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***Abstract*—Total variation (TV) is an effective tool to solve the image denoising and many other image processing problems. For the denoising problem, it is necessary to create automatic image processing methods based on parameters estimation of the corresponding models. For TV image denoising problem, almost methods focus on TV-L^2 norm. The works for parameter estimation of denoising model based on TV-L^1 norm is very little. The denoising model with TV-L^1 norm is impressive to treat the salt-and-pepper noise. In this paper, we propose a parameter estimation method based on characteristics of the salt-and-pepper noise. This method is especially effective for the images without very high contrast and with high noise level. We will handle the comparison to other salt-and-pepper denoising methods, such as TV-L^1 method and the BPDF method to prove the effectiveness of the proposed parameter estimation method for the adaptive TV-L^1 denoising model.**

Keywords—Image denoising, Salt and Pepper noise, image restoration, regularization, parameter estimation, total variation.

I. INTRODUCTION

In image acquisition systems, acquired digital images always contain noise. There are different kinds of digital image noise which are caused by many factors, such as low light, high contrast, sensor errors, transmission errors etc. The salt-and-pepper noise is a one from them. This noise can be caused by sharp and sudden disturbances in the image signal. The characteristic of the salt-and-pepper noise – the corrupted (noisy) pixels only have one from two values: the maximum or the minimum intensity value of all image domain. The maximum intensity is known as the salt pixel (white pixel) and the minimum – the pepper pixel (black pixel) [1].

To remove the salt-and-pepper noise, there are many linear and nonlinear filters were developed, such as median filter, Wiener filter, fuzzy filters and their own adaptive filters [2]. The nonlinear filters are more effective than the linear filters. In all above filters, the adaptive filters based on median filter got many achievements. Ekran et al. proposed the BPDF method based on adaptive median filter. This method is effective for salt-and-pepper noise with low and medium noise levels [2].

In recent years, the regularization has attracted a lot of attention, especially for the image restoration problem. For regularization, the total variation is a powerful tool and is widely studied.

The original TV regularization targeted image denoising under Gaussian noise. This is a popular noise. Most of the works is devoted to the L^2-norm, whereas the L^1-norm is very effective for the salt-and-pepper noise [1, 3, 4]. To remove the salt-and-pepper noise, Chan and Esedoglu proposed TV-L^1 regularization model [3]. Then, Goldstein et al. proposed the split Bregman method to handle the model [5]. Another well-known method to solve this model is the primal dual gradient method [6].

However, to create an automatic implementation, it is necessary to estimate parameter in the model. For the traditional TV-L^1 model, there is only one regularization parameter. Unfortunately, this issue is skipped. Rojas et al. developed another model based on TV-L^1 to treat the salt-and-pepper noise, that is known as spatially adaptive TV [1]. Although this model has many advantages, to evaluate the numerical solution, there are some parameters need to be configured and this issue was performed by "experience" of authors.

In this paper, we will propose the simple method to estimate the regularization in the traditional TV-L^1 to create an automatic denoising method for the salt-and-pepper noise. This method based on the characteristic of the salt-and-pepper noise: only has two intensity values for salt and pepper pixels. The acquired denoising method by the proposed parameter estimation method is totally automatic. That means we do not need to set any values for the parameters of the model in the numerical implementation. However, this method can be only applied for the salt-and-pepper (with two pixels intensities) denoising but cannot be for the impulse noise (with more pixel's intensities).

In the experiments, we handle the proposed method to process the salt-and-pepper noise with various noise levels to prove that our proposed model can work well on multiple noise levels. We also compare the denoising results with the TV-L^1 model of Chan and Esedoglu with implementation on the primal dual gradient method [3, 6] and the BPDF method [2]. This comparison is only performed on very high noise level (80%).

The rest of the paper is organized as follows. Section II presents the salt-and-pepper noise removal by TV-L^1 denoising model and the proposed parameter estimation method. Section III presents experimental results for salt-and-pepper denoising with various noise levels and the comparison. Finally, Section IV concludes.

