Replication: Rescue Policies for small business during the COVID-19 recession, accepted for publication in the Review of Economic Dynamics

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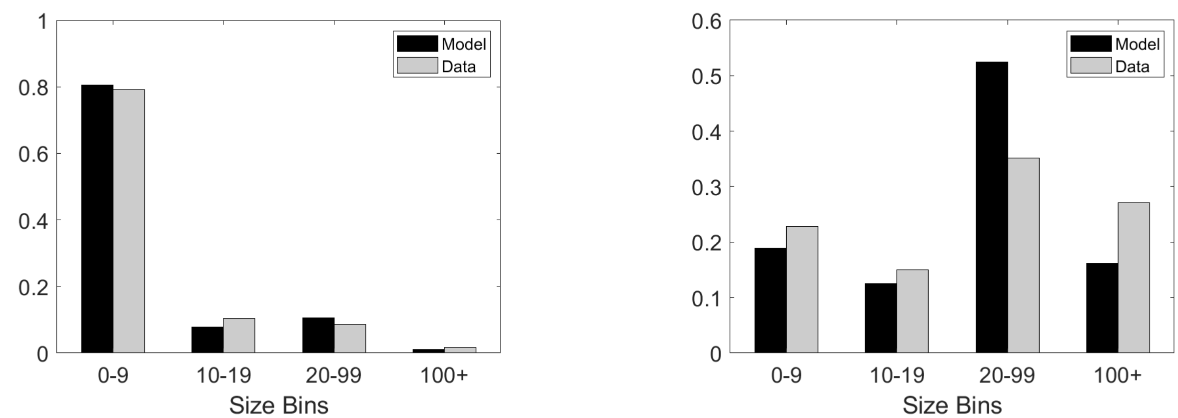
1 Steady-state

1.1 Calibration

Table 1: Computational Parameters

Parameter	Description	Value
nx	Num. of grid points for x	60
nb	Num. of grid points for b	80
nk	Num. of grid points for κ	100
T	Length of transition	180

Figure 1: Firm size and employment distributions: Model vs Data



(a) Firm size distribution

(b) Employment distribution

Table 2: External Parameters

Parameter	Description	Value
β	Subjective discount factor	0.989
σ	CRRA coefficient	2.000
α	Capital Share corporate sector	0.300
δ_k	Capital depreciation rate	0.015
λ_0	Collateral constraint parameter	1.000
γ_1	Capital Share small firms	0.318
γ_2	Span of control	0.880
A	TFP shifter	0.250
$bk0(1)$	Initial debt-asset ratio	-0.094
$bk0(2)$	Initial debt-asset ratio	0.125
$bk0(3)$	Initial debt-asset ratio	0.868
$Pr(bk0(1))$	Prob. dist.	0.250
$Pr(bk0(2))$	Prob. dist.	0.500
$Pr(bk0(3))$	Prob. dist.	0.250

Table 3: Estimated Parameter Values

Parameter	Description	Value
M	mass of potential entrants	0.045
$fixcost1$	Intercept fixed cost	0.165
$fixcost2$	Slope fixed cost	0.005
θ	Resale value of capital	0.909
ψ	Exogenous exit rate	0.004
α_κ	Shape Pareto capital	0.446
$\bar{x}$	Ln(x0) mean of AR(1)	1.074
ε_x	Dispersion of x	0.098
ρ_x	Persistence of x	0.957
ζ	marginal utility of leisure	23.420

Table 4: Steady state results

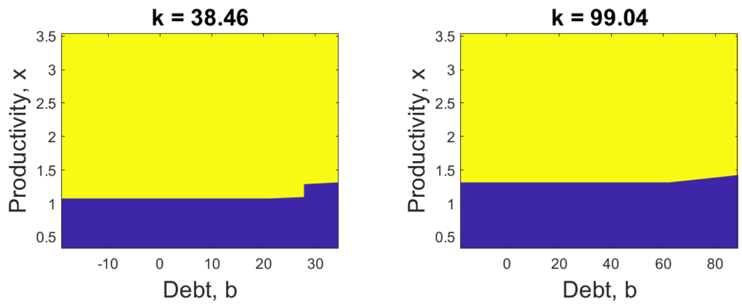
Measure of active firms:	0.0171
Measure of entrants:	0.0004
Aggregate consumption:	0.1084
Capital (corporate):	0.7526
Employment (corporate):	0.1668
Aggregate Capital:	1.2399
Employment (total):	0.3175
Output (corporate):	0.0655
Output (small firms):	0.0639
Total output:	0.1295
liq:	0.0019
entry:	0.0004
Aggregate Investment:	0.0211
btilde(1):	-1062.0793
btilde(nk):	-871.6766

Table 5: Model Fit

Moment	Data	Model
Average employment in small firms	9.2516	8.8026
Average employment, age 0	5.2935	5.3071
Small firm share of employment	0.4895	0.4746
Small firm exit rate	0.0198	0.0210
Fixed expense to revenue ratio	0.2448	0.2286
Autocorr. employment	0.9667	0.9527
Time spent in market work	0.3300	0.3175
Exit rate, emp. size 0 to 9	0.0246	0.0251
serial corr. investment rate	0.0580	0.0591
Share of firms with debt	0.3288	0.2190
Freq. positive lumpy investment	0.1860	0.2123
Share forced exit	1.0000	0.5362
Share voluntary exit	1.0000	0.3147
firmshare0to9	0.7921	0.8052
firmshare10to19	0.1039	0.0779
firmshare20to99	0.0869	0.1061
firmshare100to499	0.0171	0.0108
empshare0to9	0.2286	0.1891
empshare10to19	0.1499	0.1247
empshare20to99	0.3510	0.5244
empshare100to499	0.2705	0.1618

1.2 Steady state policy functions

Figure 2: Entry policy



Notes: Yellow = entry, blue = no entry.

