Team Reference of St. Petersburg Campus of Higher School of Economics.

SPb HSE: Abstract Economists
(Ermilov, Fedorov, Labutin)



// pollard begins

return b;

**if** (a > b)

return a-b;

return b-a;

11

12

13

14

15

18

19

21

31 32

33

34

}

const int max\_step = 4e5;

→ unsigned long long b){ if (!a) return 1;

while (a) swap(a, b%=a);

unsigned long long b){

int stage = 2, i = 0;

g = gcd(get(x, y), n);

if (i == max\_step)

if (i == stage) {

stage <<= 1;

g = gcd(get(x, y), n);

 $x = (x * (_int128)x + 1) \% n;$ 

while (g == 1) {

break;

y = x;

return g;

// pollard ends

unsigned long long gcd(unsigned long long a,

unsigned long long get (unsigned long long a,

unsigned long long pollard(unsigned long long n){

unsigned long long x = (rand() + 1) % n, y = 1,

#### pragma

```
#pragma GCC optimize(''03, no-stack-protector'')
#pragma GCC target("sse, sse2, sse4, ssse3, popcnt, abm,
```

#### Алгебра Pick

```
B + \Gamma / 2 - 1 = AREA,
```

где В — количество целочисленных точек внутри многоугольника, а  $\Gamma$  — количество целочисленных точек на границе многоугольника.

# Newton

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$\begin{aligned} & \mathbf{Catalan} \\ & C_n = \sum_{k=0}^{n-1} C_k C_{n-1-k} \\ & C_i = \frac{1}{n+1} {2n \choose n} \end{aligned}$$

### Кол-во графов

$$G_N := 2^{n(n-1)/2}$$

Количество связных помеченных графов

$$Conn_N = G_N - \frac{1}{N} \sum_{K=1}^{N-1} K\binom{N}{K} Conn_K G_{N-K}$$

Количество помеченных графов с К компонентами связности

$$D[N][K] = \sum_{S=1}^{N} {\binom{N-1}{S-1}} Conn_S D[N-S][K-1]$$

## Miller-Rabbin

```
a=a^t
FOR i = 1...s
    if a^2=1 && |a|!=1
        NOT PRIME
    a=a^2
return a==1 ? PRIME : NOT PRIME
```

# Интегрирование по формуле Симпсона $\int_a^b f(x)dx?$

$$h = \frac{b-a}{2n}$$

$$\int = (f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \dots + 4f(x_{2n-1}) + f(x_{2n})) \frac{h}{3}$$

$$O(n^4).$$

### Простые числа

 $x_i := a + ih, i = 0 \dots 2n$ 

1009,1013;10007,10009;100003,100019 1000003,1000033;10000019,10000079 100000007,100000037 10000000019,10000000033 100000000039,100000000061 10000000000031,10000000000067 10000000000000061,10000000000000069 

