Problem Set 1

Due on February 27 (Tuesday), **before class**.

Problem 1

Consider the McCall's search model seen in class. An unemployed worker receives each period an offer to work for wage w_t forever, where $w_t = w$ in the first period and $w_t = \phi^t w$ after t periods in the job. Assume $\phi > 1$, that is, wages increase with tenure. The initial wage offer is drawn from a distribution $F(w)$ that is constant over time (entry-level wages are stationary) with support $[0, B]$; successive drawings across periods are independently and identically distributed. The worker's objective function is to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t y_t \ ,$$

where $\beta \in (0,1)$ and $y_t = w_t$ if the worker is employed and $y_t = b$ if the worker is unemployed, where b is unemployment benefits. Let $V(w)$ be the lifetime utility value of a worker who is employed at wage w. Let U be the lifetime utility value of a worker who is unemployed. Assume that $\phi\beta < 1$.

1. Write down the equation for $V(w)$. *Hint*: If a worker accepts a job offer that pays an initial wage w in period t, he will earn w in t, ϕw in $t+1$, $\phi^2 w$ in $t+2$, etc.
2. Write down the equation for U.
3. Argue that the optimal policy is to accept all offers with an initial wage higher than a reservation wage.
4. Derive an equation for the reservation wage, which depends only on parameters and on the distribution F. *Hint*: in case you get stuck, have a look at Problem 2!
5. Show that the equation for the reservation wage derived in the previous question admits a unique solution.
6. Argue that, if two economies differ only in the growth rate of wages of employed workers, say $\phi_1 > \phi_2$, the economy with the higher growth rate has the smaller reservation wage. Explain the economic intuition. Note: Assume that $\phi_i\beta < 1$, for $i = 1, 2$.

Problem 2

After solving the first problem, you should have derived the following implicit equation for the reservation wage R:

$$R = \frac{b\,(1-\phi\beta)}{1-\beta} + \frac{\beta}{1-\beta}\int_R^B (w-R)\ dF(w) \equiv T(R). \tag{1}$$

Parameter	**Description**	**Value**
b	Unemp benefits	0.5
β	Discount factor	0.9
ϕ	Growth rate	1.03
λ	Exponential rate	0.2

Table 1: Parameter values

The parameter values are listed in Table 1. Assume that the wage distribution F is a Exponential with support $[0, \infty)$. Recall that the probability density function (pdf) of an exponential distribution is

$$f(w; \lambda) = \begin{cases} \lambda \exp(-\lambda w) & \text{if } w \geq 0 \\ 0 & \text{if } w < 0, \end{cases}$$

where $\lambda > 0$ is the parameter of the distribution, often called the *exponential rate*. Observe that in this case $B = \infty$, i.e. the support of the wage distribution is unbounded above (this is a slight generalization of the model seen in class).

Solve the problem numerically (e.g. using Matlab) following the questions below.

1. Solve the integral on the right-hand side of eq. (1) with a numerical routine of your choice. Matlab has a built-in function called `integral(fun,lb,ub)`, where `fun` is a function handle and `lb`, `ub` are the lower and upper bounds of integration, respectively. Observe that since the distribution is unbounded from above, you should set `ub=inf` as input to `integral`. For more information, please type `help integral` in the Matlab command window. You can also review Rafa's lecture on quadrature methods.

2. Solving equation (1) amounts to finding the fixed point $R^* = T(R^*)$. You can do so with a simple iterative algorithm:[1]

   (a) Set a fixed tolerance criterion, e.g. $tol = 10^{-6}$.

   (b) Choose an arbitrary initial guess $R_{old} \in (0, B)$.

   (c) Compute a new value R_{new} as

   $$R_{new} = \underbrace{\frac{b(1-\phi\beta)}{1-\beta} + \frac{\beta}{1-\beta} \int_{R_{old}}^{\infty} (w - R_{old}) \, dF(w)}_{=T(R_{old})}$$

   (d) Calculate the distance $dist$ between R_{old} and R_{new} as $dist = |R_{new} - R_{old}|$, where the $|\cdot|$ symbol denotes the absolute value.

   (e) If $dist$ is larger than the fixed tolerance level tol, set

   $$R_{old} = (1-\psi) \times R_{new} + \psi \times R_{old},$$

[1] This method is known as *fixed point iteration.*

where $\psi \in [0, 1)$ is a dampening factor and go back to step 2c. Otherwise, return R_{new} and stop the algorithm. *Hint*: Try setting $\psi = 0.5$. This means that every time you update you are taking the arithmetic average of the old guess and the new value. If the algorithm does not converge, try increasing ψ and/or try a different initial guess R_{old}.