II. THE SALT-AND-PEPPER NOISE REMOVAL BY TOTAL VARIATION AND PARAMETER ESTIMATION

A. Generalized Image Denoising Problem by Total Variation

Let $u_0(x), v(x), u(x) \in \mathbb{R}$ – be (grayscale) original image (without noise), noisy image and restored image, respectively, where $x = (x_1, \dots, x_n) \in \mathbb{R}^n$ – pixels. The general image restoration by total variation [7, 8] has the following form:

$$u = \underset{u}{\operatorname{argmin}} \left(\int_\Omega |\nabla u| dx + \frac{\lambda}{p} \int_\Omega |Ku - v|_p^p \, dx \right), \qquad (1)$$

where $|\nabla u| = \sqrt{\sum_i (D_{x_i} u)^2}$, $|.|_p$– the L^p-norm, p=1 or 2, $D_{x_i}u$ – the derivative of u by x_i, K – a filtering operator, in the case of denoising, K – the identity operator and λ – regularization parameter.

Rudin et al. studied model (1) with L^2-norm and this model is well-known as ROF model [9, 10] (Rudin-Osher-Fatemi):

$$u = \underset{u}{\operatorname{argmin}} \left(\int_\Omega |\nabla u| dx + \frac{\lambda}{2} \int_\Omega |u - v|_2^2 \, dx \right),$$

However, this model is only effective for Gaussian noise [11, 12] and does not preserve image contrast and geometry [3]. There are a lot of adaptive models based on ROF have been developed to remove these disadvantages. Unfortunately, those models still did not work well on the salt-and-pepper noise.

Chan and Esedoglu [3] studied the total variation denoising model with L^1-norm and this denoising model is the most effective to treat the salt-and-pepper noise.

B. TV-L^1 Regularization for the Salt-and-Pepper Denoising

The TV-L^1 denoising model that is proposed by Chan and Esedoglu is presented below [3]:

$$u = \underset{u}{\operatorname{argmin}} \left(\int_\Omega |\nabla u| dx + \lambda \int_\Omega |u - v| \, dx \right). \qquad (2)$$

With the normalized image (pixel intensity value from 0 to 1), the regularization parameter need to be $0 < \lambda \le 1$.

The TV-L^1 denoising model preserves image contrast and geometry well and this feature is very important for removing the salt-and-pepper noise. As can be seen, the model (2) is non-convex. In this work, we use the primal-dual gradient method to solve this problem [6].

C. Regularization Parameter Estimation

To perform the model (2) as the automatic denoising method, it is necessary to estimate the parameter λ. This parameter is chosen based on the characteristics of the salt-and-pepper noise.

The salt-and-pepper noise on image has following form [1]:

$$\eta(x) = \begin{cases} v_{max} \text{ with probability } q_1 \\ v_{min} \text{ with probability } q_2 \end{cases}.$$

The v_{max} – pixel intensity of salt pixels, and the v_{min} – pixel intensity of pepper pixels. The sum $q = q_1 + q_2$ – the level of the salt-and-pepper noise.

For the image without very high contrast, the pixel intensity is usually greater than 0 and less than 255 for 8-bits grayscale image. So, if the pixel intensity gets the maximum value or minimum value, it may be a pixel of corrupted parts. Especially, if the noise level is very high, this statement becomes more exact. In this case, we propose method to evaluate the regularization parameter λ as follows:

$$\lambda = \min\{\mu_v / \mathcal{L}_\eta, 1\}, \qquad (3)$$

where μ_v – the mean of corrupted pixels values with the noisy image v is normalized in interval $[0,1]$, $\mathcal{L}_\eta$ – the salt-and-pepper noise level.

Let I, I_c, I_{max}, I_{min} be set of pixels domain, set of corrupted pixels, set of pixels with maximum intensity, set of pixels with minimum intensity, respectively. In the case of the salt-and-pepper noise, we consider that $I_c = I_{max} \cup I_{min}$. For images without very high contrast and with high noise level, the mean value and noise level can be considered as:

$$\mu_v = \frac{\sum_{I_c} v_{I_c}}{card(I_c)}, \mathcal{L}_\eta = \frac{card(I_c)}{card(I)},$$

where v_{I_c} – intensities values of pixels of noisy image corresponding to I_c, the notation $card(.)$ – set cardinality (number of elements of the set).

