Efficiency and Equity of Education Tracking
A Quantitative Analysis

Suzanne Bellue[1] Lukas Mahler[2]

University of Birmingham - Macro Seminar
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[1]CREST-ENSAE

[2]KU Leuven

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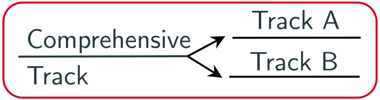

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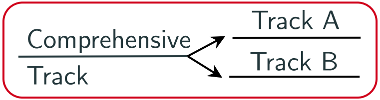

- Heterogeneity in tracking age across countries (OECD, 2020)

Tracking Age	10	11	12	13	14	15	16
Country	*AT,GER*	*SK,CZ*	*BE,NL,SG*	*BUL*	*ARG,IT*	*FR,JPN,PT*	*AUS,UK,US*

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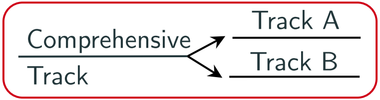

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  - Tracking can increase learning efficiency
  - Tracking (too) early → "mis-allocation" of children if development uncertain
  - (Early) tracking may consolidate inequality; hinder social mobility

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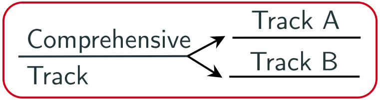

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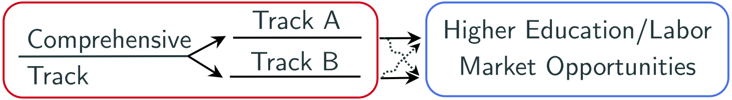

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- Build overlapping generations model of human capital formation
    - incorporate tracking during school years

      → Child skill formation affected by instruction pace and peers in school track
    - in general equilibrium
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  $\rightarrow$ Key moments: transitions through education system, inequality, mobility

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- Calibrated to early tracking education system: Germany (in 2010s)
    - $\rightarrow$ Skill formation technology estimated on data from panel of school children
    - $\rightarrow$ Key moments: transitions through education system, inequality, mobility
- Evaluate long-run effects of reforms to tracking policy

Preview of Results

$\rightarrow$ Calibrated model can rationalize observed data patterns

- Validated by untargeted, external moments

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1. Postponing tracking age entails efficiency-mobility trade-off in long-run
   - Intergenerational mobility $\uparrow$
     → Longer comprehensive schooling: skill heterogeneity $\downarrow$
   - Aggregate long-run output $\downarrow$
     → Less tailored teaching in comprehensive school: learning $\downarrow$, human capital $\downarrow$

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1. Postponing tracking age entails efficiency-mobility trade-off in long-run
    - Intergenerational mobility ↑
        → Longer comprehensive schooling: skill heterogeneity ↓
    - Aggregate long-run output ↓
        → Less tailored teaching in comprehensive school: learning ↓, human capital ↓
2. Reducing parental influence in track choice can raise mobility and efficiency
    - Frequent deviations from recommended track towards parent's education
    - Binding recommendation: mobility ↑ and output ↑
        → Track selection based more on skills: teaching becomes more efficient

Related Literature

1. Quantitative Macro Literature on inequality and mobility with education
    - Higher Education and/or Early Education (Abbott et al., 2019; Becker and Tomes, 1986; Daruich, 2022; Lee and Seshadri, 2019; Restuccia and Urrutia, 2004; Yum, 2023)
    - School Closures (Agostinelli, Doepke, et al., 2022; Fuchs-Schündeln, Krueger, Kurmann, et al., 2023; Fuchs-Schündeln, Krueger, Ludwig, et al., 2022; Jang and Yum, 2022)
    - Sec. Education in Developing Countries (Fujimoto, Lagakos, and Vanvuren, 2023)

$\rightarrow$ We add tracking in secondary school

2. Literature on Child Skill Development
    - Theory and Estimation (Agostinelli, Saharkhiz, and Wiswall, 2019; Cunha and Heckman, 2007; Cunha, Heckman, and Schennach, 2010; Duflo, Dupas, and Kremer, 2011)

$\rightarrow$ We focus on peer and instruction level effects across tracks

3. Empirical evidence on Education Tracking
    - Educational and Labor Market Outcomes (Betts, 2011; Dustmann, Puhani, and Schönberg, 2017; Hanushek and Wößmann, 2006; Matthewes, 2021) ...
    - Social Mobility (Meghir and Palme, 2005; Pekkarinen, Uusitalo, and Kerr, 2009)

$\rightarrow$ We build structural model to gauge macro effects of tracking

Outline

Model

Skill Formation during School Years

Full Quantitative Model

Calibration to Germany (2010s)

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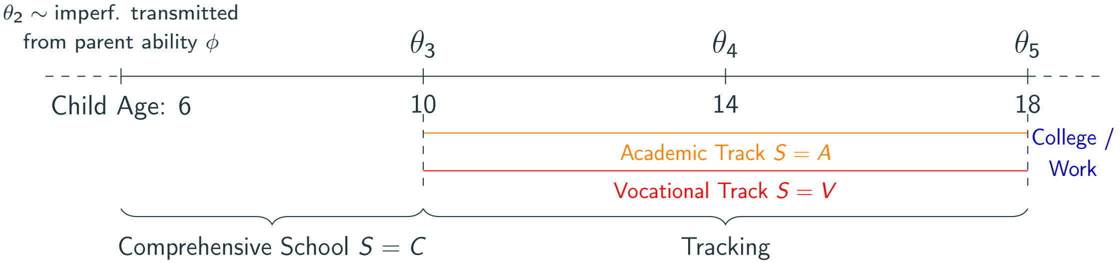
Skill Formation during School Years



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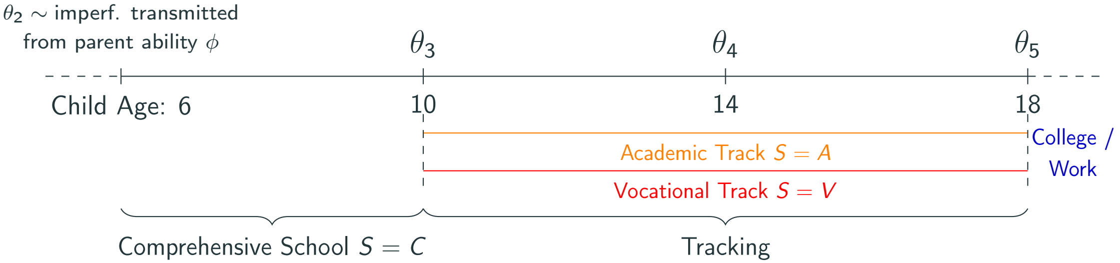

Child skill θ_j evolution in school track S *(in logs)*:

$$\theta_{j+1} = \underbrace{\kappa\theta_j + \alpha\bar{\theta}_j^S + g(\theta_j, P_j^S) + \xi E + \eta_{j+1}}_{f(\theta_j, \bar{\theta}_j^S, P_j^S, E, \eta_{j+1})} \qquad (1)$$

- $\bar{\theta}_j^S$: Avrg. skill in track S (peer effect)
- P_j^S: Instruction pace in track S
- E: Parental education ($\sim$ inputs)
- $\eta_{j+1} \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma^2_{\eta_{j+1}})$: Skill shock

Pace of Instruction

- Instruction pace affects learning via Illustration

$$g(\theta_j, P_j) = \beta P_j + \gamma \theta_j P_j - \frac{\delta}{2} P_j^2 \tag{2}$$

  - $\gamma > 0$: Complementarity btw. skills and pace (Aucejo et al., 2022; Duflo et al., 2011)

$\implies$ Optimal $P_j^*(\theta_j)$ for child with θ_j that maximizes θ_{j+1} with $\frac{\partial P_j^*}{\partial \theta_j} > 0$

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- (Education-) Policymaker can set one instruction pace per track: P_j^S
  $\rightarrow$ Set to: $\max_{P_j^S} \mathbb{E}[\theta_{j+1}]$

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$$\boxed{P_j^S = \frac{\beta + \gamma\bar{\theta}_j^S}{\delta} = P_j^*(\bar{\theta}_j^S)} \tag{3}$$

$\rightarrow$ Paces set to optimal one for average child in each S and j

$\implies$ Learning decreases in distance to track average

Optimal Tracking Policy without Skill Shocks

- Allocation of children to tracks that maximizes agg. end-of-school skills ?
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Optimal tracking policy perfectly stratifies children by skills

- Average learning gain from tracking (T) rel. to comprehensive system (C):

$$\mathbb{E}(\theta_4|T) - \mathbb{E}(\theta_4|C) = \frac{\gamma^2}{2\delta}\left(\sigma^2_{\theta_3} - \mathbb{E}(Var[\theta_3|S])\right) > 0 \qquad (4)$$

Proposition 1 Illustration

$\rightarrow$ Minimizes skill heterogeneity in tracks $\rightarrow$ teaching as efficient as possible

Optimal Tracking Policy with Skill Shocks

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Proposition 3 Illustration

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Proposition 3 Illustration

→ Uncertain skill evolution entails risk of "mis-allocating" children

→ Late tracking foregoes learning gains early, achieves more homogeneous groups later

Further Implications of Tracking for Skill Distribution

1. *Some children learn less than in C* Proposition 2 Illustration
   - Losses may be concentrated in V-Track
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$\rightarrow$ Rationalizes empirical findings on effects of tracking on learning (Hanushek and Wößmann, 2006; Matthewes, 2021)

- So far: policymaker makes "optimal" track choice to max. learning

$\rightarrow$ In reality (and model): parents make the track choice

- Take into account college education opportunities, labor market returns ...