Figure 3: Investment type. Dark blue: exit, light blue: negative investment (no exit), Green: small positive investment (investment rate less than 100%), yellow: large positive investment (investment rate greater than 100%).

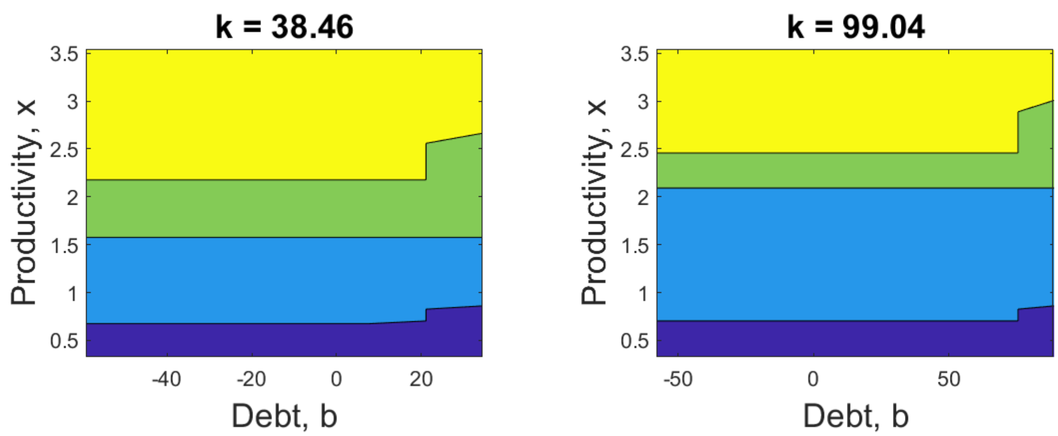
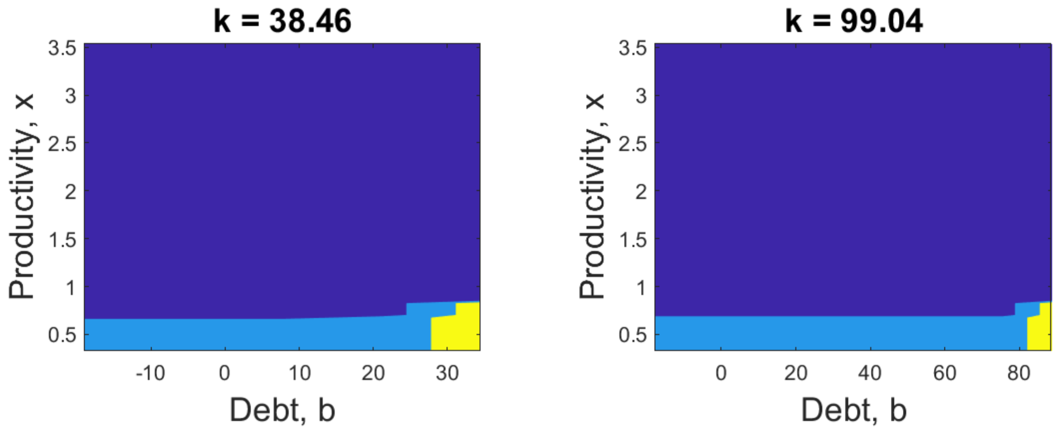


Figure 4: Exit type. Dark blue: no exit, yellow: forced exit, light blue: voluntary exit.



2 Transition

2.1 Calibration of transition

We assume that the pandemic impacts a fraction η_i of small firms. Those impacted see their productivity x goes to zero; the unimpacted firms' productivity is unaffected by the pandemic.

Table 6: Calibrated Pandemic Shock Parameters

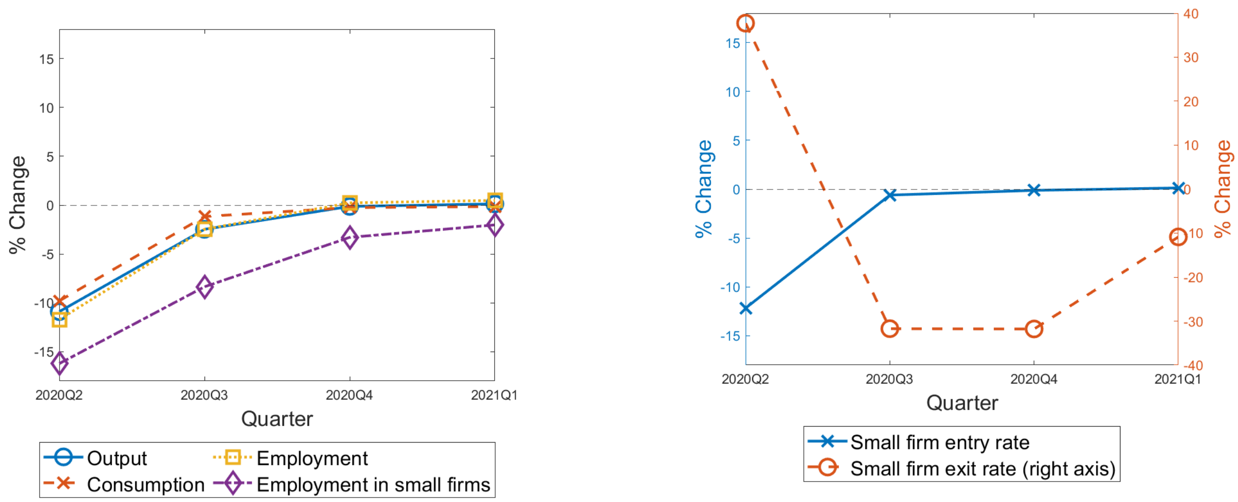
Parameter	Description	Value
η_i	Fraction of impacted small firms	0.1200
ν^n	Productivity shock on impacted firms	-1.0000
ν^λ	Credit shock on small firms	-0.1580
ν^M	Shock to the mass of potential entrants	0.2400
ν^c	Productivity shock the corporate sector	-0.0070
ν^d	Preference shock	-0.1843
ν^l	Labor supply shock	0.2500
ρ	Autocorrelation	0.1100

