Monetary Economics and the Macroeconomy

Topic 2: The Classical Monetary Model

Dr. Alessandro Di Nola

University of Birmingham

Fall 2025-26

This class

- Background on new Keynesian models
- Benchmark monetary model with flexible prices, two versions
  (i) perfect competition
  (ii) monopolistic competition, as precursor to sticky prices

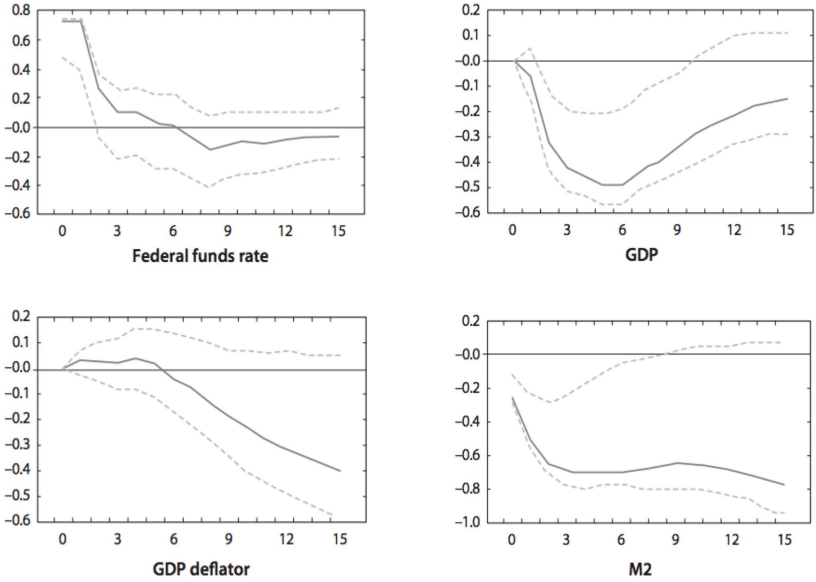
Background

- New Keynesian model builds on real business cycle model
- RBC model, key features
  - intertemporal utility maximization
  - rational expectations
  - representative agent / complete asset markets
  - perfect competition in goods and factor markets
- RBC model, key implications
  - business cycles are Pareto efficient
  - business cycles driven by exogenous productivity shocks (and other exogenous real shocks: terms-of-trade, government spending, etc)
  - money is neutral, implicitly

Background

- New Keynesian model, key features
  - intertemporal utility maximization
  - rational expectations
  - representative agent / complete asset markets
  - imperfect competition in goods and/or factor markets
  - nominal rigidities (prices are sticky)
- New Keynesian model, key implications
  - business cycles are inefficient
  - business cycles driven by mixture of exogenous productivity shocks and exogenous monetary policy shocks
  - money is not neutral in the short run
  - money is neutral in the long run

Empirical Evidence



Proportional responses to monetary policy shock. Periods in quarters.

Benchmark model with flexible prices

- An RBC-style model overlaid with nominal variables
- Simplified setup: no physical capital, no trend growth
- Begin with perfect competition in goods market
- Then monopolistic competition, firms have price-setting power

Representative household

- Maximizes expected intertemporal utility

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(C_t, N_t) \right\}, \quad 0 < \beta < 1$$

subject to sequence of budget constraints, for every t

$$P_t C_t + Q_t B_{t+1} \leq B_t + W_t N_t + D_t$$

- P_t denotes price level in units of account, W_t denotes nominal wage
- Q_t denotes nominal price of bond that delivers one unit of account next period $\implies$ *gross* return on bond is $\mathcal{I}_t \equiv 1/Q_t$
- Since households own firms, they receive dividends D_t (if any).

Representative household

- Lagrangian with non-negative multipliers λ_t

$$\mathcal{L} = \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t u\left(C_t, N_t\right) + \sum_{t=0}^{\infty} \beta^t \lambda_t \left[B_t + W_t N_t + D_t - P_t C_t - Q_t B_{t+1}\right] \right\}$$

- First order (necessary) conditions:

$$\begin{aligned}
c_t: & \quad \beta^t u_{C,t} - \beta^t \lambda_t P_t = 0 \\
N_t: & \quad \beta^t u_{N,t} + \beta^t \lambda_t W_t = 0 \\
B_{t+1}: & \quad -\beta^t \lambda_t Q_t + \beta^{t+1} \mathbb{E}_t \left\{\lambda_{t+1}\right\} = 0 \\
\lambda_t: & \quad B_t + W_t N_t + D_t - P_t C_t - Q_t B_{t+1} = 0
\end{aligned}$$

Household: Optimality conditions

- Representative household, labor supply determined by

$$-\frac{u_{N,t}}{u_{C,t}} = \frac{W_t}{P_t} \tag{H1}$$

and consumption Euler equation

$$u_{C,t} = \beta \mathbb{E}_t \left\{ u_{C,t+1} \times \frac{P_t}{P_{t+1}} \frac{1}{Q_t} \right\}$$

- Let i_t denote the nominal interest rate and π_{t+1} denote the inflation rate.
- Since $1 + \pi_{t+1} = P_{t+1}/P_t$ and $1 + i_t = 1/Q_t$, the Euler equation can be rewritten in the more familiar form

$$u_{C,t} = \beta \mathbb{E}_t \left\{ u_{C,t+1} \times \frac{1 + i_t}{1 + \pi_{t+1}} \right\} \tag{H2}$$

