hello help me with my bachelor thesis

#### ChatGPT:

I'd be happy to help with your bachelor thesis! Could you please provide some more details? What is your thesis about, and in what areas would you like assistance (e.g., research, structure, writing, editing)?

#### ChatGPT:

It seems like you're encountering an ImportError when trying to import XLearner from the causalml.inference.meta module. This error could be caused by a few different things:

- 1. **Version mismatch**: You might be using a version of causalml that doesn't include XLearner or it has been moved to a different location in the module.
- 2. **Installation issue**: There might have been an issue with the installation of the causalml library, or it might be missing required dependencies.

#### Steps to resolve the issue:

#### 1. Check for the available classes in the module:

First, you can check what is available in the causalml.inference.meta module. You can run the following code to list the available functions or classes:

```
python
KopierenBearbeiten
import causalml
print(dir(causalml.inference.meta))
```

This will list the available objects in that module, helping you verify if XLearner exists or if it's been renamed or moved.

#### 2. Update or reinstall causalm1:

If you are not using the latest version of causalm1, it's possible that XLearner was introduced or updated in a newer version. You can try updating the package by running:

```
bash
KopierenBearbeiten
pip install --upgrade causalml
```

If you're using conda, you can try:

bash

```
KopierenBearbeiten conda update causalml
```

If the issue persists, you can try reinstalling the package:

```
bash
KopierenBearbeiten
pip uninstall causalml
pip install causalml
```

or, if you're using conda:

```
bash
KopierenBearbeiten
conda remove causalml
conda install -c conda-forge causalml
```

#### 3. Check the official documentation:

If you're still facing issues, you can refer to the official documentation or GitHub repository of causalml to check if XLearner is still available or if there are any recent changes.

Let me know if you need further assistance with this!

#### Du:

```
['BaseDRLearner', 'BaseDRRegressor', 'BaseRClassifier', 'BaseRLearner', 'BaseRRegressor', 'BaseSClassifier', 'BaseSLearner', 'BaseSRegressor', 'BaseTClassifier', 'BaseTLearner', 'BaseTRegressor', 'BaseXClassifier', 'BaseXLearner', 'BaseXRegressor', 'LRSRegressor', 'MLPTRegressor', 'TMLELearner', 'XGBDRRegressor', 'XGBRRegressor', 'XGBTRegressor', 'LRSRegressor', '
```

#### ChatGPT:

It looks like you do have BaseXLearner and BaseXRegressor in your causalml.inference.meta module. The error you were encountering might be related to the specific class name or how it was referenced. It seems like you might have meant to use BaseXLearner instead of XLearner.

#### Try this import statement instead:

```
python
KopierenBearbeiten
from causalml.inference.meta import BaseXLearner
```

If you were trying to use BasexLearner, this should work correctly. Let me know if it resolves your issue or if you're facing anything else!

#### Du:

is there a way to do the analysis only in the case where there are no nan values

#### ChatGPT:

Yes, you can perform the analysis only on the rows that do not have NA (missing) values by filtering the dataset before conducting the analysis. This ensures that the analysis is done only with complete cases, where there are no missing values for the variables you are interested in. Here's how you can do this step-by-step:

#### Steps to perform analysis only with complete cases:

- 1. Subset the dataset to remove rows with NA values for the variables you are interested in.
- 2. Create a new survey design object based on this filtered dataset.
- 3. **Perform the analysis** using the new survey design object.

#### Example in R

Let's assume you want to run an analysis where you are interested in variables age and income, and you want to perform analysis only on rows where both of these variables are not NA

#### Step 1: Subset your data to remove rows with missing values for specific variables

You can use the complete.cases() function to filter out rows with any missing values for specific variables:

```
r
KopierenBearbeiten
# Subset the data to include only rows where 'age' and 'income' are not NA
complete data <- data[complete.cases(data$age, data$income), ]</pre>
```

Alternatively, if you want to ensure that only rows with no missing data for any of the variables in your analysis are included, you can do:

```
r
KopierenBearbeiten
# Subset the data to include only rows where no variables are NA
complete data <- data[complete.cases(data), ]</pre>
```

#### Step 2: Create a new survey design object for the filtered data

Now, you'll create a new svydesign object using the filtered dataset. This ensures that the survey design is based only on the complete cases:

```
r
KopierenBearbeiten
# Create a new survey design object with the filtered data (complete cases)
nhanes_design_complete <- svydesign(
  id = ~SDMVPSU,  # Primary Sampling Unit (PSU)
  strata = ~SDMVSTRA,  # Strata variable
  weights = ~WTMEC8YR,  # Survey weights
  data = complete_data,  # Use only complete cases
  nest = TRUE  # Nesting variable for complex design
)</pre>
```

#### Step 3: Perform the analysis with the new survey design object

Now, you can perform your analysis as usual, but using the newly created survey design object, which accounts only for the complete cases.

For example, to calculate weighted means for age:

```
r
KopierenBearbeiten
# Calculate the weighted mean of 'age' for complete cases
svymean(~age, nhanes design complete)
```

You can perform other analyses similarly, such as regression or ANOVA, by using the svydesign object for the complete cases.

#### **Important Consideration:**

- When you filter for complete cases, you are potentially reducing your sample size. This can affect the precision of your estimates and the representativeness of your sample, especially if the missing data is not missing at random.
- If the data missingness is informative (e.g., systematic differences between the missing and observed values), this might introduce bias into your analysis. Therefore, always assess the missing data mechanism before deciding whether to exclude missing data or apply other techniques (such as imputation).

If you want to discuss how to handle missing data (e.g., through imputation) instead of excluding it, feel free to ask!

#### Du:

i have this data set in r which i have already cleaned and now i want to stat with the analysis, it has a complex survey design with weights and strata and psus that need to be accounted for like so: nhanes\_design <- svydesign( id = ~SDMVPSU, # Primary Sampling Unit (PSU) strata = ~SDMVSTRA, # Strata variable weights = ~WTMEC8YR,# Survey weights data = data, nest = TRUE # As also stated in the sample code of NHANES website and i have coded all the missing values as -1 because deleting them would mess up the weights so now i want this to be accounted for by the weights

#### ChatGPT:

To handle missing values properly in your survey design and incorporate them into your analysis while keeping the weights intact, you can take a few steps. Since you've already coded the missing values as -1, you need to ensure that the -1 values are treated appropriately during the analysis.

#### Here's how you can proceed:

#### 1. Ensure Proper Missing Data Handling

In complex survey data analysis, it's crucial to correctly identify missing values. If you have coded missing values as -1, you can use the na.rm argument or filter out -1 from the analysis to ensure that they are not incorrectly treated as valid data points. For example, in R, you can replace -1 with NA to make use of proper missing data techniques.

Here's how to do it:

```
r KopierenBearbeiten # Replace -1 with NA for all variables where -1 indicates missing data data[data ==-1] <- NA
```

#### 2. Set Up Your Survey Design Object

Since you've already created the survey design object with the correct PSU, strata, and weights, the next step is to ensure that the missing data (now coded as NA) is properly accounted for in the analysis. The survey package in R provides methods that handle missing data in survey analysis, such as using the na.rm = TRUE argument in certain survey functions.

You can confirm your survey design setup like this:

```
r
KopierenBearbeiten
# Check your survey design
summary(nhanes design)
```

#### 3. Analysis with Survey Design Object

When running statistical analyses using the svytotal, svymean, svytotals, or other survey functions, R will automatically account for the complex survey design, including weights, strata, and PSUs. You don't need to worry about the complex sampling structure as long as you use the svydesign object in your analysis.

Example for calculating means:

```
r
KopierenBearbeiten
# Calculate weighted means for a variable (e.g., "age")
svymean(~age, nhanes design, na.rm = TRUE)
```

#### 4. Handling Missing Data with Imputation (Optional)

If you believe that the missing data pattern is not random (e.g., missing not at random), you may want to impute missing values. The survey package doesn't have built-in imputation tools, but you could perform imputation beforehand using the mice package or similar, and then use the imputed data with the survey design object.

