Monetary Economics and the Macroeconomy

Topic 3: Sticky Prices in New Keynesian Models

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Content

Literature

Key reading:

- Romer (4th ed.), chapter 7

Related reading:

Models of staggered prices/wages

- Calvo, G (1983): Staggered Prices in a Utility-Maximizing Framework, Journal of Monetary Economics 12, 383-398
- Fischer (1977): Long-Term Contracts, Rational Expectations, and the Optimal Money Supply Rule, Journal of Political Economy, 85, 191-206
- Taylor, J (1979): Staggered Wage Setting in a Macro Model, American Economic Review 69, 108-113
- Taylor, J (1980): Aggregate Dynamics and Staggered Contracts, Journal of Political Economy 88, 1-24

Literature

Models of staggered prices/wages with solid micro-foundations

- Ascari, G (2000) Optimising Agents, Staggered Wages, and Persistence in the Real Effects of Money Shocks, Economic Journal, 110 664-686
- Ascari, G (2003) Price/Wage Staggering and Persistence: A Unifying Framework, Journal of Economic Survey, 17, 511-540
- Chari, V, Kehoe, P, and McGrattan, E Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?, Econometrica, 68, 1151-1179

Empirical evidence on nominal rigidities

- Bils, M and Klenow, P (2004): Some Evidence on the Importance of Sticky Prices, Journal of Political Economy 112, 947-985

New Keynesian Phillips curve

- Roberts, J (1995): New Keynesian Economics and the Phillips Curve, Journal of Monety, Credit, and Banking 27, 975-984

Literature

An introduction to new Keynesian framework and monetary policy analysis

- Gali J and Gertler M (2007) Macroeconomic Modeling for Monetary Policy Evaluation, Journal of Economic Perspectives 21 (Fall), 25-45

Inflation inertia

- Ball, L. (1994), "What Determines the Scrifice Ratio?" In N. Gregory Mankiw, ed., Monetary Policy, 155-182. Chicago: University of Chicago Press.
- Gali, Jordi (1999), "Inflation Dynamics: A Structural Econometric Analysis," Journal of Monetary Economics, 44, 195-222.
- Christiano, Eichenbaum and Evans (2005), "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy" Journal of Political Economy, 113, 1-45.

Staggered Price Adjustment in New Keynesian Models

- The previous model with flexible prices and monopolistic competition (see Topic 2) gave some basic insights into price-setting behaviour.
- More sophisticated modeling is necessary.
- In recent years a dynamic New Keynesian workhorse model has emerged.
- It falls in the class of models also called Dynamic Stochastic General Equilibrium (DSGE) models.
- This combines micro-founded dynamic optimization theory (real-business cycle models) with price rigidities.
- It is widely used in state of the art research and policy making.

Staggered Price Adjustment in New Keynesian Models

- Previous models assumed flexible prices
- $\Rightarrow$ not realistic.
- We now evaluate dynamic models in which some but not all prices are adjusted every period.
- Prices are set in multi-period contracts.
- In each period a fraction of price contracts expire and must be renewed.
  $\Rightarrow$ Price level's adjustment to monetary policy shocks is gradual
- Monetary policy shocks may have effects on real variables.

Before we study staggered price adjustments in NK setups let us look at the basic components of these models.

New Keynesian Models - Basic Building Blocks

Three sectors:

1. Households:
   - Make consumption and savings decisions looking into the future, i.e. decide on current vs. future consumption.
   - Supply labour for production.
2. Firms:
   - Produce goods. Use labour as production input and pay wage to households to compensate for supply of labour.
   - Imperfect competition $\rightarrow$ set prices optimally (i.e. to maximize profits) **given cost/restrictions of changing prices**.
3. Monetary policy authority that controls a short-term nominal interest rate.

- In this framework all actors are forward looking.
- When firms set their price today take into account the demand and cost conditions that they may face in the future.
- Adjustment of prices (or wages) is discrete and infrequent (based on Calvo (1983), but see also Rotember (1982))

New Keynesian Models - Households

- Fixed number of infinitely lived households in the economy.
- Obtain utility from consumption and disutility from working.
- As all households are identical we can think of a representative household.

Objective function of the representative household:

$$\sum_{t=0}^{\infty} \beta^t \left[U(C_t) - H(L_t)\right], \quad 0 < \beta < 1. \tag{1}$$

where C_t is a consumption index of all goods and L_t is hours worked. β is a discount factor.

New Keynesian Models - Households

- $U(\cdot)$ has a constant relative risk aversion:

$$U(C_t) = \frac{C_t^{1-\theta}}{1-\theta}, \quad \theta > 0$$

  and hence $U'(\cdot) > 0$ and $U''(\cdot) < 0$.
  - Note: θ is the inverse of the elasticity of substitution between consumption at any two points in time: a higher $\theta \implies$ consumers less willing to change (substitute) consumption over time
- The functional form of $H(\cdot)$ is

$$H(L_t) = \frac{B}{\gamma} L_t^{\gamma} \quad B > 0, \quad \gamma > 1. \tag{2}$$

  Hence, $H'(\cdot) > 0$ and $H''(\cdot) > 0$.