Household: Optimality conditions, Intuition

- Households optimality conditions H1-H2 have quite intuitive interpretations
- Equation (H1) is an *intra-temporal* optimality condition:

$$-\frac{u_{N,t}}{u_{C,t}} = \frac{W_t}{P_t} \tag{H1}$$

  It says that, when choosing labor supply, the representative agent equates the MRS between consumption and labor to the real wage. Sounds familiar?
- Draw picture

Household: Optimality conditions, Intuition

- Equation (H2) is an *inter-temporal* optimality condition (known as Euler equation):

$$u_{C,t} = \beta \mathbb{E}_t \left\{ u_{C,t+1} \times \frac{1+i_t}{1+\pi_{t+1}} \right\} \tag{H2}$$

  The Euler equation equates the **marginal cost** and **expected marginal benefit** of saving one more unit of consumption.
  - Left-hand side: marginal utility of consumption today — the cost of saving, since saving requires giving up one unit of current consumption.
  - Right-hand side: discounted expected benefit of saving that unit. The household saves one unit today, earns a nominal return $1 + i_t$, and next period can buy $(1 + i_t)/(1 + \pi_{t+1})$ units of goods after accounting for inflation. This yield is multiplied by next period's marginal utility of consumption and discounted by the subjective factor β
- At the optimum, (Left-hand side) marginal cost of saving = marginal benefit of saving (Right-hand side)

Representative Firm

- Production function

$$Y_t = Z_t F(N_t) \tag{F1}$$

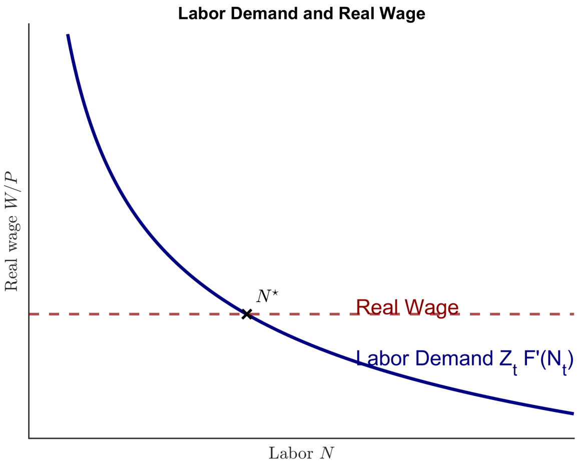
- Firm takes $\{W_t, P_t\}$ as given and choose N_t to maximize profits:

$$D_t = \max_{N_t} \{P_t Y_t - W_t N_t\}$$

- Labor demand determined by

$$\underbrace{Z_t F'(N_t)}_{\text{Marg. product labor}} = \underbrace{\frac{W_t}{P_t}}_{\text{Real wage}} \tag{F2}$$

Labor Demand



Equilibrium

- A competitive equilibrium involves:
  - Households optimize taking prices as given
  - Firms optimize taking prices as given
  - Prices such that markets clear
- Goods market clearing

$$C_t = Y_t \quad \text{for all } t \tag{M}$$

- Bond market clearing $B_t = 0$ for all t. **Why?**

Equilibrium, cont'd

- Bond market clearing $B_t = 0$ for all t. **Why?**
- Recall that $B_t > 0 =$ saving, $B_t < 0 =$ borrowing
- Suppose for a moment we have I households $i = 1, \cdots, I$
- Then in a closed system it must be that

$$\sum_{i=1}^{N} B_{i,t} = 0 \quad \text{for all } t$$

  Logic: there is a saver for every borrower, i.e. you can only borrow if someone takes the other side of this trade and lends to you
- In this model all households are alike (representative agent assumption), so it must be $B_{i,t} = 0$, for all i, i.e. in equilibrium no one will borrow and no one will save.

Equilibrium, cont'd

- Good market clearing follows from the households budget constraint in equilibrium (show this!)
- How many endogenous variables? 5 variables: $\left\{C_t, N_t, Y_t, \frac{W_t}{P_t}, r_t\right\}$, where $1 + r_t = (1 + i_t)/(1 + \pi_{t+1})$ is the gross real interest rate
- How many equations? 5 "equilibrium" equations: H1, H2, F1, F2 and M.

Standard parameterization

- Usual separable iso-elastic utility function

$$u(C, N) = \frac{C^{1-\sigma} - 1}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi}, \quad \sigma, \varphi > 0$$

  - What if $\sigma = 1$?
- And for simplicity, suppose linear production function

$$F(N) = N$$

- We have

$$u_C(C, N) = C^{-\sigma}, \quad u_N(C, N) = -N^{\varphi}, \quad F'(N) = 1$$

- Firm profits are zero, hence $D = 0$, i.e. households receive no dividends.

Equilibrium Equations

- Using the functional forms defined above, equilibrium equations for $\{C_t, N_t, Y_t, W_t/P_t, r_t\}$ become:

$$
\begin{aligned}
C_t^{\sigma} N_t^{\varphi} &= \frac{W_t}{P_t} && \text{(labor supply)} \\
C_t^{-\sigma} &= \beta \mathbb{E}_t \{ C_{t+1}^{-\sigma} (1 + r_t) \} && \text{(Euler)} \\
Z_t &= \frac{W_t}{P_t} && \text{(labor demand)} \\
Y_t &= Z_t N_t && \text{(production)} \\
C_t &= Y_t && \text{(resource constraint)}
\end{aligned}
$$

- Technology Z_t follows an exogenous stationary process
- Aim: Solve the equilibrium equations, i.e. express each endogenous variable as a function of exogenous technology shock Z_t