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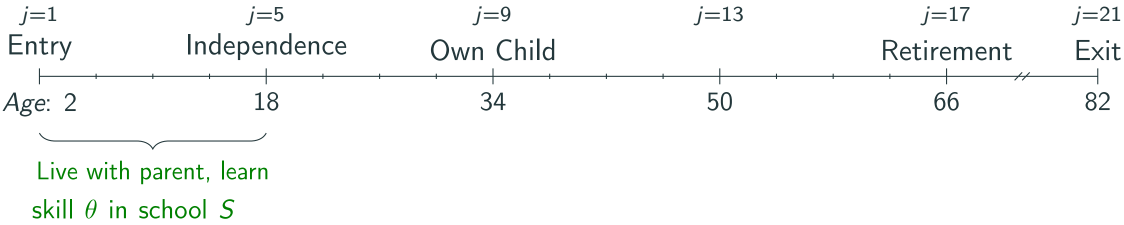

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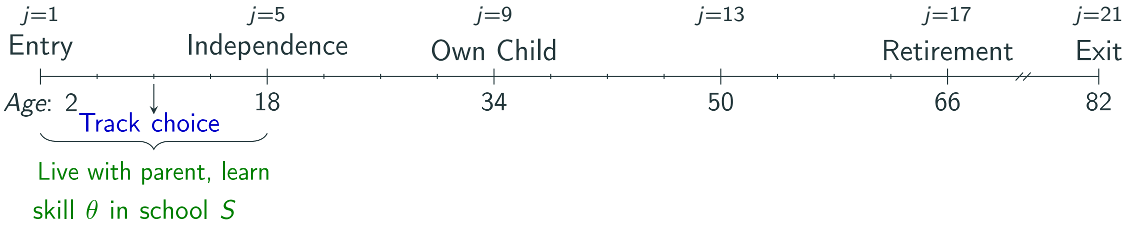

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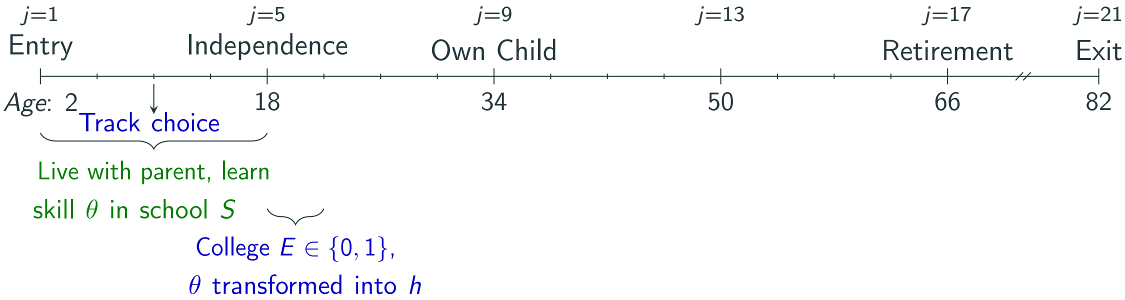

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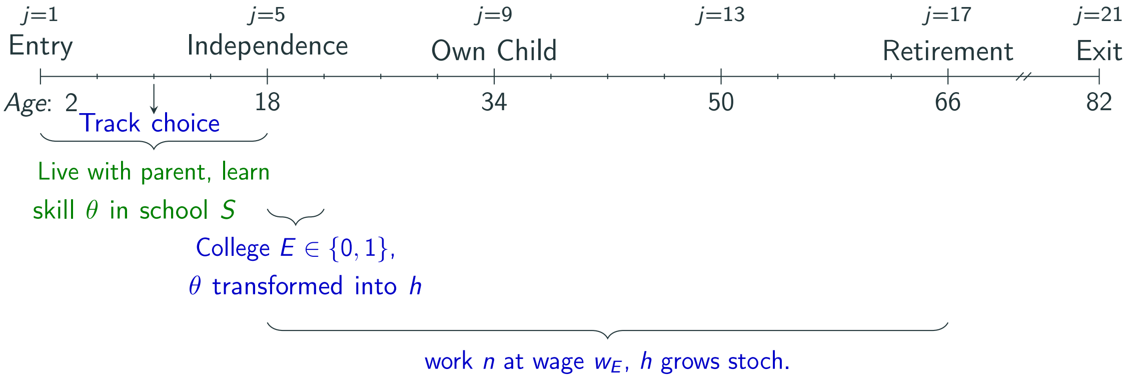

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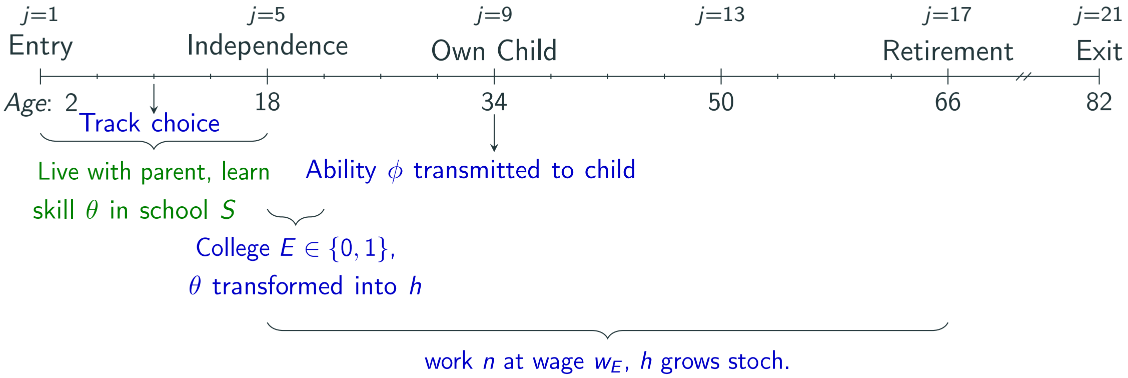

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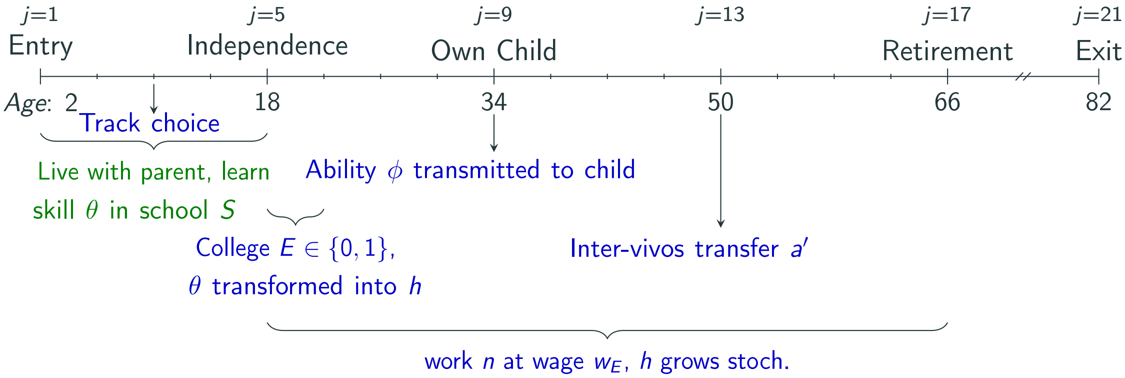

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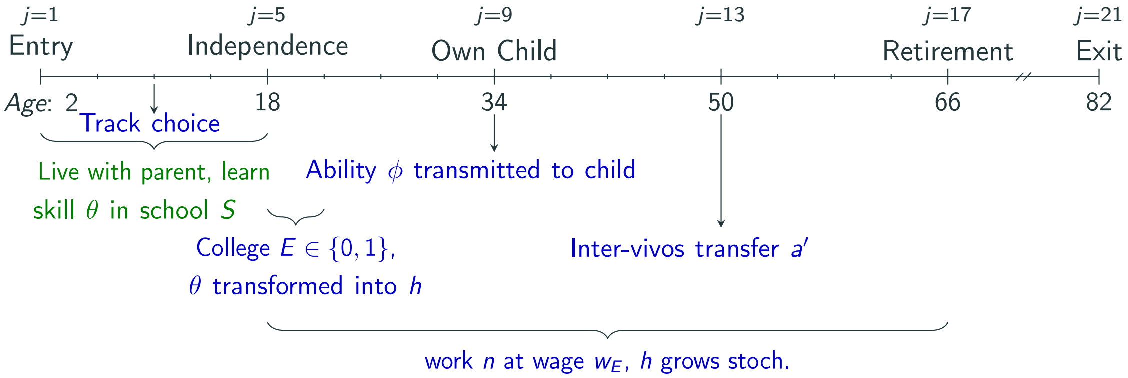

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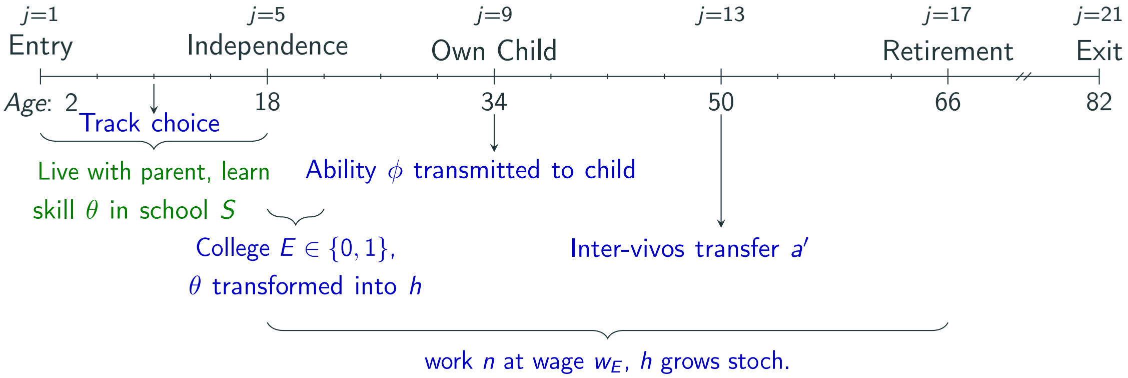

- Representative firm produces output using physical + human capital H_0, H_1
- Government taxes labor, capital income; finances pensions, transfers Details

The School Track Decision ($j = 11$, Child Age 10)

Parent with state vector $s_{11} = (E, h_{11}, a_{11}; \theta_3, \phi)$ solves

$$V_{11}(s_{11}, \chi(E)) = \max_{S} \{ W_{11}(s_{11}; S = V),\ W_{11}(s_{11}; S = A) - \chi(E) \}$$

- E, h_{11}, a_{11}: Parental education, human capital, assets; θ_3, ϕ: child skills, ability

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- $\chi(E) \sim \mathcal{N}(\mu_{\chi,E}, \sigma^2_{\chi})$: Stochastic utility cost from A-track

→ Captures why parents may (systematically) deviate from recommended track towards own educational path, e.g. information, preferences, ... Data

→ $\chi \perp\!\!\!\perp$ E: Track recommendation

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$$W_{11}(s_{11}; S) = \max_{c_{11}, a_{12}, n_{11}} \left\{ u(\frac{c_{11}}{q}, n_{11}) + \beta\, \mathbb{E}\, V_{12}(s_{12}; S) \right\}$$

$$\text{s.t. } \theta_4 = \underbrace{f(\theta_3, \bar{\theta}_3^S, E, \eta_4)}_{\text{(1) w\textbackslash optimal } P_3^S} \quad + \quad \text{Budget + Time + Borrowing Constraints}$$

- c_{11}, n_{11}: Consumption, labor supply; $q > 1$: adult consumption-equiv. scale

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- Parent has to anticipate average skill level in each track $\bar{\theta}_3^S$

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- Expectations over market luck ε_{12} and child skill shock η_4

