

# Two Party Computation

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A Thesis  
Presented to  
The Division of Mathematics and Natural Sciences  
Reed College

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Adam Groce



# Acknowledgements

I want to thank a few people.



# Preface

This is an example of a thesis setup to use the reed thesis document class.





# List of Abbreviations

You can always change the way your abbreviations are formatted. Play around with it yourself, use tables, or come to CUS if you'd like to change the way it looks. You can also completely remove this chapter if you have no need for a list of abbreviations. Here is an example of what this could look like:

|             |                                   |
|-------------|-----------------------------------|
| <b>ABC</b>  | American Broadcasting Company     |
| <b>CBS</b>  | Columbia Broadcasting System      |
| <b>CDC</b>  | Center for Disease Control        |
| <b>CIA</b>  | Central Intelligence Agency       |
| <b>CLBR</b> | Center for Life Beyond Reed       |
| <b>CUS</b>  | Computer User Services            |
| <b>FBI</b>  | Federal Bureau of Investigation   |
| <b>NBC</b>  | National Broadcasting Corporation |



# Table of Contents

|   |           |
|---|-----------|
| <b>Introduction</b>                                   | <b>1</b>  |
| 0.1 Tools for MPC                                     | 2         |
| 0.1.1 Boolean Circuit                                 | 2         |
| 0.1.2 Technical Definition of a Circuit               | 3         |
| 0.1.3 Encryption                                      | 4         |
| 0.1.4 Digital Signatures/MACS                         | 4         |
| 0.1.5 The DDH Assumption                              | 4         |
| 0.1.6 Oblivious Transfer                              | 5         |
| 0.2 Other   | 5         |
| 0.3 Classic MPC                                       | 5         |
| 0.3.1 Yao's Garbled Circuit                           | 6         |
| 0.3.2 GMW   | 6         |
| 0.4 Improving MPC                                     | 6         |
| 0.4.1 Overview of history                             | 6         |
| 0.4.2 Point and Permute                               | 6         |
| 0.4.3 Garbled Row Reduction 3                         | 6         |
| 0.4.4 Free XOR  | 6         |
| 0.4.5 Garbled Row Reduction 2                         | 6         |
| 0.4.6 FlexXOR   | 6         |
| 0.4.7 Half Gates                                      | 6         |
| 0.5 Not done yet                                      | 6         |
| 0.6 Overview of what's going to be covered            | 7         |
| <b>Chapter 1: The First</b>                           | <b>9</b>  |
| 1.1 References, Labels, Custom Commands and Footnotes | 9         |
| 1.1.1 References and Labels                           | 9         |
| 1.1.2 Custom Commands                                 | 9         |
| 1.1.3 Footnotes and Endnotes                          | 10        |
| 1.2 Bibliographies                                    | 10        |
| 1.2.1 Tips for Bibliographies                         | 10        |
| 1.3 Anything else?                                    | 11        |
| <b>Chapter 2: Mathematics and Science</b>             | <b>13</b> |
| 2.1 Math  | 13        |
| 2.2 Chemistry 101: Symbols                            | 14        |

|                    |   |           |
|--------------------|---|-----------|
| 2.2.1              | Typesetting reactions . . . . .               | 14        |
| 2.2.2              | Other examples of reactions . . . . .         | 15        |
| 2.3                | Physics . . . . .                             | 15        |
| 2.4                | Biology . . . . .                             | 15        |
| <b>Chapter 3:</b>  | <b>Tables and Graphics . . . . .</b>          | <b>17</b> |
| 3.1                | Tables . . . . .                              | 17        |
| 3.2                | Figures . . . . .                             | 19        |
| 3.3                | More Figure Stuff . . . . .                   | 21        |
| 3.4                | Even More Figure Stuff . . . . .              | 21        |
| 3.4.1              | Common Modifications . . . . .                | 21        |
| <b>Conclusion</b>  | <b>. . . . .</b>                              | <b>23</b> |
| 4.1                | More info . . . . .                           | 23        |
| <b>Appendix A:</b> | <b>The First Appendix . . . . .</b>           | <b>25</b> |
| <b>Appendix B:</b> | <b>The Second Appendix, for Fun . . . . .</b> | <b>27</b> |
| <b>References</b>  | <b>. . . . .</b>                              | <b>29</b> |

# List of Tables

|     |  |    |
|-----|--|----|
| 1   | The mapping of an XOR gate. . . . .  | 3  |
| 2   | Summary of Garbled Circuit Improvements. GRR3 stands for garbled row reduction 3 and GRR2 stands for garbled row reduction 2 . . . . | 7  |
| 3.1 | Correlation of Inheritance Factors between Parents and Child . . . .   | 17 |
| 3.2 | Chromium Hexacarbonyl Data Collected in 1998–1999 . . . . .  | 18 |



# List of Figures

|     |   |    |
|-----|---|----|
| 1   | A circuit that computes the less or equal to function, equivalent to $f$ for input of two one-bit values. <b>TODO, add truth table?</b> . . . . . | 3  |
| 2   | Semi-honest Naor-Pinkas oblivious transfer. . . . .   | 5  |
| 2.1 | Combustion of glucose . . . . .   | 14 |
| 3.1 | A Figure . . . . .  | 20 |
| 3.2 | A Smaller Figure, Flipped Upside Down . . . . .   | 21 |
| 3.3 | A Cropped Figure . . . . .  | 21 |
| 3.4 | Subdivision of arc segments . . . . .   | 21 |





# Abstract

The preface pretty much says it all.