Solving the model

- Combine labor supply and labor demand equations to eliminate the real wage:

$$C_t^{\sigma} N_t^{\varphi} = Z_t$$

$$C_t = Z_t N_t$$

- Taking logs and using lowercase letters to denote the logs of the corresponding variables (i.e. $c_t = \log C_t$), we get

$$\sigma c_t + \varphi n_t = z_t$$

$$c_t = z_t + n_t$$

- Solution for y_t, c_t and n_t is

$$y_t = c_t = \psi_{cz} z_t, \quad \psi_{cz} = \frac{1+\varphi}{\sigma+\varphi} > 0$$

and

$$n_t = \psi_{nz} z_t, \quad \psi_{nz} = \frac{1-\sigma}{\sigma+\varphi} \lessgtr 0$$

Solving the model: Economic Intuition

- Solution for y_t, c_t and n_t is

$$y_t = c_t = \psi_{cz} z_t, \quad \psi_{cz} = \frac{1+\varphi}{\sigma+\varphi} > 0$$

and

$$n_t = \psi_{nz} z_t, \quad \psi_{nz} = \frac{1-\sigma}{\sigma+\varphi} \lesseqgtr 0$$

- Coefficients ψ_{cz}, ψ_{nz} etc. can be interpreted as *elasticities*, as

$$\psi_{cz} = \frac{\partial c_t}{\partial z_t} = \frac{\partial \log C_t}{\partial \log Z_t}$$

- Output (and consumption) always rises after a productivity increase, since $\psi_{cz} > 0$
- The sign of the response of employment is ambiguous and depend on $\sigma \lesseqgtr 1$ (income vs substitution effect)

Solving the model: real wage and real interest rate

- Real wage is equal to technology shock (in logs):

$$w_t - p_t = z_t$$

- To find the real interest rate, consider Euler equation again:

$$C_t^{-\sigma} = \beta \mathbb{E}_t \left\{ C_{t+1}^{-\sigma}(1 + r_t) \right\}$$

- Letting $\rho \equiv -\log \beta$, the log-linear approximation of the Euler equation is:

$$r_t = \rho + \sigma(\mathbb{E}_t c_{t+1} - c_t)$$

- Rewrite previous equation as

$$c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\sigma}(r_t - \rho)$$

IS-curve in intermediate macro?? Economic intuition?

NK IS Curve: Intuition

$$c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\sigma}(r_t - \rho)$$

- Key mechanism is intertemporal substitution:
  - Increases in real interest rate r_t reduce current demand for consumption good
- Increases in expected future income/consumption raise current demand for consumption
- This is all very different from traditional IS curve you may have encountered in undergraduate macro!
  - Traditional IS curve is based on idea that increase in interest rate $\implies$ reduction in investment
  - Can be derived in models where consumption is completely unresponsive to interest rate
  - NK-IS curve: Presence of expectation term $\mathbb{E}_t c_{t+1}$ on the RHS

Solving the model

$$r_t = \rho + \sigma(\mathbb{E}_t c_{t+1} - c_t) \qquad \textbf{(NK IS curve)}$$

- Not a solution for the real interest rate yet...
- Since $c_t = \psi_{cz} z_t$,

$$r_t = \rho + \sigma\psi_{cz}(\mathbb{E}_t z_{t+1} - z_t)$$

- We can make further progress by assuming that $z_t = \log Z_t$ follows an exogenous AR(1) process

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z, \quad \rho_z \in [0, 1)$$

- Then the equilibrium process for the interest rate becomes

$$r_t = \rho - \sigma\psi_{cz}(1 - \rho_z) z_t$$

- How does the real interest rate r_t respond to technology z_t? What is the underlying economic intuition?

Real Interest Rate and Technology Shocks

$$r_t = \rho - \sigma\psi_{cz}(1-\rho_z)z_t \qquad \text{(Real Interest Rate)}$$

- The natural real rate is inversely related to the level of productivity: $\uparrow z_t \implies \downarrow r_t$
- High current levels of productivity $z_t > 0$ imply negative expected productivity growth $(\mathbb{E}_t z_{t+1} - z_t) < 0$ due to mean reversion
- More on AR(1) Appendix

Classical dichotomy

- A strong form of the 'classical dichotomy' holds
- **All real variables** $c_t, n_t, y_t, w_t - p_t, r_t$ **independent of nominal variables**
- In particular, given exogenous process for productivity z_t we have

$$c_t = \psi_{cz} z_t = y_t \tag{1}$$

$$n_t = \psi_{nz} z_t \tag{2}$$

$$w_t - p_t = z_t \tag{3}$$

$$r_t = \rho + \sigma \psi_{cz} (\mathbb{E}_t z_{t+1} - z_t) \tag{4}$$

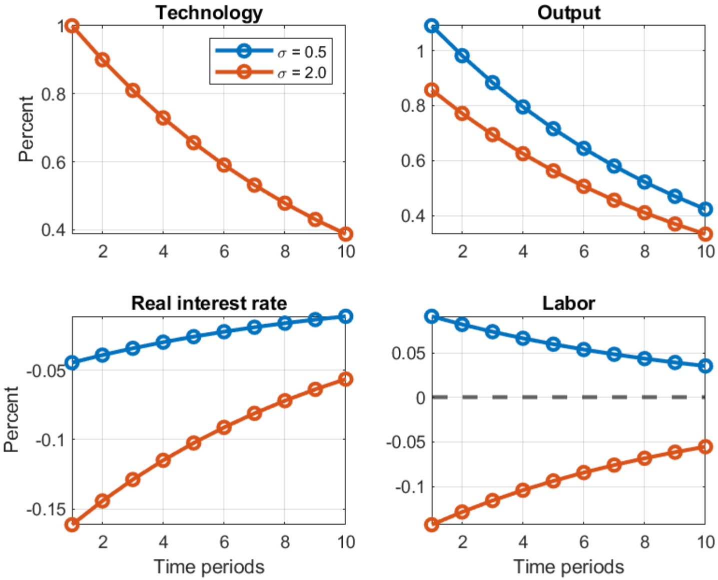
- Nominal variables π_t, i_t, p_t are merely a 'veil'

