Incomplete markets and wealth inequality

Lecture 3a

Alessandro Di Nola

University of Konstanz

Income and wealth distributions - some stylized facts

- Many industrialized countries: large income and wealth heterogeneity.
  - Gini coefficients are higher for wealth than for income.
  - USA in 2005: 0.38 for disposable household income and 0.7 for household net worth;
  - Germany in 2004: 0.34 for disposable household income and 0.69 for wealth.
- Both distributions are highly skewed to the right, and more so for wealth than for income.
- There is some mobility between different income and wealth quintiles:
  - For example, about 29 percent of households left the lowest income quintile between 1984 and 1989, and 33 percent of households left the lowest wealth quintile in this period.

Measures of concentration

- Sample $\{x_1, x_2, \dots, x_n\}$, x variable of interest (earnings, income, wealth).
- Define mean and standard deviation as:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i, \;\; std(x) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

- Commonly reported measure of dispersion, coefficient of variation:

$$cv(x) = \frac{std(x)}{\bar{x}}.$$

Measures of concentration, cont'd

- Variance of logs:

$$VL(x) = \frac{1}{n}\sum_{i=1}^{n}\left(\log x_i - \overline{\log x}\right)^2$$

  where $\overline{\log x}$ is the mean of log(x).

- VL cannot handle negative values.
- Sensitive to the bottom of the distribution.

Measures of concentration, cont'd

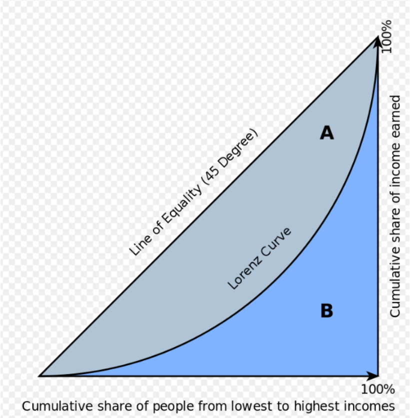
- Another very popular measure: Gini index based on Lorenz curve.
- How to derive Lorenz curve:
  - Order $\{x_1, x_2, \ldots, x_n\}$ in increasing order $\rightarrow \{y_1, y_2, \ldots, y_n\}$
  - For each $i = 1, 2, \ldots, n$ compute $\frac{i}{n}$ and

$$z_i = \frac{\sum_{j=1}^{i} y_j}{\sum_{j=1}^{n} y_j}$$

.

  - For each i, plot $\frac{i}{n}$, the percentile of household, against z_i, the fraction of total wealth/income held by this percentile.
  - By construction, $z_i \leq z_{i+1}, z_i \leq 1$ and $z_n = 1$.
  - Lorenz curve closer to 45 line $\implies$ more equal distribution.
- Gini index measures the area b/w Lorenz curve and the 45 line.
  - If $x_i \geq 0$ for all i, then $0 \leq \text{Gini} \leq 1$.

Gini index and Lorenz curve

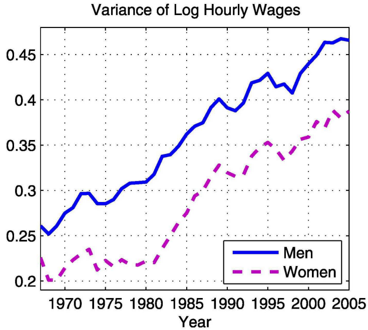


- Gini index is $A/(A+B)$. Perfect equality: Gini=0; Complete concentration: Gini=1.

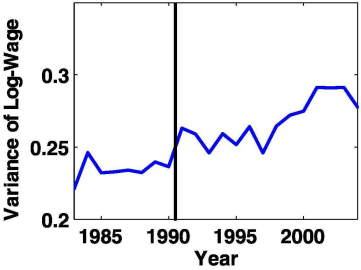
Measures of skewness (asymmetry of distribution)

- Distributions of income and wealth
  - Long, thin right tail.
  - Few rich people hold a substantial fraction of total income/wealth.
- Location of the mean.
- Mean-to-median ratio.
  - Positive skewness $\implies$ median $<$ mean.
- Ratios between various values to the median (the 99th, the 90th, etc.)
  - In a symmetric distribution, median $=$ mean and mean is located at p50.

