The search and matching model

Lecture 6

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Motivation and overview

- Trade in the labor market is decentralized and uncoordinated.
- Search for trading partners is costly. It takes time and effort.
- Need to develop a model that can account for worker and job flows.
- Search decisions of workers and firms.
- Firms decide about job creation (Pissarides 1985) and job destruction (Mortensen/Pissarides 1994).

Unemployment Across Countries and Time

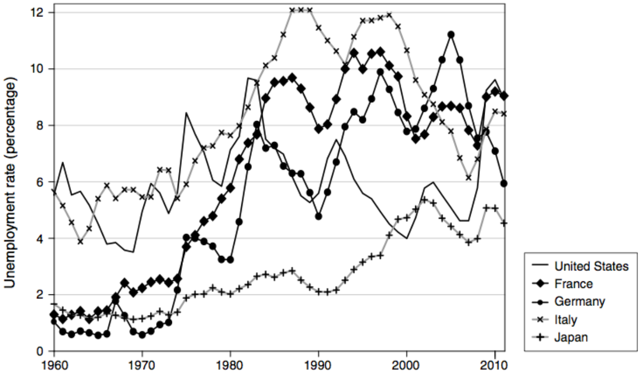

FIGURE 9.3
Unemployment rate in the United States, Japan, and continental Europe (Germany, France, Italy), 1960–2011 (persons aged 15 to 64).

Note: For Germany, estimate based on the annual growth rate for West Germany before 1991.

Source: OECD Economic Outlook database.

Job flows and worker flows (employer data)

- Job creation at time t: $JC_t = \sum_i \max(E^i_t - E^i_{t-1}, 0)$.
- Job destruction at time t: $JD_t = -\sum_i \min(E^i_t - E^i_{t-1}, 0)$.
- i is firm (establishment) index, E^i_t is employment at firm i at time t.
- Hires H_t and separations S_t can be obtained from firm surveys (JOLTS) or from household survey data (CPS).
- Net employment change $E_t - E_{t-1} = JC_t - JD_t = H_t - S_t$.
- Separations can sometimes be split into layoffs and quits.

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Job and worker flows

Job and Worker Flow Rates by Sampling Frequency and Data Source

Sampling Frequency and Data Source	*Job creation*	*Job destruction*	*Hires*	*Separations*
Monthly				
JOLTS, continuous monthly units from microdata, Dec. 2000 to Jan. 2005	1.5	1.5	3.2	3.1
Quarterly				
JOLTS, continuous quarterly units from microdata, Dec. 2000 to Jan. 2005	3.4	3.1	9.5	9.2
BED, all private establishments, 1990:2–2005:1	7.9	7.6	—	—
LEHD, all transitions, ten selected states, 1993:2–2003:3	7.0	6.0	25.0	24.0
LEHD, "full-quarter" transitions, ten selected states, 1993:2–2003:3	7.6	5.2	13.1	10.7
Annual				
BED, from Pinkston and Spletzer (2004), private establishments, 1998–2002	14.6	13.7	—	—

Sources: JOLTS is the Job Openings and Labor Turnover Survey; BED is Business Employment Dynamics data; and LEHD is Longitudinal Employer Household Dynamics data.

Source: Davis/Faberman/Haltiwanger, The flow approach to labor markets: new data sources and micro-macro links, Journal of Economic Perspectives 2006.