- **Practice**: Work out the equilibrium solution for the case $F(N) = N^{1-\alpha}$, with $\alpha \in (0,1)$.

Impulse Response Functions

- How do real variables respond to a technology shock?
- At time $t = 0$ the economy is at steady-state, i.e. $z_0 = c_0 = n_0 = y_0 = 0$
- Then, in period $t = 1$ there is a 1% unexpected shock to technology. Technology returns gradually to the steady-state as time goes by, since $|\rho_z| < 1$
- We have $z_1 = 1$, $z_2 = \rho_z$, $z_3 = \rho_z^2$, etc. so that $z_t \to 0$ as $t \to \infty$

Impulse Response Functions



Impulse Response Functions: Intuition

- Top left panel: Positive shock to technology (+1 percent deviation on impact, then slowly go back to steady-state)
- Real wage also increases and follows exactly same path as technology, since $w_t - p_t = z_t$
- If $\sigma < 1$, we know that substitution effect dominates hence household decide to work more: labor increases (see bottom right panel, blue line)
- Output increases as well, for two reasons: because technology increases and because labor increases ($y_t = z_t + n_t$)
- Real interest rate *decreases*: output is higher today but it will be lower in the future (i.e. $(\mathbb{E}_t z_{t+1} - z_t) < 0$ due to mean reversion), so household wants to save more. But in equilibrium the household cannot save since $B_t = 0$ (remember?). To make the household happy with this, the market clearing interest rate must *decrease*.

Nominal Variables

- We have seen that the 5 real variables of the model, $c_t, n_t, y_t, w_t - p_t, r_t$ are independent of nominal variables
- What about **nominal variables**, like inflation π_t or the nominal interest rate i_t?

$$i_t = r_t + \mathbb{E}_t \pi_{t+1} \tag{5}$$

$$\pi_t = p_t - p_{t-1} \tag{6}$$

- Not surprisingly, and in contrast with real variables, their equilibrium behavior cannot be determined uniquely by real forces.
    - Mathematically, r_t is determined in the real block, but here we have three endogenous variables (i_t, π_t, p_t), so we are one equation short
- Instead, it requires the specification of **how monetary policy is conducted**

A Simple Inflation-Based Interest Rate Rule

- Suppose central bank adjusts i_t based on the rule

$$i_t = \rho + \phi_\pi \pi_t \tag{7}$$

- Substituting (7) into (5) gives:

$$\pi_t = \frac{1}{\phi_\pi}\mathbb{E}_t \pi_{t+1} + \frac{1}{\phi_\pi}\widehat{r}_t \tag{8}$$

where $\widehat{r}_t \equiv r_t - \rho$.

- If $\phi_\pi > 1$, we can solve the previous difference equation by iterating forward, which yields

$$\pi_t = \lim_{T\to\infty}\left(\frac{1}{\phi_\pi}\right)^T \mathbb{E}_t \pi_{t+T} + \frac{1}{\phi_\pi}\left(\widehat{r}_t + \frac{1}{\phi_\pi}\mathbb{E}_t \widehat{r}_{t+1} + \left(\frac{1}{\phi_\pi}\right)^2 \mathbb{E}_t \widehat{r}_{t+2} + \ldots\right)$$

- Imposing the condition that inflation does not explode, we finally obtain:

$$\pi_t = \frac{1}{\phi_\pi}\sum_{k=0}^{\infty}\left(\frac{1}{\phi_\pi}\right)^k \mathbb{E}_t\{\widehat{r}_{t+k}\} \tag{9}$$

Algebra

Determination of Inflation

$$\pi_t = \frac{1}{\phi_\pi} \sum_{k=0}^{\infty} \left(\frac{1}{\phi_\pi} \right)^k \mathbb{E}_t \{ \widehat{r}_{t+k} \} \tag{9}$$

- This equation shows that inflation, and thus the price level, is fully determined by the path of the real interest rate, itself pinned down by fundamentals as in (4), reproduced here:

$$\widehat{r} = \sigma \psi_{yz} (\mathbb{E}_t z_{t+1} - z_t) \tag{4}$$

- Assume now that technology follows AR(1) process

$$z_t = \rho_z \, z_{t-1} + \varepsilon_t^z$$

where $\rho_z \in (0, 1)$. Then, (4) implies $\hat{r}_t = -\sigma \, \psi_{yz} (1 - \rho_z) \, z_t$, which combined with (9) yields the following expression for equilibrium inflation

$$\pi_t = - \frac{\sigma \, \psi_{yz} \, (1 - \rho_a)}{\phi_\pi - \rho_a} \, z_t$$

Algebra

Determination of Inflation

- Equilibrium inflation is given by

$$\pi_t = -\frac{\sigma\,\psi_{ya}\,(1-\rho_a)}{\phi_\pi - \rho_a}\,z_t \tag{10}$$

