Job Search Model

Lecture 5

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Motivation and overview

- Competitive, frictionless models cannot account for unemployment.
- We are interested in dynamic models of the labor market that can account for worker flows and job flows.
- These models can be useful for a variety of issues, for example
    - The role of labor market regulation and policy (e.g. unemployment insurance).
    - Wage inequality
    - Business cycle dynamics of the labor market
- In this lecture we start with a simple, partial model of job search.

Job search model (McCall 1970)

- Consider a risk–neutral worker; infinitely lived; discount factor β.
- Worker wants to maximize $\mathbb{E}\sum_{t\geq 0}\beta^t y_t$ where y_t is income in period t.
- In $t = 0$ worker is unemployed.
- When unemployed, the worker draws in every period $t \geq 0$ a job offer with probability λ (for a job starting in the next period). The offered wage w is drawn from cdf F.
- $F(0) = 0$ and $F(B) = 1$ for some $B < \infty$.
- An unemployed worker earns income b.

Job search model

- Decision: accept job offer or not?
- When the worker accepts a job at wage w in some period τ, he keeps the job forever: $y_t = w$ for all $t > \tau$.
- No recall of rejected offers (i.e. the worker cannot go back to a previously rejected offer).
- No quits, no layoffs: if worker accepts a job, he keeps it forever.
- Hence, the utility value of a worker employed at wage w is constant at

$$W(w) = w + \beta W(w) \implies W(w) = \frac{w}{1-\beta}$$

Job search problem

- Let U be the utility value of an unemployed worker.
- When the worker gets an offer, he decides whether to accept or not.
- Recursive problem

$$
\begin{aligned}
U &= b + \beta \Big\{ \lambda \int_0^B \max\left[W(w), U\right] dF(w) + (1-\lambda) U \Big\} \\
&= b + \beta U + \beta \lambda \int_0^B \max\left[W(w) - U, 0\right] dF(w) \ .
\end{aligned}
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Job search problem

- Value of having an offer w at hand:

$$V(w) = \max\{W(w), U\} = \max\left\{\frac{w}{1-\beta}, U\right\}$$

  where the maximization is over two actions:

  1. accept the wage offer and work forever at wage w
  2. reject the offer, receive b and sample again (with prob. λ) next period.

- Worker chooses a policy for whether to accept or reject a job: a **stopping rule.**

Reservation wage strategy

- $W(w)$ increasing in w; U does not depend on $w \implies$ unique R, called the reservation wage, such that

$$W(R) = U$$

- Solution is of the form

$$V(w) = \begin{cases} \frac{R}{1-\beta} \text{ if } w < R \\ \frac{w}{1-\beta} \text{ if } w \geq R \end{cases}$$

Reservation wage strategy

- Optimal stopping rule is a reservation wage strategy: Accept any job with

$$w \geq R \equiv (1-\beta)U \ ,$$

and reject any $w < R$.

- Rewrite the previous equation as

$$R = b + \frac{\lambda\beta}{1-\beta}\int_R^B (w-R)\,dF(w) \tag{1}$$

- Interpretation: "reservation wage = unemployment benefits + discounted expected improvement in next period offer"

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Reservation wage equation: Algebra

- The value of unemployment is

$$U = b + \beta U + \beta\lambda \int_0^B \max\left[W(w) - U, 0\right] dF(w)$$

$$U(1-\beta) = b + \beta\lambda \int_0^B \max\left[W(w) - U, 0\right] dF(w)$$

- By definition the reservation wage R satisfies

$$R \equiv (1-\beta)U$$

hence

$$R = b + \beta\lambda \int_0^B \max\left[W(w) - U, 0\right] dF(w)$$

Reservation wage equation: Algebra, cont.

- Observe that

$$\int_0^B \max\left[W(w) - U, 0\right] dF(w) = \int_0^R 0 dF(w) + \int_R^B \left(W(w) - U\right) dF(w)$$
$$= \int_R^B \left(\frac{w}{1-\beta} - \frac{R}{1-\beta}\right) dF(w)$$

- Therefore

$$R = b + \beta\lambda \int_R^B \left(\frac{w}{1-\beta} - \frac{R}{1-\beta}\right) dF(w)$$
$$= b + \frac{\beta\lambda}{1-\beta} \int_R^B (w - R)\, dF(w),$$

which is equation (1).