Table 7: Pandemic Impact and Rescue Policies

Description	Data	Model
Output, 2020Q2	-10.8570	-10.8976
Output, 2020Q3	-2.2460	-2.4738
Consumption, 2020Q2	-9.6670	-9.8018
Total employment, 2020Q2	-12.8500	-11.7066
Employment small, 2020Q2	-16.0213	-16.2132
Exit rate, 2020Q2	37.8400	37.7459
Entry rate, 2020Q2	-12.5000	-12.1711
Private investment, 2020Q2	-15.3980	-16.5231
Small firm output, 2020Q2	-15.6500	-15.8452

Notes: The pandemic shocks are calibrated so that the "Grant baseline" economy matches the data. *Data sources:* GDP, consumption, investment and aggregate employment come from FRED. Small firms output comes from Bloom et al. (2021), Figure A7. Employment by firm size comes from ADP, see Cajner et al. (2020). Exit and entry rate comes from BED.

Figure 5: Calibration of the pandemic shock

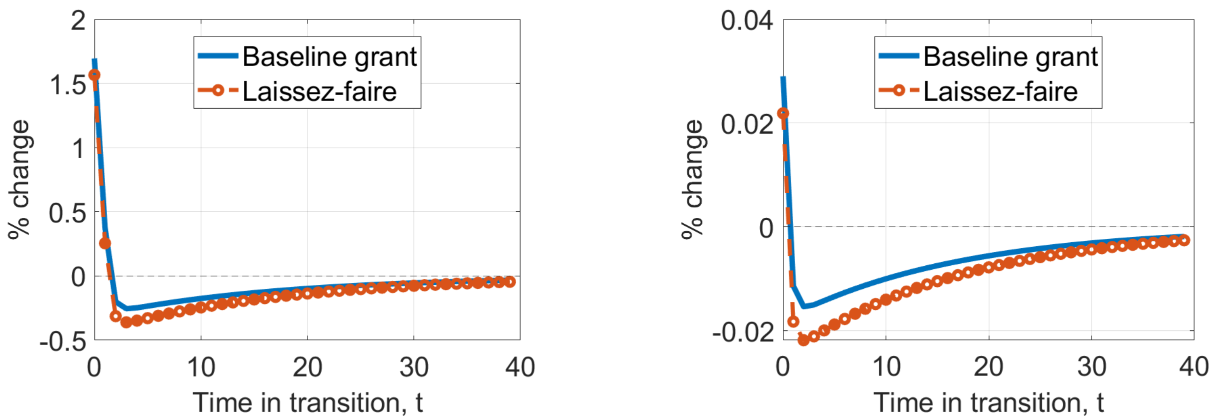


(a) Calibrated shocks

(b) Transition of targeted aggregate variables in model

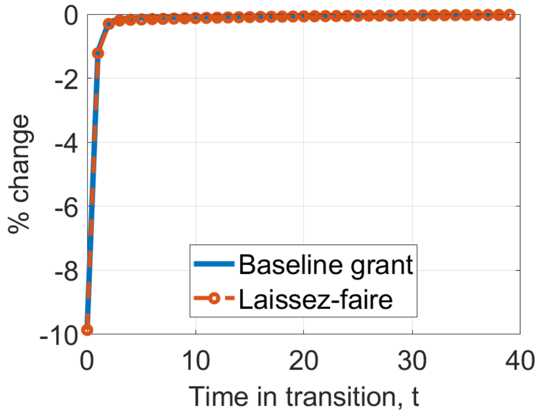
2.2 Results

Figure 6: Responses to a pandemic shock: IRFs in each policy scenario

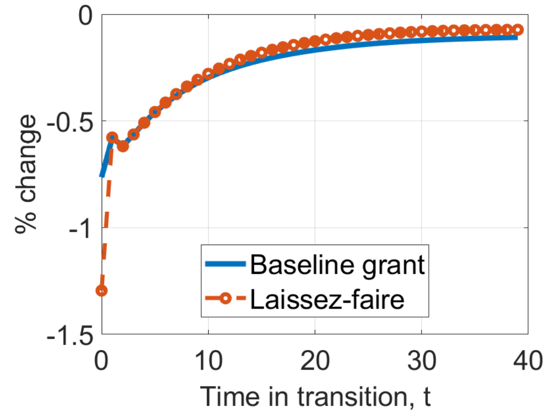


(a) Wage, w_t **(b)** Financial discount factor, q_t

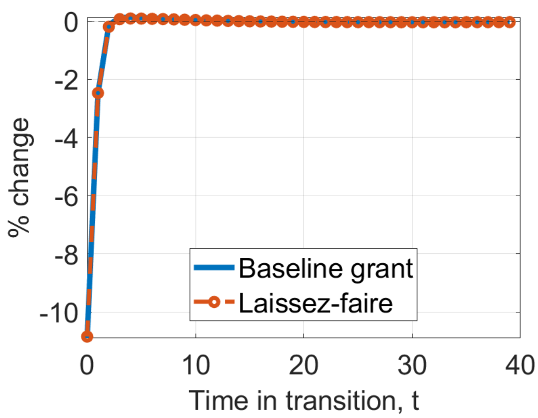
Figure 7: Responses to a pandemic shock: IRFs in each policy scenario, cont'd



