

Prove: P - Given an undirected graph G , with n vertices $\{v_0, v_1, \dots, v_n\}$, using the greedy coloring algorithm there is no edge uv s.t. $f(u) = f(v)$ where $f(v_i)$ denotes the coloring of vertex v_i .

Proof by contradiction: If we assume P is false ($\neg P$) and reach 2 contradictory assertions Q and $\neg Q$, then the assumption $\neg P$ cannot be correct, and therefore P must be true.

Assume: There is an edge uv in G such that $f(u) = f(v)$. ($\neg P$)

Assertion: $Q =$ in the i th iteration, the algorithm assigns v_i the smallest color (i.e. positive integer) k , s.t. none of its neighbors that are already colored have color k .

Assertion: $S =$ Since there is an edge uv s.t. $f(u) = f(v)$, u and v (neighbors) have been colored with the same k .

Assertion: $\neg Q =$ Since S is true given the assumption $\neg P$, Q must be false, as a vertex u or v has been colored the same as one of its already colored neighbors. $\therefore \neg Q$

Contradiction: Since Q and $\neg Q$ are both true under the assumption $\neg P$, $\neg P$ must be false because it produces a contradiction. Therefore, P must be true.