Summary

Suppose you've had a long day and just want to go home and relax. You say hi to your loving wife and kids and excuse yourself to the bathroom. You feel a tugging at your heart and turn to see a welcoming bathtub awaiting you. You fill up the tub and get in. The water's the perfect temperature and you set up a couple candles around the edge to set the mood. You relax, letting the water gently massage the stress of a particularly hard day right out of your shoulders. But before long, the tub has gone cold. You remember you're trying to cut back on your water bill from the excessive baths from last month, as your wife constantly reminds you, so you try to figure out the best way to reheat the tub while using the least amount of water possible. That's where we come in.

Our goals for this project were to create a model allowing us to explore heat dispersion, and use it to find the optimal way to heat a bath with a faucet and mixing. To do this, we created a series of models of increasing complexity to model heat dispersing throughout the bathtub. Initially, we created a very simple model which contained just one water "node", coupled with the air cooling it, and the faucet heating it. This produced great results, showing the ways that the water would reach equilibriums fairly quickly. From there, we extended the model into two water nodes, yielding a more complex system. We created this model to show that even though some of the water is far from the faucet, the bathtub can still reach equilibrium. This enabled us to introduce the notion of mixing, or the randomized spreading of heat throughout the system. After that, we used a 3D lattice based system inspired by Lattice-Boltzmann to develop the bath into a 3D model.

Once we had all of our simulations in place, we were able to restrict the water flow to a minimum so as to use the least amount of water possible while still having the faucet on. We then experimented with elements such as body position, faucet temperature, mixing frequency, bathtub size, etc. In every case, we were able to find a solution that would use the least amount of water, while still keeping the water temperature at an equilibrium close to the initial bathtub temperature.

We believe that by creating our comprehensive bath simulator, we were able to successfully solve the problem. Our lattice based 3D simulator allows us to explore how to save water, and we believe that by using our ideas you can avoid the wrath of your wife, and still have a great bath.

Contents

1 Introduction

Water is a deeply important resource that is limited, and we have less of it every day. This forces us to confront a question: What can we do to limit our water consumption? An easy solution is to take baths that require less water. But people like full bathtubs, so the best approach for this is to limit the amount of water needed to keep the bath warm after it's filled up.

In this project, we have created a comprehensive model of heat dispersion in a bathtub, including air to water heat dispersion, water to water heat dispersion, and body to water heat dispersion. This model allows us to experiment with many different parameters in order to find the optimal strategy for keeping the tub close to the original temperature. We had two primary goals: first, to develop a computational simulation system designed to be easily extensible. This flexible system would allow us to develop many possible methods of heating the tub, including multiple faucets and mixing the fluid of the tub in arbitrary ways. The second goal was to use this system to gain a greater understanding of the way that heat flows throughout a bathtub, and use that knowledge to develop methods to realistically simulate heat spreading throughout the tub.

1.1 Assumptions

For this model, we made numerous assumptions about the bathtub:

1. First, we assumed water overflow was of no importance. We modeled various levels of water flow from the faucet as differing depths that the water flow penetrated to. This allowed us to run an efficient simulation with greater fidelity. We also believe that the water flowing out of the tub is an entirely unnecessary factor in the temperature of the tub, and by abstracting away the overflow, we gained a great deal in simplicity and understanding.

2. Our second assumption concerns what materials are involved in the system. Our simulation has four forms of matter: boundary, air, water, and body. The boundary represents the tub itself: the porcelain/metal/tile surface that holds the water in. We decided early on that whatever material the tub was made of, it would have

a high enough specific heat that it would resist heating up and give off very little heat to the other materials. The air represents the material outside of the tub, and above the water. It is the primary driver of cooling for the water. The temperature of the air is fixed, and it exists to interact with the water. The body represents the person in the tub, who acts as a constant 98.6 °F heat source. The presence of the body forces the water to either lose, or gain temperature. The water fluid is the main component of what is in the tub. The water experiences temperature interchange between the other water nodes and itself, the body and itself, and the air and itself.

3. Our third assumption concerns the various forms of heat transfer that the materials experience. There are three key kinds of heat transfer that can occur between materials: convection, conduction, and radiation. Convection deals with heat that is transferred by fluid physically being moved from one area to another [1]. A classic example of this is air conditioning: pump cold air out over everything. Conduction deals with surface to surface contact: hot things heating up cold things by direct contact [1]. A great example of this is a stove warming a pan. Radiation is when heat radiates off of a surface, sometimes by using light as a medium [1]. The classic example of this interaction is the sun: heat gets to the earth by light radiation [1]. We chose to use conduction for the body to water and air to water interactions. For the water to water interactions, we chose to use conduction and convection. We believe that this accurately captures all the temperature interactions present in the system, with the possible exception of a small amount of convection from the surface water evaporating into the air.

4. Another important assumption that we made was how the air and body temperatures were always constant. We feel justified that the air temperature is constant because bathtubs will tend to be in rooms that are very large compared to the bathtub, and therefore the air has a very high capacity to absorb heat. Similarly, we feel justified in fixing the body temperature because, presumably, the person who is taking a bath is a reasonably

healthy human, and their body will stay at the same temperature.