## Числа для Фурье

```
• prime: 7340033 = 7 \cdot 2^{20} + 1; w : 5(w^{2^{20}} = 1)
```

```
• prime: 13631489 = 13 \cdot 2^{20} + 1; w: 3(w^{2^{20}} = 1)
                                                           1 // sk fast allocation begins
                                                               const int MAX_MEM = 5e8;
                                                               int mpos = 0;
                                                               char mem[MAX_MEM];
                                                               inline void * operator new ( size_t n ) {
                                                                 assert((mpos += n) <= MAX_MEM);</pre>
  • prime: 23068673 = 11 \cdot 2^{21} + 1; w: 38(w^{2^{21}} = 1)
                                                                 return (void *) (mem + mpos - n);
                                                               inline void operator delete ( void * ) noexcept {
                                                               → } // must have!
  • prime: 69206017 = 33 \cdot 2^{21} + 1; w: 45(w^{2^{21}} = 1)
                                                               // sk fast allocation ends
                                                           13
                                                               // sk fast read-write begins
  • prime: 81788929 = 39 \cdot 2^{21} + 1; w: 94(w^{2^{21}} = 1)
                                                               inline int readChar();
                                                               template <class T = int> inline T readInt();
                                                               template <class T> inline void writeInt( T x,
                                                               \rightarrow char end = 0);
  • prime: 104857601 = 25 \cdot 2^{22} + 1; w : 21(w^{2^{22}} = 1)
                                                               inline void writeChar( int x );
                                                           21
                                                               inline void writeWord( const char *s );
                                                               /** Read */
                                                               static const int buf_size = 2048;
  • prime: 113246209 = 27 \cdot 2^{22} + 1; w : 66(w^{2^{22}} = 1)
                                                               inline int getChar() {
                                                                 static char buf[buf_size];
                                                                 static int len = 0, pos = 0;
                                                                 if (pos == len)
                                                           31
  • prime: 138412033 = 33 \cdot 2^{22} + 1; w: 30(w^{2^{22}} = 1)
                                                                    pos = 0, len = fread(buf, 1, buf_size,
                                                           32
                                                                    \hookrightarrow stdin);
                                                                 if (pos == len)
                                                                    return -1;
                                                           34
                                                                 return buf[pos++];
                                                           35
  • prime: 167772161 = 5 \cdot 2^{25} + 1; w : 17(w^{2^{25}} = 1)
                                                               inline int readWord(char * buffer) {
                                                                 int c = getChar();
                                                                 while (c <= 32) {
                                                                    c = getChar();
  • prime: 469762049 = 7 \cdot 2^{26} + 1; w : 30(w^{2^{26}} = 1)
                                                                 int len = 0;
                                                                 while (c > 32) {
                                                                    *buffer = (char) c;
                                                                    c = getChar();
  • prime: 998244353 = 7.17.2^{23} + 1; w: 3^{7*17}.
                                                                    buffer++;
                                                                    len++;
                                                                 }
                                                                 return len;
  Erdős-Gallai theorem
A sequence of non-negative integers d_1 \geq \cdots \geq d_n can be
represented as the degree sequence of a finite simple graph
                                                               inline int readChar() {
on n vertices if and only if d_1 + \cdots + d_n \ d_1 + \cdots + d_n is even
                                                                 int c = getChar();
                                                                 while (c \ll 32)
                                                                    c = getChar();
  \sum_{i=1}^{k} d_i \le k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k) holds for every \frac{\partial^2}{\partial k}
                                                                 return c;
in 1 \le k \le n.
```

```
}
                                                            }
                                                            int x1, y1;
   template <class T>
                                                            int d = gcd (b%a, a, x1, y1);
61
                                                            x = y1 - (b / a) * x1;
   inline T readInt() {
     int s = 1, c = readChar();
                                                            y = x1;
     T x = 0;
                                                            return d;
                                                       12
     if (c == '-')
                                                      13
65
       s = -1, c = getChar();
66
                                                      14
     while ('^{0}' <= c && c <= '^{9}')
                                                          // extended euclid ends
       x = x * 10 + c - '0', c = getChar();
                                                          // FFT begins
     return s == 1 ? x : -x;
                                                       2
   }
70
                                                         const int LOG = 19;
71
                                                          const int N = (1 \ll LOG);
    /** Write */
72
73
                                                          typedef std::complex<double> cd;
   static int write_pos = 0;
   static char write_buf[buf_size];
                                                          int rev[N];
                                                          cd W[N];
   inline void writeChar( int x ) {
                                                       10
      if (write_pos == buf_size)
                                                      11
                                                          void precalc() {
        fwrite(write_buf, 1, buf_size, stdout),
79
                                                            const double pi = std::acos(-1);

    write_pos = 0;

                                                            for (int i = 0; i != N; ++i)
                                                      13
     write_buf[write_pos++] = x;
80
                                                              W[i] = cd(std::cos(2 * pi * i / N),
                                                       14
   }
81
                                                               \rightarrow std::sin(2 * pi * i / N));
   template <class T>
                                                            int last = 0;
                                                       16
   inline void writeInt( T x, char end ) {
                                                            for (int i = 1; i != N; ++i) {
                                                       17
      if (x < 0)
                                                              if (i == (2 << last))
                                                       18
       writeChar('-'), x = -x;
86
                                                                ++last;
     char s[24];
                                                              rev[i] = rev[i ^ (1 << last)] | (1 << (LOG -
     int n = 0;
                                                               \rightarrow 1 - last));
     while (x \mid | !n)
                                                            }
                                                      22
       s[n++] = '0' + x \% 10, x /= 10;
                                                          }
                                                      23
     while (n--)
                                                      24
       writeChar(s[n]);
                                                      25
                                                          void fft(vector<cd>& a) {
     if (end)
                                                            for (int i = 0; i != N; ++i)
       writeChar(end);
95
                                                               if (i < rev[i])
                                                      27
   }
96
                                                                 std::swap(a[i], a[rev[i]]);
                                                       28
   inline void writeWord( const char *s ) {
                                                      30
                                                            for (int lvl = 0; lvl != LOG; ++lvl)
      while (*s)
99
                                                              for (int start = 0; start != N; start += (2
                                                      31
        writeChar(*s++);
100
                                                               101
                                                                for (int pos = 0; pos != (1 << lvl); ++pos)
102
   struct Flusher {
103
                                                                   cd x = a[start + pos];
      ~Flusher() {
104
                                                                   cd y = a[start + pos + (1 << lvl)];
                                                       34
        if (write_pos)
105
          fwrite(write_buf, 1, write_pos, stdout),
106
                                                                   y *= W[pos << (LOG - 1 - lvl)];
          \rightarrow write_pos = 0;
                                                       37
     }
                                                                   a[start + pos] = x + y;
   } flusher;
108
                                                                   a[start + pos + (1 << lvl)] = x - y;
109
110
                                                          }
                                                       41
   // sk fast read-write ends
111
                                                       42
   // extended euclid begins
                                                          void inv_fft(vector<cd>& a) {
                                                       43
                                                            fft(a);
                                                      44
                                                            std::reverse(a.begin() + 1, a.end());
   int gcd (int a, int b, int & x, int & y) {
                                                      45
     if (a == 0) {
       x = 0; y = 1;
                                                            for (cd& elem: a)
                                                       47
       return b;
                                                              elem /= N;
                                                       48
```

```
if (isZero(a[el.first]))
   }
                                                                    rows[i].emplace_back(el.first, 0);
   // FFT ends
                                                                  a[el.first] = sub(a[el.first], mult(c,
                                                      55
                                                                   → el.second));
   // fast gauss begins
   using elem_t = int;
                                                                auto cond = [&a](const pair<int, elem_t>&
   // a[i][rows[i][j].first]=rows[i][j].second;

→ p) { return isZero(a[p.first]); };
    \rightarrow b[i]=a[i][n]
   bool gauss(vector<vector<pair<int, elem_t>>>
                                                                   rows[i].erase(std::remove_if(rows[i].begin(),
    → rows, vector<elem_t> &res) {
                                                                    rows[i].end(), cond), rows[i].end());
     int n = rows.size();
     res.resize(n + 1, 0);
                                                              bool ok = false;
                                                      62
     vector < int > p(n + 1);
                                                              for (auto& el: rows[i]) {
     iota(p.begin(), p.end(), 0);
10
                                                                if (el.first < n && !isZero(a[el.first])) {</pre>
     vector<int> toZero(n + 1, -1);
11
                                                                  toZero[el.first] = i;
     vector<int> zro(n + 1);
                                                                  zro[i] = el.first;
     vector<elem_t> a(n + 1);
13
                                                          //
                                                                     det = (det * a[el.first]) % MOD;
                                                      67
     // optional: sort rows
15
                                                                  elem_t c = divM(1, a[el.first]);
                                                                  for (auto& el: rows[i]) {
     sort(p.begin(), p.begin() + n, [&rows](int i,
                                                                    el.second = mult(a[el.first], c);

    int j) { return rows[i].size() <</pre>
                                                                    a[el.first] = 0;
      → rows[j].size(); });
     vector<int> invP(n + 1);
     vector<vector<pair<int, elem_t>>> rs(n);
19
                                                                  ok = true;
     for (int i = 0; i < n; i++) {
20
                                                                  break;
       invP[p[i]] = i;
21
                                                      77
       rs[i] = rows[p[i]];
22
                                                      78
23
     for (int i = 0; i < n; i++) {
                                                              if (!ok) {
       rows[i] = rs[i];
                                                                  det = 0;
                                                      81
       for (auto& el: rows[i]) {
26
                                                                return false;
                                                      82
          if (el.first < n) {</pre>
27
                                                              }
                                                      83
            el.first = invP[el.first];
                                                      85
       }
                                                            res[n] = sub(0, 1);
                                                      86
     }
31
                                                            for (int i = n - 1; i >= 0; i--) {
                                                      87
                                                              int k = zro[i];
                                                              for (auto& el : rows[i])
     for (int i = 0; i < n; i++) {
34
                                                                if (el.first != k)
                                                      90
       for (auto& el: rows[i]) {
35
                                                                  res[p[k]] = sub(res[p[k]],
                                                      91
          a[el.first] = el.second;
36
                                                                   → mult(el.second, res[p[el.first]]));
                                                      92
       while (true) {
                                                      93
          int k = -1;
                                                            return true;
          for (auto& el: rows[i]) {
            if (!isZero(a[el.first]) &&

    toZero[el.first] != -1 &&
                                                         // fast gauss ends
              (k == -1 || toZero[el.first] <</pre>
42

    toZero[k])) {

                                                          // simple geometry begins
              k = el.first;
            }
                                                          struct Point {
          }
                                                            double x, y;
          if (k == -1)
                                                            Point operator+(const Point& p) const { return
           break;
                                                            \rightarrow \{x + p.x, y + p.y\}; \}
                                                            Point operator-(const Point& p) const { return
          int j = toZero[k];
                                                            \hookrightarrow \{x - p.x, y - p.y\}; \}
          elem_t c = a[k];
                                                            Point operator*(const double d) const { return
                                                            \rightarrow {x * d, y * d}; }