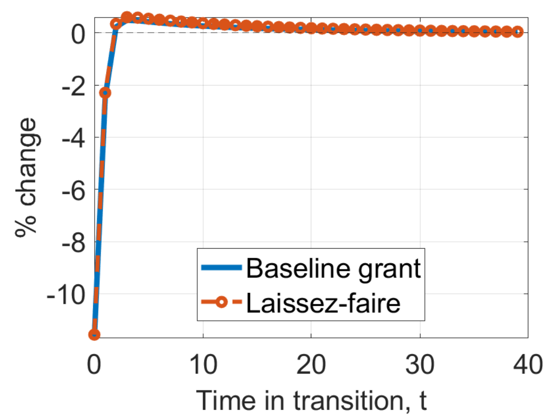
(a) Consumption



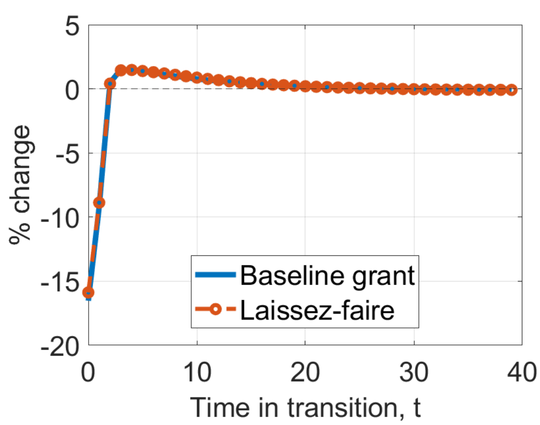
(b) Total capital, $\tilde{K}_t$



(c) Total output

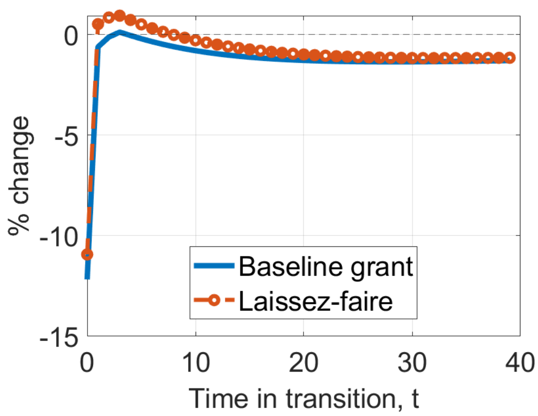


(d) Total employment

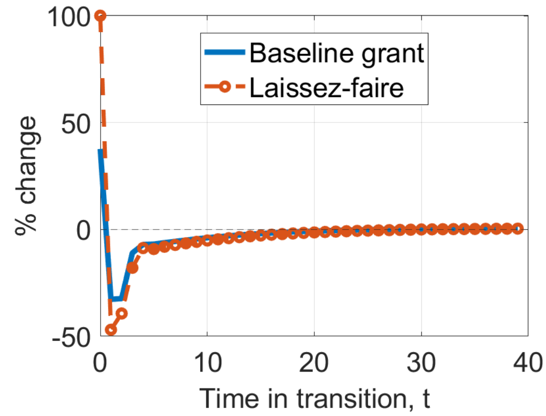


(e) Total Investment

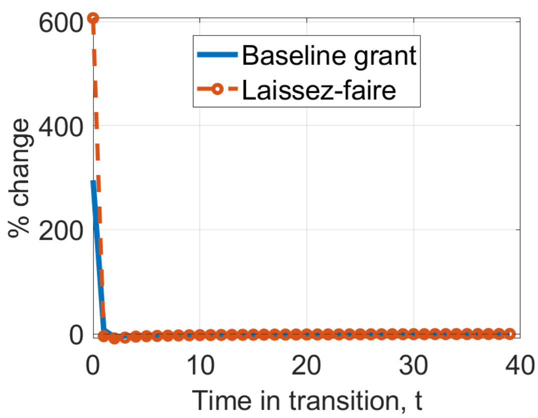
Figure 8: Responses to a pandemic shock: IRFs in each policy scenario, cont'd



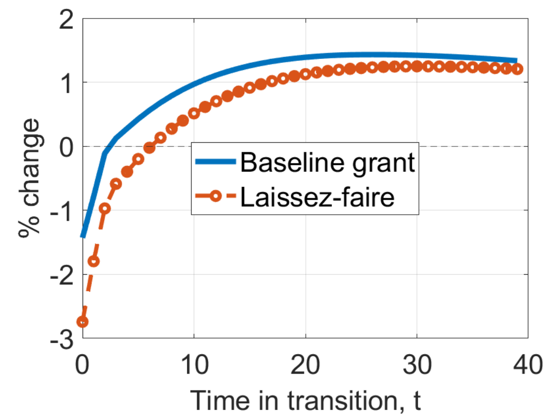
(a) Entry rate



(b) Exit (measure of exiting firms)