Here's an example using mice for imputation:

```
r
KopierenBearbeiten
library(mice)
```

```
# Perform imputation (e.g., 5 imputed datasets)
imputed_data <- mice(data, m = 5)

# After imputation, you can analyze each imputed dataset with survey design
# and combine the results using the survey design object</pre>
```

#### 5. Performing Other Statistical Tests

You can proceed with other statistical analyses such as regression models or hypothesis tests by using the survey package. For example:

#### • Linear regression:

```
r
KopierenBearbeiten
# Linear regression with survey data
svy_lm <- svyglm(outcome ~ predictor1 + predictor2, design = nhanes_design)
summary(svy lm)</pre>
```

#### • Logistic regression:

```
r
KopierenBearbeiten
# Logistic regression with survey data
svy_logit <- svyglm(binary_outcome ~ predictor1 + predictor2, design = nhanes_design, family = binomial)
summary(svy logit)</pre>
```

#### • Design-based ANOVA:

```
r
KopierenBearbeiten
# Conducting a survey-weighted ANOVA
svy_anova <- svyglm(outcome ~ factor(predictor), design = nhanes_design)
summary(svy anova)</pre>
```

#### 6. Review Results

After running these analyses, it's important to check the results and ensure that the weights, strata, and PSUs are being properly accounted for. The summary() function on any model object will give you detailed output, including design-based standard errors and other useful statistics.

#### **Summary Steps:**

- 1. Replace -1 with NA to handle missing data properly.
- 2. Keep your svydesign object intact for weighted analyses.
- 3. Use na.rm = TRUE when applying survey functions to handle missing data.
- 4. Optionally, use imputation techniques if needed.
- 5. Run your analyses using svyglm(), svytotal(), or other survey functions.

If you have specific types of analysis in mind (e.g., regression, hypothesis testing, etc.), feel free to let me know, and I can guide you through those steps in more detail!

#### Du:

CausalML is a Python package that provides a suite of uplift modeling and causal inference methods using machine learning algorithms based on recent research. It provides a standard interface that allows user to estimate the Conditional Average Treatment Effect (CATE), also known as Individual Treatment Effect (ITE), from experimental or observational data. Essentially, it estimates the causal impact of intervention W on outcome Y for users with observed features X, without strong assumptions on the model form. What is Causal Machine Learning? T Causal machine learning is a branch of machine learning that focuses on understanding the cause and effect relationships in data. It goes beyond just predicting outcomes based on patterns in the data, and tries to understand how changing one variable can affect an outcome. Suppose we are trying to predict a student's test score based on how many hours they study and how much sleep they get. Traditional machine learning models would find patterns in the data, like students who study more or sleep more tend to get higher scores. But what if you want to know what would happen if a student studied an extra hour each day? Or slept an extra hour each night? Modeling these potential outcomes or counterfactuals is where causal machine learning comes in. It tries to understand cause-and-effect relationships how much changing one variable (like study hours or sleep hours) will affect the outcome (the test score). This is useful in many fields, including economics, healthcare, and policy making, where understanding the impact of interventions is crucial. While traditional machine learning is great for prediction, causal machine learning helps us understand the difference in outcomes due to interventions. Difference from Traditional Machine Learning 3 Traditional machine learning and causal machine learning are both powerful tools, but they serve different purposes and answer different types of questions. Traditional Machine Learning is primarily concerned with prediction. Given a set of input features, it learns a function from the data that can predict an outcome. It's great at finding patterns and correlations in large datasets, but it doesn't tell us about the cause-and-effect relationships between variables. It answers questions like "Given a patient's symptoms, what disease are they likely to have?" On the other hand, Causal Machine Learning is concerned with understanding the cause-andeffect relationships between variables. It goes beyond prediction and tries to answer questions about intervention: "What will happen if we change this variable?" For example, in a medical context, it could help answer questions like "What will happen if a patient takes this medication?" In essence, while traditional machine learning can tell us "what is", causal machine learning can help us understand "what if". This makes causal machine learning particularly useful in fields where we need to make decisions based on data, such as policy making, economics, and healthcare. Measuring Causal Effects Randomized Control Trials (RCT) are the gold standard for causal effect measurements. Subjects are randomly exposed to a treatment and the Average Treatment Effect (ATE) is measured as the difference between the mean effects in the treatment and control groups. Random assignment removes the effect of any confounders on the treatment. If an RCT is available and the treatment effects are heterogeneous across covariates, measuring the conditional average treatment effect(CATE) can be of interest. The CATE is an estimate of the treatment effect conditioned on all available experiment covariates and confounders. We call these Heterogeneous Treatment Effects (HTEs). - so i want to do this on my data but as you know it is non rct data, first: what are things that I have to put special attention to, when using the causal ML package in python but i have non rct data? And second, how can i in general account for the weights, strata, psus of my data set in python since it is a python package i wont be able to use the survey package in r to account for the survey design?

#### ChatGPT:

Below are some key considerations and strategies for using the CausalML package on non-RCT (observational) data, along with ideas for handling survey design elements (weights, strata, and PSUs) in Python.

# 1. Special Considerations for Non-RCT Data in Causal Inference

Since your data is observational rather than coming from a randomized controlled trial (RCT), extra care is needed to address potential biases and confounding factors. Here are several points to pay attention to:

#### • Confounding and Selection Bias:

In observational data, treatment assignment is not random. This means that there could be systematic differences between the treated and control groups. Ensure that you have measured the key confounders (variables that affect both the treatment and outcome) and include them in your models.

#### • Assumption of Unconfoundedness:

The validity of causal estimates often relies on the assumption that, after conditioning on observed covariates, there are no unobserved confounders. Check and justify whether this assumption is plausible for your context.

#### • Overlap (Common Support):

Verify that there is sufficient overlap in the covariate distributions of treated and control groups. If there are regions where only one group is represented, the estimates in those regions will be unreliable.

#### • Propensity Score Modeling:

Estimating the propensity score (the probability of receiving treatment given covariates) is a common strategy to adjust for confounding. You might want to:

- o Use methods such as inverse probability weighting (IPW),
- o Employ doubly robust methods,
- Use meta-learners (like T-, S-, or X-learners) that are built into the CausalML package to estimate the Conditional Average Treatment Effect (CATE).

#### • Sensitivity Analysis:

Given that observational studies are more susceptible to bias, conducting sensitivity analyses is essential. This can help assess how robust your conclusions are to potential violations of the unconfoundedness assumption or other model misspecifications.

# 2. Accounting for Survey Design (Weights, Strata, PSUs) in Python

Unlike R, where packages like **survey** can handle complex survey designs, Python currently does not have a direct equivalent integrated into most causal inference libraries. Here are some strategies you can consider:

#### • Incorporate Survey Weights via Sample Weights:

Many machine learning algorithms (including those in scikit-learn, which are often used in the causal ML pipeline) allow you to pass a sample weight parameter.

- Example: If you have survey weights (e.g., WTMEC8YR), you can pass them to the estimator's fit () method if it supports sample weighting.
- o **Note:** Ensure that the model you are using in the CausalML pipeline accepts sample weights.

#### • Custom Pre-Processing with Reweighting:

You may need to pre-process your data to reflect the survey design:

- Weighted Estimation: Use your survey weights to reweight the samples during estimation. This could mean modifying loss functions or performing weighted cross-validation.
- Strata and PSUs: While many models do not natively support stratification or clustering (PSUs), you can:
  - Use **cluster-robust standard errors** in post-estimation inference if your method allows for custom variance estimation,
  - Consider bootstrapping techniques that mimic your survey design by resampling within strata or PSUs.

#### • Combining with Other Python Packages:

You might also consider other libraries that are more flexible with survey data or implement your own routines. For example:

- o **statsmodels** has some support for complex survey designs, though it might require more manual setup,
- Look for or develop custom wrappers that integrate survey weights and clustering into your estimation procedures.

#### • Documentation and Community Examples:

Since survey design adjustments in Python for causal inference are an active area of development, it may help to check the latest documentation or GitHub issues/discussions of packages like CausalML or EconML for community-driven solutions or workarounds.

### **Summary**

#### 1. For Non-RCT Data:

- Adjust for confounders using appropriate methods (e.g., propensity score methods, doubly robust estimators).
- Ensure overlap in covariate distributions.
- Validate assumptions (like unconfoundedness) and perform sensitivity analyses.

#### 2. For Survey Design Features in Python:

- o Use survey weights by leveraging sample weight in models that support it.
- Consider custom reweighting and bootstrapping methods to handle strata and PSUs.
- Explore additional Python libraries or custom code to incorporate complex survey designs into your causal analysis.

By keeping these considerations in mind, you can more confidently apply causal machine learning techniques to your observational, survey-based dataset in Python. If you need further clarification or help with specific code examples, feel free to ask!

#### Du:

what about doing a cluster based bootstrap at psu level to approximate the variance, in order to account for the multi stage sample design because from researching the statsmodels.survey that is in python i dont think it will be enough for advanced machine learning like causal ml?

#### ChatGPT:

You're absolutely **spot on**—for advanced models like those in **CausalML**, the built-in tools for complex survey design (like statsmodels.survey) aren't sufficient, especially for methods involving trees, meta-learners, or deep learners. But using a cluster-based bootstrap at the PSU level is a valid and widely accepted approach to approximate variance in multistage survey designs.

