Investigating Chaos in Schelling's Model

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Overview

Goal

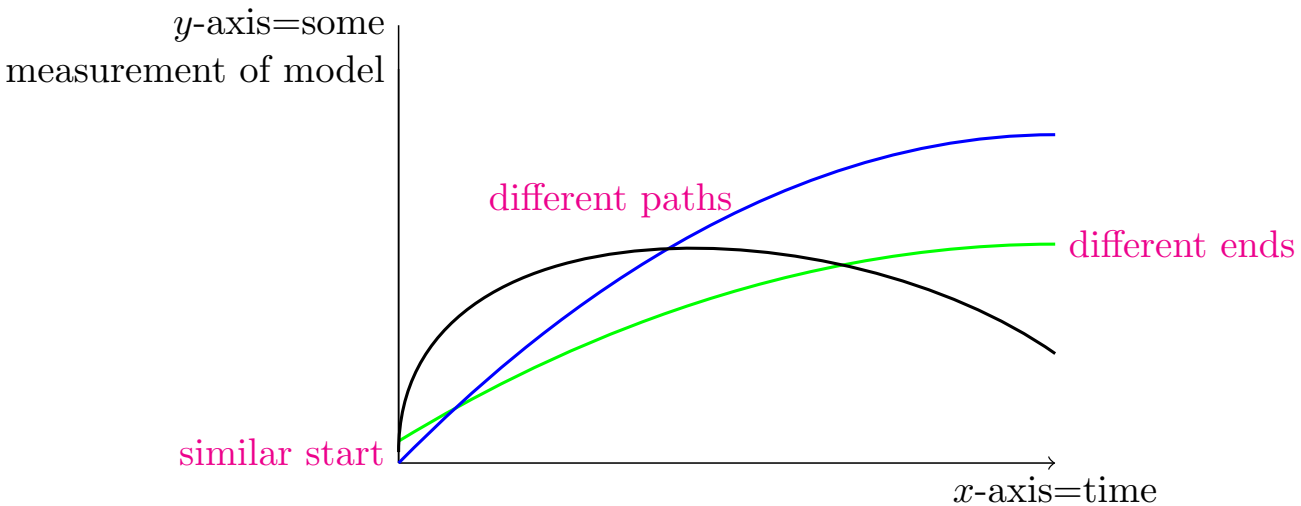
- Big question: Is society "chaotic"?
- Methodology: Investigate a system that simplifies social relations.
- See if "chaos" arises in this simpler model.

What is chaos?

There are three ways that I think about chaos.

