Optimal Control

Course Project #4

Optimal Control of a Vehicle

Vehicle models and virtual drivers are very important for virtual prototyping. This project deals with the design and implementation of an optimal control law for a simple bicycle model with static load transfer. Consider the following bicycle model.

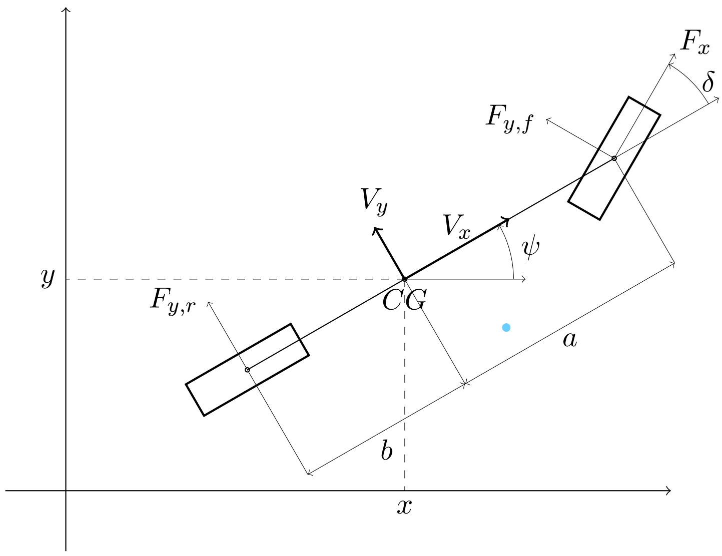


Figure 1: Bicycle vehicle

The state space consist in $x = (x, y, \psi, V_x, V_y, \dot{\psi})$, where (x, y, ψ) are the cartesian coordinates of the center of mass of the vehicle and its yaw in a global reference frame, and $(V_x, V_y, \dot{\psi})$ are vehicle velocities and yaw rate in its body fixed reference frame. The dynamic model is:

?

$$
\begin{aligned}
\dot{x} &= V_x cos(\psi) - V_y sin(\psi) \quad \text{velocity constr} \\
\dot{y} &= V_x sin(\psi) + V_y cos(\psi) \\
m(\dot{V}_x - \dot{\psi}V_y) &= F_x cos(\delta) - F_{y,f} sin(\delta) \quad \text{force along x} \\
m(\dot{V}_y + \dot{\psi}V_x) &= F_x sin(\delta) + F_{y,f} cos(\delta) + F_{y,r} \rightarrow \text{force along y} \\
I_z \ddot{\psi} &= (F_x sin(\delta) + F_{y,f} cos(\delta))a - F_{y,r} b
\end{aligned}
$$

where the control inputs are δ, the steering angle of the vehicle, and F_x, which is the force applied by the front wheel. The lateral forces F_y can be found as

$$
\begin{aligned}
F_{y,f} &= \mu F_{z,f} \beta_f \\
F_{y,r} &= \mu F_{z,r} \beta_r
\end{aligned}
$$

where β_f, β_r are the front and rear sideslip angles,

$$
\begin{aligned}
\beta_f &= \delta - \frac{V_y + a\dot{\psi}}{V_x} \\
\beta_r &= -\frac{V_y - b\dot{\psi}}{V_x}.
\end{aligned}
$$

The vertical forces on the front and rear wheel can be found as:

$$F_{z,f} = \frac{mgb}{a+b}$$
$$F_{z,r} = \frac{mga}{a+b}$$

The mechanical parameters of the robots are available in table 1.

Parameters:		
m	1480	$[Kg]$
I_z	1950	$[Kgm^2]$
a	1.421	$[m]$
b	1.029	$[m]$
μ	1	$[nodim]$
g	9.81	$[m/s^2]$

Table 1: Mechanical parameters of the vehicle

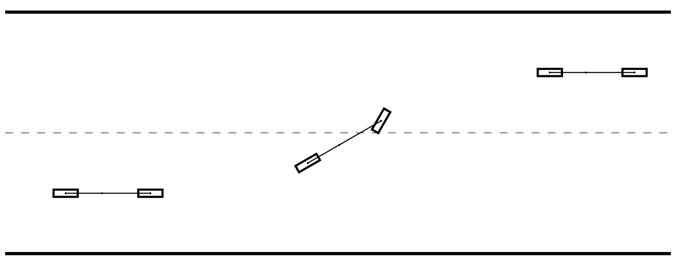
Task 0 – Problem setup

discretization

Discretize the dynamics and write the `dynamics` function.