2 Process

In the beginning of the planning process, we discussed a couple different methods, including a differential model and the Lattice-Boltzmann method. We thought about using a differential model at first, but decided we wanted a three-dimensional representation of the tub and did not want to be over encumbered by the details of complex partial-differential equations. We discovered the same effect could be achieved with a Lattice-Boltzmann-like model. We realized that we didn't need a full Boltzmann model since we were able to simplify some factors about the bath, including air temperature, body temperature, faucet temperature, etc. We found it to be important to verify our model at many steps of the process, so we first created a series of simplified models that could exhibit most of the properties that we wanted.

By using several different models, we were able to verify our results, and that our procedure was working correctly. This meant that as we developed more and more complex systems, we knew the results would be accurate, because the early models functioned properly. This let us move to complex situations with confidence, and model the complex bathtub with accuracy.

2.1 First Simple Model: One Bath temperature

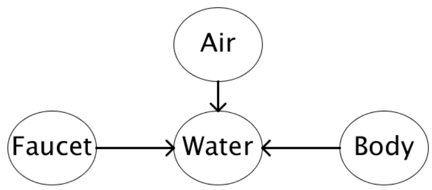


Figure 1: The temperature relationship in the first model.

In order to build understanding of the heat flow relationships between the various types of matter in our system, we built a simple model first. It consists of four variables, the temperature of the air, body, water, and faucet. The variables then evolve over time, as described by the various heat equations present in TutorVista's Heat Transfer Formulas [3]. The relationship between the various kinds of mass in our simple simulation can be seen in Figure 1, where an arrow means that the temperature of that substance affects what it is pointing at.

Figure 2 displays the temperature of the water under various conditions. The main conditions are as follows:

- No faucet flow.
- Faucet flow beginning $\frac{1}{4}$ of the way through the simulation at the water temperature, and increasing at some rate to 50 degrees Celsius.
- Faucet flow beginning $\frac{1}{4}$ of the way through the simulation at the water temperature, and increasing the faucet temperature at a linear rate to 70 degrees Celsius.
- Faucet flow beginning at $\frac{1}{4}$ of the way through the simulation increasing at a very slow rate with no cap.

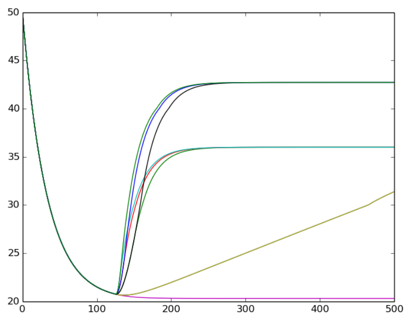


Figure 2: A graph of the temperature over time with various faucet conditions.

This model, while simple, was very helpful in our process to the final model. It allowed us to validate our understanding of heat flow, as well as validate our equations. It also shows the most important factor: our model allows equilibrium situations to arise. In every graph with non-trivial faucet temperature, our model reaches equilibrium. To reach equilibrium at a reasonable bath temperature, it would currently require a very high faucet temperature. However, we can fix this by adjusting the various heat dispersion coefficients. This is a great start, and allows us to experiment with varying constants. However, in many ways, this model was far too limited to do the necessary analysis. The notion of "mixing" the water is meaningless in this situation as the water is represented only by a single temperature. Additionally, we cannot talk about water pressure, an important part of the problem. For a more complex model, we need to have the notion of mixing, as well as water pressure.

2.2 More Complex: Two Bath Temperatures

Our next level modeling was to determine if the water could still achieve equilibrium in a two part system. This model is best represented by the following diagram:

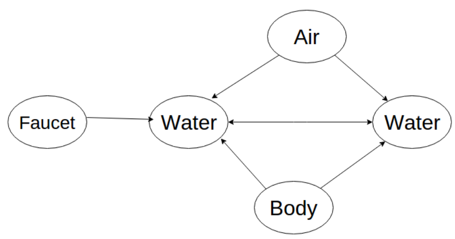


Figure 3: The graphical model displaying the organization and temperature relation of our more complex model.

In Figure 3, it shows that there are now two "waters" in the model, with only one of them connected to the faucet. Both water temperatures are still affected by the air and the body temperature, and are now connected to each other. The improvement given by this model is how only one of the water temperatures is coupled with the faucet. This introduces the notion, present in all bathtubs, of water being far from the faucet. Below is a graph of how the temperature evolves over time.

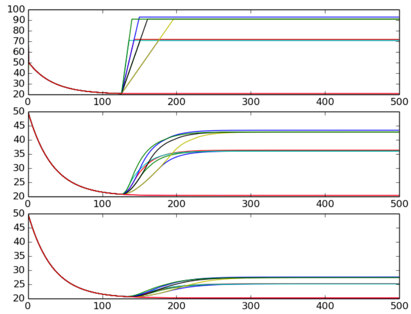


Figure 4: Temperature of the faucet, and both water temperatures over time with various faucet conditions.