- Higher technology z_t leads to lower inflation
  - Intuition: firms equate price to marginal cost, $P_t = W_t/Z_t$, hence a technological improvement decreases prices
- Central bank following the rule $i_t = \rho + \phi_\pi \pi_t$ can influence inflation **volatility** by choosing the size of ϕ_π: A larger ϕ_π $\Longrightarrow$ smaller impact of the real shock on inflation

Summary

- So far we have analyzed model with **perfect competition** and **flexible prices**
- Classical dichotomy: Real variables are determined by fundamentals alone (like technology or preference shocks) and are independent of monetary policy
- The determination of nominal variables (inflation, prices and nominal interest rate) depends on the conduct of monetary policy
- To guarantee an equilibrium, we need $\phi_\pi > 1$: the central bank must adjust nominal interest rates **more than one-for-one** in response to any change in inflation (Taylor principle).
- Monetary policy does **not** affect households welfare
    - Households care about real consumption C_t and real labor N_t whose paths are determined independently of monetary policy

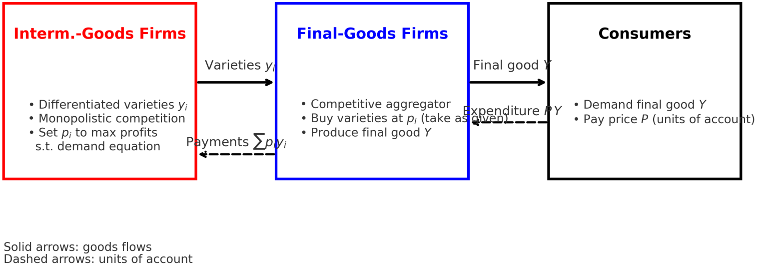
Price setting

- Benchmark model features perfect competition, firms price takers
- For nominal rigidities, need firms that have price-setting power
- Do this with **monopolistic competition**
- Intermediate firms have market power if products are not perfect substitutes
- No genuine strategic interactions

Production Side of the Model

- Consumers are exactly as before BUT now
- Two distinct production sectors:
  - Intermediate-goods sector.
  - Final-goods sector.
- Intermediate-goods sector is imperfectly competitive aka monopolistic competition
  - It consists of many firms that produce differentiated products sold to the final-goods sector.
  - Intermediate-goods producers can set prices
- Final-good firms operate in a competitive environment
  - buy differentiated goods (or "varieties") from intermediate firms taking prices as given
  - Package differentiated goods into a single good and sell it to consumers

Production Structure



- Interm.-goods firms sell varieties y_i to final-good firms
- Final-good firms "package" the many varieties into aggregate good Y, which then sell to consumers

Final-good sector

- Final-good firm:
  - operates in a competitive environment; takes prices $(P_{it})_{i\in[0,1]}$ as given.
  - buys Y_{it} units of each intermediate good $i \in [0,1]$ at price P_{it}, to produce Y_t units of the final good.
- Profit maximization:

$$\max_{Y_{it}} \left\{ P_t Y_t - \int_0^1 P_{it} Y_{it} di \right\}$$

s.t.

$$Y_t = \left[\int_0^1 Y_{it}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \text{ where } \theta > 1.$$

The last (complicated) term says that the final good Y_t is a CES (constant elasticity of substitution) aggregate of a continuum of intermediate goods

Final-good sector

- Final-good firm:
  - operates in a competitive environment; takes prices $(P_{it})_{i\in[0,1]}$ as given.
  - buys Y_{it} units of each intermediate good $i \in [0,1]$ at price P_{it}, to produce Y_t units of the final good.
- Profit maximization:

$$\max_{Y_{it}} \left\{ P_t Y_t - \int_0^1 P_{it} Y_{it} di \right\}$$

s.t.

$$Y_t = \left[\int_0^1 Y_{it}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \text{ where } \theta > 1.$$

The last (complicated) term says that the final good Y_t is a CES (constant elasticity of substitution) aggregate of a continuum of intermediate goods

Aside on CES functions

- The term
$$Y_t = \left[\int_0^1 Y_{it}^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}$$
known as "CES aggregator" looks daunting: there's an integral and complicated exponents!
- Let's take a step back and consider a CES with only two goods:
$$Y_t = \left[y_1^{\frac{\theta-1}{\theta}} + y_2^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{11}$$
- This function has constant returns to scale: if we scale both y_1 and y_2 by a factor t, we scale output by the same factor (verify this!)
- Key observation: If $\theta \to \infty$, then $\frac{\theta-1}{\theta} \to 1$. Production function (11) becomes linear and $y1$, $y2$ become perfect substitutes.

Aside on CES functions

- The term
$$Y_t = \left[\int_0^1 Y_{it}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$$
known as "CES aggregator" looks daunting: there's an integral and complicated exponents!
- Let's take a step back and consider a CES with only two goods:
$$Y_t = \left[y_1^{\frac{\theta-1}{\theta}} + y_2^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{11}$$
- This function has constant returns to scale: if we scale both y_1 and y_2 by a factor t, we scale output by the same factor (verify this!)
- Key observation: If $\theta \to \infty$, then $\frac{\theta-1}{\theta} \to 1$. Production function (11) becomes linear and $y1, \ y2$ become perfect substitutes.