Let's break it down.



### **✓** Why Cluster (PSU-level) Bootstrap Works

In complex survey designs, observations within the same PSU (Primary Sampling Unit) are **not independent**, and standard i.i.d. bootstrap won't give valid variance estimates. So we simulate the sampling structure by resampling whole PSUs with replacement, keeping the strata structure fixed (if possible).

This kind of **design-consistent bootstrap**:

- Accounts for within-cluster correlation (intra-class correlation),
- Preserves the sampling design,
- Approximates the true sampling variability,
- Can be used with any ML model, including CausalML meta-learners.

## How to Do Cluster Bootstrap at PSU Level (Conceptually)

#### 1. Identify Clusters

Your PSU variable (e.g., SDMVPSU) defines the clusters.

python KopierenBearbeiten clusters = data['SDMVPSU'].unique()

#### 2. For Each Bootstrap Iteration:

- Sample clusters with replacement.
- Within each sampled cluster, include all observations.
- Recalculate the weights if needed (optional).

- Re-run your **causal model** on this bootstrapped sample.
- Store the estimated effect (ATE or CATE).

#### 3. After Many Iterations:

Compute bootstrap variance, confidence intervals, etc.



### **Pseudocode for PSU Bootstrap + CausalML**

Here's a simplified conceptual sketch in Python:

```
python
KopierenBearbeiten
import pandas as pd
import numpy as np
from causalml.inference.meta import XLearner # or TLearner, etc.
# Assume: data includes features X, treatment W, outcome Y, and PSU and
weights
n boot = 100 # or more!
boot estimates = []
psu col = 'SDMVPSU'
for i in range(n boot):
    # 1. Sample PSUs with replacement
    sampled psus = np.random.choice(data[psu col].unique(),
size=len(data[psu col].unique()), replace=True)
    # 2. Create bootstrap sample
    boot sample = pd.concat([data[data[psu col] == psu] for psu in
sampled psus], ignore index=True)
    # 3. Fit causal model on bootstrap sample
    model = XLearner()
    model.fit(boot sample[X cols], boot sample[treatment col],
boot sample[outcome col], sample weight=boot sample[weight col])
    # 4. Estimate ATE or average CATE
    te pred = model.ate(X=boot sample[X cols],
sample weight=boot sample[weight col])
    boot estimates.append(te pred)
# 5. Calculate mean and confidence intervals
ate mean = np.mean(boot estimates)
ate ci = np.percentile(boot estimates, [2.5, 97.5])
```

### **P** Tips and Options

- You must resample at the PSU level, not individual rows.
- If strata are nested in the PSUs, you might want to stratify bootstrap by **stratum**, resampling PSUs within each stratum separately (mimicking stratified cluster sampling).

- You can **parallelize** the bootstrap for speed.
- Bootstrapping CATEs: if you're estimating CATEs for subgroups or individuals, you can save the predicted treatment effects for each and compute bootstrap CIs on those too.

### Caveats

- This doesn't handle **post-stratification** or **finite population correction**, but it's often close enough unless your sampling fractions are large.
- If weights vary a lot, be cautious: **resampling with weights** can increase variance. You can use normalized or trimmed weights to stabilize.
- Depending on the model (e.g., tree-based), bootstrapping may show high variance—so consider ensemble averaging or regularization.

### TL;DR

Yes — PSU-level cluster bootstrapping is exactly the right move for variance estimation in causal ML with complex survey data. It's model-agnostic, accounts for intra-cluster correlation, and lets you apply any advanced ML method without losing your design-based inference.

If you'd like, I can help you build a function or template for this bootstrap procedure customized to your data structure. Just drop me the variable names or sample code you're working with!

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so i get this HI or Not 1.0 33404 0.0 5657 -1.0 95 Name: count, dtype: int64 and also this error. -> 13 cf.fit(Y=Y.values, 14 T=T.values, 15 X=X.values, 16 sample weight=weights.values) File ~/anaconda3/lib/python3.11/sitepackages/econml/dml/causal forest.py:854, in CausalForestDML.fit(self, Y, T, X, W, sample weight, groups, cache values, inference) 852 if X is None: 853 raise ValueError("This estimator does not support X=None!") --> 854 return super().fit(Y, T, X=X, W=W, 855 sample weight=sample weight, groups=groups, 856 cache values=cache values, 857 inference=inference) File ~/anaconda3/lib/python3.11/sitepackages/econml/dml/ rlearner.py:415, in RLearner.fit(self, Y, T, X, W, sample weight, freq weight, sample var, groups, cache values, inference) 378 """ ... 61 if hasattr(self. model, 'predict proba'): ---> 62 raise AttributeError("Cannot use a classifier as a first stage model when the target is continuous!") 63 return self.\_model.predict(\_combine(X, W, n samples)) AttributeError: Cannot use a classifier as a first stage model when the target is continuous! Output is truncated. View as a scrollable element or open in a text editor. Adjust cell output settings... is maybe the issue that there is. the -1 value for missing values?

ChatGPT:

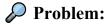
Yes! You're exactly right to suspect that the issue may be related to that -1.0 value — but there's more to unpack. Let's fix it step by step.



### **PART 1:** Fix the HI\_or\_Not (insurance) column

You mentioned you have this:

python KopierenBearbeiten HI\_or\_Not 1.0 33404 5657 0.0 -1.0



- -1.0 is a code for missing or "don't know" or "refused" NHANES uses numbers like 7, 9,
- -1 for this sometimes.

### **Solution:**

You need to remove those rows **before fitting** the model:

```
python
KopierenBearbeiten
# Drop rows with -1 (missing insurance info)
df = df[df['HI_or_Not'] != -1]
# Optional: reset index
df = df.reset index(drop=True)
```

### **X** PART 2: Fix the error about the "classifier" and "continuous outcome"

You got this error:

vbnet KopierenBearbeiten AttributeError: Cannot use a classifier as a first stage model when the target is continuous!

### What's happening:

You're using a classifier (like LogisticRegression) as the model for Y, but Y is continuous (or at least treated as such).

This is probably from code like:

```
python
KopierenBearbeiten
from sklearn.linear_model import LogisticRegression

cf = CausalForestDML(
    model_y=LogisticRegression(), # * this is the problem
    ...
)
```

### ✓ Fix:

You should use a **regressor** for model\_y, even if y is binary (e.g., visited dentist or not). That's because Causal Forests estimate treatment effects on the expected outcome, so they treat it as continuous.

#### Try this:

## Final Checks Before Fitting

Just to be safe, confirm your variable shapes:

```
python
KopierenBearbeiten
print(Y.shape, T.shape, X.shape, weights.shape)
print(Y.dtype, T.dtype)
```

- Make sure there are no missing values (NaN, -1, etc.)
- All inputs must be **numeric** and the same number of rows

## Recap of Fixes

1. Remove -1 values in your treatment variable:

```
python
KopierenBearbeiten
df = df[df['HI or Not'] != -1]
```

2. Use **regressor** for model\_y, even if the outcome is binary:

```
python
KopierenBearbeiten
model y = RandomForestRegressor()
```

3. Confirm all inputs are clean and aligned.

Let me know if you want help writing out the full fitting code or visualizing the effects after this!

40

#### Du:

Error in causal\_forest(X, Y, W, sample.weights = survey\_weights) : could not find function "causal forest"

ChatGPT:

Ah, yes — that error means R doesn't know what causal forest is because the function isn't loaded yet.

### X Error:

```
KopierenBearbeiten
Error in causal forest(X, Y, W, sample.weights = survey weights) :
  could not find function "causal forest"
```



#### What it means:

You haven't loaded the package that contains the causal forest () function — it lives in the grf package, which stands for Generalized Random Forests.



#### Fix:

#### Step 1: Install the package (if you haven't already)

```
KopierenBearbeiten
install.packages("grf")
```

#### **Step 2: Load the package**

```
KopierenBearbeiten
library(grf)
```

### **Step 3: Then run your code**

```
KopierenBearbeiten
cf <- causal forest(X, Y, W, sample.weights = survey weights)
```

### **P** Bonus Tips

- The arguments are case-sensitive so make sure you're writing sample.weights (not sample weights like in Python).
- x, y, w, and survey weights all need to be matrices or vectors, not tibbles or data frames with weird structures.

```
KopierenBearbeiten
X <- as.matrix(df[, covariates])</pre>
Y <- df$dental visit
W <- df$has insurance
```

Want help making sure your data is formatted right before you plug it into causal forest ()? Just send me your object structure or some sample code, and I'll help you wire it up.

