Macroeconomics II

Lesson 01 — Preliminaries. Job Search and Unemployment

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Basic Information

- Lecturer: Alessandro Di Nola
- Office: 412 (Diagonal 696 Building)
- E-mail: dinola@ub.edu
- Office hours by appointment (send an email)
- Course material regularly uploaded on Campus Virtual
- Class times:
  - Tuesday, 10:00-12:00 (Room 1030)
  - Thursday, 10:00-12:00 (Room 1030)
  - End half an hour earlier? Short break in the middle?

Grading

- Final Exam 60%
- Three Problem Sets 40%
- However, if the grade of the final exam is higher than the grade of the problem sets, only the final exam matters
- Students are allowed to cooperate with each other in small groups

Topic 1: Job Search and Unemployment

Motivation and overview

- Competitive, frictionless models cannot account for unemployment
  - In the basic RBC model, each agent has perfect information about all the jobs on offer $\implies$ only decides how many hours to work given the single wage prevailing in the labor market
  - The only alternative to waged work is leisure (or home production), there is no need to look for a job
- In the real world, unemployed workers do devote effort to looking for work, since they have imperfect information about jobs and wages

Motivation and overview, cont'd

- We are interested in dynamic models of the labor market that can account for worker flows and job flows
- These models can be useful for a variety of issues, for example
  - The role of labor market regulation and policy (e.g. unemployment insurance, firing costs)
  - Wage inequality
  - Business cycle dynamics of the labor market
- In this lecture we start with a simple, partial model of job search

Job search model (McCall 1970)

- Consider a risk–neutral worker; infinitely lived; discount factor $\beta < 1$
- Worker wants to maximize $\mathbb{E} \sum_{t \geq 0} \beta^t y_t$ where y_t is income in period t
- In $t = 0$ worker is unemployed
- When unemployed, the worker draws in every period $t \geq 0$ a job offer (for a job starting in the next period). The offered wage w is drawn from cdf F
- $F(0) = 0$ and $F(B) = 1$ for some $B < \infty$. The assumption $B < \infty$ can be relaxed.
- Review basic facts about random variables and their distribution
- An unemployed worker earns income b

Job search model

- Decision: accept job offer or not?
- When the worker accepts a job at wage w in some period τ, he keeps the job forever: $y_t = w$ for all $t > \tau$
- No recall of rejected offers (i.e. the worker cannot go back to a previously rejected offer)
- No quits, no layoffs: if worker accepts a job, he keeps it forever
- Main **trade-off**:
  - Waiting for too long for a good offer is costly, since the future is discounted ($\beta < 1$)
  - Accepting too early is costly, since the worker might draw better offers in the future

Job search problem

- The utility value of a worker employed at wage w is constant at

$$V(w) = w + \beta V(w) \implies V(w) = \frac{w}{1-\beta}$$

- When the worker gets an offer, he decides whether to accept or not.
- Let U be the utility value of an unemployed worker
- The unemployed worker utility value U satisfies the following equation:

$$\begin{aligned} U &= b + \beta \int_0^B \max\left[V(w), U\right] dF(w) \\ &= b + \beta U + \beta \int_0^B \max\left[V(w) - U, 0\right] dF(w) \end{aligned}$$

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Job search problem

- Value of having an offer w at hand:

$$\mathcal{V}(w) = \max\{V(w), U\} = \max\left\{\frac{w}{1-\beta}, U\right\}$$

where the maximization is over two actions:

1. accept the wage offer and work forever at wage w
2. reject the offer, receive b and sample again next period.