Young Adult Makes College Choice ($j = 5$, Age 18) other periods

Newly independent adult with states $s_5 = (a_5, S, \theta_5, \phi)$ solves

$$V_5(s_5, \nu(E^p)) = \max_{E} \{ W_5(s_5, E = 0),\ W_5(s_5, E = 1) - \psi(S, \theta_5, \nu(E^p)) \}$$

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- a_5 : Inter-vivos transfer from altruistic parent; S: school track, θ_5: end-of-school skills, ϕ: ability

Transfer Decision

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- $\psi(S, \theta_5, \nu(E^p))$: "Psychic" college costs depend on school track S, skills θ_5, and parent-specific preference shock $\nu(E^p) \sim G^{E^p}(\nu)$

$\rightarrow$ "second-chance" opportunity

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$$\text{s.t. } c_5 + a_6 = w_0 h_5 n_5 + (1 + r)a_5 - T(y_5, a_5)$$

$$h_5 = \exp(\theta_5), \quad h_6 = \gamma_{5,E} h_5 \varepsilon_6 \quad + \quad \text{Borrowing Constraint}$$

- College ($E = 1$) pays market wage rate w_1 in future, affects human capital $\gamma_{j,E}$

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- $\bar{n}(E = 1) < 1$: Time available for work when going to college (paying w_0)

Production

- Cobb-Douglas aggregate production function

$$Y = AK^{\alpha}H^{1-\alpha}, \tag{5}$$

where H is aggregated using a CES technology:

$$H = \left\{ \varphi \underbrace{H_0^{\sigma_f}}_{\text{Non-CL Labor}} + (1-\varphi) \underbrace{H_1^{\sigma_f}}_{\text{CL Labor}} \right\}^{\frac{1}{\sigma_f}} \tag{6}$$

Equilibrium

- Stationary equilibrium: $\rightarrow$ cross-sectional distribution over states of any cohort of age-period j is constant across time (Lee and Seshadri, 2019) Definition
- Households and firm optimize
- Aggregate prices r, w_0, and w_1 clear markets
- Parents' expectations about average skills in each track are consistent with actual distribution

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Calibration Steps and Data

German Education System | Parameterization

1. Estimate child skill formation technology on *German National Education Panel Study (NEPS)* Details

   → Treat skills as latent variables and use achievement test scores as measures (Agostinelli, Doepke, et al., 2023; Cunha, Heckman, and Schennach, 2010)

   → Evidence that learning decreases in distance to track average

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   - → Evidence that learning decreases in distance to track average
2. Set "standard" parameters exogenously or estimate directly from *German Socio-Economic Panel (SOEP)* External Parameters
   - → Human capital growth rates $\{\gamma_{j,E}\}$ (Lagakos et al., 2018) Details

Calibration Steps and Data

German Education System · Parameterization

1. Estimate child skill formation technology on *German National Education Panel Study (NEPS)* Details
   - → Treat skills as latent variables and use achievement test scores as measures (Agostinelli, Doepke, et al., 2023; Cunha, Heckman, and Schennach, 2010)
   - → Evidence that learning decreases in distance to track average
2. Set "standard" parameters exogenously or estimate directly from *German Socio-Economic Panel (SOEP)* External Parameters
   - → Human capital growth rates $\{\gamma_{j,E}\}$ (Lagakos et al., 2018) Details
3. Method of simulated moments calibration of 26 parameters to match 26 data targets:
   - → Transitions through education system, inequality, mobility

MSM Results: Non-Standard Parameters and Moments

Parameterization Remaining Parameters

Parameter	Value	Description	Target	Data	Model
A-Track Costs					
$\mu_{\chi,A}$	0.048	Uniform Costs	Share A-Track Recommend.	0.44	0.44
χ_0	0.0020	Mean Costs if $E = 0$	Dev. A-Track Recom. if $E = 0$	0.16	0.16
χ_1	-0.0036	Mean Costs if $E = 1$	Dev. V-Track Recom. if $E = 1$	0.23	0.23
σ_χ	$0.17 \cdot 10^{-3}$	Std. Cost Shock	Reg. S on θ_3: var(resid.)	0.17	0.17

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College Costs					
ψ	0.77	Intercept	Share in CL from A-Track	0.66	0.65
ψ_V	0.16	Costs for V-Track	Share in CL from V-Track	0.11	0.11
ψ_θ	-0.35	Coefficient on θ_5	Reg. E on θ_5 & S: coef. θ_5	0.40	0.50
$\Delta(\mu_{\nu,E})$	0.034	Diff. in Means by E	Share in CL from Non-CL HH	0.20	0.28
σ_ν	0.008	Std. Taste Shock	Reg. E on θ_5 & S: var(residuals)	0.14	0.14

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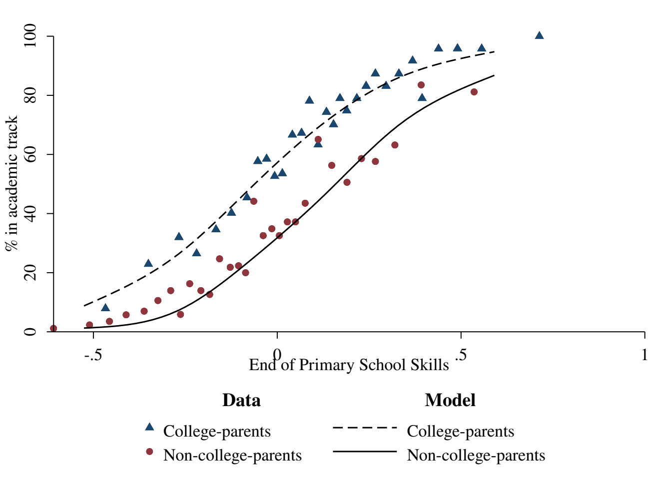
Child Skill Shocks					
σ_{η_3}	0.052	Std. Skill Shock $j = 3$	$\text{Rank}_{j=2}$-$\text{Rank}_{j=3}$	0.72	0.73
σ_{η_4}	0.030	Std. Skill Shock $j = 4$	$\text{Rank}_{j=3}$-$\text{Rank}_{j=4}$	0.79	0.80
σ_{η_5}	0.032	Std. Skill Shock $j = 5$	$\text{Rank}_{j=4}$-$\text{Rank}_{j=5}$ if $S = 1$	0.74	0.75

Model Validation - Untargeted School Track Choice



- S-shaped probability of A-track given skills, parental education

Model Validation - Other Untargeted Moments

Model produces realistic:

- Distribution of skills across school tracks and parental education Moments
- Intergenerational mobility and cross-sec. inequality Inequality

Moment	(Dodin et al., 2024)	**Model**
Regression of A-track on parental income rank (x10)	0.52	0.32
A-track share if parental income in bottom quintile	0.34	0.30
A-track share ratio: top to bottom income quintile	2.13	1.82

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- Small long-term labor market effects of track choice for children at the margin between school tracks
  (Dustmann, Puhani, and Schönberg, 2017) Details

Lifetime Variance Decomposition

Outline

Model

Skill Formation during School Years

Full Quantitative Model

Calibration to Germany (2010s)

Policy Experiments

Postponing the Tracking Age

- Experiment: Postpone tracking from Age 10 to Age 14 (*OECD Avrg.*)
  - $\rightarrow$ Comprehensive school before Age 14
  - $\rightarrow$ New steady-state equilibrium

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Outcome (%-change rel. to early tracking)	
Output	-0.11
Inequality: Gini Earnings	-0.4
Interg. Mobility: -(Income rank-rank)	2.2
Consumption Equiv. Variation Def.	-0.05

- Output ↓, but inequality ↓ and intergenerational mobility ↑

→ Trade-off between efficiency and mobility

Sources of Efficiency Losses from Late Tracking

- Longer time in comprehensive school:

$\rightarrow$ Less efficient teaching in 1st period of secondary school

$\rightarrow$ Avrg. skills at age 14 drop relative to early tracking ($\bar{\theta}_4 \downarrow$ -2.1%) Figure

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- Losses can't be recovered by more efficient teaching after (late) tracking

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  $\rightarrow$ Skill shock variance during secondary school not sufficiently high

  $\rightarrow$ "Inefficient" parental track choice deviations from recommendation $\uparrow$ 2%

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$\rightarrow$ Avrg. end-of-school skills also drop rel. to early tracking ($\bar{\theta}_5 \downarrow$ -1.6%)

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2. from different parental backgrounds

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1. by (later) tracks
2. from different parental backgrounds

%-change in late tracking rel. to early tracking	
Reg. of A-track on parental income rank	-6.8
Share College after V-track	0.5
Share College after A-track	-1.7

→ Initial conditions less predictive for end-of-school skills, college, income Details

Postponing the Tracking Age without GE Effects

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Outcome (%-change rel. to early tracking)	GE	fixed (w_0, w_1, r)
Output	-0.11	0.19
Inequality: Gini Earnings	-0.4	-0.4
Interg. Mobility: -(Income rank-rank)	2.2	3.5
Consumption Equiv. Variation	-0.05	0.18
A-Track Share	1.8	5.9
College Share	-0.3	4.6

- Less efficient learning triggers A-track ↑ and thus College share ↑: Output ↑

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→ Adjustment in wages in GE crucial for efficiency-mobility trade-off (long-run)

Economy without Tracking

Tracking Age Outcome (%-change rel. to early tracking)	14 GE	Never GE
Output	-0.11	-0.24
Inequality: Gini Earnings	-0.4	-0.8
Interg. Mobility: -(Income rank-rank)	2.2	23.9
Consumption Equiv. Variation	-0.05	-0.08
Mean end-of-school skills	-1.9	-2.7
Variance of end-of-school skills	-0.2	-2.1

- Never tracking strongly improves mobility, but decreases human capital and output even more than late tracking → trade-off intensifies

Consequences of Parental Influence on Track Choice

- Parental education important driver of track choice even cond. on skills
  - → One reason: Asymmetric A-track costs $\chi_0 > \chi_1$ Reasons
  - → May create "mis-allocation" of children across tracks

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- Experiment 1: Eliminate asymmetry in χ $\approx$ make track recommend. binding

Outcome (%-change rel. to baseline)	$\chi_0 = \chi_1 = 0$
Output	0.04
Inequality: Gini Earnings	0.0
Interg. Mobility: -(Income rank-rank)	0.9
Consumption Equiv. Variation	0.04
Mean end-of-school skills	0.8
Reg. Coeff: A-Track on Skills	5.7

- Track choice more skill-based $\rightarrow$ teaching efficiency $\uparrow$ and mobility $\uparrow$ Details

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- Experiment 2: Impose strict skill cut-off that governs track choice

Outcome (%-change rel. to baseline)	$\chi_0 = \chi_1 = 0$	Skill Cutoff
Output	0.04	0.12
Inequality: Gini Earnings	0.0	0.8
Interg. Mobility: -(Income rank-rank)	0.9	-6.5
Consumption Equiv. Variation	0.04	-0.01
Mean end-of-school skills	0.8	3.0
Reg. Coeff: A-Track on Skills	5.7	59.4

- Purely skill-based track choice: learning $\uparrow\uparrow$, but equality, mobility $\downarrow$ Details

Conclusion

Long-term aggregate and distributional effects of education tracking?

