Problem Set 2

Due on Tuesday, May 7, h 13:00. Mail box F230.

Hansen's RBC model, continued (30 points)

Consider the basic RBC model analysed in Problem Set 1. Assume the following values for the parameters:

Parameter	Value	Description
β	0.99	discount factor
γ	0.0045	disutility of hours worked
η	1.0039	gross rate of labor augmenting techn. progress
θ	0.2342	capital share in prod. function
A	6.0952	constant term in AR(1) for TFP shock
δ	0.025	depreciation rate (capital)
ρ	0.9983	persistence parameter in AR(1) for TFP shock
σ^2	0.00025	error variance in AR(1) for TFP shock.

1. Given the parameter values listed above, compute numerically the steady state of the model, i.e. provide values for the 6 variables $\overline{a}, \overline{k}, \overline{i}, \overline{c}, \overline{h}, \overline{y}$. As usual, a bar over a variable denotes its value in the steady-state.

2. Categorize the model endogenous variables in either *(i)* state variables, denoted as s_t, *(ii)* control or forward-looking variable, denoted as u_t and *(iii)* static or redundant variables, denoted as f_t. The vectors s_t, u_t and f_t have dimensions $n_s \times 1$, $n_c \times 1$ and $n_f \times 1$, respectively, with $n_s + n_c + n_f = 6$.

3. The log-linearized model equations (derived in Problem Set 1 and restated here) are:

$$\widetilde{y}_t = \widetilde{a}_t + \theta \widetilde{k}_t + (1-\theta)\widetilde{h}_t, \tag{1}$$

$$\widetilde{a}_t = \rho \widetilde{a}_{t-1} + \varepsilon_t, \tag{2}$$

$$\kappa \widetilde{y}_t = (\kappa - \theta\lambda)\widetilde{c}_t + \theta\lambda \widetilde{i}_t, \tag{3}$$

$$\eta \widetilde{k}_{t+1} = (1-\delta)\widetilde{k}_t + \lambda \widetilde{i}_t, \tag{4}$$

$$\widetilde{c}_t + \widetilde{h}_t = \widetilde{y}_t, \tag{5}$$

$$(\eta/\beta)\widetilde{c}_t - (\eta/\beta) E_t \widetilde{c}_{t+1} + \kappa E_t \widetilde{y}_{t+1} - \kappa \widetilde{k}_{t+1} = 0, \tag{6}$$

for all $t = 0, 1, 2, \ldots$ To ease the notation, I have defined the additional parameters $\kappa \equiv \eta/\beta - 1 + \delta$ and $\lambda \equiv \eta - 1 + \delta$. As usual, a tilde over variables denotes log deviation around the steady state: $\widetilde{x}_t \equiv \log x_t - \log \overline{x}$, for $x_t \in \{k_t, a_t, c_t, y_t, i_t, h_t\}$.

4. Rewrite the equations above in matrix form as follows: $AE_t x_{t+1} = Bx_t$, where $x_t = \begin{bmatrix} k_t & a_t & c_t & y_t & i_t & h_t \end{bmatrix}'$.

5. (Try to) solve the model using the Blanchard-Kahn method. *Hint*: call the function `fun_blanchard_kahn1` as explained in my lecture notes. You will get an error message. What is the problem here?

6. Define the vectors

$$x_t^0 = \begin{bmatrix} k_t & a_t & c_t \end{bmatrix}'$$

and

$$f_t = \begin{bmatrix} y_t & i_t & h_t \end{bmatrix}'.$$

Vector x_t^0 contains the model dynamic variables (both predetermined and non-predetermined), whereas vector f_t contains the model static/redundant variables. Likewise, equations (1), (3) and (5) are **intra-temporal** relationships, i.e. contain only variables dated at t, and therefore can be written as:

$$Af_t = Bx_t^0. \tag{7}$$

Equations (2), (4) and (6) describe instead **inter-temporal** relationships, i.e. contain variables with leads and/or lags, and can be written as:

$$DE_t x_{t+1}^0 + FE_t f_{t+1} = Gx_t^0 + Hf_t. \tag{8}$$

Your job is to write a matlab file where the matrices A, B, D, F, G and H are defined, as functions of model parameters and (possibly) steady-state values.

7. Equations (7) are known as the *static block* of the model, whereas equations (8) are known as the *dynamic block* of the model. Rewrite (7) as

$$f_t = A^{-1}Bx_t^0.$$

Substitute this result into (8), thus obtaining:

$$DE_t x_{t+1}^0 + FA^{-1}BE_t x_{t+1}^0 = Gx_t^0 + HA^{-1}Bx_t^0$$

$$\underbrace{\left(D + FA^{-1}B\right)}_{\widetilde{A}} E_t x_{t+1}^0 = \underbrace{\left(G + HA^{-1}B\right)}_{\widetilde{B}} x_t^0$$

or more simply

$$\widetilde{A}E_t x_{t+1}^0 = \widetilde{B}x_t^0$$

Compute these matrices with Matlab.

8. Solve the model using the Blanchard-Kahn method. *Hint*: call the function `fun_blanchard_kahn1` as explained in my lecture notes.

9. Report the solution of the model in state space form i.e.

$$\begin{bmatrix} \widetilde{k}_{t+1} \\ \widetilde{a}_{t+1} \end{bmatrix} = \begin{bmatrix} ?? & ?? \\ 0 & \rho \end{bmatrix} \begin{bmatrix} \widetilde{k}_t \\ \widetilde{a}_t \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \varepsilon_{t+1},$$

$$\begin{bmatrix} \widetilde{c}_t \\ \widetilde{y}_t \\ \widetilde{i}_t \\ \widetilde{h}_t \end{bmatrix} = \begin{bmatrix} ?? & ?? \\ ?? & ?? \\ ?? & ?? \\ ?? & ?? \end{bmatrix} \begin{bmatrix} \widetilde{k}_t \\ \widetilde{a}_t \end{bmatrix}.$$

10. (Optional) Generate impulse responses of all variables w.r.t. a technology shock $\varepsilon_1 = 0.01$, $\varepsilon_t = 0$, $\forall t > 1$.

Remarks. Most of the questions require you to write a simple matlab file. Please print out your code and submit it together with your analytical results. Do **not** send your m file by e-mail!