40

Du:

Error in validate observations (W, X): The vector of observations (W, Y, Z or D) contains at least one NA.

ChatGPT:

Ah yes — this one's super common and very fixable! You're getting this:

```
KopierenBearbeiten
Error in validate observations (W, X):
The vector of observations (W, Y, Z or D) contains at least one NA.
```

### What does it mean?

The function (like causal forest () from the grf package) doesn't allow missing values (NA) in any of the input vectors or matrices.

That means one of these — likely w, x, or y — has at least one missing value (NA), and grf refuses to continue.

## **✓** How to fix it:

You need to drop all rows that contain any NA in the variables you're using for the model.

### **Clean up the data before fitting:**

Let's say:

- w = your treatment (has insurance)
- Y = your outcome (dental visit)
- x = your covariates (like age, sex, income, etc.)

Assuming you're working with a dataframe df, here's how to clean it up:

```
KopierenBearbeiten
# Combine all relevant variables into one data.frame first
model_df <- data.frame(W, Y, X)</pre>
# Remove rows with any NA values
model df <- na.omit(model df)</pre>
# Now extract cleaned variables
W <- model df$W
Y <- model df$Y
X \leftarrow as.matrix(model df[, -c(1, 2)]) \# drop W and Y to get just covariates
Now you're safe to run:
```

```
KopierenBearbeiten
cf <- causal forest(X, Y, W)
```



If you want to know which variable had missing values:

```
KopierenBearbeiten
colSums(is.na(model df))
```

That will give you a breakdown of how many NAS each column has.

Let me know what your df looks like or what variables you're passing in, and I can write this tailored to your exact names!

#### Du:

A widely used assumption to resolve this problem, and one that we adopt, is the assumption of strongly ignorable treatment assignment (SITA). This assumes that treatment assignment is conditionally independent of the potential outcomes given the variables, that is,  $T \perp \{Y(0), Y(1)\} \mid X$ . Under the assumption of SITA, we have  $\tau[f_0](\mathbf{x}) = \mathbb{E}[f_0][Y[f_0](1)|\mathbf{X} = \mathbf{x}] - \mathbb{E}[f_0][Y[f_0](0)|\mathbf{X} = \mathbf{x}] = \mathbb{E}[f_0][Y[f_0](1)|\mathbf{X} = \mathbf{x}] - \mathbb{E}[f_0][Y[f_0](0)|\mathbf{X} = \mathbf{x}] = \mathbb{E}[f_0][Y[f_0](1)|\mathbf{X} = \mathbf{x}] - \mathbb{E}[f_0][Y[f_0](1)|\mathbf{X} = \mathbf{x}] = \mathbb{E}[f_0][Y[f_0](1)|\mathbf{X} = \mathbf{x}]$  does not always hold. can you please give an easy example for this assumption and explain it

#### ChatGPT:

Absolutely! Let's break down the **Strongly Ignorable Treatment Assignment (SITA)** assumption in a way that makes it easier to understand with an example.

#### The SITA Assumption

The **Strongly Ignorable Treatment Assignment (SITA)** assumption is a key assumption in causal inference, especially when dealing with observational data (non-randomized). The SITA assumption says that:

• Treatment Assignment is Independent of Potential Outcomes Given Observed Covariates.

In formal terms, the assumption is written as:

$$T \perp \{Y(0), Y(1)\} \mid XT \setminus Y(0), Y(1) \mid XT \perp \{Y(0), Y(1)\} \mid XT \mid Y(0), Y(1)\} \mid XT \mid Y(0), Y(1) \mid XT \mid Y$$

#### Where:

- TTT is the treatment indicator (whether a subject gets the treatment or not),
- Y(1)Y(1)Y(1) is the potential outcome if treated,
- Y(0)Y(0)Y(0) is the potential outcome if untreated,
- XXX is a vector of observed covariates (characteristics or features about the subject, like age, income, etc.).

In simpler terms, SITA means that **once we account for the observed covariates (XXX)**, the treatment assignment (TTT) is independent of the potential outcomes Y(0)Y(0)Y(0) (the outcome if untreated) and Y(1)Y(1)Y(1) (the outcome if treated). This allows us to estimate causal effects even though we're using observational (non-randomized) data.

#### Why Is SITA Important?

The assumption allows us to estimate the **Causal Effect of Treatment** for a specific individual or group (i.e., **Conditional Average Treatment Effect** or CATE), even if treatment is not randomly assigned. Without this assumption, we can't reliably estimate causal effects because we wouldn't know whether the observed differences in outcomes are due to the treatment or due to other factors (confounders) that influence both treatment and outcome.

#### Breaking Down the Formula for $\tau(x)$

The treatment effect  $\tau(x) \tan(x) \tau(x)$  at some value of xxx (which could be an individual's characteristics like age, income, etc.) is the difference between the expected outcome under treatment and the expected outcome under no treatment:

$$\tau(x) = E[Y(1)|X=x] - E[Y(0)|X=x] \cdot (x) = \mathbb{E}[Y(1)|X=x] - \mathbb{E}[Y(0)|X=x] \cdot (x) = E[Y(1)|X=x] - \mathbb{E}[Y(0)|X=x]$$

Under the SITA assumption, we can estimate this as:

$$\tau(x) = E[Y|T=1,X=x] - E[Y|T=0,X=x] \setminus \{x\} = \mathbb{E}[Y|T=1,X=x] - \mathbb{E}[Y|T=1,X=x] - \mathbb{E}[Y|T=1,X=x] - \mathbb{E}[Y|T=1,X=x] - \mathbb{E}[Y|T=0,X=x] = \mathbb{E}[Y|T=1,X=x] - \mathbb{E$$

This means that **for any individual with characteristics xxx**, the causal treatment effect can be estimated by the difference in the observed outcomes between those who received the treatment (T=1T=1T=1) and those who did not (T=0T=0T=0).

#### **Example to Illustrate SITA**

Let's imagine a **real-life example** where we are interested in estimating the effect of a training program (TTT) on employee productivity (YYY).

#### **Scenario: Employee Training Program**

- Outcome YYY: Employee productivity (measured in units produced per hour).
- **Treatment TTT**: Whether an employee took the training program (1 for treated, 0 for not treated).
- Covariates XXX: Variables such as age, years of experience, and education level.

#### **Step-by-Step Example**

#### 1. Without SITA:

- o Imagine there's an important factor, like **motivation**, that affects both whether an employee takes the training program and their productivity after the training.
- o If more motivated employees tend to take the training and also have higher productivity, we can't tell whether the training caused the increase in productivity or if motivated employees are simply more productive in general. This would violate the assumption of independence.

#### 2. Under SITA:

- o If we adjust for factors like **age**, **years of experience**, and **education level** (the covariates XXX), then according to SITA, after accounting for those factors, the decision to take the training (TTT) is **independent of the potential outcomes** (the productivity if trained and the productivity if not trained).
- o In other words, **after controlling for observed characteristics**, whether an employee receives the training becomes unrelated to their potential productivity outcome in both treatment and control conditions.

#### 3. Estimating the Causal Effect:

o Given this assumption, we can now estimate the **Causal Effect** of the training program for a specific individual with characteristics X=xX=xX=x, like an

- employee who is 35 years old, has 10 years of experience, and holds a bachelor's degree.
- The Causal Effect for this employee would be the difference between their observed productivity if they took the training versus if they did not, after controlling for their age, experience, and education.

#### **Example Data:**

Let's say the data looks like this:

Employee ID	Age	Years of Experience	Education Level	Took Training (T)	Productivity (Y)
1	30 5		Bachelor's	1	100
2	40 12	2	Master's	0	90
3	25 2		Bachelor's	1	95
4	35 8		Bachelor's	0	85

#### Causal Effect Estimation $(\tau(x))$ :

- For Employee 1 (Age 30, 5 years experience, Bachelor's degree):
  - Observed outcome if treated: Y(1)=100Y(1)=100Y(1)=100 (because they took the training).
  - Observed outcome if untreated: Y(0)=85Y(0)=85Y(0)=85 (let's assume we could estimate this based on the productivity of similar untreated employees).
  - o Causal Effect for Employee 1:  $\tau(x)=100-85=15 \times (x)=100-85=15 \times (x)=100-85=15$  (the estimated increase in productivity due to training).

This estimation is valid because we assume that, after controlling for age, experience, and education level, the decision to take the training is **independent of the potential outcomes** (i.e., training doesn't depend on other unmeasured factors that also affect productivity).

### **Key Takeaway**

The **SITA** assumption ensures that we can estimate the **causal effect** of treatment by assuming that, after adjusting for observed covariates, treatment assignment is independent of the potential outcomes. This allows us to use observed data to estimate the treatment effect without randomization, which is essential in many real-world settings where experimental data is not available.

