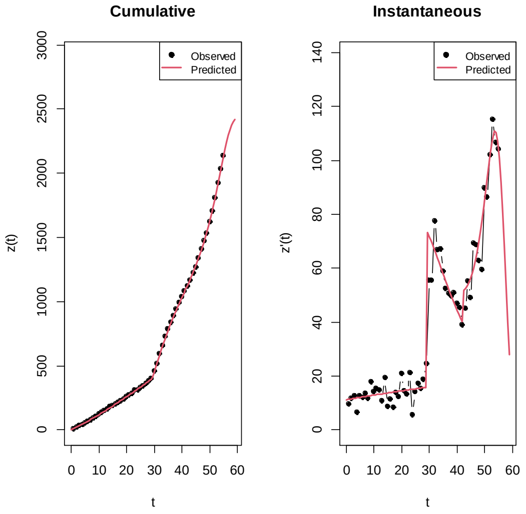


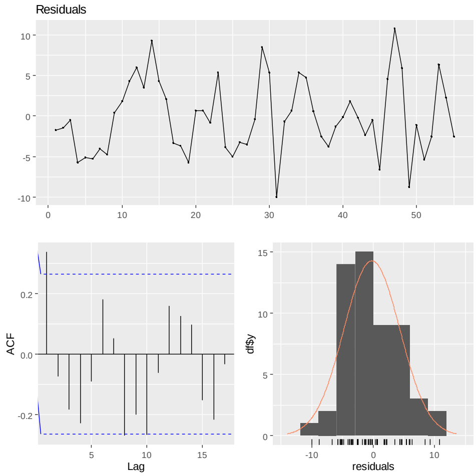




GBM Australia

```
Call: ( Generalized Bass model with 2  Exponential  shock )

  GBM(series = lit.aus.exp.quarter.kton, shock = "exp", nshock = 2,
    prelimestimates = c(5742.86, 0.001144887, 0.06183144, 27,
        0.1, 0.1, 43, 0.1, 0.1), oos = 4)

Residuals:
     Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
-10.0016   -3.5782   -0.5029   -0.2060   2.9150   10.8381

Coefficients:
           Estimate     Std.Error          Lower          Upper  p-value
m    2451.91235646 7.604703e+01  2.302863e+03  2.600962e+03 3.17e-33 ***
p       0.00461427 1.633521e-04  4.294106e-03  4.934434e-03 1.04e-30 ***
q       0.01920551 1.511792e-03  1.624245e-02  2.216857e-02 1.20e-16 ***
a1     29.25114472 9.603027e-02  2.906293e+01  2.943936e+01 1.13e-77 ***
b1     -0.08381038 7.952976e-03 -9.939793e-02 -6.822284e-02 7.48e-14 ***
c1      3.63499244 1.657908e-01  3.310049e+00  3.959936e+00 5.43e-26 ***
a2     42.39489870 4.367424e-01  4.153890e+01  4.325090e+01 7.21e-55 ***
b2      0.23871050 1.630093e-02  2.067613e-01  2.706597e-01 6.17e-19 ***
c2      0.71115920 9.841783e-02  5.182638e-01  9.040546e-01 4.18e-09 ***
---
 Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

 Residual standard error  4.964882  on  46  degrees of freedom
 Multiple R-squared:   0.999943  Residual sum of squares:  1133.903
```

