Notes on Schelling's Model

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1 Mark's Advice

1. Try to answer some philosophical question:
   (a) To what extent can a simple model explain the real world?
      i. What is life
      ii. Is life complex?
      iii. phase transitions?
      iv. Is the behavior of the model exhibited in the real world?
   (b) What kinds of emergence are possible?

2 Is the world chaotic

2.1 My Paper

1. Question: Is life Chaotic?
2. Proposed methodology: Explore a simple system that is a simplification of life.
   (a) I will look at Schelling's Model and see what chaotic behavior exist in Schelling's model. I will look at the various metrics of chaos and see how they are in Schelling's Model.
      i. Sensitivity to initial conditions
      ii. Boundedness - ecosystems/our world is bounded (yes and no)
         A. Normal model: bounds on map so not really tested.
         B. Try running Schelling's model in a big map while starting in a small area and see if agents reach the edge of the map.
      iii. Lyapunov's exponent
      iv. Neural Net prediction
      v.
3. Important points about Schelling's Model:
   (a) Parameters of Schelling's Model
      i. Assume two races; lattice model; 4 neighbor - vision; swapping? selection of who gets to move.
      ii. Preferences: lower bound and upper bound.
      iii. Size of map.
   (b) Wolfram class?
   (c) Turing Complete?

(d) Behavior exhibited in the real world?
(e) Local behavior?
(f) Weak Emergence?
(g) Self-Organization?

2.2 My thoughts

1. Address the question: Is the world chaotic?
2. Chaos has various definitions:
   (a) Sensitivity to initial conditions
   (b) Stochastic in nature
   (c) Random result
   (d) Weak emergence
3. I want to explore the notion of weak chaos.
4. The intuition behind weak chaos is that...

2.3 Wolfram Paper

1. Direct mathematical analysis of 2D-CA is of little help
2. Resort direct simulation and random sampling.
3. Class 1,2,3,4
4. http://www.timswast.com/blog/2013/03/31/an-exploration-of-cellular-automata-as-dynamical-systems/
5. http://bactra.org/notebooks/cellular-automata.html
6. https://class.coursera.org/modelthinking/lecture
7. http://www.math.tamu.edu/ mpilant/math614/
8. Notes from lecture:
   (a) Some definition of dimension:
   (b) Define orbit $\{x_i\}_{i=1}^{N}$.
   (c) not that if $i \neq j$ then $x_i \neq x_j$ because consdiering noperiodic orbital
   (d) In an orbit: define $val1 = \#\{(x_i, x_j) \mid d(x_i, x_j) < \epsilon, i \neq j\}$.
   (e) $val2 = \#\{(x_i, x_j) \mid i \neq j\} = \frac{N(N+1)}{2}$
   (f) ratio $= c(\epsilon) = \frac{val1}{val2}$
   (g) $\epsilon \to \infty \implies c(\epsilon) \to 1$.
   (h) $\epsilon \to 0 \implies c(\epsilon) \to 0$.
   (i) correlation exponent $= \alpha = \lim_{\epsilon \to 0} |\frac{\ln c(\epsilon)}{\ln \epsilon}|$

(j) Correlation exponent defined in terms of orbit; not the space.
(k) Therefore the correlation exponent is easy to calculate.
(l) Loop over $N(N+1)/2$ pairs of x_i, x_j where $i \neq j$.
(m) calculate $d(x_i, x_j)$. If $dist < \epsilon$, increment counter
(n) implies n^2 operations. implies very efficient.
(o) $N(\epsilon)$ =number of boxes of width ϵ that contain a point in orbit.
(p) define α the same way for $N(\epsilon)$.
(q) Haussdorff Dimension
(r) given set S. cover with balls of diamter ϵ.
(s) define $H_m(S) = \lim_{\epsilon \to 0} \sum_{\text{number balls}} (\epsilon)^m$
(t) example curve of length L takes L/ϵ balls of diamter ϵ so $H_1(S) = \lim(L/\epsilon)\epsilon = L$.
(u) $H_2(S) = \lim(L/\epsilon)\epsilon^2 = 0$.
(v) So Haussdorf dimension = largest d such that $H_d(S) \neq 0$.
(w) equivalent to smallest d such that $H_d(S) = 0$.
(x) expensive to compute.