Figure 4 contains three graphs. The top graph is the faucet temperature with respect to time. During the initial phase, the "cool down period", the faucet temperature is set to the water temperature. This means that the faucet has no effect on the water temperature. At time $t = 125$, the faucet "turns on" and begins affecting the water, whose graph is represented as the middle graph in Figure 4. As you can see, at a variety of faucet temperature rates, the water temperature increases. This then affects the temperature in the second water. The temperature of the second water increases as the faucet temperature increases as well, but in a less dramatic way, because the faucet can only increase it in a detached way.

As you can see in Figure 4, the system reaches an equilibrium over time. It is not however, a balanced equilibrium: the two water temperatures are not close to each other. We are pleased with this result. It reflects how real life bath tubs have temperature distributions throughout the tub instead of constant temperatures. This allows us to introduce the notion of mixing. Since we have two water temperatures, we can periodically mix them together to change the equilibrium.

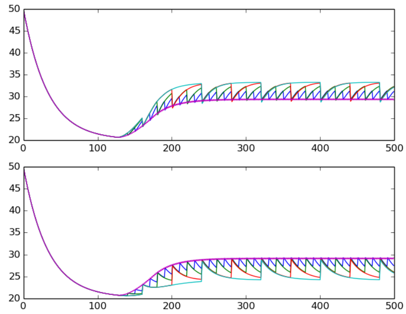


Figure 5: Graphs displaying the effect of mixing.

Figure 5 shows the effect of mixing at various frequencies of both the water near the faucet (top) and far from the faucet (bottom). As you can see, the mixing decreases the difference between the two temperatures by a significant factor. The following picture shows that with frequent mixing, the temperatures stay very close together, and reach a pseudo equilibrium.

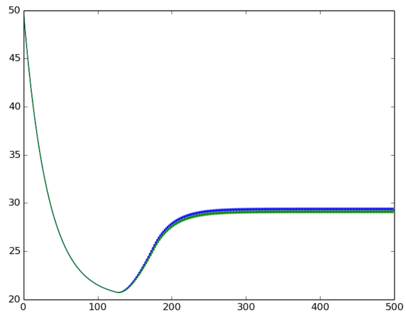


Figure 6: Graph displaying the equilibrium of the model with very frequent mixing.

As can be seen in the Figure 6, with mixing at various frequencies, the water is able to reach a very stable equilibrium where the two bath temperatures are close together.

This new, more complex model introduces a difficulty: adding the second water temperature causes the system to be significantly colder. This means that to reach a desirable bath temperature throughout the bath, we need to increase the faucet temperature, or increase the air temperature. It is certainly possible that the air currently has a dominate effect on the bath temperature, thus making a pleasant bath temperature difficult.

These difficulties, however, illuminate that our model is very successful, and that we are heading down the right path. The next step in our modeling is to extend it to a 3D lattice structure.

This model is still incomplete. The 3 features we need to correctly model a hot bath are:

1. Faucet Temperature
2. Mixing the water
3. Faucet Volume

Because this model is unable to account for faucet volume, we need to make our model more complex. This led to our 3D Lattice based model.

2.3 Lattice Structure Bath Model

Our previous models never had any structure beyond the temperature relations, but in this third and final model, we created a three dimensional bathtub simulation using the techniques developed in the previous models.

For this simulation, we created a lattice based structure, wherein nodes are connected only by the nodes directly adjacent to them. This means that the temperature of a node is only dependent on its neighbors, creating a structure where the heat (or cold) must propagate through the system at a relatively slow pace. It also allows for the introduction of geometry for a simulated person, faucet location, and bathtub geometry. This model is much more general, and creates a situation where we can experiment with different structures, and procedures and expect to have results that approximately match real life results.

As stated in a previous section, there are three main components necessary for a complete simulation of the bath. We will address each of those issues, and talk about how this model implements them.

1. **Faucet Temperature**
   This model incorporates the faucet temperature in a very natural way: We have a faucet that is set to a particular temperature, and it warms the water near it.

2. **Mixing the water**
   The simulation is programmed to randomly shuffle the temperatures of the water based on certain conditions, be it time, temperature variations, etc.

3. **Faucet Volume**
   The faucet volume is represented by how large the region of water designated "faucet" is. This allows us to understand the effect of a large faucet pressure without having to deal with the complexity of computational fluid dynamics.

3 Model Design

In this section we present several novel modeling techniques that we employed in our system that led to a powerful, accurate, and useful simulation.

3.1 Heat Transfer

Our model is primarily concerned with heat transfer by convection and conduction. We used the classical conduction and convection equations.

This is the equation we used for conduction:

$$Q = \frac{kA(T_{Hot} - T_{Cold})t}{d} \text{ [3].}$$

Where A is the cross sectional area of the surface through which the masses interact, t is the time that the energy has to transfer, and d is the thickness of the material that the material is transferring to [3]. The value k is the thermal conductivity of the material, a value that can change with temperature [3]. This will be addressed later in this section.

The convection equation that we used is:

$$Q = H_c A(T_{Hot} - T_{Cold}) \text{ [3].}$$

The variable H_c is the heat transfer coefficient [3].