# Dedication

You can have a dedication here if you wish.



# Introduction

Multiparty computation (MPC) is the study and creation of protocols for computing a function between multiple parties, such that no party learns the input of any other party.

The idea is best communicated through an example: suppose Alice and Bob are millionaires and wish to determine who is wealthier, but Alice and Bob are also secretive, and do not want to disclose their exact amount of wealth. Is there some method by which they can determine who has more money?

The goal of MPC is to design a protocol which will help Alice and Bob solve their problem. The desired properties of a secure MPC scheme can be informally described as follows:

- **Privacy:** Each party's input is kept secret.
- **Correctness:** The correct answer to the computation is computed.

Originally, the goal was to come up with a protocol that was secure and prove that the protocol was secure. In more recent times, the focus has shifted to making the MPC faster, fast enough that it can be used regularly in the real world.

If MPC can be made fast enough, it could serve a wide range of applications. For example, imagine that two companies who operate in a similar industry want to work together, but they don't want to disclose any company research which the other doesn't know. These companies could run a set intersection function (a function that given two inputs finds their intersection, or overlap), to determine what information they can disclose without giving away important information.

Another interesting example of MPC is to improve the outsourcing of computation. As it is right now, cloud computing companies, such as Amazon and Google, have really nice computers which they will rent out to you. You can pay them someone money, write a program, and run it on their computers. The problem is that you may not trust the cloud computing company, and you want some guarantees that they are going to respect the privacy of your computation. An MPC protocol, in this setting, would allow you to run computation in the cloud, with the guarantee that the inputs to your computation are disguised.

Since the research into MPC has focused on creating a method by which an arbitrary function can be computed securely, the application of MPC beyond what we can presently conceive of. It's not unlikely that MPC protocols will become a standard in the internet, where when you access the internet, behind the scenes your access is being plugged into an MPC protocol, sent off to another computer to

do some processing. As cryptography improves, research in MPC and other areas of cryptography, the hope is that the security of our computer systems will improve as well. However, there is no guarantee. The modern cryptography needs to be implemented and used, perhaps in some cases built into low-level standards, and used correctly. At this point, the outlook of cryptography is bright, but the future will only be realized positively if it is actively worked towards.

## 0.1 Tools for MPC

Imagine again that millionaires Alice and Bob wish to determine who has more wealth. Suppose that Alice has  $x$  dollars and Bob has  $y$  dollars. Then the function that they wish to compute is  $f : \{0, 1\}^n \rightarrow \{0, 1\}$  as defined by:

$$f(x, y) = \begin{cases} 0, & x \leq y; \\ 1, & \text{otherwise.} \end{cases} \quad (1)$$

**Why is this here?** If Alice and Bob successfully compute  $f(x, y)$  and each gets the output, then each party has gained some knowledge about the nature of the other's input. For example, if  $f(x, y)$  outputs 0, then Alice and Bob know that Alice has more money. Alice has learned that Bob has less than  $x$  dollars, and Bob has learned that Alice has more  $y$  dollars. Therefore it's not possible for MPC to guarantee that *no* information about the other parties' inputs is learned, only that nothing more is learned than what can be inferred from a single party's input and the output of  $f$ .

MPC requires a combination of several cryptographic tools. The next few sections will describe these basic tools, and then MPC will be described in section  $x$ . tools discussed are

1. Boolean Circuit
2. Encryption
3. Oblivious Transfer (need to explain semihonest and honest)

### 0.1.1 Boolean Circuit

A function for an MPC protocol is represented by a boolean circuit. A boolean circuit takes as input a sequence of  $n$  0s and 1s, (i.e. a value in  $\{0, 1\}^n$ ), performs a series of small operations on the inputs, and outputs a sequence of  $m$  0s and 1s (i.e. a value in  $\{0, 1\}^m$ ). You may have encountered circuits and logical operators in another context, where the inputs and outputs were True and False. For our usage, True will correspond to the value 1, and False will correspond to the value 0.

The small operations performed inside of a circuit are performed by an object called a *gate*. A gate is composed of three wires: two input wires and one output wire, where a *wire* can have a value either 0 or 1. A gate performs a simpler operation on

| x | y | xor(x,y) |
|---|---|----------|
| 1 | 1 | 0        |
| 1 | 0 | 1        |
| 0 | 1 | 1        |
| 0 | 0 | 0        |

Table 1: The mapping of an XOR gate.

the two inputs, resulting in a single output bit. Table 0.1.1 gives the mapping of an XOR gate.

A circuit is a combination of gates. In fact, a circuit built out of only AND gates and XOR gates can compute any function. **Find details and citation** In other words, if there's some algorithm that do it, then there is some circuit that can do it as well. Hence, a circuit is a sufficient representation of the function  $f$ . Figure 0.1.1 shows the circuit representation of a circuit that computes the less than function, the function  $f$ , specified in equation 1 that millionaires Alice and Bob wanted to compute.