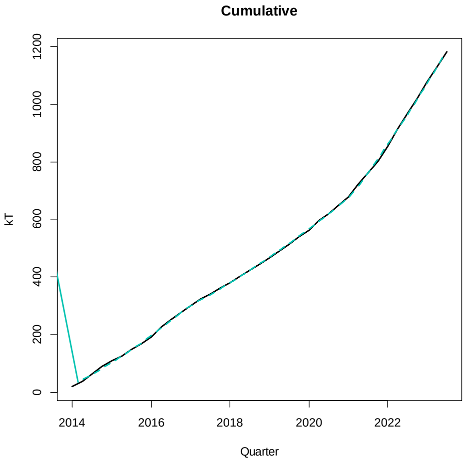


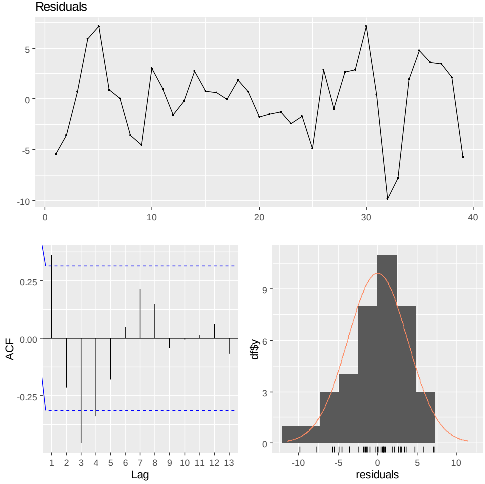


GBM Chile

```
Call: ( Generalized Bass model with 2  Mixed  shock )

  GBM(series = lit.chl.exp.quarter.kton, shock = "mixed", nshock = 2,
    prelimestimates = c(56729.61, 0.0002953943, 0.02662497, 25,
        0.1, 0.1, 7, 14, 0.1), oos = 4)

Residuals:
      Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
-9.835813 -1.744298  0.650401  0.005721  2.684263  7.171730

Coefficients:
           Estimate     Std.Error          Lower         Upper  p-value
m     6.034528e+04 1.409382e+06 -2.701993e+06 2.822684e+06 9.66e-01
p     1.850806e-04 4.336374e-03 -8.314055e-03 8.684216e-03 9.66e-01
q     2.623386e-02 1.183852e-02  3.030781e-03 4.943694e-02 3.44e-02   *
a1    2.941646e+01 3.407102e-01  2.874868e+01 3.008424e+01 1.60e-37 ***
b1   -3.484008e-02 5.799911e-02 -1.485162e-01 7.883609e-02 5.53e-01
c1    5.840652e-01 7.861523e-02  4.299822e-01 7.381483e-01 2.80e-08 ***
a2   -9.363961e-01 6.961145e-01 -2.300755e+00 4.279632e-01 1.89e-01
b2    1.240923e+01 4.155060e-01  1.159486e+01 1.322361e+01 7.12e-24 ***
c2    5.559315e-01 1.083900e-01  3.434910e-01 7.683721e-01 1.62e-05 ***
---
 Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

 Residual standard error  4.283943  on  30  degrees of freedom
 Multiple R-squared:   0.999864  Residual sum of squares:  550.5651
```

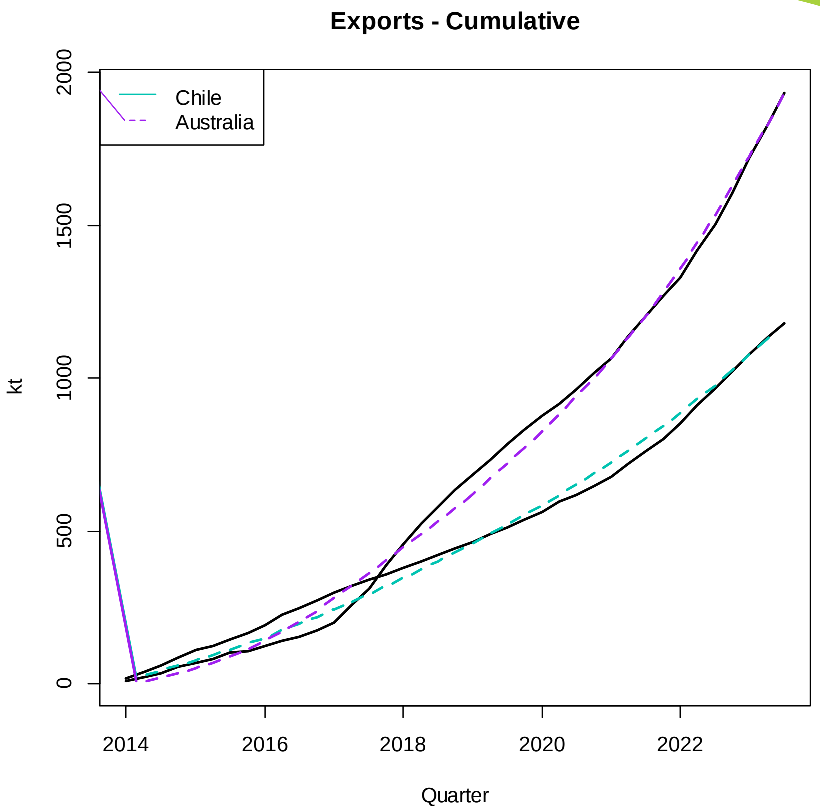
```
        Ljung-Box test

data:  Residuals
Q* = 26.922, df = 8, p-value = 0.0007294

Model df: 0.   Total lags used: 8
```





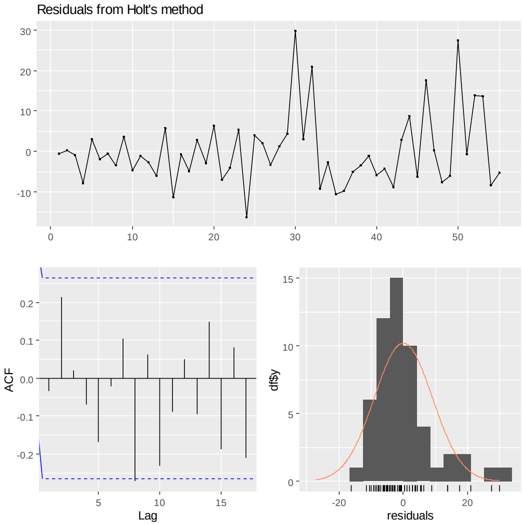
Competition model Australia vs. Chile

```
Call: ( UCRCD Model )

  UCRCD(series1 = lit.aus.exp.quarter.kton2, series2 = lit.chl.exp.quarter.kton,
    display = T)

Residuals Series 1:
      Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
-19.71753 -11.67947   0.60355   0.00006   6.90625  36.25851

Residuals Series 2:
       Min.    1st Qu.     Median       Mean    3rd Qu.       Max.
-11.480068  -6.052674   0.283861   0.000297   4.896704  16.228050

Coefficients:
            Estimate    Std.Error         Lower         Upper p-value
mc     2.619344e+08 4.664402e+12 -9.141798e+12  9.142322e+12 1.00000
p1c    9.718295e-09 1.747179e-04 -3.424310e-04  3.424505e-04 1.00000
p2     4.927435e-08 8.859117e-04 -1.736306e-03  1.736404e-03 1.00000
q1c    1.472441e-01 4.531378e-02  5.843069e-02  2.360575e-01 0.00177  **
q2     4.123059e-02 4.470854e-02 -4.639653e-02  1.288577e-01 0.36000
delta -1.839038e-01 6.964760e-02 -3.204106e-01 -4.739701e-02 0.01020   *
gamma  4.543773e-02 6.961459e-02 -9.100436e-02  1.818798e-01 0.51600
---
 Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

 Residual standard error Series 1: 14.46583  on  32  degrees of freedom
 Residual standard error Series 2: 7.359257  on  32  degrees of freedom
 Multiple R-squared:   0.824858  Residual sum of squares:  8429.404


          mc           p1c     q1c+delta          q1c            p2
  2.619344e+08  9.718295e-09 -3.665973e-02  1.472441e-01  4.927435e-08
            q2      q2-gamma
  4.123059e-02 -4.207138e-03
```