- Worker chooses a policy for whether to accept or reject a job: a **stopping rule.**

Reservation wage strategy

- $V(w)$ increasing in w; U does not depend on $w \implies$ unique R, called the reservation wage, such that

$$V(R) = U$$

- Solution is of the form

$$\mathcal{V}(w) = \begin{cases} \frac{R}{1-\beta} \text{ if } w < R \\ \frac{w}{1-\beta} \text{ if } w \geq R \end{cases}$$

Reservation wage strategy

- Optimal stopping rule is a **reservation wage strategy**: Accept any job with

$$w \geq R \equiv (1-\beta)U \ ,$$

and reject any $w < R$.
- **Intuition**: continue to hunt for a job as long as incoming job offers give wages below the reservation wage
- We would like to derive an equation for the reservation wage R that does not depend on U but only on model parameters

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Reservation wage strategy

- Recall the equation for U:

$$U = b + \beta U + \beta \int_0^B \max\left[V(w) - U, 0\right] dF(w)$$

- Rewrite the previous equation as

$$R = b + \frac{\beta}{1-\beta} \int_R^B (w - R)\, dF(w) \tag{1}$$

Derivation

- Interpretation: "reservation wage = unemployment benefits + discounted expected value of what the job search can give above the reservation wage"

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Reservation wage

- Reservation wage is "a" solution of

$$R - b = \frac{\beta}{1-\beta} \int_R^B (w - R) \; dF(w) \equiv h(R) \tag{2}$$

- The LHS is a line with positive slope and vertical intercept $-b$.
- The RHS is s.t. $h(0) = \mathbb{E}(w)\frac{\beta}{1-\beta} > 0$ and is downward sloping with

$$\begin{aligned} h'(R) &= \frac{\beta}{1-\beta} \frac{\partial}{\partial R} \left[\int_R^B (w - R) \; dF(w) \right] \\ &= -\frac{\beta}{1-\beta} [1 - F(R)] \leq 0 \; . \end{aligned}$$

Leibniz's rule

- Hence equation (2) has a **unique** positive solution R.

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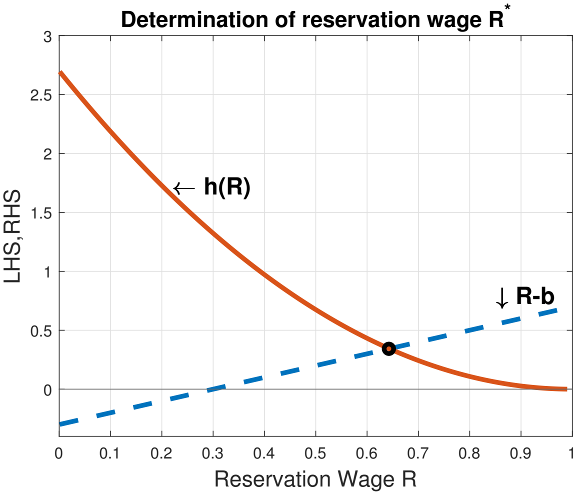
Reservation Wage: Numerical Example

Example with $F(w)$ uniform on $[0, B]$, $B = 1$, $b = 0.3$ and $\beta = 0.9$. It follows that $h(R) = \frac{\beta}{1-\beta}\left[(1/2B)R^2 - R + \mathbb{E}(w)\right]$, with $\mathbb{E}(w) = 1/2$.



Job Finding Rate and Unemployment Duration

- The job–finding rate (*outflow* rate from unemployment) is

$$\Pr\{w \geq R\} = [1 - F(R)] \equiv \phi(R)$$

- Probability of being unemployed for t periods:

$$[1 - \phi(R)]^{t-1}\,\phi(R)$$

i.e. prob. of remaining unemployed for $t-1$ periods and finding a job in period t

- Expected unemployment duration is then

$$ED = \sum_{t=1}^{\infty} t\,[1 - \phi(R)]^{t-1}\,\phi(R) = \frac{1}{\phi(R)}$$

Math

Comparative statics

$$\boxed{R - b = \frac{\beta}{1-\beta}\int_R^B (w-R)\;dF(w) \equiv h(R)}$$

- How does R vary with b?
- By implicit differentiation:

$$\begin{aligned} \frac{dR}{db} - 1 &= h'(R)\frac{dR}{db} \\ \iff \frac{dR}{db} &= \frac{1}{1-h'(R)} > 0 \end{aligned}$$

since $h'(R) = -\frac{\beta}{1-\beta}[1-F(R)] \leq 0$.