In the experiments, we will show that the proposed estimation method (3) gets more effective if the salt-and-pepper noise level is greater than 50%. We expect that in the case of very high noise level from 70% and above, the denoising result of the proposed method will be good enough to compare to other state-of-the-art methods.

III. EXPERIMENTAL RESULTS AND DISCUSSIONS

We handle the experiments of the proposed parameter estimation method for the salt-and-pepper denoising on MATLAB. The configuration of the computing system is Windows 10 Pro with Intel Core i5, 1.6GHz, 4GB 2295MHz DDR3 RAM memory.

A. Parameter Values and Error Metrics

There is no parameter that affects the final restoration quality in the adaptive denoising method, because the unique parameter λ was estimated. To handle the test, we set the tolerance $\epsilon = 10^{-5}$ between two successive images in L^2-norm, $\|u^{[i+1]} - u^{[i]}\| < \epsilon$, where i – index of iteration step. Otherwise, we also limit the maximum number of iterations – 500. This setting is helpful if the convergence is slow. However, in all our tests, the loop has not reach this condition. That means the convergence speed is fast enough. These settings are like for implementing TV-L^1 method of Chan and Esedoglu [3]. The BPDF is non-iterative manner, so these settings are unnecessary.

To quantitatively test the restoration performance, we utilize the following error metrics that are widely used in the image processing literature [8, 11, 12, 13, 14, 15]:

$$PSNR = 10\log_{10}\left(\frac{u_{max}^2}{MSE}\right)\ dB$$

where $MSE = (u - u_0)/(mn)$ is the means square error with u_0 is the original (latent) image, u_{max} denotes the maximum value, for e.g for 8-bit images $u_{max} = 255$. Note that a difference of 0.5dB is visible and higher PSNR (measured in decibels – dB) indicate better quality image. Structural similarity (SSIM) is a proven to be a better error metric for comparing the image quality and it is in the range [0, 1] with value closer to one indicating better structure preservation. The SSIM is computed between two windows and of common size $N \times N$,

$$SSIM = \frac{(2\mu_{\omega_1}\mu_{\omega_2} + c_1)(2\sigma_{\omega_1\omega_2} + c_2)}{(\mu_{\omega_1}^2 + \mu_{\omega_2}^2 + c_1)(\sigma_{\omega_1}^2 + \sigma_{\omega_2}^2 + c_2)},$$

where μ_{ω_i} – the average of ω_i, $\sigma_{\omega_i}^2$ – the variance of ω_i, $\sigma_{\omega_1\omega_2}$ – the covariance, and c_1, c_2 stabilization parameters. The mean value of SSIM (MSSIM) is taken as the single value of quality indicating the structural similarity of the two images compared. For image denoising, in the synthetic noise added case, the original noise-free and restored image is compared to find out how much structures are preserved by the restored method while removing noise.

B. Synthetic Images and Test Cases

We test the denoising performance of the proposed method with following cases:

Fig. 1. The original images without noise for test cases.

(a) *The first case*: we implement the proposed method on two images of open dataset with various noise levels. In this case, we will evaluate the image quality metrics PSNR/SSIM after denoising. The original images are taken from the open dataset of the UC Berkeley *https://www2.eecs.berkeley.edu/Research/Projects/CS/vision/bsds/BSDS300/html/dataset/images.html*. We will generate the salt-and-pepper noise by using the built-in function *imnoise* of MATLAB for the tests.

(b) *The second case*: the proposed method will be compared to other salt-and-pepper denoising methods. The dataset is like above. In this case, we only use the salt-and-pepper noise level 80% for all the tests.

Figure 1 shows the selected images of the UC Berkeley dataset that we use for the tests. We choose 20 images. All images are stored in JPEG, grayscale and size 481x321 pixels.

For the first case, we will perform the proposed method on two images with ID 3096 and 42049. Firstly, we add salt-and-pepper noise to image with 20%, 50%, 60% and 70% to the original images. Secondly, we implement the proposed method to remove noise on these noisy input images. The figure 2 shows the denoising result on the plane image with ID 3096. The figure 3 – the result after denoising on the eagle image with ID 42049.