Holt's exponential smoothing Australia

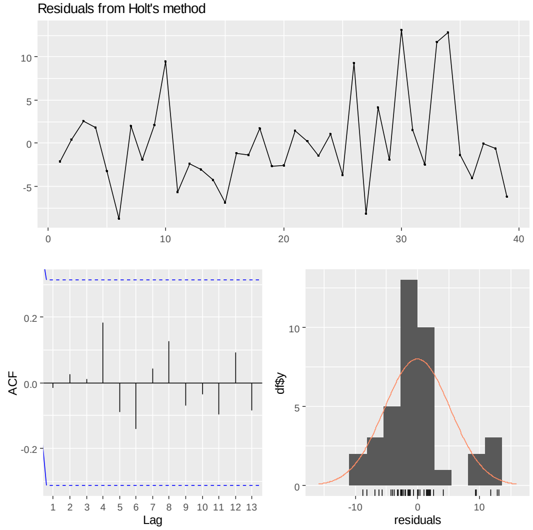
```
Forecast method: Holt's method

Model Information:
Holt's method

Call:
 holt(y = lit.aus.exp.quarter.kton, h = 4)

  Smoothing parameters:
    alpha = 0.8402
    beta  = 1e-04

  Initial states:
    l = 8.4038
    b = 1.7611

  sigma:  9.273

     AIC     AICc      BIC
471.2322 472.4567 481.2689

Error measures:
                        ME     RMSE      MAE       MPE     MAPE      MASE       ACF1
Training set -0.0008214128 8.929441 6.418935 -11.96011 26.68084 0.9941299 -0.0340883

Forecasts:
   Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
56       106.8669 94.98309 118.7508 88.69216 125.0417
57       108.6281 93.10596 124.1502 84.88905 132.3671
58       110.3892 91.93197 128.8465 82.16129 138.6171
59       112.1504 91.16399 133.1367 80.05448 144.2462

                    Ljung-Box test

        data:  Residuals from Holt's method
        Q* = 14.547, df = 10, p-value = 0.1495

        Model df: 0.   Total lags used: 10
```



Holt's exponential smoothing Chile

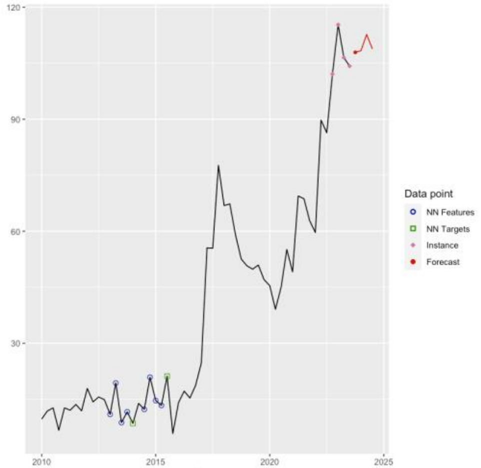
```
Forecast method: Holt's method

Model Information:
Holt's method

Call:
 holt(y = lit.chl.exp.quarter.kton, h = 4)

  Smoothing parameters:
    alpha = 0.6064
    beta  = 1e-04

  Initial states:
    l = 19.6842
    b = 0.8529

  sigma:  5.5312

     AIC     AICc      BIC
282.0701 283.8883 290.3880

Error measures:
                       ME     RMSE      MAE       MPE     MAPE      MASE        ACF1
Training set -0.009837826 5.239877 3.890208 -3.173123 13.64065 0.8893848 -0.01554943

Forecasts:
   Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
40       53.52751 46.43900 60.61603 42.68656 64.36847
41       54.38041 46.09023 62.67058 41.70167 67.05914
42       55.23330 45.89450 64.57209 40.95084 69.51576
43       56.08619 45.80488 66.36750 40.36228 71.81009
```

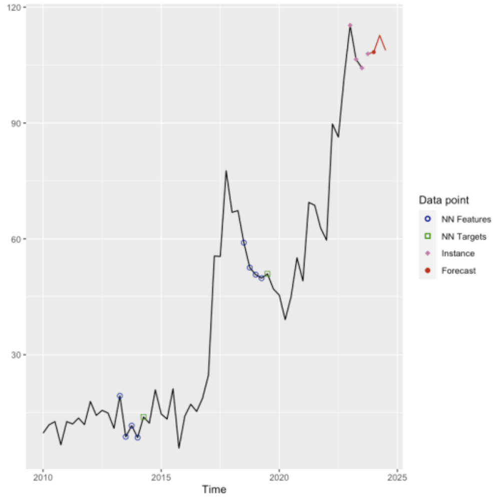
```
        Ljung-Box test

data:  Residuals from Holt's method
Q* = 3.8423, df = 8, p-value = 0.8711

Model df: 0.   Total lags used: 8
```