         for (auto el: rows[j]) {
                                                            Point rotate() const { return {y, -x}; }
```

```
double operator*(const Point& p) const { return
                                                            Line 11(p1, p2), 12(q1, q2);
      \rightarrow x * p.x + y * p.y; }
                                                            if (11 || 12) return false;
     double operator^(const Point& p) const { return
                                                            x = 11 ^ 12;
      \rightarrow x * p.y - y * p.x; }
                                                            return on_segment(p1, p2, x, false);
     double dist() const { return sqrt(x * x + y * 59
      \rightarrow y); }
   };
                                                          // in case circles are not equal
12
                                                          inline bool intersect_circles(const Circle& c1,
13
   struct Line {
                                                          double a, b, c;
                                                            double d = (c2.c - c1.c).dist();
15
                                                            if (d > c1.r + c2.r + EPS \mid \mid d < fabs(c1.r -
     Line(const Point& p1, const Point& p2) {
16
                                                            \leftrightarrow c2.r) - EPS)
       a = p1.y - p2.y;
       b = p2.x - p1.x;
                                                              return false;
                                                            double cosa = (sqr(d) + sqr(c1.r) - sqr(c2.r))
       c = -a * p1.x - b * p1.y;
19
                                                            \rightarrow / (2 * c1.r * d);
       double d = sqrt(sqr(a) + sqr(b));
                                                            double 1 = c1.r * cosa, h = sqrt(sqr(c1.r) -
21
       a /= d, b /= d, c /= d;
                                                            \rightarrow sqr(1));
                                                            Point v = (c2.c - c1.c) * (1 / d), p = c1.c + v
23
     bool operator | | (const Line& 1) const { return
                                                            → * 1:
24
      \rightarrow fabs(a * 1.b - 1.a * b) < EPS; }
                                                            p1 = p + v.rotate() * h, p2 = p - v.rotate() *
     double dist(const Point& p) const { return
                                                             \hookrightarrow h;
      \rightarrow fabs(a * p.x + b * p.y + c); }
                                                            return true;
                                                      70
     Point operator (const Line& 1) const {
                                                      71
26
       return \{(1.c * b - c * 1.b) / (a * 1.b - 1.a_2)\}
        \rightarrow * b),
                                                          inline bool intersect_circle_and_line(const
            (1.c * a - c * 1.a) / (1.a * b - a *
                                                          → Circle& c, const Line& l, Point& p1, Point&
            \rightarrow 1.b)};
                                                          → p2) {
     }
                                                            double d = 1.dist(c.c);
     Point projection(const Point& p) const {
                                                            if (d > c.r + EPS)
                                                      75
30
       return p - Point{a, b} * (a * p.x + b * p.y \pi
                                                              return false;
31
                                                            Point p = 1.projection(c.c);

→ c);
                                                      77
     }
                                                            Point n{1.b, -1.a};
   };
                                                            double h = sqrt(sqr(c.r) - sqr(l.dist(c.c)));
33
                                                            p1 = p + n * h, p2 = p - n * h;
34
   struct Circle {
                                                            return true;
                                                      81
     Point c;
     double r:
37
     Circle(const Point& c, double r) : c(c), r(r) 84
                                                          // simple geometry ends
38
      → {}
     Circle(const Point& a, const Point& b, const
                                                          // convex hull begins
      → Point& c) {
       Point p1 = (a + b) * 0.5, p2 = (a + c) * 0.5;
                                                          struct Point {
       Point q1 = p1 + (b - a).rotate(), q2 = p2 + 3
                                                            int x, y;
        \rightarrow (c - a).rotate();
                                                            Point operator-(const Point& p) const { return
       this->c = Line(p1, q1) ^ Line(p2, q2);
42
                                                            \rightarrow \{x - p.x, y - p.y\}; \}
       r = (a - this->c).dist();
43
                                                            int64_t operator^(const Point& p) const {
     }
44
                                                            → return x * 111 * p.y - y * 111 * p.x; }
   };
45
                                                            int64_t dist() const { return x * 111 * x + y *
46
                                                            \rightarrow 111 * y; }
   inline bool on_segment(const Point& p1, const
47
                                                            bool operator<(const Point& p) const { return x
    → Point& p2, const Point& x, bool strictly) {
                                                            \rightarrow != p.x ? x < p.x : y < p.y; \}
     if (fabs((p1 - x) ^ (p2 - x)) > EPS)
48
                                                          };
       return false;
49
     return (p1 - x) * (p2 - x) < (strictly ? - EPS^{10}
50
                                                          // all point on convex hull are included

→ : EPS);

                                                          vector<Point> convex_hull(vector<Point> pt) {
   }
51
                                                            int n = pt.size();
                                                            Point p0 = *std::min_element(pt.begin(),
   // in case intersection is not a segment
                                                            \rightarrow pt.end());
   inline bool intersect_segments(const Point& p1,
                                                            std::sort(pt.begin(), pt.end(), [&p0](const

→ const Point& p2, const Point& q1, const

                                                            → Point& a, const Point& b) {
    \rightarrow Point& q2, Point& x) {
                                                              int64_t cp = (a - p0) ^ (b - p0);
```

```
return cp != 0 ? cp > 0 : (a - p0).dist() < 34
                                                                  add_line(v + v + 2, m, r, a, b, nw);
           (b - p0).dist();
                                                                  return;
     });
18
                                                        37
     int i = n - 1;
                                                                bool lef = f(nw, 1) < f(line[v], 1);
     for (; i > 0 && ((pt[i] - p0) ^ (pt[i - 1] -
                                                                bool mid = f(nw, m) < f(line[v], m);</pre>
21
      \rightarrow p0)) == 0; i--);
     std::reverse(pt.begin() + i, pt.end());
                                                                if (mid) swap(line[v], nw);
22
     vector<Point> ch;
                                                                if (1 == r - 1)
                                                        43
24
     for (auto& p : pt) {
                                                                  return;
                                                        44
       while (ch.size() > 1) {
                                                                if (lef != mid)
          auto& p1 = ch[(int) ch.size() - 1];
          auto& p2 = ch[(int) ch.size() - 2];
                                                                  add_line(v + v + 1, l, m, a, b, nw);
                                                        47
          int64_t cp = (p1 - p2) ^ (p - p1);
                                                                else
                                                        48
          if (cp >= 0) break;
                                                                  add_line(v + v + 2, m, r, a, b, nw);
                                                        49
          ch.pop_back();
                                                             }
       }
                                                        51
                                                              ftype get(int v, int 1, int r, int x) {
        ch.push_back(p);
                                                        52
                                                                if(1 == r - 1)
                                                        53
                                                        54
                                                                  return f(line[v], x);
     return ch;
                                                                int m = (1 + r) / 2;
                                                        55
36
                                                                if(x < m) {
37
                                                        56
                                                                  return min(f(line[v], x), get(v + v + 1, 1,
   // convex hull ends
                                                                  \rightarrow m, x));
                                                                } else {
                                                        58
   // convex hull trick begins
                                                                  return min(f(line[v], x), get(v + v + 2, m,
                                                                  \rightarrow r, x));
   typedef long long ftype;
                                                        60
   typedef complex<ftype> point;
                                                              }
                                                        61
   #define x real
   #define y imag
                                                           } cdt(maxn);
                                                        64
   ftype dot(point const& a, point const& b) {
                                                           // convex hull with stack
                                                        65
     return (conj(a) * b).x();
10
                                                           ftype cross(point a, point b) {
                                                        67
11
                                                             return (conj(a) * b).y();
                                                        68
   ftype f(point const& a, int x) {
                                                        69
     return dot(a, {compressed[x], 1});
                                                        70
      //return\ dot(a, \{x, 1\});
14
                                                           vector<point> hull, vecs;
                                                        72
16
                                                           void add_line(ftype k, ftype b) {
                                                        73
   int pos = 0;
17
                                                             point nw = \{k, b\};
                                                        74
18
                                                              while(!vecs.empty() && dot(vecs.back(), nw -
   // (x, y) \rightarrow (k, b) \rightarrow kb + x
                                                              \rightarrow hull.back()) < 0) {
   struct li_chao { // for min
20
                                                               hull.pop_back();
                                                        76
     vector<point> line;
21
                                                                vecs.pop_back();
                                                        77
                                                             }
                                                        78
     li_chao(int maxn) {
                                                              if(!hull.empty()) {
                                                        79
        line.resize(4 * maxn, \{0, inf\});
24
                                                                vecs.push_back(1i * (nw - hull.back()));
25
                                                             hull.push_back(nw);
     void add_line(int v, int 1, int r, int a, int _{83}
      \rightarrow b, point nw) {
        if (r <= a || b <= 1) return; // remove if n_{s5}
                                                           int get(ftype x) {
           [a, b) query
                                                             point query = \{x, 1\};
                                                             auto it = lower_bound(vecs.begin(), vecs.end(),
        int m = (1 + r) >> 1;
30
                                                              → query, [](point a, point b) {
31
                                                                return cross(a, b) > 0;
        if (!(a \le 1 \&\& r \le b)) \{ // remove if no
                                                              });
        \rightarrow [a, b) query
                                                             return dot(query, hull[it - vecs.begin()]);
          add_line(v + v + 1, l, m, a, b, nw);
```

```
}
                                                                a = par[upper[a]];
                                                              } else {
   // convex hull trick ends
                                                                back.push_back({t_in[upper[b]], t_in[b],

  false});
   // heavy-light begins
                                                                b = par[upper[b]];
                                                              }
                                                      62
   int sz[maxn];
                                                            }
                                                      63
                                                      64
   void dfs_sz(int v, int par = -1) {
                                                            front.push_back({min(t_in[a], t_in[b]),
     sz[v] = 1;
                                                            \rightarrow max(t_in[a], t_in[b]), t_in[a] > t_in[b]});
     for (int x : gr[v])
                                                            // for edge-weighted hld add:
       if (x != par) {
                                                            dfs_sz(x, v);
                                                            front.insert(front.end(), back.rbegin(),
         sz[v] += sz[x];
10

→ back.rend());
       }
11
     for (int i = 0; i < gr[v].size(); i++)</pre>
                                                            return front;
       if (gr[v][i] != par)
13
                                                         }
       if (sz[gr[v][i]] * 2 >= sz[v]) {
14
                                                      71
         swap(gr[v][i], gr[v][0]);
15
                                                         // heavy-light ends
         break;
       }
                                                          // max flow begins
17
   }
18
                                                         struct edge{
                                                            int from, to;
   int rev[maxn];
                                                            int c, f, num;
   int t_in[maxn];
21
   int upper[maxn];;
                                                            edge(int from, int to, int c, int
                                                            \rightarrow num):from(from), to(to), c(c), f(0),
   int par[maxn];
                                                            \rightarrow num(num){}
   int dep[maxn];
                                                            edge(){}
                                                         };
   int T = 0;
   void dfs_build(int v, int uppr, int pr = -1) { 10
                                                         const int max_n = 600;
     rev[T] = v;
29
                                                          edge eds[150000];
     t_in[v] = T++;
                                                      12
30
     dep[v] = pr == -1 ? 0 : dep[pr] + 1;
                                                          int num = 0;
                                                      13
31
                                                          int it[max_n];
     par[v] = pr;
                                                      14
                                                      15
                                                         vector<int> gr[max_n];
     upper[v] = uppr;
                                                          int s, t;
                                                          vector<int> d(max_n);
                                                      17
     bool first = true;
                                                      18
     for (int x : gr[v])
                                                          bool bfs(int k) {
37
       if (x != pr) {
                                                      20
                                                            queue<int> q;
38
                                                            q.push(s);
