Problem Set 2

Due on Monday, May 13, h 13:00. Mail box F230.

Problem 1 (10 points)

Consider the standard OLG model with money seen in Lecture 2. Let the preferences of the representative agent of cohort $t \geq 1$ be given by the following utility function:

$$U(c_t, c_{t+1}) = \frac{c_t^{1-\gamma}}{1-\gamma} + \frac{c_{t+1}^{1-\gamma}}{1-\gamma},$$

with $\gamma > 0$ and $\gamma \neq 1$. Each agent of generation t has endowment vector (e^Y, e^O), with $e^Y > e^O$. The initial old at time 1 have endowment e^O and are also given a stock of fiat money equal to $M > 0$.

1. Compute analytically the saving function of a young agent.
2. Define a competitive equilibrium.
3. Compute all stationary equilibria.
4. Describe briefly the non-stationary equilibria.

Problem 2 (10 points)

Consider a Diamond growth model with Cobb–Douglas production function $F(K_t, A_tL_t) = K_t^{\alpha}(A_tL_t)^{1-\alpha}$. Population grows at rate n and technology A_t grows at rate g. Capital depreciates at rate δ. Two–period lived households have logarithmic utility and discount factor β.

1. Define a competitive equilibrium.
2. Characterize all competitive equilibria by a dynamic equation in $k_t = K_t/(A_tL_t)$.
3. Calculate the unique steady state k^* in terms of the parameters $(n, g, \beta, \delta, \alpha)$. Calculate the steady–state interest rate.

4. Calibrate parameters (n, g, δ, α) when the period length is thirty years to match suitable long-run features of the United States (or some other country of your choice).

5. For what values of the parameter β is the economy dynamically inefficient? That is, for what parameters is the interest rate $f'(k) - \delta$ smaller than the long-run output growth rate $n + g + ng$?

6. Choose a value for β to match a 2% annual real interest rate. Suppose the economy's capital stock is 50% smaller than its steady-state level. Simulate the growth path of the economy. How many years does it take until the economy's capital stock is only 5% below the long-run level?

Problem 3 (10 points)

Consider a Diamond growth model with Cobb–Douglas production function $F(K_t, L_t) = K_t^{\alpha} L_t^{1-\alpha}$. Population grows at rate n and capital depreciates at rate $\delta = 0$. Two–period lived households have logarithmic utility and discount factor β. Suppose the government runs a pay–as–you–go pension scheme that collects a contribution τ of every young person which is used to finance a pension b for every old person. The government budget is balanced: $b = \tau(1 + n)$.

1. Determine savings of a young person.

2. Derive the equation describing the evolution of k_t.

3. How does the steady state level of k depend on τ?

4. Can the introduction of a pay–as–you–go system implement the Golden–Rule level of capital?