- Introduce theory of child learning in school with tracking into quantitative GE overlapping generations framework

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- Parental influence in track choice beyond skills affects mobility and efficiency

  → Potential of (mentoring) programs that alleviate influence (Falk et al., 2020)

  → Debate about binding track recommendations

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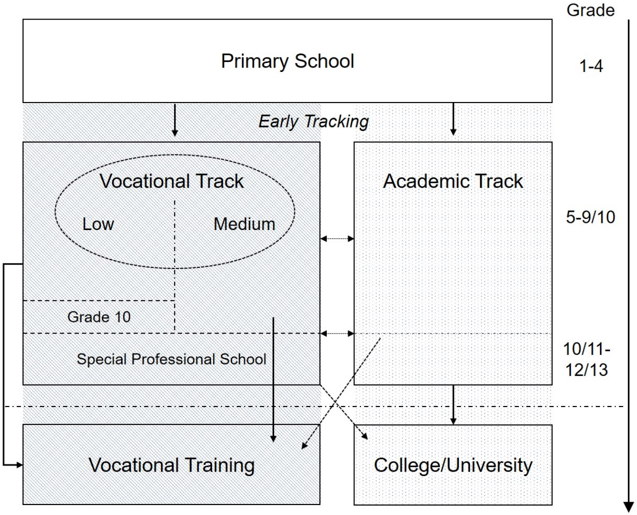
Thank you!

Back-up

German Education System Sketch Back



→ We aggregate: Academic track = "Gymnasium", Vocational Track = other schools

Details on German Education System Back

- Some states have a three-tier system (most of former West Germany), others have a two-tier system (former East) with integrated schools
- Academic track (*Gymnasium*) ends in university-entry qualification, other tracks prepare for vocational career
- Switches between tracks are possible but rare ($\approx 2.5\%$)
- Majority of states track after 4 years of primary school (i.e. children are aged 9-10), some (Berlin, Brandenburg) track after 6 years
- In most states, track selection is done by parents, in some states academic track is only possible with min grades in German and Math in primary school (Bavaria, Saxony, Thuringia, Brandenburg)
- Teacher recommendations based on grades and subjective assessment of primary school teacher should "inform" track choice

Main Differences in School Tracks

	Vocational Track	Academic Track
Teaching Level		
Intensity	32 hrs/week	35 hrs/week
Quality of Peers		
Avrg. Reading Score	-0.491	0.558
Avrg. Math Score	-0.583	0.663

- Main differences: Teaching level and peer exposure (Dustmann, Puhani, and Schönberg, 2017)
- Average per-pupil expenditures and teacher quality do not differ much between tracks

Details on Proposition 1

The allocation of children across tracks is characterized by a skill threshold $\tilde{\theta}_3$, such that all children with initial skills below $\tilde{\theta}_3$ go to one track and all children with initials skills above $\tilde{\theta}_3$ go the other track.

- If the policymaker does the track allocation, the optimal skill threshold corresponds to the average initial skill level $\tilde{\theta}_3^* = \mathbb{E}[\theta_3] = 0$.
- If parents do the track allocation, the endogenous skill threshold that emerges from this game depends on the direct peer externality α. With $\alpha > 0$, the threshold is smaller than $\tilde{\theta}_3^*$.

Back

Details on Proposition 2

- Aggregate end-of-school skills in a full tracking system are larger than in a full comprehensive system. This holds regardless of who makes the track decision, i.e. regardless of the tracking skill threshold. $\tilde{\theta}_1$. The expected gain from tracking is

$$\mathbb{E}(\theta_4|T) - \mathbb{E}(\theta_4|C) = \frac{\gamma^2}{2\delta}\left(\sigma^2_{\theta_3} - \mathbb{E}(Var[\theta_3|S])\right) \tag{7}$$

- The end-of-school skill distribution in a full tracking system has a "fatter" right tail. In case of tracking at the optimal skill threshold $\tilde{\theta}_3 = \mathbb{E}(\theta_3)$, the variance of end-of-school skills in a full tracking system is larger than the variance in a full comprehensive system iff

$$\alpha^2 + 2\alpha\left(1 + \frac{\beta\gamma}{\delta}\right) - (8-\pi)\frac{\gamma^4}{\pi\delta^2}\sigma^2_{\theta_3} > 0. \tag{8}$$

- Children with initial skills inside an non-empty interval lose from a full tracking system in terms of their end-of-school skills relative to a full comprehensive system. With $\alpha = 0$ the losses are symmetric in both tracks. With $\alpha > 0$, the losses are concentrated in the track with the lower average skill level. Back

Details on Proposition 3

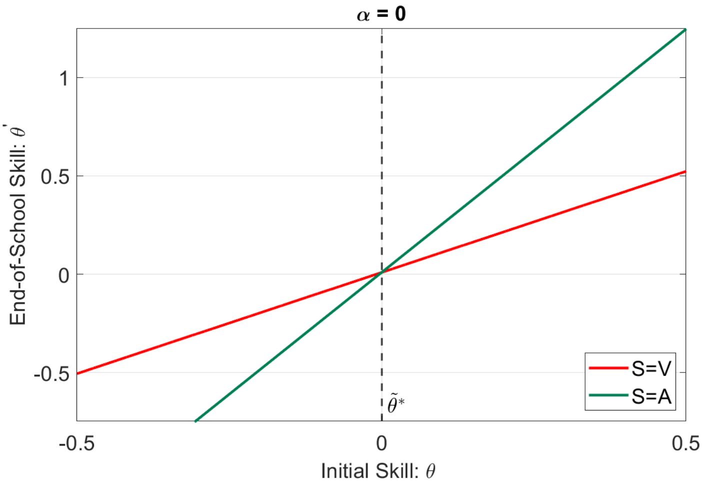
Proposition 3

Average end-of-school skills are larger in an optimal late tracking system than in an optimal early tracking system iff

$$\frac{\sigma^2_{\eta_4}}{\sigma^2_{\theta_3}} > \alpha + \left(\alpha + (1 + \frac{\beta\gamma}{\delta})\right)^2 - \frac{\beta\gamma}{\delta}\left(1 + \frac{\beta\gamma}{2\delta}\right) + \frac{\gamma^4}{2\delta^2\pi}\sigma^2_{\theta_3}.$$

Back

Illustration: Early Tracking vs Comprehensive System



- Child with $\theta = \tilde{\theta}^* = \tilde{\theta}^p$ is indifferent between tracks

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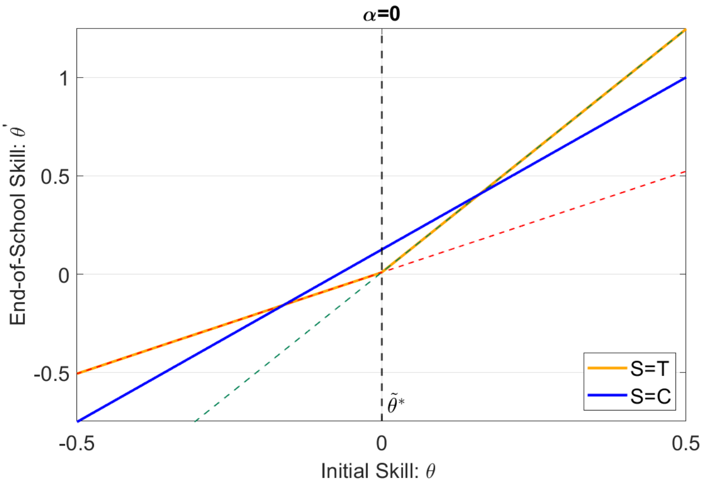

- Children around $\tilde{\theta}^*$ lose from tracking (symmetrically)

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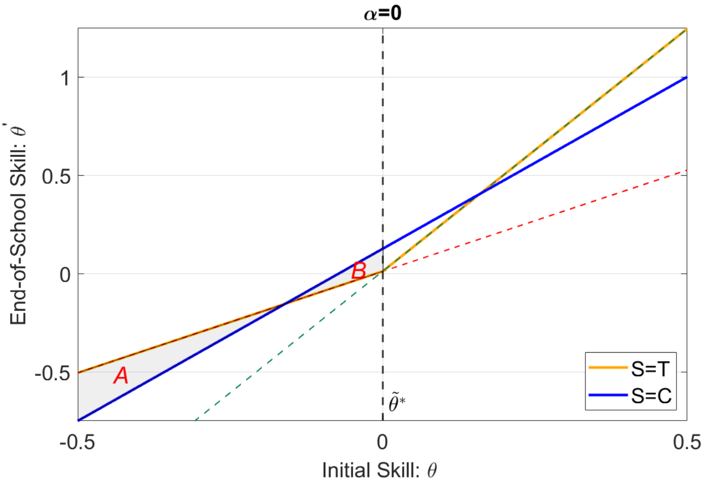

- But gains (A) from tracking outweigh losses (B) in both tracks

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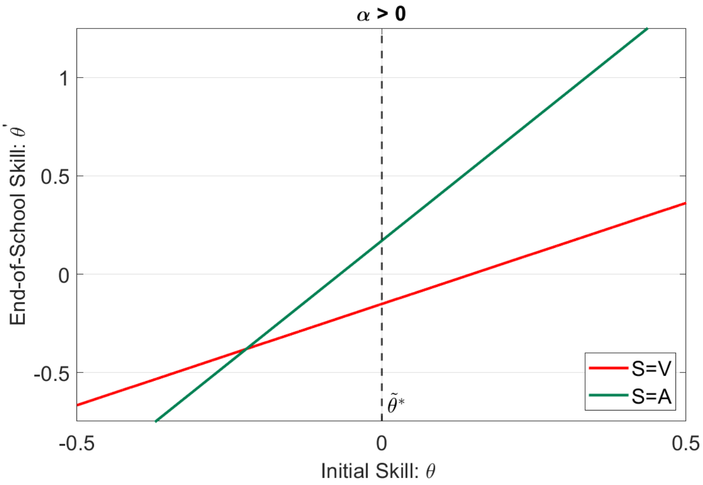

- $\alpha > 0 \implies \tilde{\theta}^*$ is not "incentive-compatible"