         dfs_build(x, first ? upper[v] : x, v);
                                                      21
                                                            fill(d.begin(), d.end(), -1);
          first = false;
                                                      22
40
                                                            d[s] = 0;
41
                                                            while (!q.empty()) {
   }
                                                      24
42
                                                              int v = q.front();
                                                      25
                                                              q.pop();
   struct interval {
44
     int 1;
                                                              for (int x : gr[v]) {
45
                                                                int to = eds[x].to;
                                                      28
                                                                if (d[to] == -1 \&\& eds[x].c - eds[x].f >=
     bool inv; // should direction be reversed
   };
                                                                    (1 << k)){}
48
                                                                  d[to] = d[v] + 1;
                                                                  q.push(to);
   // node-weighted hld
                                                      31
                                                                }
   vector<interval> get_path(int a, int b) {
                                                      32
51
                                                              }
     vector<interval> front;
                                                      33
52
                                                            }
     vector<interval> back;
53
                                                            return (d[t] != -1);
     while (upper[a] != upper[b]) {
                                                      36
       if (dep[upper[a]] > dep[upper[b]]) {
         front.push_back({t_in[upper[a]], t_in[a], 38
                                                          int dfs(int v, int flow, int k) {

    true});
```

```
if (flow < (1 << k))
                                                          }
       return 0;
41
                                                          while (decomp(s, 1e9 + 5));
     if (v == t)
42
                                                      100
       return flow;
     for (; it[v] < gr[v].size(); it[v]++) {</pre>
                                                          // max flow ends
        int num = gr[v][it[v]];
                                                          // min-cost flow begins
       if (d[v] + 1 != d[num].to])
46
         continue;
47
                                                          long long ans = 0;
                                                       3
       int res = dfs(eds[num].to, min(flow,
                                                          int mx = 2 * n + 2;
        \rightarrow eds[num].c - eds[num].f), k);
       if (res){
49
                                                          memset(upd, 0, sizeof(upd));
          eds[num].f += res;
                                                          for (int i = 0; i < mx; i++)
          eds[num ^ 1].f -= res;
                                                            dist[i] = inf;
          return res;
52
                                                          dist[st] = 0;
53
                                                          queue<int> q;
     }
54
                                                          q.push(st);
                                                       11
     return 0;
                                                          upd[st] = 1;
                                                       12
56
                                                          while (!q.empty()){
57
                                                            int v = q.front();
   void add(int fr, int to, int c, int nm) {
                                                             q.pop();
                                                       15
     gr[fr].push_back(num);
                                                             if (upd[v]){
     eds[num++] = edge(fr, to, c, nm);
                                                       16
60
                                                               for (int x : gr[v]) {
     gr[to].push_back(num);
                                                       17
61
                                                                 edge &e = edges[x];
     eds[num++] = edge(to, fr, 0, nm); //corrected 18
62
                                                                 if (e.c - e.f > 0 && dist[v] != inf &&
   }
63
                                                                 \rightarrow dist[e.to] > dist[v] + e.w) {
64
                                                                   dist[e.to] = dist[v] + e.w;
   int ans = 0;
                                                                   if (!upd[e.to])
     for (int k = 30; k \ge 0; k--)
                                                                     q.push(e.to);
       while (bfs(k)) {
67
                                                                   upd[e.to] = true;
          memset(it, 0, sizeof(it));
                                                                   p[e.to] = x;
          while (int res = dfs(s, 1e9 + 500, k))
            ans += res;
       }
                                                              upd[v] = false;
                                                       27
                                                       28
                                                          }
                                                       29
   // decomposition
                                                       30
75
                                                          for (int i = 0; i < k; i++){
   int path_num = 0;
                                                       31
76
                                                            for (int i = 0; i < mx; i++)
                                                       32
   vector<int> paths[max_n];
                                                               d[i] = inf;
                                                       33
   int flows[max_n];
                                                            d[st] = 0;
                                                       34
                                                            memset(used, false, sizeof(used));
                                                       35
   int decomp(int v, int flow) {
                                                            set<pair<int, int> > s;
     if (flow < 1)
                                                             s.insert(make_pair(0, st));
                                                       37
       return 0;
                                                             for (int i = 0; i < mx; i++){
     if (v == t) {
                                                       38
83
                                                               int x;
       path_num++;
                                                       39
84
                                                               while (!s.empty() && used[(s.begin() ->
       flows[path_num - 1] = flow;

    second)]){
       return flow;
                                                                 s.erase(s.begin());
     }
                                                       41
                                                               }
     for (int i = 0; i < gr[v].size(); i++) {</pre>
                                                               if (s.empty())
       int num = gr[v][i];
                                                                 break:
                                                       44
        int res = decomp(eds[num].to, min(flow,
90
                                                               x = s.begin() -> second;
                                                       45
        \rightarrow eds[num].f));
                                                               used[x] = true;
       if (res)
                 {
91
                                                               s.erase(s.begin());
         eds[num].f -= res;
                                                               for (int i = 0; i < gr[x].size(); i++){</pre>
                                                       48
         paths[path_num -
                                                                 edge &e = edges[gr[x][i]];
                                                       49
          → 1].push_back(eds[num].num);
                                                                 if (!used[e.to] && e.c - e.f > 0){
                                                       50
         return res;
                                                                   if (d[e.to] > d[x] + (e.c - e.f) * e.w +

    dist[x] - dist[e.to]){
     }
96
                                                                     d[e.to] = d[x] + (e.c - e.f) * e.w +
                                                       52
     return 0;
97

→ dist[x] - dist[e.to];
```

```
p[e.to] = gr[x][i];
                                                            while (pos != -1){
              s.insert(make_pair(d[e.to], e.to));
                                                              int nxt = par2[prev[pos]];
                                                              par[pos] = prev[pos];
55
                                                      43
         }
                                                              par2[prev[pos]] = pos;
                                                      44
       }
                                                              pos = nxt;
       dist[x] += d[x];
                                                            }
                                                      46
                                                         }
                                                      47
59
                                                         cout << ans << "\n";
     int pos = t;
                                                      48
60
                                                         for (int i = 0; i < n; i++)
     while (pos != st){
       int id = p[pos];
                                                            cout << par[i] + 1 << "" << i + 1 << "\n";
62
       edges[id].f += 1;
       edges[id ^ 1].f -= 1;
                                                         // bad hungarian ends
       pos = edges[id].from;
66
                                                         // Edmonds O(n^3) begins
67
68
                                                         vector<int> gr[MAXN];
   // min-cost flow ends
                                                         int match[MAXN], p[MAXN], base[MAXN], q[MAXN];
   // bad hungarian begins
                                                         bool used[MAXN], blossom[MAXN];
                                                         int mark[MAXN];
   fill(par, par + 301, -1);
                                                         int C = 1;
   fill(par2, par2 + 301, -1);
                                                         int lca(int a, int b) {
   int ans = 0;
                                                           C++;
   for (int v = 0; v < n; v++){
                                                           for (;;) {
                                                      11
     memset(useda, false, sizeof(useda));
                                                              a = base[a];
                                                      12
     memset(usedb, false, sizeof(usedb));
                                                              mark[a] = C;
                                                      13
     useda[v] = true;
10
                                                              if (match[a] == -1) break;
     for (int i = 0; i < n; i++)
11
                                                              a = p[match[a]];
       w[i] = make_pair(a[v][i] + row[v] + col[i], 16
12
        \rightarrow v);
     memset(prev, 0, sizeof(prev));
                                                            for (;;) {
13
     int pos;
                                                              b = base[b];
                                                      19
     while (true){
                                                              if (mark[b] == C) return b;
15
                                                      20
       pair<pair<int, int>, int> p =
16
                                                              b = p[match[b]];

→ make_pair(make_pair(1e9, 1e9), 1e9);
       for (int i = 0; i < n; i++)
                                                      23
         if (!usedb[i])
                                                      24
           p = min(p, make_pair(w[i], i));
                                                         void mark_path(int v, int b, int children) {
                                                      25
       for (int i = 0; i < n; i++)
                                                      26
                                                            while (base[v] != b) {
         if (!useda[i])
                                                              blossom[base[v]] = blossom[base[match[v]]] =
                                                      27
           row[i] += p.first.first;

    true;

       for (int i = 0; i < n; i++)</pre>
                                                              p[v] = children;
23
         if (!usedb[i]){
                                                              children = match[v];
24
                                                      29
           col[i] -= p.first.first;
                                                              v = p[match[v]];
                                                      30
            w[i].first -= p.first.first;
                                                            }
                                                      31
         }
                                                         }
                                                      32
       ans += p.first.first;
                                                      33
       usedb[p.second] = true;
                                                          int find_path(int root) {
                                                      34
       prev[p.second] = p.first.second; //us emopous
                                                           memset(used, 0, sizeof(used));
30
        ⇔ в первую
                                                           memset(p, -1, sizeof p);
       int x = par[p.second];
                                                            for (int i = 0; i < N; i++)
                                                      37
       if (x == -1){
                                                              base[i] = i;
                                                      38
         pos = p.second;
                                                      39
         break;
                                                            used[root] = true;
       }
                                                            int qh = 0, qt = 0;
                                                      41
       useda[x] = true;
                                                            q[qt++] = root;
                                                      42
       for (int j = 0; j < n; j++)
                                                           while (qh < qt) {
         w[j] = min(w[j], \{a[x][j] + row[x] +
                                                              int v = q[qh++];
                                                      44
          \rightarrow col[j], x});
                                                              for (int to : gr[v]) {
                                                      45
                                                                if (base[v] == base[to] || match[v] == to)
                                                      46
     }
40

→ continue;
```

```
if (to == root || match[to] != -1 &&
                                                          vector<int> getP(string s){
          \rightarrow p[match[to]] != -1) {
                                                            vector<int> p;
            int curbase = lca(v, to);
                                                            p.resize(s.size(), 0);
48
                                                      22
            memset(blossom, 0, sizeof(blossom));
                                                            int k = 0;
                                                      23
            mark_path(v, curbase, to);
                                                            for (int i = 1; i < s.size(); i++){
                                                      24
            mark_path(to, curbase, v);
                                                               while (k > 0 \&\& s[i] == s[k])
                                                      25
51
            for (int i = 0; i < N; i++)
                                                                k = p[k - 1];
                                                      26
52
              if (blossom[base[i]]) {
                                                              if (s[i] == s[k])
                                                      27
53
                base[i] = curbase;
                                                                k++;
                                                              p[i] = k;
                if (!used[i]) {