Job creation and job destruction rates

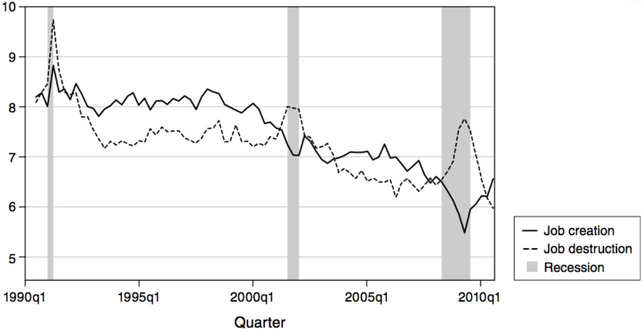
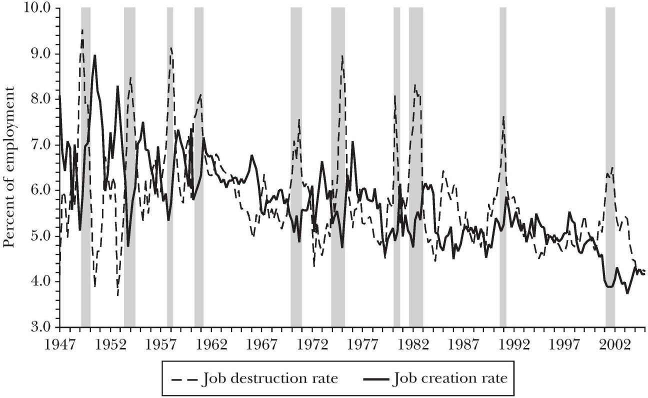


FIGURE 9.9
Quarterly job flows in the United States. Private sector, 1990q2–2010q2. Job creation rate and job destruction rate in percentage of employment.

Source: Davis et al., (2012) database.

Job creation and job destruction rates, cont.

Quarterly Job Flows in Manufacturing, 1947–2005



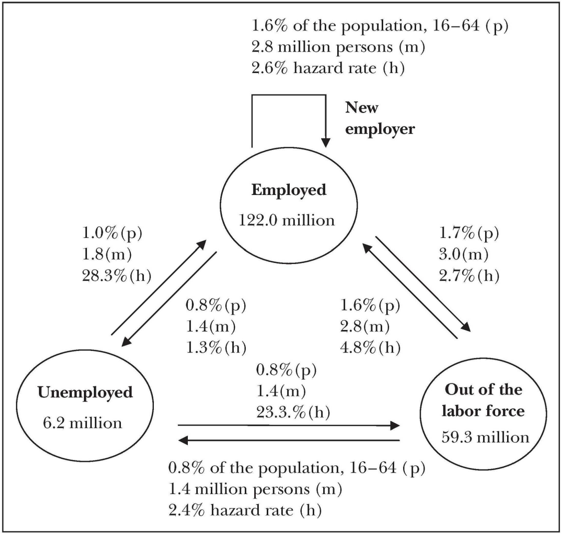
Source: Davis/Faberman/Haltiwanger, The flow approach to labor markets: new data sources and micro-macro links, Journal of Economic Perspectives 2006.

Worker flows (household data)

- Population surveys (CPS, G-SOEP)
- There are substantial flows between the three labor market states employment, unemployment and non-participation.
- Flows between employment and unemployment are more cyclical than the other flows.

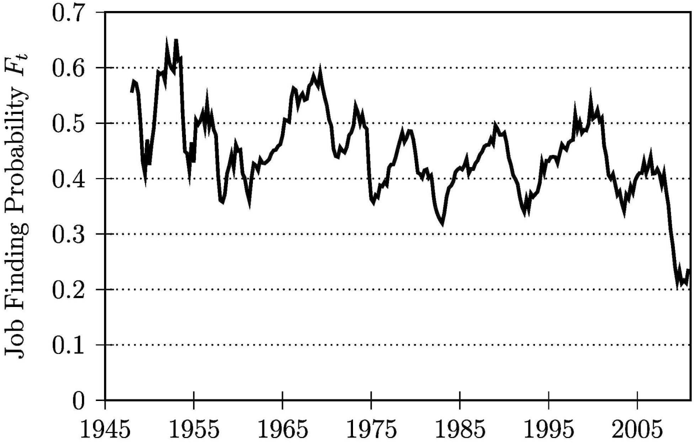
Worker flows in the U.S.