The way that the model works is simple: We gather up all the surrounding nodes of a selected node, and calculate the various q values, and use the simple equation:

$$t_{new} = t_{old} + \frac{q}{mass} \text{ .}$$

This captures the traditional notion that 1 unit of energy is enough to increase 1 unit of mass by 1 degree. Using this equation and iterating over all the lattice nodes, we are able to model temperature flowing throughout the bathtub, temperature flowing out of the body, and temperature flowing into the air.

In addition, this was fast. It allowed us to simulate large tubs for long periods of time, without worrying about the need to spend hours waiting for the results. This allowed us to iterate on our model, getting successively better results.

3.2 Interpolation Array

As stated above, the value of the k variable from the conduction equation changes with the temperature [3]. This means that it was very important that we had valid data for the thermal conductivity of water, otherwise our model would display behavior that simply would not match with reality. However, we had a limited amount of data, present in Table 1 and Figure 7 below:

t °C	Thermal Conductivity mW/Km
0	561.0
10	580.0
20	598.4
30	615.4
40	630.5
50	643.5
60	654.3
70	653.1
80	670.0
90	675.3
100	679.1

Table 1: [2]

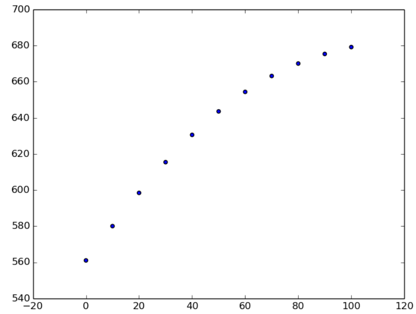


Figure 7: Graph displaying the thermal conductivity of the water as temperature goes up.

As is visible in Table 1 and Figure 7, the data for the thermal conductivity, while spreading over a wide range, lacks fidelity. There are 10 degree gaps prevalent throughout the data set.

In order to fill these gaps, we wrote a data structure called an interpolation array. This is a data structure that allows the user to infer missing values from a data set in an efficient and natural way. The interpolation array also allows us to access data points between a range as if they already exist, making the interface very simple. The core idea of the data structure is that between individual data points, the data is approximately linear, and therefore can be estimated using linear interpolation.

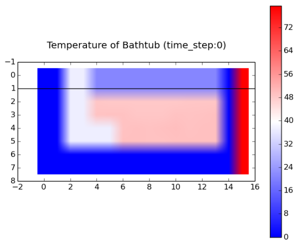
This elegant abstraction allowed us to proceed as if we knew the thermal conductivity to arbitrary precision, and not need to use complex equations in our code to calculate the conduction heat flow.

4 Simulation

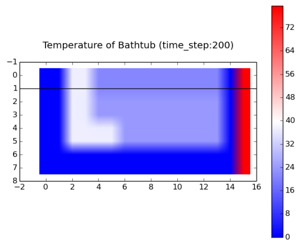
Our comprehensive bathtub simulation was a great success. In this section, we will present several different scenarios and the method in which the heat flowed through them. We will use these scenarios to discuss the power that our system exhibits, and use it to demonstrate some of our results.

4.1 No Faucet

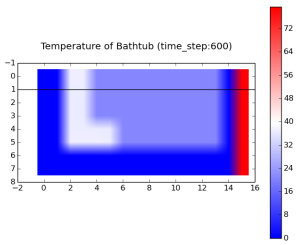
When we were first running our simulation, we were curious about how the system would evolve without the faucet acting as a heat source. The system will cool rapidly, eventually reaching an equilibrium.



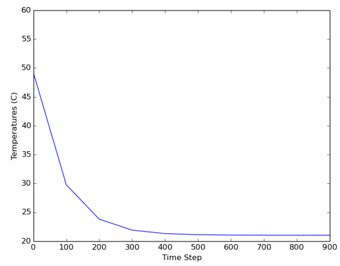
(a) After $t = 0$.



(b) After $t = 300$.



(c) After $t = 700$.



(d) A plot of the mean temperature vs time.

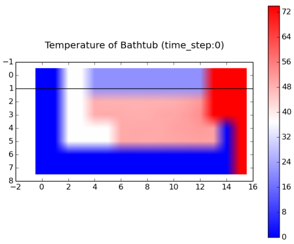
Figure 8: Images depicting the cooling of the water temperature in the absence of a faucet.

Figure 8 shows the system evolving over time from a side perspective. The large white “L” is where the body is. As you can see from the scale, at first, the temperature of the bath is approximately 50 °C. However, very rapidly, the heat from the bathtub is drained by the air above it. You can see, that even after $t = 0$ in Figure 8b, the water near the tub is already getting cold. In Figure 8d, you can see the progression of the mean temperature over time, and note that it reaches equilibrium. This is the point where the water is close enough in temperature to the air that the air no longer has any meaningful effect on the water temperature.

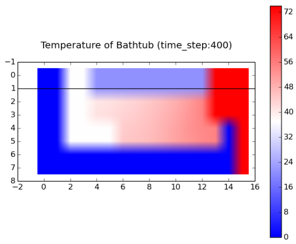
This result is important to see, as it shows that our large, generalized lattice based system is able to express the same kind of relationship that our other, more simple models have with the faucet.