                                                      29
                                                            }
                  used[i] = true;
                                                      30
                  q[qt++] = i;
                                                      31
                                                            return p;
                }
                                                          }
                                                      32
              }
                                                      33
          } else if (p[to] == -1) {
                                                          vector<int> getH(string s){
                                                      34
            p[to] = v;
                                                            vector<int> h;
                                                      35
            if (match[to] == -1)
                                                            h.resize(s.size() + 1, 0);
              return to;
                                                            for (int i = 0; i < s.size(); i++)
                                                      37
            to = match[to];
                                                               h[i + 1] = ((h[i] * 111 * pow) + s[i] - 'a' +
                                                      38
                                                               \rightarrow 1) % mod;
            used[to] = true;
            q[qt++] = to;
                                                       39
                                                            return h;
                                                       40
67
       }
68
                                                       41
     }
                                                          int getHash(vector<int> &h, int 1, int r){
69
                                                            int res = (h[r + 1] - h[1] * p[r - 1 + 1]) %
70
     return -1;
                                                             \rightarrow mod;
71
   }
                                                            if (res < 0)
72
                                                              res += mod;
   memset(match, -1, sizeof match);
                                                            return res;
74
     for (int i = 0; i < N; i++) {
75
                                                      47
       if (match[i] == -1 && !gr[i].empty()) {
                                                      48
76
          int v = find_path(i);
                                                          // string basis ends
          while (v != -1) {
            int pv = p[v], ppv = match[pv];
                                                       1
                                                          // min cyclic shift begins
            match[v] = pv; match[pv] = v;
                                                          string min_cyclic_shift (string s) {
            v = ppv;
                                                            s += s;
82
                                                            int n = (int) s.length();
83
                                                             int i=0, ans=0;
     }
84
                                                            while (i < n/2) {
                                                               ans = i;
   // Edmonds O(n^3) ends
                                                               int j=i+1, k=i;
   // string basis begins
                                                               while (j < n \&\& s[k] <= s[j]) {
                                                       10
                                                                 if (s[k] < s[j])
                                                      11
   vector<int> getZ(string s){
                                                                   k = i;
                                                       12
     vector<int> z;
                                                                 else
     z.resize(s.size(), 0);
                                                                   ++k:
                                                       14
     int 1 = 0, r = 0;
                                                       15
                                                                 ++j;
     for (int i = 1; i < s.size(); i++){
       if (i <= r)
                                                               while (i \le k) i += j - k;
         z[i] = min(r - i + 1, z[i - 1]);
                                                            }
       while (i + z[i] < s.size() \&\& s[z[i]] == s[i_{19}]
                                                            return s.substr (ans, n/2);
        \rightarrow + z[i]])
         z[i]++;
11
       if (i + z[i] - 1 > r){
                                                          // min cyclic shift ends
         r = i + z[i] - 1;
          1 = i;
                                                          // suffix array O(n) begins
15
                                                       2
     }
                                                          typedef vector<char> bits;
16
                                                       3
     return z;
17
   }
                                                          template < const int end>
18
                                                          void getBuckets(int *s, int *bkt, int n, int K) {
19
```

```
fill(bkt, bkt + K + 1, 0);
                                                                if (SA[i] >= 0)
     forn(i, n) bkt[s[i] + !end]++;
                                                                  SA[j--] = SA[i];
     forn(i, K) bkt[i + 1] += bkt[i];
                                                             int *s1 = SA + n - n1;
                                                        64
   }
                                                             if (name < n1)</pre>
10
   void induceSAl(bits &t, int *SA, int *s, int
                                                                SA_IS(s1, SA, n1, name - 1);
    → *bkt, int n, int K) {
                                                        67
                                                             else
     getBuckets<0>(s, bkt, n, K);
                                                                forn(i, n1)
                                                        68
12
     forn(i, n) {
                                                                  SA[s1[i]] = i;
13
                                                        69
        int j = SA[i] - 1;
                                                             getBuckets<1>(s, bkt, n, K);
        if (j >= 0 && !t[j])
                                                             for (i = 1, j = 0; i < n; i++)
15
                                                        71
          SA[bkt[s[j]]++] = j;
                                                                if (isLMS(i))
16
                                                        72
                                                                  s1[j++] = i;
17
                                                        73
   }
                                                             forn(i, n1)
18
                                                        74
   void induceSAs(bits &t, int *SA, int *s, int
                                                                SA[i] = s1[SA[i]];
                                                        75
19
    \rightarrow *bkt, int n, int K) {
                                                             fill(SA + n1, SA + n, -1);
                                                        76
     getBuckets<1>(s, bkt, n, K);
                                                             for (i = n1 - 1; i \ge 0; i--) {
20
                                                        77
     for (int i = n - 1; i >= 0; i--) {
                                                                j = SA[i], SA[i] = -1;
        int j = SA[i] - 1;
                                                               SA[--bkt[s[j]]] = j;
                                                        79
        if (j >= 0 \&\& t[j])
                                                             induceSAl(t, SA, s, bkt, n, K);
         SA[--bkt[s[j]]] = j;
25
     }
                                                             induceSAs(t, SA, s, bkt, n, K);
   }
26
                                                        83
                                                           // suffix array O(n) ends
27
   void SA_IS(int *s, int *SA, int n, int K) { //
    → require last symbol is 0
                                                           // suffix array O(n log n) begins
   #define isLMS(i) (i \&\&\ t[i]\ \&\&\ !t[i-1])
                                                           string str;
     int i, j;
                                                           int N, m, SA [MAX_N], LCP [MAX_N];
     bits t(n);
                                                           int x [MAX_N], y [MAX_N], w [MAX_N], c [MAX_N];
     t[n-1] = 1;
32
     for (i = n - 3; i \ge 0; i--)
                                                           inline bool cmp (const int a, const int b, const
33
                                                            \rightarrow int 1) { return (y [a] == y [b] && y [a + 1]
       t[i] = (s[i] < s[i+1] \mid | (s[i] = s[i+1] \&\&
34
        \rightarrow t[i+1]==1));
                                                               == y [b + 1]); }
     int bkt[K + 1];
35
                                                        92
                                                           void Sort () {
     getBuckets<1>(s, bkt, n, K);
                                                       93
                                                             for (int i = 0; i < m; ++i) w[i] = 0;
     fill(SA, SA + n, -1);
                                                       94
                                                             for (int i = 0; i < N; ++i) ++w[x[y[i]]];
     forn(i, n)
        if (isLMS(i))
                                                             for (int i = 0; i < m - 1; ++i) w[i + 1] +=
         SA[--bkt[s[i]]] = i;
                                                              → w[i];
40
     induceSAl(t, SA, s, bkt, n, K);
                                                             for (int i = N - 1; i \ge 0; --i)
41
     induceSAs(t, SA, s, bkt, n, K);
                                                              \hookrightarrow SA[--w[x[y[i]]] = y[i];
     int n1 = 0;
43
                                                       98
     forn(i, n)
        if (isLMS(SA[i]))
                                                           void DA () {
                                                       100
                                                             for (int i = 0; i < N; ++i) x[i] = str[i], y[i]
          SA[n1++] = SA[i];
     fill(SA + n1, SA + n, -1);
                                                              \hookrightarrow = i;
47
     int name = 0, prev = -1;
                                                             Sort ();
48
                                                       102
     forn(i, n1) {
                                                             for (int i, j = 1, p = 1; p < N; j <<= 1, m =
49
        int pos = SA[i];
                                                              → p) {
                                                               for (p = 0, i = N - j; i < N; i++) y[p++] =
        bool diff = false;
51
        for (int d = 0; d < n; d++)
          if (prev == -1 || s[pos+d] != s[prev+d] | l_{05}
                                                                for (int k = 0; k < N; ++k) if (SA[k] >= j)
          \rightarrow t[pos+d] != t[prev+d])
                                                                \rightarrow y[p++] = SA[k] - j;
            diff = true, d = n;
                                                                Sort();
                                                       106
          else if (d > 0 && (isLMS(pos+d) ||
                                                                for (swap (x, y), p = 1, x[SA[0]] = 0, i = 1;
55
                                                       107

    isLMS(prev+d)))

                                                                \rightarrow i < N; ++i) x[SA [i]] = cmp (SA[i - 1],
            d = n;
                                                                \rightarrow SA[i], j) ? p - 1 : p++;
        if (diff)
57
                                                       108
                                                           }
          name++, prev = pos;
                                                       109
        SA[n1 + (pos >> 1)] = name - 1;
                                                           // common for all algorithms
60
                                                       111
     for (i = n - 1, j = n - 1; i \ge n1; i--)
                                                           void kasaiLCP () {
                                                       112
61
```

```
for (int i = 0; i < N; i++) c[SA[i]] = i;
                                                                     //v[b].sum_in += v[cur].sum_in;
113
      for (int i = 0, j, k = 0; i < N; LCP [c[i++]] 46
                                                                     cur = v[cur].suff;
        if (c [i] > 0) for (k ? k-- : 0, j = SA[c[i]_{48}]
                                                                   v[a].suff = b;
115
                                                                 }
        \rightarrow -1]; str[i + k] == str[j + k]; k++);
        else k = 0;
                                                                 return;
116
                                                       51
117
                                                            }
118
   void suffixArray () { // require last symbol is 53
     m = 256;
                                                          // bad suffix automaton ends
120
     N = str.size();
121
                                                          // pollard begins
      DA ();
122
     kasaiLCP ();
123
                                                          const int max_step = 4e5;
124
    // suffix array O(n \log n) ends
125
                                                          unsigned long long gcd(unsigned long long a,
    // bad suffix automaton begins
                                                           → unsigned long long b){
                                                            if (!a) return 1;
    struct node{
                                                            while (a) swap(a, b\%=a);
      map<char, int> go;
                                                            return b;
      int len, suff;
      long long sum_in;
                                                       10
      node(){}
                                                          unsigned long long get (unsigned long long a,
                                                       11
   };

    unsigned long long b){
                                                            if (a > b)
                                                       12
   node v[max_n * 4];
10
                                                              return a-b;
11
                                                       14
    int add_node(int max_len){
12
                                                              return b-a:
                                                       15
      //v[number].sum_in = 0;
13
                                                          }
                                                       16
      v[number].len = max_len;
14
                                                       17
      v[number].suff = -1;
                                                          unsigned long long pollard(unsigned long long n){
                                                       18
      number++;
                                                            unsigned long long x = (rand() + 1) \% n, y = 1,
                                                       19
      return number - 1;
17

→ g;
18
                                                            int stage = 2, i = 0;
19
                                                            g = gcd(get(x, y), n);
                                                       21
    int last = add_node(0);
                                                            while (g == 1) {
                                                       22
                                                               if (i == max_step)
                                                       23
    void add_char(char c) {
                                                       24
                                                                break;
      int cur = last;
                                                               if (i == stage) {
                                                       25
      int new_node = add_node(v[cur].len + 1);
                                                                y = x;
                                                       26
      last = new_node;
25
                                                                 stage <<= 1;
      while (cur != -1){
26
                                                               }
        if (v[cur].go.count(c) == 0){
27
                                                              x = (x * (__int128)x + 1) \% n;
          v[cur].go[c] = new_node;
                                                              i++:
          //v[new_node].sum_in += v[cur].sum_in;
                                                               g = gcd(get(x, y), n);
                                                       31
          cur = v[cur].suff;