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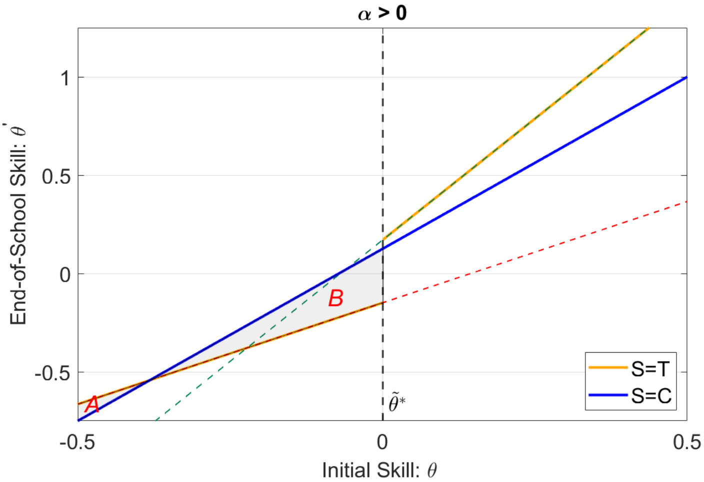
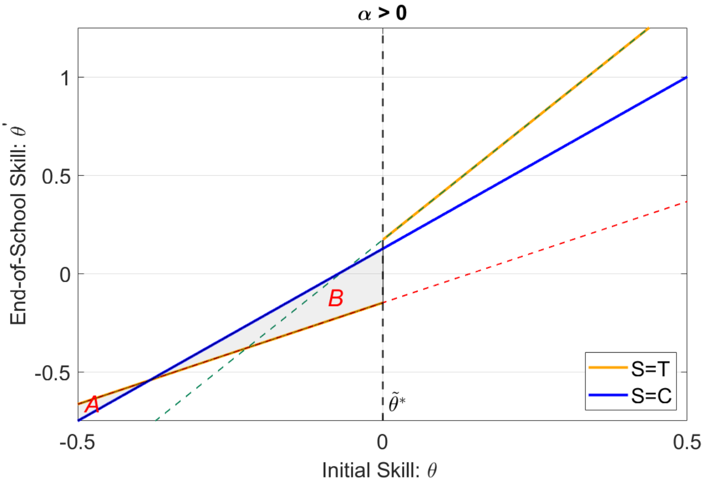


- Children in V-Track lose on average more from tracking than children in A-Track

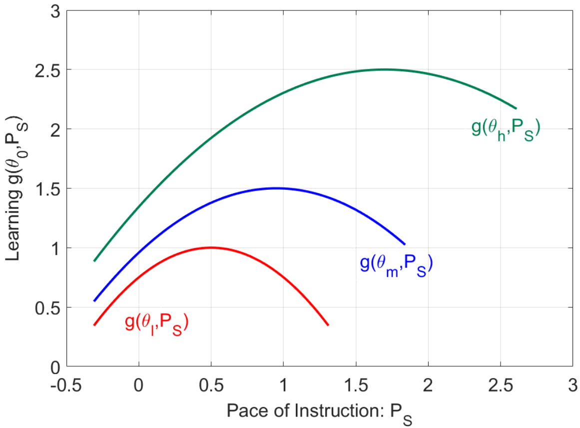
Illustration: Early Tracking vs Comprehensive System



- Children in V-Track may even, on average, learn less than in comprehensive system Back

Illustration of Pace of Instruction Back

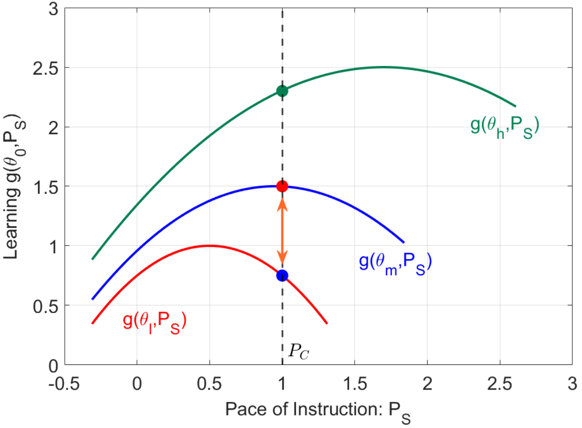
- Sketch of learning for three initial skill levels: $\theta_l < \theta_m < \theta_h$



$\implies \forall\ \theta$, there is one optimal pace, $P^*(\theta)$ with $\frac{\partial P^*}{\partial \theta} > 0$ that maximizes learning

Illustration: Tracking vs Comp. School after Shock Realization

Learning in Comprehensive Track when θ_l is shocked to θ_m and θ_m is shocked to θ_l

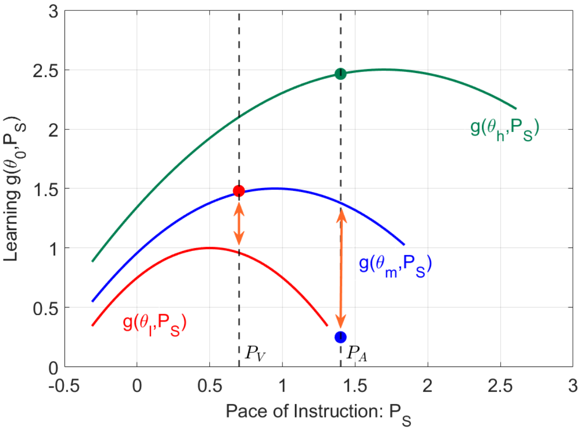


- Aggregate learning remains unaffected

Back

Illustration: Tracking vs Comp. School after Shock Realization

Learning in Tracking System when θ_l is shocked to θ_m and θ_m is shocked to θ_l

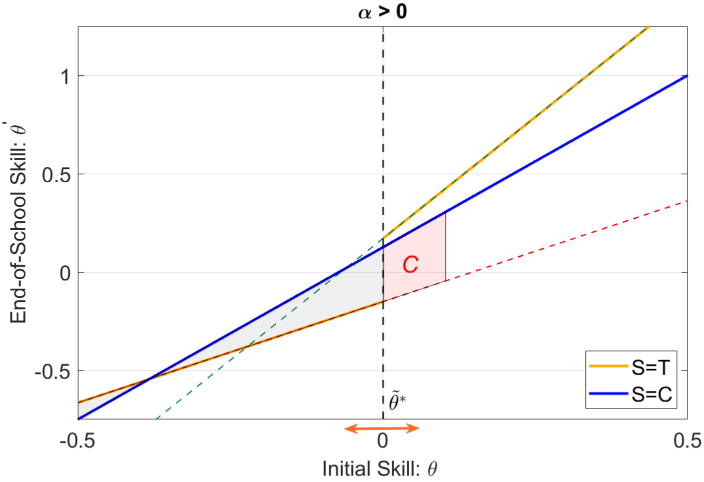


- Shocks can lead to aggregate learning losses

Back

Illustration: Child Skill Shocks with direct peer effects

Learning in terms of initial θ in tracks V and A



- Misallocation losses concentrated among children in V-Track

School Track Selection by Parental Education

Back

	Academic Track	
High SES	0.35***	0.24***
	0.02	0.02
Controls:		
Age & Gender	yes	yes
Tests	no	yes
R^2	0.2	0.36
N	2,480	2,475

Deviations from Teacher Recom.	High SES	Low SES
Academic Recom.		
Follow	94%	81%
Deviate	6%	19%
Vocational Recom.		
Follow	78%	91%
Deviate	22%	9%

Notes: Data from NEPS Starting Cohort 3. High SES $= 1$ if at least one parent has an academic school degree and household income $\geq$ 2,000 EUR/month.

- Significant conditional SES-gap in academic track attendance in grade 5

  (Falk, Kosse, and Pinger, 2020)

- Parents bias track selection towards their own educational background

Teacher Recommendations and Deviations

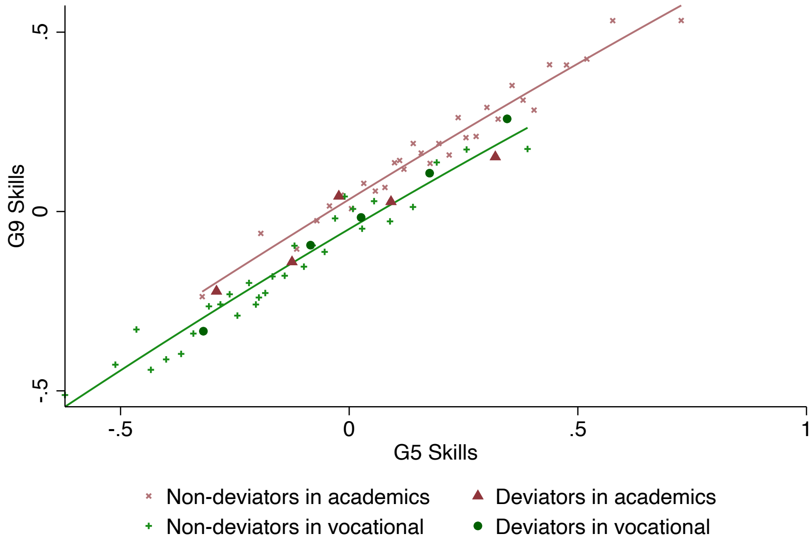
	Academic Track Recom.	
	Cohort 2	*Cohort 3*
High SES	0.11***	0.14***
	0.01	0.02
Controls:		
Age & Gender	Yes	Yes
Tests	Grades (4)	Test (5)
R^2	0.44	0.36
N	3,575	2,634

Back

Dependent Variable: Grade 9 Skills	
Panel A: Cohort 3 - Academic Track Recommendations	
Grade 5 Skills	0.757***
	(0.026)
Downward Deviators (n = 84)	-0.062***
	(0.023)
Obs.	1,101
Panel B: Cohort 3 - Vocational Track Recommendations	
Grade 5 Skills	0.760***
	(0.033)
Upward Deviators (n = 84)	0.031
	(0.022)
Obs.	591

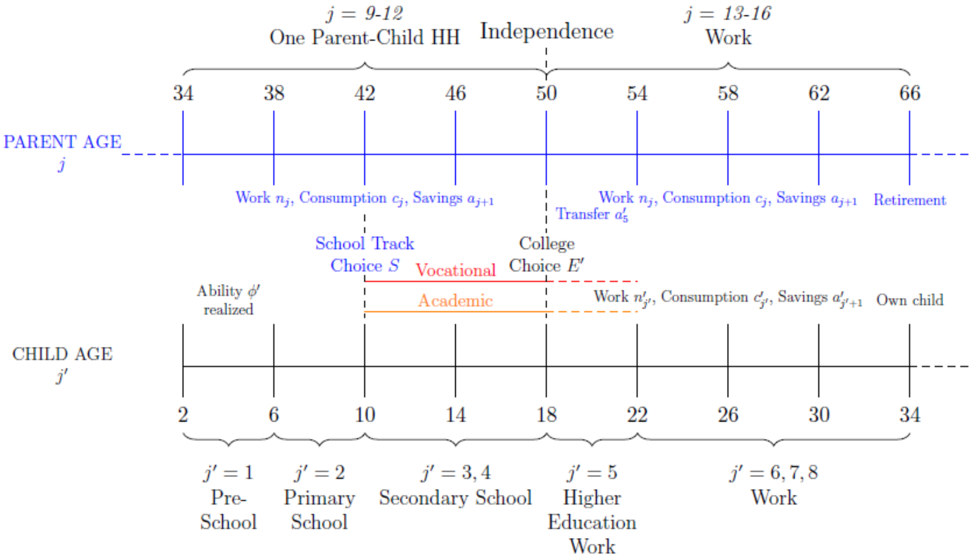
Notes: Regressions of skills in grade 9 on skills in grade 5 and deviation status for children with academic track teacher recommend. (Panel A) and children with vocational track teacher recommend. (Panel B). Models control for parental education.