New Keynesian Models - Households

- Households' maximization problem:

$$\max_{\{C_t, B_t, L_t\}} \sum_{t=0}^{\infty} \beta^t \left[U\left(C_t\right) - H\left(L_t\right) \right]$$

  subject to

$$P_t C_t + B_t \leq (1 + i_{t-1}) B_{t-1} + W_t L_t$$

  and no-Ponzi-game condition.

- We have already encountered very similar problem in Topic 2: recall Lagrangian method to find FOCs.
- Define gross real interest rate as

$$1 + r_t \equiv (1 + i_t) \frac{P_t}{P_{t+1}} \equiv \frac{1 + i_t}{1 + \pi_{t+1}}$$

New Keynesian Models - Households

The FOC with respect to savings

$$C_t^{-\theta} = \beta \left(1 + r_t\right) C_{t+1}^{-\theta}$$
$$\Leftrightarrow \frac{C_{t+1}}{C_t} = \left\{\beta \left(1 + r_t\right)\right\}^{\frac{1}{\theta}} \tag{3}$$

yields the standard Euler equation, where r is the real interest rate. If the elasticity of substitution $1/\theta$ is very high, small changes in r_t can produce large changes in the rate of growth of consumption. Take logs on both sides:

$$\ln C_t = \ln C_{t+1} - \frac{1}{\theta} \ln \left(1 + r_t\right)$$

using the fact that $\ln(1 + r) \approx r$ and that consumption equals output in equilibrium

$$\ln Y_t = \ln Y_{t+1} - \frac{1}{\theta} r_t \tag{4}$$

This is the new Keynesian IS curve (already seen in Topic 2). Note: We ignored $\ln(\beta)$ as it is constant.

New Keynesian Models - Households

Household's choose labour optimally so that the real wage equals the marginal rate of substitution between consumption and hours worked [derivation not shown]

$$\frac{W_t}{P_t} = \frac{H'(L_t)}{U'(C_t)}.$$

Given that labour is the only production input and the production function is homogeneous of degree 1, $C_t = L_t = Y_t$, hence

$$\begin{aligned} \frac{W_t}{P_t} &= \frac{H'(Y_t)}{U'(Y_t)} \\ \Leftrightarrow \frac{W_t}{P_t} &= BY_t^{\gamma+\theta-1} \end{aligned} \tag{5}$$

shows the real wage for a given level of output.

New Keynesian Models - Firms

Firm i's production function

$$Y_{i,t} = L_{i,t}.$$

Firm faces the demand function

$$Y_{i,t} = Y_t \left(P_{i,t}/P_t\right)^{-\eta}$$

Then the firm's real profits are

$$\begin{aligned} R_t &= \left(\frac{P_{i,t}}{P_t}\right) Y_{i,t} - \left(\frac{W_t}{P_t}\right) Y_{i,t} \\ &= Y_t \left[\left(\frac{P_{i,t}}{P_t}\right)^{1-\eta} - \left(\frac{W_t}{P_t}\right)\left(\frac{P_{i,t}}{P_t}\right)^{-\eta}\right]. \end{aligned} \tag{6}$$

New Keynesian Models - Firms

Price setting:

- Assume firm sets price in period $t = 0$.
- Let q_t denote the probability that the price the firm sets in $t = 0$ is in effect in period t.
- Each firm is owned equally by all households and transfers profits back to these.
- Hence, firms value their profits according to the utility they provide for households.
- The marginal utility of household's consumption in period t relative to period 0 is $\beta^t U'(C_t) / U'(C_0) \equiv \lambda_t$.

The representative firm chooses its price in period $0, P_i$, to maximise

$$V \equiv \sum_{t=0}^{\infty} q_t \lambda_t R_t$$

$$= \sum_{t=0}^{\infty} q_t \lambda_t \left\{ Y_t \left[\left(\frac{P_i}{P_t} \right)^{1-\eta} - \left(\frac{W_t}{P_t} \right) \left(\frac{P_i}{P_t} \right)^{-\eta} \right] \right\} \tag{7}$$

Where we used (6) and R_t is the firm's profit in period t if P_i is still in effect.

Firm's price setting

Which price P_i will firm set?
Rewrite eq (7) as

$$V = \sum_{t=0}^{\infty} q_t \lambda_t Y_t P_t^{\eta-1} \left(P_i^{1-\eta} - W_t P_i^{-\eta} \right)$$

... and recall from Topic 2 that under monopolistic competition...

- ...the profit maximising price, P_t^* is a constant times W_t.
- So one can write ($P_i^{1-\eta} - W_t P_i^{-\eta}$) as a function of P_i and P_t^*.
- Let the lower case letters p_i and p_t^* denote variables in logs, we can call this function $F(p_i, p_t^*)$.