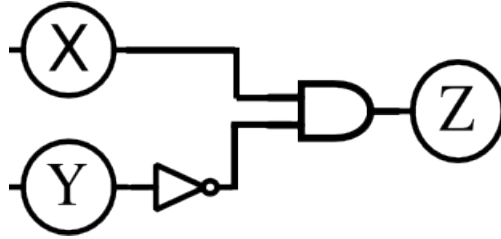


Figure 1: A circuit that computes the less or equal to function, equivalent to  $f$  for input of two one-bit values. **TODO, add truth table?**

A circuit **with what constraints???** can compute any function.

### 0.1.2 Technical Definition of a Circuit

A circuit, which I will refer to as  $g$ , is formalized by a 6-tuple  $g = (n, m, a, A, B, G)$ , where  $n$  is the number of inputs,  $m$  is the number of outputs and  $q$  is the number of gates. We define  $r = n + q$  to be the number of wires inside of the circuit. We let  $\text{Wires} = \{1, \dots, n + q\}$ ,  $\text{InputWires} = \{1, \dots, n\}$ ,  $\text{OutputWires} = \{n + 1 - m + 1, \dots, n + q\}$ , and  $\text{Gates} = \{n + 1, \dots, n + q\}$ . Then  $A : \text{Gates} \rightarrow \text{Wires} \setminus \text{OutputWires}$  identifies each gate's first incoming wire. And  $B : \text{Gates} \rightarrow \text{Wires} \setminus \text{OutputWires}$  identifies each gate's second incoming wire. Finally,  $G : \text{Gates} \times \{0, 1\}^2 \rightarrow \{0, 1\}$  identifies the functionality of each gate.

For example, the less than circuit shown in figure 0.1.1 has values:

### 0.1.3 Encryption

Encryption is the process of encoding a message such that only parties with the key can read the message. An encryption protocol is composed of two parts. The first part is the encryption function which disguises the message, and the second part is the decryption function which unobfuscates the disguised message. We notate encryption and decryption with

$$\begin{aligned} \text{Enc}_k(pt) &= ct \\ \text{Dec}_k(ct) &= pt \end{aligned} \tag{2}$$

where  $pt$  stands for plaintext and is the original message,  $ct$  stands for ciphertext and is the encrypted message,  $k$  is the secret key, that only authorized readers of the message hold, and is a random sequence of  $\lambda$  0s and 1s, and  $\lambda$  is a security parameter of our protocol (As  $\lambda$  increases, the size of the key increases, and so the encryption becomes harder to break.)

The encryption described above is more precisely called symmetric-key encryption, as opposed to public-key encryption. The difference between the two is that for symmetric-key encryption, there is a single key and all communicating must have the same key in order to achieve secure communication, whereas in public-key encryption, the encryption key is published publicly, and anyone can send a message using the public key, and the message can only be decrypted by those with the secret key. For more information on encryption, we encourage the reader to peruse the many great online resources.

The MPC protocols that will be examined here exclusively use symmetric-key encryption.

### 0.1.4 Digital Signatures/MACS

kA Do we need this?

### 0.1.5 The DDH Assumption

When a cryptographic scheme is said to be *secure*, cryptographers actually mean something much more precise. When a cryptographic scheme is considered secure, it actually means that an adversary can beat the scheme only if the adversary can tackle the hardness assumptions on which the scheme is based. A common hardness assumption in cryptography is the Decisional Diffie-Hellman Assumption (DDH Assumption), an assumption about solving a problem concerning discrete logs. Formally, the DDH assumption is:

The *Decisional Diffie-Hellman Assumption* (DDH assumption) is hardness assumption about solving a problem about discrete logs. Many cryptographic protocols can be reduced down to the DDH problem, meaning that if the DDH problem can be solved, then the cryptographic schemes are insecure. But as long as DDH remains hard, the cryptographic protocols remain hard. When we say that cryptograph



| Alice  |  |  |               | Bob              |                           |   |
|--|--|--|---------------|------------------|---------------------------|---|
| Secret   | Public                                   | Calculus   |               | Secret           | Public                    | Calculus  |
| $m_0, m_1$   |  | Messages to be sent  |               |                  |                           |   |
| $d$  | $N, e$                                   | Generate RSA key pair and send public portion to Bob         | $\Rightarrow$ |                  | $N, e$                    | Receive public key  |
|  | $x_0, x_1$                               | Generate two random messages                                 | $\Rightarrow$ |                  | $x_0, x_1$                | Receive random messages   |
|  |  |  |               | $k, b$           |                           | Choose $b \in \{0, 1\}$ and generate random $k$                         |
|  | $v$                                      |  | $\Leftarrow$  |                  | $v = (x_b + k^e) \bmod N$ | Compute the encryption of $k$ , blind with $x_b$ and send to Alice      |
| $k_0 = (v - x_0)^d \bmod N$<br>$k_1 = (v - x_1)^d \bmod N$ |  | One of these will equal $k$ , but Alice does not know which. |               |                  |                           |   |
|  | $m'_0 = m_0 + k_0$<br>$m'_1 = m_1 + k_1$ | Send both messages to Bob                                    | $\Rightarrow$ |                  | $m'_0, m'_1$              | Receive both messages   |
|  |  |  |               | $m_b = m'_b - k$ |                           | Bob decrypts the $m'_b$ since he knows which $x_b$ he selected earlier. |

Figure 2: Semi-honest Naor-Pinkas oblivious transfer.