4.2 Faucet on but without mixing

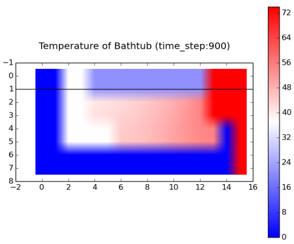
After performing these simulations, we ran the simulation with a variety of faucet temperatures in an attempt to pull the equilibrium to the initial bath temperature. In our first attempt, we set the faucet water temperature to 74 °C and only added a small amount of faucet water. It is important to note here that the initial water temperature was at 50 °C, and the body temperature at 37 °C. Our goal with this simulation was to have the system equilibrium lie at approximately 50 °C.



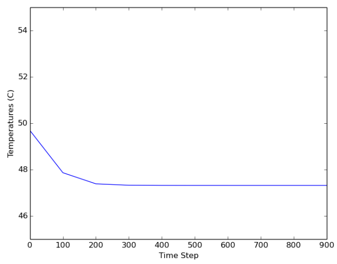
(a) After $t = 0$.



(b) After $t = 500$.



(c) After $t = 1000$.



(d) A plot of the mean temperature vs time.

Figure 9: Several images taken from a simulation run with a running faucet.

This simulation features the same structure of images as the previous simulation. As you can see in Figure 9d, the tub remains relatively warm, but unfortunately, it is well below our desired equilibrium. As evident in Figure 9c and Figure 10, there are a wide range of temperatures in the final time-step. We try to restrict the temperature range in future simulations.

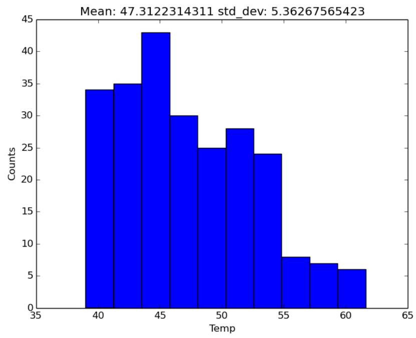
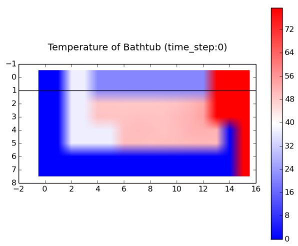


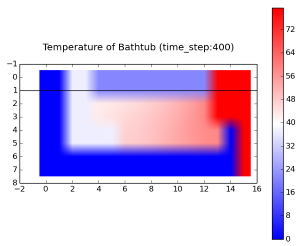
Figure 10: Histogram displaying the variations in temperatures.

This histogram is much wider than we would like in our bathtub, as parts of it would be very cold, and other parts are too warm.

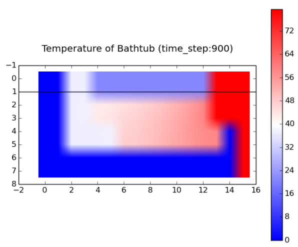
In Figure 11, we present a model with a higher faucet temperature. Once again, our goal with this simulation was to reach an equilibrium close to the desired water temperature, but with little variation in the temperature.



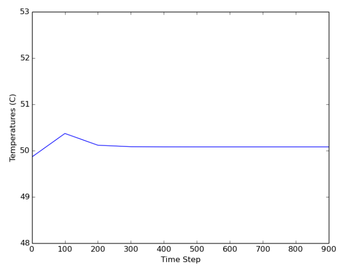
(a) After $t = 0$.



(b) After $t = 500$.



(c) After $t = 1000$.



(d) A plot of the mean temperature vs time.

Figure 11: Several images taken during a simulation run with a higher faucet temperature.

Looking at 11,you can see the bath temperature reaches our desired equilibrium temperature. However, the temperature through the bathtub is not uniform, as depicted in Figure 11 and Figure 12. While we are unable to achieve the desired equilibrium in this case, it illuminates an important concept: There is a great temperature gradient along the tub, which means the heat is from the faucet is spreading throughout the tub in a uniform fashion, becoming less influential as it gets farther from the faucet. This is the reason we decided to implement mixing.

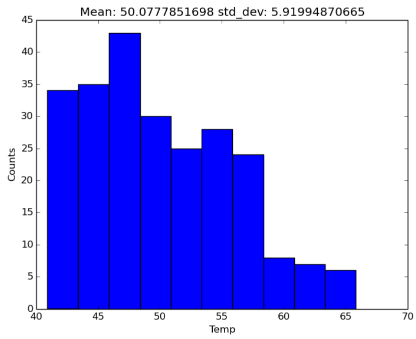


Figure 12: Histogram displaying the range of temperatures.

In Figure 12, it is very clear that the range of temperatures is quite high. We hope to tighten this range using mixing, our next step.

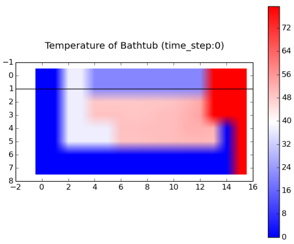
4.3 Faucet on with Mixing

In order to achieve uniform temperature through the system, we implemented a mixing procedure. After a significant amount of trial and error, we decided to mix the fluid every n time steps, rather than a more complex procedure involving the temperature of the fluid. The mixing is simple: it permutes the temperatures of all of the water nodes using a randomized algorithm. This was very effective. We believe it to be justified, as it models the person in the tub vigorously moving all of their arms and legs.