Average Monthly Worker Flows, Current Population Survey, 1996–2003



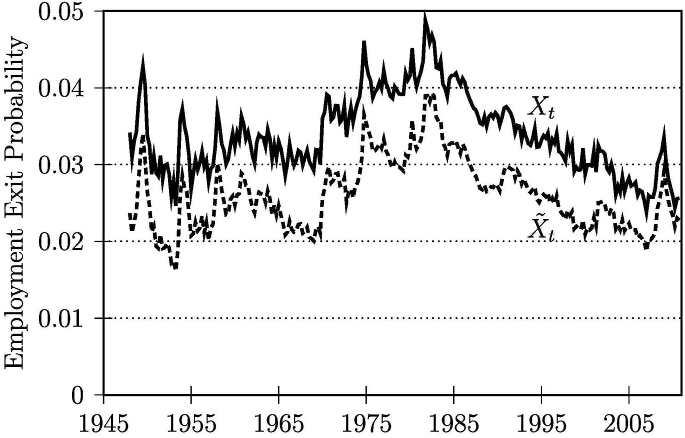
Source: Fallick and Fleischman (2004).

U.S. job-finding probability



Source: Shimer, Reassessing the Ins and Outs of Unemployment, Review of Economic Dynamics 2012.

U.S. separation probability



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The search and matching model

- This follows Pissarides (2000, Ch.1).
- Describe trade in a decentralized labor market.
- Search is costly for workers and firms.
- Matching probabilities depend on aggregate search activity by workers and firms (proxied by unemployment and vacancy measures) $\Rightarrow$ Reduced-form matching function.
- Firms decide about job creation. Job destruction is exogenous.
- Wages are bargained between the worker and the employer.
- Risk-neutral workers and firms. Common discount factor β.

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The matching function

- Meetings or contacts determined through a matching function.
- Suppose that at some point in time there are
  - u_t unemployed workers searching for a job
  - v_t vacancies posted by firms
- Matches per unit of time given by a matching technology

$$m_t = m(u_t, v_t)$$

- m is increasing and concave (congestion externalities for workers and firms) and

$$m(0, v_t) = m(u_t, 0) = 0 \text{ for all } (u_t, v_t)$$

The matching function

- m has constant returns to scale.
- Write $\theta_t = \frac{v_t}{u_t}$ for the vacancy-unemployment ratio (market tightness).
- The job-finding rate

$$f_t = \frac{m_t}{u_t} = m(1, \theta_t) \equiv f(\theta_t)$$

  is increasing and concave.
- The job-filling rate

$$q_t = \frac{m_t}{v_t} = m(1/\theta_t, 1) \equiv q(\theta_t) = f(\theta_t)/\theta_t$$

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- Average duration of unemployment is $1/f_t$, average duration of an open vacancy is $1/q_t$.

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The matching function, cont.

- Typically m is assumed to be Cobb-Douglas:

$$m = \sigma_m u^{\sigma} v^{1-\sigma}$$

where $\sigma_m > 0$ captures the efficiency of the matching process and $\sigma \in (0, 1)$ is the constant elasticity of matches to unemployed.

- The job finding rate is given by:

$$f(\theta) = \frac{\sigma_m u^{\sigma} v^{1-\sigma}}{u} = \sigma_m \theta^{1-\sigma}$$

- The vacancy filling rate is given by:

$$q(\theta) = \frac{\sigma_m u^{\sigma} v^{1-\sigma}}{v} = \sigma_m \theta^{-\sigma}$$

- An alternative proposed by Den Haan et al. (2000): $m = \frac{uv}{(u^{\nu}+v^{\nu})^{1/\nu}}$

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Unemployment dynamics

- Normalize the size of the labor force to one: $1 = u_t + e_t$.
- A fraction δ of jobs is destroyed every period. (Note here: job destruction = separations).
- Then $u_{t+1} = \delta(1 - u_t) + (1 - f_t)u_t$.
- Consider steady states:

$$\boxed{u = \frac{\delta}{\delta + f(\theta)},\ \theta = v/u}$$

- This is a downward-sloping relation between v and u (Beveridge curve).
- **Example**: if $m(u, v) = \sigma_m u^{\sigma} v^{1-\sigma}$, then

$$v = \left(\left(\frac{\delta}{\sigma_m}\right)\left(\frac{1-u}{u^{\sigma}}\right)\right)^{1/(1-\sigma)}$$