Wage inequality (USA and Germany)



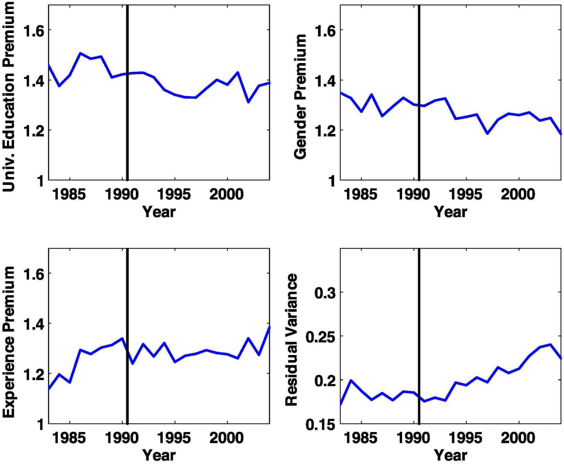
(a) United States



(b) Germany

Sources:
Left: Heathcote/Perri/Violante, Unequal we stand: An empirical analysis of economic inequality in the United States, Review of Economic Dynamics (2010)
Right: Fuchs-Schuendeln/Krueger/Sommer, Inequality trends for Germany in the last two decades: A tale of two countries, Review of Economic Dynamics (2010).

Decomposition of Wage Inequality (Germany)



Source:
Fuchs-Schuendeln/Krueger/Sommer, Inequality trends for Germany in the last two decades: A tale of two countries, Review of Economic Dynamics (2010).

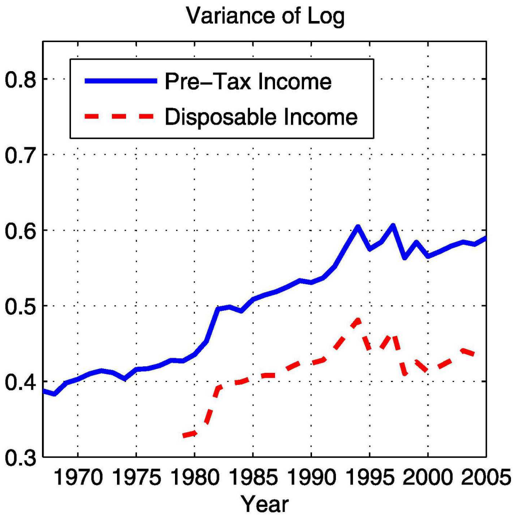
Wage Inequality Across Countries

Wage inequality and wage premia.

Country	Level in year 2000				Change				
	Var. log w	College premium	Exp. premium	Gender premium	College premium	Exp. premium	Gender premium	Var. log w	Period
Canada	0.45	1.80	1.32	1.33	0.22	0.31	−0.11	0.17	1978–2006
Germany	0.27	1.38	1.27	1.28	−0.08	0.22	−0.15	0.05	1983–2003
Italy	0.17*	1.51	1.34	1.03	−0.08	0.11	−0.05	0.03	1987–2006
Mexico	0.62	1.88	1.23	1.21	0.40	0.22	−0.06	0.04	1989–2002
Russia	0.77*	1.50	1.05*	1.49	−0.06	0.05*	−0.07	−0.13*	1998–2005
Spain[a]	0.23	1.48	1.43	1.16	−0.33	0.07	−0.21	−0.18	1985–1996
Sweden[b]	0.18	1.61	1.20	1.22	0.14	−0.02	−0.05	−0.09	1990–2001
UK	0.33	1.62*	1.25*	1.32	0.12*	0.20*	−0.21	0.10	1978–2005
USA	0.44*	1.80*	1.38*	1.36	0.40*	0.28*	−0.25*	0.21*	1980–2006
Average	**0.38**	**1.62**	**1.27**	**1.27**	**0.11**	**0.17**	**−0.10**	**0.04**	