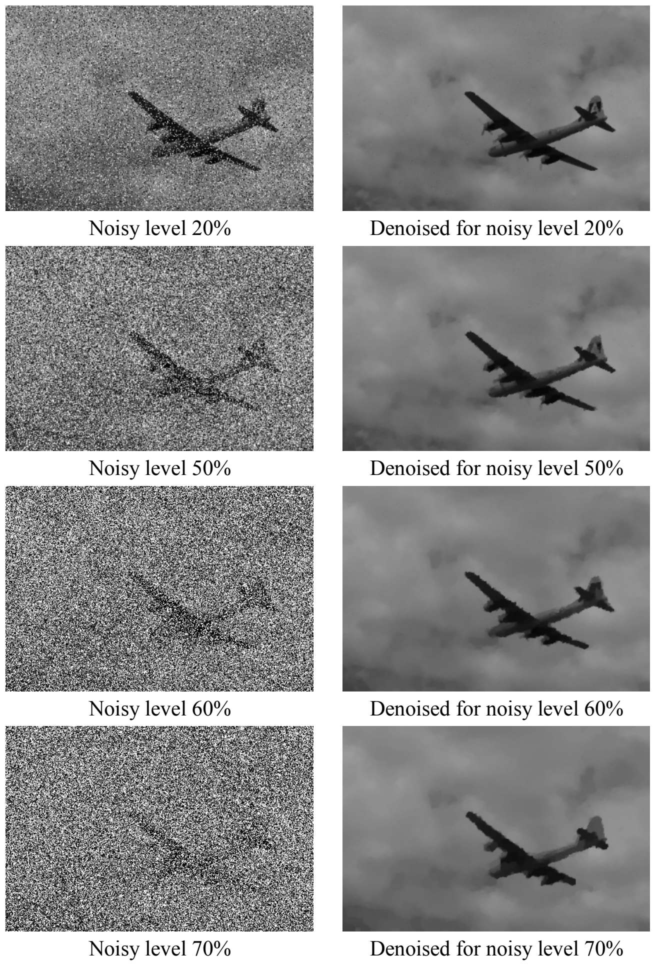

Fig. 2. Denoising result of the proposed method for various noise levels on image with ID 3096.

As we can see on figure 2, the detail of plane is preserved well after denoising with various noise level from 20% to 70%. For the eagle image, with the noise level up to 50%, the denoising result is very good. With high noise level (60%, 70%), some tiny details of the branches and of eagle face are lost. This happens not only for our proposed method, but also for many other denoising methods. The reason is that the tiny details on the images with very high noise levels are almost impossible to see. So, the denoising process cannot restore the tiny details fully. In this case, machine learning based methods are could be helpful to predict the tiny detail. However, in this work, we only focus on non-learning-based methods.

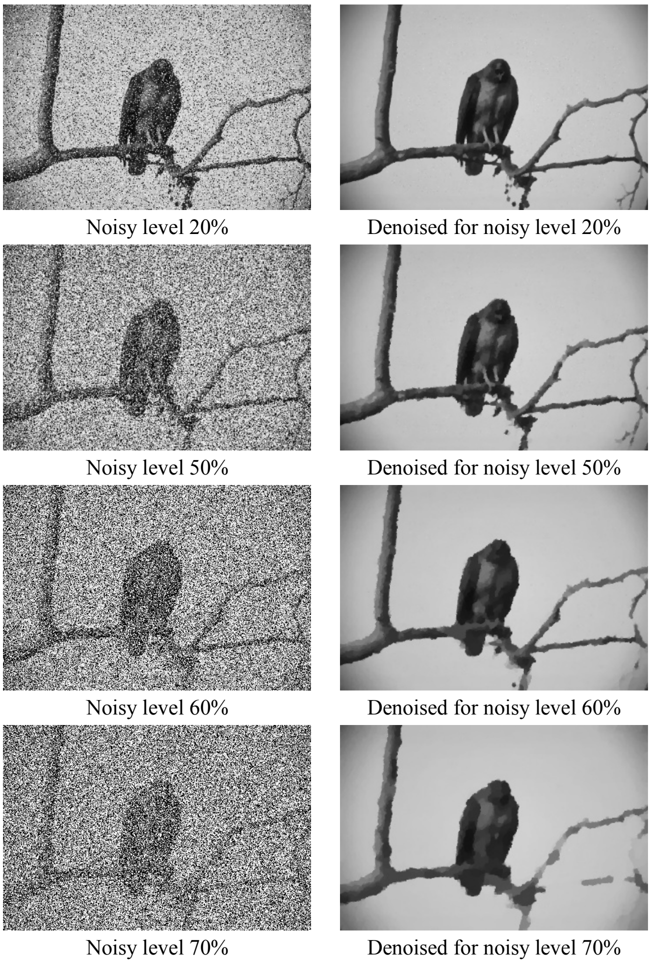

Fig. 3. Denoising result of the proposed method for various noise levels on image with ID 3096.

