Dynamic Macroeconomics: Topic I

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One-period general equilibrium model

- We introduce a one-period, general equilibrium, macroeconomic model
- Representative consumer and representative firm
  - Under certain assumptions, equivalent to economy with many identical consumers and many identical firms
- Despite its simplicity, useful to introduce key notions
- **Competitive equilibrium**
  - Agents optimize *taking prices as given*
  - All markets clear
- **Pareto optimality**
  - Under certain assumptions, competitive equilibrium allocation is Pareto optimal

Preferences, endowments and technology

Whenever we set up a macroeconomic model, we have to specify **three fundamental components**:

- Preferences of consumers
- Technology available to firms
- Endowments of resources available to consumers and firms

Preferences

- Preferences of consumers: utility function

$$u(c, \ell)$$

  where c is consumption and ℓ is leisure

- Here $u(c, \ell)$ is increasing and concave in both arguments. This implies that
  - Marginal utility of consumption is *positive* and *decreasing* in consumption:

$$u_c(c, \ell) > 0, \;\; u_{cc}(c, \ell) < 0$$

  - Marginal utility of leisure is *positive* and *decreasing* in leisure:

$$u_\ell(c, \ell) > 0, \;\; u_{\ell\ell}(c, \ell) < 0$$

- Note: u_c is the partial derivative of u with respect to c, u_{cc} is the *second* partial derivative, etc.

Technology

- The representative firm produces consumption goods according to technology
$$y = zf\left(k, n\right)$$
where y is output, k is capital input, n is labor input and z is total factor productivity
- Production function f has positive and diminishing returns to capital and labor, and is homogeneous of degree one (constant returns to scale), so that
$$f\left(\lambda k, \lambda n\right) = \lambda f\left(k, n\right)$$
for all $\lambda > 0$. By Euler's theorem (see Appendix),
$$f\left(k, n\right) = f_k\left(k, n\right)k + f_n\left(k, n\right)n$$
- **Economic implication**. If k and n are paid their marginal products, then payments to production factors are equal to total output $\implies$ pure economic profits are zero

Endowments

- The representative consumer is endowed with one unit of time, which can be split between **work** and **leisure**, i.e.

$$n + \ell = 1$$

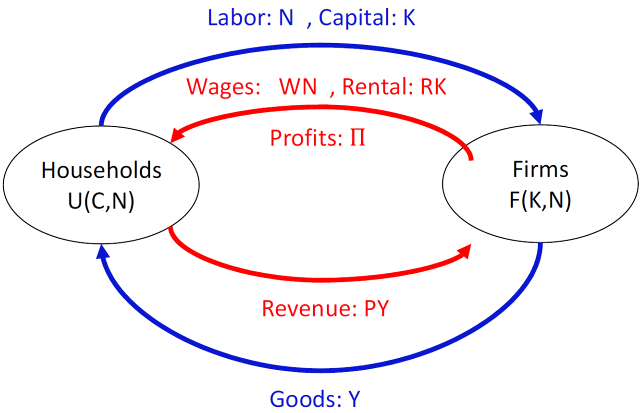
  where n is work and ℓ is leisure
- Consumer also owns k_0 units of **capital**, which can be rented to firm.
- Consumer owns the firm, so firm's profits (if any) are distributed to consumer as **dividends**
- Firm buys capital and labor from the consumer.

Competitive Equilibrium: Road-map

- We will see that in a competitive equilibrium
  - Consumers maximize utility subject to budget constraint
  - Firms maximize profits subject to technological constraint
  - All markets clear
- What about prices?
  - From microeconomics, we know that in general equilibrium we can at most determine all *relative prices* $\Longrightarrow$ price of one good can be set equal to 1 without loss of generality
  - Three markets: consumption goods, capital and labor
  - Normalize the price of consumption to 1 (so, consumption is the *numeraire*) and let r, w denote price of capital and labor respectively.

Flows of Goods and Payments

We normalize price of consumption goods, $P = 1$. Furthermore, $N = 1 - \ell$, so wage income is $W * N = W * (1 - \ell)$, where ℓ is leisure



Consumer's Problem

- Each consumer treats $\{w, r\}$ as fixed (price-taking assumption!) and solves

$$\max_{c,\ell,k_s} u(c,\ell)$$

subject to

$$c = w(1-\ell) + rk_s + \pi$$

$$0 \leq k_s \leq k_0$$

$$c \geq 0,\ 0 \leq \ell \leq 1$$

Consumer's Problem, cont'd

- Here k_s is the quantity of capital that the consumer rents to firms
- Consumer receives labor income $w(1-\ell)$, capital income rk_s and dividends from ownership of firms π
  - In equilibrium $\pi = 0$ (see this later)
- We must have

$$c > 0, \; 0 < \ell < 1 \text{ and } k_s = k_0$$

Why?

