Problem Set 3

Due on Tuesday, May 21, h 13:00. Mail box F230.

Hansen's RBC model, continued (20 points)

Consider again the RBC model analysed in problem sets 1 and 2. In this problem set you will go again through the solution of the model, but this time using Dynare. Dynare greatly simplifies some of the tasks (you don't have to log-linearize equations and write down complicated matrices) but the interpretation of the results requires some care.

Assume the same parameterization as before:

Parameter	**Value**	**Description**
β	0.99	discount factor
γ	0.0045	disutility of hours worked
η	1.0039	gross rate of labor augmenting techn. progress
θ	0.2342	capital share in prod. function
A	6.0952	constant term in AR(1) for TFP shock
δ	0.025	depreciation rate (capital)
ρ	0.9983	persistence parameter in AR(1) for TFP shock
σ^2	0.00025	error variance in AR(1) for TFP shock.

Symbol	**Variable name**	**Description**
k	Capital stock	endogenous state
a	Technology (TFP)	exogenous state
c	Consumption	control or forward-looking
y	Output	static
i	Investment	static
h	Labor	static
ε	Innovation AR1	Exogenous shock

1. Assign numerical values to the parameters in your file, i.e. write `beta = 0.99`, etc. *Hint*: you can either declare the parameters in matlab and then pass the values to Dynare, or just write directly the parameter values inside the mod file.

2. In Dynare, declare the endogenous variables, the exogenous shocks and the parameters.

3. Given the parameter values listed above, compute numerically the steady state of the model, i.e. provide values for the 6 endogenous variables $\{k, a, c, y, i, h\}$. You can follow one of the methods described in the slides. Specifically:

(a) *Method 1.* Find the steady state analytically and type the relevant equations in `steady_state_model` block. As a template, have a look at the files `main1.m` and `growth1.mod` in ILIAS.

(b) *Method 2.* Compute the steady state in a separate *.m file. Call the steady state values as `varname_ss`, i.e. $\overline{k}$ is `k_ss`, $\overline{a}$ is `a_ss`, and so on. You can compute these values either analytically or numerically with the help of `fsolve`. Then pass the steady-state values computed in the Matlab script to Dynare. To do so, save these values in the data file `paramfile.dat` and then load `paramfile` in Dynare. As a template, have a look at the files `main2.m` and `growth2.mod` in ILIAS.

4. Write the model block in Dynare. In Problem set 1 you had derived the equilibrium equations (after detrending for labor augmenting progress). There are six equations for the six endogenous variables $\{k, a, c, y, i, h\}$. I repeat them here for convenience:

$$y_t = a_t k_t^{\theta} h_t^{1-\theta} \tag{1}$$

$$\log(a_t) = (1-\rho)\log(A) + \rho \log(a_{t-1}) + \varepsilon_t \tag{2}$$

$$y_t = c_t + i_t \tag{3}$$

$$\eta k_{t+1} = (1-\delta) k_t + i_t \tag{4}$$

$$\gamma c_t h_t = (1-\theta) y_t \tag{5}$$

$$\frac{1}{c_t} = \frac{\beta}{\eta} E_t \left\{ \frac{1}{c_{t+1}} \left[\theta \frac{y_{t+1}}{k_{t+1}} + 1 - \delta \right] \right\} \tag{6}$$

and $\varepsilon_t \sim N(0, \sigma^2)$.

5. Solve the model using Dynare (i.e. type `dynare yourfile.mod noclearall` in the Matlab command window or in a matlab script). Report the policy function matrix, moments and correlations and plot IRF to an innovation to technology for 40 periods.

6. After running Dynare, save in Matlab the matrices g_x and g_u (see slides for details). Note that Dynare orders the coefficients of the policy function in a different way from the declaration order. The declaration order that you followed is $\{k, a, c, y, i, h\}$. What is the DR-order? Write the model in state-space form notation as follows:

$$x_t = \overline{x} + g_x (s_{t-1} - \overline{s}) + g_u \varepsilon_t$$

where x_t is a vector containing all 6 endogenous variables and s_t is a subset of x_t containing only the state variables. What are the dimensions (i.e. number of rows and columns) of g_x and g_u?

7. Generate simulated time series for all endogenous variables for $T = 100000$. Discard the first 1000 observations. For each variable $x \in \{k, a, c, y, i, h\}$, compute the standard deviation relative to output, i.e. report the following table:

Symbol	**Variable name**	**Std(x)/std(y)**
y	Output	1
k	Capital stock	??
a	Technology (TFP)	??
c	Consumption	??
i	Investment	??
h	Labor	??

   Note: to check your results, you can inspect the table of standard deviations generated by Dynare. You should be able to get close.

8. Suppose you want to compare the dynamics of this RBC model for different levels of the shock persistence. Plot **in the same graph** the IRF of technology, consumption, investment, labor, capital and output to an innovation to technology, for $\rho = 0.5$ and $\rho = 0.95$. What is the economic intuition behind your findings? Focus in particular on the initial response of labor to the shock. Does labor react on impact more strongly if $\rho = 0.5$ or if $\rho = 0.95$?

New-Keynesian model (10 points)

Consider the following New-Keynesian model:

$$
\begin{aligned}
x_t &= E_t x_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} - r\right) + u_t && (7)\\
\pi_t &= \beta E_t \pi_{t+1} + \kappa x_t && (8)\\
i_t &= r + \delta \pi_t + v_t && (9)
\end{aligned}
$$

where x_t is the output gap, π_t is the inflation rate, i_t is the nominal interest rate,[1] u_t is a productivity shock and v_t is a policy shock. The model parameters satisfy the restrictions $\sigma > 0$, $\kappa > 0$, $\delta > 0$, $0 < \beta < 1$.

1. Eliminate i_t from the model by substituting (9) into (7).

2. Rewrite equations (7) and (8) in matrix form as follows:

$$
\begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = N \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \frac{1}{\sigma} v_t - u_t \\ 0 \end{bmatrix},
$$

   where N is a 2×2 matrix. Find out what are the 4 elements of N.

[1] Therefore $i_t - E_t \pi_{t+1}$ is the *real* interest rate.

3. Compute the characteristic polynomial of N:

$$P(\lambda) = \lambda^2 - tr(N) \cdot \lambda + \det(N),$$

where of course $tr(N)$ is the trace of the matrix N and $\det(N)$ is the determinant. The eigenvalues of N, λ_1 and λ_2, are the roots of the characteristic polynomial. In this case you can safely assume that the two eigenvalues are *real* and *distinct*. (See Mathematical Appendix of Lecture 2 if you need some background information on eigenvalues, characteristic roots, etc).

4. Show that both eigenvalues are positive.

5. Show that if $\delta \in (0,1)$, then $0 < \lambda_1 < 1 < \lambda_2$. Are the Blanchard-Kahn conditions satisfied in this case?

6. Show that if $\delta > 1$, then $1 < \lambda_1 < \lambda_2$. Are the Blanchard-Kahn conditions satisfied in this case?

Remarks. Most of the questions require you to write a simple matlab file. Please print out your code and submit it together with your analytical results (if any). Do **not** send your m file by e-mail!