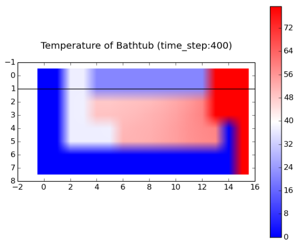
Once again, we desire uniform temperature throughout the tub, and a mean temperature of around
50 °C.

In Figure 13, you can see a bathtub with an approximately uniform temperature distribution throughout the tub. It also is very close to the temperature that we desired. We consider this to be a very successful model, as it simply uses hot water, with relatively low pressure and vigorous mixing to produce the desired result.

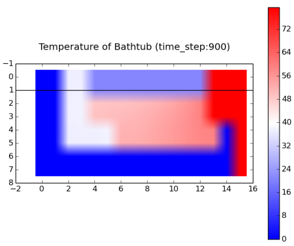
As you can see in Figure14, the water is an excellent temperature, with only a small part of it being hot. We are happy with this result, because the hot water comes almost exclusively from the water very close to the faucet, which the individual bathing is far from, and even though it is too hot for the individual, it doesn't matter, because it is so far away.



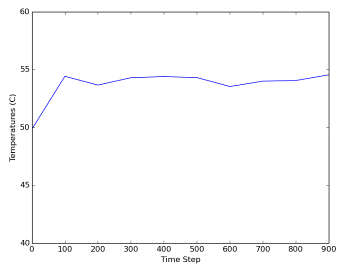
After $t = 0$



(a) After $t = 500$.



(b) After $t = 1000$.



(c) A plot of the mean temperature vs time.

Figure 13: Several images taken from a simulation run with a faucet temperature of 79.5 °C with frequent mixing.

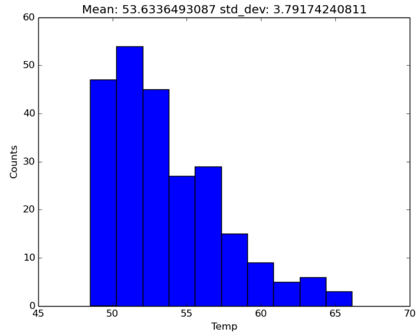


Figure 14: Histogram displaying the variations in temperatures.

4.4 Body Position and Temperature

One thing we did not anticipate, but none the less fits in with how the world works, is the way that the bath temperature varies with the position of the person. The way that it works is simple: when the person is closer to the faucet, they block the heat from spreading throughout the tub.

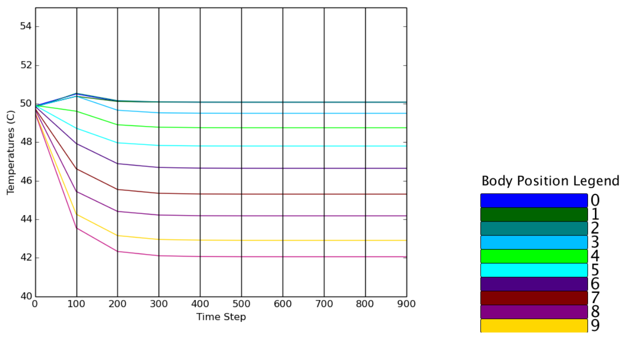


Figure 15: Parallel Coordinates showing the variation in mean temperature over time, with the body position as a third variable.

Figure 15 shows 10 different colors, representing 10 different body positions. From the top to the bottom is a series of increasing distance from body to faucet. As you can see, when the body gets closer to the faucet, the mean temperature goes down. This is an interesting result, and contributes an important idea to proper bath temperature stability: put your back against the back wall.

5 Results

The goal of our project was to develop an extensive model to determine the optimal way to warm a bathtub. In this section, we shall present the results of our models. Additionally, we shall discuss the issue of bubble soap, and how we could integrate it into our model.

5.1 Our Suggestion

Our comprehensive bath simulator allowed us to explore many options for warming the tub. These included using multiple faucets in strategic locations, very high faucet temperatures, very high faucet pressures, and much more. We found that the solution we are about to present allowed for the best conversation of water while still keeping the temperatures approximately uniform throughout the bathtub.

Our solution involves 4 factors:

1. The temperature of the faucet
2. The amount of water coming from the faucet
3. The frequency of mixing the water
4. The position of the individual in the bath tub.

Our recommendations for each of these elements are the following. We will discuss our process that obtained these results.

1. The temperature of the faucet should be between 25 °C and 35 °C hotter than the bathwater for reasonable bath temperatures between 40 °C and 60 °C.
2. The faucet should be at a sufficiently high pressure to output roughly .1% of the volume of the tub every second, meaning that the tub is refilled every 25 minutes.
3. The bathtub should be vigorously mixed with both arms and legs every 20 seconds.
4. The individual in the tub should be as far from the faucet as possible

We chose this suggestion because during our simulation, our best simulation, involving in excess of 3200 nodes reaches a great equilibrium with these inputs. We ended up upon the

25 °C-35 °C hotter because using more with that could potentially cause burns against the individual in the bathtub. We believe that this range is sufficiently small for experimentation by the individual bathing.