Consumer's Problem: Solution

- Since constraints $c \geq 0,\ 0 \leq \ell \leq 1$ can be ignored and $k_s = k_0$, $\pi = 0$, we can write Lagrangian

$$L(c, \ell, \lambda) = u\left(c, \ell\right) + \lambda\left[w\left(1 - \ell\right) + rk_0 - c\right]$$

where $\lambda \geq 0$ is Lagrange multiplier.

- First-order conditions for optimality are:

$$\frac{\partial L}{\partial c} = u_c\left(c, \ell\right) - \lambda = 0$$
$$\frac{\partial L}{\partial \ell} = u_\ell\left(c, \ell\right) - \lambda w = 0$$
$$\frac{\partial L}{\partial \lambda} = w\left(1 - \ell\right) + rk_0 - c = 0$$

Consumer's Problem: Solution

- Eliminating the multiplier from the first two conditions, we get

$$wu_c(c,\ell) = u_\ell(c,\ell)$$

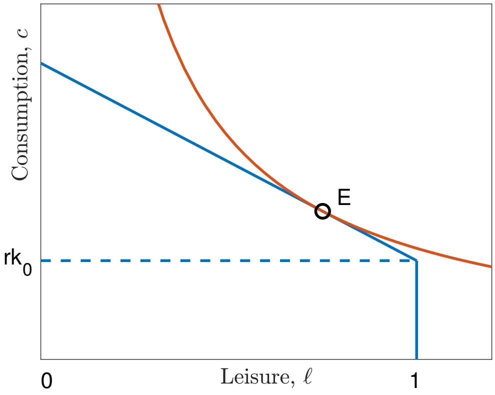
- Interpretation:
    - If the consumer allocates one unit of time to leisure he enjoys $u_\ell(c,\ell)$
    - If he spends that time at work, he increases consumption by w, so his utility increases by $wu_c(c,\ell)$
    - At the optimum, he must be indifferent b/w allocating marginal unit of time b/w two alternatives.
- Another interpretation:

$$\frac{u_\ell(c,\ell)}{u_c(c,\ell)} = w$$

i.e. the *marginal rate of substitution* between leisure and consumption equals the *wage* rate.

Consumer's Problem: Graph

- The consumer maximizes utility at E, where the budget constraint, which has slope $-w$, is tangent to the highest indifference curve, where an indifference curve has slope $-\frac{u_\ell}{u_c}$. There is a kink at $\ell = 1$, $c = rk_0$



Consumer's Problem: Solution

- The optimal quantities (c, ℓ) chosen by the consumer satisfy the two equations:

$$wu_c(c, \ell) - u_\ell(c, \ell) = 0$$

$$w(1-\ell) + rk_0 - c = 0$$

- We can substitute the second equation in the first and obtain the following single equation

$$wu_c(w(1-\ell) + rk_0, \ell) - u_\ell(w(1-\ell) + rk_0, \ell) = 0$$

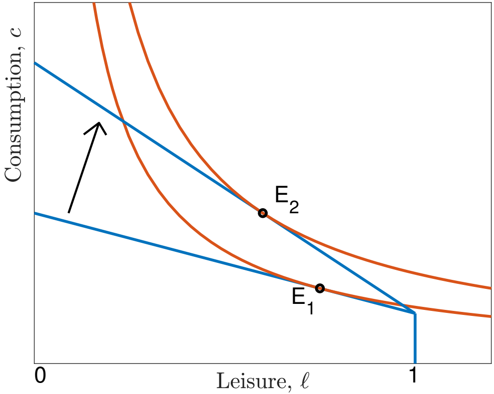
This defines optimal leisure ℓ as an *implicit function* of (w, r, k_0).

Comparative Statics

- How does the consumer respond to a change in non-labor income rk_0 ?
  - An increase in rk_0 induces a pure income effect: wage remains the same while disposable income increases
  - Since c and ℓ are *normal* goods, an increase in income implies that the consumer chooses more c and more ℓ, hence labor supply falls
- How does the consumer respond to a change in real wage w ?
  - **Substitution effect**: w increases $\implies$ leisure becomes more expensive relative to consumption $\implies$ consumer substitutes away from leisure *implies* $\uparrow c$, $\downarrow \ell$
  - **Income effect**: w increases $\implies$ budget constraint shifts out $\implies$ consumer consumes more consumption goods *and* more leisure, $\uparrow c$, $\uparrow \ell$
  - Quantity of c increases but effect on ℓ and labor supply is ambiguous

Comparative Statics: Increase in Real Wage

- Increase in w makes budget constraint steeper. Kink in budget constraint remains fixed. In figure, substitution effect dominates: leisure decreases and labor supply increases (optimal bundle moves from E1 to E2)



Firm's Problem

- Each firm chooses k and n to maximize profits π, treating r and w as given (*price-taking* assumption again):

$$\pi = \max_{k \geq 0, n \geq 0} \{y - rk - wn\}$$

subject to

$$y = zf(k, n)$$

Firm's Problem, cont'd

- First-order conditions for an optimum:

$$\begin{aligned} zf_k(k,n) &= r \\ zf_n(k,n) &= w \end{aligned}$$

- Since f is homogeneous of degree 1, partial derivatives are homogeneous of degree 0, hence

$$\begin{aligned} f_k(k,n) &= f_k\left(\frac{k}{n},1\right) \\ f_n(k,n) &= f_n\left(\frac{k}{n},1\right) \end{aligned}$$

- Only the capital-to-labor ratio k/n is determined, not k and n separately $\implies$ Optimal scale of operation of the firm is *indeterminate*.