Reservation wage

- Reservation wage is "a" solution of

$$R - b = \frac{\lambda\beta}{1-\beta}\int_R^B (w - R)\ dF(w) \equiv h(R) \tag{2}$$

- The LHS is a line with positive slope and vertical intercept $-b$.
- The RHS is s.t. $h(0) = \mathbb{E}(w)\frac{\lambda\beta}{1-\beta} > 0$ and is downward sloping with

$$\begin{aligned} h'(R) &= \frac{\lambda\beta}{1-\beta}\frac{\partial}{\partial R}\left[\int_R^B (w - R)\ dF(w)\right] \\ &= -\frac{\lambda\beta}{1-\beta}[1 - F(R)] \leq 0\ . \end{aligned}$$

- Hence equation (2) has a **unique** positive solution R.

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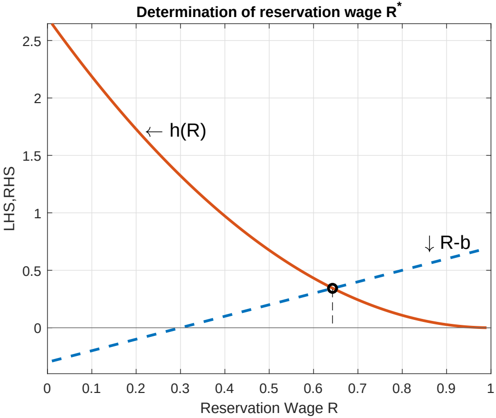
Reservation Wage

Example with $F(w)$ uniform on $[0, B]$, $B = 1$, $b = 0.3, \beta = 0.9$ and $\lambda = 0.6$. It follows that $h(R) = \frac{\lambda\beta}{1-\beta}\left[(1/2B)R^2 - R + \mathbb{E}(w)\right]$



Job Finding Rate and Unemployment Duration

- The job–finding rate (unemployment hazard rate) is

$$\lambda \cdot \Pr\{w \geq R\} = \lambda[1 - F(R)] \equiv \phi(R)$$

- Probability of being unemployed for t periods:

$$[1 - \phi(R)]^{t-1} \phi(R)$$

i.e. prob. of remaining unemployed for $t - 1$ periods and finding a job in period t

- Expected unemployment duration is then

$$ED = \sum_{t=1}^{\infty} t \, [1 - \phi(R)]^{t-1} \phi(R) = \frac{1}{\phi(R)}$$

Math

Comparative statics

$$\boxed{R - b = \frac{\lambda\beta}{1-\beta}\int_R^B (w - R)\ dF(w) \equiv h(R)}$$

- How does R vary with b?
- By implicit differentiation:

$$\begin{aligned}\frac{dR}{db} - 1 &= h'(R)\frac{dR}{db}\\ \iff \frac{dR}{db} &= \frac{1}{1-h'(R)} > 0\end{aligned}$$

since $h'(R) = -\frac{\lambda\beta}{1-\beta}[1 - F(R)] \leq 0$.

- Therefore, higher b increases R, reduces job-finding rate $\phi(R)$ and increases duration of unemployment $1/\phi(R)$.
- Intuition: worker becomes choosier.

Comparative statics, cont.

- Higher λ increases R; the effect on $\phi(R)$ is (generally) ambiguous.
- Higher β increases R.
- What is the effect of a more risky wage–offer distribution?