                                                       32
          if (cur == -1)
                                                            return g;
                                                       33
            v[new_node].suff = 0;
                                                      34
        }else{
33
          int a = v[cur].go[c];
                                                          // pollard ends
          if (v[a].len == v[cur].len + 1){
            v[new_node].suff = a;
                                                          // linear sieve begins
            int b = add_node(v[cur].len + 1);
                                                          const int N = 1000000;
            v[b].go = v[a].go;
            v[b].suff = v[a].suff;
                                                          int pr[N + 1], sz = 0;
                                                          /* minimal prime, mobius function, euler function
            v[new_node].suff = b;
            while (cur != -1 && v[cur].go.count(c) !=
                                                          → */
42
             \rightarrow 0 && v[cur].go[c] == a){
                                                          int lp[N + 1], mu[N + 1], phi[N + 1];
              v[cur].go[c] = b;
              //v[a].sum_in -= v[cur].sum_in;
                                                       p[1] = mu[1] = phi[1] = 1;
44
```

```
for (int i = 2; i <= N; ++i) {
                                                           if (x*x>pp)
     if (lp[i] == 0)
                                                             break;
11
       lp[i] = pr[sz++] = i;
                                                           if (pp \% x == 0){
12
     for (int j = 0; j < sz && pr[j] <= lp[i] && i *
                                                             while (pp \% x == 0)

    pr[j] <= N; ++j)
</pre>
                                                               pp /= x;
       lp[i * pr[j]] = pr[j];
                                                             v.push_back(x);
15
     mu[i] = lp[i] == lp[i / lp[i]] ? 0 : -1 * mu[i_{33}]
                                                        }
16
     → / lp[i]];
                                                         if (pp != 1){
     phi[i] = phi[i / lp[i]] * (lp[i] == lp[i /
                                                     35
                                                           v.push_back(pp);
17
      → lp[i]] ? lp[i] : lp[i] - 1);
                                                     36
                                                        while (true){
18
                                                     37
                                                           long long a = primes[rand()%5000]%n;
19
   // linear sieve ends
                                                           if (gcd(a, n) != 1)
                                                     39
                                                             continue;
                                                     40
   // discrete log in sqrt(p) begins
                                                          bool bb = false;
                                                     41
                                                           for (long long x:v)
   int k = sqrt((double)p) + 2;
                                                             if (pow(a, phi/x) == 1){
                                                     43
                                                               bb = true;
                                                     44
   for (int i = k; i >= 1; i--)
                                                     45
                                                               break;
     mp[bin(b, (i * 111 * k) \% (p-1), p)] = i;
                                                             }
                                                           if (!bb){
                                                     47
   bool answered = false;
                                                             cout << a << "\n";
   int ans = INT32_MAX;
                                                             break:
   for (int i = 0; i <= k; i++){
10
                                                           }
     int sum = (n * 111 * bin(b, i, p)) % p;
11
                                                        }
                                                     51
     if (mp.count(sum) != 0){
       int an = mp[sum] * 111 * k - i;
13
                                                        // prime roots mod n ends
       if (an < p)
                                                        // simplex begins
         ans = min(an, ans);
16
                                                        const double EPS = 1e-9;
   }
17
18
                                                        typedef vector<double> vdbl;
   // discrete log in sqrt(p) ends
   // prime roots mod n begins
                                                        // n variables, m inequalities
                                                        // Ax <= b, c*x -> max, x >= 0
   int num = 0;
                                                        double simplex( int n, int m, const vector<vdbl>
   long long phi = n, nn = n;
                                                         for (long long x:primes){
                                                         ← {
     if (x*x>nn)
                                                          // Ax + Ez = b, A[m]*x -> max
       break;
                                                     11
                                                           // x = 0, z = b, x >= 0, z >= 0
     if (nn \% x == 0){
                                                          vector\langle vdbl \rangle a(m + 2, vdbl(n + m + 2));
                                                     12
       while (nn \% x == 0)
                                                           vector<int> p(m);
                                                     13
        nn /= x;
                                                           forn(i, n)
       phi -= phi/x;
                                                             a[m + 1][i] = c[i];
11
                                                     15
       num++;
                                                           forn(i, m) {
12
                                                     16
                                                             forn(j, n)
13
                                                     17
   }
                                                               a[i][j] = a0[i][j];
14
                                                     18
   if (nn != 1){
                                                             a[i][n + i] = 1;
15
                                                     19
                                                             a[i][m + n] = -1;
     phi -= phi/nn;
16
                                                     20
17
     num++;
                                                             a[i][m + n + 1] = b[i];
   }
                                                             p[i] = n + i;
   if (!((num == 1 && n % 2 != 0) || n == 4 || n ==_{23}
   → 2 | | (num == 2 && n % 2 == 0 && n % 4 != 0))2
                                                           // basis: enter "j", leave "ind+n"
     cout << "-1\n";
                                                           auto pivot = [&]( int j, int ind ) {
                                                             double coef = a[ind][j];
     continue;
                                                     27
21
                                                             assert(fabs(coef) > EPS);
   }
22
                                                     28
                                                             forn(col, n + m + 2)
   vector<long long> v;
                                                     29
                                                               a[ind][col] /= coef;
   long long pp = phi;
                                                     30
   for (long long x:primes){
                                                             forn(row, m + 2)
                                                     31
```

```
if (row != ind && fabs(a[row][j]) > EPS) \{_{89}
                                                          double result = simplex(n, m, a, b, c, x);
            coef = a[row][j];
                                                           if (isinf(result))
            forn(col, n + m + 2)
                                                             puts(''Unbounded'');
              a[row][col] -= a[ind][col] * coef;
                                                          else if (isnan(result))
                                                       92
            a[row][j] = 0; // reduce precision errom3
                                                             puts(''No solution'');
         }
                                                           else {
                                                       94
                                                             printf("".9f :", result);
       p[ind] = j;
                                                       95
38
     };
                                                             forn(i, n)
                                                       96
39
                                                               printf(" %.9f", x[i]);
     // the Simplex itself
                                                             puts('"');
41
     auto iterate = [&]( int nn ) {
42
                                                       99
       for (int run = 1; run; ) {
                                                      100
          run = 0;
                                                           // simplex ends
          forn(j, nn) {
45
                                                           // sum over subsets begins
            if (a[m][j] > EPS) { // strictly positive
                                                           // fast subset convolution O(n 2^n)
              run = 1;
                                                          for(int i = 0; i < (1<<N); ++i)
              double mi = INFINITY, t;
                                                             F[i] = A[i];
              int ind = -1;
                                                          for(int i = 0; i < N; ++i) for(int mask = 0; mask
              forn(i, m)
                                                           \hookrightarrow < (1<<N); ++mask){
                if (a[i][j] > EPS && (t = a[i][n + m])
                                                             if(mask & (1<<i))
                 \rightarrow + 1] / a[i][j]) < mi - EPS)
                                                               F[mask] += F[mask^(1<< i)];
                                                       7
                  mi = t, ind = i;
              if (ind == -1)
53
                                                          // sum over subsets ends
                return false;
              pivot(j, ind);
                                                          // algebra begins
         }
                                                          Pick
       }
                                                          B + \Gamma / 2 - 1,
       return true;
59
                                                           где В - количество целочисленных точек внутри
     };
60
                                                               многоугольника, а Г - количество
61
                                                               целочисленных точек на границе
     int mi = min_element(b.begin(), b.end()) -
                                                               многоугольника.
      → b.begin();
     if (b[mi] < -EPS) {
                                                          Newton
       a[m][n + m] = -1;
                                                           x_{i+1}=x_{i-f}(x_i)/f'(x_i)
       pivot(n + m, mi);
       assert(iterate(n + m + 1));
                                                           Catalan
       if (a[m][m + n + 1] > EPS) // optimal value 10
67
                                                           C_n = \sum_{k=0}^{n-1} C_{k} C_{n-1-k}
        \hookrightarrow is positive
                                                           C_i = \frac{1}{n+1}  binom 2n n
         return NAN;
       forn(i, m)
                                                           G_N:=2^{n(n-1)/2}
          if (p[i] == m + n) {
                                                       14
                                                           Количество связных помеченных графов
            int j = 0;
            while (find(p.begin(), p.end(), j) !=
                                                           Conn_N = G_N - \sqrt{frac 1 N \sqrt{sum}}
            \rightarrow p.end() || fabs(a[i][j]) < EPS)
                                                           Limits_{K=1}^{N-1} K binom N K Conn_K
              j++, assert(j < m + n);
                                                           \hookrightarrow G_{N-K}
            pivot(j, i);
74
                                                       17
                                                           Количество помеченных графов с К компонентами
     }
76
                                                               связности
     swap(a[m], a[m + 1]);
                                                           D[N][K] = \sum_{sum} \lim_{s \to \infty} S=1 \sinom \{N-1\} \{S-1\}
     if (!iterate(n + m))
                                                           \hookrightarrow Conn_S D[N-S][K-1]
       return INFINITY;
     x = vdbl(n, 0);
80
                                                          Miller-Rabbin
                                                       21
     forn(i, m)
81
                                                          a=a^t
                                                       22
        if (p[i] < n)
                                                          FOR i = 1...s
          x[p[i]] = a[i][n + m + 1];
                                                             if a^2=1 && |a|!=1
                                                       24
     return -a[m][n + m + 1];
                                                               NOT PRIME
                                                       25
   }
                                                             a=a^2
                                                       26
                                                          return a==1 ? PRIME : NOT PRIME
   // simplex usage:
87
                                                       28
   vdbl x(n);
                                                       29
```

```
Интегрирование по формуле Симпсона
                                                                 //(r-rb) is amt of nos from first (r) nos
                                                                 → that go to right
   \int int_a^b f(x) dx - ?
31
                                                                 //so [l-lb, r-rb] represents nos from [l, r]
   x_i := a+ih, i=0 ldots 2n
                                                                 → that go to right
   h = \frac{b-a}{2n}
                                                                 return this->r->kth(l-lb, r-rb, k-inleft);
                                                        45
   \sqrt{\text{int}} = (f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) +
                                                            };
    \Rightarrow 2f(x_4) + \frac{1}{2n-1} + + \frac{2n-1}{2n-1} + \frac{2n-1}{2n-1}
    \rightarrow f(x<sub>{2n}))) \frac h 3</sub>
                                                            // wavelet tree ends
   Погрешность имеет порядок уменьшения как O(n^4).
                                                            // berlecamp-massey begins
   // algebra ends
                                                            const int SZ = MAXN;
                                                            11 qp(11 a, 11 b) {
   // wavelet tree begins
                                                              11 x = 1;
                                                              a \%= MOD;
   struct wavelet_tree{
                                                              while (b) {
     int lo, hi;
                                                                 if (b & 1) x = x * a \% MOD;
     wavelet_tree *1, *r;
                                                                a = a * a % MOD;
     vi b;
                                                        10
                                                        11
                                                                b >>= 1;
                                                              }
     //nos are in range [x,y]
                                                              return x;
     //array indices are [from, to)
                                                        13
     wavelet_tree(int *from, int *to, int x, int y).{
        lo = x, hi = y;
11
        if(lo == hi or from >= to) return;
                                                            namespace linear_seq {
12
                                                              inline vector<int> BM(vector<int> x) {
                                                        17
                                                                 vector<int> ls, cur;
        int mid = (lo+hi)/2;
                                                        18
                                                                 int lf, ld;
        auto f = [mid](int x){
                                                        19
15
                                                                 for (int i = 0; i < int(x.size()); ++i) {