Our suggested volume comes from our model. To explain how we got this number, we need to explain a little bit about the way that our model deals with faucet pressure/volume.

As discussed earlier in the paper, our 3D lattice system has a number of different node types (water, body, Boundary, air). The faucet operates by marking certain water nodes as being faucet nodes. These faucet nodes are bound to the current faucet temperature, and exchange energy/heat with the others. We define the number of nodes which are currently marked as faucet to be the faucet size. This faucet size is proportional to the volume of water that is flowing into the bathtub. The relationship is simple: the faucet size is the the faucet volume spread out over several seconds of flow. We say several seconds of flow because it is changes depending on the faucet size and the faucet shape. In our large model, we found that the faucet size was approximately .2% of the tub size. We used a quick Fermi calculation to estimate that the water would take 2 time-steps to flow into this volume, using reasonable water speeds. We therefore got that the volume per time-step would be .1% of the volume of the tub. We believe this to be a reasonable amount of water flow, conserving water while still allowing for sufficiently toasty baths.

The frequency of mixing is close to the optimal for keeping the bath in equilibrium. At this frequency of mixing, the marginal utility of increasing the frequency is very low, so we have attempted to optimize for both comfort of bathing (not having to move vigorously) and comfort of the temperature (even temperature balance throughout the bathtub). An individual bathing in the bathtub could choose to mix less frequency, and still achieve a good bathtub temperature through. Our solution is *dependent* on mixing, but the *rate* of mixing is unessential, as long as the tub is well mixed after the tub is mixed.

The body position is essential in our solution. We have found, through experimentation with our model, that body position is a significant factor (as seen in Figure 15). We consider this to be a very essential part of the model, and a very reasonable request to the bather. We have found, through accidental lifelong experimentation, that the comfiest place in a bathtub is leaning against the back wall, and we have found that this also results in a pleasant temperature for the tub, an excellent result.

5.2 Bubbles

During our research concerning bubble soap and bubbles [4], we found that bubbles have two primary effects on the way that heat changes overtime throughout the bath tub.

1. The amount of heat energy that the bathtub has for a given temperature is higher, so it is harder to heat soapy water. This effect is not particularly powerful.

2. The bubbles resting at the top of the bath can only hold so much heat before popping. This means they will absorb only as much heat as they can before popping, limiting heat loss for the water.

This means that to implement the bubbles in our simulation, we would need to have limited the Q (the heat transferred) from the water to the air (now representing the bubbles) every time step. This would cause the bathtub to be warmer in simulations with the bubble present. However, the faucet would be less effective, since the bath can now absorb more heat. This two results combine into a simple fact: In order to have a bath at a nice temperature with bubble bath, simply pretend that the bubbles are not there, as we believe that they have a net zero effect on the thermal properties of the bathtub.

5.3 Sensitivity Analysis

We shall now present analysis of the sensitivity of the equilibrium causing faucet temperatures for varying desirable bath temperatures. We have experimented with many different possible bath temperatures, and present the best faucet temperature to result in an equilibrium with all the bath temperatures. We then compare it to our suggestion above.

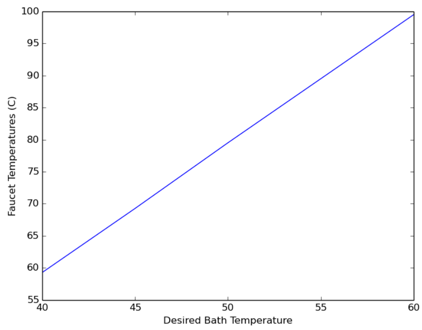


Figure 16: Graph of desired bath temperatures and their required faucet temperature.

In testing sensitivity analysis, we took five different starting bath temperatures and found their required faucet temperature in order to keep the temperature at the initial value. Figure 16 shows the five different bath temperatures and the shockingly linear relationship the desired temperature has with the optimal faucet temperature. This illustrates that a change in bath temperature is directly proportional to the required faucet temperature to maintain the desired equilibrium, which mirrors reality. Since it is linear, it means that our model has a linear sensitivity to temperature, an ideal result.

This also shows that our suggestion of the faucet temperature being 25 to 35 degrees C hotter than the bath temperature is excellent for all temperatures, a great result. This shows that our results are very general, and very effective.

6 Conclusion

In our introduction, we described the goal of this paper as finding the best way to heat a bath tub to insure even temperature close the equilibrium you desire. We have described the way that our solution relies upon the faucet temperature, pressure, body position, and mixing frequency. We believe that our model has lived up to our promise. In our conclusion, we talk about some strengths and some weaknesses that our project has, and some future work necessary for our model.

6.1 Strengths and Weaknesses

Our model has many strengths and many weaknesses, which we shall present here.

The best thing about our model is its adaptability. We created our simulation to be very adaptable, allowing us to customize it throughout the process. This meant that we could add anything we could think of to the system, we could improve it, and we can add new elements. This adaptability meant that finding our solution took only our minds, because we already had the tool that would let us find it.