- Therefore, higher b increases R, reduces job-finding rate $\phi(R)$ and increases duration of unemployment $1/\phi(R)$.

Comparative statics, cont.

- We have shown that an increase in unemployment benefits b lengthens the duration of unemployment
- Intuition: If unemployed workers get higher compensation, they will be more demanding in terms of the wage they hope to get $\implies$ on average they will spend more time looking for a job
- Higher β increases R (Proof: exercise)
- Intuition: Higher β $\implies$ unemployed workers place more value on the future $\implies$ they have a higher reservation wage $\implies$ they spend more time looking for a job

Comparative statics, cont.

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Comparative statics, cont.

- What is the effect of a more risky wage–offer distribution?
- We need to define formally what "more risky" means for a distribution
- Use a concept known as **mean preserving spread**

Mean Preserving Spread

A distribution G is a mean–preserving spread of F if it is obtained by a process that shifts probability towards the tails of the distribution while keeping the mean constant.

Digression: a useful formula

- For any $R \leq B$ and distribution F on $[0, B]$:

$$\int_R^B (w - R)\, dF(w) = \int_R^B [1 - F(w)]\, dw = \mathbb{E}_F(w) - R + \int_0^R F(w)\, dw,$$

  where

$$\mathbb{E}_F(w) = \int_0^B w \; dF(w) = \int_0^B [1 - F(w)]\, dw.$$

- Proof: see notes on probability.

Mean–preserving spread: formal definition

- Distribution G (also defined on $[0, B]$) is a mean–preserving spread of F if there exists $\hat{w} \in (0, B)$ such that

$$
\begin{aligned}
G(w) &\geq F(w)\ , \ \text{for } w \leq \hat{w}\ , \\
G(w) &\leq F(w)\ , \ \text{for } w \geq \hat{w}\ ,
\end{aligned}
$$

  and $\mathbb{E}_F(w) = \mathbb{E}_G(w) \Leftrightarrow \int_0^B [G(w) - F(w)]\ dw = 0$.

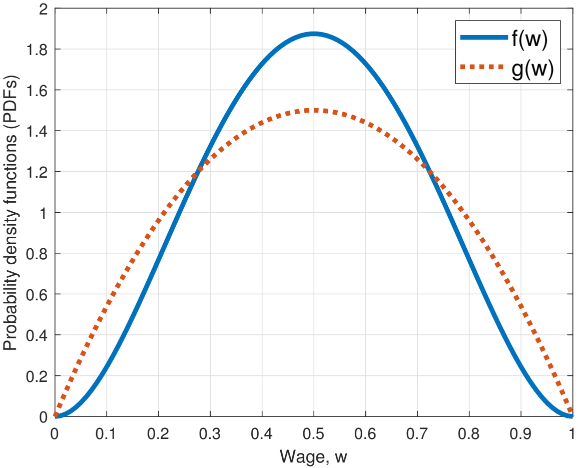
- **Intuition**: Distribution G has the same mean as F but is riskier, puts more weight on the "tails".

- Equivalent is the following feature:

$$
\int_0^y [G(w) - F(w)]\ dw \geq 0 \ \text{ for all } y \in [0, B]\ , \qquad (3)
$$

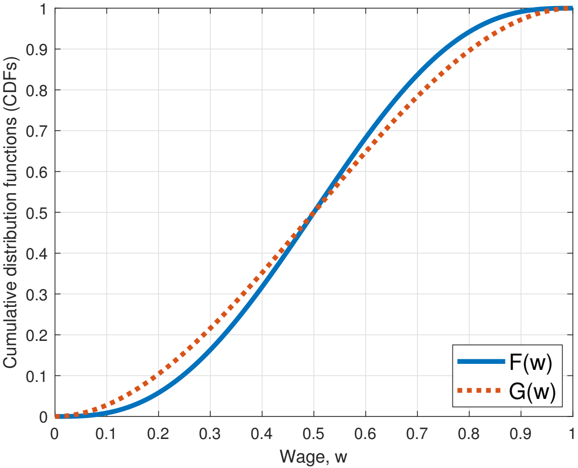
  with equality for $y = B$.