Firm's Problem, cont'd

- Moreover, since f has constant returns to scale, Euler's theorem (see Appendix) implies

$$f(k,n) = f_k(k,n)\,k + f_n(k,n)\,n$$

hence

$$\begin{aligned}\pi &= zf(k,n) - rk - wn \\ &= zf(k,n) - zf_k(k,n)\,k - zf_n(k,n)\,n \\ &= 0\end{aligned}$$

- Constant returns to scale *and* perfect competition $\Longrightarrow$ equilibrium profits are zero
- No need to be concerned with how firm's profits are distributed

Competitive Equilibrium

- Exogenous vs endogenous variables
- Exogenous variables, determined *outside* of the model:

$$z, k_0$$

- Endogenous variables, determined *inside* of the model

$$c, \ell, n, k, w, r$$

Competitive Equilibrium

- A competitive equilibrium is a set of quantities $\{c, \ell, n, k, y\}$ and prices $\{w, r\}$ which satisfy
    1. *Given* (w, r), consumers choose (c, ℓ) optimally

    $$\max_{c \geq 0, \ell \in [0,1]} u(c, \ell) \text{ s.t. } c = w(1 - \ell) + rk_0$$

    2. *Given* (r, w), firms choose (k, n) optimally

    $$\pi = \max_{k \geq 0, n \geq 0} [y - wn - rk] \text{ s.t. } y = zf(k, n)$$

    3. All markets clear (demand = supply):

    $$\begin{aligned} n &= 1 - \ell \ \text{(Labor)} \\ k &= k_0 \ \text{(Capital)} \\ c &= y \ \text{(Consumption goods)} \end{aligned}$$

Walras' Law

- **Property**. The total value of *excess demand* across markets is identically equal to zero, i.e.

$$1 \cdot (c - y) + r\,(k - k_0) + w\,[n - (1 - \ell)] = 0 \text{ (Walras' Law)}$$

  - To see this, recall that consumer's budget constraint implies:

$$c - w\,(1 - \ell) - rk_0 = 0$$

  - Firm's profit maximization implies (zero profits):

$$-y + rk + wn = 0$$

  - Summing up the two equations above yields

$$c - y + w\,(n - (1 - \ell)) + r\,(k - k_0) = 0$$

Walras' Law

- **Key observation:** By Walras' law, if there are 3 markets and 2 of these markets are in equilibrium, then the additional market is also in equilibrium

$$c - y + w\left(n - (1 - \ell)\right) + r\left(k - k_0\right) = 0$$

- Indeed, suppose labor and capital markets clear:

$$\begin{aligned} n &= 1 - \ell \\ k &= k_0 \end{aligned}$$

- Then

$$c = y$$

  is automatically satisfied

- Important: We can omit the market clearing condition for consumption goods!

Counting Equations

- We have 7 endogenous variables $\{c, \ell, n, k, y, w, r\}$ to pin down $\implies$ We need 7 independent equations
- 2 optimality conditions for consumer:

$$wu_c(c,\ell) - u_\ell(c,\ell) = 0 \tag{E1}$$

$$c = w(1-\ell) + rk_0 \tag{E2}$$

- 3 Optimality conditions and technological constraint for the firm:

$$zf_k(k,n) = r \tag{E3}$$

$$zf_n(k,n) = w \tag{E4}$$

$$zf(k,n) = y \tag{E5}$$

- 2 market clearing conditions (we discard the third one due to Walras' law):

$$1-\ell = n \tag{E6}$$

$$k_0 = k \tag{E7}$$

Counting Equations

- To make the system of equations more manageable, we can substitute out c and w in (E1) obtaining an equation that depends only on ℓ

$$zf_n\left(k_0, 1-\ell\right) u_c\left(zf\left(k_0, 1-\ell\right), \ell\right) - u_\ell\left(zf\left(k_0, 1-\ell\right), \ell\right) = 0 \tag{CE}$$

- Given the solution for ℓ, solve for the remaining variables

$$r = zf_k\left(k_0, 1-\ell\right)$$

$$w = zf_n\left(k_0, 1-\ell\right)$$

$$n = 1-\ell$$

$$c = zf\left(k_0, 1-\ell\right)$$

Pareto Optimality

- Assume that instead of markets there is a **social planner** who:
  - controls all resources in the economy
  - is benevolent: her objective is to make the representative consumer as well off as possible
  - does not have to deal with markets: she can simply order the representative firm to hire a given quantity of labor and produce a given quantity of consumption goods
  - has the power to coerce the consumer into supplying the required amount of labor and capital

