Awesome-state Aiyagari

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- Young individual's problem:

  $$V^Y(a,e) = \max_{l,a'} \left\{ \frac{\left(c^\gamma (1-l)^{1-\gamma}\right)^{1-\sigma}}{1-\sigma} + \beta\,(1-p_{ret}) \sum_{e'} V^Y(a',e')\Pi^e(e,e') + \beta p_{ret} V^R(a') \right\}$$

  subject to

  $$c + a' = wel + (1+r)a,$$

  $$a' \geq 0.$$

  Policy functions: $a' = g^Y_{a'}(a,e)$ and $l = g^Y_l(a,e)$.

- Old individual's problem:

  $$V^R(a) = \max_{a'} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \beta p_{death} \sum_{e'} V^Y(a',e')G^e(e') + \beta\,(1-p_{death})\, V^R(a') \right\}$$

  subject to

  $$c + a' = \omega_{ret} + (1+r)a,$$

  $$a' \geq 0.$$

  Here Π^e denotes the transition matrix of the idiosyncratic shock e, G^e denotes the *initial* distribution of e. Pension benefits are denoted as ω_{ret}. Policy function: $a' = g^R_{a'}(a)$.

- Stationary distribution $\left[\mu^Y(a,e), \mu^R(a)\right]$ satisfies

  $$\mu^Y(a',e') = (1-p_{ret}) \sum_a \sum_e \mathbb{I}_{\left\{a'=g^Y_{a'}(a,e)\right\}} \Pi^e(e,e')\mu^Y(a,e) + p_{death} \sum_a \mathbb{I}_{\left\{a'=g^R_{a'}(a)\right\}} G^e(e')\mu^R(a)$$

  $$\mu^R(a') = p_{ret} \sum_a \sum_e \mathbb{I}_{\left\{a'=g^Y_{a'}(a,e)\right\}} \mu^Y(a,e) + (1-p_{death}) \sum_a \mathbb{I}_{\left\{a'=g^R_{a'}(a)\right\}} \mu^R(a)$$

- **Calibration**. Guvenen et al. (2023) assume that newborn individuals draw their initial labor efficiency from the initial distribution G^e. Guvenen et al. (2023) set $n_e = 4$, and take the values e_1,e_2, e_3 and the 4×4 transition matrix Π^e from Castaneda, Diaz-Gimenez and Rios-Rull (2003, Tables 4-5). They calibrate e_{n_e}, the value of the "awesome" state, to match a share of wealth held by the top 1% equal to 30%. They calibrate ω_{ret} so that the ratio of pension benefits to GDP is equal to 4.9%. For simplicity, they set G^e equal to the stationary distribution of labor efficiency, γ^e.

- **VFI toolkit**. We combine $e = [e_1, \ldots, e_4]$ and $age = [Y, R]$ into

$$z\_grid = [e\_grid; age\_grid]$$

obtaining a vector with $2n_e$ points for the exogenous state. Note that this wastes grid points since we need only $n_e + 1$ values for the exogenous state z!. We set the transition matrix for the exogenous state z as follows

$$\underbrace{\Pi^z}_{2n_e \times 2n_e} = \begin{bmatrix} (1 - p_{ret}) \underbrace{\Pi^e(e, e')}_{n_e \times n_e} & p_{ret} I_{n_e} \\ p_{death} \underbrace{ones(n_e, 1) G^e}_{n_e \times n_e} & (1 - p_{death}) I_{n_e} \end{bmatrix}$$

where I_{n_e} denotes the $n_e \times n_e$ identity matrix and $ones(n_e, 1)G^e$ denotes a matrix with G^e in every row. Let $n_z = 2n_e$. The toolkit will compute the arrays V, $Policy$ and $StationaryDist$

$$V : n_a \times n_e \times n_{age} \text{ array}$$

$$Policy : 2 \times n_a \times n_e \times n_{age} \text{ array}$$

$$StationaryDist : n_a \times n_e \times n_{age} \text{ array}$$

Note that

$$\begin{aligned} g_l(a, e, age) &= Policy(1, a, e, age), \\ g_{a'}(a, e, age) &= Policy(2, a, e, age), \end{aligned}$$

where $a = 1, .., n_a$, $e = 1, .., n_e$ and $age = Y, R$. Note that e.g. $V(a, e, R)$ must be the same for all e, since once the individual is retired the only relevant state variable is a, not labor productivity.

- **VFI toolkit with correlated shocks**. TBA.