Advanced Macroeconomics II

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Basic Information

- Lecturer: Alessandro Di Nola
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- Office hours: Friday, 14:00-15:00
- Class times: Lecture (weekly) on Monday, 13:30-15:00 (G 309)
- Tutorial (biweekly) on Monday, 15:15-16:45 (G 309)

Grading

- Final exam 60%
- Problem sets 40%
- Submit all 5 assignments and be ready to present their solutions in the tutorials
- Printouts (also computer code and figures if required) of the assignments are to be submitted before the respective tutorial
- Students are allowed to cooperate with each other but must submit an individual copy of each problem set
- Only best 4 problem sets will be graded

Outline of the course

- Go beyond benchmark model with complete and frictionless markets (representative agent)
- Emphasize the role of heterogeneity and market imperfections
- **Topic 1**: Overlapping-generation model (OLG)
  - Agents of different ages live at the same time in the economy
  - Analyze social security, retirement, public debt, etc.
- **Topic 2**: Heterogeneous agent model (HA)
  - Income and wealth inequality
- **Topic 3**: Search and matching in the labor market
  - Analyze unemployment and labor market dynamics

OLG: Pure exchange

Overlapping Generations Model

- The standard neoclassical growth model has a representative infinitely lived household.
- Overlapping generations models have an infinite number of finitely–lived households.
- Interesting applications, such as social security, life cycle consumption/savings profiles, distributional effects of government debt etc.
- Several predictions differ from standard general equilibrium models.
    - Failure of the first welfare theorem
    - Equilibrium indeterminacy (existence of a continuum of equilibria, sunspot equilibria).
    - Positively valued outside money (speculative bubble).

Economic Environment

- Time $t = 1, 2, 3, \ldots$. Generations $i = 0, 1, 2, \ldots$.
- In each period, there is a single, non–storable consumption good.
- Generation $i = t \geq 1$ lives in periods $t, t+1$ and has endowments $e_t^t \geq 0$, $e_{t+1}^t \geq 0$. No endowments in other periods. Note: superscript $\rightarrow$ date of birth, subscript $\rightarrow$ time period.
- Preferences are $U\left(c_t^t, c_{t+1}^t\right) \equiv u(c_t^t) + \beta u(c_{t+1}^t)$.
- $u' > 0$ and $u'' \leq 0$.
- The initial old generation $i = 0$ has endowment e_1^0 and preferences $u(c_1^0)$.
- Consider a stationary environment:

$$e_t^t = e^Y \equiv 1 - \varepsilon \ , \ e_{t+1}^t = e^O \equiv \varepsilon \ , \text{ with } \varepsilon \in [0, 1] \ .$$

- To fix ideas, we will often assume that $\varepsilon < 0.5$, hence young would like to save, and $\beta = 1$

Demographic Structure

GENERATIONS	TIME 1	2	3	$\cdots$
0	old (initial)			$\cdots$
1	young	old		$\cdots$
2		young	old	$\cdots$
3			young	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

- Each agent lives for two periods. Therefore at each point in time the economy is populated by two generations, the young and the old. At each point in time a new generation appears.
- At time 1, there are old people (the initial old) around.

Markets

- Individuals trade their endowments using one period lending and borrowing arrangements, to maximize their lifetime utility.
- Market clearing determines prices and interest rates.
- Write p_t, $t \geq 1$, for the price of the consumption good.
- Let R_t be the *gross* interest rate between t and $t+1$

$$R_t \equiv \frac{p_t}{p_{t+1}}$$

- *Net* interest rate is r_t, where $1 + r_t = R_t$.
- Let s_t be the savings of generation t from t to $t+1$

Important Definitions

- **Feasible allocation**. An allocation is feasible if $c_t^{t-1} \geq 0, c_t^t \geq 0$ for all $t \geq 1$ and
$$c_t^{t-1} + c_t^t = e_t^{t-1} + e_t^t \text{ for all } t \geq 1.$$
- **Pareto optimal allocation**. An allocation $\left\{c_1^0, (c_t^t, c_{t+1}^t)_{t\geq 1}\right\}$ is Pareto optimal (PO) if it is feasible and there is no other feasible allocation $\left\{\widetilde{c}_1^0, (\widetilde{c}_t^t, \widetilde{c}_{t+1}^t)_{t\geq 1}\right\}$ such that
$$\begin{aligned} U\left(\widetilde{c}_t^t, \widetilde{c}_{t+1}^t\right) &\geq U\left(c_t^t, c_{t+1}^t\right), \text{ for all } t \geq 1 \\ u(\widetilde{c}_1^0) &\geq u(c_1^0) \end{aligned}$$
with *strict* inequality for at least one $t \geq 0$.
- **Autarkic allocation**. An allocation $\left\{c_1^0, (c_t^t, c_{t+1}^t)_{t\geq 1}\right\}$ is autarkic if
$$c_1^0 = e_1^0,\ c_t^t = e_t^t,\ c_{t+1}^t = e_{t+1}^t$$
i.e. every agent consume all of his endowment and does not trade.