Perhaps the greatest weakness of the model is the way that it does not incorporate any computational fluid dynamics into it. Water does not flow in our model, only heat does.

Another weakness of the model is that we have never verified if the way the heat dissipates from the surface of the water is realistic. However, this weakness is not much of a problem, as what we care about is scale, and relative heat loss. We believe that our model deals with this elegantly, because no matter what we pick for faucet, air, and body temperatures, we always reach equilibrium fairly quickly. This means that it accurately reflects reality. While nominally a weakness, in reality it has very little effect on the power for our model to make meaningful predictions.

A great strength of our model is described in the above paragraph: Our model always achieves equilibrium. This means that we can make excellent predictions about the way that faucet temperature and faucet strength affects the bath temperature. In addition, it means that we can simulate the bath for short periods of time, and have meaningful results. This has allowed us to iterate on our faucet temperature and faucet placement, letting us find the optimal solutions for the problem.

Another strength to our model is the way that the body acts as a pseudo boundary between regions of the tub. We have experimented with different positions for the body, and have found that positioning the body near the middle of the tub excellently blocks heat flow from one side of the body to the other. While useless in attempting to create bathtubs with little variation in the heat, it is an excellent verification that our body is working correctly, and that it has the perceived benefit that we anticipated that it would have.

One weakness we have is in estimating how much water is flowing in for a given "faucet depth". The way we model high faucet flow is by using the depth that the faucet reaches. This gives us a good estimation of how much fluid is flowing, but it is not exact. If we had more time, we would implement this using a Lattice-Boltzmann based system, allowing us to actually cause fluid to flow around in the tub.

6.2 Future Work

With more time, we would have optimized our algorithm so we could simulate sized bathtubs with multiple people. Currently, the system takes longer than twenty minutes to complete a multiple simulation runs that iterate between different variables with a full sized tub. It is not so much a matter of the speed of our algorithm, it is a matter of just how much data is used in the computation. One solution would be to paralyze the computation so that instead of iterating through every node sequentially, different threads would take different slices of nodes and complete the computation separately. This would allow the algorithm to run in a fraction of the time it would take to complete unoptimized.

Another optimization method would be the use of a look-up table. A look-up table would allow the algorithm to store commonly used values so those values could be accessed through memory, sparing an unnecessary computation.

By including a 3D computational fluid dynamics system, we could have potentially more accurate results, at the cost of great speed. This would make our results more general however, and in future work is something we shall look into.

A final improvement of the system would be three-dimensional visualization. Currently our simulation displays a two-dimensional slice of three-dimensional data. Visualizing the data in three-dimensions would allow us to have a better grasp on how uniformly the heat disperses. We are currently at a disadvantage since we can only visualize heat movement from two-dimensions.

6.3 Concluding Remarks

Our model is very strong, because it successful reaches equilibrium no matter the starting conditions. This gives our model great strength, because it means that by adjusting parameters, we can produce any kind of result we want. We believe that this means that our results are very powerful, because they respect reality in meaningful ways. While our model could be extended in many ways, it has many of the most important features.

Most importantly, we believe that the things that we have implemented are the most important elements of a successful bath simulation. We consider our solution to be a successful one.

Dear Users of Our Bathtub

You have just purchased the Bathtub 3000, a very advanced bathtub that is revolutionizing the bathtub industry.

As you take your first bath, you might notice that while it stays at a perfect temperature at the beginning, it sometimes begins to cool down. While we have created what is perhaps the optimal bathtub, we still aren't able to fix that simple problem. This is caused by your bathtub losing heat to the air. Luckily, thanks to the advanced technology of the simple modern faucet, it is possible to heat up your bath. We present a simple, step by step model for keeping your bath nice and toasty! This will let you save water and have a great time!

1. Turn your faucet on to a gentle trickle, just enough pressure that it will heat up the region of the tub near the faucet.

2. Make sure your faucet is at a nice temperature! You want it to be hotter than your bath is. Luckily, the Bathtub 3000 comes with an advanced system for detecting the temperature of your tub. You want faucet to be between 25 and 35 degrees Celsius hotter than you want the bath to be.

3. Now vigorously mix the bathtub every now and then! Simply do your favorite dance in the tub, and it will mix nicely. This will mix the bathtub, spreading the heat throughout the tub.

4. It's very important that you sit on the back wall of the bathtub, as far from the faucet as possible. By doing this, you let the heat spread throughout the tub.

As you can see, this is a simple instruction set that will allow you to have a very nice bath with little complications.

Now, I know your family is probably wondering why you need to do all this. Isn't the Bathtub 3000 the most powerful and best bathtub of all time? Well, unfortunately even though the Bathtub 3000 is a very advanced bathtub, it isn't able to prevent some fundamental thermodynamic events from occurring.

Tragically, since the bathtub is exposed to the air, it will eventually cool to close to the air temperature. This means that you need to turn the faucet on to keep the Bathtub 3000 nice and toasty. But when the faucet is on, the tub has uneven temperatures throughout. When that happens, just mix the tub, and it'll be just right.

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