TABLE 1. IMAGE QUALITY METRICS **PSSR/SSIM** OF DENOISED IMAGES WITH ID **3096** AND **42049** ON VARIOUS NOISE LEVELS.

Noise Level	**Metrics**	**ID 3096**		**ID 42049**	
		Corrupted	**Denoised**	**Corrupted**	**Denoised**
20%	PSNR	12.8021	37.8500	12.0064	29.7937
	SSIM	0.0373	0.9659	0.8095	0.9121
50%	PSNR	8.8442	34.2140	8.0347	25.8639
	SSIM	0.0125	0.9606	0.0328	0.8901
60%	PSNR	8.0544	32.1811	7.2079	24.2787
	SSIM	0.0094	0.9536	0.0232	0.8743
70%	PSNR	7.3748	29.9554	6.5257	21.5455
	SSIM	0.0075	0.9424	0.0168	0.8397

The table 1 shows the image quality metrics PSSR and SSIM of the noisy and denoised versions of the images with ID 3096 and 42049. The deterioration image level is proportional to the raise of noisy level. For the corrupted images, the PSNR metric in every noise level is very low (under 13), that means the corrupted images has very low quality. For the SSIM is same, but there is an exception for image with ID 42049 and noise level 20%, the SSIM is not too bad (SSIM=0.8095). However, after denoising by the proposed method, the reconstructed images have very good quality with high PSNR (greater than 20) and SSIM (greater than 0.8) metrics.

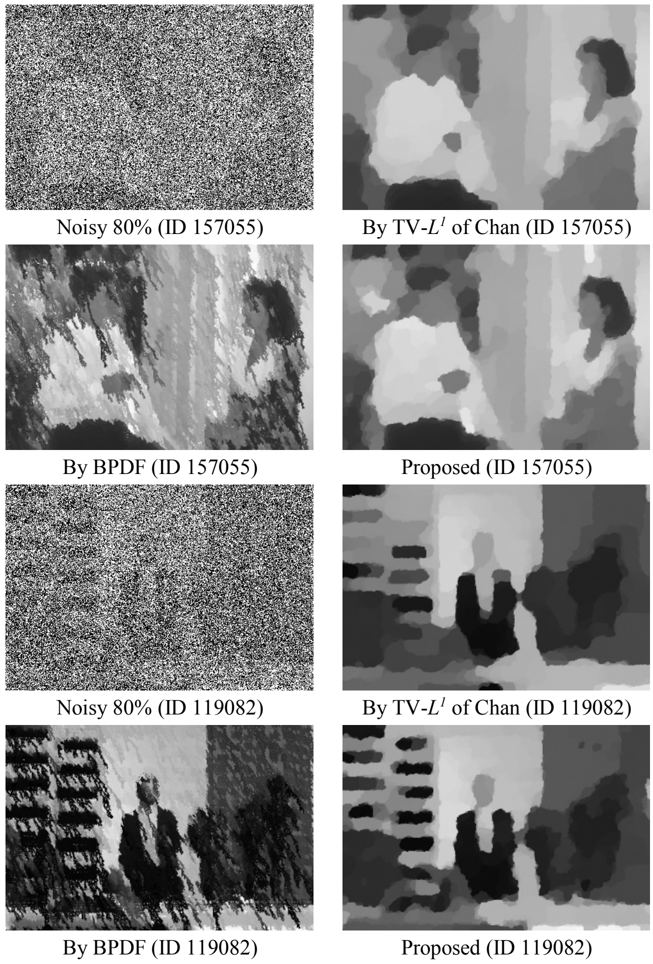

Fig. 4. Result of denoising methods for the salt-and-pepper noise level 80%.

