Linear Algebra Problem Set

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Problem 1. True or False?

a. True. This is almost word for word a corollary to theorem 2.32. Thm 2.32 states For any diffential operator $p(D)$ of order n, the null space of $p(D)$ is an n-dimensional subspace of C^∞. Thereby the solution-space is an n-dimensional of C^∞.
b. True. This also also follows from Theorem 2.32 which is stated above. c. False - the solutions to the auxiliary polynomial correspond to the solutions of the differential equations.
d. False. Counterexample is example 1 on page 129 where sine and cosine are used.
e. True. This is true because the solutions to a homogeneous linear differential equation form a basis, wherein linear combinations form the solutions to the equation. Follows from a combination of theorem 2.31 and 2.33
f. False. This is only true for homogenous linear differential equations of order k. See corollary to theorem 2.33.
g. True. $p(t) \in P(C)$ corresponds to the coefficients of the homogeneous linear differential equation. A term of a homogenous linear differential equation can have any coefficient in C. Therefore any term of $p(t)$ correspond to any term in homogenous linear differential equation, and there exists a homogeneous differential equation for all possible $p(t)$ over C.