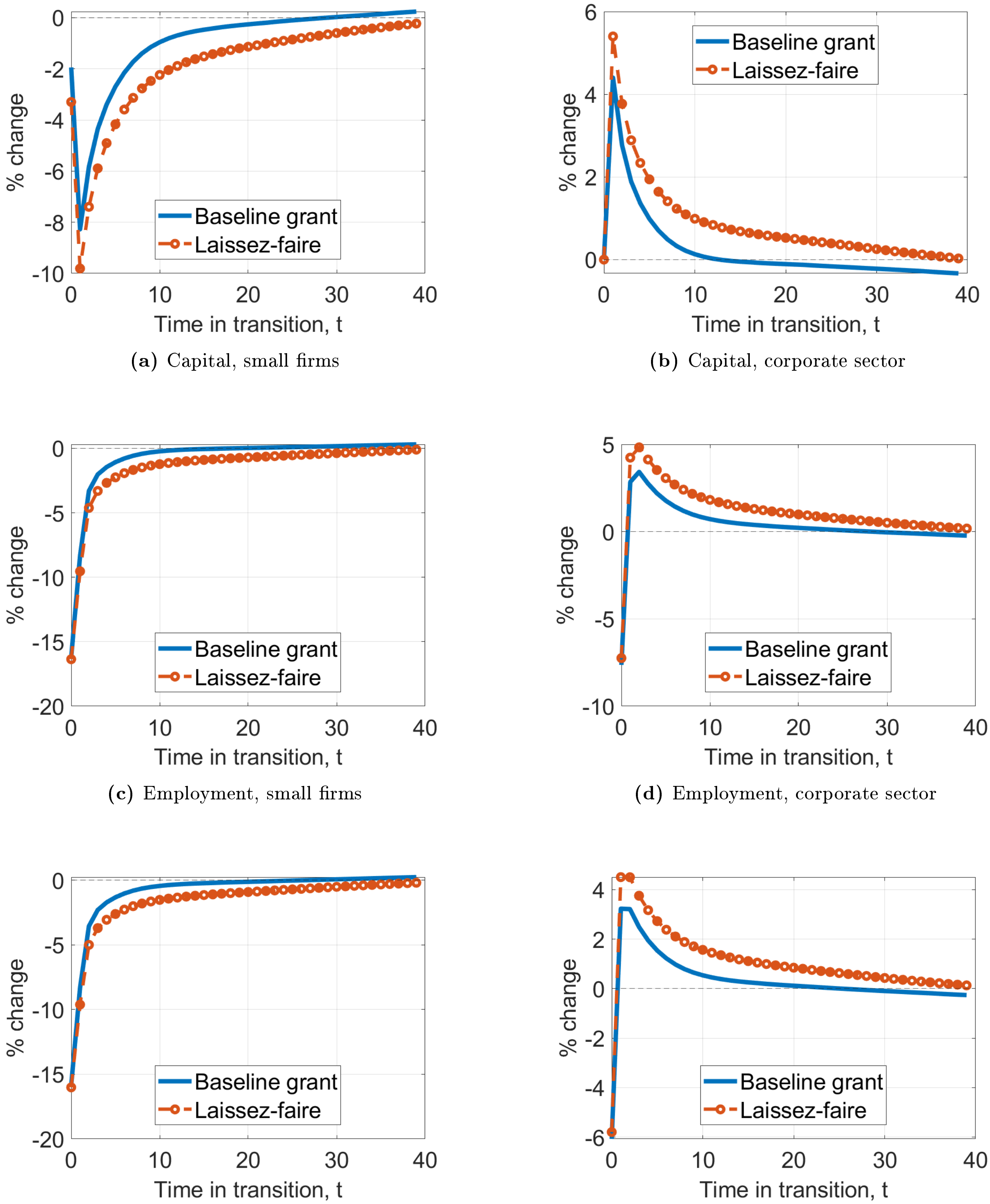


(c) Liquidation value of small firms



(d) Measure of small firms

Figure 9: Responses to a pandemic shock: IRFs in each policy scenario, cont'd



(a) Capital, small firms

(b) Capital, corporate sector

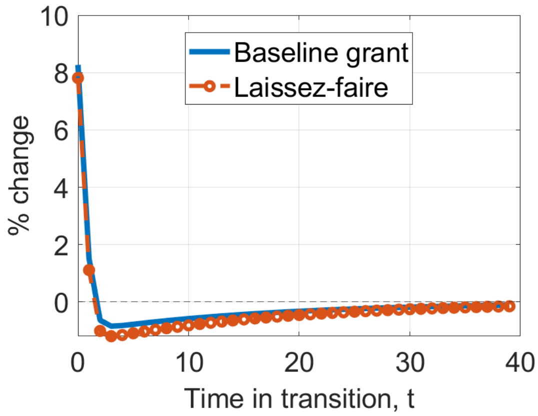
(c) Employment, small firms

(d) Employment, corporate sector

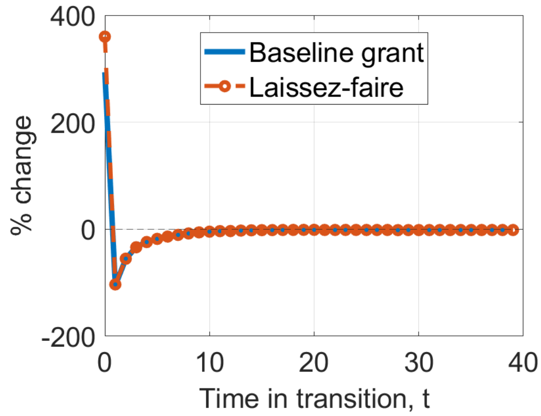
(e) Output, small firms

(f) Output, corporate sector

Figure 10: Responses to a pandemic shock: IRFs in each policy scenario, cont'd

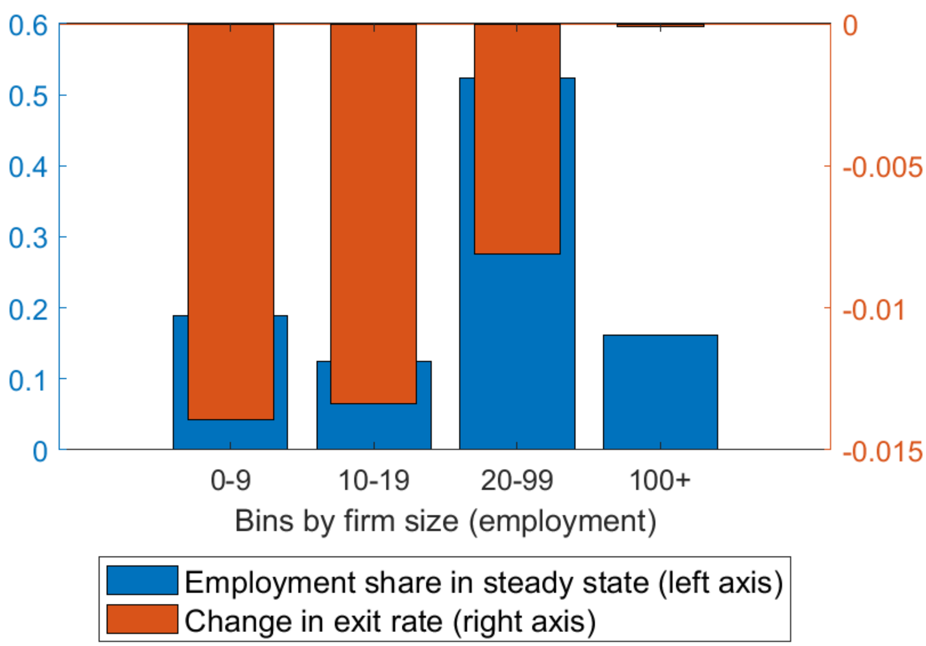


(a) Capital-to-Labor ratio, corporate, sector



(b) Investment, corporate sector

Figure 11: Employment share by deciles of firm size and change in exit rate due to grant



Notes: Average firm size in the first bin is ?

2.3 Policy effects on small firm investments

In addition to saving small firms from a total liquidation, the rescue grant also plays a role in firms' investment decisions. The pandemic shocks may force small firms to liquidate part of their capital in order to pay a non-negative dividend. Due to the partial irreversibility of capital, such liquidation may be inefficient. The rescue policy can prevent firms from liquidating their capital. For firms that are unimpacted and not forced to liquidate their capital, the rescue grant may lead to an increase in capital investment. We quantify the impact of the rescue policy on investment by compute the amount of inefficient capital liquidation prevented and the amount of additional investment boosted by the policy.

There are in total four types of capital adjustments: upward and downward adjustments by active firms, capital buying by entrants, and capital selling by exiters. We compute the aggregate value of adjustments.

- Upward capital adjustment by active firms:

$$\mathcal{A}_u = \int k'_u(x,b,k)d\mu(x,b,k)$$

  where $k'_u(x,b,k) \equiv \max\{k'(x,b,k) - (1-\delta)k, 0\}$.

- Downward capital adjustment by active firms:

$$\mathcal{A}_d = \int k'_d(x,b,k)d\mu(x,b,k)$$

  where $k'_d(x,b,k) \equiv \min\{k'(x,b,k) - (1-\delta)k, 0\}$.

- Capital bought by entrants:

$$\mathcal{A}_{ue} = M \int k d^e(x,b,k)d\Phi(x,b,k).$$

- Capital sold by exiters:

$$\mathcal{A}_{de} = -\int k(\psi + (1-\psi)d^l(x,b,k))d\mu^0(x,b,k).$$