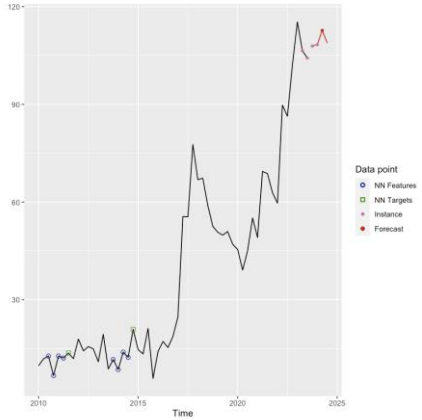


KNN Regression Australia

```
Call:  knn_forecasting(timeS = aus.exp.ts, h = 4, lags = 1:4, k = 2)

Multiple-Step Ahead Strategy: recursive
K (number of nearest neighbors): 2
Autoregressive lags: 1 2 3 4
Number of examples: 51
Targets are combined using the mean function.
Forecasting horizon: 4
Forecast:
         Qtr1     Qtr2     Qtr3     Qtr4
2023                            107.9178
2024 108.3608 112.7049 108.8447
```

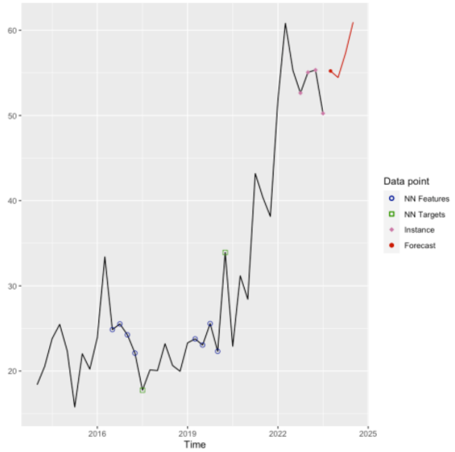


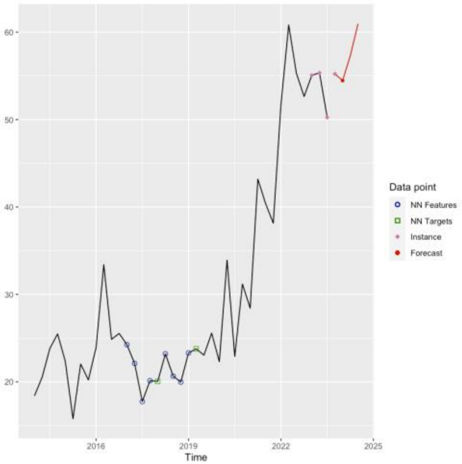


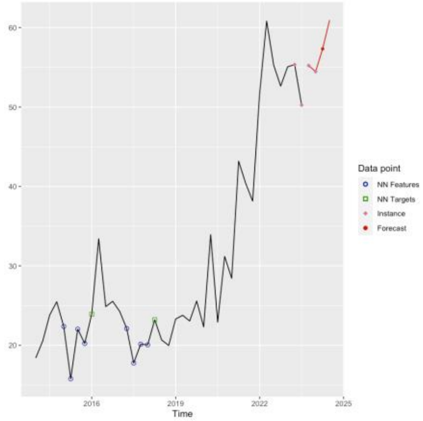


KNN Regression Chile

```
Call:  knn_forecasting(timeS = ch.exp.ts, h = 4, lags = 1:4, k = 2,
    msas = "recursive", transform = "additive")

Multiple-Step Ahead Strategy: recursive
K (number of nearest neighbors): 2
Autoregressive lags: 1 2 3 4
Number of examples: 35
Targets are combined using the mean function.
Forecasting horizon: 4
Forecast:
          Qtr1     Qtr2     Qtr3     Qtr4
2023                             55.22379
2024 54.45693 57.32102 60.95001
```