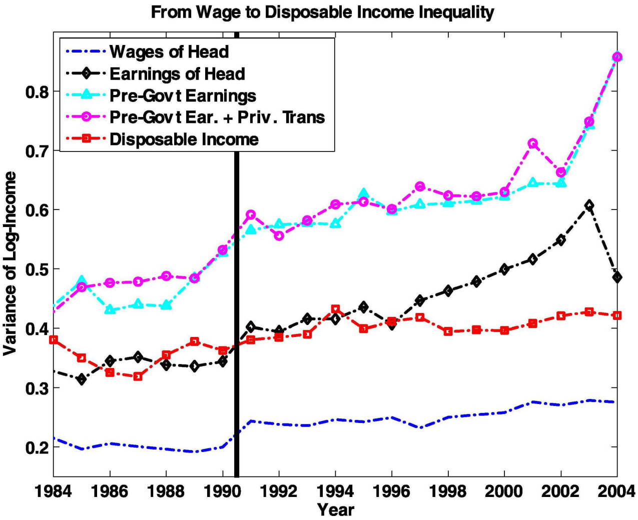
Source: Krueger/Perri/Pistaferri/Violante, Cross-sectional facts for macroeconomists, Review of Economic Dynamics (2010).

Disposable household income in the US



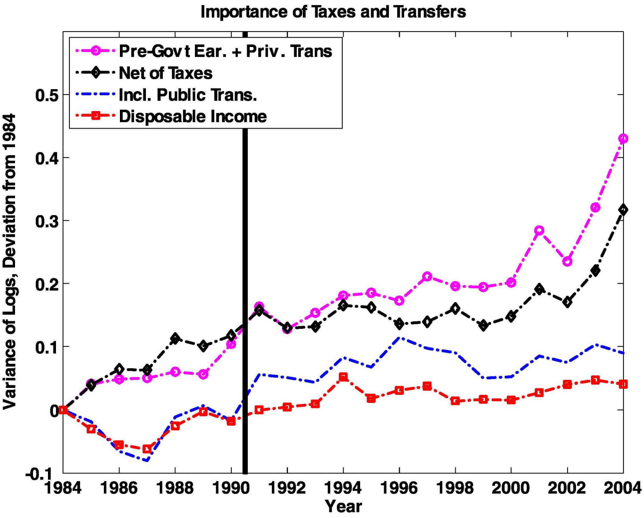
Source: Heathcote/Perri/Violante, Unequal we stand: An empirical analysis of economic inequality in the United States, Review of Economic Dynamics (2010).

Disposable household income in Germany



Source: Fuchs-Schuendeln/Krueger/Sommer, Inequality trends for Germany in the last two decades: A tale of two countries, Review of Economic Dynamics (2010).

Income redistribution in Germany



Source: Fuchs-Schuendeln/Krueger/Sommer, Inequality trends for Germany in the last two decades: A tale of two countries, Review of Economic Dynamics (2010).

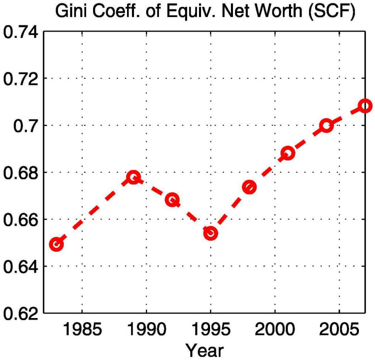
Income Redistribution Across Countries

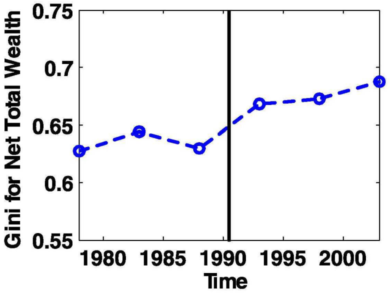
Inequality in pre- and post-government household income (variance of the log).

Country	Level in year 2000		Change		Period
	Pre-gov. income	Post-gov. income	Pre-gov. income	Post-gov. income	
Canada	0.50	0.25	0.16	0.05	1978–2005
Germany	0.63	0.40	0.42	0.04	1984–2004
Italy[a]	0.72	0.73	0.06	0.07	1987–2006
Mexico	2.10	1.70	1.15	0.75	1989–2002
Russia[b]	0.86	0.60	−0.11	−0.09	1994–2005
Spain[c]	0.73	0.56	−0.20	−0.09	1993–2000
Sweden	0.95	0.38	0.36	0.05	1978–2004
UK[d]	0.55	0.32	0.22	0.13	1978–2005
USA	0.67	0.41	0.11	0.11	1979–2005
Average	**0.86**	**0.59**	**0.24**	**0.11**	

Source: Krueger/Perri/Pistaferri/Violante, Cross-sectional facts for macroeconomists, Review of Economic Dynamics (2010).