Equilibrium

A competitive equilibrium (CE) is an allocation $\left\{\hat{c}_1^0, (\hat{c}_t^t, \hat{c}_{t+1}^t, \hat{s}_t)_{t\geq 1}\right\}$ and prices $(R_t)_{t\geq 1}$, such that:

1. Given $(R_t)_{t\geq 1}$, for all $t \geq 1$, $(\hat{c}_t^t, \hat{c}_{t+1}^t, \hat{s}_t)$ solves

$$\max u(c_t^t) + \beta u(c_{t+1}^t)$$

   subject to

$$\begin{aligned} c_t^t + s_t &\leq e_t^t, \\ c_{t+1}^t &\leq e_{t+1}^t + R_t s_t. \end{aligned}$$

   For $i = 0$ (the initial old), $\hat{c}_1^0$ maximizes $u(c_1^0)$ subject to $c_1^0 \leq y_1^0$.

2. For all $t \geq 1$,

$$\hat{c}_t^{t-1} + \hat{c}_t^t = e_t^{t-1} + e_t^t \ .$$

- Q: Do we need to impose a constraint on borrowing, i.e. $s_t \geq -A$, in order to prevent Ponzi schemes?

Trade?

- Given the OLG structure, will the members of any generation engage in transactions with members of another generation?

Trade?

- Suppose $e^Y = 1 - \varepsilon$, $e^O = \varepsilon$ with $\varepsilon < \frac{1}{2}$
- There is an intertemporal consumption smoothing problem for a young agent who has income now but wants to save for old age when she has low income.
- She can't lend to someone of her own generation since they are all identical.
- she can't lend to an old agent since they will be dead next period (and hence cannot repay the loan).
- she can't lend to the next periods young since they aren't even born yet.
- This is a source of **endogenous market incompleteness** (i.e. the environment rules out a private loan market).
- Things would be better if consumers lived for three or more periods...

Trade?

Lemma

The only competitive equilibrium has

$$\widehat{c}_1^0 = e^O,\ \ \widehat{c}_t^t = e^Y,\ \widehat{c}_{t+1}^t = e^O, \widehat{s}_t = 0,$$

i.e. autarky is the only equilibrium.

Proof.

- Since initial old has no outside asset, then $c_1^0 = e^O$.
- Consider market clearing in $t = 1$

$$c_1^0 + c_1^1 = e^O + e^Y$$

It follows that $c_1^1 = e^Y$, i.e. the first young generation must consume their endowment when young.

□

Proof, cont'd

- Consider BC of young living in $t = 1$ and $t = 2$

$$
\begin{aligned}
c_1^1 + s_1 &= e^Y \\
c_2^1 &= e^O + R_1 s_1
\end{aligned}
$$

Since $c_1^1 = e^Y$, it follows that $c_2^1 = e^O$.

- Consider market clearing in $t = 2$

$$c_2^1 + c_2^2 = e^O + e^Y.$$

Hence also the second young generation (born in $t = 2$) is forced to consume their endowments, and so on (formally, using induction).

- Trades break down completely.

Autarkic interest rate

- How can we determine equilibrium prices?
- Equilibrium prices (or interest rates) will make households happy to consume their endowment.
- Optimality conditions (Euler equation):

$$\frac{u'(c_t^t)}{\beta u'(c_{t+1}^t)} = \frac{p_t}{p_{t+1}} = R_t.$$

- Prices that support autarky are s.t.

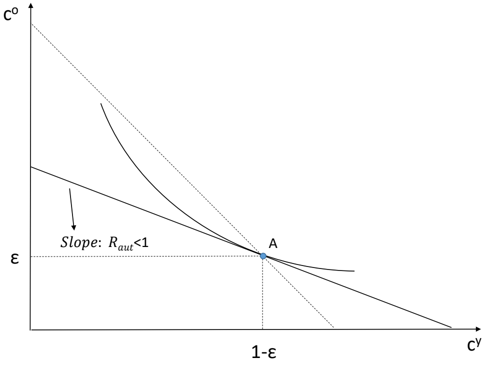
$$R_t = \frac{u'(e^Y)}{\beta u'(e^O)}.$$

- We call this (constant) interest rate R_{aut}.