Final-good sector, cont'd

- Now that we are more confident on CES functions, let's go back to the problem of our final-good firm

$$\max_{Y_{it}} \left\{ P_t Y_t - \int_0^1 P_{it} Y_{it} di \right\}$$

s.t.

$$Y_t = \left[\int_0^1 Y_{it}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \text{ where } \theta > 1.$$

- First-order condition w.r.t. $Y_{it} \implies$ Demand function for variety $i \in [0, 1]$:

$$Y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\theta} Y_t \tag{12}$$

Algebra

Final-good sector, cont'd

- Note that θ is the elasticity of substitution b/w intermediate goods. A high substitution reduces the market power of the intermediate firms $\implies$ if they increase the price of the good, demand will fall more
- Therefore, θ is also the elasticity of demand: if the relative price of good i, P_{it}/P_t, increases by 1%, then relative demand Y_{it}/Y_t will *decrease* by θ%
- Zero profit condition implies an expression for price aggregator:

$$P_t = \left[\int_0^1 P_{it}^{1-\theta} di \right]^{\frac{1}{1-\theta}}$$

To see this, substitute demand equation (12) into profit and impose zero-profit condition.

Interm. Goods sector: Monop. Competition

- **Intermediate-goods producers**, indexed by $i \in [0, 1]$, maximize real profits:

$$\frac{R_{it}}{P_t} = \frac{P_{it} Y_{it} - W_t N_{it}}{P_t}$$

subject to:

$$Y_{it} = Z_t N_{it} \quad \text{(Production)}$$

$$Y_{it} = Y_t \left(\frac{P_{it}}{P_t} \right)^{-\theta} \quad \text{(Demand)}$$

- Substituting constraints into real profit:

$$\frac{R_{it}}{P_t} = \left(\frac{P_{it}}{P_t} \right)^{1-\theta} Y_t - \frac{W_t}{P_t} \frac{1}{Z_t} \left(\frac{P_{it}}{P_t} \right)^{-\theta} Y_t$$

Interm. Goods sector: Monop. Competition

- **Intermediate-goods producers**, indexed by $i \in [0,1]$, maximize real profits:

$$\frac{R_{it}}{P_t} = \frac{P_{it} Y_{it} - W_t N_{it}}{P_t}$$

subject to:

$$Y_{it} = Z_t N_{it} \quad \text{(Production)}$$

$$Y_{it} = Y_t \left(\frac{P_{it}}{P_t}\right)^{-\theta} \quad \text{(Demand)}$$

- Substituting constraints into real profit:

$$\frac{R_{it}}{P_t} = \left(\frac{P_{it}}{P_t}\right)^{1-\theta} Y_t - \frac{W_t}{P_t}\frac{1}{Z_t}\left(\frac{P_{it}}{P_t}\right)^{-\theta} Y_t$$

Equilibrium with monopolistic competition

- The first-order condition for P_{it}/P_t delivers

$$(1-\theta)\left(\frac{P_{it}}{P_t}\right)^{-\theta} Y_t + \theta \frac{W_t}{P_t Z_t}\left(\frac{P_{it}}{P_t}\right)^{-\theta-1} Y_t = 0$$

- Divide both sides by Y_t and $(P_{it}/P_t)^{-\theta-1}$ to solve for P_{it}/P_t:

$$\frac{P_{it}}{P_t} = \underbrace{\frac{\theta}{\theta-1}}_{\text{mark-up}} \frac{W_t}{P_t}\frac{1}{Z_t} \qquad \text{(Optimal Pricing)}$$

- Optimal relative price is equal to a constant mark-up over marginal cost
- The size of the mark-up is determined by the elasticity of demand θ

Equilibrium with monopolistic competition

$$\frac{P_{it}}{P_t} = \underbrace{\frac{\theta}{\theta - 1}}_{\text{mark-up}} \frac{W_t}{P_t} \frac{1}{Z_t} \qquad \text{(Optimal Pricing)}$$

- Remember initial assumption $\theta > 1$?
  - It guarantees existence of a profit maximizing price
- The higher is the elasticity of demand θ, the lower is the producers' market power, the lower is the mark-up
- In the limiting case $\theta \to \infty$, intermediate goods are perfect substitutes (remember CES?) and intermediate producers don't have any market power $\implies$ perfect competition with "price = marginal cost"

Equilibrium with monopolistic competition

$$\frac{P_{it}}{P_t} = \frac{\theta}{\theta - 1} \frac{W_t}{P_t} \frac{1}{Z_t} \qquad \text{(Optimal Pricing)}$$

- In symmetric equilibrium, $P_{it} = P_t$ for all $i \in [0, 1]$, hence

$$1 = \frac{\theta}{\theta - 1} \frac{W_t}{P_t} \frac{1}{Z_t}$$

- Real wage

$$w_t \equiv \frac{W_t}{P_t} = \frac{\theta - 1}{\theta} Z_t < Z_t$$

  Note: Labor is **paid less** than its marginal product!

- What if intermediate producers don't have market power (perfect competition)?