We can also compute the counterparts of the above for each period on the transition path and compare them to the steady state. For each t on the transition path, we can compute the four types of capital adjustment as follows:

- Upward capital adjustment by active firms:

$$\mathcal{A}_{u,t+1} = \int k'_{u,t}(x,b,k)d\mu_t(x,b,k)$$

  where $k'_{u,t}(x,b,k) \equiv \max\{k'_t(x,b,k) - (1-\delta)k, 0\}$. Note that for $K'_{u,t=0} = K'_u$, the steady state value. This is because capital of continuing firms in $t = 0$ is determined by the investment decisions made in the previous period, before the pandemic shock hits the economy.

- Downward capital adjustment by active firms:

$$\mathcal{A}_{d,t+1} = \int k'_{d,t}(x,b,k)d\mu_t(x,b,k)$$

where $k'_{d,t}(x,b,k) \equiv \min\{k'_t(x,b,k) - (1-\delta)k, 0\}$. Note that for $K'_{d,t=0} = K'_d$, the steady state value.

- Capital bought by entrants:

$$\mathcal{A}_{ue,t} = M_t \int k d^e_t(x,b,k) d\Phi(x,b,k).$$

- Capital sold by exiters:

$$\mathcal{A}_{de,t} = -\int k(\psi + (1-\psi)d^l_t(x,b,k))d\mu^0_t(x,b,k).$$

In Table 8, we compute steady state capital adjustment rates, which are defined as capital adjustments divided by the steady state capital in the small firm sector.

Figure **??** shows the paths of capital adjustments. The capital adjustments in these graphs are shown as a percentage of the steady state capital in the small firm sector.

Next, we decompose cumulative changes in small firm capital in the pandemic into the four types of capital adjustments. We consider the following time spans: (a) Short run: change from the pre-pandemic time to the end of Q2; (b) medium run: Q3-Q8; (c) long run: Q9-Q40; (d) Q1-Q40. We show the results in Figure 12. For example, consider the upward capital adjustment by small firms. We compute the cumulative excess upward adjustment in each time span as follows:

- Short run:

$$\Delta\mathcal{A}_{u,sr} = \sum_{t=1}^{T_{sr}} (\mathcal{A}_{u,t} - \mathcal{A}_u)$$

- Medium run:

$$\Delta\mathcal{A}_{u,mr} = \sum_{t=T_{sr}+1}^{T_{mr}} (\mathcal{A}_{u,t} - \mathcal{A}_u)$$

- Long run:

$$\Delta\mathcal{A}_{u,lr} = \sum_{t=T_{mr}+1}^{T_{lr}} (\mathcal{A}_{u,t} - \mathcal{A}_u)$$

- Overall:

$$\Delta\mathcal{A}_{u,all} = \sum_{t=1}^{T_{lr}} (\mathcal{A}_{u,t} - \mathcal{A}_u)$$