Samuelson vs Classical case

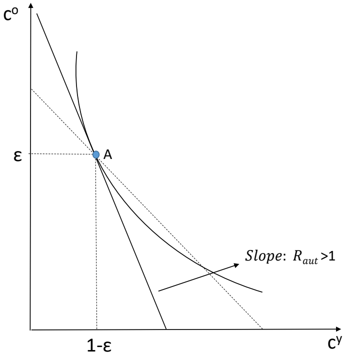
- Samuelson case: If $\beta e^{Y} > e^{O}$ (young want to save) then $R_{aut} < 1$.
- Classical case: If $\beta e^{Y} < e^{O}$ (young want to borrow) then $R_{aut} > 1$.

Samuelson case



- $\varepsilon < 1/2$. Feasibility line (dashed line): $c^y + c^o = 1$. Budget line (solid line): $c^o = R_t(1-\varepsilon) + \varepsilon - R_t c^y$.

Classical case



- $\varepsilon > 1/2$. Feasibility line (dashed line): $c^y + c^o = 1$. Budget line (solid line): $c^o = R_t(1-\varepsilon) + \varepsilon - R_t c^y$.

Example: Equilibrium

- $u(c) = \log c$, $\beta = 1$, $\mathbf{e} = \left(e^Y, e^O\right) = (3, 1)$
- The initial old simply consume their endowment: $c_1^0 = 1$.
- Generation t solves

$$\max_{\left\{c_t^t, c_{t+1}^t, s_t\right\}} \log\left(c_t^t\right) + \log\left(c_{t+1}^t\right)$$

subject to

$$c_t^t + \frac{c_{t+1}^t}{R_t} = 3 + \frac{1}{R_t}. \tag{IBC}$$

- IBC is lifetime/intertemporal budget constraint.
- The Euler equation is

$$\frac{c_{t+1}^t}{c_t^t} = R_t. \tag{EE}$$

Example: Equilibrium, cont'd

- Combine IBC and EE to solve for *demand functions* $c_t^t(R_t)$ and $c_{t+1}^t(R_t)$:

$$c_t^t(R_t) = \frac{1}{2}\left(3 + \frac{1}{R_t}\right),\ c_{t+1}^t(R_t) = \frac{1}{2}(3R_t + 1).$$

  Note: can be written also in terms of p_t / p_{t+1}.
- Intuition: As R_t increases, consumption goods tomorrow become cheaper (i.e. p_{t+1} falls). This leads the household to substitute away from current consumption into the less costly future consumption (*substitution effect*).
- *Income effect* leads the household to consume more of c_t^t and c_{t+1}^t.
- Given demand functions and the budget constraints, write savings function as $s(R_t) = 3 - c_t^t(R_t)$. Hence

$$s(R_t) = \frac{3}{2} - \frac{1}{2R_t}.$$

Example: Equilibrium, cont'd

- Here $s'(R_t) > 0$ (SE dominates, for all R) but this need not be the case.
- The OLG literature calls the case where SE outweigh IE "gross substitutability".
- Since in equilibrium $s(R_t) = 0$ (no lending and borrowing),

$$R_t = \frac{1}{3}.$$

- A negative *net* interest rate is needed in order to make autarky optimal for the agents.

▸ Multiple Equilibria

Example: efficiency

- Is the equilibrium allocation in this example Pareto efficient?
- Can the planner redistribute resources in the economy in order to make everyone better off?
- Planner is constrained only by feasibility $\Longrightarrow$ she can divide the total resources available for consumption $(e^Y + e^O)$ between the old and the young in any manner.
- The planner can take away 1 unit of endowment from each young generation and pass it to the old generation alive in the same period.
- Clearly the initial old are strictly better off.
- Any generation $i \geq 1$ is also strictly better off since:

$$\log(2) + \log(2) > \log(3) + \log(1)$$

- Conclusion: the equilibrium allocation $\{1, (3, 1)_{t\geq 1}\}$ is Pareto dominated by the alternative, feasible allocation $\{2, (2, 2)_{t\geq 1}\}$.

Some Remarks 1

- The backwards redistribution scheme shown in the previous example worked since $T = \infty$.
- If $T < \infty$, then the autarkic equilibrium would be PO. Why?
- (*Heuristic proof*). Suppose the economy ends at $t = T$, with $T \geq 2$.
- Agents of generation T die young.
- For each generation $t < T$, agents give up consumption when young but are compensated when old.
- But generation T dies young, hence the "chain" breaks and generation T is made worse off.
- <u>Conclusion</u>: in the finite economy case, any equilibrium is PO.