### 0.1.6 Oblivious Transfer

Oblivious Transfer is a special method of communicating a message between two parties where a sender sends one of two messages the receive, and the sender remains oblivious as which message was sent. The setup is the following: Alice has two messages,  $m_1$  and  $m_2$ , and he wants to send a message to Bob under the following conditions: First, Alice sends either  $m_1$  or  $m_2$  but not both. Second, Alice does not know which message he sent to Bob. Third, Bob selects which message he wants to receive.

Here we give the Naor-Pinkas protocol of 1 – 2 oblivious transfer. The protocol relies on the DDH assumption (see section 0.1.5) and is secure in the semi-honest setting (see section ??).

Researchers have also developed methods for performing k-out-of-n oblivious transfer, where the sender sends exactly  $k$  messages out of a possible  $n$ . The method described above is called 1 – 2 oblivious transfer, and is the OT used in MPC protocols.

Since the first proposal of OT in **year**, several improvements have been developed. Preprocessing. OT extension. Semi-honest/Malicious.

## 0.2 Other

**TODO** Paper has topological circuit next to real circuit change arbitrary functoin to arbitrary boolean function.

## 0.3 Classic MPC

MPC was first proposed by Andrew Yao in an oral presentation about secure function evaluation. Resulting from Yao's presentation, two methods for performing MPC were produced. One method was contributed by Yao himself, and the other was contributed by a group of researchers, Beaver, Micali and Widgerson, (GMW). The two methods are premised on a similar idea: encrypt a circuit by encrypting its gates, with has since been termed garbled circuit. At this point, it is unclear which method is better, both in terms of security of speed, and research is still being done on both protocols. We will present an overview of both protocols, going more in depth into

Yao's garbled circuit as it is used for the research in the remainder of the thesis.

### 0.3.1 Yao's Garbled Circuit

### 0.3.2 GMW

## 0.4 Improving MPC

### 0.4.1 Overview of history

working on specific function verse general functions working on number of parties  
making it work for sugarbeet auction election potential? auctions

### 0.4.2 Point and Permute

### 0.4.3 Garbled Row Reduction 3

### 0.4.4 Free XOR

### 0.4.5 Garbled Row Reduction 2

### 0.4.6 FlexXOR

### 0.4.7 Half Gates

## 0.5 Not done yet

Imagine again that Alice and Bob from the beginning of the introduction are trying to compute the maximum amount of money between them. If Alice makes a circuit that computes the max, plugs her input encoded as a binary number into the circuit and gives the circuit to Bob, then Bob can see Alice inputs - they are sitting right there hardcoded in the circuit! But Alice doesn't want Bob to know how much money she has. So simply passing around a circuit and plugging in inputs is insecure, because the inputs of each party are sitting in plain sight of the subsequent parties who need to plug in their inputs.

And hence a method is needed for disguising Alice and Bob's input. Two methods for disguising inputs have been developed. The first method was developed by Goldreich, Micali and Widgerson, Slipping them into the circuit, so that nobody sees them.

In some year Yao did something that was cool for some reason.

Around the same time, a group, whose names are ..., going by GMW did a similar thing like this.

Next, we will walk through Yao and GMW's techniques for garbling circuits (or garbled gates?).

## 0.6 Overview of what's going to be covered

| Mode Transition Times<br>Duration (Typical) | Size ( $x\lambda$ ) |      | Eval Cost |         | Garble Cost |     | Assumption |
|---|---------------------|------|-----------|---------|-------------|-----|------------|
|   | XOR                 | AND  | XOR       | AND     | XOR         | AND |            |
| Classical                                   | 1365                | 1260 | 1024      | $\mu$ s | a           | b   | a          |
| Point and Permute                           | TBD                 | TBD  | TBD       | $\mu$ s | a           | b   | a          |
| GRR3  | TBD                 | TBD  | TBD       | $\mu$ s | a           | b   | a          |
| Free XOR                                    | TBD                 | TBD  | TBD       | $\mu$   | a           | bs  | a          |
| GRR2 XOR                                    | TBD                 | TBD  | TBD       | $\mu$   | a           | bs  | a          |
| FleXOR                                      | TBD                 | TBD  | TBD       | $\mu$   | a           | bs  | a          |
| Half Gates                                  | TBD                 | TBD  | TBD       | $\mu$   | a           | bs  | a          |

Table 2: Summary of Garbled Circuit Improvements. GRR3 stands for garbled row reduction 3 and GRR2 stands for garbled row reduction 2



# Chapter 1

## The First

This is the first page of the first chapter. You may delete the contents of this chapter so you can add your own text; it's just here to show you some examples.