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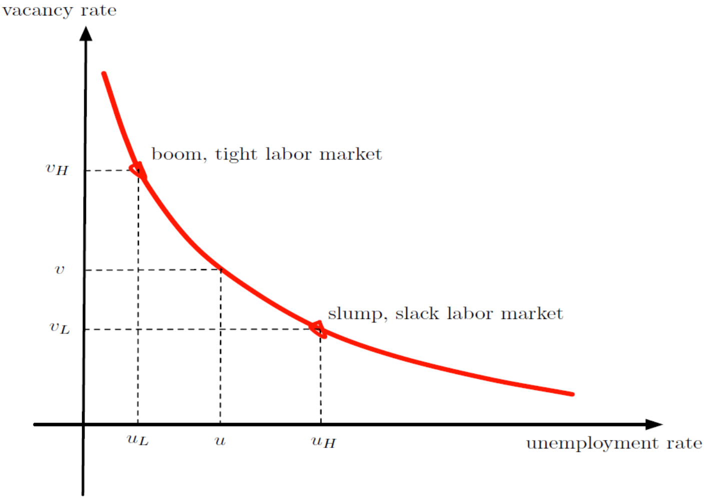
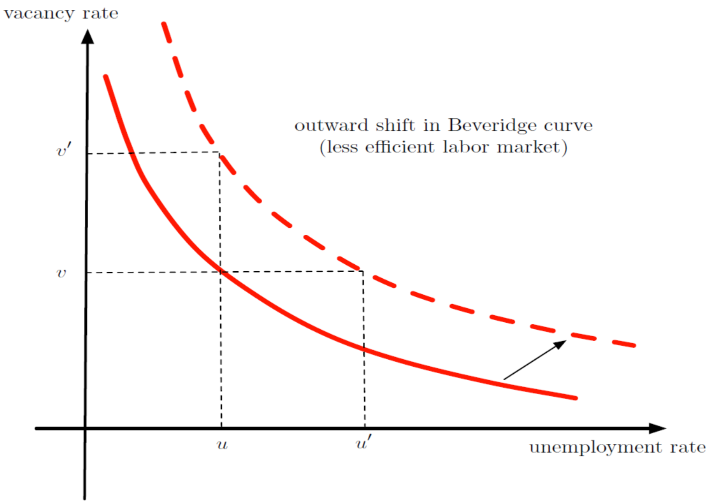
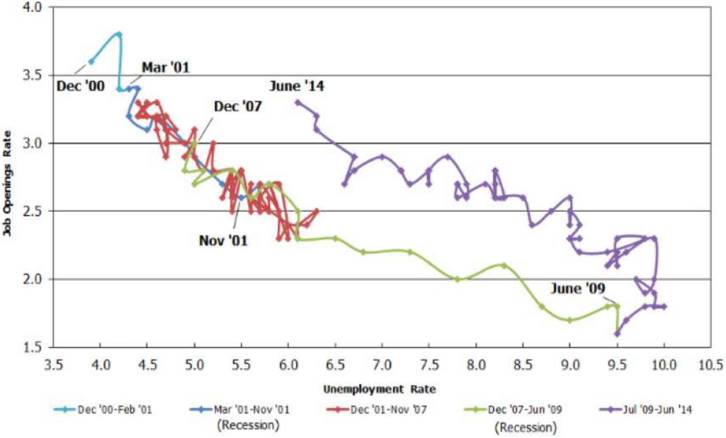
Beveridge Curve



Shifts in the Beveridge Curve



U.S. Beveridge curve (12/2000-06/2014)



Source: P.Diamond and A.Sahin, Shifts in the Beveridge Curve, Research in Economics 2015.