Social Planner's Problem

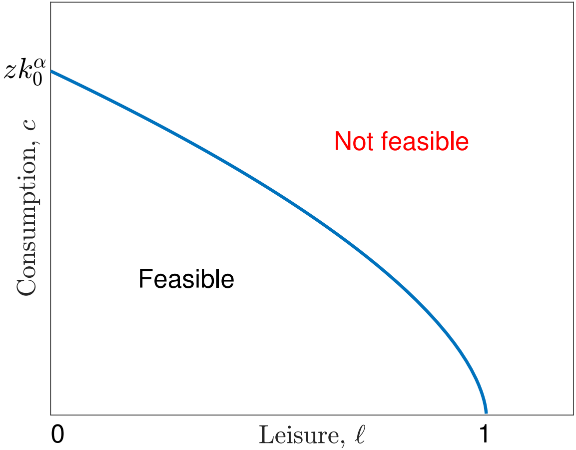
- The social planner's problem is to choose c and ℓ, given the technology for converting ℓ into c, to make the representative consumer as well off as possible:

$$\max_{c\geq 0, \ell\in[0,1]} u\left(c,\ell\right) \quad \text{s.t. } c \leq zf\left(k_0, 1-\ell\right)$$

- Inequality $c \leq z\ f\left(k_0, 1-\ell\right)$ describes the production possibility set: this is the set of all *feasible* $\left(c,\ell\right)$
- Social planner does not deal with markets but cannot violate this constraint
  - If planner wishes to give more c, leisure ℓ must decrease, i.e. the representative consumer has to work more

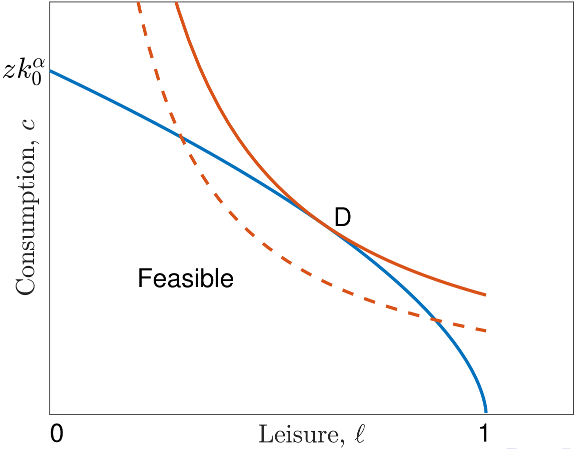
Production Possibility Set: Figure

- Assume $f(k_0, 1-\ell) = k_0^{\alpha}\ (1-\ell)^{1-\alpha}$, $\alpha \in (0,1)$
- Production possibility frontier (blue): $c = zk_0^{\alpha}(1-\ell)^{1-\alpha}$
- All allocations that lie either *on* or *below* the PPF are *feasible*



Production Possibility Set: Figure

- Production possibility frontier (blue): $c = zk_0^\alpha(1-\ell)^{1-\alpha}$
- Planner's indifference curves (red): (c,ℓ) s.t. $u(c,\ell) = \overline{u}$
- **Allocation D is the optimal choice for the planner.**



Social Planner's Problem: Solution

- Set up the Lagrangian

$$L = u\left(c, \ell\right) + \lambda\left[zf\left(k_0, 1-\ell\right) - c\right]$$

- First-order conditions:

$$\frac{\partial L}{\partial c} = u_c\left(c, \ell\right) - \lambda = 0$$
$$\frac{\partial L}{\partial \ell} = u_\ell\left(c, \ell\right) - \lambda z f_n\left(k_0, 1-\ell\right) = 0$$
$$\frac{\partial L}{\partial \lambda} = zf\left(k_0, 1-\ell\right) - c = 0$$

- Therefore, solution $\left(c^{SP}, \ell^{SP}\right)$ characterized by

$$u_\ell\left(c^{SP}, \ell^{SP}\right) = u_c\left(c^{SP}, \ell^{SP}\right) \times zf_n\left(k_0, 1-\ell^{SP}\right) \quad \text{(SP1)}$$
$$c^{SP} = zf\left(k_0, 1-\ell^{SP}\right) \quad \text{(SP2)}$$

Social Planner's Problem: Solution

$$u_\ell\left(c^{SP},\ell^{SP}\right) = u_c\left(c^{SP},\ell^{SP}\right) \times zf_n\left(k_0, 1-\ell^{SP}\right)$$
$$c^{SP} = zf\left(k_0, 1-\ell^{SP}\right)$$

- We can rewrite the first condition as

$$\underbrace{\frac{u_\ell\left(c^{SP},\ell^{SP}\right)}{u_c\left(c^{SP},\ell^{SP}\right)}}_{MRS} = \underbrace{zf_n\left(k_0, 1-\ell^{SP}\right)}_{MRT}$$

- This states the the marginal rate of substitution of leisure for consumption is equal to the marginal product of labor (the marginal rate of transformation) at the Pareto optimum.