### 1.1 References, Labels, Custom Commands and Footnotes

It is easy to refer to anything within your document using the `label` and `ref` tags. Labels must be unique and shouldn't use any odd characters; generally sticking to letters and numbers (no spaces) should be fine. Put the label on whatever you want to refer to, and put the reference where you want the reference. `LATEX` will keep track of the chapter, section, and figure or table numbers for you.

#### 1.1.1 References and Labels

Sometimes you'd like to refer to a table or figure, e.g. you can see in Figure 3.2 that you can rotate figures. Start by labeling your figure or table with the `label` command (`\label{labelvariable}`) below the caption (see the chapter on graphics and tables for examples). Then when you would like to refer to the table or figure, use the `ref` command (`\ref{labelvariable}`). Make sure your label variables are unique; you can't have two elements named "default." Also, since the reference command only puts the figure or table number, you will have to put "Table" or "Figure" as appropriate, as seen in the following examples:

As I showed in Table 3.1 many factors can be assumed to follow from inheritance. Also see the Figure 3.1 for an illustration.

#### 1.1.2 Custom Commands

Are you sick of writing the same complex equation or phrase over and over?

The custom commands should be placed in the preamble, or at least prior to the first usage of the command. The structure of the `\newcommand` consists of the name of the new command in curly braces, the number of arguments to be made in square

brackets and then, inside a new set of curly braces, the command(s) that make up the new command. The whole thing is sandwiched inside a larger set of curly braces.

In other words, if you want to make a shorthand for  $\text{H}_2\text{SO}_4$ , which doesn't include an argument, you would write: `\newcommand{\hydro}{H$_2$SO$_4$}` and then when you needed to use the command you would type `\hydro`. (sans verb and the equals sign brackets, if you're looking at the .tex version). For example:  $\text{H}_2\text{SO}_4$

### 1.1.3 Footnotes and Endnotes

You might want to footnote something.<sup>1</sup> Be sure to leave no spaces between the word immediately preceding the footnote command and the command itself. The footnote will be in a smaller font and placed appropriately. Endnotes work in much the same way. More information can be found about both on the CUS site.

## 1.2 Bibliographies

Of course you will need to cite things, and you will probably accumulate an armful of sources. This is why BibTeX was created. For more information about BibTeX and bibliographies, see our CUS site ([web.reed.edu/cis/help/latex/index.html](http://web.reed.edu/cis/help/latex/index.html))<sup>2</sup>. There are three pages on this topic: *bibtex* (which talks about using BibTeX, at [/latex/bibtex.html](http://latex/bibtex.html)), *bibtexstyles* (about how to find and use the bibliography style that best suits your needs, at [/latex/bibtexstyles.html](http://latex/bibtexstyles.html)) and *bibman* (which covers how to make and maintain a bibliography by hand, without BibTeX, at [/latex/bibman.html](http://latex/bibman.html)). The last page will not be useful unless you have only a few sources. There used to be APA stuff here, but we don't need it since I've fixed this with my apa-good natbib style file.

### 1.2.1 Tips for Bibliographies

1. Like with thesis formatting, the sooner you start compiling your bibliography for something as large as thesis, the better. Typing in source after source is mind-numbing enough; do you really want to do it for hours on end in late April? Think of it as procrastination.
2. The cite key (a citation's label) needs to be unique from the other entries.
3. When you have more than one author or editor, you need to separate each author's name by the word "and" e.g.  
`Author = {Noble, Sam and Youngberg, Jessica},.`
4. Bibliographies made using BibTeX (whether manually or using a manager) accept LaTeX markup, so you can italicize and add symbols as necessary.

---

<sup>1</sup>footnote text

<sup>2</sup>Reed College (2007)

5. To force capitalization in an article title or where all lowercase is generally used, bracket the capital letter in curly braces.
6. You can add a Reed Thesis citation<sup>3</sup> option. The best way to do this is to use the phdthesis type of citation, and use the optional “type” field to enter “Reed thesis” or “Undergraduate thesis”. Here’s a test of Chicago, showing the second cite in a row<sup>4</sup> being different. Also the second time not in a row<sup>5</sup> should be different. Of course in other styles they’ll all look the same.

## 1.3 Anything else?

If you’d like to see examples of other things in this template, please contact CUS (email [cus@reed.edu](mailto:cus@reed.edu)) with your suggestions. We love to see people using L<sup>A</sup>T<sub>E</sub>X for their theses, and are happy to help.

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<sup>3</sup>Noble (2002)

<sup>4</sup>Noble (2002)

<sup>5</sup>Reed College (2007)





# Chapter 2

## Mathematics and Science

### 2.1 Math

T<sub>E</sub>X is the best way to typeset mathematics. Donald Knuth designed T<sub>E</sub>X when he got frustrated at how long it was taking the typesetters to finish his book, which contained a lot of mathematics.