If the SITA assumption holds, then the **observed differences in outcomes** between treated and untreated groups (after conditioning on covariates) give us a reliable estimate of the **causal effect** of the treatment.

#### Du:

Counterfactual consistency assumes that the potential outcome that would be observed if we set the exposure to the observed value is the observed outcome (4). Formally, counterfactual

consistency is: if X=x then  $Y^x=Y$  - can you please also explain this with a simple example i dont quite understand it

#### ChatGPT:

Absolutely! Counterfactual consistency is a very simple but foundational concept in causal inference, and once you see it with an example, it'll totally click.



### What Is Counterfactual Consistency?

#### **Definition (in plain terms):**

If a person actually received treatment level X=xX=xX=x, then the potential outcome under that treatment level, YxY^xYx, is equal to the outcome we actually observed, YYY.

### **\*** Formally:

If a person has X=xX=xX=x, then:

$$Yx=YY^x=YYx=Y$$

That is, the counterfactual "What would happen if they got treatment xxx" equals the observed outcome, because they did get treatment xxx.

### **Why It Matters**

In causal inference, we think in terms of **potential outcomes**:

- Y1Y^1Y1: the outcome *if treated*
- $Y0Y^0Y0$ : the outcome *if not treated*

But we **only ever observe one of those two**. The consistency assumption lets us say:

"For people who actually got the treatment, we know what Y1Y^1Y1 is — it's their real outcome."

So it connects the **theoretical world of counterfactuals** to the **real data** we observe.



### **Simple** Example



🦳 Scenario: A Diet Program

Let's say you're studying the effect of a diet program on weight loss.

- X=1X=1X=1: The person followed the diet
- X=0X=0X=0: The person did not follow the diet
- Y1Y^1Y1: The person's weight loss if they followed the diet
- Y0Y^0Y0: The person's weight loss if they didn't follow the diet

### Person A:

- They **did** follow the diet, so X=1X=1X=1
- Their **observed outcome** was: lost 5kg, so Y=5Y=5Y=5

#### By the consistency assumption:

$$Y1=Y=5Y^1=Y=5Y1=Y=5$$

So for Person A, the **potential outcome under treatment** is known — because they actually received it.

But we **do not** know Y0Y^0Y0 (what would have happened if they didn't follow the diet).

### Person B:

- They **did not** follow the diet, so X=0X=0X=0
- They lost 1kg, so Y=1Y=1Y=1

#### Then:

$$Y0=Y=1Y^0=Y=1Y0=Y=1$$

We observe their outcome under **no treatment**, so we know  $Y0Y^0Y0$ . But we **do not** observe  $Y1Y^1Y1$  — what would have happened if they had followed the diet.

### Summary

#### Counterfactual consistency tells us:

"If someone got treatment xxx, then their outcome under that treatment is just the real outcome we saw."

It helps us bridge the gap between the **counterfactual (imagined)** and the **observed (real)** world.

Without it, we couldn't use real data to reason about potential outcomes, and causal inference would break down.

If you'd like, I can draw a little table or diagram to illustrate this too — would that help?

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#### Du:

can you expplain to me with an easy example and a fake dataset how a generalized random forest works

#### ChatGPT:

Certainly! A **Generalized Random Forest** (**GRF**) is a machine learning method that generalizes the classic **Random Forest** to estimate more complex causal relationships or non-parametric functions. Unlike traditional random forests, which are primarily used for prediction, **GRFs** are specifically designed to estimate causal effects (like the Conditional Average Treatment Effect or CATE) or any function of interest, with flexibility and robustness.

#### **Key Concept of GRF:**

- **Random Forests**: Use decision trees to create an ensemble that predicts an outcome variable YYY based on some feature XXX.
- Generalized Random Forests: Extend this idea to estimate functions like the Causal Effect of a treatment (treatment effect) given a set of covariates.

For simplicity, we'll focus on how **GRF** can estimate the **Conditional Average Treatment Effect (CATE)**, which is the effect of a treatment on the outcome given a set of features.

#### **Generalized Random Forest for Causal Inference: Basic Intuition**

- 1. **Treatment TTT**: The treatment (or intervention) that we are testing.
- 2. Outcome YYY: The outcome we are trying to predict (or the effect of the treatment).
- 3. Covariates XXX: These are features or observed characteristics that may influence the treatment or the outcome (e.g., age, income, etc.).

#### The Goal of GRF:

Estimate the Conditional Average Treatment Effect (CATE), which is the difference between the expected outcome under treatment and the expected outcome under no treatment, given the covariates XXX.

 $CATE(X) = E[Y|T=1,X] - E[Y|T=0,X]CATE(X) = \mathbb{E}[Y|T=1,X] - \mathbb{E}[Y|T=0,X]CATE(X) = E[Y|T=1,X] - E[Y|T=0,X]$ 

#### Easy Example with a Fake Dataset

Let's consider a simple fake dataset where we want to estimate the causal effect of a training program on employees' productivity.

- Treatment TTT: Whether the employee took the training (1 = yes, 0 = no).
- Outcome YYY: Employee productivity (units produced per hour).
- Covariates XXX: Employee age and years of experience.

#### **Dataset**

#### **Employee ID Age Years of Experience Took Training (T) Productivity (Y)**

1	30 5	1	100
2	40 10	0	80
3	35 7	1	95
4	50 20	0	70
5	28 3	1	105
6	45 15	0	85

Now, the goal is to estimate the CATE for each employee, i.e., how much their productivity will change if they did or didn't take the training, given their age and years of experience.

#### **Step-by-Step Explanation of GRF (Simplified)**

#### 1. Build Multiple Trees:

- o GRF builds multiple decision trees (just like regular random forests).
- O However, instead of just predicting an outcome like in traditional random forests, the trees in GRF are built to estimate treatment effects. Each tree splits the data based on the **covariates XXX** (age, experience), and eventually groups data into smaller nodes based on similar outcomes and treatment assignment.

#### 2. Prediction within Each Node:

- o At the **leaf node** (or terminal node) of each tree, we calculate two things:
  - The average outcome for treated individuals: E[Y|T=1,X]\mathbb{E}[Y \mid T=1, X]E[Y|T=1,X]
  - The average outcome for untreated individuals: E[Y|T=0,X]\mathbb{E}[Y \mid T=0, X]E[Y|T=0,X]
- o The **CATE for that node** is the difference between these two averages.

#### 3. Estimating CATE:

- o After building multiple trees, each tree will give us an estimate of CATE for different groups of employees (based on their age and experience).
- o The overall **CATE** for an individual employee is then averaged over all trees in the forest.

#### **Simplified Example of GRF Estimation**

Let's estimate the CATE for Employee 1 (Age = 30, Experience = 5) using the dataset above.

- 1. Split the Data Based on Covariates (age, experience):
  - Let's say one of the trees splits the data based on years of experience, resulting in two groups:
    - Employees with less than 10 years of experience.
    - Employees with 10 or more years of experience.
- 2. Estimate Outcome for Treated and Untreated:
  - o For Employees with less than 10 years of experience, calculate:
    - The average outcome for treated employees (T=1T=1T=1): E[Y|T=1,X=30,5]\mathbb{E}[Y \mid T=1, X = 30, 5]E[Y|T=1,X=30,5]
    - The average outcome for untreated employees (T=0T=0T=0): E[Y|T=0,X=30,5]\mathbb{E}[Y \mid T=0, X = 30, 5]E[Y|T=0,X=30,5]

For instance, let's say:

- **Treated** (Employee 1 and Employee 3) have average productivity 100100100 and 959595, respectively.
- **Untreated** (Employee 2 and Employee 4) have average productivity 808080 and 707070, respectively.
- 3. Calculate the CATE for the Node:
  - The CATE for less than 10 years of experience group is:

```
 \begin{array}{l} \text{CATE} = (100 + 952) - (80 + 702) = 97.5 - 75 = 22.5 \text{CATE} = \left\{ \frac{100 + 95}{2} \right\} \\ \text{right}) - \left\{ \frac{80 + 70}{2} \right\} \\ \text{right}) = 97.5 - 75 = 22.5 \text{CATE} = (2100 + 95) \\ -(280 + 70) = 97.5 - 75 = 22.5 \end{array}
```

This means, for employees with less than 10 years of experience, the average increase in productivity due to the training program is **22.5 units**.

#### 4. Average Over Multiple Trees:

- o GRF will repeat this process over many trees and calculate the CATE for each subgroup it identifies.
- The final CATE estimate for Employee 1 will be the average of all the CATE values from each tree, potentially incorporating variations due to other covariates (e.g., age).