Mean Preserving Spread: Example



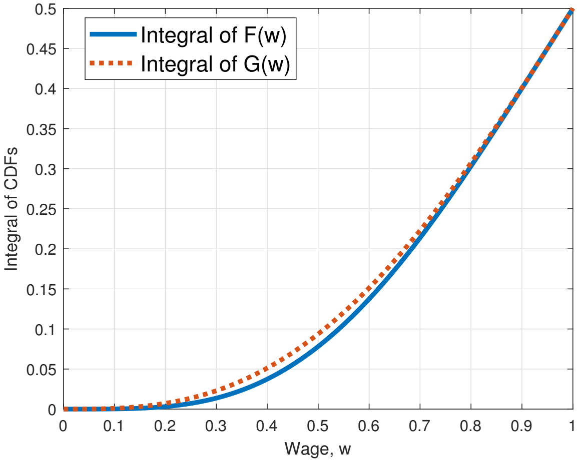
- Distribution G is a mean–preserving spread of F: Comparison of the **probability density functions**

Mean Preserving Spread: Example, cont'd



- Distribution G is a mean–preserving spread of F: Comparison of the **cumulative density functions**

Mean Preserving Spread: Example, cont'd



- Distribution G is a mean–preserving spread of F, hence by (3)

$$\int_0^y G(w)dw \geq \int_0^y F(w)dw \quad \text{for all } y \in [0, B]$$

Effect of mean–preserving spread on reservation wage

$$\boxed{R - b = \frac{\beta}{1-\beta} \int_R^B (w - R) \;\, dF(w) \equiv h(R)}$$

- If G is a mean-preserving spread of F,

$$\begin{aligned} h_G(R) &= \frac{\beta}{1-\beta}\left[\mathbb{E}_G(w) - R + \int_0^R G(w)\; dw\right] \,, \\ &\geq \frac{\beta}{1-\beta}\left[\mathbb{E}_F(w) - R + \int_0^R F(w)\; dw\right] = h_F(R) \;. \end{aligned}$$

  where the inequality follows from (3).
- This shows that $R_G \geq R_F$.

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Mean–preserving spread: intuition for result

- We have shown that if G is a mean–preserving spread of F, then the reservation wage under G is higher than under F
- More risk raises the *option value of waiting*.
- Under a mean–preserving increase in risk
  - More very good wage offers increase the value of waiting
  - More very bad wage offers: irrelevant, since they will be rejected
- Compare to a well-know result in finance: value of an option is increasing in the variance of the price of the underlying assets

Extending the model

- So far we have presented a basic model to describe the behavior of a job-seeker in a situation with imperfect information
- Model gives sharp predictions about the effects of a change in policy (e.g. unemployment benefits b) or in the environment (e.g. mean-preserving-spread of the offer distribution F)
- The basic model is however grounded on a number of unrealistic assumptions, such as
  1. The probability of getting an offer is fixed (equal to one in the basic model) and does not depend on the intensity of search
  2. Employed workers keep their job forever (i.e. no quits and no layoffs)
  3. Workers cannot look for (another) job while they are employed

Allowing quits

- Suppose workers are allowed to quit a job in which case they must be unemployed for at least one period.
- They don't do it!
- For any $w \geq R$, a quit would be optimal if $V(w) < U$.
- But $U = \dfrac{R}{1-\beta} \leq \dfrac{w}{1-\beta} = V(w)$

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Separations and Stochastic Offers

