Buera & Shin (2013) - Financial Friction and the Persistence of History: A Quantitative Exploration

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Brief description of replication of Buera and Shin (2013). I denote the exogenous shock z, which they call e, I also call γ what they call σ. I use μ as the agent distribution (while they had it as the distribution of just the exogenous process, they used G for the agent distribution).

Note, the codes use $\beta = 0.8$, overwriting the value of $\beta = 0.904$, for reasons explained near end.

The model is about solving a transition path in an infinite horizon model. Here I just write the steady state problem as I can't be bothered writing all the time subscripts involved in the transition path.

Buera and Shin (2013) present the household problem as a sequence problem on their page 232, we first just rewrite this as a value function problem,

$$
\begin{aligned}
V(a,z) = & \max_{c,a'} \frac{c^{1-\gamma}}{1-\gamma} + \beta E[V(a',z')] \\
\text{subject to } & c + a' \leq max\{w, \pi(a,z|r,w)\} + (1+r)a \\
& \pi(a,z|r,w) = \max_{l,k}\{f(z,k,l) - wl - (r+\delta)k \\
& f(z,k,l) = z(k^{\alpha} l^{1-\alpha})^{1-\upsilon} \\
& a' \geq 0, \ k \leq \lambda a
\end{aligned}
$$

In this form the problem looks tricky, but fortunately the part relating to profit (π) maximization can be solved analytically, and so the model reduces to

$$
\begin{aligned}
V(a,z) = & \max_{c,a'} \frac{c^{1-\gamma}}{1-\gamma} + \beta E[V(a',z')] \\
\text{subject to } & c + a' \leq max\{w, \pi(a,z|r,w)\} + (1+r)a \\
& k_1 = \left[\left(\frac{1}{r+\delta} \alpha(1-\upsilon) z \right)^{1-(1-\alpha)(1-\upsilon)} ((1/w)(1-\alpha)(1-\upsilon)z)^{(1-\alpha)(1-\upsilon)} \right]^{1/\upsilon} \\
& k^* = min\{k_1, \lambda a\} \\
& l^* = [(1/w)(1-\alpha)(1-\upsilon) z k^{\alpha(1-\upsilon)}]^{\frac{1}{1-(1-\alpha)(1-\upsilon)}} \\
& \pi(a,z|r,w) = z((k^*)^{\alpha}(l^*)^{1-\alpha})^{1-\upsilon} - wl^* - (\delta + r)k^* \\
& a' \geq 0, \ k \leq \lambda a
\end{aligned}
$$

I put all the derivations of this in the appendix. This looks a mess, but the equations for k_1, $k*$, $l*$ and π are all trivial to evaluate.

We can now see that the problem is just a value function problem with one endogenous state a and one exogenous state z, and that the only action to be taken is choosing a'.

Of course we will actually want to solve a transition path for this model so we should in some sense have time subscripts on everything.

One of the tricks in Buera and Shin (2013) that keeps this model so simple is that being a worker/entrepreneur is just something that you 'do for the day'. The decision to be an entrepreneur or worker is entirely in the $max\{w, \pi(a, z|r, w)\}$ term (if the max is w you are a worker today, if the max is $\pi(a, z|r, w)$ you are an entrepreneur today. Hence there is no persistence of entrepreneur/worker decisions after we condition on (a, z) and so it disappears from the state space of the computational problem. In the codes I generate a figure to help make clear who is an entrepreneur/worker, Buera and Shin (2013) explain this obliquely in the bottom paragraph of their page 232.

The exogenous process on entrepreneurial ability z is that with probability ψ they keep the same value of z, while with probability $1 - \psi$ they draw from an iid random variable.

The other part of the model we need before we go to the code is the general equilibrium conditions.

We have the labor market clearance, which is that the labor demand of entrepreneurs equals the mass of workers (implicitly Buera and Shin (2013) have all workers provide an endowment of one unit of labor, while entrepreneurs don't work). That is,

$$\int l^* \mathbb{I}_{entrepreneur} d\mu - \int \mathbb{I}_{worker} d\mu \tag{1}$$

We have capital market clearance, which is that capital used by entrepreneurs equals the total assets of all agents

$$\int k^* \mathbb{I}_{entrepreneur} d\mu - \int a d\mu \tag{2}$$

Note that actually the above value function problem is incomplete, as Buera and Shin (2013) on page 236 also add 'individual or idiosyncratic taxes/subsidies/wedges on output (τ_{yi}) that distort the static profit-maximization problem of entrepreneur i into $\pi(a, z|r, w) = \max_{l,k}\{((1 - \tau_{yi}) z k^{\alpha} l^{1-\alpha})^{1-\upsilon} - wl - (r + \delta)k$. τ is a 'random variable' and τ is governed by the same ψ that governs z, so individuals draw a new τ exactly when they draw a new z.

So we will need to add τ as a shock. The 'exactly when' for changing τ and z is actually a bit of a pain for us, as it means we need another bivariate exogenous state that reflects the ψ probabilities so that we can enforce this 'exactly when'; we denote ψ_v the value of this state, which is 1 with probability ψ and 0 with probability $1 - \psi$, when it is one the z and τ remain fixed, when it is zero we draw a new z and τ.

