Problem set 1

Optimization and Decision Analytics 2025/26

UC3M — *Master on Statistics for Data Science*

Due date: Friday September 26. Value: 50% of the final grade.

Note: This is an individual assignment. Evidence of plagiarism will be penalized. Hand in the assignment as a pdf file through the Assignment module in Aula Global, with Gurobi–Python code printouts in pdf and all required explanations.

Problem 1 (50 points). Consider the following linear optimization problem, formulating the optimal allocation of two resources to five economic activities: with $n = 5$,

$$\begin{aligned}
&\text{maximize } \sum_{j=1}^{n} r_j x_j \\
&\text{subject to:} \\
&\sum_{j=1}^{n} a_{1j} x_j = 30 \\
&\sum_{j=1}^{n} a_{2j} x_j = 50 \\
&x_j \geqslant 0, \;\; j = 1, \ldots, n.
\end{aligned}$$

The coefficient vectors $\mathbf{r} = (r_j)_{j=1}^n$, $\mathbf{a}_1 = (a_{1j})_{j=1}^n$, $\mathbf{a}_2 = (a_{2j})_{j=1}^n$ are to be randomly generated as independent n-vectors, with elements between 1 and 10 included. To ensure that each student gets a different set of vectors, use a time-dependent seed with the pseudo-random number generator you use. In Python, you can use the following code, replacing "vec" as needed.

```
import time, random

seed = int(time.time())          # time-dependent seed (seconds)
random.seed(seed)

vec = [random.randint(1, 10) for _ in range(5)]
```

(a, 7 points) Formulate the problem you get (which should be different for each of you), formulate the dual problem, and find all its optimal solutions using the graphical method.

(b, 7 points) Formulate the optimality conditions that must be satisfied by any optimal primal solution in relation with a dual optimal solution $\boldsymbol{\pi}^*$. Apply them, along with part (a), to find all optimal solutions to the primal problem.

(c, 6 points) Obtain all possible optimal dual solutions, as well as all possible values for the reduced costs. Are they unique ? Interpret the reduced costs obtained.

(d, 15 points) Carry out a sensitivity analysis with respect to simultaneous changes of contraint right-hand sides for the primal problem. Contrast the results with those obtained with Gurobi–Python.

(e, 15 points) Carry out a sensitivity analysis with respect to simultaneous changes of objective coefficients for the primal problem. Contrast the results with those obtained with Gurobi–Python.

Problem 2 (50 points). In a CSI investigation, the crime suspect left both his/her hand and shoe imprints in the crime scene. From that evidence the investigators want to infer the suspect's height. For that purpose, they plan to obtain a prediction equation for height based on hand and shoe size based on the following data:

Hand size (cm)	**Shoe size (cm)**	**Height (cm)**
16.9	29.7	175.3
17.1	30.9	177.8
19.3	33.8	185.4
16.8	31.8	175.3
15.3	27.6	172.7
25.2	35.9	198.5

(a, 16 points) Formulate the Linear Optimization model seen in class for estimating the best prediction equation under the Mean Absolute Error (MAE) criterion, and implement it in Gurobi–Python.

(b, 17 points) Solve the model and give the optimal solution (prediction equation). Is it unique? Why? Characterize all possible prediction equations if there is more than one.

(c, 17 points) Obtain the optimal dual solution and discuss its interpretation.