          return x <= mid;
       };
                                                                   11 t = 0;
                                                        21
                                                                   for (int j = 0; j < int(cur.size()); ++j)</pre>
       b.reserve(to-from+1);
                                                        22
                                                                     t = (t + x[i - j - 1] * (11) cur[j]) %
       b.pb(0);
19
                                                                     \hookrightarrow MOD;
        //b[i] = no of elements from first "i"
20
                                                                   if ((t - x[i]) \% MOD == 0) continue;
        \rightarrow elements that go to left node
                                                        24
                                                                   if (!cur.size()) {
        for(auto it = from; it != to; it++)
                                                        25
                                                                     cur.resize(i + 1);
          b.pb(b.back() + f(*it));
                                                                     lf = i;
                                                                     ld = (t - x[i]) \% MOD;
        //see how lambda function is used here
                                                                     continue;
        auto pivot = stable_partition(from, to, f); 29
        1 = new wavelet_tree(from, pivot, lo, mid); 30
26
        r = new wavelet_tree(pivot, to, mid+1, hi); 31
                                                                   11 k = -(x[i] - t) * qp(1d, MOD - 2) % MOD;
27
                                                                   vector<int> c(i - lf - 1);
     }
                                                                   c.pb(k);
29
                                                                   for (int j = 0; j < int(ls.size()); ++j)</pre>
     //kth smallest element in [l, r]
30
                                                                     c.pb(-ls[j] * k \% MOD);
     int kth(int 1, int r, int k){
                                                        35
31
                                                                   if (c.size() < cur.size())</pre>
        if(l > r) return 0;

    c.resize(cur.size());

        if(lo == hi) return lo;
33
                                                                   for (int j = 0; j < int(cur.size()); ++j)</pre>
        //how many nos are there in left node from
34
                                                                     c[j] = (c[j] + cur[j]) \% MOD;
        \hookrightarrow [l, r]
                                                        38
                                                                   if (i - lf + (int) ls.size() >= (int)
        int inleft = b[r] - b[1-1];
                                                        39

    cur.size())

        int lb = b[1-1]; //amt of nos from first
                                                                     ls = cur, lf = i, ld = (t - x[i]) % MOD;
        \rightarrow (l-1) nos that go in left
                                                                   cur = c;
        int rb = b[r]; //amt of nos from first (r) 41
        \rightarrow nos that go in left
                                                                 for (int i = 0; i < int(cur.size()); ++i)</pre>
        //so [lb+1, rb] represents nos from [l, r] ^{43}
                                                                   cur[i] = (cur[i] % MOD + MOD) % MOD;
        \hookrightarrow that go to left
                                                                 return cur;
        if(k <= inleft) return this->l->kth(lb+1, rb<sup>15</sup>
        \rightarrow , k);
        //(l-1-lb) is amt of nos from first (l-1) nows
                                                              11 a[SZ], h[SZ], t_[SZ], s[SZ], t[SZ];

→ that go to right
```

```
104 // berlecamp-massey ends
50
      inline void mull(ll* p, ll* q) {
        52
        for (int i = 0; i < m; ++i)</pre>
                                                         void fast_fourier(vector<int>& a) { // AND-FFT.
          if (p[i])
                                                           for (int k = 1; k < SZ(a); k *= 2)
            for (int j = 0; j < m; ++j)
                                                           for (int start = 0; start < (1 << K); start +=</pre>
              t_{i} = (t_{i} + j) + p[i] * q[j]) %
56
                                                            \rightarrow 2 * k) {
               \hookrightarrow MOD;
                                                             for (int off = 0; off < k; ++off) {</pre>
        for (int i = m + m - 1; i >= m; --i)
                                                             int a_val = a[start + off];
          if (t_[i])
                                                             int b_val = a[start + k + off];
            for (int j = m - 1; ~j; --j)
              t_{i} - j - 1 = (t_{i} - j - 1) + t_{i}
                                                             a[start + off] = b_val;
               → * h[j]) % MOD;
                                                             a[start + k + off] = add(a_val, b_val);
        for (int i = 0; i < m; ++i) p[i] = t_[i];</pre>
61
                                                      12
62
                                                      13
63
                                                         }
                                                      14
      inline ll calc(ll K) {
        for (int i = m; ~i; --i)
                                                      15
                                                         void inverse_fast_fourier(vector<int>& a) {
          s[i] = t[i] = 0;
                                                           for (int k = 1; k < SZ(a); k *= 2)
                                                      17
        s[0] = 1;
                                                           for (int start = 0; start < (1 << K); start +=</pre>
        if (m != 1) t[1] = 1; else t[0] = h[0];
                                                      18
                                                            \rightarrow 2 * k) {
        while (K) {
69
                                                             for (int off = 0; off < k; ++off) {</pre>
          if (K & 1) mull(s, t);
                                                             int a_val = a[start + off];
                                                      20
          mull(t, t);
                                                             int b_val = a[start + k + off];
                                                      21
          K >>= 1;
        }
                                                             a[start + off] = sub(b_val, a_val);
        11 su = 0;
                                                             a[start + k + off] = a_val;
        for (int i = 0; i < m; ++i) su = (su + s[i] **
                                                              }

→ a[i]) % MOD;
        return (su % MOD + MOD) % MOD;
                                                      26
76
                                                         }
77
                                                         // AND-FFT ends
      inline int work(vector<int> x, ll n) {
        if (n < int(x.size())) return x[n];</pre>
                                                         // 2-chinese begins
        vector < int > v = BM(x);
        m = v.size();
                                                         template <typename Info>
        if (!m) return 0;
                                                         class DSU {
        for (int i = 0; i < m; ++i) h[i] = v[i], a[i]
                                                           public:
        \rightarrow = x[i];
                                                           DSU ( int n ) : jump (new int[n]), rank (new
        return calc(n);

→ int [n]), info (new Info [n]) {
      }
                                                             for (int i = 0; i < n; i++) {
                                                              jump[i] = i;
      //b = a0/(1-p)
                                                             rank[i] = 0;
      inline void calc_generating_function(const
                                                              }

  vector<int>& b, vector<int>& p,

                                                           }
                                                      11
      \hookrightarrow vector<int>& a0) {
                                                           Info& operator [] ( int x ) {
                                                      12
       p = BM(b);
                                                              return info[get (x)];
                                                      13
        a0.resize(p.size());
                                                      14
        for (int i = 0; i < a0.size(); ++i) {</pre>
                                                           void merge ( int a, int b, const Info &comment
                                                      15
          a0[i] = b[i];
                                                            → ) {
          for (int j = 0; j < i; ++j) {
                                                             a = get(a);
            a0[i] += MOD - (p[j] * 111 * b[i - j -
                                                             b = get (b);

→ 1]) % MOD;

                                                              if (rank[a] <= rank[b]) {</pre>
                                                      18
            if (a0[i] > MOD) {
96
                                                              jump[a] = b;
                                                      19
              a0[i] -= MOD;
                                                             rank[b] += rank[a] == rank[b];
                                                      20
            }
                                                              info[b] = comment;
          }
                                                      22
                                                             } else {
        }
100
                                                              jump[b] = a;
                                                      23
      }
101
                                                              info[a] = comment;
102
                                                      25
103
                                                           }
                                                      26
```

```
private:
                                                         Treap* treap_merge ( Treap *x, Treap *y ) {
     int *jump, *rank;
                                                           if (!x)
     Info *info;
                                                           return y;
                                                     85
                                                           if (!y)
     int get ( int x ) {
                                                           return x;
       return jump[x] == x ? x : (jump[x] = get
                                                           if (x->height < y->height) {
                                                     88
32
           (jump[x]));
                                                           x->push();
                                                     89
     }
                                                           x->right = treap_merge (x->right, y);
33
   };
                                                           x->recalc ();
34
                                                           return x;
35
                                                     92
                                                           } else {
                                                     93
   struct Treap {
                                                           y->push ();
                                                     94
                                                           y->left = treap_merge (x, y->left);
     int value, add;
     int source, target, height;
                                                           y->recalc ();
                                                     96
39
     int min_value, min_path;
                                                           return y;
                                                     97
40
41
                                                           ŀ
                                                         }
     Treap *left, *right;
     Treap ( int _source, int _target, int _value )01
                                                         Treap* treap_getmin ( Treap *x, int &source, int

⇒ : value (_value), add (0), source

                                                         assert (x);
     height = rand ();
                                                           x->push();
                                                     103
45
     min_value = value, min_path = 0;
                                                           if (x->min_path == 0) {
46
                                                     104
     left = right = 0;
                                                           // memory leak, sorry
47
                                                           source = x->source;
                                                           target = x->target;
                                                     107
     Treap& operator += ( int sub ) {
                                                           value = x->value + x->add;
                                                     108
     add += sub;
                                                           return treap_merge (x->left, x->right);
51
     return *this;
                                                           } else if (x->min_path == -1) {
                                                     110
52
     }
                                                           x->left = treap_getmin (x->left, source,
53
                                                     111

    target, value);

54
     void push () {
                                                           value += x->add;
     if (!add)
                                                           x->recalc ();
                                                     113
       return;
                                                           return x;
                                                     114
57
                                                           } else if (x->min_path == +1) {
     if (left) {
                                                     115
       left->add += add;
                                                           x->right = treap_getmin (x->right, source,

    target, value);

     if (right) {
                                                           value += x->add;
61
                                                     117
                                                           x->recalc ();
       right->add += add;
62
                                                     118
                                                           return x;
     value += add;
                                                           } else
                                                     120
     min_value += add;
                                                           assert (0);
                                                     121
     add = 0;
                                                         }
                                                     122
                                                     123
                                                         Treap* treap_add ( Treap *x, int add ) {
                                                     124
     void recalc () {
                                                           if (!x)
69
                                                     125
     min_value = value;
                                                           return 0;
70
                                                     126
     min_path = 0;
                                                           return \&((*x) += add);
                                                     127
     if (left && left->min_value + left->add <
                                                     128

    min_value) {
                                                     129
       min_value = left->min_value + left->add;
       min_path = -1;
                                                         int main () {
                                                     131
74
                                                     132
                                                           int n, m;
75
     if (right && right->min_value + right->add < 133
                                                           while (scanf (''%d%d'', &n, &m) == 2) {
76

    min_value) {
                                                           Treap * g[n + 1];
       min_value = right->min_value + right->add; 135
                                                           for (int i = 0; i <= n; i++)
                                                             g[i] = 0;
       min_path = +1;
                                                     136
     }
                                                           for (int i = 1; i <= n; i++) {
                                                     137
     }
                                                             int a;
                                                     138
   };
                                                             assert (scanf (''%d'', &a) == 1);
                                                     139
81
```