Hence,

$$V = \sum_{t=0}^{\infty} q_t \lambda_t Y_t P_t^{\eta-1} F(p_i, p_t^*)$$

Firm's price setting

$$V = \sum_{t=0}^{\infty} q_t \lambda_t Y_t P_t^{\eta-1} F\left(p_i, p_t^*\right)$$

Some simplifying assumptions:

- Variation in $\lambda_t Y_t P_t^{\eta-1}$ is relatively small compared to the variation in q_t and p_t^* and therefore negligible.
- $F(\cdot)$ can be well approximated by a second-order Taylor approximation around $p_i = p_t^*$.
- Then it follows that $F\left(p_i, p_t^*\right) \simeq F\left(p_t^*, p_t^*\right) - K\left(p_i - p_t^*\right)^2$ with $K > 0$.

The assumptions allow us to formulate the problem of choosing P_i to max V as

$$\max_{p_i} \sum_{t=0}^{\infty} q_t F\left(p_i, p_t^*\right) \Leftrightarrow \max_{p_i} \sum_{t=0}^{\infty} q_t \left[F\left(p_t^*, p_t^*\right) - K\left(p_i - p_t^*\right)^2\right]$$

$$\Leftrightarrow \min_{p_i} \sum_{t=0}^{\infty} q_t \left(p_i - p_t^*\right)^2$$

Digression - Taylor Approximation

The second-order Taylor approximation of a function h at $x = a$ is given as

$$h(x) \simeq h(a) + \left.\frac{\partial h(x)}{\partial x}\right|_{x=a} (x-a) + \frac{1}{2}\left.\frac{\partial^2 h(x)}{\partial x^2}\right|_{x=a} (x-a)^2$$

Applying this to our case, a second-order Taylor approximation for of $F(p_i, p_t^*)$ around $p_i = p_t^*$ is given as

$$F(p_i, p_t^*) \simeq F(p_t^*, p_t^*) + \left.\frac{\partial F(p_i, p_t^*)}{\partial p_i}\right|_{p_i = p_t^*} (p_i - p_t^*) + \frac{1}{2}\left.\frac{\partial^2 F(p_i, p_t^*)}{\partial p_i^2}\right|_{p_i = p_t^*} (p_i - p_t^*)^2$$

For our specific case we know that

- Profits in period t are maximised at $p_i = p_t^* \ldots$
- ... thus at $p_i = p_t^*$, $\frac{\partial F(p_i, p_t^*)}{\partial p_i} = 0$
- ... and $\frac{\partial^2 F(p_i, p_t^*)}{\partial p_i^2} < 0$.

Hence,

$$F(p_i, p_t^*) \simeq F(p_t^*, p_t^*) - K(p_i - p_t^*)^2 \quad \text{with} \quad K > 0$$

Firm's price setting - FOC

$$
\begin{aligned}
&\sum_{t=0}^{\infty} 2q_t \left(p_i - p_t^*\right) \overset{!}{=} 0 \\
\Leftrightarrow & q_0 \left(p_i - p_0^*\right) + q_1 \left(p_i - p_1^*\right) + q_2 \left(p_i - p_2^*\right) + \ldots = 0 \\
\Leftrightarrow & q_0 p_i - q_0 p_0^* + q_1 p_i - q_1 p_1^* + q_2 p_i - q_2 p_2^* + \ldots = 0 \\
\Leftrightarrow & p_i \left(q_0 + q_1 + q_2 + \ldots\right) = q_0 p_0^* + q_1 p_1^* + q_2 p_2^* + \ldots \\
\Leftrightarrow & p_i \sum_{\tau=0}^{\infty} q_\tau = \sum_{t=0}^{\infty} q_t p_t^* \\
\Leftrightarrow & p_i = \sum_{t=0}^{\infty} \frac{q_t}{\sum_{\tau=0}^{\infty} q_\tau} p_t^* = \sum_{t=0}^{\infty} \omega_t p_t^*, \quad \text{with} \quad \omega_t \equiv q_t / \sum_{\tau=0}^{\infty} q_\tau \qquad (8)
\end{aligned}
$$

- ω_t is the probability that the price the firm sets in $t = 0$ will be in effect in period t; divided by a normalizing factor.
- Hence, ω_t measures the importance of period t to the choice of p_i.

Firm's price setting

$$p_i = \sum_{t=0}^{\infty} \omega_t p_t^*$$

The price set by firm i is a weighted average of the profit-maximising prices during the time the price will be in effect. Account for uncertainty about future: Firm i will base the decision about p_i on expectations about the $p_t^{*\prime}$ s:

$$p_i = \sum_{t=0}^{\infty} \omega_t E_0 \left[p_t^* \right] \tag{9}$$

where $E_0[\cdot]$ denotes expectations in period $t = 0$.