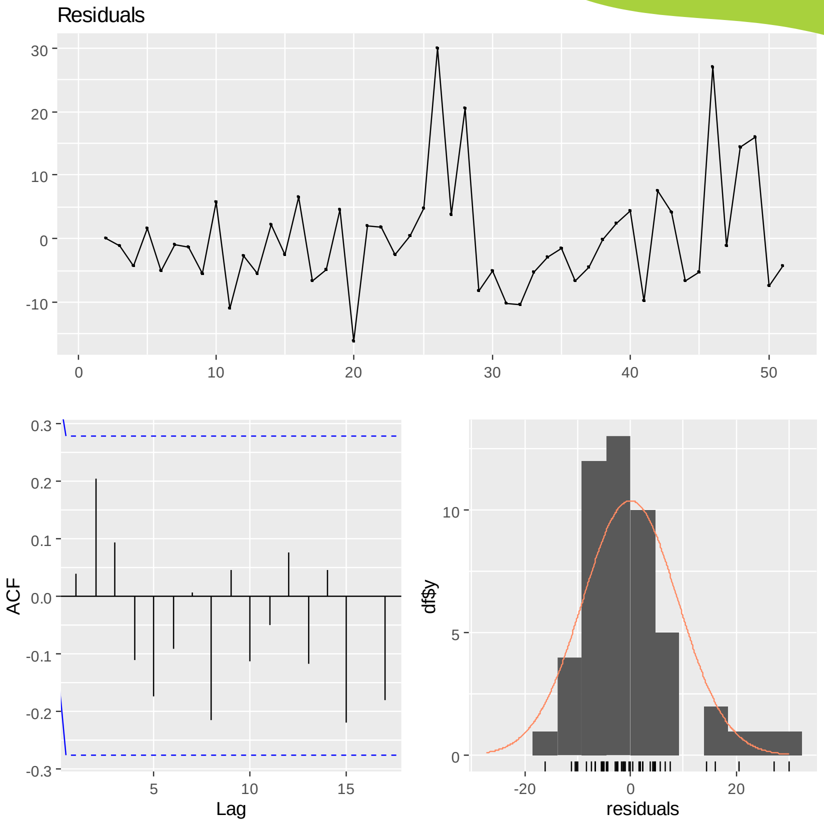






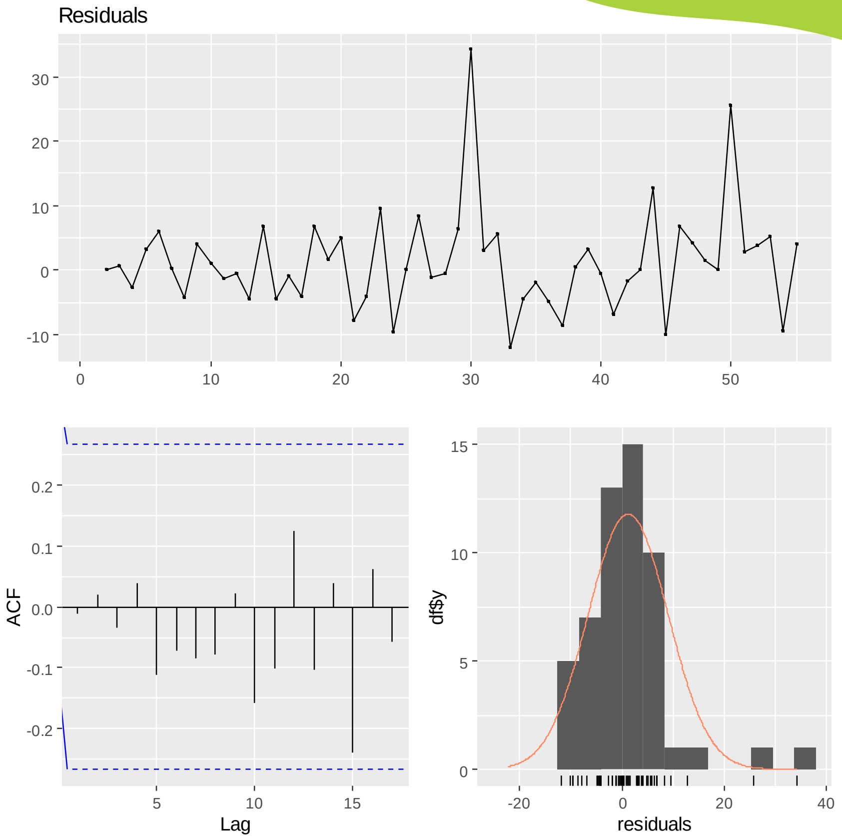
ARMAX Australia: GDP Variation

```
Series: df$lit_aus_exp_kton_met
Regression with ARIMA(1,1,0) errors

Coefficients:
          ar1   drift     var_0    var_1
      -0.2407  1.9400  107.6573  21.3806
s.e.   0.1374  1.0453   63.6846  64.3608

sigma^2 = 87.05:  log likelihood = -177.44
AIC=364.88   AICc=366.27   BIC=374.34

Training set error measures:
                       ME     RMSE      MAE       MPE     MAPE      MASE
Training set -0.004740116 8.949036 6.407176 -11.60387 24.77131 0.9600486
                   ACF1
Training set 0.03749308
```