Equilibrium with monopolistic competition

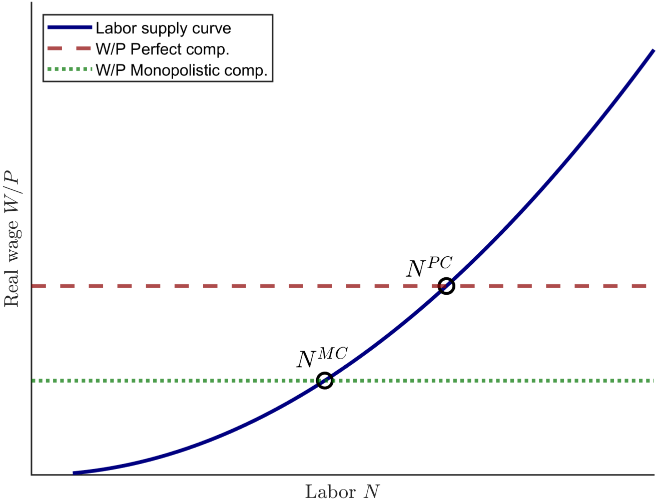
- Equilibrium behavior of 5 endogenous "real" variables $\left\{C_t, N_t, Y_t, \frac{W_t}{P_t}, R_t\right\}$ is pinned down by 5 equations:

$$
\begin{aligned}
C_t^{\sigma} N_t^{\varphi} &= \frac{W_t}{P_t} && \text{(labor supply)} \\
C_t^{-\sigma} &= \beta \mathbb{E}_t \left\{ C_{t+1}^{-\sigma} (1 + r_t) \right\} && \text{(Euler)} \\
\frac{W_t}{P_t} &= \frac{\theta - 1}{\theta} Z_t && \text{(labor demand)} \\
Y_t &= Z_t N_t && \text{(production)} \\
C_t &= Y_t && \text{(resource constraint)}
\end{aligned}
$$

- With monopolistic competition, only equation for labor demand changed

Equilibrium with monopolistic competition

Monopolistic comp.: firms market power $\implies$ mark-down on real wage $\implies$ less labor supply $\implies$ lower output and consumption



Equilibrium with monopolistic competition

- Take logs; Using labor market and goods market clearing

$$\log C_t = \frac{1+\varphi}{\sigma+\varphi}\log Z_t - \frac{1}{\sigma+\varphi}\log\left(\frac{\theta}{\theta-1}\right)$$
$$\log N_t = \frac{1-\sigma}{\sigma+\varphi}\log Z_t - \frac{1}{\sigma+\varphi}\log\left(\frac{\theta}{\theta-1}\right)$$

- Levels of output and employment less than in perfectly competitive benchmark but response to fluctuations in z_t unchanged
- In the new Keynesian model, this flexible price outcome correspond to the underlying trend or 'natural' level of output
- With sticky prices, actual output fluctuates around this natural level, there is an 'output gap'

Appendix

Discount Rate and Discount Factor

- Recall that $\log(1+x) \approx x$ for x small (first-order Taylor approximation).
- The relation between the discount rate ρ and the discount factor β:

$$\beta = \frac{1}{1+\rho}, \quad \rho > 0$$

  so that $\beta \in (0,1)$
- Taking logs,

$$\log \beta = -\log(1+\rho) \approx -\rho$$

- Hence the approximation

$$\rho \approx -\log \beta$$

Brief Review of AR(1) Process

- Imagine we want to model evolution of productivity as an exogenous, stochastic process
- We want the process to be *stationary* and show some *persistence*; common choice is to assume the form

$$y_t = \rho y_{t-1} + \varepsilon_t, \quad |\rho| < 1$$

- Since this holds for all t, we also have that

$$y_{t-1} = \rho y_{t-2} + \varepsilon_{t-1}$$

- Substituting for y_{t-1} in the first equation yields:

$$\begin{aligned} y_t &= \rho(\rho y_{t-2} + \varepsilon_{t-1}) + \varepsilon_t \\ &= \varepsilon_t + \rho\varepsilon_{t-1} + \rho^2 y_{t-2} \end{aligned}$$

Brief Review of AR(1) Process

- Continuing in this way gives

$$y_t = \varepsilon_t + \rho\varepsilon_{t-1} + \rho^2\varepsilon_{t-2} + \rho^3\varepsilon_{t-3} + \cdots$$

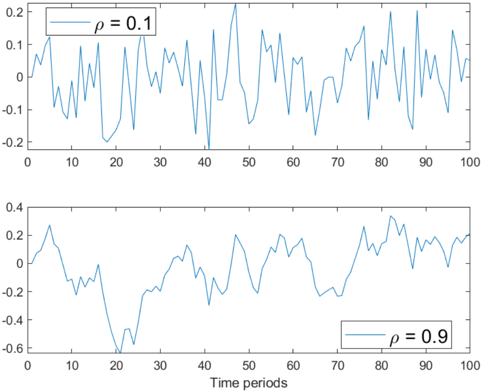
  or equivalently

$$y_t = \sum_{j=0}^{\infty} \rho^j \varepsilon_{t-j}$$

- It is clear that ρ determines the *persistence* of ε_t: the closer is ρ to 1, the stronger is the influence of ε_{t-j} on y_t
- Some examples in the next slides

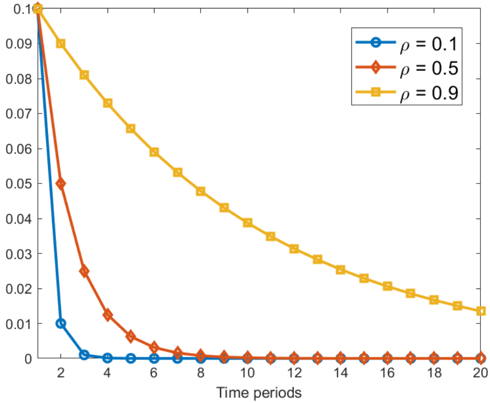
AR(1) Simulated paths

Here we simulate two sample paths for $\{y_t\}_{t=1}^{100}$ assuming low persistence (top panel) and high persistence (bottom panel)