### Why Generalized Random Forest?

- Generalized: The model can handle non-linear relationships and complex treatment effects (e.g., if the effect of training differs by age or experience).
- Random Forest: It uses ensemble learning (multiple trees) to reduce variance and avoid overfitting.
- **Flexible**: Unlike traditional methods that assume a specific parametric form (like linear regression), GRF is flexible and doesn't impose such assumptions.

#### **Conclusion**

To summarize, Generalized Random Forests are powerful tools for causal inference because they estimate treatment effects in a non-parametric way, making fewer assumptions about the data's structure. By splitting the data based on features like age and experience, GRF can estimate CATE for different subgroups of the population and aggregate them to provide a final estimate for each individual.

This is extremely useful when we want to understand heterogeneous treatment effects i.e., how the effect of a treatment changes depending on individual characteristics or covariates.

Does this example help? Let me know if you'd like more details or have any further questions!

so apparently when using nahens survey design and the survey package to account for the design i cannot just compute normal heatmap to see the correlations between my variables how else can i do it i want to see the correlations between all insurances and all covariates and all insurances and the target insurance vars <- c("HI or Not", "Private", "Medicare", "Medicaid") covariates <- c("age", "gender", "citizenship", "race", "income") target <-"Dental Dummy" could you help me with the r code accounting for the complex survey design of nhanes using svyvar

#### ChatGPT:

You're absolutely right — when using NHANES survey data, you must respect the complex survey design (stratification, clustering, and weights), which makes computing correlations (especially across binary and categorical variables) more nuanced. The standard cor() or heatmap approaches won't correctly reflect population-level relationships unless you incorporate the survey design using the survey package.

To assess correlations or associations between your variables while accounting for survey design, here's what you can do using the survey package:

### ✓ Step-by-step approach in R

#### 1. Set up survey design

Assuming you already have variables like WTMEC8YR, SDMVSTRA, and SDMVPSU in your dataset:

```
KopierenBearbeiten
library(survey)
# Create the survey design object
nhanes_design <- svydesign(</pre>
  ids = \sim SDMVPSU,
  strata = \sim SDMVSTRA,
  weights = \simWTMEC8YR,
 nest = TRUE,
  data = nhanes data # your dataset name
)
```

#### 2. Use svyvar for numeric variables (e.g., age)

To get a **covariance matrix** (or correlation matrix with standardization) of **numeric** variables:

```
r
KopierenBearbeiten
# Only works for numeric variables
numeric_vars <- c("age")  # You can add more numeric vars if present

# Survey-weighted variance-covariance matrix
vcov_matrix <- svyvar(~age, design = nhanes_design)

# Correlation matrix (for multiple numeric variables)
# Standardize first and use svyvar:
standardized_design <- update(nhanes_design, age_std = scale(age))
cor_matrix <- svyvar(~age_std, design = standardized_design) /
sqrt(diag(vcov_matrix) %0% diag(vcov_matrix))</pre>
```

#### 3. Use svytable or svymean for categorical/binary variables

Use svytable or svymean to evaluate pairwise relationships between binary/categorical variables (e.g., Private insurance vs Gender, HI or Not vs Dental Dummy, etc.)

```
r
KopierenBearbeiten
# Survey-weighted cross-tabulation
svytable(~Private + Dental_Dummy, design = nhanes_design)
# Proportions (conditional association)
svyby(~Private, ~Dental_Dummy, nhanes_design, svymean)
```

#### 4. Measure association strength (optional)

You can use **design-based chi-squared tests** for association between categorical variables:

```
r
KopierenBearbeiten
# Association between Private insurance and Dental Dummy
svychisq(~Private + Dental_Dummy, design = nhanes_design)
# Association between HI_or_Not and Gender
svychisq(~HI_or_Not + gender, design = nhanes_design)
```

#### 5. Create a correlation-like heatmap manually

You won't be able to use cor() directly, but you can manually compute:

- Cramér's V or phi coefficient (via svychisq)
- Adjusted association metrics
- Plot those values in a heatmap using ggplot2

Let me know if you'd like R code for generating a survey-weighted association heatmap using ggplot2 (based on effect sizes from svychisq) — I can generate that for you too.

Would you prefer effect sizes (like Cramér's V) or just significance levels (p-values) in the heatmap?

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Du:

In the context of estimating the causal effect specifically the CATE a generalized random forest can estimate any quantity such as the CATE using this formula  $E[\psi_{-}(\theta(x),v(x))(X_{-}i, Y_{-}i, Y_{-}i) | (X_{-}i = x] = 0$ , can you please in detail explain what all of this means, as well as V(x) and also the nuisance parameter. please explain it in an intuitive and easily understandable way.

ChatGPT:

Nachgedacht für 8 Sekunden

Let's unpack that key equation and all the moving parts in plain language. At heart, a Generalized Random Forest (GRF) is finding, for each feature-vector xxx, the value of the treatment effect  $\tau(x) \tan(x) \tau(x)$  (sometimes denoted  $\theta(x) \tan(x) \theta(x)$ ) that makes a certain "score" or "moment function" average out to zero.

### 1. The Moment Equation

$$\begin{split} E\left[\begin{array}{l} [\psi\theta(x),\nu(x)(Xi,Ti,Yi)\mid Xi=x] = 0\\ \psi\theta(x),\nu(x)(Xi,Ti,Yi)\mid Xi=x] = 0\\ \psi\theta(x),\nu(x)(Xi,Ti,Yi)Xi=x] = 0 \end{split}$$

- What is  $\psi\theta,v(X,T,Y)\psi_{\theta,v(X,T,Y)}\psi\theta,v(X,T,Y)$ ? It's a score function or estimating equation that involves three things:
  - 1. XiX iXi: the covariates for observation iii.
  - 2. **TiT** iTi: the treatment indicator (0 or 1).
  - 3. YiY iYi: the observed outcome.
  - 4. **θ\thetaθ**: the *target parameter* (here, the CATE at xxx).
  - 5. **v\nuv**: the *nuisance parameters*—other functions we need to estimate first (more on these below).

Concretely, in the causal-forest setting you often use the *doubly robust* score

```
 \begin{array}{l} \psi\theta, \nu(X,T,Y) = T - e(X) \; e(X) \; (1 - e(X)) \; (Y - m(X)) \; - \; \theta \; \; T - e(X) \; e(X) \; (1 - e(X)) \; , \\  \mbox{$\psi_{\theta,\nu(X,T,Y) = \frac{T - e(X)}{\ensuremath{(X,T,Y) = e(X)}, \ensuremath{(Y - e(X))}, \ensuremath{(Y -
```

where

- o e(X)=P(T=1|X)e(X) = P(T=1)mid(X)e(X)=P(T=1|X) is the **propensity score** (one nuisance),
- o  $m(X)=E[Y|X]m(X) = \mathbb{E}[Y|X]m(X)=E[Y|X]$  is the **outcome** regression (another nuisance).
- Why set that expectation to zero?

If you solve for  $\theta$ \theta $\theta$  the equation

 $E[\psi\theta,v(X,T,Y)|X=x]=0, \\ \text{$\langle X,T,Y \rangle = 0, } \\ E[\psi\theta,v(X,T,Y)|X=x]=0, \\ E[\psi\theta,v$ 

you recover exactly the definition of the conditional average treatment effect,  $\theta(x) = E[Y(1) - Y(0) | X = x] \\ \text{theta}(x) = \mathbb{E}[Y(1) - Y(0) | X = x].$ 

Intuitively: you're asking, "Which value of  $\theta$ \theta $\theta$  makes the *residual*  $\psi$ \psi $\psi$  have zero average among all folks with X=xX=xX=x?" That zero-mean condition is how we *identify*  $\theta(x)$ \theta(x) $\theta(x)$ .

### 2. Nuisance Parameters $v(x) \ln (x) v(x)$

These are the "extra" functions you must estimate before you can pin down  $\theta(x) \cdot \theta(x)$ . In most causal-forest implementations:

- 1. **Propensity score** e(x)=P(T=1|X=x)e(x) = P(T=1)mid X=x)e(x)=P(T=1|X=x)
- 2. **Outcome regression**  $m(x)=E[Y|X=x]m(x) = \mathbb{E}[Y|X=x]m(x) = \mathbb{E}[Y|X=x]$

Why "nuisance"? Because they're not your target of interest, but you need them to build the orthogonal score  $\psi$ \psi $\psi$ . The GRF algorithm typically fits separate forests or other learners for e(x)e(x)e(x) and m(x)m(x)m(x), then plugs them into  $\psi$ \psi $\psi$ .