Firm's price setting

We know:

- The profit maximising real price P^*/P is proportional to the real wage W/P.
- The real wage increases in Y (see for example eq (5)).

Keeping the log-linear structure of the model, the (log) profit maximising price takes the form

$$p_t^* = p_t + \text{ constant } + \phi y_t, \quad \phi > 0. \tag{10}$$

Let m_t denote log nominal GDP ($p_t + y_t$) and setting the constant to zero for simplicity yields

$$\begin{aligned} p_t^* &= \phi y_t + p_t \\ &= \phi(y_t + p_t) - \phi p_t + p_t \\ &= \phi m_t + (1 - \phi)p_t. \end{aligned} \tag{11}$$

Firm's price setting

Substitute eq (11) into eq (9) gives us

$$p_i = \sum_{t=0}^{\infty} \omega_t E_0 \left[\phi m_t + (1 - \phi) p_t \right] \tag{12}$$

This is the key equation for firm's price setting and therefore aggregate supply in the model.
The New Keynesian IS curve (4) describes aggregate demand for a given real interest rate.
The real interest rate is set by the monetary policy authority.

New Keynesian Models - Central Bank

How is monetary policy conducted in New Keynesian models?

- Monetary policy is often described to follow a rule for how it sets the real interest rate.
- ...can depend on output, inflation and future expectations of these.
- We will look at monetary policy rules in detail later.

For now:

- We assume that the central bank has a target path for nominal GDP, m_t, and conducts monetary policy to achieve it.
- For now we only want to focus on the aggregate supply side of the economy.
- Hence, we will take the path of nominal GDP, m_t, as given and examine the economy's behaviour in response to shocks to nominal GDP.
- This allows us to abstract from the money market and from the IS equation

New Keynesian Models - Extensions

This is one of the most simplest versions of the NK model. Extensions may include

- A role for capital and fixed investment. Hence, consumption does not equal output anymore ($C + I = Y$) and we have capital as production input in addition to labour.
- A more prominent role for the impact of expectations. We mostly neglected uncertainty.
- A role for government - fiscal policy.
- Explicit modelling of a financial sector with frictions in financial intermediation.

This model is - in various forms - used in research and for policy making. For more extensions see Romer chapter 7 and the references therein.

Staggered Price Adjustment in New Keynesian Models

There are two different pricing rules that have been introduced into NK models:

- State-dependent rules: price changes as a function of state, i.e. prices change due to certain developments in the economy. For example if deviation from price's optimal level is too large.
- Example for price adjustment in retail stores.
- Time-dependent rules: price changes as a function of time, i.e. length of price contract is determined when price is set.
- Example for union contracts or prices in catalogues.
- Distinction within the class of time-dependent rules:
  i Predetermined prices (Fisher, 1977): For a given length of period, the path of prices is predetermined, but within this path price setters can set different prices for different periods.
  ii Fixed prices (Taylor, 1979, 1980): For a certain time, prices are not only predetermined but also constant during the length of the predetermined period.

Taylor's Model of Staggered Price Adjustment

We exemplify staggered price adjustment in a NK model using a time-dependent rule introduced by Taylor.
Model assumptions:

- Firm can set a price in period t that is in place for periods t and $t + 1$. Hence, prices are predetermined and fixed.
- For simplicity: m_t is given by a random walk: $m_t = m_{t-1} + u_t$
- ... where the disturbance u_t is white noise, hence $E_t[u_{t+1}] = 0$.

Taylor's Model of Staggered Price Adjustment

Let x_t denote the price chosen by firms that set their prices in period t. Then the equation for firm's price setting (9) in the NK model and eq (11) imply

$$
\begin{aligned}
x_t &= \frac{1}{2}\left(p^*_{i,t} + E_t p^*_{i,t+1}\right) \\
&= \frac{1}{2}\left\{[\phi m_t + (1-\phi)p_t] + [\phi E_t m_{t+1} + (1-\phi)E_t p_{t+1}]\right\}
\end{aligned}
$$

We also know that:

- Since half of the prices are set every period, p_t is the average of x_t and x_{t-1}.
- Due to the random walk structure of $m_t : E_t m_{t+1} = E_t m_t + E_t u_{t+1} = m_t$.

Using this in the equation above yields

$$
\begin{aligned}
x_t &= \phi m_t + \frac{1}{2}(1-\phi)p_t + \frac{1}{2}(1-\phi)E_t p_{t+1} \\
&= \phi m_t + \frac{1}{2}(1-\phi)\frac{1}{2}\left(x_t + x_{t-1}\right) + \frac{1}{2}(1-\phi)\frac{1}{2}\left(E_t x_{t+1} + x_t\right) \\
&= \phi m_t + \frac{1}{4}(1-\phi)\left(x_{t-1} + 2x_t + E_t x_{t+1}\right)
\end{aligned}
$$

Taylor's Model of Staggered Price Adjustment

Solving for x_t yields

$$x_t = A\left(x_{t-1} + E_t x_{t+1}\right) + (1-2A)m_t, \qquad \text{with } A \equiv \frac{1}{2}\frac{1-\phi}{1+\phi} \quad (13)$$

This is the key equation for firm's price setting in the model.