For the second case, we test on 20 selected images with very high noise level (80%). The noisy images have very low quality (PSNR is under 7, SSIM is under 0.03). All of noisy images are fully corrupted. In figure 4, we compare the denoising result of the proposed method to the TV-L^1 method of Chan et al., the BPDF method of Erkan et al. The BPDF has very good performance with low noise level (up to 40%). With higher noise level, the BPDF creates a raindrops effect. Our proposed method preserved details better than TV-L^1 of Chan et al.

The full detail of image quality after denoising by all methods by PSNR and SSIM metrics is presented in table 2. The first value of every cell in table 2 is the PSNR value, the second one – the SSIM value. As we can see, our proposed method gives better denoising result than other methods by both PSNR and SSIM metrics. The average PSNR of denoised images by our proposed method is greater than 20. With very high noise level (80%), the acquired result is really impressive. With very high noise level, it is impossible to restore the tiny details (e.g. the face details) were corrupted by noise in the input images with non-learning-based methods.

TABLE 2. THE COMPARISON OF IMAGE QUALITY METRICS **PSSR/SSIM** OF DENOISING METHODS WITH NOISE LEVEL **80%**.

Image ID \ Method	Corrupted	TV-L^1 (Chan et al.)	BPDF	Proposed
42049	5.9795, 0.0119	18.519, 0.7905	16.4907, 0.7204	***19.4292, 0.8025***
3096	6.7864, 0.006	24.552, 0.9132	23.1412, 0.8423	***26.3411, 0.9206***
300091	6.179, 0.0092	20.9335, 0.6226	18.9197, 0.6044	***21.8633, 0.6325***
16077	6.0822, 0.0136	19.1862, 0.4381	16.3922, 0.4196	***19.9615, 0.4573***
38092	5.9617, 0.0155	19.1217, 0.4481	15.1844, 0.456	***19.6729, 0.4639***
43074	5.9662, 0.0072	24.5578, 0.6464	21.6799, 0.5524	***24.9183, 0.6557***
76053	6.5264, 0.0097	22.3041, 0.468	21.1269, 0.4757	***22.9042, 0.4785***
103070	6.3028, 0.0089	21.6052, 0.5571	19.6269, 0.4948	***22.3067, 0.5741***
119082	5.6603, 0.0219	14.727, 0.3271	12.3868, 0.3438	***16.2289, 0.371***
126007	6.2344, 0.0096	19.7746, 0.6378	18.8031, 0.6448	***20.7662, 0.6468***
157055	6.0205, 0.0149	18.7844, 0.4794	15.6388, 0.4298	***19.3516, 0.4964***
163085	6.2495, 0.0094	21.6652, 0.5385	19.6901, 0.5031	***22.3726, 0.5553***
170057	6.6378, 0.0102	21.8757, 0.5134	19.0199, 0.4846	***22.2866, 0.5243***
182053	6.2957, 0.0181	17.5257, 0.5129	16.6074, 0.5215	***17.8687, 0.5239***
219090	6.2705, 0.0129	19.4204, 0.568	17.7219, 0.5539	***20.0866, 0.5801***
220075	5.9046, 0.0151	16.7594, 0.5133	15.7823, 0.4682	***17.4347, 0.5358***
253027	6.506, 0.0221	16.9442, 0.4211	15.6788, 0.4167	***17.0059, 0.4295***
291000	6.3189, 0.0264	16.7097, 0.14	14.2091, 0.1455	***16.9998, 0.1495***
295087	6.2187, 0.01	22.1957, 0.5382	20.4098, 0.5429	***22.488, 0.5455***
296007	6.7653, 0.0068	25.508, 0.6607	22.589, 0.6227	***26.5729, 0.6693***
$\overline{PSNR}$ $\overline{SSIM}$	6.2433, 0.013	20.1335, 0.5367	18.0549, 0.5122	***20.843, 0.5506***

From denoising result of test cases, we can see that the proposed method can remove the salt-and-pepper noise with various noise level well. Specifically, on the very high noise level, our proposed method gives impressive result to compare to other salt-and-pepper denoising methods.

