Problem Set 4

Due on Tuesday, June 4, h 13:00. Mail box F230. The aim of this Problem Set is to familiariaze yourself with some nonlinear solution/simulation methods.

Simulation and Pruning (10 points)

Suppose the dynamics of your system is given by:

$$y_t = \rho y_{t-1} + \alpha y_{t-1}^2 + \varepsilon_t \tag{1}$$

where $\rho = 0.8$, $\alpha = 0.5$ and $\varepsilon_t \sim N(0, \sigma)$, $\sigma = 0.1$. You can think of equation (1) as a simplified version of the 2nd-order approximation to a DSGE model, with only one endogenous variable y_t. Compare also to the example in Lecture 6.

1. Compute the deterministic steady-state(s) of equation (1). Comment on your findings.
2. To get an idea of the stochastic dynamics that (1) can potentially generate, plot in a single graph $y_t = \rho y_{t-1} + \alpha y_{t-1}^2 + 2\sigma$, $y_t = \rho y_{t-1} + \alpha y_{t-1}^2 - 2\sigma$ and the 45 line $y_t = y_{t-1}$. What can be inferred from the graph?
3. Now assume the standard deviation of ε_t is $\sigma/10$ (very small) and simulate two time series: one for the original process in (1), the second one for $y_t = \rho y_{t-1} + \varepsilon_t$ (we set $\alpha = 0$ here). Plot the two simulated series for $t = 1, 2, .., 500$. *Hint:* draw the i.i.d. shocks $\{\varepsilon_t\}_{t=1}^T$ using the Matlab command `sigma*randn(T,1)`, where $T = 500$. `sigma` is either $\sigma/10$ or σ.
4. Redo the exercise in the previous point but this time assume that the standard deviation of ε_t is σ. Do you notice anything strange?
5. (Pruning). In order to avoid explosive paths in the simulation of a 2nd order system, researchers often use a procedure called *pruning*. This is also what Dynare does when doing simulations if order>1. Follow these steps:
   (a) First, draw a sequence $\{\varepsilon_t\}_{t=1}^T$, with $T = 500$.
   (b) Next, solve for $\{\widetilde{y}_t\}_{t=1}^T$ in the linear component of the process:
   $$\widetilde{y}_t = \rho \widetilde{y}_{t-1} + \varepsilon_t$$
   (c) The *pruned* solution to the 2nd-order equation (1) is $\{y_1, \ldots, y_T\}$ in
   $$y_t = \rho y_{t-1} + \alpha \widetilde{y}_{t-1}^2 + \varepsilon_t$$
   Note that such procedure ensures that the y_t sequence cannot explode!

Plot in a single graph the two series $\widetilde{y}_t$ and y_t (pruned), for $t = 1, .., 500$. Comment on your findings.

Remark. I suggest that you always use the same random numbers throughout the exercise. You can fix the seed for the random number generator using `rng('default')`.

Hansen's RBC model, with VFI (20 points)

Consider again the RBC model analyzed in the previous problem sets. In this exercise you will go again through the solution of the model, but this time using VFI on a discrete grid. Assume the same parametrization as before, but do note that I changed the value of ρ to 0.95[1]

Parameter	Value	Description
β	0.99	discount factor
γ	0.0045	disutility of hours worked
η	1.0039	gross rate of labor augmenting techn. progress
θ	0.2342	capital share in prod. function
A	6.0952	constant term in AR(1) for TFP shock
δ	0.025	depreciation rate (capital)
ρ	0.95	persistence parameter in AR(1) for TFP shock
σ^2	0.00025	error variance in AR(1) for TFP shock.

Symbol	Variable name	Description
k	Capital stock	endogenous state
a	Technology (TFP)	exogenous state
c	Consumption	control or forward-looking
y	Output	static
i	Investment	static
h	Labor	static
ε	Innovation AR1	Exogenous shock

The VFI algorithm requires that you discretize the continuous AR(1) process for the TFP shock into a finite Markov chain. You can use the Matlab function `[Tran,s]=markovapprox(rho,sigma,m,N)` uploaded on ILIAS. This function takes as inputs the persistence ρ and the standard deviation σ of the AR(1) process. Furthermore you also have to specify N which the number of points of the discretize Markov chain. I would recommend that you set $N = 7$, but if your code is too slow, you can set a lower value. m is a technical parameter: set $m = 3$. The function will then deliver the transition matrix, called `Tran`, and a discretized vector for a, called `s` (read also the comments inside the function file).

[1] Ireland (2004) estimates the model with maximum likelihood after solving the model with log-linearization. He finds a very high value for the persistence of the TFP shocks which is not ideal when you discretize the AR(1) process.

1. Find the deterministic steady-state (you have found it already in the previous problem sets).

2. Write down the Bellman equation for the social planner's problem. The state variables are of course capital k and technology a.

3. Solve the model using VFI. You can follow the algorithm outlined in Lecture 6 (steps 1 to 8), without the Howard improvement step (for now). Remark: here the TFP shock is called a_t, whereas in Lecture 6 is called z_t. Pay attention to (i) how you define the grid for capital and to (ii) endogenous labor supply!

4. Plot (in separate graphs) the value function $V(k,a)$, the policy functions $g_k(k,a), g_h(k,a)$ and $g_c(k,a)$ as functions of k, for the *lowest* and the *highest* value of the shock a. For example, you have to make a graph with $g_k(k, a_{\min})$, $g_k(k, a_{\max})$, etc.

5. (Optional) Code the Howard improvement step. Make sure you get the same results. How much time do you save with respect to baseline VFI?

6. (Optional, harder). Plot the impulse response functions to a 1 percent positive technology shock.

Remark: Robert Kirkby has a very useful toolkit to implement value function iteration on a discrete grid, similar is spirit to Dynare. You may want to check out his toolkit for future reference at: vfitoolkit.