Job creation (I)

- A job produces revenue p per period. Let w denote the wage.
- A vacant job costs c per period (e.g. recruitment cost, capital rental cost).
- Let V denote the *value of a vacancy* to a firm. Satisfies the steady state Bellman equation:

$$V = -c + \beta[q(\theta)J + (1 - q(\theta))V]$$

hence

$$V = -c + \beta V + q(\theta)\beta(J - V)$$

- Let J denote the *value of a filled job* to a firm. Satisfies the steady state Bellman equation:

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Job creation (II)

- **Free entry**: $V = 0$. Write $J = J(w)$ for the profit value.
- Imposing $V = 0$ in the Bellman equation for J implies

$$J(w) = \frac{p - w}{1 - \beta(1 - \delta)}$$

- Imposing $V = 0$ in the Bellman equation for V implies

$$c = q(\theta)\beta J(w)$$

- Combining the two expressions above yields:

$$\boxed{\frac{c}{q(\theta)} = \beta \frac{p - w}{1 - \beta(1 - \delta)}}$$

- This defines a negative relation between w and θ.
- **Job-creation condition**: plays the role of a labor demand schedule.

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Workers

- Flow income of unemployed (employed) workers is b (w).
- Let U denote the *value of being unemployed*. Steady state Bellman equation is

$$U = b + \beta[f(\theta)W + (1 - f(\theta))U]$$

  hence

$$U = b + \beta U + f(\theta)\beta(W - U)$$

- Let W denote the value of a job to a worker. Steady state Bellman equation is

$$W = w + \beta[\delta U + (1 - \delta)W]$$

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$$W = w + \beta W + \delta\beta(U - W)$$

- Write $W = W(w)$. It follows

$$W(w) - U = \frac{w - (1 - \beta)U}{1 - \beta(1 - \delta)} .$$

Wage bargaining

- Match between unemployed worker and firm with vacancy creates a **mutual profit opportunity**. How should these profits be split?
- Payments $p - w$ to firm, w to worker.
- Wage w determined by **bargaining** between worker and firm.
- Choice of w affects job value to individual firm $J(w)$ and to individual worker $W(w)$ taking as given aggregate market conditions.
- At a wage of w, the firm's surplus from a match is $J(w) - V$ and the worker's surplus is $W(w) - U$
- Joint surplus of a match is $S = J(w) + (W(w) - U)$. Here I used free entry condition $V = 0$.

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Nash Bargaining

- Wage w maximizes the *Nash product*

$$(W(w) - U)^{\gamma} J(w)^{1-\gamma}$$

  where the parameter $\gamma \in [0, 1]$ denotes the workers' bargaining power.

- The F.O.C. for this problem can be written

$$\gamma \frac{W'(w)}{W(w) - U} = -(1-\gamma)\frac{J'(w)}{J(w)}$$

- Note that

$$W'(w) = \frac{1}{1-\beta(1-\delta)}, \qquad J'(w) = -\frac{1}{1-\beta(1-\delta)}$$

- So we can write $\gamma J = (1-\gamma)(W - U)$, which implies $J = (1-\gamma)S$

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Nash Bargaining, cont.

- Observe that the joint surplus S satisfies

$$S = J(w) + W(w) - U = \frac{p - (1-\beta)U}{1 - \beta(1-\delta)} \; .$$

- Recall that

$$J(w) = \frac{p - w}{1 - \beta(1-\delta)}$$

- Then, given surplus splitting rule $J = (1-\gamma)S$, we obtain

$$w = \gamma p + (1-\gamma)(1-\beta)U$$

- The term $(1-\beta)U$ is the flow value of unemployment (also: reservation wage)
- **Intuition**: Bargained wage is a weighted average of productivity p and reservation wage $(1-\beta)U$.

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$$w = \gamma p + (1-\gamma)(1-\beta)U$$

- The term $(1-\beta)U$ is the flow value of unemployment (also: reservation wage)
- **Intuition**: Bargained wage is a weighted average of productivity p and reservation wage $(1-\beta)U$.

Wage Equation

- The reservation wage satisfies

$$\begin{aligned}(1-\beta)U &= b+\beta f(\theta)[W(w)-U] = b+\beta f(\theta)\frac{\gamma}{1-\gamma}J(w) \\ &= b+\beta f(\theta)\frac{\gamma}{1-\gamma}\frac{c}{\beta q(\theta)} = b+\frac{\gamma c}{1-\gamma}\theta\end{aligned}$$

where the last equality uses the relation $f(\theta)=\theta q(\theta)$.