Past and Future Skills by School Track and Deviator Status



Detailed Timeline of Life-Cycle Events

Back



Constraints affecting the Decision Problems each Period Back

- Budget Constraint (during work years)

$$c_j + a_{j+1} = w_E h_j n_j + (1+r)a_j + T(y_j, a_j), \tag{9}$$

  where $T(y_j, a_j)$ gives lump-sum transfers g net of progressive labor income taxes and linear capital income taxes; during retirement, agents receive pension benefits $\pi_j(h_{17}, E) = \Omega h_{17} w_E$

- Borrowing Constraint: $a_{j+1} \geq \frac{g}{1+r}$, where r is the interest rate
- Time Constraint (during working years): $n_j \in [0, 1]$
- Human Capital Growth (during working years):

$$h_{j+1} = \gamma_{j,E} h_j \varepsilon_{j+1}, \quad \log \varepsilon_j \sim \mathcal{N}(0, \sigma_\varepsilon^2), \tag{10}$$

  where $\gamma_{j,E}$ are age- and education-specific deterministic growth rates and ε_{j+1} are market luck shocks

Government

- Labor income (y) taxation according to $y_{net} = \lambda y^{1-\tau_n}$
- Capital income taxation according to $\tau_a r a_j$
- Finances retirement benefits π_j and lump-sum transfers g

Back

Value of young Parent without Child $(j = 6, 7, 8)$

$$V_j(E, h_j, a_j, \phi) = \max_{c_j > 0, a_{j+1}, n_j} \left\{ u(c_j) + \beta\, \mathbb{E}_{\varepsilon_{j+1}} V_{j+1}(E, h_{j+1}, a_{j+1}, \phi) \right\}$$

$$h_{j+1} = \gamma_{j,s} h_j \varepsilon_{j+1}$$

BC + Time Constraint + Borrowing Constraint

- In $j = 8$, parents takes expectations over future child's ability ϕ'

$\rightarrow$ $\log \phi' = \rho_\phi \log \phi + \varepsilon_\phi, \quad \varepsilon_\phi \sim \mathcal{N}(0, \sigma_\phi^2)$

Value of Parent with young Child $(j = 9, 10)$

$$V_j(E, h_j, a_j; \theta_{j'}, \phi') = \max_{c_j, a_j, n_j} \left\{ u(\frac{c_j}{q}, n_j) + \beta\, \mathbb{E}_{\varepsilon_{j+1}, \eta_{j'+1}}\, V_{j+1}(E, h_{j+1}, a_{j+1}, \theta_{j'+1}, \phi') \right\}$$

$$\text{s.t. } \theta_3 = f(\theta_2, \bar{\theta}_2, E, \eta_3)$$

$$\theta_2 = \log(\phi')$$

BC + Time Constraint + Borrowing Constraint

- In $j = 9$ no expectation over child skill uncertainty

Value of Parent with Child in Secondary School $(j = 12)$

$$V_j(E, h_j, a_j; \theta_{j'}, \phi, S) = \max_{c_j, a_j, n_j} \left\{ u(\frac{c_j}{q}, n_j) + \beta\, \mathbb{E}_{\varepsilon_{j+1}, \eta_{j'+1}}\, V_{j+1}(E, h_{j+1}, a_{j+1}, \theta_{j'+1}, \phi, S) \right\}$$

$$\text{s.t. } \theta_5 = f(\theta_4, \bar{\theta}_4, E, \eta_5)$$

BC + Time Constraint + Borrowing Constraint

Value when own Child becomes independent $(j = 13)$

Parent makes transfer decision a_5' just before child becomes independent, not knowing college taste shock $\nu'(E) \sim G^E(\nu)$

$$V_{13}(E, h_{13}, a_{13}, \phi, \theta_5, S) = \max_{a_5' \geq 0} \left\{ \widetilde{V}_{13}(E, h_{13}, a_{13} - a_5') + \Lambda \, \mathbb{E}_{\nu'} \, V_{j'=5}(\theta_5, a_5', \phi, S, E, \nu'(E)) \right\}$$

$\widetilde{V}_{13}$ is the value for a parent with savings a_{13} after the inter-vivos transfer has been made

$$\widetilde{V}_{13}(E, h_{13}, a_{13}) = \max_{c_{13}, a_{14}, n_{13}} \{u(c_{13}, n_{13}) + \beta \, \mathbb{E}_{\varepsilon_{14}} \, V_{14}(E, h_{14}, a_{14})\}$$

BC + Time Constraint + Borrowing Constraint

- Λ: weight governing dynastic altruism

Back

Values during Work before Retirement and Retirement

Model period: $j = 14, 15, 16$; Age: 54-65

$$V_j(E, h_j, a_j, \phi) = \max_{c_j, a_{j+1}, n_j} \left\{ u(c_j, n_j) + \beta \, \mathbb{E}_{\varepsilon_{j+1}} \, V_{j+1}(E, h_{j+1}, a_{j+1}, \phi) \right\}$$

BC + Time Constraint + Borrowing Constraint

- In $j = 16$, no expectation over market luck shock

Model period: $j = 17, 18, 19, 20$; Age: 66-81

$$V_j(E, h_{17}, a_j) = \max_{c_j > 0, a_{j+1} \geq \underline{a}} \{ u(c_j, 0) + \beta V_{j+1}(E, h_{17}, a_{j+1}) \}$$

$$\text{s.t. } c_j + a_{j+1} = \pi_j(h_{17}) + (1 + r) a_j - T(0, a_j).$$

Back

Equilibrium Definition i

Let $x_j \in X_j$ be the age-specific state vector of an individual of age j, as defined by the recursive representation of the individual's problems. Let its stationary distribution be $\Theta(X)$. Then, a stationary recursive competitive equilibrium for this economy is a collection of: (i) decision rules for college graduation $\{d^E(x_5)\}$, for school track $\{d^S(x_{11})\}$, consumption, labor supply, and assets holdings $\{c_j(x_j), n_j(x_j), a_j(x_j)\}$, and parental transfers $\{a_5\,(x_j)\}$; value functions $\{V_j\,(x_j)\}$; (iii) aggregate capital and labor inputs $\{K, H_0, H_1\}$; (iv) prices $\{r, w_0, w_1\}$; and (v) average skill levels among children in school track S $\{\bar{\theta}_{j,S}\}$ such that:

1. Given prices and average skill levels among children in each school track, decision rules solve the respective household problems and $\{V_j\,(x_j)\}$ are the associated value functions.
2. Given prices, aggregate capital and labor inputs solve the representative firm's problem, i.e. it equates marginal products to prices.

Equilibrium Definition ii

3. Given average skill levels among children in each school track, allocation of children in school track solves the parent's problem, i.e. actual average skill levels are consistent with parents' prior.

4. Labor market for each education level clears.
   For high-school level:

$$H_0 = \sum_{j=5}^{J_r} \int_{X_j} n_j(x_j)\ h_j\,(x_j)\, d\Theta(X \mid E = 0) + \sum_{j=5}^{5} \int_{X_j} n_j(x_j)\ h_j\,(x_j)\, d\Theta(X \mid E = 1)$$

   where the first summation is the supply of high-school graduates while the second is that labor supply of college students while studying in $j = 5$.

Equilibrium Definition iii

For college level:

$$H_1 = \sum_{j=6}^{J_r} \int_{X_j} n_j(x_j)\; h_j\,(x_j)\, d\Theta(X \mid E = 1).$$

5. Asset market clears

$$K = \sum_{j=J_e}^{J_d} \int_{X_j} a_j(x_j) d\Theta(X),$$

which implies that the goods market clears;

6. The distribution of X is stationary: $\Theta(X) = \int \Gamma(X) d\Theta(X)$.

Back

Welfare Measure Back i

- $\mathcal{V}_5^C(\theta_5, a_5, \phi, S, E^p, \Delta)$ = welfare of agents in the initial state of the economy in counterfactual C ($j = 5$) if their consumption (and that of their descendants) were multiplied by $(1 + \Delta)$

$$\mathcal{V}_5^C(\theta_5, a_5, \phi, S, E^p, \Delta) = \mathbb{E}^C \sum_{j=5}^{j=20} \beta^{j-5} v_j \left(c_j^{*C}(1+\Delta), n_j^{*C}, E^{*C}, \theta_5, S, E^p\right) \\ + \beta^{13-5} \delta \mathcal{V}_{j5}^C \left(\theta_5', a_5', \phi', S', E^{*C}, \Delta\right) \tag{11}$$

where E^p is the education of the parent, and for $j = 6, ..., 10, 12, ..., 20$

$$v_j(c_j, n_j, E, \theta_5, S, E^p) = \frac{(c_j/q)^{1-\sigma}}{1-\sigma} - b\frac{n_j^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}}, \tag{12}$$

Welfare Measure Back ii

for $j = 5$

$$v_j(c_j, n_j, E, \theta_5, S, E^p) = \frac{(c_j/q)^{1-\sigma}}{1-\sigma} - b\frac{n_j^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} - 1\{E = 1\}\, \psi(S, \theta_5, \nu(E^p)), \quad (13)$$

and for $j = 11$

$$v_j(c_j, n_j, E, \theta_5, S, E^p) = \frac{(c_j/q)^{1-\sigma}}{1-\sigma} - b\frac{n_j^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} - 1\{S = A\}\, \chi(E) \qquad (14)$$

- Policy functions are assumed to be unchanged when Δ is introduced

Welfare Measure Back iii

- Average welfare:

$$\bar{\mathcal{V}}^C(\Delta) = \sum_{S,E^p} \int_{\theta_5, a_5, \phi} \mathcal{V}^C(\theta_5, a_5, \phi, S, E^p, \Delta)\mu_C(\theta_5, a_5, \phi, S, E^p)$$

where μ_C is the distribution of initial states $\{\theta_5, a_5, \phi, S, E^p\}$ in the economy C.