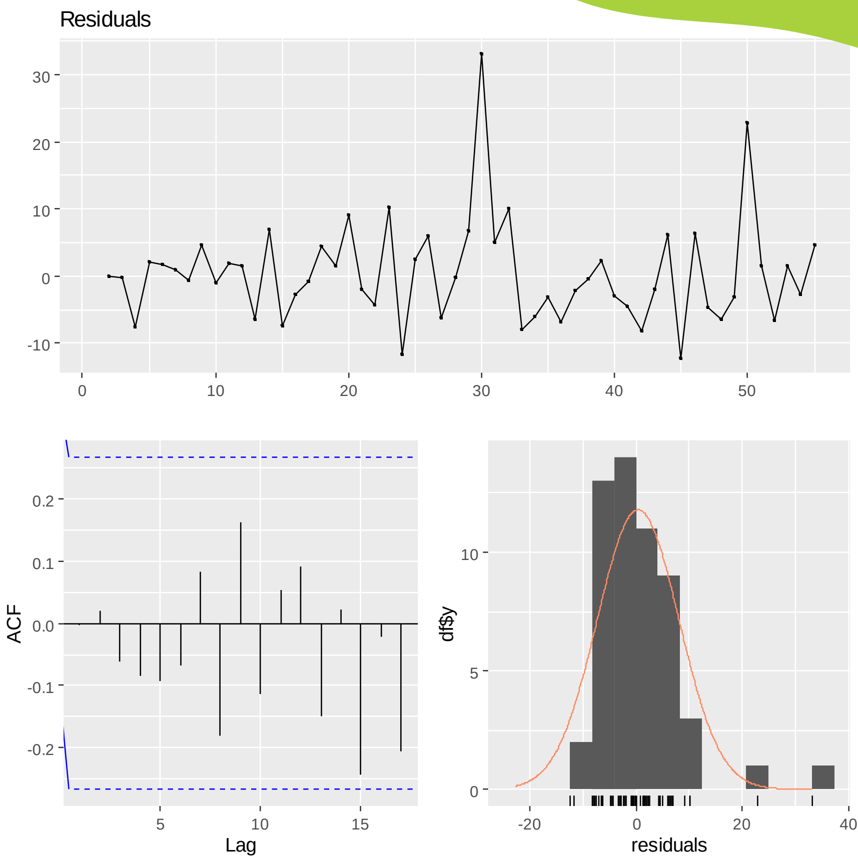
ARMAX Australia: Lithium Batteries World Trends

```
Series: df$lit_aus_exp_kton_met
Regression with ARIMA(0,1,2) errors

Coefficients:
          ma1     ma2    xreg
      -0.2043  0.4470  2.8502
s.e.   0.1314  0.1176  0.8257

sigma^2 = 65.34:  log likelihood = -185.18
AIC=378.35   AICc=379.18   BIC=386.23

Training set error measures:
                    ME     RMSE      MAE      MPE     MAPE      MASE
Training set 1.230656 7.855367 5.192002 -1.49854 20.84089 0.8041091
                    ACF1
Training set -0.01072171
```



ARMAX Australia: Mineral Resources

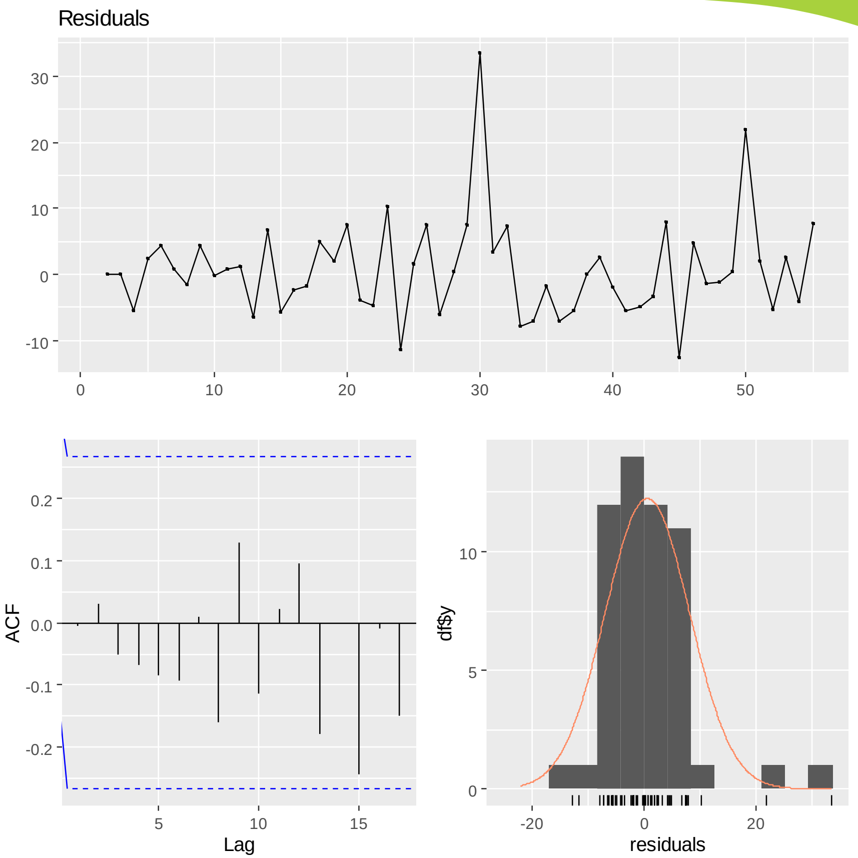
```
Series: df$lit_aus_exp_kton_met
Regression with ARIMA(2,1,0) errors

Coefficients:
          ar1     ar2   var_0   var_1
      -0.2598  0.2144  0.7207  0.3937
s.e.   0.1343  0.1335  0.2070  0.2140

sigma^2 = 62.66:  log likelihood = -183.42
AIC=376.83   AICc=378.11   BIC=386.68

Training set error measures:
                    ME     RMSE      MAE       MPE     MAPE
Training set 0.4077493 7.617112 5.283478 -4.858721 22.48089
                  MASE         ACF1
Training set 0.8182764 -0.002910985
```