AR(1) Impulse response function

If ρ is close to 0, shock decays very quickly; as ρ gets closer to 1, the effect of the shock lasts longer



Back

Solving Difference Equation by Forward Iteration

We have to solve this first-order difference equation:

$$\pi_t = \left(\frac{1}{\phi_\pi}\right) E_t \pi_{t+1} + \left(\frac{1}{\phi_\pi}\right) \widehat{r}_t$$

We know $\widehat{r}_t$ but how do we pin down the expected inflation term $E_t \pi_{t+1}$?
One way is to use forward iteration as follows.
Shift one-period ahead and take expectations:

$$E_t \pi_{t+1} = \left(\frac{1}{\phi_\pi}\right) E_t \pi_{t+2} + \left(\frac{1}{\phi_\pi}\right) E_t \widehat{r}_{t+1}$$

Forward Iteration, cont'd

Substituting back into the first equation:

$$\begin{aligned}\pi_t &= \left(\frac{1}{\phi_\pi}\right)\left(\left(\frac{1}{\phi_\pi}\right)E_t\pi_{t+2} + \left(\frac{1}{\phi_\pi}\right)E_t\widehat{r}_{t+1}\right) + \left(\frac{1}{\phi_\pi}\right)\widehat{r}_t \\ &= \left(\frac{1}{\phi_\pi}\right)^2 E_t\pi_{t+2} + \left(\frac{1}{\phi_\pi}\right)\left(\widehat{r}_t + \left(\frac{1}{\phi_\pi}\right)E_t\widehat{r}_{t+1}\right)\end{aligned}$$

Continuing in this fashion:

$$\pi_t = \lim_{T\to\infty}\left(\frac{1}{\phi_\pi}\right)^T E_t\pi_{t+T} + \left(\frac{1}{\phi_\pi}\right)\left[\widehat{r}_t + \left(\frac{1}{\phi_\pi}\right)E_t\widehat{r}_{t+1} + \left(\frac{1}{\phi_\pi}\right)^2 E_t\widehat{r}_{t+2} + \dots\right]$$

The assumption $\phi_\pi > 1$ is crucial otherwise the first term does not converge

Back

Derivation of Inflation Equation

Real interest rate

$$\widehat{r}_t = \sigma\psi_{yz}(E_t z_{t+1} - z_t)$$

AR1 for technology:

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z, \quad \rho_z \in (0,1)$$

Expected future technology is $E_t z_{t+1} = \rho_z z_t$ since $E_t \varepsilon_{t+1}^z = 0$. Therefore

$$E_t z_{t+1} - z_t = -(1-\rho_z) z_t$$

Substituting this into the equation for the real interest rate gives:

$$\begin{aligned} \widehat{r}_t &= \sigma\psi_{yz}(E_t z_{t+1} - z_t) \\ &= -\sigma\psi_{yz}(1-\rho_z) z_t \end{aligned}$$

Derivation of Inflation Equation, cont'd

Then

$$
\begin{aligned}
E_t \widehat{r}_{t+k} &= -\sigma\psi_{yz}(1-\rho_z)E_t z_{t+k} \\
&= -\sigma\psi_{yz}(1-\rho_z)\rho_z^k z_t
\end{aligned}
$$

Plugging this expression for $E_t \widehat{r}_{t+k}$ into the equation for inflation delivers

$$
\begin{aligned}
\pi_t &= \frac{1}{\phi_\pi}\sum_{k=0}^{\infty}\left(\frac{1}{\phi_\pi}\right)^k E_t \widehat{r}_{t+k} \\
&= -\frac{1}{\phi_\pi}\sum_{k=0}^{\infty}\left(\frac{\rho_z}{\phi_\pi}\right)^k \sigma\psi_{yz}(1-\rho_z)z_t
\end{aligned}
$$

Derivation of Inflation Equation, cont'd

Provided that $\frac{\rho_z}{\phi_\pi} < 1$, we can apply the formula for geometric series and get

$$\sum_{k=0}^{\infty} \left(\frac{\rho_z}{\phi_\pi} \right)^k = \frac{1}{1 - \frac{\rho_z}{\phi_\pi}}$$

we finally have

$$\begin{aligned} \pi_t &= -\frac{1}{\phi_\pi} \frac{1}{1 - \frac{\rho_z}{\phi_\pi}} \sigma \psi_{yz} (1 - \rho_z) z_t \\ &= -\frac{\sigma \psi_{yz} (1 - \rho_z)}{\phi_\pi - \rho_z} z_t \end{aligned}$$

which is exactly equation (10) in the main text.

Back

Final-goods Firm: Algebra

- The profit maximization problem is:

$$\max_{p_{it}} p_t \left[\int_0^1 y_{it}^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}} - \int_0^1 p_{it} y_{it} di$$

- FOC w.r.t. p_{it}:

$$p_t \left(\frac{\theta}{\theta-1}\right) \left[\int_0^1 y_{it}^{\frac{\theta-1}{\theta}} di\right]^{\frac{1}{1-\theta}} \left(\frac{\theta-1}{\theta}\right) y_{it}^{-\frac{1}{\theta}} = p_{it}$$

for all $i \in [0,1]$ and for all $t \geq 0$.

- Note:

$$\left[\int_0^1 y_{it}^{\frac{\theta-1}{\theta}} di\right]^{\frac{1}{1-\theta}} = y_t^{\frac{1}{\theta}}.$$

- Using the above result and rearranging delivers

$$y_{it} = y_t \left(\frac{p_{it}}{p_t}\right)^{-\theta}.$$

Back