- Δ^C: consumption equivalence that makes individuals indifferent between being born in the baseline economy policy $C \neq 0$, such that:

$$\bar{\mathcal{V}}^0(\Delta^C) = \bar{\mathcal{V}}^C(0).$$

Calibration Details

Preferences

Back

- Flow utility in each period j:

$$u(c_j, n_j) = \frac{(c_j/q)^{1-\sigma}}{1-\sigma} - b\frac{n_j^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}}, \tag{15}$$

  - $q > 1$ whenver a child is present in the household

Parameterization of Academic School Track Costs Back

- Stochastic school track utility costs: $\chi(E) \sim H^E(\chi) \equiv \mathcal{N}(\mu_{\chi,E}, \sigma_\chi^2)$, with

$$\mu_{\chi,E} = \mu_{\chi,A} + \begin{cases} \chi_1 & \text{if } E = 1 \\ \chi_0 & \text{if } E = 0, \end{cases} \tag{16}$$

  - $\mu_{\chi,A} > 0$: uniform utility cost from A-track attendance $\rightarrow$ match share of A-track recommendations (0.44)
  - χ_0, χ_1: asymmetric preferences for A-track by parental education $\rightarrow$ match share of deviations from track recommendations by E
  - σ_χ^2: Variance of the residuals from a regression of school track on end-of-primary-school skills

Parameterization of College Costs Back

- "Psychic" college cost function (Daruich, 2022):

$$\psi(S, \theta_5, \nu(E)) = \exp(\psi_0 + \psi_{S=V} + \psi_\theta \theta_5 + \nu(E)) \\ \nu(E) \sim G^E(\nu) \equiv \mathcal{N}(\mu_{\nu,E}, \sigma_\nu^2). \tag{17}$$

  - ψ_0, $\psi_{S=V}$: shares of academic (vocational) track who go to college
  - ψ_θ: Regression coefficient of college dummy on end-of-school test scores, controlling for school track
  - $\mu_{\nu,E=1} = \Delta(\mu_{\nu,E})$ and $\mu_{\nu,E=0} = -\Delta(\mu_{\nu,E})$: share of college children from college parents over share of college children from non-college parents
  - σ_ν^2: variance of residuals from regression of college education on end-of-school skills and school track

Human Capital Growth $\{\gamma_{j,E}\}$, $j = 5, ..., 16$ Back

- Create four-year work experience bins in *SOEP* data for each education group E
- Mincer regressions of wages on years of schooling and potential work experience, controlling for time and cohort effects (Lagakos et al., 2018)
- Assume no experience effect on wage growth in last 8 years of work to disentangle time from cohort effects

Experience (Years)	Wage Growth Non-College	College
0	1.00	1.00
4	0.96	1.15
8	1.09	1.19
12	1.10	1.11
16	1.04	1.06
20	1.02	1.01
24	1.00	0.99
28	1.01	0.97
32	0.99	0.98
36	1.01	0.99
40	0.99	1.01

1. Estimation of Child Skill Technology Back

- Technology with optimal pace implies estimation equation:

$$\theta_{i,j+1} = \omega_{0,j} + \omega_{1,j}\theta_{i,j} + \omega_{2,j}\bar{\theta}^S_{-i,j} + \omega_{3,j}\theta^2_{i,j} + \omega_{4,j}\left(\theta_{i,j} - \bar{\theta}^S_j\right)^2 + \omega_{5,j}E_i + \eta_{i,j+1}, \quad (18)$$

  - $\bar{\theta}^S_{-i,j}$: Average (log) skills of child i's *classroom* peers
  - $\bar{\theta}^S_j$: Average (log) skills in school track S
  - E_i : Parental-college dummy variable
  - $\omega_{0,j}$: function of age and gender

$\rightarrow$ Implies the restriction $\omega_{3,j} = -\omega_{4.j}$

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- Use (covariances of) three test score measures to identify latent skills (Agostinelli, Saharkhiz, and Wiswall, 2019; Cunha, Heckman, and Schennach, 2010)

Details on Measurement and Identification

1. Child Skill Technology Estimates

Back

	Grade 9 on Grade 5	
Coefficient	Variable	
$\hat{\omega}_{1,3}$	$\theta_{i,j}$	0.664***
		(0.022)
$\hat{\omega}_{2,3}$	$\bar{\theta}^S_{-i,j}$	0.003
		(0.020)
$\hat{\omega}_{3,3}$	$\theta^2_{i,j}$	0.008*
		(0.004)
$\hat{\omega}_{4,3}$	$(\theta_{i,j} - \bar{\theta}_{j,S})^2$	-0.011*
		(0.006)
$\hat{\omega}_{5,3}$	$E = 1$	0.034***
		(0.010)

Controls: Year-of-Birth, Gender, School FE.
SE clustered at school level. $N = 1{,}847$.

- Small, often stat. insignificant direct peer effects ($\hat{\omega}_2 > 0$)
- Learning decreases in distance to track average ($\hat{\omega}_4 < 0$)
- Cannot reject hypothesis $\hat{\omega}_3 = -\hat{\omega}_4$

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Back

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- Cannot reject hypothesis $\hat{\omega}_3 = -\hat{\omega}_4$

$\rightarrow$ Take these estimates for secondary school skill parameters

Externally set Parameters

Back

Parameter	Value	Description	Source
Household			
σ	2.0	Inverse EIS	Lee and Seshadri (2019)
γ	0.5	Frisch Elasticity	Fuchs-Schündeln, Krueger, Ludwig, et al. (2022)
q	1.56	Equiv. Scale	Jang and Yum (2022)
$\bar{n}(E' = 1)$	0.40	Time Cost of College	
Firm			
σ_f	1/3	E.o.S (H_0, H_1)	Ciccone and Peri (2005)
δ_f	6%	Annual Depreciation	Kindermann, Mayr, and Sachs (2020)
Government			
τ_n	0.128	Labor Tax Progressivity	Kindermann, Mayr, and Sachs (2020)
λ	0.679	Labor Tax Scale	Kindermann, Mayr, and Sachs (2020)
τ_a	0.25	Capital Tax Rate	Tax Rate on Capital Gains in Germany
g	0.06	Lump-sum Transfers	

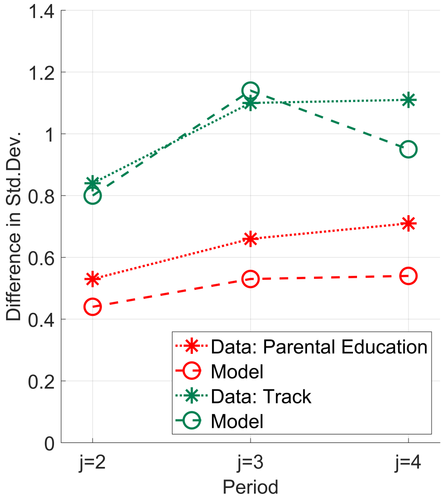
MSM Results: Remaining Parameters and Moments

Back

Parameter	Value	Description	Target	Data	Model
Preferences					
β	0.935	Discount Factor	Annl. Interest Rate	0.04	0.04
b	20.7	Labor Disutility	Avrg. Labor Supply	0.36	0.36
Λ	0.31	Parental Altruism	Transfer/Income	0.49	0.49
Child Skill Technology					
$\omega_{1,2}$	0.65	Own Skill Elasticity ($j = 2$)	Reg. θ_3 on θ_2 & E: coef. θ_2	0.649	0.649
$\omega_{5,2}$	0.072	Coefficient on E ($j = 2$)	Reg. θ_3 on θ_2 & E: coef. E	0.072	0.072
$\omega_{1,4}$	0.81	Own Skill Elasticity ($j = 4$)	$S = 1$, Reg. θ_5 on θ_4 & E: coef. θ_4	0.825	0.812
$\omega_{5,4}$	0.032	Coefficient on E ($j = 4$)	$S = 1$, Reg. θ_5 on θ_4 & E: coef. E	0.033	0.032
Initial Skills and Ability Transmission					
σ_ϕ	0.032	Std. of Ability	Variance initial skills	0.10	0.12
ρ_ϕ	0.9	Persistence of Ability	IGE (income rank)	0.24	0.23
Miscellaneous					
Ω	0.1	Pension Anchor	Replacement Rate	0.40	0.40
A	3.31	TFP	Avrg. Labor Earnings	1.0	1.0
φ	0.543	Weight V. Human Capital	College Share	0.35	0.35
σ_ε	0.011	Std. Luck Shock	Std(Log Labor Income)	0.84	0.86

Model Verification: Untargeted Child Skill Moments

Back



Moment	**Data**	**Model**
Rank-rank coefficients		
$\text{Rank}_{j=3} - \text{Rank}_{j=4}$ if S = 1	0.68	0.70
$\text{Rank}_{j=3} - \text{Rank}_{j=4}$ if S = 0	0.74	0.71
Skill evolution during secondary school		
Reg. θ_4 on θ_3 and E: coef. θ_4	0.81	0.66
Reg. θ_4 on θ_3 and E: coef. E	0.04	0.04

Model Verification: Education Choices and Inequality

Back

Moment	Data	Model
Education Choices		
% in academic track if college parents	66%	70%
% in academic track if non-college parents	32%	32%
Reg. S on θ_3: coef. θ_3	0.90	1.04
Reg. E on θ_5 & S: coef. S	0.41	0.39
Inequality		
Gini Coefficient of Income	0.29	0.26
College Wage Premium	1.35	1.46

Model Verification: Effect of Track on Labor Market Outcomes

- Dustmann, Puhani, and Schönberg (2017): Track choice has no long-term labor market effects for children at the *margin* between school tracks

Model Verification: Effect of Track on Labor Market Outcomes

- Dustmann, Puhani, and Schönberg (2017): Track choice has no long-term labor market effects for children at the *margin* between school tracks

→ Compare children with *same states* $(E, h_{11}, a_{11}, \phi, \theta_3)$ at time of tracking who go to different school tracks (because of taste shocks χ)

→ Academic track choice for these *model-marginal* children yields 6.6% higher PV of lifetime earnings, 4.4% higher PV of lifetime wealth

Back

Details on Estimation of Child Skill Formation

Log-Linear Measurement System

- For each j, we have (at least) three measures $m = 1, 2, 3$ for latent skills, given by:

$$M_{i,j,m} = \mu_{j,m} + \lambda_{j,m}\theta_{i,j} + \epsilon_{i,j,m} \tag{19}$$