Competitive Equilibrium is Pareto Optimal

- Recall that the competitive equilibrium quantities of consumption and leisure $\left(c^{CE}, \ell^{CE}\right)$ are given by

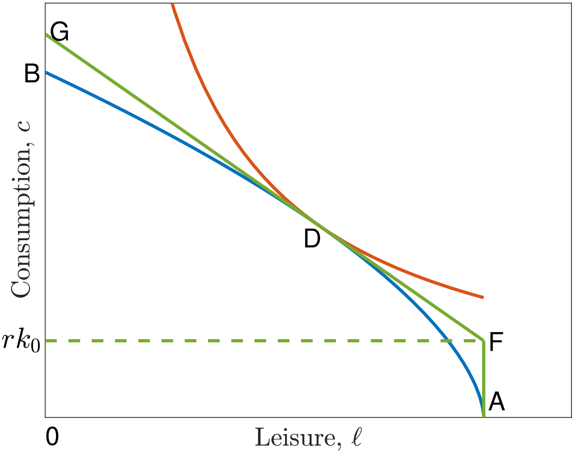
$$zf_n\left(k_0, 1-\ell^{CE}\right)u_c\left(c^{CE}, \ell^{CE}\right) - u_\ell\left(c^{CE}, \ell^{CE}\right) = 0$$

$$c^{CE} = zf\left(k_0, 1-\ell^{CE}\right)$$

- But the equations above are identical to (SP1) and (SP2), which solve for Pareto-optimal quantities $\left(c^{SP}, \ell^{SP}\right)$
- As a result, the competitive equilibrium and the Pareto optimum are the same thing in this economy!
- Graphical representation $\rightarrowtail$ Next slide

Competitive equilibrium and Pareto optimum are identical

- AB is the production frontier and the Pareto optimum is at D, where highest indifference curve is tangent to the production frontier.
- In competitive equilibrium, consumer faces budget constraint AFG and maximizes at point D where slope of budget line, $-w$, is equal to $-\frac{u_\ell}{u_c}$



Pareto Optimality: Welfare Theorems

- Competitive equilibrium and Pareto optimum are identical here. Not a coincidence!
- *Under some restrictions*, it is true that
  1. A competitive equilibrium is Pareto optimal (First Welfare Theorem)
  2. Any Pareto optimum can be supported as a competitive equilibrium with an appropriate choice of endowments (Second Welfare Theorem)
- The non-technical assumptions for (1) and (2) to hold, include
  - Absence of externalities
  - Absence of distorting taxes
  - Completeness of markets

Pareto Optimality: Implications

- **Normative implication**
  - Say we can explain a particular phenomenon (e.g. business cycles) using a competitive equilibrium model where the first welfare theorem holds.
  - We can argue that the existence of such a phenomenon does not require government intervention
- "**Computational**" **implication**
  - In general, it is easier to solve the social planner's problem than the competitive equilibrium, but we are interested in the latter
  - We can find the competitive equilibrium by first solving for (c, ℓ) from the social planner's problem
  - Then we find the prices (w, r) from the marginal pricing conditions of the firm

The road so far.. and the way ahead

- So far we have built a model with
  - Optimizing consumers
  - Optimizing firms
  - A notion of equilibrium
- Let's begin to ask the model questions!
- We run **experiments** with the model
  - What are the effects of a change in Total Factor Productivity (i.e. $z \uparrow$)?
  - What are the effects of a change in Government Expenditures? (we need to introduce government into model..)
- First experiment: change in TFP $\rightarrow$ next

Experiment

- Suppose consumer has utility function

$$u(c, \ell) = \log c + \ell$$

- The production technology is given by

$$f(k, n) = k^{\alpha} n^{1-\alpha},\ 0 < \alpha < 1$$

- The social planner's problem is

$$\max_{c \geq 0, \ell \in [0,1]} [\log c + \ell] \text{ s.t. } c = z k_0^{\alpha} (1 - \ell)^{1-\alpha}$$

or

$$\max_{\ell \in [0,1]} \left[\log \left(k_0^{\alpha} (1 - \ell)^{1-\alpha} \right) + \ell \right]$$

(Alternatively, use Lagrangian)

Experiment, cont'd

- The first-order condition for a maximum is

$$-\frac{1-\alpha}{1-\ell}+1=0 \implies \ell^*=\alpha$$

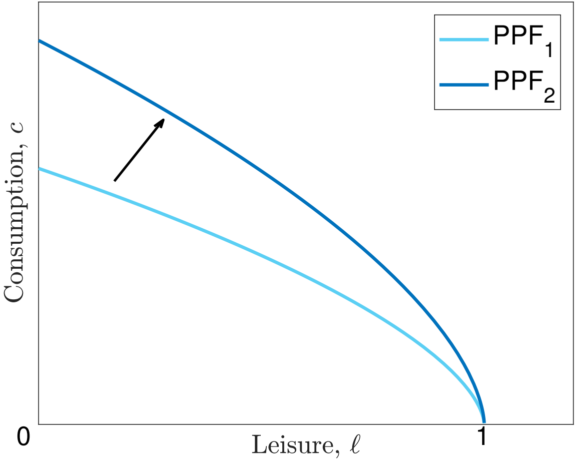
- Therefore,

$$n^* = 1-\alpha$$
$$c^* = y^* = (1-\alpha)^{1-\alpha} z k_0^{\alpha}$$
$$w^* = (1-\alpha)^{1-\alpha} z k_0^{\alpha}$$