82

```
g[i] = treap_merge (g[i], new Treap (i, 0, 196
                                                                 while (dsu[u].first) {
140
         → a));
                                                                 int next = jump[dsu[u].first];
                                                                 finish.push_back (dsu[u].first);
141
                                                        198
                                                                 dsu.merge (u, 0, make_pair (0, (Treap *)0));
      n++;
                                                        199
142
      for (int i = 0; i < m; i++) {
                                                                 u = next:
143
        int a, b, c;
                                                                 }
144
        assert (scanf (''%d%d%d'', &a, &b, &c) == 3); 202
145
        g[b] = treap\_merge (g[b], new Treap (b, a, 203)
                                                               bool ok[k];
146
                                                               int res[n];

    c));
      }
                                                               memset (ok, 0, sizeof (ok[0]) * k);
147
                                                        205
      DSU <pair <int, Treap*> > dsu (n + 1);
                                                               memset (res, -1, sizeof (res[0]) * n);
148
                                                        206
                                                               function <void (int, int)> add_edge = [&ok,
      for (int i = 0; i < n; i++) {
149
                                                        207
        dsu[i] = make_pair (i, g[i]);
                                                                   &parent, &res, &n] (int a, int b) {
150
                                                                 assert (0 \leq a && a \leq n);
151
                                                        208
                                                                 assert (0 \leq b && b \leq n);
152
                                                        209
      int ans = 0, k = n;
                                                                 assert (res[a] == -1);
153
      int jump[2 * n], jump_from[2 * n], parent[2 * 211
                                                                 res[a] = b;
      \rightarrow n], c[n];
                                                                 while (a != -1 \&\& !ok[a]) {
      vector <int> children[2 * n];
                                                                 ok[a] = true;
155
                                                        213
      memset (c, 0, sizeof (c[0]) * n);
                                                                 a = parent[a];
156
                                                        214
      memset (parent, -1, sizeof (parent[0]) * 2 *
                                                                 }
      \rightarrow n);
                                                               };
                                                        216
      vector <int> finish;
                                                               function < void (int) > reach = [&ok, &reach,
158
                                                        217
      for (int i = 0; i < n; i++) {

→ &children, &jump, &jump_from, &add_edge](
159
        if (dsu[i].first == 0)
                                                                \hookrightarrow int u ) {
160
                                                                 if (!ok[u])
        continue:
161
                                                        218
        int u = i;
                                                                 add_edge (jump_from[u], jump[u]);
                                                        219
                                                                 for (auto x : children[u])
        c[u] = 1;
                                                        220
        while (true) {
                                                                 reach (x);
                                                        221
164
        int source, target, value;
                                                               };
165
        dsu[u].second = treap_getmin (dsu[u].second;23
                                                               for (auto x : finish)
166

→ source, target, value);
                                                                  reach (x);
        if (dsu[target] == dsu[u])
                                                               printf (''%d\n'', ans);
                                                        225
167
                                                               for (int i = 1; i < n; i++)
          continue;
                                                        226
        treap_add (dsu[u].second, -value);
                                                                 printf (''%d%c'', res[i] ? res[i] : -1, ''\n ''[i
                                                        227
        ans += value;
                                                                  \rightarrow < n - 1]);
                                                               }
        jump_from[dsu[u].first] = source;
                                                        228
171
        jump[dsu[u].first] = target;
                                                               return 0;
172
                                                        229
        if (dsu[target].first == 0)
                                                        230
173
          break;
                                                        231
        if (!c[target]) {
                                                             // 2-chinese ends
175
                                                        232
          c[target] = 1;
                                                             // general max weighet match begins
                                                         1
          u = target;
                                                         2
           continue;
                                                             #define DIST(e)
        }
179
                                                             \rightarrow (lab[e.u]+lab[e.v]-q[e.u][e.v].w*2)
        assert (k < 2 * n);
180
                                                            using namespace std;
        int node = k++, t = target;
181
                                                            typedef long long 11;
        parent[dsu[u].first] = node;
182
                                                            const int N = 1023, INF = 1e9;
        children[node].push_back (dsu[u].first);
183
                                                            struct Edge {
        dsu[u].first = node;
                                                               int u, v, w;
        Treap *v = dsu[u].second;
                                                            } g[N][N];
        while (dsu[t].first != node) {
186
                                                            int n, m, n_x, lab[N], match[N], slack[N], st[N],
           int next = jump[dsu[t].first];
187

→ pa[N], flower_from[N][N], S[N], vis[N];

          parent[dsu[t].first] = node;
188
                                                            vector < int> flower[N];
          children[node].push_back (dsu[t].first);
189
                                                            deque < int> q;
                                                         12
          v = treap_merge (v, dsu[t].second);
                                                            void update_slack(int u, int x) {
                                                         13
          dsu.merge (u, t, make_pair (node, v));
191
                                                               if (!slack[x] || DIST(g[u][x]) <</pre>
                                                         14
          t = next;
                                                                \hookrightarrow DIST(g[slack[x]][x])) slack[x] = u;
        }
                                                            }
                                                         15
194
                                                            void set_slack(int x) {
                                                         16
        u = i;
195
                                                               slack[x] = 0;
                                                         17
```

```
for (int u = 1; u <= n; ++u)
                                                              flower[b].clear();
        if (g[u][x].w>0 \&\& st[u] != x \&\& S[st[u]] ==_{72}
                                                              flower[b].push_back(lca);

→ 0) update_slack(u, x);
                                                              for (int x = u, y; x != lca; x = st[pa[y]])
                                                        73
   }
                                                                flower[b].push_back(x), flower[b].push_back(y
                                                        74
20
   void q_push(int x) {
                                                                 \rightarrow = st[match[x]]), q_push(y);
21
     if (x <= n) return q.push_back(x);</pre>
                                                              reverse(flower[b].begin() +1, flower[b].end());
22
     for (int i = 0; i < flower[x].size(); i++)</pre>
                                                              for (int x = v, y; x != lca; x = st[pa[y]])
                                                        76
23
                                                                flower[b].push_back(x), flower[b].push_back(y

¬ q_push(flower[x][i]);

                                                        77
   }
                                                                 \rightarrow = st[match[x]]), q_push(y);
   void set_st(int x, int b) {
                                                              set_st(b, b);
25
     st[x] = b;
                                                              for (int x = 1; x \le n_x; ++x) g[b][x].w =
     if (x <= n) return;</pre>
                                                              \rightarrow g[x][b].w = 0;
     for (int i = 0; i < flower[x].size(); ++i)</pre>
                                                              for (int x = 1; x <= n; ++x) flower_from[b][x]</pre>

    set_st(flower[x][i], b);

                                                              \rightarrow = 0;
   }
                                                              for (int i = 0; i < flower[b].size(); ++i) {</pre>
29
                                                        81
   int get_pr(int b, int xr) {
                                                                int xs = flower[b][i];
30
                                                        82
     int pr = find(flower[b].begin(),
                                                                for (int x = 1; x \le n_x; ++x)

→ flower[b].end(), xr)-flower[b].begin();
                                                                  if (g[b][x].w == 0 \mid \mid DIST(g[xs][x]) <
     if (pr % 2 == 1) {
                                                                   \hookrightarrow DIST(g[b][x]))
                                                                    g[b][x] = g[xs][x], g[x][b] = g[x][xs];
       reverse(flower[b].begin() +1,
        → flower[b].end());
                                                                for (int x = 1; x \le n; ++x)
       return (int) flower[b].size()-pr;
                                                                  if (flower_from[xs][x]) flower_from[b][x] =
                                                        87
34
     }
35
                                                              }
     else return pr;
36
   }
                                                              set_slack(b);
37
   void set_match(int u, int v) {
                                                        90
     match[u] = g[u][v].v;
                                                           void expand_blossom(int b) // S[b] == 1 {
                                                        91
                                                              for (int i = 0; i < flower[b].size(); ++i)</pre>
     if (u <= n) return;</pre>
                                                        92
     Edge e = g[u][v];
                                                                set_st(flower[b][i], flower[b][i]);
                                                        93
41
     int xr = flower_from[u][e.u], pr = get_pr(u,
                                                              int xr = flower_from[b][g[b][pa[b]].u], pr =
42
      \rightarrow xr);

    get_pr(b, xr);

     for (int i = 0; i < pr; ++i)
                                                              for (int i = 0; i < pr; i += 2) {

    set_match(flower[u][i], flower[u][i^1]);

                                                                int xs = flower[b][i], xns = flower[b][i+1];
     set_match(xr, v);
                                                                pa[xs] = g[xns][xs].u;
44
     rotate(flower[u].begin(), flower[u].begin()
                                                                S[xs] = 1, S[xns] = 0;
45
                                                                slack[xs] = 0, set_slack(xns);
      → +pr, flower[u].end());
   }
                                                                q_push(xns);
46
                                                        100
   void augment(int u, int v) {
47
                                                       101
     int xnv = st[match[u]];
                                                              S[xr] = 1, pa[xr] = pa[b];
                                                       102
     set_match(u, v);
                                                              for (int i = pr+1; i < flower[b].size(); ++i) {</pre>
                                                                int xs = flower[b][i];
     if (!xnv) return;
                                                       104
     set_match(xnv, st[pa[xnv]]);
                                                                S[xs] = -1, set_slack(xs);
51
                                                       105
                                                              }
     augment(st[pa[xnv]], xnv);
                                                       106
   }
                                                              st[b] = 0;
53
                                                       107
   int get_lca(int u, int v) {
54
                                                       108
     static int t = 0;
                                                           bool on_found_Edge(const Edge &e) {
55
                                                       109
     for (++t; u || v; swap(u, v)) {
                                                              int u = st[e.u], v = st[e.v];
                                                       110
        if (u == 0) continue;
                                                              if (S[v] == -1) {
                                                       111
        if (vis[u] == t) return u;
                                                                pa[v] = e.u, S[v] = 1;
                                                       112
        vis[u] = t;
                                                                int nu = st[match[v]];
                                                       113
                                                                slack[v] = slack[nu] = 0;
        u = st[match[u]];
                                                       114
                                                                S[nu] = 0, q_push(nu);
        if (u) u = st[pa[u]];
                                                       115
61
     }
62
                                                       116
                                                              else if (S[v] == 0) {
     return 0:
63
                                                       117
                                                                int lca = get_lca(u, v);
64
   void add_blossom(int u, int lca, int v) {
                                                                if (!lca) return augment(u, v), augment(v,
                                                       119
     int b = n+1;
                                                                 \rightarrow u), 1;
     while (b \leq n_x && st[b]) ++b;
                                                                else add_blossom(u, lca, v);
                                                       120
     if (b>n_x) ++n_x;
                                                              }
                                                       121
     lab[b] = 0, S[b] = 0;
                                                              return 0;
                                                       122
69
     match[b] = match[lca];
                                                           }
                                                       123
```

```
bool matching() {
                                                                for (int u = 0; u \le n; ++u) st[u] = u,
124
      fill(S, S+n_x+1, -1), fill(slack, slack+n_x+1,

    flower[u].clear();

125
      \rightarrow 0);
                                                                int w_max = 0;
                                                                for (int u = 1; u <= n; ++u)
      q.clear();
126
                                                          179
                                                                  for (int v = 1; v \le n; ++v) {
      for (int x = 1; x \le n_x; ++x)
         if (st[x] == x \&\& !match[x]) pa[x] = 0, S[x]_{81}
                                                                    flower_from[u][v] = (u == v?u:0);
128
         \rightarrow = 0, q_push(x);
                                                                    w_max = max(w_max, g[u][v].w);
      if (q.empty()) return 0;
                                                                  }
129
                                                          183
                                                                for (int u = 1; u <= n; ++u) lab[u] = w_max;</pre>
      for (;;) {
130
        while (q.size()) {
                                                                while (matching()) ++n_matches;
                                                         185
131
           int u = q.front();
                                                                for (int u = 1; u <= n; ++u)
                                                         186
132
                                                                  if (match[u] \&\& match[u] < u)
           q.pop_front();
                                                          187
           if (S[st[u]] == 1) continue;
                                                                    tot_weight += g[u][match[u]].w;
                                                         188
           for (int v = 1; v \le n; ++v)
                                                                return make_pair(tot_weight, n_matches);
135
                                                         189
             if (g[u][v].w>0 \&\& st[u] != st[v]) {
                                                         190
136
               if (DIST(g[u][v]) == 0) {
                                                              int main() {
137
                                                         191
                 if (on_found_Edge(g[u][v])) return 19;
                                                                cin>>n>>m;
                                                                for (int u = 1; u <= n; ++u)
                                                         193
139
                                                                  for (int v = 1; v <= n; ++v)</pre>
               else update_slack(u, st[v]);
140
                                                          194
                                                                    g[u][v] = Edge \{u, v, 0\};
             }
                                                          195
        }
                                                                for (int i = 0, u, v, w; i < m; ++i) {
        int d = INF;
                                                                  cin>>u>>v>>w;
                                                          197
143
        for (int b = n+1; b \le n_x; ++b)
                                                                  g[u][v].w = g[v][u].w = w;
144
                                                          198
           if (st[b] == b \&\& S[b] == 1) d = min(d,
145
           \rightarrow lab[b]/2);
                                                                cout << weight_blossom().first << '\n';</pre>
                                                         200
        for (int x = 1; x \le n_x; ++x)
                                                                for (int u = 1; u \le n; ++u) cout << match[u]
146
                                                         201
           if (st[x] == x \&\& slack[x]) {
                                                                 if (S[x] == -1) d = min(d,
                                                             }
                                                         202
             → DIST(g[slack[x]][x]));
                                                         203
             else if (S[x] == 0) d = min(d,
                                                              // general max weighet match ends
149
                                                         204
             \rightarrow DIST(g[slack[x]][x])/2);
                                                             // slow min circulation begins
           }
150
        for (int u = 1; u <= n; ++u) {
151
                                                             struct Edge {
           if (S[st[u]] == 0) {
152
                                                                int a;
             if (lab[u] <= d) return 0;</pre>
                                                                int b:
             lab[u] - = d;
                                                                int cost;
           }
155
                                                             };
           else if (S[st[u]] == 1) lab[u] += d;
156
157
                                                              vector<int> negative_cycle(int n, vector<Edge>
        for (int b = n+1; b \le n_x; ++b)
                                                              if (st[b] == b) {
159
                                                                // O(nm), return ids of edges in negative cycle
             if (S[st[b]] == 0) lab[b] += d*2;
             else if (S[st[b]] == 1) lab[b] - = d*2;
                                                                vector<int> d(n);
           }
                                                                vector<int> p(n, -1); // last edge ids
        q.clear();
163
        for (int x = 1; x \le n_x; ++x)
164
                                                                const int inf = 1e9;
            \mbox{if } (\mbox{st}[\mbox{x}] \ == \mbox{x \&\& slack}[\mbox{x}] \ \&\& \mbox{st}[\mbox{slack}[\mbox{x}]]^{15} 
           \rightarrow != x && DIST(g[slack[x]][x]) == 0)
                                                                int x = -1;
             if (on_found_Edge(g[slack[x]][x])) return
166
                                                                for (int i = 0; i < n; i++) {
             x = -1;
                                                          19
        for (int b = n+1; b \le n_x; ++b)
                                                                  for (int j = 0; j < edges.size(); j++) {</pre>
           if (st[b] == b && S[b] == 1 && lab[b] == 0)
168
                                                                    Edge &e = edges[j];

→ expand_blossom(b);
      }
169
                                                                    if (d[e.b] > d[e.a] + e.cost) {
                                                          23
      return 0;
170
                                                                       d[e.b] = max(-inf, d[e.a] + e.cost);
                                                          24
171
                                                                       p[e.b] = j;
                                                          25
    pair < 11, int> weight_blossom() {
172
                                                                       x = e.b;
                                                          26
      fill(match, match+n+1, 0);
173
                                                          27
      n_x = n;
                                                                  }
                                                          28
      int n_matches = 0;
175
                                                                }
                                                          29
      11 tot_weight = 0;
176
                                                          30
```

```
if (x == -1)
31
       return vector<int>(); // no negative cycle 92
                                                          const int max_n = 200;
     for (int i = 0; i < n; i++)
                                                          vector<int> gr[max_n];
                                                      94
       x = edges[p[x]].a;
                                                          vector<edge> edges;
                                                      95
                                                          void add(int fr, int to, int c, int cost) {
     vector<int> result;
                                                      97
37
     for (int cur = x; ; cur = edges[p[cur]].a) {
                                                            gr[fr].push_back(edges.size());
       if (cur == x && result.size() > 0) break;
                                                            edges.push_back({fr, to, c, 0, cost});
       result.push_back(p[cur]);
                                                            gr[to].push_back(edges.size());
                                                      100
40
                                                            edges.push_back({to, fr, 0, 0, -cost}); //
                                                      101
     reverse(result.begin(), result.end());

    single

                                                          }
                                                      102
     return result;
44
                                                      103
   }
                                                          void calc_min_circulation(int n) {
45
                                                      104
                                                            while (true) {
46
                                                      105
   vector<int> min_avg_cycle(int n, vector<Edge>
                                                              vector<Edge> eds;
    vector<int> origin;
                                                      107
     const int inf = 1e3;
48
                                                      108
                                                              for (int i = 0; i < edges.size(); i++) {</pre>
                                                      109
                                                                edge &e = edges[i];
     for (auto &e : edges)
                                                      110
       e.cost *= n * n;
                                                                if (e.c - e.f > 0) {
                                                      111
51
                                                                  eds.push_back({e.from, e.to, e.cost});
52
                                                      112
     int 1 = -inf;
                                                                  origin.push_back(i);
53
                                                      113
                                                                }
     int r = inf;
                                                      114
     while (l + 1 < r) {
                                                              }
                                                      115
       int m = (1 + r) / 2;
                                                      116
       for (auto &e : edges)
                                                              vector<int> cycle = negative_cycle(n, eds);
                                                      117
          e.cost -= m;
                                                      118
                                                              if (cycle.empty())
                                                      119
       if (negative_cycle(n, edges).empty())
                                                                break;
                                                      120
60
         1 = m;
       else
                                                              for (auto id : cycle) {
                                                      122
                                                                int x = origin[id];
         r = m;
                                                      123
                                                                edges[x].f += 1;
                                                      124
       for (auto &e : edges)
                                                                edges[x ^1].f = 1;
                                                      125
          e.cost += m;
                                                      126
                                                            }
67
                                                      127
                                                      128
     if (r \ge 0) // if only negative needed
       return vector<int>();
                                                          // slow min circulation ends
70
     for (auto &e : edges)
       e.cost -= r;
                                                          // fast hashtable begins
74
     vector<int> result = negative_cycle(n, edges);
75
                                                          #include <ext/pb_ds/assoc_container.hpp>
                                                          using namespace __gnu_pbds;
                                                          gp_hash_table<int, int> table;
     for (auto &e : edges)
       e.cost += r;
                                                          const int RANDOM = chrono ::
                                                          → high_resolution_clock ::
     for (auto &e : edges)
                                                          → now().time_since_epoch().count();
81
       e.cost /= n * n;
82
                                                          struct chash {
83
                                                            int operator()(int x) { return hash<int>{}(x ^
     return result;
                                                            → RANDOM); }
   }
85
                                                          };
                                                          gp_hash_table<key, int, chash> table;
                                                      11
   struct edge {
                                                      12
     int from, to;
                                                          // fast hashtable ends
     int c, f, cost;
89
   };
90
```

 $\mathbf{D}\mathbf{M}$ 

xmodmap -e 'keycode 94=' setxkbmap us

Кол-во корневых деревьев:

$$t(G) = \frac{1}{n}\lambda_2 \dots \lambda_n \ (\lambda_1 = 0)$$

$$t(O) = {}_n \lambda_2 \dots \lambda_n \ (\lambda_1 = 0)$$
 Кол-во эйлеровых циклов:  $e(D) = t^-(D,x) \cdot \prod_{y \in D} (outdeg(y) - 1)!$  Наличие совершенного паросочетания:

T — матрица с нулями на диагонали. Если есть ребро

$$(i,j)$$
, to  $a_{i,j} := x_{i,j}$ ,  $a_{j,i} = -x_{i,j}$ 

 $\det(T) = 0 \Leftrightarrow$  нет совершенного паросочетания.