ARMAX Australia: Multivariable Model

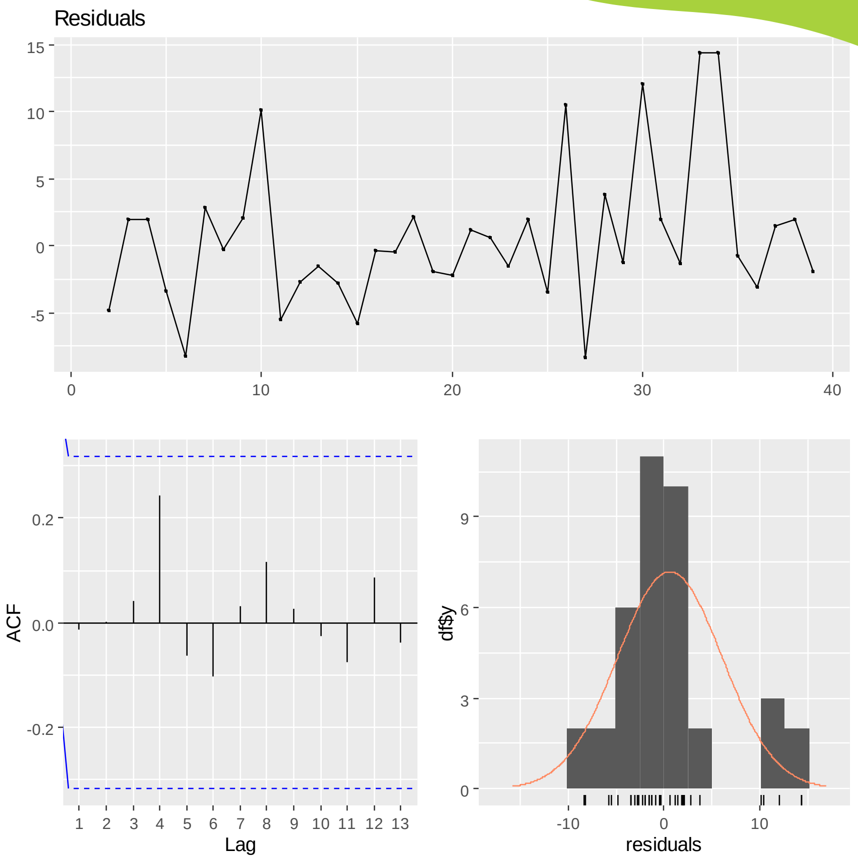
Variable	Time	Forecasting
Mineral Resources	t, t-1	Exponential Smoothing
Lithium Bateries World Trends	t-1	Exponential Smoothing

```
Series: data_trends_aux$lit_aus_exp_kton_met
Regression with ARIMA(2,1,0) errors

Coefficients:
          ar1     ar2    var0    var1    var2
      -0.2280  0.2983  0.5803  0.2256  1.5752
s.e.   0.1329  0.1453  0.2268  0.2457  1.1635

sigma^2 = 61.7:  log likelihood = -182.51
AIC=377.03   AICc=378.85   BIC=388.85

Training set error measures:
                    ME     RMSE      MAE       MPE     MAPE
Training set 0.6490937 7.482392 5.097209 -3.704196 21.87899
                  MASE         ACF1
Training set 0.789428 -0.005767621
```



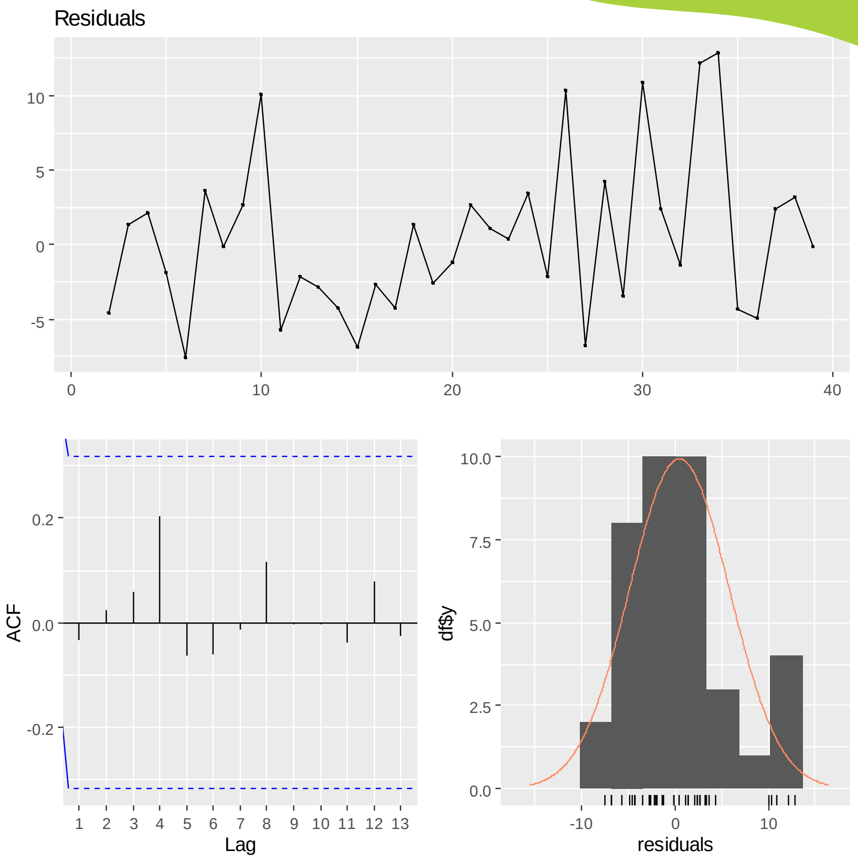
ARMAX Chile: Lithium Batteries World Trends

```
Series: df$lit_chl_exp_kton_met
Regression with ARIMA(2,0,0) errors

Coefficients:
         ar1     ar2  intercept    xreg
      0.6076  0.3227    27.2214  0.4743
s.e.  0.1600  0.1601    13.1100  0.6923

sigma^2 = 32.01:  log likelihood = -119.14
AIC=248.28   AICc=250.15   BIC=256.47

Training set error measures:
                    ME     RMSE      MAE       MPE
Training set 0.6168364 5.429898 3.876251 -1.409716
                 MAPE     MASE        ACF1
Training set 13.29638 0.886194 -0.01413027
```