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Digression: a useful formula

- Expected value of positive random variable. For any distribution F on $[0, B]$:

$$\mathbb{E}_F(w) = \int_0^B w \ dF(w) = \int_0^B [1 - F(w)] \ dw \ .$$

Proof

- Using the above result, can show that for any $R \leq B$ and distribution F on $[0, B]$:

$$\int_R^B (w - R \ ) \, dF(w) = \mathbb{E}_F(w) - R + \int_0^R F(w) \ dw$$

Proof

Mean–preserving spread

- Distribution G (also defined on $[0, B]$) is a mean–preserving spread of F if there exists $\hat{w} \in (0, B)$ such that

$$\begin{aligned} G(w) &\geq F(w)\ ,\ \text{for } w \leq \hat{w}\ , \\ G(w) &\leq F(w)\ ,\ \text{for } w \geq \hat{w}\ , \end{aligned}$$

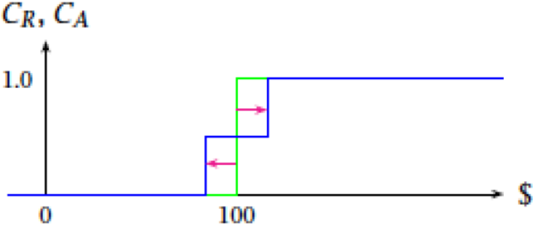
  and $\mathbb{E}_F(w) = \mathbb{E}_G(w) \Leftrightarrow \int_0^B [G(w) - F(w)]\ dw = 0$.

- Equivalent is the following feature:

$$\int_0^y [G(w) - F(w)]\ dw \geq 0 \ \text{ for all } y \in [0, B]\ , \tag{3}$$

  with equality for $y = B$.

Lottery example



- We know that the blue lottery (G) is a mean-preserving spread of the green lottery (F) because
  - They have the same mean (i.e. 100 \$)
  - The CDFs cross exactly once
  - G lies on or above F to the left of the crossing point.

Mean–preserving spread

$$\boxed{R - b = \frac{\lambda\beta}{1-\beta}\int_R^B (w - R)\ dF(w) \equiv h(R)}$$

- If G is a mean-preserving spread of F,

$$\begin{aligned} h_G(R) &= \frac{\lambda\beta}{1-\beta}\left[\mathbb{E}_G(w) - R + \int_0^R G(w)\ dw\right] , \\ &\geq \frac{\lambda\beta}{1-\beta}\left[\mathbb{E}_F(w) - R + \int_0^R F(w)\ dw\right] = h_F(R) \ . \end{aligned}$$

  where the inequality follows from (3).

- This shows that $R_G \geq R_F$.
- More risk raises the *option value of waiting*.

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Quits

- Suppose workers are allowed to quit a job in which case they must be unemployed for at least one period.
- They don't do it!
- For any $w \geq R$, a quit would be optimal if $W(w) < U$.
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Separations

- Suppose a job ends with exogenous separation/firing probability δ in which case the worker becomes unemployed.
- Then
$$W(w) = w + \beta\Big[\delta U + (1-\delta)W(w)\Big] \ ,$$
hence
$$W(w) - U = \frac{w - (1-\beta)U}{1 - \beta(1-\delta)}.$$
- Optimal policy takes again the form of a reservation wage strategy.
- Reservation wage is such that $W(R) = U \implies R = (1-\beta)U$.
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$$(1-\beta)U = b + \beta\lambda \int_R^B \frac{w-R}{1-\beta(1-\delta)} dF(w)$$

- Modified reservation–wage equation is

$$R - b = \frac{\lambda\beta}{1-\beta(1-\delta)} \int_R^B (w-R)\, dF(w) = h(R) \ .$$

- The RHS is smaller with firings $\implies$ the reservation wage R is strictly lower.
- Moreover, an increase of δ (less stable jobs) reduces R and hence increases the job–finding rate $\phi(R) = \lambda\Big[1 - F(R)\Big]$.
- **Implication**: positive (cross-country) correlation between unemployment inflows and outflows.