Wealth Inequality (USA and Germany)





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Heterogeneity in macroeconomics

- Complete markets: Income heterogeneity plays no role for macroeconomic aggregates. Individual consumption is perfectly correlated with aggregate income. Representative household describes aggregate behavior.
- Matters are quite different with incomplete markets.
- OLG model: agents that differ by age, hence differ in consumption and savings choices.
- With only two groups of agents, very limited heterogeneity.
- Need to build models that feature a nontrivial distribution of income and wealth across agents.
- Households cannot perfectly insure against idiosyncratic income risk.
- They accumulate assets in good times and decumulate assets in bad times (endogenous wealth distribution).
- Individual consumption is correlated with individual income.

A simple two-period model

Economic Environment

- Follows Aiyagari (QJE 1994) and Davila et al. (ECTA 2012).
- Continuum (measure 1) of two-period-lived workers/consumers.
- Workers are ex-ante identical: in period 1, each agent is endowed with ω units of output.
- Make consumption/savings decision.
- In $t = 2$, random labor endowment: e_L (prob. π), $e_H > e_L$ (prob. $1 - \pi$).
- Law of large numbers: total labor L in period 2 is

$$\pi e_L + (1 - \pi) e_H.$$

- Neoclassical production using CRS F in period 2. Prices r and w.
- No insurance markets: just *precautionary* savings with capital.

Workers problem

- Discount factor $\beta \in (0, 1]$, preferences s.t. $u' > 0$, $u'' < 0$ (risk-aversion), $u''' > 0$ (prudence).
- Workers solve

$$\max_{a \in \mathcal{A}} u(c_1) + \beta \mathbb{E}[u(c_2)]$$

s.t.

$$c_1 + a = \omega, \tag{1}$$

$$c_{2,L} = Ra + we_L, \tag{2}$$

$$c_{2,H} = Ra + we_H, \tag{3}$$

where $R \equiv 1 + r$, $a \in \mathcal{A}$ is savings/borrowing.

- von Neumann-Morgenstern expected utility:

$$\mathbb{E}[u(c_2)] = \pi u(c_{2,L}) + (1 - \pi) u(c_{2,H}).$$

Natural Borrowing Limit

- How should we specify constraint set $\mathcal{A}$?
- Upper bound on savings:

$$a \leq \omega.$$

- We do not want to restrict borrowing unnecessarily, but we do assume that loans are risk-free.
- Risk-free loans (no default) $\implies$ it must be feasible for workers to repay debt with prob. 1
- Can borrow any amount as long as they are able to pay back in all states of the world.
- Q: What constraint should we impose?

Natural Borrowing Limit

- A: Consumption has to be non-negative even in the "worst case scenario", i.e.

$$c_{2,L} \geq 0 \implies a \geq -\frac{w}{R} e_L. \tag{4}$$

  If this is true, then also $c_{2,H} \geq 0$ (indeed, $c_{2,H} > 0$ given that $e_H > e_L$).

- In the next few slides we are going to see a generalization of the natural borrowing limit (4) in a model with infinite horizon.

Euler Equation

- We can restate the workers problem as follows:

$$\max_{a \in \left[-\frac{w}{R} e_L, \omega\right]} u\left(\omega - a\right) + \beta\left[\pi u\left(Ra + we_L\right) + \left(1 - \pi\right) u\left(Ra + we_H\right)\right]$$

- If Inada conditions on u hold, a unique interior optimum exists:

$$-\frac{w}{R} e_L < a^* < \omega.$$

- Optimal savings a^* satisfy the Euler equation:

$$u'(\omega - a^*) = \beta R\left[\pi u'(Ra^* + we_L) + (1 - \pi) u'(Ra^* + we_H)\right].$$

Precautionary Savings: A Digression (1/2)

- Suppose for a moment that labor endowment in $t = 2$ is not risky:
  - workers are paid wL with certainty
  - L is equal to its unconditional mean $\pi e_L + (1 - \pi) e_H$.
- Assume further that
  - $\beta = R = 1$,
  - $\omega = wL$.
- Q: What is the optimal consumption/savings choice?
- A: $a^* = 0$, $c_1^* = c_2^* = \omega$
- Indeed, from the Euler equation,

$$u'(\omega - a) = u'(Ra + \omega) \implies a^* = 0.$$

Precautionary Savings: A Digression (2/2)

- $\beta = R = 1$ eliminates *intertemporal motive* for saving $\implies c_1^* = c_2^*$.
- $\omega = wL$ (flat income profile over time) eliminates the *smoothing motive*.
- Since future income is certain, there is no *precautionary motive*.
- Go back now to the assumption that labor endowment is risky.
- Q: What is the optimal consumption/savings choice?
- A: $a^{**} > 0 = a^*$, $c_1^{**} < c_1^* = \omega$
- *How much* depends on *variance* of e_s and on *prudence*.