- The measures for skills constitute test scores for different domains
- $\epsilon_{i,j,m}$: Measurement errors with $\mathbb{E}[\epsilon_{j,m}] = 0 \ \forall \ m, j$

Assumptions

1. $\lambda_{j,1} = 1 \ \forall \ j$
2. $\mathbb{E}[\theta_{i,j}] = 0 \ \forall \ j$
3. Measurement errors are independent contemporaneously across measures, and from latent variables

Identification

- Under the assumptions, we can identify $\mu_{j,m}$ and $\lambda_{j,m}$ from ratios of covariances of the measures

$\rightarrow$ identify latent skills up to measurement error

$$\theta_{i,j} = \frac{M_{i,j,m} - \mu_{j,m}}{\lambda_j} - \frac{\epsilon_{i,j,m}}{\lambda_{j,m}} = \tilde{M}_{i,j,m} - \frac{\epsilon_{i,j,m}}{\lambda_{j,m}} \tag{20}$$

- Rewrite empirical analogue of (18) in terms of observed $\tilde{M}_{i,j,m}$ such that it can be estimated from data

$\rightarrow$ Residuals contain structural errors η and measurement errors ϵ

$\rightarrow$ Aggregate measure into unbiased factor using Bartlett scores (Agostinelli, Doepke, et al., 2023)

Back

Data

- German National Educational Panel Survey (NEPS), Starting Cohorts 2,3,4: 2011-2018

$\rightarrow$ longitudinal data on child competencies and school, classroom and home environments

$\rightarrow$ independent ($\sim$ biannual) tests on math, reading, scientific, and other domains

- We estimate using test measures $\tilde{M}_{i,j,m}$ between four periods:

  $j^c = 2$ primary school (ages 6-10)

  $j^c = 3$ first stage of secondary school (ages 10-14)

  $j^c = 4$ second stage of secondary school (ages 14-18)

- Assume latent variables and errors are normally distributed

Back

Details on Results

Lifetime Inequality Decomposition by Age Back

- How much of inequality in lifetime earnings (LFE) and wealth (LFW) is accounted for at earlier life-stages? Hugget et al. (2011), Lee and Seshadri (2019)

$\rightarrow$ Calculate
$$\frac{Var(\mathbb{E}[LFE|\text{States at Age } j])}{Var(LFE)}$$

Lifetime Inequality Decomposition by Age

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→ Calculate

$$\frac{Var(\mathbb{E}[LFE|\text{States at Age } j])}{Var(LFE)}$$

	Explained Variance	
Life Stage	*LFE*	*LFW*
Independence (Age 18)	69%	65%
School Track Choice (Age 10)	30%	33%
Pre-Birth (Parent Age 30)	14%	21%

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	Explained Variance	
Life Stage	LFE	LFW
Independence (Age 18)	69%	65%
School Track Choice (Age 10)	30%	33%
Pre-Birth (Parent Age 30)	14%	21%

- $\sim$ 2/3 of lifetime inequality explained at age 18

$\rightarrow$ Human capital differences most important determinant *(comparable to U.S.)*

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$\rightarrow$ Calculate
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Life Stage	Explained Variance *LFE*	*LFW*
Independence (Age 18)	69%	65%
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Pre-Birth (Parent Age 30)	14%	21%

- $\sim$ 1/3 of inequality explained at school tracking age

$\rightarrow$ Significant role of skill development during secondary school

Lifetime Inequality Decomposition by Age

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$\rightarrow$ Calculate

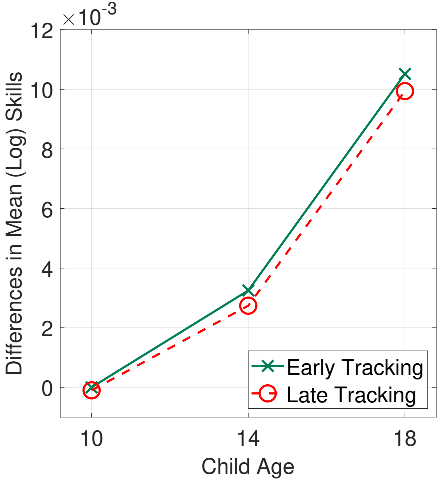
$$\frac{Var(\mathbb{E}[LFE|\text{States at Age } j])}{Var(LFE)}$$

Life Stage	Explained Variance LFE	Explained Variance LFW
Independence (Age 18)	69%	65%
School Track Choice (Age 10)	30%	33%
Pre-Birth (Parent Age 30)	14%	21%

- $\sim 1/5$ of inequality in lifetime wealth explained by parent states before birth

$\rightarrow$ $> 1/3$ of variance in transfers

Mean Skills in Early versus Late Tracking

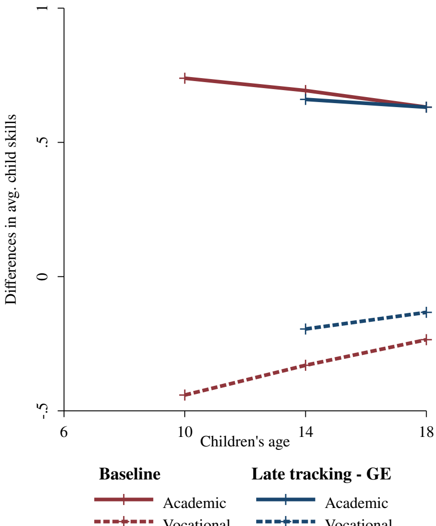


- Changes relative to log skills at age 10 in ET (normalized to mean 0) Back

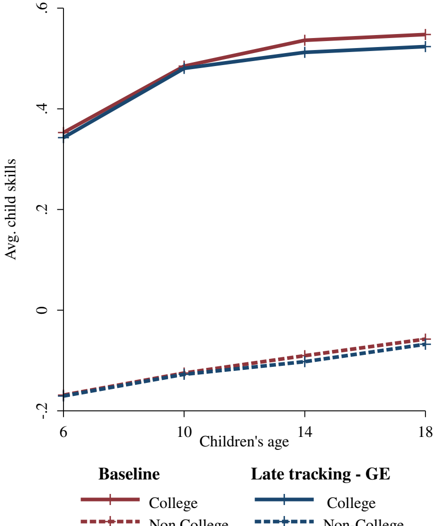
Skill Differences by Track and Parental Education: ET vs. LT

Back

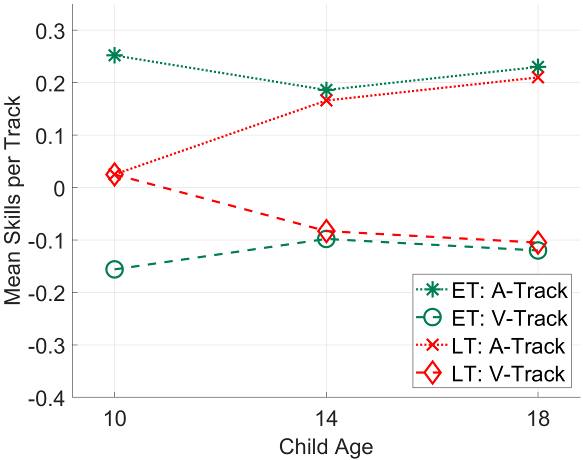
(a) by school track



(b) by parental education



Change in Average Skills for 1st Cohort after Late Tracking



Back

Parental Influence on School Track Choice Back

- Parental education second most important driver of track choice
  1. Direct effect of parental education in child skill development: $\omega_{5,j=3,4} > 0$
  2. Asymmetric A-track utility costs: $\chi_0 > \chi_1$
  3. Tastes for college: $\Delta(\mu_{\nu,E}) \neq 0$

Parental Influence on School Track Choice Back

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  1. Direct effect of parental education in child skill development: $\omega_{5,j=3,4} > 0$
  2. Asymmetric A-track utility costs: $\chi_0 > \chi_1$
  3. Tastes for college: $\Delta(\mu_{\nu,E}) \neq 0$
- Utility costs and college tastes are quantitatively more important

	Coefficient estimates: Regression of A-track on states			
	Baseline	$\omega_{5,j=3,4} = 0$	$\chi_0 = \chi_1 = 0$	$\Delta(\mu_{\nu,E}) = 0$
θ_3	0.78	0.79	0.82	0.86
$E = 1$	0.42	0.38	0.32	0.18

Notes: Results from counterfactual steady state distributions. Controls for ϕ, h_{11}, a_{11}

Timing of Tracking Results - Educational Outcomes

Back

Economy Tracking Age	PE 14	GE 14	GE Never
% Academic track	+5.4	+2.0	
. . . if college parents	+2.2	+1.5	
. . . if non-college parents	+6.8	+2.4	
% College	+3.9	-0.3	-0.2
. . . if college parents	+2.9	+0.5	-18.1
. . . if non-college parents	+3.4	-0.9	+16.7
. . . if academic track	-0.4	-1.7	
. . . if vocational track	+3.0	+0.5	
Correlation between academic track and initial skills	-20	-14	
Correlation between end-of-school skills and initial skills	-0.5	-0.1	-3.3
Correlation between college graduation and initial skills	-18	-12.6	-69.5
Correlation between college parents and end-of-school skills	-6.0	-6.0	-26.2
Correlation between college graduation and end-of-school skills	-4.1	-3.5	-12.9

Parental Influence on Track - Educational Outcomes

Back

	$\chi_0 = \chi_1 = 0$	Skill Threshold
% Acadmic track	-0.7	0.0
... if college parents	-8.6	-12
... if non-college parents	+9.6	+15.6
% College	0.3	0.0
... if college parents	-3.5	-8.8
... if non-college parents	+ 3.9	+7.2
... if academic track	+0.5	-3.5
... if vocational track	0.0	+14.4
Average end-of-school skills ($\bar{\theta}_5$)	+0.8	+3.0
Average skills in V-Track upon tracking ($\bar{\theta}_3 \| S = V$)	-0.4	-50.0
Average skills in A-Track upon tracking ($\bar{\theta}_3 \| S = A$)	+1.2	+38.5
Variance of end-of-school skills ($Var(\theta_5)$)	+0.2	+1.9
Variance in V-Track upon tracking ($Var(\theta_3 \| S = V)$)	-0.4	-39.1
Variance in A-Track upon tracking ($Var(\theta_3 \| S = A)$)	-0.4	-18.1
Correlation between A-Track and Skills in period 3	+5.7	+59.4
Correlation between academic track and initial skills	+5.4	+79.5
Correlation between end-of-school skills and initial skills	-0.6	+0.9
Correlation between college graduation and initial skills	+0.9	+54.1