1. Sensitivity to Initial Conditions

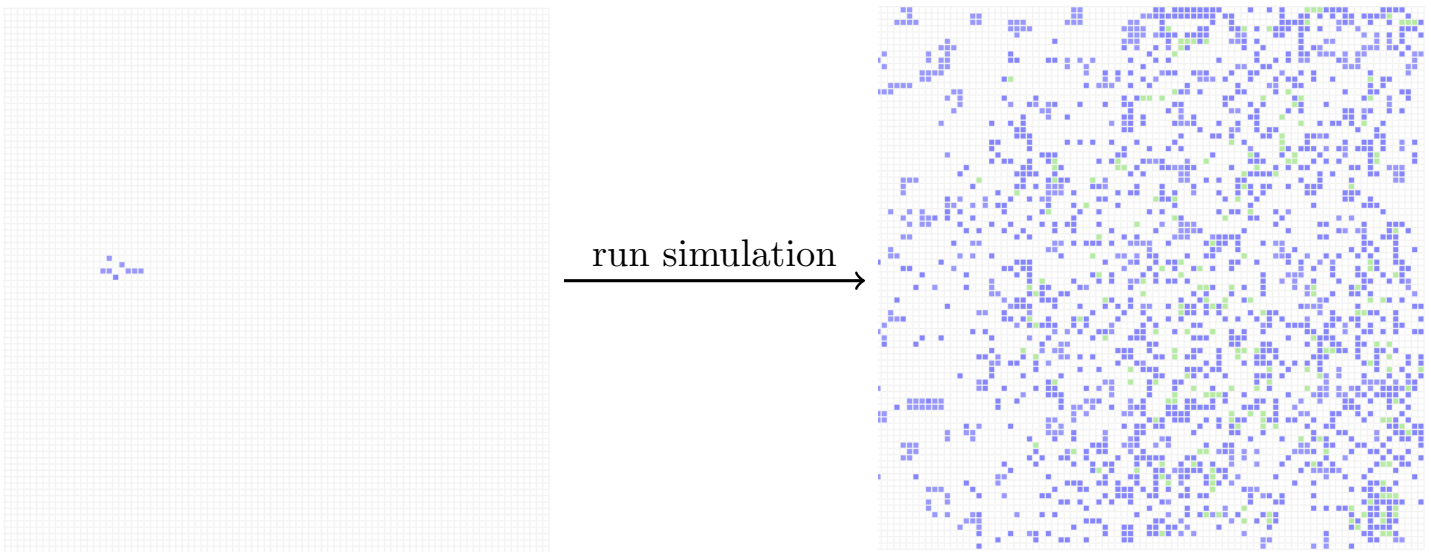
1. Aka the "butterfly effect"
2. If the initial conditions are changed by a marginal amount, how much does that change the output?
3. e.g. Game of Life: changing one cell in Game of Life can dramatically change the output; hence GoL is chaotic.
4. e.g. Weather: a slight difference in pressure can lead to dramatically different weather patterns.



Double Pendulum Video

2. Unboundedness

1. A system is unbounded if it grows uncontrollably.
2. e.g. a cellular automata where it's easy to birth cell and hard to kill cells. Then cells grow indiscrimately into infinitude.



start in small radius

grows outwards unboundedly

3. Unpredictability

1. More similar to weak emergence than chaos, but still applicable.
2. Applicable because unpredictability implies that there is no easy way to descrive the behavior of the system (thereby the system is "chaotic" in some sense)
3. Interesting because Economists are so bad at predicting trends. Perhaps we can show that it's because the model is inherently chaotic.
4. More than showing that the model is chaotic, I want to pinpoint precisely some attributes that make reality chaotic.

When, why and how does society exhibit Chaos?

When, why and how does society exhibit Chaos?

Does society exhibit sensitivity to initial conditions?

1. First you have to define initial conditions: what are society's initial conditions?
2. Ignore that question: say the initial conditions are the state of society right now.
3. Then would society, after many years, be significantly different if I had been sitting instead of standing?
4. It really depends on what condition you change in the initial configuration.
5. Example 1:
   - 5.1 If in one world a baby dies and another the same baby lives, how different are the world?
   - 5.2 No idea. Classic argument: what is that baby is Einstein?
   - 5.3 However, most likely no major effect.

Does society exhibit unboundedness?

1. Seems like the answer depends on your scope.
2. Example 1: A big society - The World.
   - 2.1 Our population is growing exponentially
   - 2.2 We are using more and more resources. Likely to eventually go to Space.
3. Example 2: A smaller society - Reed College
   - 3.1 Reed College is a local ecosystem of social relations.
   - 3.2 And it hasn't outgrown its bounds yet.
   - 3.3 I would say the Reed college ecosystem is bounded.
4. Worthwhile to ask: what are the key differences between Reed College and all of Mankind that explain the differences in unboundedness?
5. Perhaps Schelling's Model can elucidate some possible answers.

Does society exhibit unpredictability?

1. Well economics and social trends are notoriously hard to predict.
2. So it seems like yeah, to a large extent society is not predictable.

Does society exhibit chaos?

1. Does society exhibit chaos?
2. If it does, why does it exhibit chaos?
3. If it doesn't, why doesn't it exhibit chaos?
4. I decided to tackle this question by looking at a "simpler" version of society: Schelling's model.
5. Instead address the question:
6. If, when, and why does Schelling's model exhibit chaos?