- Next: Effects of an increase in productivity z

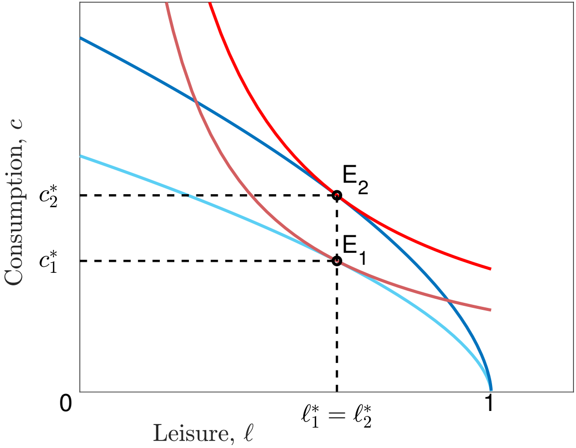
Effects of an increase in productivity z

- Production possibility frontier shifts outward from PPF_1 to PPF_2



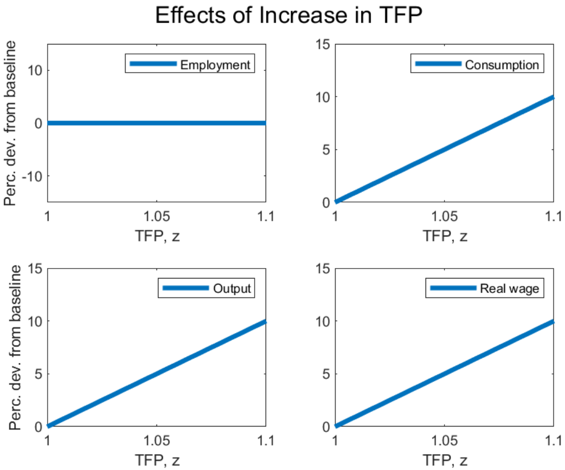
Effects of an increase in productivity z

- After increase in productivity, equilibrium moves from E_1 to E_2, leading to $\uparrow c$, $\uparrow y$, but no effect on leisure/labor supply



Effects of an increase in productivity z

- Gradually increase z by 10%, starting from $z = 1$. Show evolution of equilibrium employment, consumption, output and real wage



Experiment, cont'd

- Endogenous variables $\{n^*, c^*, w^*\}$ as a function of z

$$n^* = 1 - \alpha, \quad c^* = (1-\alpha)^{1-\alpha} z k_0^{\alpha}, \quad w^* = (1-\alpha)^{1-\alpha} z k_0^{\alpha}$$

- An increase in productivity z causes $\uparrow c^*$, $\uparrow w^*$ but no effect on employment n^*, why?
- If consumer faces a higher wage
  - leisure becomes more expensive, so consumer substitutes away from it $\implies \uparrow n$ (substitution effect)
  - Endowment income increases, want more leisure (normal good) $\implies \downarrow n$ (income effect)

Comparing to the data

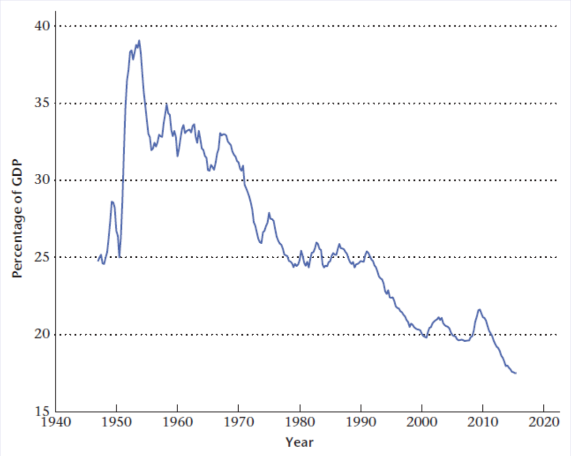
- In our experiment we find that an increase in total factor productivity leads to higher consumption, higher output and higher real wages. The effect on employment is however muted.
- What do we see in the data?
- Let's look at **long-run** economic trends.
    - After World War II, many technological innovations. In data, aggregate output, consumption and wages all increased steadily. Hours worked per employed person remained roughly constant or slightly decreased
    - If income and substitution effects roughly cancel over in the long-run, then the model predictions are consistent with the data