- x_t depends on x_{t-1} and m_t which are known, and it depends on the expectations x_{t+1}.
- To solve the model we need to eliminate $E_t x_{t+1}$ from this expression.
- To do this we use the Method of Undetermined Coefficients.

Method of Undetermined Coefficients

Idea:

- Make a sophisticated guess for the general form of the solution...
- ...then use the model to determine the precise coefficients.

Our guess:

- In period t, the money stock, m_t, and prices set in the previous period, x_{t-1}, are known.
- We know that the model is linear.

It is therefore a reasonable guess that the price set by the firm, x_t, is a linear function of m_t and x_{t-1} :

$$x_t = \lambda x_{t-1} + (1-\lambda)m_t. \tag{15}$$

Our goal:

- Find value for λ that yield a solution of the model.

Taylor's Model of Staggered Price Adjustment

What was our goal and how will we progress:

- Reminder: Overall goal was to find an expression for $E_t x_{t+1}$ in equation (13) $[x_t = A\left(x_{t-1} + E_t x_{t+1}\right) + (1 - 2A)m_t]$.
- Plan 1: We will use eq (15) to find an expression for $E_t x_{t+1}$ - however, it will depend on λ.
- Plan 2: Then we will try to find appropriate values for λ.

Plan 1: Find Expression for $E_t x_{t+1}$

Since equation eq (15) [$x_t = \lambda x_{t-1} + (1-\lambda) m_t$] holds in each period, iterate forward to write

$$
\begin{aligned}
x_{t+1} &= \lambda x_t + (1-\lambda) m_{t+1} \\
E_t x_{t+1} &= \lambda E_t x_t + (1-\lambda) E_t m_{t+1} \\
E_t x_{t+1} &= \lambda x_t + (1-\lambda) m_t \\
E_t x_{t+1} &= \lambda \left[\lambda x_{t-1} + (1-\lambda) m_t \right] + (1-\lambda) m_t \\
E_t x_{t+1} &= \lambda^2 x_{t-1} + \left(1-\lambda^2\right) m_t. \qquad (16)
\end{aligned}
$$

take expectations... and since $E_t m_{t+1} = m_t \ldots$ use eq (15) to substitute for $x_t \ldots$ and rearrange...
Plan 1: done!

Taylor's Model of Staggered Price Adjustment

Substitute this expression for $E_t x_{t+1}$ into eq (13)

$$
\begin{aligned}
x_t &= A\left(x_{t-1} + \lambda^2 x_{t-1} + \left(1-\lambda^2\right) m_t\right) + (1-2A)m_t, \\
&= \left(A + A\lambda^2\right) x_{t-1} + \left[A\left(1-\lambda^2\right) + (1-2A)\right] m_t
\end{aligned} \tag{17}
$$

This is a solution for x_t independent of $E_t x_{t+1}$. But what is λ ?
If we have found a solution of the model, then this solution, eq (17), and the assumed solution eq (15) must be the same.

Plan 2: Finding Values for λ

Compare eq (15) and eq (17):

$$\begin{array}{llllll} x_t = & \left(A + A\lambda^2\right) & x_{t-1} & + \left[A\left(1-\lambda^2\right) + (1-2A)\right] & m_t \\ x_t = & \lambda & x_{t-1} & +(1-\lambda) & m_t \end{array}$$

If we have found a solution of the model these two equations must be the same. They are the same if

$$\begin{aligned} A + A\lambda^2 &= \lambda \\ &\text{and} \\ A\left(1-\lambda^2\right) + (1-2A) &= 1-\lambda \end{aligned} \tag{19}$$

Eq (19) simplifies to (18), thus we only need to consider (18). This is a quadratic in λ and solves to

$$\lambda = \frac{1 \pm \sqrt{1-4A^2}}{2A}. \tag{20}$$

Plan 2: Finding a Value for λ

Use our definition for A in the equations for λ yields

$$\lambda_1 = \frac{1-\sqrt{\phi}}{1+\sqrt{\phi}} \quad \text{and} \quad \lambda_2 = \frac{1+\sqrt{\phi}}{1-\sqrt{\phi}}.$$

- When $\lambda = \lambda_2, |\lambda| > 1$.
  $\Rightarrow$ One can see that the economy is unstable (from eq (21) below). The slightest disturbance sends output off towards $\pm\infty$.
- When $\lambda = \lambda_1, |\lambda| < 1$.
  $\Rightarrow$ The economy is stable. Hence, we focus on $\lambda = \lambda_1$.

Plan 2: done!
Thus, equation (15) with $\lambda = \lambda_1$ solves the model.

