HW Solution

CS/ECE 374: Algorithms & Models of Computation, Spring 2019 Version: **1.0**

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(100 PTS.) Construct This

Let L_1 and L_2 be regular languages over Σ accepted by DFAs $M_1 = (Q_1, \Sigma, \delta_1, s_1, A_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, s_2, A_2)$, respectively.

A (30 PTS.)

Describe a DFA $M = (Q, \Sigma, \delta, s, A)$ in terms of M_1 and M_2 that accepts $L = L_1 \cup \overline{L_2} \cup \{\epsilon\}$. Formally specify the components Q, δ, s, and A for M in terms of the components of M_1 and M_2.

B (30 PTS.)

Let $H_1 \subseteq Q_1$ be the set of states q such that there exists a string $w \in \Sigma^*$ where $\delta_1^*(q, w) \in A_1$.

Consider the DFA $M' = (Q_1, \Sigma, \delta_1, s_1, H_1)$. What is the language $L(M')$? Formally prove your answer!

C (40 PTS.) Suppose that for every $q \in A_2$ and $a \in \Sigma$, we have $\delta_2(q, a) = q$. Prove that $\epsilon \in L_2$ if and only if $L_2 = \Sigma^*$.

6 Solution:

A DFA $M = (Q, \Sigma, \delta, s, A)$ where:

- $Q = Q_1 X Q_2$
- $\Sigma = \Sigma$
- $\delta = (\delta(q1, a), \delta(q2, a)) \forall all p \in Q_1, q \in Q_2, a \in \Sigma$
- $s = (s_1, s_2)$
- $A = \{(p, q) |\ p \in A_1,\ or\ q \in \overline{A_2}\} = A_1 X Q_2 \cup Q_2 X \overline{A_2} \cup s \subseteq Q_1 X Q_2$

B $L(M') = \{w \in \Sigma^* \mid \delta^*(s, w) \in A\}$

to define and formally prove L(M'), we must show that L(M') $\subseteq L$ and $L \subseteq L(M')$ where L =

C We will prove $\epsilon \in L_2 \iff L_2 = \Sigma^*$. It is given that $q \in A_2$, $a \in \Sigma$, and $\delta(q, a) = q$. To prove our claim, we must show that $L_2 \subseteq \Sigma^*$ and $\Sigma^* \subseteq L_2$ only under the condition given.

We assert $L_2 \subseteq \Sigma^*$ because L_2 consists of strings made of alphabet Σ. Because it is given that any input a where $a \in \Sigma$ leads to an accepting state q where $q \in A_2$ from the given $\delta(q, a) = q$, we know any non-empty string generated from Σ leads to some state $q \in A_2$, therefore $\Sigma^+ \subseteq L_2$.

By the nature of the $+$ and $*$ operators, $\Sigma^+ \cup \epsilon = \Sigma^*$. Therefore, if and only if $\epsilon \in L_2$, we can be certain that $\Sigma^* \subseteq L_2$, and in addition to our previous assertion, that $L_2 = \Sigma^*$.