- Plugging this into our expression for wages and collecting terms:

$$\boxed{w=(1-\gamma)b+\gamma(p+c\theta)}$$

- The equation above is the *wage setting* rule. It plays the role of a labor supply schedule.

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Equilibrium in the search and matching model

Equilibrium values (θ, w, u) solve

1. Job-creation condition

$$\frac{c}{q(\theta)} = \beta \frac{p - w}{1 - \beta(1 - \delta)} \tag{JC}$$

2. Wage-setting condition

$$w = (1 - \gamma)b + \gamma(p + c\theta) \tag{WS}$$

3. Beveridge curve

$$u = \frac{\delta}{\delta + f(\theta)} \tag{BC}$$

(i) Solve (JC) and (WS) simultaneously for (θ, w). *(ii)* Recover u from (BC) and $v = \theta u$.

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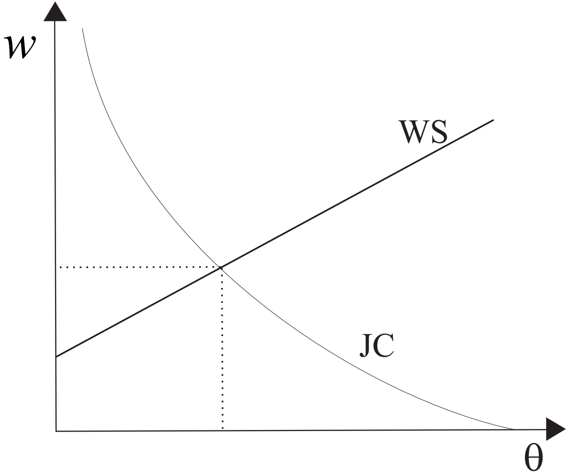
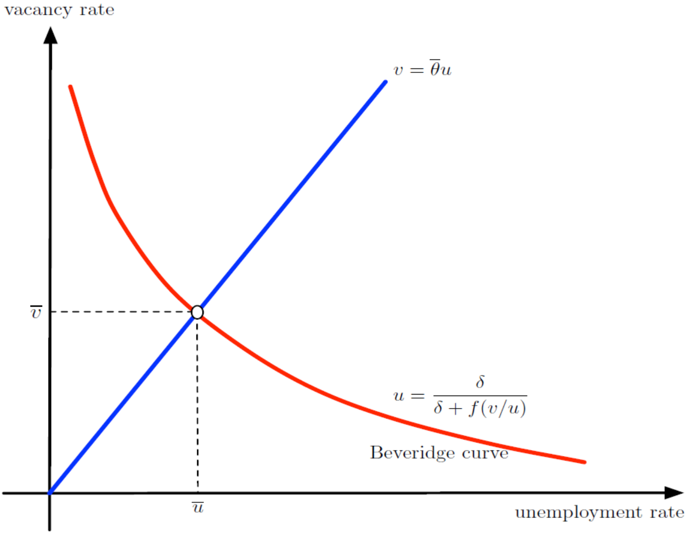
Equilibrium in the search and matching model

(i) Find steady state $(\overline{\theta}, \overline{w})$ by solving simultaneously (JC) and (WS)



Equilibrium in the search and matching model

(ii) Given $\overline{\theta}$, recover $\overline{u}$ from the Beveridge curve (BC)



Equilibrium market tightness

- It is convenient to combine (JC) and (WS) to find an equation that gives equilibrium labor market tightness θ

$$\frac{c}{\beta q(\theta)} = \frac{(1-\gamma)(p-b) - \gamma c\theta}{1-\beta(1-\delta)}$$

- Using the notation

$$\beta = \frac{1}{1+r}$$

for the discount factor, the equilibrium equation for θ can be rewritten as

$$\boxed{\frac{c}{q(\theta)} = \frac{(1-\gamma)(p-b) - \gamma c\theta}{r+\delta}} \qquad (*)$$

which is identical to the condition that would arise in a continuous time version of the model.