Taylor's Model - Output Behaviour

Given our solution for λ we can now describe the behaviour of output. Recall that real output can be written as

$$\begin{aligned} y_t &= m_t - p_t \\ &= m_t - \left(x_{t-1} + x_t\right)/2 \end{aligned}$$

Using the solution for x_t in equation (15):

$$\begin{aligned} y_t &= m_t - \left([\lambda x_{t-2} + (1-\lambda)m_{t-1}] + [\lambda x_{t-1} + (1-\lambda)m_t]\right)/2 \\ &= m_t - \left[\lambda \frac{1}{2}\left(x_{t-2} + x_{t-1}\right) + (1-\lambda)\frac{1}{2}\left(m_{t-1} + m_t\right)\right]. \end{aligned}$$

Knowing: $m_t = m_{t-1} + u_t$ and $\left(x_{t-1} + x_{t-2}\right)/2 = p_{t-1}$ we can further simplify:

$$\begin{aligned} y_t &= m_{t-1} + u_t - \left[\lambda p_{t-1} + (1-\lambda)m_{t-1} + (1-\lambda)\frac{1}{2}u_t\right)\right] \\ &= \lambda\left(m_{t-1} - p_{t-1}\right) + \frac{1+\lambda}{2}u_t \\ &= \lambda y_{t-1} + \frac{1+\lambda}{2}u_t \end{aligned} \tag{21}$$

Taylor's Model of Staggered Price Adjustment

Analysis of output's behaviour:

- Equation (21) implies for $\lambda > 0$ (which is true for $\phi < 1$), that shocks to aggregate demand (u_t) have long-lasting effects on output.
- Effects on y typically persist even after all firms have changed their prices.
- The speed with which the shock's impact diminshes over time is negatively correlated with the size of λ.
  $\Rightarrow$ Monetary shocks are non-neutral when price adjustment is staggered as suggested by Taylor, they have effects on the real variables in the economy.

Reasons for non-neutrality of monetary shocks in Taylor's model:

- Given $p_t^* = \phi m_t + (1 - \phi) p_t$, for $\phi < 1$ agents put weight on the overall price level and so the fact that not all firms are able to adjust their prices mutes their adjustment.
- Nominal rigidity (the fact that not all prices adjust every period) leads to gradual adjustment of the price level.

The Calvo Model and the New Keynesian Phillips Curve

- Problem with staggered price setting a-la Taylor: Moving beyond two-periods of fixed prices becomes intractable very quickly.
- Calvo (1983) proposes a slightly different way of staggered price setting.
- Similarities between Taylor's and Calvo's price setting:
- Prices are predetermined and fixed between the times they are adjusted.
- What is new?
- Prices change stochastically rather than deterministically.
- Hence, rather than to predetermine the number of periods the price stays fixed (as Taylor suggests), Calvo assumes that the opportunity for a firm to change prices follows a Poisson process...
- ...i.e. the probability of a firm being able to change price is the same in each period, regardless when it was last able to change price.

The Calvo Model and the New Keynesian Phillips Curve

Implications of Taylor's and Calvo's way of price setting:

- Qualitative predictions of the two models are the same:
  - Non-neutrality of monetary shocks.
  - Nominal rigidity leads to gradual adjustment of the price level, and real rigidity magnifies the effects of nominal rigidity.
- Advantages of the Calvo way of price setting:
  - Model can easily accommodate any degree of price stickiness. Important advantage over Taylor model for use in practice.
  - Leads to simple expression for the New Keynesian Phillips curve showing the dynamics of inflation.

Staggered price setting in the Calvo Model

Average price:

- Each period, a fraction $\alpha(0 < \alpha < 1)$ of firms sets new prices, with the firms chosen randomly.
- Average price in period t : α times the price set by firms that set new prices in t, x_t; plus $(1 - \alpha)$ times the average price charged by firms in period t that do not change their prices.
- Because the firms that change prices are chosen at random and because the number of firms is large, the average price charged by firms that do not change price in period t equals the average price charged by all firms in $t - 1$.

Thus, the average price, p_t, is given by

$$p_t = \alpha x_t + (1 - \alpha) p_{t-1}$$

where x_t is the price set by firms that are able to change their prices.

Staggered price setting in the Calvo Model

Subtracting p_{t-1} from both sides will give an expression for inflation

$$\pi_t = \alpha \left(x_t - p_{t-1} \right) \tag{22}$$

Recap from the outline of the NK model of firm's price setting:

- We derived the price firms set to be a weighted average of the profit maximising price while the price is in effect:
- See eq (8): $x_i = \sum_{j=0}^{\infty} q_j / \sum_{k=0}^{\infty} q_k E_t p^*_{t+j}$.
- For simplicity we assumed that the discount factor is nearly unity and therefore abstracted from it.
- With prices only in effect for two periods this was not a problem, but now we look indefinitely in the future, so we cannot just abstract from the discount factor.