Some remarks, 2

- Given the demographic structure, in equilibrium there is no trade *between* generations.
- However, if agents *within* generations are heterogeneous, there may be *intra*-generational trade.
- Consider the previous example with $u(c) = \log c$, $\beta = 1$, but let each generation be populated by two types of agents that differ in their endowments:

$$\mathbf{e}^1 = (1, 1), \mathbf{e}^2 = (2, 1).$$

- Now type-2 agents want to save when they are young, and might be able to lend to type-1 agents.
- Type-1 agent will repay the loan when they are old.
- Now the market clearing condition for assets is

$$s_t^1 + s_t^2 = 0.$$

- What is the equilibrium interest rate? $\implies$ Problem Set.

Pareto optimality: a general criterion

- **Proposition**. Consider a stationary and symmetric OLG economy with endowments $\left(e^Y, e^O\right)$. Then the (autarkic) equilibrium is PO iff $R_{aut} \geq 1$.
- Note: in equilibrium we have

$$R_{aut} = \frac{u'(e^Y)}{\beta u'(e^O)}.$$

- **Proof**. (Only if) Suppose that $R_{aut} = \frac{u'(e^Y)}{\beta u'(e^O)} < 1$. WTS that the equilibrium allocation $\left(c^Y = e^Y, c^O = e^O\right)$ is not PO.
- Consider the alternative, feasible allocation

$$\begin{aligned} \widetilde{c}^Y &= e^Y - \varepsilon, \\ \widetilde{c}^O &= e^O + \varepsilon. \end{aligned}$$

- The initial old agents are better off.
- Agents in any generation $i \geq 1$ are also better off since $u'(e^Y) < \beta u'(e^O)$.

Proof, cont'd

- Formally, let $U(\varepsilon) = u(e^Y - \varepsilon) + \beta u(e^O + \varepsilon)$. Then $U'(\varepsilon) = -u'(e^Y - \varepsilon) + \beta u'(e^O + \varepsilon) > 0$ for ε small.
- (If part). Suppose $R_{aut} \geq 1$. WTS that the equilibrium allocation $\left(c^Y = e^Y, c^O = e^O\right)$ is PO.
- By contradiction, assume there exists an alternative feasible allocation $\left(\widetilde{c}_t^t, \widetilde{c}_{t+1}^t\right)$ that Pareto dominates the equilibrium allocation.
- The only way this can happen is if goods are redistributed from the young to the old:

$$\widetilde{c}_t^t = e^Y - \varepsilon_t,$$
$$\widetilde{c}_{t+1}^t = e^O + \varepsilon_{t+1}.$$

  Since $u'(e^Y) \geq \beta u'(e^O)$, we need $\varepsilon_{t+1} > \varepsilon_t$.
- Eventually $\varepsilon_{t+k} > e^Y$, for some $k > 1$, violating feasibility.

Moving forward

- The fact that autarky is the only equilibrium is a rather unpleasant feature of the economy.
- We have seen that under some conditions young agents may be willing to give up some of their consumption in exchange of an increase in their consumption when old.
- How is it possible to implement such exchanges? E.g.
  1. Introducing a pay-as-you-go social security system.
  2. Introducing an intrinsically worthless piece of paper, called money, that allows *inter*-generational trade.
- Note: both (1) and (2) require some form of "social contrivance".
- Let's see first PAYGO social security.

Social Security

- Consider the pure-exchange OLG economy as before
- Assume population N_t grows at rate $n > 0$

$$N_t = (1 + n) N_{t-1}.$$

- Market clearing reads as

$$c_t^{t-1} + (1 + n) c_t^t = e_t^{t-1} + (1 + n) e_t^t.$$

- All else is the same. W/out any government intervention, the unique equilibrium is autarky.
- We introduce in this economy a pay-as-you-go social security system, such that young agents pay a tax $\tau \in (0, e^Y)$, the proceeds of which are distributed as benefit b among the old agents around.
- The social security system runs a balanced budget, hence

$$b = \tau (1 + n).$$

Social Security

- The only possible equilibrium is again autarky, i.e.