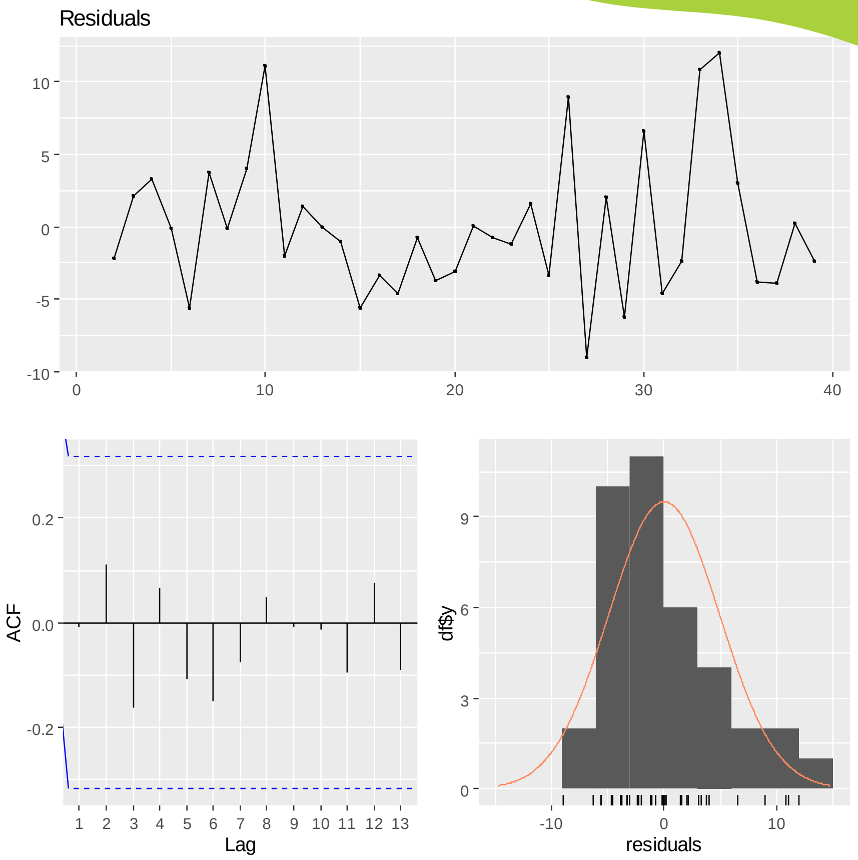
ARMAX Chile: SQM stock

```
Series: df$lit_chl_exp_kton_met
Regression with ARIMA(1,0,1) errors

Coefficients:
         ar1      ma1  intercept    xreg
      0.9313  -0.3262    24.0333  0.1960
s.e.  0.0719   0.1609     8.6713  0.1105

sigma^2 = 30.32:  log likelihood = -117.88
AIC=245.77   AICc=247.64   BIC=253.95

Training set error measures:
                    ME     RMSE      MAE       MPE
Training set 0.4679963 5.284258 4.131787 -1.724194
                 MAPE      MASE        ACF1
Training set 14.31525 0.9446149 -0.03275982
```



ARMAX Chile: Mineral Resources

```
Series: df$lit_chl_exp_kton_met
Regression with ARIMA(1,0,0) errors

Coefficients:
         ar1  intercept   var_0  var_1
      0.3841    17.3434  0.3085  0.195
s.e.  0.1530     1.9247  0.1432  0.146

sigma^2 = 25.62:  log likelihood = -114.06
AIC=238.12   AICc=239.99   BIC=246.31

Training set error measures:
                     ME     RMSE      MAE      MPE
Training set 0.02938578 4.857458 3.707437 -2.47511
                 MAPE      MASE        ACF1
Training set 12.76616 0.8475995 -0.00855879
```



ARMAX Chile: Multivariable Model

Variable	Time	Forecasting
SQM	t-1	Exponential Smoothing
Lithium Bateries World Trends	t-1	Exponential Smoothing
Mineral Resources	t, t-1	Exponential Smoothing

```
Series: data_trends_aux$lit_chl_exp_kton_met
Regression with ARIMA(1,0,0) errors

Coefficients:
         ar1  intercept    var0    var1     var2     var3
      0.3337    27.1546  0.4289  0.2201  -0.7636  -0.0836
s.e.  0.1849     8.2837  0.1878  0.1499   0.6629   0.1205

sigma^2 = 26.2:  log likelihood = -113.35
AIC=240.69   AICc=244.43   BIC=252.16

Training set error measures:
                     ME     RMSE      MAE       MPE
Training set 0.03755148 4.769805 3.694767 -2.350516
                 MAPE      MASE        ACF1
Training set 12.47628 0.8447029 0.01180559
```