Staggered price setting in the Calvo Model

The formulation in eq (8) with a discount factor would be

$$x_i = \sum_{j=0}^{\infty} \frac{\beta^j q_j}{\sum_{k=0}^{\infty} \beta^k q_k} E_t p^*_{t+j}$$

where β is the discount factor and q_j is the probability that the price will still be in effect in period $t+j$.
Calvo's assumption that price changes follow a Poisson process implies that $q_j = (1-\alpha)^j$. Then the above formulation becomes

$$\begin{aligned} x_t &= \sum_{j=0}^{\infty} \frac{\beta^j (1-\alpha)^j}{\sum_{k=0}^{\infty} \beta^k (1-\alpha)^k} E_t p^*_{t+j} \\ &= [1-\beta(1-\alpha)] \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j E_t p^*_{t+j} \qquad (23) \end{aligned}$$

where $\sum_{k=0}^{\infty} \beta^k (1-\alpha)^k = 1/[1-\beta(1-\alpha)]$ follows as it is an infinite series.

Digression 1: Infinite Sums

Digression 1

For an infinite series one can show that $\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$ if $|r| < 1$. Then in our case:

$$\sum_{k=0}^{\infty}(\beta(1-\alpha))^k = \frac{1}{[1-\beta(1-\alpha)]} \quad \text{with} \quad a = 1 \quad \text{and} \quad r = (\beta(1-\alpha))$$

as $0 < \beta < 1$ and $0 < (1-\alpha) < 1$.

Digression 2: Recall that

$$x_t = [1-\beta(1-\alpha)] \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j E_t p^*_{t+j}$$

Iterating this one period forward and taking expectations, E_t, yields

$$E_t x_{t+1} = [1-\beta(1-\alpha)] \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j E_t p^*_{t+1+j}$$

Staggered price setting in the Calvo Model

Back to

$$x_t = [1-\beta(1-\alpha)]\sum_{j=0}^{\infty}\beta^j(1-\alpha)^j E_t p^*_{t+j}$$

We can write the RHS of this separately for period t and all following periods:

$$x_t = [1-\beta(1-\alpha)]\beta^0(1-\alpha)^0 E_t p^*_{t+0} + [1-\beta(1-\alpha)]\left[\sum_{j=1}^{\infty}\beta^j(1-\alpha)^j E_t p^*_{t+j}\right]$$

$$= [1-\beta(1-\alpha)]p^*_t + \beta(1-\alpha)[1-\beta(1-\alpha)]\left[\sum_{j=0}^{\infty}\beta^j(1-\alpha)^j E_t p^*_{t+1+j}\right]$$

$$= [1-\beta(1-\alpha)]p^*_t + \beta(1-\alpha)E_t x_{t+1}$$

To relate this with eq (22) subtract p_t from both sides:

$$x_t - p_t = [1-\beta(1-\alpha)]\left(p^*_t - p_t\right) + \beta(1-\alpha)\left(E_t x_{t+1} - p_t\right) \Leftrightarrow$$

$$\left(x_t - p_{t-1}\right) - \left(p_t - p_{t-1}\right) = [1-\beta(1-\alpha)]\left(p^*_t - p_t\right) + \beta(1-\alpha)\left(E_t x_{t+1} - p_t\right) \tag{24}$$

The Calvo Model and the New Keynesian Phillips Curve

Recall from eq (22) that $\pi_t = \alpha\left(x_t - p_{t-1}\right)$. It follows that

- $x_t - p_{t-1} = \pi/\alpha$ and $E_t x_{t+1} - p_t = E_t \pi_{t+1}/\alpha$.
- and we know that
- $p_t - p_{t-1} = \pi_t$ and $p_t^* - p_t = \phi y_t$

Using this in equation (24)

$$
\begin{aligned}
(\pi_t/\alpha) - \pi_t &= [1 - \beta(1-\alpha)]\phi y_t + \beta(1-\alpha)\left(E_t \pi_{t+1}/\alpha\right) \\
\Leftrightarrow \pi_t &= \frac{\alpha}{1-\alpha}[1 - \beta(1-\alpha)]\phi y_t + \beta E_t \pi_{t+1} \\
\Leftrightarrow \pi_t &= \kappa y_t + \beta E_t \pi_{t+1}, \quad \text{with} \quad \kappa \equiv \frac{\alpha[1 - (1-\alpha)\beta]\phi}{1-\alpha}. \qquad (25)
\end{aligned}
$$

Equation (25) is the Standard New Keynesian Phillips Curve. It states that inflation depends positively on output and expected inflation.

The Calvo Model and the New Keynesian Phillips Curve

The Standard New Keynesian Phillips Curve is often rewritten as

$$E_t\pi_{t+1} - \pi_t = \frac{1-\beta}{\beta}\pi_t - \frac{\kappa}{\beta}\left(y_t - \bar{y}_t\right) \tag{26}$$

where $\bar{y}_t$ is the economy's natural level of output which we had set to zero for simplicity in the analysis above.