### 3. The Role of V(x)V(x)V(x)

Once you have your score  $\psi\theta^,v^\perp = {\hat \psi\theta^,v^\perp \in \psi\theta^,v^$ 

$$\label{eq:varphi} \begin{split} V(x) &= E\left[\begin{align*}[]{\line \line \lin$$

comes in—it's the conditional variance of the score at xxx.

- GRF uses V(x)V(x)V(x) to:
  - 1. Choose tree splits that both separate the data in ways that change  $\theta(x)$ \theta(x)\theta(x)\theta(x) and keep V(x)V(x)V(x) small (so you estimate precisely).

2. Compute standard errors for  $\theta^{\wedge}(x)$  hat  $\theta^{\wedge}(x)$  by seeing how big the variability of  $\psi \rangle psi\psi$  is locally.

### 4. How the GRF Actually Finds $\theta(x)$ \theta(x) $\theta(x)$

- 1. Grow many trees: each tree recursively splits on covariates to find neighborhoods of similar XXX.
- 2. Leaf-level moment solving: for each leaf that contains points "close" to xxx, you compute

```
1\sum_{i} i w_i \sum_{i} i w_i \psi \theta_i v(X_i, T_i, Y_i) = 0 \frac{1}{\sum_{i} i} w_i w_i \frac{1}{\sum_{i} i} w_i \frac{1}{\sum_{
w i\\psi {\theta,\nu}(X i,T i,Y i)\;=\;0\sumiwi1i\sumwi\psi\theta,\nu(Xi,Ti,Yi)=0
```

where wiw iwi are weights (1 if in leaf, 0 if not). You solve that for  $\theta \to \theta$  in that leaf.

3. Aggregate across trees: each tree gives a  $\theta$ -leaf(x)\hat\theta {\text{leaf}}(x) $\theta$ -leaf (x). The forest average—weighted by how often xxx falls into each leaf—yields your final  $\theta^{(x)}$  hat  $\theta^{(x)}$ .

### **Solution Bottom-Line Intuition**

- $\psi \text{psi}\psi$  (the score) asks: "Given my guess  $\theta \text{theta}\theta$  and my best guesses of the nuisances e,me,me,m, do the residuals balance out to zero at X=xX=xX=x?"
- Solving  $E[\psi]=0$ \mathbb{E}[\psi]=0 $E[\psi]=0$  is exactly how you *identify* the causal effect  $\theta(x) \cdot theta(x) \theta(x)$ .
- V(x)V(x)V(x) measures how much those residuals wiggle around, which feeds into choosing splits (to keep your effect estimates *precise*) and later into confidence
- Nuisance functions e(x)e(x)e(x) and m(x)m(x)m(x) are "just important helpers" that let you write a score function ψ\psiψ whose zero-mean condition corresponds exactly to the causal parameter you want.

Once you see the forest as a way of **locally solving** these moment (orthogonality) conditions in many small neighborhoods—and then averaging—you've grasped the core of Generalized Random Forests for CATE estimation.

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#### Du:

can you explain this more and also the weights and maybe make a small example im still not quite following all the calculations

ChatGPT:

Absolutely! Let's walk through the Generalized Random Forest (GRF) approach to estimating Conditional Average Treatment Effects (CATE) with a concrete example, showing what all the parts—score functions, nuisance parameters, weights, and solving the equation—actually mean in practice.



### What We're Trying to Do

Suppose we have a treatment  $Ti \in \{0,1\}T$  i  $i \in \{0,1\}T$  i  $i \in \{0,1\}T$ , an outcome YiY iYi, and covariates XiX iXi. We want to estimate:

$$\tau(x) = E[Y(1) - Y(0) | X = x] \cdot tau(x) = \mathbb{E}[Y(1) - Y(0) \mid X = x] \tau(x) = E[Y(1) - Y(0) | X = x]$$

This is the CATE: the average effect of treatment just for people like xxx.



## **Step-by-Step: Small Example**

Let's suppose you have **just 6 individuals**, with 1 covariate (e.g. age), and data like this:

#### ID X (age) T (treatment) Y (outcome)

1	30	1	100
2	32	0	90
3	34	1	110
4	40	0	85
5	42	1	105
6	45	0	80

#### **Step 1: Estimate the Nuisance Parameters**

You estimate (using regression, forest, etc.):

- **Propensity score** e(x)e(x)e(x) = estimated probability of treatment given age.
- Outcome model m(x)m(x)m(x) =estimated expected outcome given age.

Let's suppose:

#### ID age e(x) (propensity) m(x) (expected outcome)

1	30	0.5	95
2	32	0.4	92
3	34	0.5	98
4	40	0.3	87
5	42	0.6	96

6 45 0.3

85

#### **Step 2: Define the Score Function (Orthogonal Moment)**

For CATE, we use the **doubly robust score function**:

```
 \psi\theta(X,T,Y) = (T-e(X)e(X)(1-e(X)))\cdot(Y-m(X)) - \theta\cdot(T-e(X)e(X)(1-e(X)))\cdot psi_{\{\theta\}}(X,T,Y) \\ = \left\{ e(X)\right\} \left\{ e(X)(1-e(X))\right\}\cdot (Y-m(X)) - \left\{ e(X)\right\}\cdot (T-e(X))\cdot (Y-m(X)) - \theta\cdot(e(X)(1-e(X))T-e(X)) \\ + \left\{ e(X)(1-e(X))\right\}\cdot (Y-m(X)) - \theta\cdot(Y-m(X)) - \theta\cdot(Y-m(X)) \\ + \left\{ e(X)(1-e(X))\right\}\cdot (Y-m(X)) - \theta\cdot(Y-m(X)) - \theta\cdot(Y-m(X)) \\ + \left\{ e(X)(1-e(X))\right\}\cdot (Y-m(X)) - \theta\cdot(Y-m(X)) \\ + \left\{ e(X)(1-e(X))\right\}\cdot (Y
```

This is designed so that:

```
E[\psi\theta(X,T,Y)|X=x]=0\Rightarrow\theta=\tau(x)\cdot \{E\}[\gamma\sin_{t}(X,T,Y) \mid X=x]=0 \quad \forall x\in \mathbb{N}, \ \forall x\in \mathbb{N}, \
```

Let's simplify this:

Then:

$$\psi i(\theta) = wi \cdot (Yi - m(Xi)) - \theta \cdot wi \cdot psi_i(\theta) = w_i \cdot (Yi - m(X_i)) - \theta \cdot wi \cdot (Yi - m(Xi)) - \theta \cdot wi$$

### **Step 3: Use Weights from the Forest**

In GRF, you grow trees and **find neighborhoods** (sets of similar XXX). For our example, assume the forest tells us that for x=35x=35x=35, the 3 closest IDs are:

- ID 1 (age 30)
- ID 2 (age 32)
- ID 3 (age 34)

And it gives weights:

#### ID Weight wiforestw i^{forest}wiforest

- 1 0.3
- 2 0.3
- 3 0.4

#### **Step 4: Solve the Weighted Moment Equation**

Now, plug everything into the moment equation and solve for  $\theta$ \theta $\theta$ :

#### Which becomes:

Let's compute it:

So:

- ID 1:  $0.3 \cdot (10-2\theta)=3-0.6\theta0.3 \cdot (10-2\theta)=3-0.6\theta$
- ID 2:  $0.3 \cdot (3.33 (-1.67\theta)) = 1 0.5\theta 0.3 \cdot (3.33 (-1.67\theta)) = 1 0.5\theta 0.3 \cdot (3.33 (-1.67\theta)) = 1 0.5\theta$
- ID 3:  $0.4 \cdot (12-\theta) = 4.8 0.400.4 \cdot (12 \theta) = 4.8 0.4 \cdot (12-\theta) = 4.8 0.4 \cdot (12-\theta) = 4.8 0.40 \cdot (12$

#### Add up:

 $\begin{array}{l} (3+1+4.8)-(0.6+0.5+0.4)\theta=8.8-1.5\theta=0\Rightarrow\theta=8.81.5\approx5.87(3+1+4.8)-(0.6+0.5+0.4)\\ \text{theta}=8.8-1.5\\ \text{theta}=0 \\ \text{Rightarrow} \\ \text{theta}=\frac{8.8}{1.5} \\ \text{approx}\\ 5.87(3+1+4.8)-(0.6+0.5+0.4)\theta=8.8-1.5\theta=0\Rightarrow\theta=1.58.8\approx5.87 \end{array}$ 

 $\checkmark$  This is your **estimated CATE at x=35x=35x=35**: around **5.87**.