If you are doing a thesis that will involve lots of math, you will want to read the following section which has been commented out. If you're not going to use math, skip over this next big red section. (It's red in the .tex file but does not show up in the .pdf.)

$$\sum_{j=1}^n (\delta\theta_j)^2 \leq \frac{\beta_i^2}{\delta_i^2 + \rho_i^2} \left[ 2\rho_i^2 + \frac{\delta_i^2 \beta_i^2}{\delta_i^2 + \rho_i^2} \right] \equiv \omega_i^2$$

From Informational Dynamics, we have the following (Dave Braden):  
After  $n$  such encounters the posterior density for  $\theta$  is

$$\pi(\theta|X_1 < y_1, \dots, X_n < y_n) \propto \pi(\theta) \prod_{i=1}^n \int_{-\infty}^{y_i} \exp\left(-\frac{(x-\theta)^2}{2\sigma^2}\right) dx$$

Another equation:

$$\det \begin{vmatrix} c_0 & c_1 & c_2 & \dots & c_n \\ c_1 & c_2 & c_3 & \dots & c_{n+1} \\ c_2 & c_3 & c_4 & \dots & c_{n+2} \\ \vdots & \vdots & \vdots & & \vdots \\ c_n & c_{n+1} & c_{n+2} & \dots & c_{2n} \end{vmatrix} > 0$$

Lapidus and Pindar, Numerical Solution of Partial Differential Equations in Science and Engineering. Page 54

$$\int_t \left\{ \sum_{j=1}^3 T_j \left( \frac{d\phi_j}{dt} + k\phi_j \right) - kT_e \right\} w_i(t) dt = 0, \quad i = 1, 2, 3.$$

L&P Galerkin method weighting functions. Page 55

$$\sum_{j=1}^3 T_j \int_0^1 \left\{ \frac{d\phi_j}{dt} + k\phi_j \right\} \phi_i dt = \int_0^1 k T_e \phi_i dt, \quad i = 1, 2, 3$$

Another L&P (p145)

$$\int_{-1}^1 \int_{-1}^1 \int_{-1}^1 f(\xi, \eta, \zeta) = \sum_{k=1}^n \sum_{j=1}^n \sum_{i=1}^n w_i w_j w_k f(\xi, \eta, \zeta).$$

Another L&P (p126)

$$\int_{A_e} (\cdot) dx dy = \int_{-1}^1 \int_{-1}^1 (\cdot) \det[J] d\xi d\eta.$$

## 2.2 Chemistry 101: Symbols

Chemical formulas will look best if they are not italicized. Get around math mode's automatic italicizing by using the argument  `$\mathrm{formula here}$` , with your formula inside the curly brackets.

So,  $\text{Fe}_2^{2+}\text{Cr}_2\text{O}_4$  is written  `$\mathrm{Fe_2^{2+}Cr_{20_4}}$`

Exponent or Superscript:  $\text{O}^-$

Subscript:  $\text{CH}_4$

To stack numbers or letters as in  $\text{Fe}_2^{2+}$ , the subscript is defined first, and then the superscript is defined.

Angstrom:  $\text{\AA}$

Bullet:  $\text{CuCl} \bullet 7\text{H}_2\text{O}$

Double Dagger:  $\ddagger$

Delta:  $\Delta$

Reaction Arrows:  $\longrightarrow$  or  $\xrightarrow{\text{solution}}$

Resonance Arrows:  $\longleftrightarrow$

Reversible Reaction Arrows:  $\rightleftharpoons$  or  $\xrightleftharpoons{\text{solution}}$  (the latter requires the chemarr package)

### 2.2.1 Typesetting reactions

You may wish to put your reaction in a figure environment, which means that LaTeX will place the reaction where it fits and you can have a figure legend if desired:

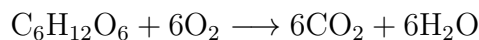
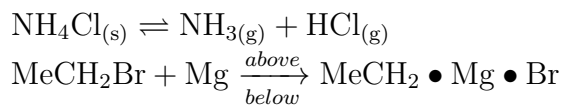


Figure 2.1: Combustion of glucose

### 2.2.2 Other examples of reactions



## 2.3 Physics

Many of the symbols you will need can be found on the math page (<http://web.reed.edu/cis/help/latex/math.html>) and the Comprehensive L<sup>A</sup>T<sub>E</sub>X Symbol Guide (enclosed in this template download). You may wish to create custom commands for commonly used symbols, phrases or equations, as described in Chapter 1.1.2.

## 2.4 Biology

You will probably find the resources at <http://www.lecb.ncifcrf.gov/~toms/latex.html> helpful, particularly the links to bst's for various journals. You may also be interested in TeXShade for nucleotide typesetting (<http://homepages.uni-tuebingen.de/beitz/txe.html>). Be sure to read the proceeding chapter on graphics and tables, and remember that the thesis template has versions of Ecology and Science bst's which support webpage citation formats.



# Chapter 3

## Tables and Graphics

### 3.1 Tables

The following section contains examples of tables, most of which have been commented out for brevity. (They will show up in the .tex document in red, but not at all in the .pdf). For more help in constructing a table (or anything else in this document), please see the LaTeX pages on the CUS site.