Comparative Statics

- Some comparative statics can be easily derived, since many model parameters affect one curve but not the others.
- **Higher unemployment benefits** b shifts (WS) curve up: wages increase and market tightness falls.
  - Workers claim a higher wage when b is higher b/c the cost of unemployment is lower, and with higher wages firms create fewer jobs.
  - The straight line $\overline{\theta}$ rotates clockwise, reducing vacancies and increasing unemployment.
- Consider a **permanent increase in productivity**
  - A higher p shifts the (JC) line to the right and the (WS) curve up.
  - Wages unambiguously increase. What is the *net effect* on θ?

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Comparative Statics

- To determine the net effect on θ of a permanent increase in p, need to consider equation (*) to get θ as an implicit function of p

$$g(\theta, p) \equiv \frac{c}{q(\theta)} - \frac{(1-\gamma)(p-b) - \gamma c\theta}{r+\delta} = 0$$

- Then by implicit differentiation

$$\frac{d\theta}{dp} = -\frac{g_p(\theta, p)}{g_\theta(\theta, p)}$$

Comparative Statics

- Calculating the partial derivative with respect to p

$$g_p(\theta, p) = -\frac{1-\gamma}{r+\delta} < 0$$

as long as $\gamma < 1$.

- Calculating the partial derivative with respect to θ

$$g_\theta(\theta, p) = -\frac{c}{q(\theta)^2} q'(\theta) + \frac{\gamma c}{r+\delta} > 0$$

since $q'(\theta) < 0$.

- Hence

$$\frac{d\theta}{dp} = -\frac{g_p(\theta, p)}{g_\theta(\theta, p)} > 0$$

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Productivity shock

Steady-state

- Permanent increase in $p \implies w$ and θ increase.
- This rotates the straight line $\overline{\theta}$ anti-clockwise, so that u falls.
- $f(\theta)$ increases and $q(\theta)$ decreases: in the new steady-state, the duration of unemployment is lower but it takes more time to fill a vacancy.

Transitional dynamics

- In the transition, θ and w immediately jump to their new steady-state values, whereas u adjusts sluggishly.
- Counter-clockwise movement in (u, v) space.

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Business cycles

- Shimer (2005) considers a business-cycle version of this model.
- θ_t and f_t are pro-cyclical and volatile in U.S. data.
- Davis, Faberman, and Haltiwanger (2010) construct a measure of the job-filling rate q_t ("aggregate vacancy yield") and find that it is counter-cyclical.
- But a reasonably calibrated version of the search and matching model produces too little amplification (and no propagation) of productivity shocks. Compare Tables 1 and 3 in the next slides.
- The failure of the (baseline) search and matching model to replicate U.S. labor market data is known as the **"Shimer puzzle"**.

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U.S. Business Cycle (1951-2003)

TABLE 1—SUMMARY STATISTICS, QUARTERLY U.S. DATA, 1951–2003

		u	v	v/u	f	s	p
Standard deviation		0.190	0.202	0.382	0.118	0.075	0.020
Quarterly autocorrelation		0.936	0.940	0.941	0.908	0.733	0.878
Correlation matrix	u	1	−0.894	−0.971	−0.949	0.709	−0.408
	v	—	1	0.975	0.897	−0.684	0.364
	v/u	—	—	1	0.948	−0.715	0.396
	f	—	—	—	1	−0.574	0.396
	s	—	—	—	—	1	−0.524
	p	—	—	—	—	—	1

Notes: Seasonally adjusted unemployment u is constructed by the BLS from the Current Population Survey (CPS). The seasonally adjusted help-wanted advertising index v is constructed by the Conference Board. The job-finding rate f and separation rate s are constructed from seasonally adjusted employment, unemployment, and mean unemployment duration, all computed by the BLS from the CPS, as explained in equations (1) and (2). u, v, f, and s are quarterly averages of monthly series. Average labor productivity p is seasonally adjusted real average output per person in the non-farm business sector, constructed by the Bureau of Labor Statistics (BLS) from the National Income and Product Accounts and the Current Employment Statistics. All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5.

Source: Shimer, The Cyclical Behavior of Equilibrium Unemployment and Vacancies, American Economic Review 2005.