Government

Data

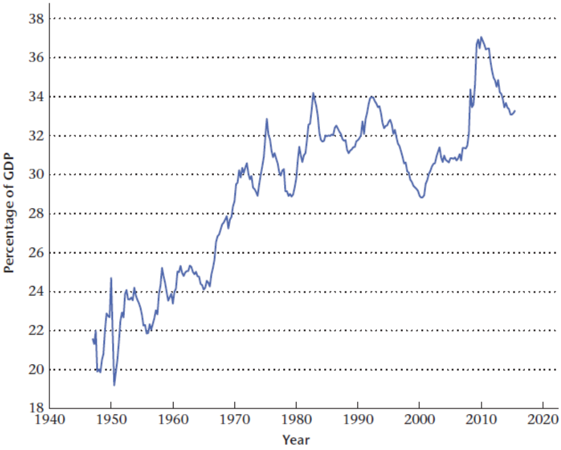
- Total government spending includes:
  - Government expenditures on goods and services (i.e. the "G" that you see in macro textbooks!) such as roads and bridges, defense, public schools, etc.
  - Social transfers (Social security, unemployment insurance)
  - Interest on government debt

Government Expenditures as a Percentage of GDP: G/Y



- Government expenditures have fallen since 1950s, as a percentage of GDP

Total Government Outlays as a Percentage of GDP



- Total government outlays have grown on trend $\implies$ growth in role of transfers

Government

- Introduce government into simple model $\implies$ analyze fiscal policy
- Assume government purchases consumption goods and finances these purchases with lump-sum taxes levied on the consumer
- Budget constraint

$$G = T$$

  where G is government purchases and T is lump-sum taxes
  - G is government expenditures (public school, hospitals, etc.), not public transfers (social security, unemployment benefits, etc.)
- Implicit assumptions:
  - Budget must be balanced (i.e. government cannot issue debt)
  - Government expenditures are wasteful

Equilibrium with Government

- A competitive equilibrium consists of quantities $\{c, \ell, n, k, T\}$ and prices $\{w, r\}$ such that
  1. Given $\{w, r\}$ and T, the representative consumer chooses c, ℓ to solve
  $$\max_{c \geq 0, \ell \in [0,1]} u(c, \ell) \text{ s.t. } c = w(1-\ell) + rk_0 - T$$
  2. Given $\{w, r\}$ the representative firm chooses k, n to maximize profits,
  $$\pi = \max_{k \geq 0, n \geq 0} [zf(k, n) - rk - wn]$$
  3. Markets for labor and capital clear
  $$n = 1 - \ell, \; k = k_0$$
  4. **NEW**. The government budget constraint is satisfied
  $$G = T$$

Aggregate Resource Constraint

- What is the aggregate resource constraint for the economy?
- Starting with the consumer budget constraint:

$$
\begin{aligned}
c &= w(1-\ell) + rk_0 - T \\
&= wn + rk - T && \text{(Market clearing)} \\
&= wn + rk - G && \text{(Government budget)} \\
&= zf_n(k,n)n + zf_k(k,n)k - G && \text{(Firm optimality)} \\
&= zf(k,n) - G && \text{(Constant returns scale)}
\end{aligned}
$$

- Then, we obtain the aggregate resource constraint:

$$c = y - G$$

which is also the income-expenditure identity for this economy.

Finding the equilibrium

- Since taxes are **not** *distortionary*, the competitive equilibrium and the Pareto optimum are equivalent
- The social planner problem is

$$\max_{c\geq 0,\ell\in[0,1]} u(c,\ell) \text{ s.t. } c = zf(k_0, 1-\ell) - G$$

- Optimal quantities of consumption and leisure (c,ℓ) are given by

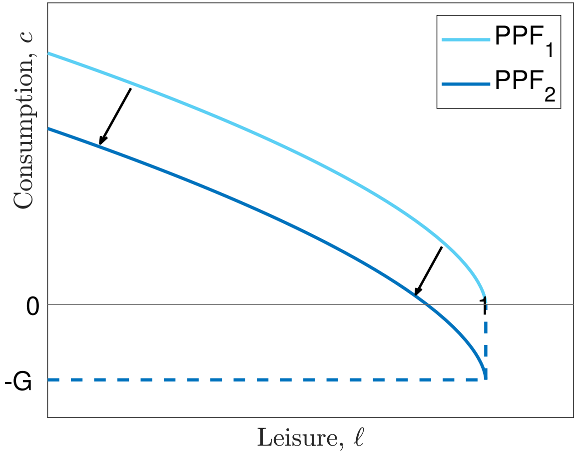
$$-zf_n(k_0, 1-\ell)\,u_c(c,\ell) + u_\ell(c,\ell) = 0$$