- We extend the baseline model along two dimensions:
  - **Separations**: A job ends with exogenous separation/firing probability δ in which case the worker becomes unemployed
  - **Stochastic offers**: When unemployed, a worker draws in every period a job offer with probability λ
- The value for employed worker becomes

$$V(w) = w + \beta\Big[\delta U + (1-\delta)V(w)\Big]$$

- The value for unemployed worker becomes

$$U = b + \beta(1-\lambda)U + \beta\lambda \int_0^B \max\left[V(w), U\right] dF(w)$$

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Separations and Stochastic Offers

- Subtract U from $V(w)$:

$$V(w) - U = \frac{w - (1-\beta)U}{1 - \beta(1-\delta)}.$$

- Optimal policy takes again the form of a reservation wage strategy
- Reservation wage is such that $W(R) = U \implies R = (1-\beta)U$.
- But now the value of being unemployed, U, is different $\implies R$ will be different from the previous case.

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Separations and Stochastic Offers

- Value of unemployment is

$$U = b + \beta U + \beta\lambda \int_0^B \max\left[V(w) - U, 0\right] dF(w)$$

- Modified reservation–wage equation is

$$R - b = \frac{\lambda\beta}{1-\beta(1-\delta)} \int_R^B (w - R)\, dF(w) \equiv h(R) \qquad (4)$$

- The RHS is smaller with firings and stochastic offers $\implies$ the reservation wage R is strictly lower
- Moreover, an increase of δ (less stable jobs) reduces R and hence increases the job–finding rate $\phi(R) = \lambda\Big[1 - F(R)\Big]$.
- **Implication**: positive (cross-country) correlation between unemployment inflows and outflows.

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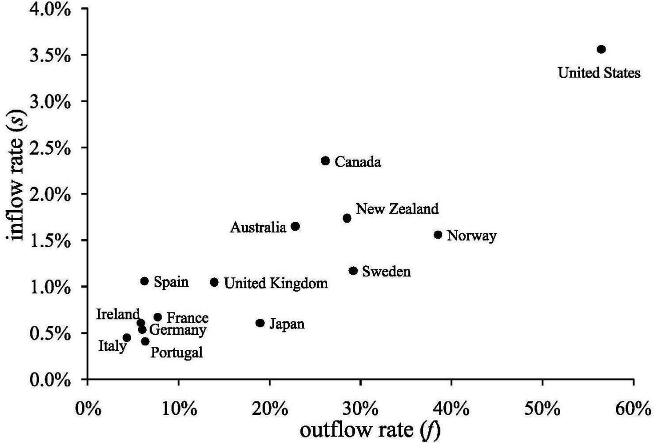
Unemployment inflow and outflow rates



Source: Elsby/Hobijn/Sahin, Unemployment Dynamics in the OECD, Review of Economics and Statistics 2013.

The unemployment rate

- Unit measure of individuals $L = 1$.
- When $u_t \in [0, 1]$ workers are unemployed in t, unemployment in $t + 1$ is

$$u_{t+1} = [1 - \phi(R)]u_t + \delta[1 - u_t]$$

or

$$u_{t+1} - u_t = \underbrace{\delta[1 - u_t]}_{\text{inflow to } u} - \underbrace{\phi(R)u_t}_{\text{outflow from } u}$$

- u_t converges to the stationary unemployment rate

$$u^* = \frac{\delta}{\delta + \phi(R)}$$

which is increasing in R.

- Impact of higher δ on u^* ?

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The Diamond (1971) paradox

Diamond (1971)

If the optimizing behavior of firms is introduced into the job search model, the equilibrium distribution of wages is concentrated at a single point.