IV. CONCLUSIONS

In this paper, we proposed a parameter estimation method for the TV-L^1 denoising model to remove the salt-and-pepper noise. Our method is simple and easy to perform. This proposed method is effective only for salt-and-pepper noise without very high contrast. Because the proposed parameter estimation method bases on characteristics of the salt-and-pepper noise, it is inefficient for the impulse noise (the general noise of the salt-and-pepper noise).

As can be seen, the proposed method can work well on various noise levels, but it gets more effective on corrupted images with high noise level (from 50% and above). On images with very high noise level (from 80% and above), the proposed method can be a competitive method to other state-of-the-arts salt-and-pepper denoising methods.

In future works, we can extend the proposed method to apply to the image inpainting problem, the salt-and-pepper noise removal problem for color images and/or 3D images.

REFERENCES

[1] R. Rojas and P. Rodriguez, "Spatially Adaptive Total Variation Image Denoising Under Salt and Pepper noise," in *19th European Signal Processing Conference EUSIPCO*, pp 278-282, Barcelona, 2011.

[2] U. Erkan and L. Gokrem, "A new method based on pixel density in salt and pepper noise removal," *Turkish Journal of Electrical Engineering & Computer Sciences,* vol. 26, pp. 162-171, 2018.

[3] T. F. Chan and S. Esedoglu, "Aspects of Total Variation Regularized L1 Function Approximation," *SIAM Journal on Applied Mathematics,* vol. 65, no. 5, p. 1817–1837, 2005.

[4] P. Rodriguez, "Total Variation Regularization Algorithms for Images Corrupted with Different Noise Models: A Review," *Journal of Electrical and Computer Engineering,* vol. 2013, pp. 1-18, 2013.

[5] T. Goldstein and S. Osher, "The split Bregman method for L1 regularized problem," *SIAM Journal on Imaging Sciences,* vol. 2, no. 2, pp. 323-343, 2009.

[6] T. F. Chan and J. Shen, Image Processing and Analysis: Variational, PDE, Wavelet and Stochastic Methods, SIAM, 2005.

[7] V. B. S. Prasath, D. Vorotnikov, R. Pelapur, S. Jose, G. Seetharaman and K. Palaniappan, "Multiscale Tikhonov-Total Variation Image Restoration Using Spatially Varying Edge Coherence Exponent," *IEEE Transactions*

on Image Processing, vol. 24, no. 12, pp. 5220-5235, 2015.

[8] D. N. H. Thanh and S. Dvoenko, "A method of total variation to remove the mixed Poisson-Gaussian noise," *Pattern Recognition and Image Analysis,* vol. 26, no. 2, pp. 285-293, 2016.

[9] L. I. Rudin, S. Osher and E. Fatemi, "Nonlinear total variation based noise removal algorithms," *Physica D,* vol. 60, p. 259–268, 1990.

[10] D. N. H. Thanh and S. Dvoenko, "Image noise removal based on Total Variation," *Computer Optics,* vol. 39, no. 4, pp. 564-571, 2015.

[11] V. B. S. Prasath, D. N. H. Thanh and N. H. Hai, "On Selecting the Appropriate Scale in Image Selective Smoothing by Nonlinear Diffusion," in *IEEE 7th International Conference of Communications and Electronics*, pp. 267-272, Hue, 2018.

[12] V. B. S. Prasath, D. N. H. Thanh and N. H. Hai, "Regularization Parameter Selection in Image Restoration with Inverse Gradient: Single Scale or Multiscale," in *IEEE 7th International Conference on Communications and Electronics*, pp. 278-282, Hue, 2018.

[13] V. B. S. Prasath, D. N. H. Thanh, N. H. Hai and N. X. Cuong, "Image Restoration With Total Variation and Iterative Regularization Parameter Estimation," in *The Eighth International Symposium on Information and Communication Technology SoICT*, pp. 378-384, NhaTrang, 2017.

[14] D. N. H. Thanh, S. Dvoenko and D. V. Sang, "Colour image denoising based on a combined model," in *Seventh Symposium on Information and Communication Technology*, pp. 308-315, Ho Chi Minh city, 2016.

[15] D. N. H. Thanh and S. Dvoenko, "A denoising of biomedical images," *The International Archives of Photogrammetry, Remote Sensing and Spatial Information Sciences,* vol. XL, no. 5, pp. 73-78, 2015.