Firms

- In $t = 2$ output is produced by a representative firm with CRS technology:
$$F(K, L).$$
- Firm rents capital and labor from workers/households.
- Maximization problem:
$$F(K, L) - RK - wL.$$
- Marginal factor pricing:
$$R = F_K(K, L),$$
$$w = F_L(K, L).$$
- Equilibrium profits are zero.

Equilibrium

Definition. A CE is a list $(a^*, K^*, L^*, R^*, w^*)$ such that

1. Household maximization:

$$a^* = \arg \max_{a \in \left[-\frac{w}{R} e_L, \omega\right]} u(\omega - a) + \beta \left[\pi u(R^* a + w^* e_L) + (1 - \pi) u(R^* a + w^* e_H)\right]$$

2. Firms maximization:

$$R^* = F_k(K^*, L^*),$$
$$w^* = F_L(K^*, L^*).$$

3. All markets clear:

   - Labor

$$\pi e_L + (1 - \pi) e_H = L^*.$$

   - Capital

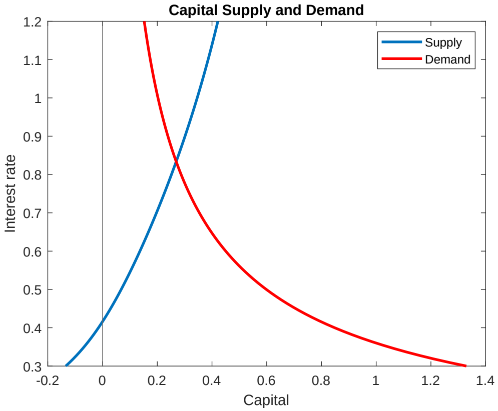
$$a^* = K^*.$$

Walras' Law

- Q: What about the market clearing condition for consumption good?
  - In this simple economy: $C = F(K, L)$, where $C = \pi c_{2,L} + (1-\pi) c_{2,H}$.
- By Walras' law, if N-1 markets clear, also the Nth market clears.
- Indeed,

$$\begin{aligned}
C &= \pi (Ra + we_L) + (1-\pi)(Ra + we_H) && (5)\\
&= Ra + w[\pi e_L + (1-\pi) e_H] && (6)\\
&= RK + wL && (7)\\
&= F_K(K, L) K + F_L(K, L) L && (8)\\
&= F(K, L). && (9)
\end{aligned}$$

Equilibrium: Graphical Illustration



- Capital supply: $K^s = a^*(R)$, Capital demand: $K^d = F_k^{-1}(R)$

Mean Preserving Spread

- Consider a mean preserving spread (MPS) on labor endowment: $(\widetilde{e}_L, \widetilde{e}_H)$.
- Variance of the process increases but average remains constant:

$$\widetilde{e}_L < e_L$$

$$\widetilde{e}_H > e_H$$

$$L = \pi \widetilde{e}_L + (1 - \pi)\, \widetilde{e}_H.$$

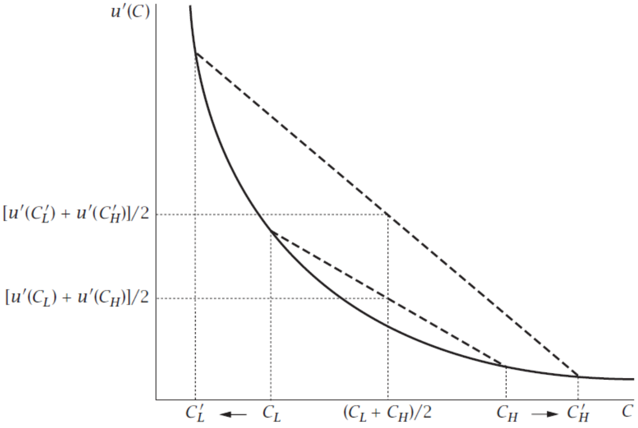
- Q: What happens to workers' optimal savings?
- A: If $u''' > 0$, savings will increase (Jensen's inequality). Math