Table 3.1: Correlation of Inheritance Factors between Parents and Child

| Factors               | Correlation between Parents & Child | Inherited |
|-----------------------|-------------------------------------|-----------|
| Education             | -0.49                               | Yes       |
| Socio-Economic Status | 0.28                                | Slight    |
| Income                | 0.08                                | No        |
| Family Size           | 0.19                                | Slight    |
| Occupational Prestige | 0.21                                | Slight    |

If you want to make a table that is longer than a page, you will want to use the longtable environment. Uncomment the table below to see an example, or see our online documentation.

Table 3.2: Chromium Hexacarbonyl Data Collected in 1998–1999

| Chromium Hexacarbonyl |                  |            |  |
|-----------------------|------------------|------------|--|
| State                 | Laser wavelength | Buffer gas | Ratio of $\frac{\text{Intensity at vapor pressure}}{\text{Intensity at 240 Torr}}$ |
| $z^7P_4^\circ$        | 266 nm           | Argon      | 1.5  |
| $z^7P_2^\circ$        | 355 nm           | Argon      | 0.57   |
| $y^7P_3^\circ$        | 266 nm           | Argon      | 1  |
| $y^7P_3^\circ$        | 355 nm           | Argon      | 0.14   |
| $y^7P_2^\circ$        | 355 nm           | Argon      | 0.14   |
| $z^5P_3^\circ$        | 266 nm           | Argon      | 1.2  |
| $z^5P_3^\circ$        | 355 nm           | Argon      | 0.04   |
| $z^5P_3^\circ$        | 355 nm           | Helium     | 0.02   |
| $z^5P_2^\circ$        | 355 nm           | Argon      | 0.07   |
| $z^5P_1^\circ$        | 355 nm           | Argon      | 0.05   |
| $y^5P_3^\circ$        | 355 nm           | Argon      | 0.05, 0.4  |
| $y^5P_3^\circ$        | 355 nm           | Helium     | 0.25   |
| $z^5F_4^\circ$        | 266 nm           | Argon      | 1.4  |
| $z^5F_4^\circ$        | 355 nm           | Argon      | 0.29   |
| $z^5F_4^\circ$        | 355 nm           | Helium     | 1.02   |
| $z^5D_4^\circ$        | 355 nm           | Argon      | 0.3  |
| $z^5D_4^\circ$        | 355 nm           | Helium     | 0.65   |
| $y^5H_7^\circ$        | 266 nm           | Argon      | 0.17   |
| $y^5H_7^\circ$        | 355 nm           | Argon      | 0.13   |
| $y^5H_7^\circ$        | 355 nm           | Helium     | 0.11   |
| $a^5D_3$              | 266 nm           | Argon      | 0.71   |
| $a^5D_2$              | 266 nm           | Argon      | 0.77   |
| $a^5D_2$              | 355 nm           | Argon      | 0.63   |
| $a^3D_3$              | 355 nm           | Argon      | 0.05   |
| $a^5S_2$              | 266 nm           | Argon      | 2  |
| $a^5S_2$              | 355 nm           | Argon      | 1.5  |
| $a^5G_6$              | 355 nm           | Argon      | 0.91   |
| $a^3G_4$              | 355 nm           | Argon      | 0.08   |
| $e^7D_5$              | 355 nm           | Helium     | 3.5  |
| $e^7D_3$              | 355 nm           | Helium     | 3  |
| $f^7D_5$              | 355 nm           | Helium     | 0.25   |
| $f^7D_5$              | 355 nm           | Argon      | 0.25   |
| $f^7D_4$              | 355 nm           | Argon      | 0.2  |
| $f^7D_4$              | 355 nm           | Helium     | 0.3  |
| Propyl-ACT            |                  |            |  |

| State          | Laser wavelength | Buffer gas | Ratio of $\frac{\text{Intensity at vapor pressure}}{\text{Intensity at 240 Torr}}$ |
|----------------|------------------|------------|--|
| $z^7P_4^\circ$ | 355 nm           | Argon      | 1.5  |
| $z^7P_3^\circ$ | 355 nm           | Argon      | 1.5  |
| $z^7P_2^\circ$ | 355 nm           | Argon      | 1.25   |
| $z^7F_5^\circ$ | 355 nm           | Argon      | 2.85   |
| $y^7P_4^\circ$ | 355 nm           | Argon      | 0.07   |
| $y^7P_3^\circ$ | 355 nm           | Argon      | 0.06   |
| $z^5P_3^\circ$ | 355 nm           | Argon      | 0.12   |
| $z^5P_2^\circ$ | 355 nm           | Argon      | 0.13   |
| $z^5P_1^\circ$ | 355 nm           | Argon      | 0.14   |
| Methyl-ACT     |                  |            |  |
| $z^7P_4^\circ$ | 355 nm           | Argon      | 1.6, 2.5   |
| $z^7P_4^\circ$ | 355 nm           | Helium     | 3  |
| $z^7P_4^\circ$ | 266 nm           | Argon      | 1.33   |
| $z^7P_3^\circ$ | 355 nm           | Argon      | 1.5  |
| $z^7P_2^\circ$ | 355 nm           | Argon      | 1.25, 1.3  |
| $z^7F_5^\circ$ | 355 nm           | Argon      | 3  |
| $y^7P_4^\circ$ | 355 nm           | Argon      | 0.07, 0.08   |
| $y^7P_4^\circ$ | 355 nm           | Helium     | 0.2  |
| $y^7P_3^\circ$ | 266 nm           | Argon      | 1.22   |
| $y^7P_3^\circ$ | 355 nm           | Argon      | 0.08   |
| $y^7P_2^\circ$ | 355 nm           | Argon      | 0.1  |
| $z^5P_3^\circ$ | 266 nm           | Argon      | 0.67   |
| $z^5P_3^\circ$ | 355 nm           | Argon      | 0.08, 0.17   |
| $z^5P_3^\circ$ | 355 nm           | Helium     | 0.12   |
| $z^5P_2^\circ$ | 355 nm           | Argon      | 0.13   |
| $z^5P_1^\circ$ | 355 nm           | Argon      | 0.09   |
| $y^5H_7^\circ$ | 355 nm           | Argon      | 0.06, 0.05   |
| $a^5D_3$       | 266 nm           | Argon      | 2.5  |
| $a^5D_2$       | 266 nm           | Argon      | 1.9  |
| $a^5D_2$       | 355 nm           | Argon      | 1.17   |
| $a^5S_2$       | 266 nm           | Argon      | 2.3  |
| $a^5S_2$       | 355 nm           | Argon      | 1.11   |
| $a^5G_6$       | 355 nm           | Argon      | 1.6  |
| $e^7D_5$       | 355 nm           | Argon      | 1  |