$$c_t^t = e^y - \tau,\ c_{t+1}^t = e^o + \tau\left(1+n\right).$$

- The equilibrium interest rate satisfies:

$$R_t = R = \frac{u'(e^y - \tau)}{\beta u'(e^o + \tau\left(1+n\right))}.$$

- Q: Does the introduction of pension system improve welfare?
- Clearly, initial old generation receives a windfall gain of $\tau\left(1+n\right)$, w/out paying any contribution.
- For all other generations, lifetime utility is

$$U\left(\tau\right) \equiv u\left(e^y - \tau\right) + \beta u\left(e^o + \tau\left(1+n\right)\right)$$
$$U'\left(\tau\right) = -u'\left(e^y - \tau\right) + \beta\left(1+n\right)u'\left(e^o + \tau\left(1+n\right)\right).$$

- PAYGO system is welfare improving if

$$U'\left(0\right) > 0 \iff 1+n > \frac{u'(e^Y)}{\beta u'(e^O)} = R_{aut}.$$

Intuition

- The introduction of a pension system is welfare imprioving iff population growth rate exceeds autarkic interest rate.
- This happens whenever autarky is not PO.
- Social security is an institution that transfers resources b/w generations (backward in time) that do not trade among each other in equilibrium.
- It is important to note that these social security taxes and transfers are “lump sum”. In the real world, the taxes are proportional to one’s earnings and can distort the labor supply decision.
    - In that case, there is a trade-off between efficiency losses when young and consumption smoothing when old.

Social Security: Optimal Size

- We have shown that if $(1+n) > R_{aut}$, a marginal rise in social security taxes τ (starting from 0) is welfare-improving.
- Is there an "optimal" level of τ?
- Let τ^* be the unique size of the social security system that maximizes the lifetime utility of the representative generation: $U'(\tau^*) = 0$.
- If, for example, $u(c) = \log c$, we can solve explicitly for τ^*:

$$\tau^*(\beta, n, e^Y, e^O) = \frac{\beta}{1+\beta} e^Y - \frac{e^O}{(1+\beta)(1+n)}.$$

- Note: the optimal size of social security is an increasing function of young income, the population growth rate n, and the time discount factor β; and a decreasing function of the old income.

Some Examples

Example 1: Growing endowment

- Consider pure-exchange OLG economy w/out money and w/out social security.
- Assume $\beta > 0$, $u(c) = \dfrac{c^{1-\sigma}}{1-\sigma}$, with $\sigma \neq 1$.
- Each generation $t \geq 1$ receives an income stream:

$$\left(e_t^t, e_{t+1}^t\right) = \left(e_1(1+\gamma)^t, e_2(1+\gamma)^{t+1}\right).$$

- Income grows at constant rate $\gamma > 0$.
- Find the equilibrium interest rate and call it R_{aut}.
- Discuss how consumer patience (i.e. β) and the steepness of the income profile (i.e. γ) affect the interest rate in equilibrium.

Example 2: Heterogeneity and Credit Constraints

- Suppose the economy consists of three households (indexed by h, with $h = 1, \ldots, H$ and $H = 3$) with endowment vectors:

$$\mathbf{e}^1 = (4, 2), \mathbf{e}^2 = (0, 2a), \mathbf{e}^3 = (0, 2b),$$

  with $0 < a < b < 1$.
- All households have the same preferences, $U^h(c_t^t, c_{t+1}^t) = c_t^t c_{t+1}^t$, for $h = 1, 2, 3$.
- Carefully define a competitive equilibrium for this pure-exchange OLG economy. (Make sure you define all new variables that you introduce).
- Find the equilibrium interest rate, call it $\bar{R}$, and the amount of funds borrowed and lent in equilibrium, call it $\bar{B}$.
- Now assume that consumers face the following exogenous constraint on borrowing:

$$c_t^t - e_t^t \leq a/R.$$

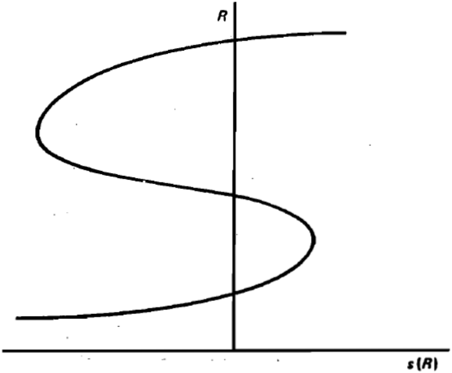
Example 2: Heterogeneity and Credit Constraints

- Find the equilibrium interest rate, call it $\bar{R}_c$, and the amount of funds borrowed and lent in equilibrium, call it $\bar{B}_c$.
- Discuss the economic intuition. Specifically, how do $\bar{R}$ and $\bar{R}_c$ compare?

Appendix

Multiple Equilibria

- $s(R)$ is the aggregate saving function.
- Figure below shows a case where "gross substitutability" condition is violated $\implies$ three equilibria.



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