Mahler and Yum (2024)

Alessandro Di Nola, Robert Kirkby and Haomin Wang

1 Model description

Brief description of replication of Mahler and Yum (2024).

- State variables:
  - Asset holdings a
  - Health status h
  - Labor productivity z
  - Permanent types:
    - Fixed education $e \in \{0, 1\}$ (non-college, college)
    - Patience $\beta \in \{0, 1\}$ (low, high)
    - Ability $\theta \in \{0, 1\}$ (low, high)
    - Fixed health $\eta \in \{0, 1\}$ (bad, good)
- Choice variables:
  - Labor supply $n \in \{0, n_{PT}, n_{FT}\}$
  - Health effort $f \in [0, 1]$
  - Next-period assets a'
- Individual problem

  $$V_j\left(a, h, z, \overline{e}, \overline{\beta}, \overline{\theta}, \overline{\eta}\right) = \max_{c,a',n,f} \left\{u\left(c\right) + \overline{\beta} s_j(h, e) E\left[V_{j+1}(a', h', z', \overline{e}, \overline{\beta}, \overline{\theta}, \overline{\eta})\right]\right\}$$

  subject to

  $$c + a' = (1+r)a + T(c, h, n) + w_j\left(h, z, \overline{e}, \overline{\theta}\right) n - \mathcal{T}\left(w_j\left(h, z, \overline{e}, \overline{\theta}\right) n, y_{bar}\right), \text{ if } j < J_R$$

  and

  $$c + a' = (1 + r)a + P(e), \text{ if } j \geq J_R$$

where $T(\cdot)$ denotes transfers and $\mathcal{T}(\cdot)$ denotes income taxes. The expected value function is defined as

$$E\left[V_{j+1}(a',h',z',\overline{e},\overline{\beta},\overline{\theta},\overline{\eta})\right] = \sum_{z'}\sum_{h'} V_{j+1}(a',h',z',\overline{e},\overline{\beta},\overline{\theta},\overline{\eta}) \Pr(z'|z) \Pr(h'|h,f,\overline{e},\overline{\eta},j)$$

Note that the transition probability for health h depends on health effort f, a decision variable. Therefore h is a semi-exogenous state. Moreover, the transition probability for health is also affected by fixed education and health types, and by age.

1.1 Functional Forms

- Transition probability for health status $h \in \{0,1\}$, where $h=0$ is "unhealth" and $h=1$ is "healthy". The probability of bein healthy in the next period is an increasing function of effort $f \in [0,1]$ modelled as a logistic curve:

$$\Pr(h'=1|) = \frac{1}{1+\exp\left(-\left(\pi_j^0 + \lambda_1 f + \delta h + \gamma_1 \overline{e} + \gamma_2 \overline{\eta}\right)\right)}$$

  Note that π_j^0 is a shifter that depends on age, f is health effort, h is health in the current period, $\overline{e}$ is fixed education type (0 if non-college, 1 if college) and $\overline{\eta}$ is fixed health type (0 if low, 1 if high). Plot $f(x) = \frac{1}{1+\exp(-x)}$: it is defined over $x \in (-\infty,\infty)$, increasing, convex for $x<0$, concave for $x>0$, inflexion point at $x=0$ where $f(0)=0.5$. As $x \to \infty$, $f(x) \to 1$).

- Tax function $\mathcal{T}(y, y_{bar})$ is defined as

$$\mathcal{T}(y, y_{bar}) = y - (1-\tau_s) y^{1-\tau_p} (y_{bar})^{\tau_p}$$

  where y is taxable income and includes only labor earnings $w_j\left(h,z,\overline{e},\overline{\theta}\right)n$.

- Social transfers are defined as $T(c,h,n) = T_1 + T_2$ where

$$T_1 = \begin{cases} \widetilde{c} - c & \text{if } c < \widetilde{c} \\ 0 & \text{otherwise} \end{cases}$$

  and

$$T_2 = \begin{cases} \widetilde{T} > 0 & \text{if } h=0 \text{ and } n=0 \\ 0 & \text{otherwise} \end{cases}$$

- The wage function (i.e. the wage per unit of labor supplied) is defined as

$$w_j\left(h, z, \overline{e}, \overline{\theta}\right) = \exp\left(\lambda_j\left(h, \overline{e}\right) + \overline{\theta} + z\right)$$

  where $\lambda_j\left(h, \overline{e}\right)$ is the age-dependent component, $\overline{\theta}$ is a fixed effect (unobserved ability) and z is an AR(1) productivity shock. The only non-standard assumption is that $\lambda_j\left(h, \overline{e}\right)$ depends also on health: unhealthy individuals (with $h = 0$) suffer a wage penalty $w_p^{\overline{e}}$ which depends on education:

$$\lambda_j\left(h, \overline{e}\right) = \zeta_0^{\overline{e}} \exp\left(\zeta_1^{\overline{e}}\left(j-1\right) + \zeta_2^{\overline{e}}\left(j-1\right)^2\right)\left(1 - w_p^{\overline{e}} I_{\{h=0\}}\right)$$

- Preferences. The per-period utility function takes the form:

$$u(c, n, f, h, \overline{e}) = \kappa\left(h\right)\left(\frac{c^{1-\sigma}}{1-\sigma} + b\right) - \phi\left(n; h, \overline{e}\right) - \varphi\left(f; h, \overline{e}\right)$$

  where

  - Utility of consumption is

$$\kappa\left(h\right)\left(\frac{c^{1-\sigma}}{1-\sigma} + b\right)$$

  - Disutility of labor is

$$\phi\left(n; h, \overline{e}\right) = v_j\left(h\right) \exp\left(v_e I_{\{\overline{e}=0\}}\right) \frac{n^{1+1/\gamma}}{1+1/\gamma}$$

    where v_e is an extra disutility of working incurred by individual with non-college education (i.e. $\overline{e} = 0$). $v_j\left(h\right)$ is an age-dependent shifter that depends also on health status (to match employment rates by age and by health).

  - Disutility of health effort is

$$\varphi\left(f; h, \overline{e}\right) = \iota_j\left(h, \overline{e} = 0\right) \frac{f^{1+1/\psi}}{1+1/\psi}$$

    where $\iota_j\left(h, \overline{e} = 0\right)$ is an age-specific shifter that depends also on health state and on education fixed type (to match mean health effort by age, health status and education).