Schelling's Model

My Implementation of Schelling's model:

1. Two types of people (white and black) located on an $n \times n$ board.
   - (picture a chessboard)
2. Each type of person has a preference for who they like to live near.
3. If a person's preferences are not met, then they are unhappy.
4. During each iteration of the model, one unhappy person is randomly selected.
5. Then that person moves to a random, empty space such that they are happy.

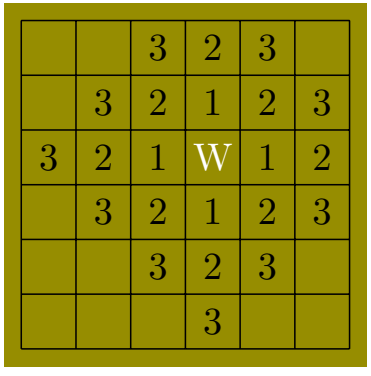
Schelling's Model

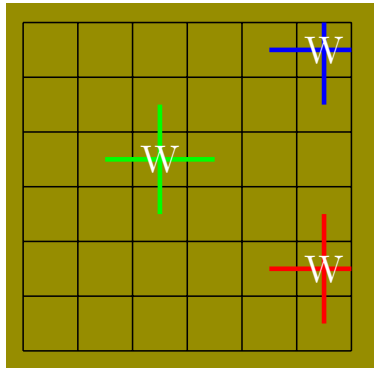
Fixed Parameters to Schelling's Model

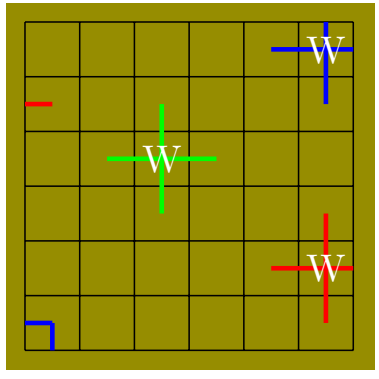
1. Two races
2. Lattice, checkerboard space
3. Selecting a person to move randomly

Variable Parameters to Schelling's Model

1. Vision radius
2. Moving radius
3. Selection of who gets to move
4. Size of board
5. Edge behavior of board (true edges vs. torus) (TODO add figure)







Justification for Using Schelling's Model

1. Schelling's model is a simplification of many social relations.
2. Schelling's model obvious misses a lot of important aspects of social relations, but for the most part those are beside the point.
3. If Schelling's model under realistic parameters does *not* exhibit chaotic behavior, then
   - 3.1 Society is not chaotic
   - 3.2 Society is chaotic
     - 3.2.1 since Schelling's model is a simplification of reality, reality may also be chaotic in the same way.
   - 3.3 Schelling's Model does not sufficiently model society and we can make no claim about whether society is chaotic.
4. Conversely, if Schelling's model under realistic parameters does exhibit chaotic behavior, then there are three possibilities
   - 4.1 Society is chaotic, but the chaos stems from phenomena that Schelling's model is not capturing.
   - 4.2 Society is not chaotic
   - 4.3 Schelling's Model does not sufficiently model society and we can make no claim about whether society is chaotic.

Overview of the Tests

1. Sensitivity to inital conditions
   - 1.1 Computed Lyapunov's Exponent
2. Unboundedness
   - 2.1 Idea: initialize to a 20×20 board but only place people in the middle 8×8 square. Do the people move outwards to the edges of the 20×20 board?
     - 2.1.1 Given the model that I decribed, of course they do!
     - 2.1.2 The people move the random empty spaces.
   - 2.2 Change the model such that people only move within their "vision".
   - 2.3 I.e. people have a radius that they use to calculate the composition of their neighbors, and they have a radius that if they are unhappy, they move a random empty square inside of that radius.
   - 2.4 Actually makes the model more realistic, because it's unlikely that people move long, random distances when unhappy; they would be more likely to stay as close as possible while still being happy.
   - 2.5
3. Unpredictability

Lyapunov's Exponents Test and Results

Unboundedness Test and Results

Unpredictability Test and Results

Analysis

Conclusions and Further Questions