$$c + G = zf(k_0, 1-\ell)$$

- Graphical representation $\rightarrowtail$ next slide

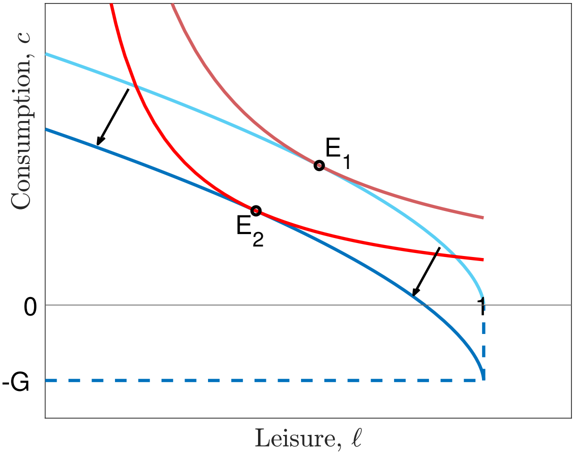
Comparative Statics: an Increase in G

- Assume the economy starts with $G = 0$. The production possibility frontier is PPF_1 given by $c = zf(k_0, 1 - \ell)$. With $G > 0$, the production possibility frontier becomes PPF_2 given by $c = zf(k_0, 1 - \ell) - G$



Comparative Statics: an Increase in G

- Initial equilibrium with $G = 0$ is at E1. New equilibrium with $G > 0$ is at E2, where both c and ℓ are lower and labor supply is higher



Comparative Statics: an Increase in G

- Effects of higher government spending on consumption, employment and real wages?
- $\uparrow G$ shifts down the production possibility frontier $c = zf(k_0, n) - G$
  - For each level of employment, consumer can afford lower c
- Given government budget constraint $G = T$, $\uparrow T \implies$ negative income effect
- Since c and ℓ are both normal goods, $\downarrow c$ and $\downarrow \ell$, so that $\uparrow n$
- A higher labor supply leads to lower wages (firm will hire more labor only if wage falls)
- Hence $\uparrow G$ *crowds out* private consumption, but output increases since employment is higher

$$\downarrow c + \Uparrow G = \uparrow y$$

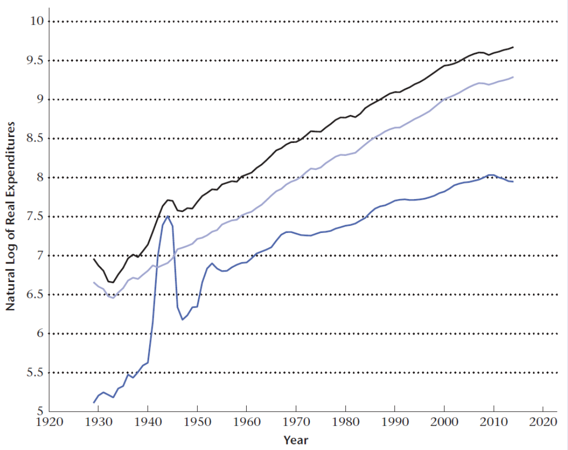
Comparison to Data

Let's confront the model predictions with the data

- Model predictions: when $\uparrow G$, aggregate output and employment increase, consumption and real wage decrease
- In data, many "shocks" besides changes in G happen at the same time
- Use World War II as a "natural experiment" to test empirically the predictions of the model
- Huge increase in G happened during World War II, sharp increase in total GDP and slight decrease in private consumption

Comparison to Data

- Huge increase in G (dark blue line) happened during World War II, sharp increase in total GDP (black) and slight decrease in private consumption (light blue) $\implies$ consistent with our model



Seminar Classes

First Seminar

Seminar on this topic (one-period, general equilibrium model) will have two analytical problems. I suggest that you try and solve these problem **before** the class: by doing so, you will learn more and hopefully you will make the most out of the class discussion.

- **Problem 1**. Competitive equilibrium is not Pareto optimal with distortionary taxes. Laffer curve.
- **Problem 2**. Consider the model with government, but now assume that government spending makes private firms more productive. Explore the effects of an increase in G on output, consumption and labor supply.

Appendix

Constant Returns to Scale and Euler's Theorem

- **Constant Returns to Scale**. The function $f(k, n)$ has constant returns to scale (i.e. is homogeneous of degree 1) if

$$f(\lambda k, \lambda n) = \lambda f(k, n), \text{ for all } \lambda > 0 \tag{1}$$

- **Euler's Theorem**. If function f is homogeneous of degree 1, then

$$f(k, n) = f_k(k, n)\, k + f_n(k, n)\, n$$

  - To see this, differentiate both sides of (1) with respect to λ:

$$f_k(\lambda k, \lambda n)\, k + f_n(\lambda k, \lambda n)\, n = f(k, n)$$

Setting $\lambda = 1$ yields

$$f(k, n) = f_k(k, n)\, k + f_n(k, n)\, n$$

Constant Returns to Scale implies that marginal products are homogeneous of degree 0

- **Property**. Constant returns to scale imply that the marginal products of capital and labor are homogeneous of degree 0.
- To see this, differentiate

$$f(\lambda k, \lambda n) = \lambda f(k, n)$$

with respect to one of the inputs, e.g. k, to obtain

$$\lambda f_k(\lambda k, \lambda n) = \lambda f_k(k, n)$$

$$f_k(\lambda k, \lambda n) = f_k(k, n)$$

and thus the marginal product of capital is homogeneous of degree 0 in its arguments (same can be done for MPN).