## 3.2 Figures

If your thesis has a lot of figures, L<sup>A</sup>T<sub>E</sub>X might behave better for you than that other word processor. One thing that may be annoying is the way it handles “floats” like tables and figures. L<sup>A</sup>T<sub>E</sub>X will try to find the best place to put your object based on the text around it and until you’re really, truly done writing you should just leave it where it lies. There are some optional arguments to the figure and table environments

to specify where you want it to appear; see the comments in the first figure.

If you need a graphic or tabular material to be part of the text, you can just put it inline. If you need it to appear in the list of figures or tables, it should be placed in the floating environment.

To get a figure from StatView, JMP, SPSS or other statistics program into a figure, you can print to pdf or save the image as a jpg or png. Precisely how you will do this depends on the program: you may need to copy-paste figures into Photoshop or other graphic program, then save in the appropriate format.

Below we have put a few examples of figures. For more help using graphics and the float environment, see our online documentation.

And this is how you add a figure with a graphic:

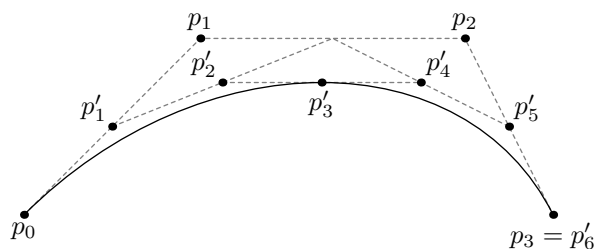


Figure 3.1: A Figure



### 3.3 More Figure Stuff

You can also scale and rotate figures.

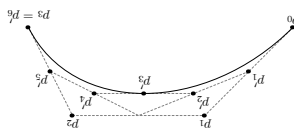


Figure 3.2: A Smaller Figure, Flipped Upside Down

### 3.4 Even More Figure Stuff

With some clever work you can crop a figure, which is handy if (for instance) your EPS or PDF is a little graphic on a whole sheet of paper. The viewport arguments are the lower-left and upper-right coordinates for the area you want to crop.

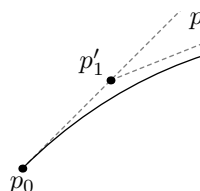


Figure 3.3: A Cropped Figure

#### 3.4.1 Common Modifications

The following figure features the more popular changes thesis students want to their figures. This information is also on the web at [web.reed.edu/cis/help/latex/graphics.html](http://web.reed.edu/cis/help/latex/graphics.html).

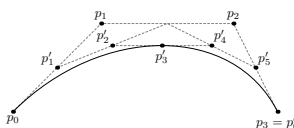


Figure 3.4: Subdivision of arc segments. You can see that  $p_3 = p_6'$ .



# Conclusion

Here's a conclusion, demonstrating the use of all that manual incrementing and table of contents adding that has to happen if you use the starred form of the chapter command. The deal is, the chapter command in L<sup>A</sup>T<sub>E</sub>X does a lot of things: it increments the chapter counter, it resets the section counter to zero, it puts the name of the chapter into the table of contents and the running headers, and probably some other stuff.

So, if you remove all that stuff because you don't like it to say "Chapter 4: Conclusion", then you have to manually add all the things L<sup>A</sup>T<sub>E</sub>X would normally do for you. Maybe someday we'll write a new chapter macro that doesn't add "Chapter X" to the beginning of every chapter title.

## 4.1 More info

And here's some other random info: the first paragraph after a chapter title or section head *shouldn't be* indented, because indents are to tell the reader that you're starting a new paragraph. Since that's obvious after a chapter or section title, proper typesetting doesn't add an indent there.



# Appendix A

## The First Appendix



## Appendix B

The Second Appendix, for Fun





# References

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