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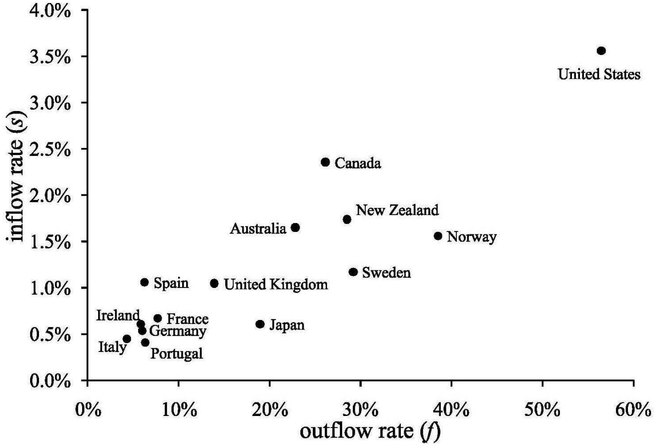
Unemployment inflow and outflow rates



Source: Elsby/Hobijn/Sahin, Unemployment Dynamics in the OECD, Review of Economics and Statistics 2013.

The unemployment rate

- Unit measure of individuals $L = 1$.
- When $u_t \in [0,1]$ workers are unemployed in t, unemployment in $t+1$ is

$$u_{t+1} = [1 - \phi(R)]u_t + \delta[1 - u_t]$$

or

$$u_{t+1} - u_t = \underbrace{\delta[1 - u_t]}_{\text{inflow to } u} - \underbrace{\phi(R)u_t}_{\text{outflow from } u}$$

- u_t converges to the stationary unemployment rate

$$u^* = \frac{\delta}{\delta + \phi(R)}$$

which is increasing in R.

- Impact of higher δ on u^* ?

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The Diamond (1971) paradox

- Suppose workers are homogeneous (as in the previous model).
- Distribution $F(w)$ is exogenously given: not very appealing feature of the model.
- What is the support of F? That is, what wages do profit–maximizing firms offer?
- No firm offers $w < R$ because no worker would accept.
- No firm offers $w > R$ because it would lose profit.
- Hence $R = b$ (Bertrand competition) and F degenerates at that point.
- Therefore the model is internally inconsistent.

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Ways out of the Diamond paradox

- How to break the paradox?
- On–the–job search (Burdett–Mortensen 1998).
  - On–the–job search $\implies$ trade-off b/w profit-margin and the size of the workforce
  - For employed workers, reservation wage is their current wage.
  - If a firm posts a high wage, it attracts more workers but makes less profit per worker.
- If firms must search for workers (like workers search for jobs), two matched partners split a surplus. Hence the wage is not set unilaterally by firms but is bargained between worker and firm (**Diamond–Mortensen–Pissarides** model) → topic of **Lecture 6**.

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Appendix

Geometric Series

- Recall the formula for a geometric series:

$$\sum_{n=1}^{\infty} nx^{n-1} = \frac{1}{(1-x)^2}$$

for $|x| < 1$.

- To derive expression for ED in the main text, apply the above formula by letting $x = 1 - \phi(R)$ and $t = n$:

$$\begin{aligned}\sum_{t=1}^{\infty} t(1-\phi(R))^{t-1}\phi(R) &= \phi(R)\sum_{t=1}^{\infty} t(1-\phi(R))^{t-1} \\ &= \phi(R)\frac{1}{[1-(1-\phi(R))]^2} \\ &= \frac{1}{\phi(R)}.\end{aligned}$$

Back

Expected value formula

- Recall formula for integration by parts:

$$\int_0^B u dv = uv|_0^B - \int_0^B v du$$

- Let $u = w$ and $dv = dF(w)$. Then

$$\begin{aligned}\int_0^B w dF(w) &= wF(w)|_0^B - \int_0^B F(w) dw \\ &= B - \int_0^B F(w) dw \\ &= \int_0^B [1 - F(w)]\, dw.\end{aligned}$$

Q.E.D.

Back

Algebra

- Derivation:

$$\begin{aligned}\int_R^B (w - R\ )\, dF(w) &= \int_0^B (w - R\ )\, dF(w) - \int_0^R (w - R\ )\, dF(w) \\ &= \left[\int_0^B w dF(w) - R\right] - \int_0^R (w - R\ )\, dF(w) \\ &= \mathbb{E}_F(w) - R + \int_0^R F(w)\ dw\end{aligned}$$

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Back