- Note that with β close to 1 , the term $((1-\beta)/\beta)$ is small. $\Rightarrow$ Anticipated disinflation is associated with a boom in output (implies anti-inertia...compare to the following slides).

The Calvo staggered price setting and the associated NKPK is an important part of most state of the art New Keynesian Models used by policy makers and researchers.

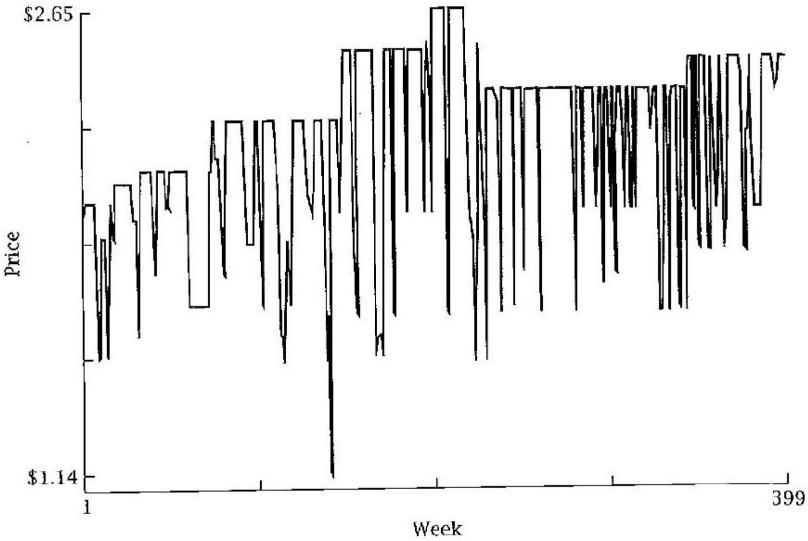
Microeconomic Price Adjustment

NK model reasonably matches aggregate behaviour in terms of money non-neutrality. How well does it account for microeconomic price setting? Microeconomic evidence about price adjustment

- Temporary sale prices are common for many products, i.e. price often falls sharply and is then quickly raised again, often to its previous level.
- Other prices are in place for exactly one year.
- Price setting varies in several dimensions:
- sizes of the reductions during sales vary
- intervals between adjustments of the regular price are heterogeneous
- the regular price sometimes rises and sometimes falls
- the sizes of the changes in the regular price vary
  $\Rightarrow$ Price adjustment does not just follow any simple pattern - if at all.
- If a pattern can be detected for one product, further aggregation difficulties as tremendous heterogeneity across products in the frequency of adjustment.

Sticky Prices - Empirical Evidence

Figure: Price of a 9.5 ounce box of Triscuits, 2000.



Microeconomic Price Adjustment vs. NK Models

Thus the microeconomic evidence does not show clearly what assumptions about price adjustment we should use in building a macroeconomic model.

- Time-dependent models:
  - Contradicted by the data which implies mainly irregular intervals between price adjustments
- State-dependent models:
  - Contradicted by frequent tendency for some prices to be in effect for exactly one year.
  - Contradicted by strong tendency for prices to revert to their original level after a sale.

  $\Rightarrow$ No perfect solution for modeling the heterogeneous microeconomic price adjustment in a simple macroeconomic model.

Further information about microeconomics of price adjustment, see Romer chapter 7.6.

The New Keynesian Phillips Curve - Inflation Inertia

Another empirical observation related to price developments is inflation inertia.

- Ball (1994) identifies 28 episodes over the period 1960-1990 in nine industrialised countries in which inflation fell substantially.
- He reports that the disinflation is in all cases attributed to monetary policy.
- Output was below normal in these episodes.
- View that high inflation has a tendency to continue unless there is a period of low output is refered to as inflation inertia.
- Inflation inertia does not refer to high persistence (serial correlation) of inflation...
- ...but to it being costly to reduce.

Ample empirical evidence for inflation inertia. For more details see for example Romer Chapter 7.6. and the references therein.

The New Keynesian Phillips Curve - Inflation Inertia

Does our model with the NKPC match the empirical observations of inflation inertia?
Recall that the NKPC can be written as:

$$E_t\left[\pi_{t+1}\right] - \pi_t = \frac{1-\beta}{\beta}\pi_t - \frac{\kappa}{\beta}\left(y_t - \bar{y}_t\right).$$

- As β is typically close to 1 , the term $((1-\beta)/\beta)$ is small.
- In response to an anticipated fall in inflation, the NKPC implies an increase in output.
- The NKPC implies anti-inertia! Inconsistent with empirical observations.
- Although the NKPC implies that prices display inertia, the inflation rate does not exhibit intrinsic inertia.
  $\Rightarrow$ The standard form of NKPC is not without a problem.