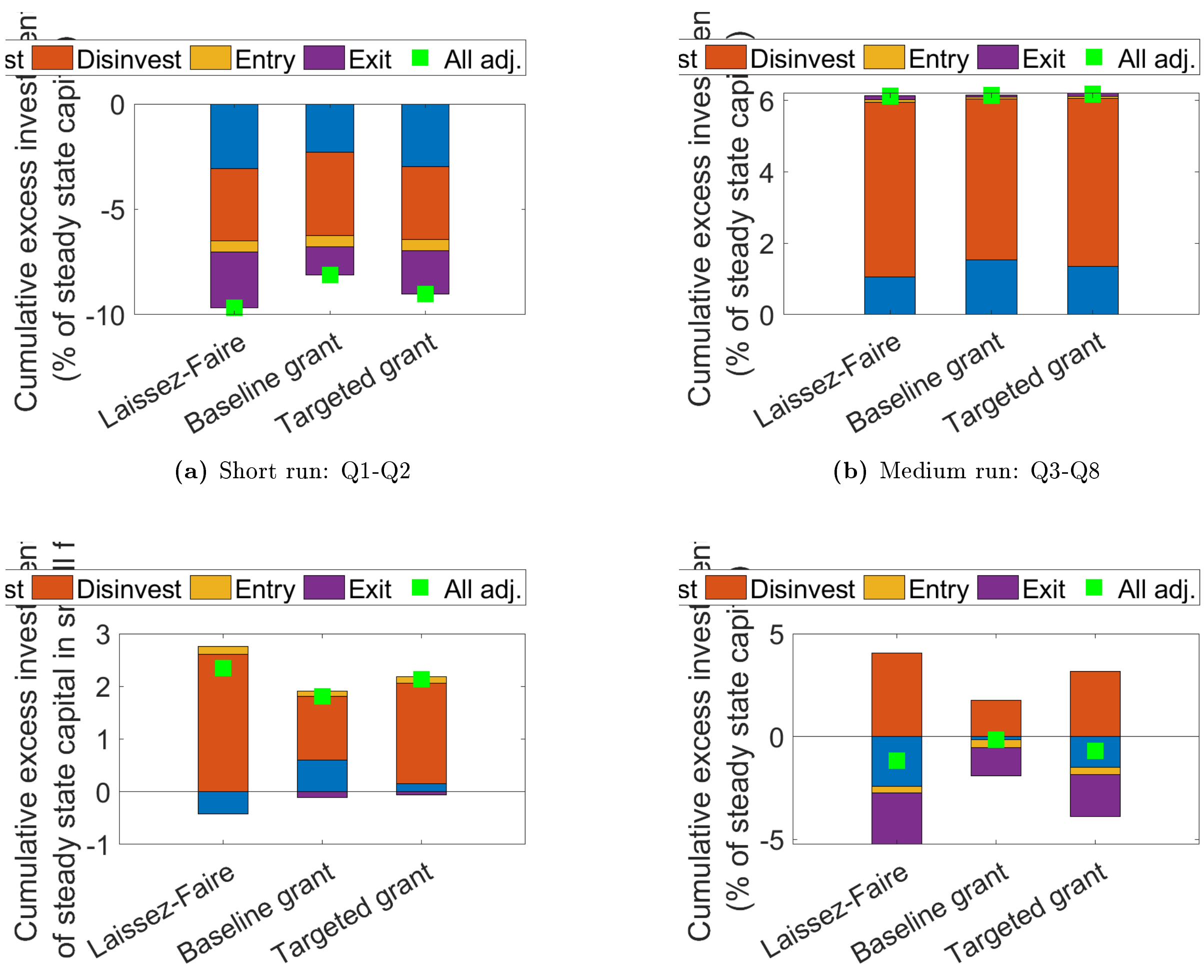
In Figure 12, we express the cumulative excess capital adjustments as a percentage of the steady state capital in small firms. For example, we plot $\frac{\Delta\mathcal{A}_{u,sr}}{K_{small}}$ in panel A. In the figure, we also plot the sum of all four types of excess capital adjustments. For example, the sum in the short run is equal to $\Delta\mathcal{A}_{u,sr} + \Delta\mathcal{A}_{d,sr} + \Delta\mathcal{A}_{ue,sr} + \Delta\mathcal{A}_{de,sr}$.

Table 8: Four types of capital adjustment rates in the steady state

Up. adj. by active firms	0.056
Down. adj. by active firms	-0.052
Capital bought by entrants	0.015
Capital sold by exiters	-0.004
Overall small-firm investment rate	0.015

Notes: Capital adjustment rates and the overall investment rate are computed relative to steady state small firm capital.

Figure 12: Cumulative excess investment by types of capital adjustment



(a) Short run: Q1-Q2

(b) Medium run: Q3-Q8

(c) Long run: Q9-Q40

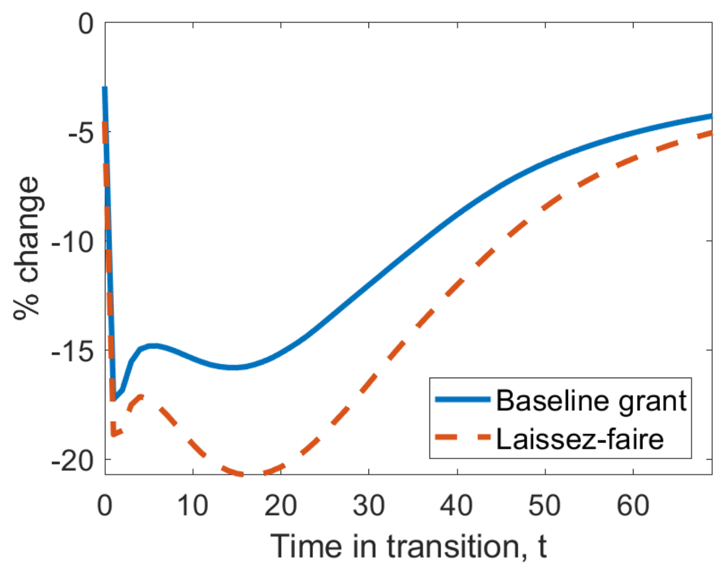
(d) Q1-Q40

Notes: Invest. = upward capital adjustment by active firms; Disinvest. = downward capital adjustment by active firms; Entry = capital bought by entrants; Exit = capital sold by exiters.

