Problem Set 1

Due on Monday, April 29, h 13:00. Mail box F230.

Problem 1 (10 points)

Consider the simple model seen in Lecture 1 (pure-exchange OLG economy w/out money and w/out social security). Assume $\beta > 0$, $u(c) = \dfrac{c^{1-\sigma}}{1-\sigma}$, with $\sigma \neq 1$. Each generation $t \geq 1$ receives an income stream:

$$\left(e_t^t, e_{t+1}^t\right) = \left(e_1(1+\gamma)^t, e_2(1+\gamma)^{t+1}\right).$$

Income grows at constant rate $\gamma > 0$.

1. Find the equilibrium interest rate and call it R_{aut}.
2. Discuss how consumer patience (i.e. β) and the steepness of the income profile (i.e. γ) affect the interest rate in equilibrium.

Problem 2 (10 points)

Consider the simple model seen in Lecture 1 (pure-exchange OLG economy w/out money and w/out social security) but let each generation be populated by two types of agents, $h = 1, 2$, that differ in their endowments. Let $\mathbf{e}^h = \left(e_t^t(h), e_{t+1}^t(h)\right)$ denote the endowment vector of a type-h agent. Specifically,

$$\mathbf{e}^1 = (1,1)\,, \mathbf{e}^2 = (2,1)\,.$$

Intuitively, type-2 agents would like to save when they are young, and might be able to lend to type-1 agents. Type-1 agents will repay the loan when they are old. The discount factor is $\beta = 1$ and agents have the same preferences, i.e. $U^h(c_t^t(h), c_{t+1}^t(h)) = \log c_t^t(h) + \log c_{t+1}^t(h)$, for $h = 1, 2$. Let $\left(c_t^t(h), c_{t+1}^t(h), s_t^h\right)$ denote the allocation chosen by a type-h agent of generation $t \geq 1$.

1. Carefully define a competitive equilibrium for this pure-exchange OLG economy, including the market clearing conditions for consumption good and for assets. (Make sure you define all new variables, if any, that you introduce).
2. What is the equilibrium interest rate?
3. What is the equilibrium allocation?

Problem 3 (10 points)

Suppose the economy consists of three households (indexed by h, with $h = 1, \dots, H$ and $H = 3$). Let $\mathbf{e}^h = \big(e^t_t(h), e^t_{t+1}(h)\big)$ denote the endowment vector of a type-h agent. The endowment vectors are:

$$\mathbf{e}^1 = (4, 2), \mathbf{e}^2 = (0, 2a), \mathbf{e}^3 = (0, 2b),$$

with $0 < a < b < 1$. All households have the same preferences, $U^h(c^t_t(h), c^t_{t+1}(h)) = c^t_t(h) \cdot c^t_{t+1}(h)$, for $h = 1, 2, 3$.

1. Let $s^h_t(R_t)$ denote the savings function of a type-h agent. Write down the market clearing condition for assets in terms of $s^h_t(R_t)$.
2. Find the equilibrium interest rate, call it $\bar{R}$, and the amount of funds borrowed and lent in equilibrium, call it $\bar{B}$.
3. Now assume that consumers face the following exogenous constraint on borrowing:
$$c^t_t(h) - e^t_t(h) \leq a/R_t,$$
for all $h = 1, 2, 3$.
4. Find the equilibrium interest rate, call it $\bar{R}_c$, and the amount of funds borrowed and lent in equilibrium, call it $\bar{B}_c$.
5. Discuss the economic intuition. Specifically, how do $\bar{R}$ and $\bar{R}_c$ compare?