Task 1 – Trajectory exploration: lane change maneuver

The first test for this car will be to perform a lane change maneuver. What characterizes this maneuver is that, while the position of the car changes, its yaw with respect to the road at the start and at the end of the maneuver must remain the same (zero).



DDP algorithm

Figure 2: Lane change maneuver

Choose two equilibria and define a step between these two configurations. Compute the optimal transition for the vehicle to move from one equilibrium to another exploiting the DDP algorithm.

Task 2 – Trajectory optimization: skidpad

The vehicle will start this manuver in the center of a 8-shaped track. At first, it must turn right and follow the right circle. When it has finished, the vehicle has to turn left and complete the left circle. Define a simple trajectory to perform this maneuver (e.g., the centerline of the track). Exploit the DDP algorithm to compute the optimal trajectory.

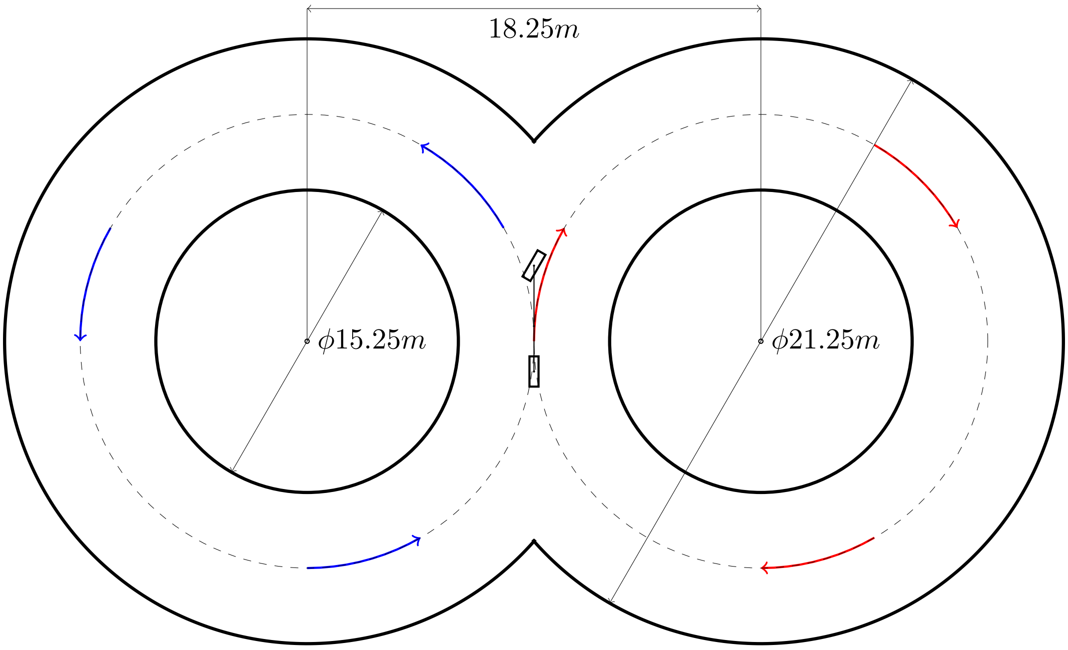


Figure 3: Skidpad

Task 3 – Trajectory tracking

Linearizing the vehicle dynamics about the (optimal) trajectory $(\mathbf{x}^*, \mathbf{u}^*)$ computed in Task 2, exploit the LQR algorithm to define the optimal feedback controller to track this reference trajectory. In particular, you need to solve the LQ Problem

$$
\begin{aligned}
\min_{\substack{\Delta x_1,\ldots,\Delta x_T \\ \Delta u_0,\ldots,\Delta u_{T-1}}} \quad & \sum_{t=0}^{T-1} \Delta x_t^\top Q_t \Delta x_t + \Delta u_t^\top R_t \Delta u_t + \Delta x_T^\top Q_T \Delta x_T \\
\text{subj.to} \quad & \Delta x_{t+1} = A_t^* \Delta x_t + B_t^* \Delta u_t \qquad t = 0, \ldots, T-1 \\
& x_0 = 0
\end{aligned}
$$

where A_t^*, B_t^* represent the linearization of the (nonlinear) system about the optimal trajectory. The cost matrices of the regulator are a degree-of-freedom you have.

Task 4 – Animation

Produce a simple animation of the vehicle executing Task 3. You can use MATLAB, PYTHON or whatever software you like.

Notes

1. Any other information and material necessary for the project development will be given during project "meetings".
2. The project report must be written in LaTeX and follow the main structure of the attached template.
3. Any email for project support must have the subject:

   "[OPTCON2021]-Group X: rest of the subject".
4. All the developed code must be handled in a `zip` folder.