2.4 Firm indebtedness

On the transition path, the firm exit rate drops below the steady state level after the initial impact in both the baseline economy and the laissez-faire economy. To understand why the exit rate drops, it is helpful to look at changes in firm indebtedness on the transition path. Figure 13 shows that firms are on average less indebted in both policy scenarios. Partly this is because of the credit shock, which tightens the debt limit of firms. There are also reasons specific to each policy scenario that lead to a decrease in firm indebtedness. In the baseline economy, firms are less indebted because of the grant. In the laissez-faire economy, many highly indebted firms exit on impact and continuing firms make less capital investment (see Figure **??** (a)).

Figure 13: Firm indebtedness



2.5 Zombie firms

Table 9: Zombie firm analysis. Impact period only.

Baseline grant	
Fraction of saved firms	0.4713
Fraction of zombie firms	0.0135
Targeted grant	
Fraction of saved firms	0.6850
Fraction of zombie firms	0.0036
Targeted grant (large)	
Fraction of saved firms	0.8979
Fraction of zombie firms	0.0180

Notes: Fraction of saved firms is the measure of saved firms in period $t = 0$ divided by the measure of firms that exit under the laissez-faire environment, all conditional on receiving the grant. Fraction of zombie firms is the measure of zombie firms divided by the measure of saved firms.

2.6 Welfare analysis

Inputs: Let C_{ss} and L_{ss} denote consumption and employment in the steady-state. Let $\{C_t, L_t\}_{t=1}^{T+1}$ denote consumption and labor along the transition. We assume that the economy has reached the steady-state for $t \geq T + 2$.

The utility function is

$$U(C, L) = \frac{C^{1-\sigma}}{1-\sigma} + \zeta(1 - L).$$

During the transition it is

$$U(C_t, L_t; D_t, \zeta_t) = \frac{D_t C_t^{1-\sigma}}{1-\sigma} + \zeta_t \zeta(1 - L_t)$$

The value in the steady-state is

$$V_{ss} = \frac{1}{1-\beta} U(C_{ss}, L_{ss}) = \frac{1}{1-\beta} \left[\frac{C_{ss}^{1-\sigma}}{1-\sigma} + \zeta(1 - L_{ss}) \right] = V_{ss}^C + V_{ss}^L$$

where $V_{ss}^C = \frac{1}{1-\beta} \frac{C_{ss}^{1-\sigma}}{1-\sigma}$ and $V_{ss}^L = \frac{1}{1-\beta} \zeta(1 - L_{ss})$.

The value in the first period of the transition is

$$\begin{aligned} V_{t=1} =& U(C_1, L_1; D_1, \zeta_1) + \beta U(C_2, L_2; D_2, \zeta_2) + \ldots + \beta^T U(C_{T+1}, L_{T+1}; D_{T+1}, \zeta_{T+1}) \\ &+ \sum_{j=T+2}^{\infty} \beta^{j-1} U(C_j, L_j; D_j, \zeta_j) \end{aligned}$$

where $C_j = C_{ss}$, $L_j = L_{ss}$, $D_j = 1$ and $\zeta_j = 1$ for all $j \geq T + 2$. Therefore the equation above can be simplified to

$$V_{t=1} = U(C_1, L_1; D_1, \zeta_1) + \beta U(C_2, L_2; D_2, \zeta_2) + \ldots + \beta^T U(C_{T+1}, L_{T+1}; D_{T+1}, \zeta_{T+1}) + \frac{\beta^{T+1}}{1-\beta} U(C_{ss}, L_{ss})$$

The CEV (welfare gain or loss) comparing the laissez-faire to the scenarios (i) baseline grant, (ii) targeted grant, etc. is defined as

$$CEV = \left(\frac{V_{t=1} - V_{t=1}^{LF}}{V^C} + 1 \right)^{\frac{1}{1-\sigma}} - 1.$$

where $V_{t=1}^{LF}$ is the value under the laissez-faire economy at $t = 1$, and V^C is the value derived from consumption under the laissez-faire economy at $t = 1, 2, 3, 4$, i.e.

$$V^C = \sum_{t=1}^{4} \beta^{t-1} \frac{(C_t^{LF})^{1-\sigma}}{1 - \sigma}.$$

Table 10: Consumption equivalent variation of grant policies, relative to laissez-faire.

Baseline grant	0.007897
Targeted grant	0.004422

2.7 Cost per job saved in small firms

Table 11: Cost per job saved in small firms

	Baseline grant	Targeted grant
Cost (Frac. GDP)	0.0499	0.0078
Emp. save (Frac. Emp)	0.0076	0.0039
Cost per perc. jobs saved	0.0654	0.0202

Notes: The fiscal cost is computed as a fraction of GDP in the steady state. Employment saved is computed as the per-quarter difference in small-firm employment relative to the laissez-faire economy over a 10-year period from the onset of the pandemic.

Table 12: Cost per job saved in small firms (Robustness)

	Baseline grant	Targeted grant	Targeted grant (large)	Targeted grant (small)
Cost (Frac. GDP)	0.0499	0.0078	0.0499	0.0039
Emp. save (Frac. Emp)	0.0076	0.0039	0.0064	0.0032
Cost per perc. jobs saved	0.0654	0.0202	0.0775	0.0123