- Suppose workers are homogeneous (as in the previous model).
- Distribution $F(w)$ is exogenously given: not very appealing feature of the model.
- What is the support of F? That is, what wages do profit–maximizing firms offer?
- Workers' strategy: accept any offer with $w \geq R$, *without distinction*

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The Diamond (1971) paradox, cont'd

- Since workers accept all wages equal or higher than R, no firm would choose $w > R$ because it would lose profit.
- No firm offers $w < R$ because no worker would accept.
- In equilibrium all firms will set $w = R \implies$ wage distribution F is degenerate at $w = R$
- From the reservation wage equation

$$R = b + \frac{\beta}{1-\beta} \int_R^B (w - R)\, dF(w) = b \tag{5}$$

- Hence $R = b$ and F degenerates at that point.
- Therefore the model is internally inconsistent.

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Ways out of the Diamond paradox

- How to break the paradox?
- On–the–job search (Burdett–Mortensen 1998).
  - On–the–job search $\Longrightarrow$ trade-off b/w profit-margin and the size of the workforce
  - For employed workers, reservation wage is their current wage.
  - If a firm posts a high wage, it attracts more workers but makes less profit per worker.
- If firms must search for workers (like workers search for jobs), two matched partners split a surplus. Hence the wage is not set unilaterally by firms but is bargained between worker and firm (**Diamond–Mortensen–Pissarides** model) $\rightarrow$ Second topic of the course.

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  - For employed workers, reservation wage is their current wage.
  - If a firm posts a high wage, it attracts more workers but makes less profit per worker.
- If firms must search for workers (like workers search for jobs), two matched partners split a surplus. Hence the wage is not set unilaterally by firms but is bargained between worker and firm (**Diamond–Mortensen–Pissarides** model) $\rightarrow$ Second topic of the course.

On–the–job search

- Suppose that unemployed (employed) workers receive job offers with probability λ_0 (λ_1). All wage offers are drawn from the same distribution F
- Employed workers follow a simple reservation wage strategy: change a job if (and only if) the new wage offer w' is greater than the current wage w
- The timing of events is now:
  1. Employed workers start the period with wage w
  2. They get fired with probability δ and become unemployed. They draw a new offer w' with probability λ_1 and decide whether to take it or not. With complementary probability $(1 - \delta - \lambda_1)$ they remain employed at wage w.

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On–the–job search

- The Bellman equation for an employed worker is

$$\begin{aligned} V(w) &= w + \beta\delta U + \beta(1-\delta-\lambda_1)V(w) \\ &\quad + \beta\lambda_1 \int_0^B \max[V(w'), V(w)]dF(w') \end{aligned}$$

- Using the property that $V(w') \geq V(w)$ iff $w' \geq w$ and rearranging, we obtain

$$\begin{aligned} V(w) &= w + \beta V(w) + \beta\delta(U - V(w)) \\ &\quad + \beta\lambda_1 \int_w^B [V(w') - V(w)]dF(w') \ . \end{aligned}$$

On–the–job search

- Differentiate to obtain

$$W'(w) = \frac{1}{1 - \beta[1 - \delta - \lambda_1(1 - F(w))]} \tag{6}$$

- The Bellman equation of an unemployed worker is

$$U = b + \beta U + \beta\lambda_0 \int_R [V(w) - U] \, dF(w) \, .$$

- Use $W(R) = U$ to obtain

$$R - b = \beta\left(\lambda_0 - \lambda_1\right) \int_R [V(w) - U] \, dF(w) \, .$$

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On–the–job search

- Partial integration and (6) yield

$$R - b = \beta(\lambda_0 - \lambda_1) \int_R \left(\frac{1 - F(w)}{1 - \beta\left[1 - \delta - \lambda_1(1 - F(w))\right]} \right) dw$$

- This equation implicitly defines the reservation wage R as a function of parameters λ_0, λ_1 and the distribution $F(\cdot)$
- If $\lambda_1 = 0$, i.e. no on-the-job search, we go back to the reservation wage of the basic model with separations (see eq. 4)
- If instead $\lambda_1 > 0$, workers take into account the value of on-the-job search and the reservation wage is lower
  - If $\lambda_1 > \lambda_0$, $R < b$ (foot-in-the-door effect).
  - if $\lambda_0 > \lambda_1$, $R > b$ (unemployment has positive search value).