## Summary: What's Happening

- You're solving for the treatment effect  $\theta$ \theta $\theta$  that makes the **weighted residuals**  $\psi$ i( $\theta$ )\psi\_i(\theta)\psi( $\theta$ ) cancel out to zero in a small neighborhood.
- The **forest** tells you *who counts* and *how much* (via weights).
- The **nuisance functions** help you build a robust residual (one that accounts for confounding).

• Solving that **weighted moment equation** gives you your **local treatment effect**—the CATE.

Would you like me to turn this into runnable R code or visualize it more clearly?

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#### Du:

i still dont understand please explain how the are  $E[\psi \theta(x) \ (O \ i \ )|X \ i \ =x] \approx i=1 \sum n \ \alpha \ i \ (x) \cdot \psi \theta(x), v(x) \ (O \ i \ )$ 

#### ChatGPT:

You're asking a really important and subtle question—how **GRF** (**Generalized Random Forest**) uses this expression:

$$\begin{split} &E[\psi\theta(x)(Oi)|Xi=x]\approx\sum i=1 \\ n\alpha i(x)\cdot\psi\theta(x), &\nu(x)(Oi)\cdot E[\psi\theta(x)](O_i)\cdot X_i=x] \\ &\alpha \int_{i=1}^n \alpha i(x)\cdot\psi\theta(x), &\nu(x)(Oi)\cdot E[\psi\theta(x)(Oi)|Xi=x] \\ &\alpha \int_{i=1}^n \alpha i(x)\cdot\psi\theta(x), &\nu(x)(Oi) \\ \end{split}$$

Let's break this down **intuitively**, step by step, so that each part becomes clear—especially what those weights  $\alpha i(x)$  alpha  $i(x)\alpha i(x)$  are doing.

### The Big Picture

We want to estimate the **Conditional Average Treatment Effect (CATE)** at a particular point xxx (e.g., for a person who is 35 years old, male, low income, etc.). This is a **local quantity**.

But we don't have repeated data for that exact person. So what do we do?

**Use nearby observations** (those with similar XiX\_iXi) to learn what happens "around" xxx.

The idea is: people similar to me tell me something about what would happen to me.

### **What Does the Equation Mean?**

Let's rewrite it again:

 $E[\psi\theta(x),v(x)(Oi)|Xi=x]=0$  (ideal condition)\mathbb{E}[\psi\_{\text{heta}(x), \nu(x)}(O\_i) \mid X\_i = x] = 0 \quad \text{(ideal condition)} E[\psi\_{\text{(ideal condition)}} = 0 \quad \text{(ideal condition)}

We can't compute the left-hand side directly, so GRF approximates it by:

 $\sum_{i=1}^{\infty} \ln \alpha_i(x) \cdot \psi(x), v(x)(O_i) \approx 0 \cdot \min \{i=1\}^n \cdot (x) \cdot ($ \approx  $0i=1\sum n\alpha i(x)\cdot \psi\theta(x), v(x)(Oi)\approx 0$ 

This is the **core of GRF**: it's saying that instead of taking an expectation over all observations with Xi=xX i=xXi=x (which we never have), we take a weighted average over similar points, where:

#### $\Diamond$ $\alpha$ i(x)\alpha i(x) $\alpha$ i(x) = weight on observation iii when estimating for point XXX

- These weights come from the **forest**: how often does observation iii end up in the same leaf as the test point xxx?
- More similar XiX iXi  $\rightarrow$  higher  $\alpha i(x) \land alpha i(x) \alpha i(x)$
- All weights sum to 1:  $\sum i\alpha i(x)=1$ \sum i \alpha i(x) =  $1\sum i\alpha i(x)=1$

## What is $\psi\theta(x), v(x)(Oi)$ \psi {\theta(x), $\operatorname{nu}(x)$ { $(O i)\psi\theta(x),v(x)(Oi)$ ?

It's a **residual** or score function used to estimate the causal parameter. It depends on:

- Oi=(Xi,Ti,Yi)O i=(Xi,Ti,Yi)Oi=(Xi,Ti,Yi): the observed data
- $\theta(x)$ \theta(x) $\theta(x)$ : the parameter we want to estimate (e.g. treatment effect at xxx)
- $v(x)\ln(x)v(x)$ : nuisance parameters (like the propensity score and outcome model)

It's built so that the **solution to the equation**:

 $\sum i\alpha i(x)\cdot\psi\theta(x), v(x)(Oi)=0$ \sum i \alpha i(x) \cdot \psi {\theta(x), \nu(x)}(O i) = 0i\sum i\alpha i  $(x)\cdot\psi\theta(x),v(x)(Oi)=0$ 

will give you an honest, bias-reduced estimate of the treatment effect at xxx.

### What's Actually Happening

- 1. The forest groups the data into neighborhoods (tree leaves).
- 2. For a given test point xxx, GRF finds all observations that fall in the same leaf nodes across many trees.
- 3. It assigns weights  $\alpha i(x) \cdot alpha i(x) \cdot \alpha i(x)$  based on how often each observation ends up in the same leaf as xxx.
- 4. It uses those weights to compute a **local moment condition**.
- 5. Solving that local equation gives you  $\theta(x)$  theta(x) $\theta(x)$ , the CATE at point xxx.



### **Concrete Example (Tiny Forest)**

Suppose we're estimating at  $x=age 35x = \text{text}\{age 35\}x=age 35, and the forest gives us:$ 

#### ID Age Treatment TiT\_iTi Outcome YiY\_iYi Weight αi(x)\alpha\_i(x)αi(x)

1	30	1	100	0.4
2	32	0	90	0.3
3	34	1	110	0.3

Then GRF uses this weighted data to solve:

$$0.4 \cdot \psi\theta(x), v(x)(O1) + 0.3 \cdot \psi(O2) + 0.3 \cdot \psi(O3) = 00.4 \cdot \phi(x), v(x)(O1) + 0.3 \cdot \phi(O2) + 0.3 \cdot \phi(O3) = 00.4 \cdot \psi\theta(x), v(x)(O1) + 0.3 \cdot \psi(O3) = 00.4 \cdot \psi\theta(x), v(x)(O1) + 0.3 \cdot \psi(O3) = 00.4 \cdot \psi\theta(x), v(x)(O1) + 0.3 \cdot \psi(O3) = 00.4 \cdot \psi\theta(x), v(x)(O1) + 0.3 \cdot \psi(O3) = 00.4 \cdot \psi\theta(x), v(x)(O1) + 0.3 \cdot \psi(O3) = 00.4 \cdot \psi\theta(x), v(x)(O1) + 0.3 \cdot \psi(O3) = 00.4 \cdot \psi\theta(x), v(x)(O1) + 0.3 \cdot \psi(O3) = 00.4 \cdot \psi\theta(x), v(x)(O1) + 0.3 \cdot \psi(O3) = 00.4 \cdot \psi\theta(x), v(x)(O1) + 0.3 \cdot \psi(O3) = 00.4 \cdot \psi\theta(x), v(x)(O1) + 0.3 \cdot \psi(O3) = 00.4 \cdot \psi\theta(x), v(x)(O1) + 0.3 \cdot \psi(O3) = 00.4 \cdot \psi\theta(x), v(x)(O1) + 0.3 \cdot \psi(O3) = 00.4 \cdot \psi\theta(x), v(x)(O1) + 0.3 \cdot \psi(O3) = 00.4 \cdot \psi\theta(x), v(x)(O1) + 0.3 \cdot \psi(O3) = 00.4 \cdot \psi\theta(x), v(x)(O1) + 0.3 \cdot \psi(O3) = 00.4 \cdot \psi\theta(x), v(x)(O1) + 0.3 \cdot \psi(O3) = 00.4 \cdot \psi\theta(x), v(x)(O1) + 0.3 \cdot \psi(O3) = 00.4 \cdot \psi\theta(x), v(x)(O1) + 0.3 \cdot \psi(O3) = 00.4 \cdot \psi\theta(x), v(x)(O1) + 0.3 \cdot \psi(O3) = 00.4 \cdot \psi\theta(x), v(x)(O3) = 00.4 \cdot \psi\theta(x), v(x)(C3) =$$

This is like saying: "Average the treatment effect signals from people like me and solve for the effect that makes them balance out."

## Summary (Simple Analogy)

- **GRF** is like asking: "What's the treatment effect for someone like me?"
- You look at similar people, determined by the random forest.
- Each similar person iii gets a weight  $\alpha i(x) \wedge alpha i(x) \alpha i(x)$ .
- You use their **residual signals** ψ\psiψ to solve an equation.
- That solution is the **CATE** at xxx.

Would it help if I wrote this as a short simulation in R to make the steps clearer?