(a) Question: when is an orbit chaotic?
(b) Take finite orbit $\{x_i\}_i^N$. and orbit z_i
(c) $d(x(t), z(t)) \sim d_o e^{\alpha t} \implies$ chaotic if $\alpha > 0$.
(d) Lyapunov exponent $= \alpha = \frac{1}{1} \ln |\frac{x(t)-z(t)}{x_0 - z_0}|$
(e) Different orbits can have different exponents.
(f) Also might be dependent on component/dimension that you are looking (x-component verse y-component)
(g) Wolf Algorithm (1983) for computing approximate lyapunov exponents of an orbit:
    i. Pick initial point x_0.
    ii. Iterate till close to attractor. Say choose $N >> 1$
    iii. Fix x_N, find another point (x_m) in the orbit such that x_m is close to x_N. i.e. $d(x_m, x_N) \sim 10^4\epsilon$ where ϵ is machine ϵ i.e. smalleest floating point number.
    iv. i.e. $\epsilon = 10^{-16}$ in double precision. 10^{-8} in single precision.
    v. do stuff with sequences. slide 10/11

3 Thesis

Introduction

1. Check out sugarscape model.j

   (a) An agent based model for Schelling
   (b) from 1969. There exists java implementations somewhere.
2. Bak et al. use statistical mechanics upon he self-organized systems.
3. Bak: global emergent properties have the property that they are scale-free.
4. Author follows that with the claim: Therefore we can use global emergent properties to measure the system.
5. Zhang uses Game Theory to model Schelling' Model
6. each agent has fixed income. each agent has π that correlates to the happiness of an agent.
7. Gives locations a price, number white nbrs, number black nbrs. Then uses simply supply and demand to determine prices
8. page 21 discusses some research on Schelling's model
9. Schelling focuses on preferences of agents
10. other research on the neighborhood of agents,up to a distance R, calling this parameter an agent's vision
11. Herb Simon's 'situated ant'
12. An ant following a random walk on a beach produces a complex path. But it is the complexity of the environment that is creating the complex path rather than the ant
13. So complexity of environment is as important as complexity of individual.
14. My Analysis of section:
    (a) Author points out a variety of systems:
        i. Agent Based Model (sugarscape)
        ii. Statistical Mechanics (sandpiles)
        iii. Game Theory (Zhang's Model with happiness and incomes)
        iv. Complex Environment verse Complex Ant

3.1 On Schelling

1. Three models:
2. Spatial Proximity Model
3. Bounded Neighborhood Model = characters consider proportions within system rather than in a local neighborhood of the system.
4. Tipping Model
5. Author says Schelling's Models too simplistic to model real world
6. Author says need to make environment more complex

3.2 Further Work

1. Heterogenous Populations: explore different types within population. (e.g. each white person has a different race tolerance)
2. Space: Explore nonuniform spaces. Change perceptions of Space
   (a) This is done to some extent in the thesis.
   (b) The author explores a model where agents have a random variable for their "vision", i.e. they can see all neighbors with r spaces from them where r is a random variable.
3. Timescales: Explore the effects of agents moving at different timescales, either individually or by type.
4. Time decay (or growth) for social bonds: would a social bond that decayed (or strengthened) over time affect the results of the model?
5. Ratio Calculations: Instead of agents recalculating ratio every iteration, only calculate it every once in a while.
6. **Population Adaptation: have the population change over time. This turns Schelling's model into an evolutionary model**
7. Little work on Bounded Neighborhood Model
8. Stoica and Flache apply Schelling's residential segregation to school segregation by incorporating Zhang's utility (measuring distance from a school instead of price)