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Burdett-Mortensen model (sketch)

- A filled job ends with probability $\delta + \lambda_1(1 - F(w))$.

  Higher wage $\Rightarrow$ Less quits

- A random worker accepts job offer w with probability $u + (1-u)G(w)$ where u is the unemployment rate and $G(w)$ is the (endogenous) earnings distribution (in steady state).

  Higher wage $\Rightarrow$ More hires

- So firms trade off the profit loss of higher wage offers against these two profitable effects.

- The unique equilibrium has a continuous distribution of wage offers with no mass points. Why?

- Implications:
  - Wage dispersion between equally productive firms.
  - Job-to-job transitions in equilibrium.

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Appendix

Reservation wage equation: Algebra

- The value of unemployment is

$$U = b + \beta U + \beta \int_0^B \max\left[V(w) - U, 0\right] dF(w)$$

$$U(1-\beta) = b + \beta \int_0^B \max\left[V(w) - U, 0\right] dF(w)$$

- By definition the reservation wage R satisfies

$$R \equiv (1-\beta)U$$

hence

$$R = b + \beta \int_0^B \max\left[V(w) - U, 0\right] dF(w)$$

Reservation wage equation: Algebra, cont.

- Observe that

$$\begin{aligned}\int_0^B \max\left[V(w)-U,0\right] dF(w) &= \int_0^R 0 dF(w) + \int_R^B \left(V(w)-U\right) dF(w)\\ &= \int_R^B \left(\frac{w}{1-\beta} - \frac{R}{1-\beta}\right) dF(w)\end{aligned}$$

- Therefore

$$\begin{aligned}R &= b + \beta \int_R^B \left(\frac{w}{1-\beta} - \frac{R}{1-\beta}\right) dF(w)\\ &= b + \frac{\beta}{1-\beta}\int_R^B \left(w-R\right) dF(w),\end{aligned}$$

which is equation (1).

Back

Geometric Series

- Recall the formula for a geometric series:

$$\sum_{n=1}^{\infty} nx^{n-1} = \frac{1}{(1-x)^2}$$

  for $|x| < 1$.

- To derive expression for *ED* in the main text, apply the above formula by letting $x = 1 - \phi(R)$ and $t = n$:

$$\begin{aligned}\sum_{t=1}^{\infty} t(1-\phi(R))^{t-1}\phi(R) &= \phi(R)\sum_{t=1}^{\infty} t(1-\phi(R))^{t-1} \\ &= \phi(R)\frac{1}{[1-(1-\phi(R))]^2} \\ &= \frac{1}{\phi(R)}.\end{aligned}$$

Back

Expected value formula

- Recall formula for integration by parts:

$$\int_0^B u dv = uv|_0^B - \int_0^B v du$$

- Let $u = w$ and $dv = dF(w)$. Then

$$\begin{aligned}\int_0^B w dF(w) &= wF(w)|_0^B - \int_0^B F(w) dw \\ &= B - \int_0^B F(w) dw \\ &= \int_0^B [1 - F(w)] dw.\end{aligned}$$

Q.E.D.

Back

Leibniz's rule

- To take derivatives of an integral we apply Leibniz's rule. Let

$$I(t) = \int_{\alpha(t)}^{\beta(t)} f(x,t)dx$$

  for $t \in [c,d]$. Assume that f and f_t are continuous and that α, β are differentiable on $[c,d]$.

- Then $I(t)$ is differentiable on $[c,d]$ and

$$I'(t) = \beta'(t) \times f(\beta(t),t) - \alpha'(t) \times f(\alpha(t),t) + \int_{\alpha(t)}^{\beta(t)} f_t(x,t)dx.$$

- To apply this formula to the equation in the text, let R play the role of t.

Back