Model Simulations

TABLE 3—LABOR PRODUCTIVITY SHOCKS

		u	v	v/u	f	p
Standard deviation		0.009	0.027	0.035	0.010	0.020
		(0.001)	(0.004)	(0.005)	(0.001)	(0.003)
Quarterly autocorrelation		0.939	0.835	0.878	0.878	0.878
		(0.018)	(0.045)	(0.035)	(0.035)	(0.035)
Correlation matrix	u	1	−0.927	−0.958	−0.958	−0.958
			(0.020)	(0.012)	(0.012)	(0.012)
	v	—	1	0.996	0.996	0.995
				(0.001)	(0.001)	(0.001)
	v/u	—	—	1	1.000	0.999
					(0.000)	(0.001)
	f	—	—	—	1	0.999
						(0.001)
	p	—	—	—	—	1

Notes: Results from simulating the model with stochastic labor productivity. All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5. Bootstrapped standard errors—the standard deviation across 10,000 model simulations—are reported in parentheses. The text provides details on the stochastic process for productivity.

Source: Shimer, The Cyclical Behavior of Equilibrium Unemployment and Vacancies, American Economic Review 2005.

Efficiency

- Is the search equilibrium efficient? That is, do firms create too many or too few jobs?
- Clearly inefficient relative to a first-best alternative, i.e. a social planner that can avoid the matching frictions $\rightarrow$ not very interesting.
- Consider a social planner subject to the same matching constraints as are the private agents in the economy.
- Planner can decide job creation each period (i.e. can choose θ or v) so as to maximize discounted aggregate surplus, subject to the LoM of unemployment.
- Planner internalizes that choosing θ affects the labor transition rates whereas firms and workers in the market economy take $f(\theta)$ and $q(\theta)$ as given.

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Efficiency

- Write $S(u)$ for the discounted surplus value if current unemployment is u.
- **Recursive problem**:

$$S(u) = \max_{\theta} p(1-u) + bu - c\theta u + \beta S(u_+)$$

  subject to $u_+ = (1 - f(\theta))u + \delta(1-u)$.

- State variable (current value): u. Future state: u_+.
- Control variable: θ or (v).
- Current payoff of the planner is the sum of three terms:

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- Interpretation?

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Efficiency

- Substitute the LoM for u in the objective:

$$S(u) = \max_{\theta} p(1-u) + bu - c\theta u + \beta S[\underbrace{(1-f(\theta))u + \delta(1-u)}_{u_+}]$$

- Envelope condition: $S'(u) = -p + b - c\theta + \beta S'(u_+)[1 - f(\theta) - \delta]$.
- First-order condition: $S'(u_+) = -c/[\beta f'(\theta)]$
- Combining the two and imposing steady state implies

$$c[1-\beta(1-\delta)] = \beta\left[f'(\theta)(p-b+c\theta) - f(\theta)c\right].$$

- Recall that the job creation condition in the market equilibrium is

$$\frac{c}{\beta q(\theta)} = \frac{(1-\gamma)(p-b)-\gamma c\theta}{1-\beta(1-\delta)} \qquad (*)$$

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Efficiency

- The efficient outcome coincides with the market equilibrium defined by (*) if, and only if

$$\gamma = \underbrace{1 - \frac{f'(\theta)\theta}{f(\theta)}}_{1-\varepsilon_{f,\theta}} = \underbrace{-\frac{q'(\theta)\theta}{q(\theta)}}_{-\varepsilon_{q,\theta}}$$

- **This is the "Hosios condition"**. $\varepsilon_{f,\theta}$ is the elasticity of the job finding rate $f(\theta)$ wrt θ and $\varepsilon_{q,\theta}$ is the elasticity of the vacancy filling probability $q(\theta)$ wrt θ.
- The RHS measures the congestion externality from the firms' perspective. If it is larger, wages must be higher to discourage excessive job creation.
- If $m(u, v) = \sigma_m u^{\sigma} v^{1-\sigma}$, then the Hosios condition becomes $\gamma = \sigma$, i.e. bargaining power of workers must equal the elasticity of matching with respect to unemployment.