Hence the value function problem now becomes,

$$
\begin{aligned}
&V(a,z,\tau,\psi_v) = \max_{c,a'} \frac{c^{1-\gamma}}{1-\gamma} + \beta E[V(a',z',\tau')] \\
&\text{subject to } c + a' \leq max\{w, \pi(a,z|r,w)\} + (1+r)a \\
&\quad k_1 = \left[\left(\frac{1}{r+\delta}\alpha(1-\upsilon)(1-\tau)z \right)^{1-(1-\alpha)(1-\upsilon)} ((1/w)(1-\alpha)(1-\upsilon)(1-\tau)z)^{(1-\alpha)(1-\upsilon)} \right]^{1/\upsilon} \\
&\quad k^* = min\{k_1, \lambda a\} \\
&\quad l^* = [(1/w)(1-\alpha)(1-\upsilon)(1-\tau)z(k^*)^{\alpha(1-\upsilon)}]^{\frac{1}{1-(1-\alpha)(1-\upsilon)}} \\
&\quad \pi(a,z|r,w) = (1-\tau)z((k^*)^{\alpha}(l^*)^{1-\alpha})^{1-\upsilon} - wl^* - (\delta+r)k^* \\
&\quad a' \geq 0, \; k \leq \lambda a
\end{aligned}
$$

note that we essentially get $(1-\tau)z$ wherever we previously had z, and that τ is now an exogenous state variable.

The baseline transition of the paper of Buera and Shin (2013) is to elimate the taxes. This is easy to do but a touch awkward for how the VFI Toolkit sets up a transition path. Specifically, the toolkit sets up a ParamPath, but here the path is on the exogenous shock grid, so instead we will just change the exogenous shock grid, and then set up a placebo ParamPath (in the codes this is on λ but it could be any parameter that does not change between initial and final distribution).

That is the model. Now just something that was problematic when solving. Buera and Shin (2013) set $\beta = 0.904$, but when I solve the model with this the initial equilibrium had a negative interest rate, whereas their Table 1 reports that $\beta = 0.904$ was specifically chosen so that the model interest rate is 0.045 (which equals their data interest rate). It is not really clear why this is the case (without the original codes it is not possible to know why, my guess is that there was an issue either with their max allowed value of assets, or with their approximation method for the value function, and that this caused issues at high asset levels).[1] For this reason the codes instead use $\beta = 0.8$ which is not enough to get the interest rate up to 0.045, but does make it positive (in both the initial and final eqm).[2]

For the economics of the model see Buera and Shin (2013).

References

Francisco Buera and Yongseok Shin. Financial frictions and the persistence of history: A quantitative exploration. Journal of Political Economy, 121(2):221–272, 2013.

A Profit Maximization derivations

The firm profit maximization problem is given by,

$$
\pi(a,z|r,w) = \max_{l,k}\{z(k^{\alpha}l^{1-\alpha})^{1-\upsilon} - wl - (r+\delta)k
$$

[1] Of course, I may have just made an error in my own codes, but I can't see it :).

[2] I didn't bother recalibrating β to target $r = 0.045$, but that would be the obvious thing to do.

subject to $k \leq \lambda a$. Note that I already substituted in the production function.

We can start by just ignoring the 'subject to' to see what would happen if the firm is unconstrained, and taking the first order conditions of the remaining unconstraint maximization problem we get

$$\frac{\partial \pi}{\partial l} = 0; \quad (1-\alpha)(1-\upsilon) z k^{\alpha(1-\upsilon)} l^{(1-\alpha)(1-\upsilon)-1} - w = 0 \tag{3}$$

$$\frac{\partial \pi}{\partial k} = 0; \quad \alpha(1-\upsilon) z k^{\alpha(1-\upsilon)-1} l^{(1-\alpha)(1-\upsilon)} - (r+\delta) = 0 \tag{4}$$

$$\tag{5}$$

We can rewrite the first FOC to get

$$l^* = [(1/w)(1-\alpha)(1-\upsilon) z k^{\alpha(1-\upsilon)}]^{\frac{1}{1-(1-\alpha)(1-\upsilon)}} \tag{6}$$

which is the expresion we will use for labor, and we will call it $l*$.

We can rewrite the second FOC to get

$$k^{1-\alpha(1-\upsilon)} = \frac{1}{r+\delta} \alpha(1-\upsilon) z l^{(1-\alpha)(1-\upsilon)} \tag{7}$$

and we can then substitute our earlier expresion for l that we got from the first FOC into this and rearrange and clean to get

$$k_1 = \left[\left(\frac{1}{r+\delta} \alpha(1-\upsilon) z \right)^{1-(1-\alpha)(1-\upsilon)} ((1/w)(1-\alpha)(1-\upsilon) z)^{(1-\alpha)(1-\upsilon)} \right]^{1/\upsilon} \tag{8}$$

which is the capital firms would use if they could ignore the capital constraint, and we will call it k_1.

Hence we have that the actual capital firms use will be is

$$k^* = \min\{k_1, \lambda a\} \tag{9}$$

which is to say they will use k_1 if possible, but that if they collateral constraint binds then they use the binding quantity.

Note that if we were to use the constrained version of the firm problem, we would still get the same solution for l^* so we can use that whether or not the collateral constraint binds (you can write it out as a Karusk-Kuhn-Tucker problem, and observe that the derivative w.r.t. l just ends up the same) so that applies whether the firm is constrained or unconstrained. (You could have done all of the above using the KKT, but the unconstrained problem is just easier to work with :)

With this we can replace the firm maximization problem with the three equations (6), (8) and (9).