Intuition

- MPS of e_s increases uncertainty about future income.
- In $t = 2$, consumption in the low state, $c_{2,L}$, falls, whereas consumption in the high state, $c_{2,H}$, rises.
- Expected value of $c_{2,s}$ is unchanged, i.e. $\mathbb{E}_s(c_{2,s}) = Ra + wL$.
- Since marginal utility u' is convex, the increase in uncertainty raises expected marginal utility of c_2, for given expected second-period consumption.
- This implies that $\mathbb{E}[u'(c_2)] > u'(c_1)$ so it is optimal to reduce c_1 by saving more.

Intuition

Graphical proof



MPS $\Longrightarrow$ increase in expected utility of future consumption $\Longrightarrow$ higher savings.

Formal Proof

- Define MPS $\widetilde{e} = e + \varepsilon$, where $\mathbb{E}(\varepsilon) = 0$ and $V(\varepsilon) > 0$
- Euler equation:
$$u'(c_1) = \mathbb{E}\left[u'(c_2)\right]$$
or
$$u'(\omega - a) = \mathbb{E}\left[u'(Ra + w\widetilde{e})\right].$$
- LHS is increasing in a since $u'' < 0$, and the RHS is decreasing for the same reason, hence a^* is uniquely determined.

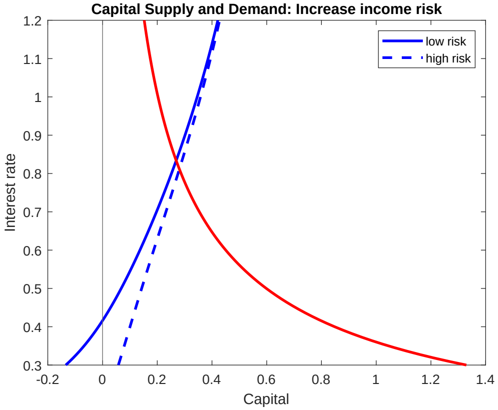
Formal Proof

- Observe:

$$
\begin{aligned}
\mathbb{E}\left[u'\left(Ra + w\tilde{e}\right)\right] &= \mathbb{E}\left[u'\left(Ra + we + w\varepsilon\right)\right] && (10)\\
&= \mathbb{E}\left\{\mathbb{E}\left[u'\left(Ra + we + w\varepsilon\right)\right]|e\right\} && (11)\\
&> \mathbb{E}\left\{u'\left(\mathbb{E}\left[Ra + we + w\varepsilon\right]|e\right)\right\} && (12)\\
&= \mathbb{E}\left\{u'(Ra + we)\right\}. && (13)
\end{aligned}
$$

- Hence a MPS of e increases the value of the RHS, for all possible values of a.
- RHS shifts up $\implies$ optimal savings increase, current consumption falls.
- What about general equilibrium effects through r, w?

New Equilibrium



- Effect of a mean preserving spread of labor endowment e.

Takeaways

Partial equilibrium

- If marginal utility is convex (i.e. $u''' > 0$), a rise in future income risk leads to a rise in savings and a decline in current consumption.
- If marginal utility is linear (i.e. $u''' = 0$), future income risk has no effect on savings: certainty equivalence.

General equilibrium

- A rise in future income risk leads to lower interest rate and higher capital accumulation.

Limitations of two-periods analysis

- Model delivers consumption inequality but wealth inequality is absent (unless we assume initial wealth heterog.).
- Wealth inequality arises endogenously when $T > 2$.

Next:

The infinite horizon model

Appendix

Jensen's inequality

- Given a real convex function f, for any $t \in [0,1]$

$$tf(x_1) + (1-t)f(x_2) \geq f\left(tx_1 + (1-t)x_2\right)$$

- Geometric interpretation: A secant line of a convex function lies above the graph
- In a probabilistic setting, if X is a r.v. and f is a convex function, then:

$$E\left[f(X)\right] \geq f\left(E\left[X\right]\right)$$

- The same holds for conditional expectation:

$$E\left[f(X) \mid Y\right] \geq f\left(E\left[X \mid Y\right]\right)$$

Back