Problem Set 5

Due on Monday, July 8, h 13:00. Mail box F230.

Job Search (15 points)

An unemployed worker samples wage offers from a cumulative distribution $F(w)$ with compact support $[0, B]$, and density f. Successive drawings across periods are independently and identically distributed. The worker chooses a strategy to maximize

$$\mathbf{E}\sum_{t=0}^{\infty}\beta^t u\left(c_t\right),$$

where $\beta \in (0,1)$, $c_t = w$ if the worker is employed at the wage w, and $c_t = b$ if the worker is unemployed. Here b is the unemployment compensation ($0 < b < B$). The utility function u is increasing and *concave*, with $u(0) = 0$. Assume that, having accepted a job offer at wage w, then in each period the worker might either be fired with probability $\delta \in (0,1)$, or, with probability $(1-\delta)$, remain in the job (earning the same wage w). When the worker is fired he faces the same distribution of offers F as we described above.

a) Write the problem of the worker in recursive form, identifying the states, the controls and the low of motion for states.

b) (Optional) Formally argue that the Bellman equation you reported in (a) defines a contraction.

c) Show that the optimal policy is defined by a wage level w^*, such that the worker accepts all wage offers $w \geq w^*$, and she reject all offers below w^*. Draw a graph of the value function $V(\cdot)$, where $V(w)$ represents the lifetime utility of an unemployed worker who just received a wage offer w.

d) Derive the (implicit) equation that defines the "reservation wage" w^* in terms of parameters only. Briefly comment on your equation.

e) Once both the optimal reservation wage policy and the probability of being fired have been taken into account, one can summarize the dynamics of the worker as follows. The worker can be in two states: state u (unemployed) or state e (employed). The probability of a previously unemployed worker to move into employment is $\phi := \int_{w^*}^{B} dF(w)$; while the probability of an employed worker of moving into unemployment is δ. We want to see such dynamics as a Markov chain.

i) Let the state space be $X := \{e, u\}$. Describe the Markov chain on X that is induced by the description above [Hint: Write the associated transition matrix carefully explaining the meaning of all entries of the matrix].

ii) Compute all stationary distributions of the chain. Carefully explain your result.

iii) Suppose the parameters of the model are such that the optimal reservation wage policy of the worker implies $\phi = 0.05$, and that $\delta = .25$. Compute the fraction of his life that an infinitely lived worker would spend unemployed according to the stationary distribution.

iv) Formally explain how the answer to the previous point would change if we increased the unemployment compensation level b [Hint: You might want to make reference to the expression you obtained for question (d)].

DMP model with aggregate productivity shocks (15 points)

Consider the baseline DMP model with exogenous job destruction $\delta \in (0, 1)$. Households are risk-neutral and discount the future at rate $\beta = 1/R$, where $R = 1 + r$. The change wrt the model seen in Lecture 6 is that aggregate productivity z is not constant at p but is drawn from a Markov chain $\Gamma(z, z')$ with N values.[1] Employed workers receive a wage $w(z)$ and separate at rate δ. Firm profits are $z - w(z)$. Unemployed workers receive benefits b and search for vacancies. There is an aggregate CRS matching function $m(u, v)$ with $f(\theta) = m/u$ and $q(\theta) = m/v$. As usual, let $\theta = v/u$. The flow cost of posting a vacancy is c. Finally, the law of motion for unemployment is $u_{t+1} = u_t + \delta(1 - u_t) - f(\theta_t)u_t$.

a) Let $U(z)$ and $W(z)$ denote the value of being unemployed or employed, when the state of the economy is z. Write down the two Bellman equations for $U(z)$ and $W(z)$. Hint: the Bellman equation for the unemployed can be written as:

$$U(z) = b + \beta \mathbb{E}_z \left[f(\theta)W(z') + (1 - f(\theta))U(z') \right]$$

b) Let $V(z)$ and $J(z)$ denote the values for vacant and matched firm. Write down the two Bellman equations.

[1] The term $\Gamma(z, z')$ represents the probability that tomorrow's productivity is z' given that today's productivity is z.

c) Assuming the free entry condition $V(z) = 0, \ \forall z$ derive the job creation equation.

d) The wage $w(z)$ is determined by Nash bargaining between worker and firm, i.e. wage solves

$$\max_{w(z)} [W(z) - U(z)]^{\gamma} [J(z) - V(z)]^{1-\gamma}$$

where γ is the bargaining power of worker. Show that the solution is the following:

$$w(z) = (1-\gamma)b + \gamma\left(z + c\theta(z)\right) \tag{1}$$

Remark: eq (1) is very similar to the (WS) equation derived in Lecture 6 but has one notable difference: market tightness θ now depends on stochastic productivity z.

e) Combine the job creation condition derived in point (c) with the wage equation (1) to obtain a single equation that determines $\theta(z)$. It should look similar (but not identical) to equation (*) in Lecture 6.

f) Detail a solution algorithm to solve the model numerically. Remark: you don't have to *actually* solve the model numerically, just provide a verbal explanation.