HW Solution

CS/ECE 374: Algorithms & Models of Computation, Spring 2019 Version: **1.0**

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(100 PTS.) Aberrant

1 (25 PTS.) Prove that the following language is not regular by providing a fooling set. You need to prove an infinite fooling set and also prove that it is a valid fooling set. For $\Sigma = \{a, b\}$, the language is

$$L = \left\{ ww \mid w \in \Sigma^+ \right\}.$$

2 (25 PTS.) Same as (A) for the following language. Recall that a ***run*** in a string is a maximal non-empty substring of identical symbols. Let L be the set of all strings in Σ^* that contains two distinct runs of equal length. A few examples about L:

- L contains any string of the form $b^i a^+ b^+ a^i$.
- L contains any string of the form $b^i a^+ b^i$.
- L does not contain the strings $abbaaa$, $abbaaabbbb$.

3 (25 PTS.) Suppose you are given two languages L, L' that are not regular but such that $L' \setminus L$ is regular. Prove that $L \cup L'$ is not regular. (Hint: Use closure properties of regular languages.)

4 (15 PTS.) Provide a counter-example for the following claim:

Claim: Consider two languages L and L'. If $\overline{L}$ is not regular, L' is regular, and $L \cup L'$ is regular, then $L \cap L'$ is regular.

5 (10 PTS.) (Slightly harder[1]) Same as (A) for $L = \left\{ 0^{n^4} \mid n \geq 3 \right\}$.

10 Solution:

[1]Feel free to use IDK.

1. Let F be the language $w|w \in \Sigma$.

   Let x and y be arbitrary strings in F.

   Then x = w = a and y = w = b for some w.

   Let z = w = a.

   Then xw = aa is in language L, and yw = ab is not in language L.

   Thus F is a fooling set of L.

   Because F is infinite, L cannot be regular.

2. Let L be put into two cases:

   $w_2^* w_1^i w_2^+ w_1^i w_2^*$

   or

   $w_2^* w_1^i w_2^k w_1^m w_2^i w_1^*$

   where $w \in \Sigma^+$, $w_1 \neq w_2$, and $(k = m = 0)|(k \geq 1 \ \& \ m \geq 1)$.

   Let F be the language $w_2^* w_1^i w_2^+ \mid w \in \Sigma^+$, $w_1 \neq w_2$.

   Then let x, y, u, and v be arbitrary strings in F.

   Let z = $w_2^* w_1^i w_2^k$,

   x = $w_1^i w_2^*$,

   y = $w_1^j w_2^*$,

   u = $w_1^m w_2^i w_1^*$,

   v = $w_1^m w_2^j w_1^*$

   where $i \neq j$, $k \geq 1$ if there is only k and no m, and $(k = m = 0)|(k \geq 1 \ \& \ m \geq 1)$ if there exists both k and m.

   Then zx = $w_2^* w_1^i w_2^k \ w_1^i w_2^*$ is in language L, and zy = $w_2^* w_1^i w_2^k \ w_1^j w_2^*$ is not in language L. We also see that zu = $w_2^* w_1^i w_2^k \ w_1^m w_2^i w_1^*$ is in language L, and zv = $w_2^* w_1^i w_2^k \ w_1^m w_2^j w_1^*$ is not in language L.

   Thus F is a fooling set of L.

   Because F is infinite, L cannot be regular.

3. Claim: $L \cup L'$ is not regular, given L and L' are not regular, and $L' \setminus L$ is regular.

   Proof: We will prove this claim by contradiction. Assume $L \cup L'$ is regular.

   Since $L \cup L'$ is regular, and $L \cap L' \subseteq L \cup L'$, therefore $L \cap L'$ must also be regular.

   If $L \cap L'$ is regular, and $L' \setminus L$ is regular as well, then by set closure properties the union of these two sets is regular. The union of these two sets: $(L \cap L') \cup (L' \setminus L) = L'$, is therefore regular.

   But L' cannot be regular, as we are told in the problem description that it is not regular. We have arrived at a contradiction. Therefore, our original claim that $L \cup L'$ is not regular must be true.

4. Let $L = 0^i 1^i \mid i \geq 1$.

   Let $L' = 0^* 1^*$.

   L is not regular (as seen in class), therefore $\bar{L}$ is also not regular by closure properties (cond. i). L' is regular, as seen in class (cond. ii).

   $L \cup L'$ is equivalent to L', as $L \subseteq L'$. Since L' is regular, $L \cup L'$ must be regular (cond. iii).

   $L \cap L'$ is equivalent to L, as it is a subset of L'. Since L is not regular, therefore $L \cap L'$ is not regular as well, and (cond. iv) fails.

5. IDK