Team Reference of St. Petersburg Campus of Higher School of Economics.

SPb HSE: Abstract Economists
(Ermilov, Fedorov, Labutin)



// pollard begins

return b;

if (a > b)

return a-b;

return b-a;

11

12

13

14

15

18

19

21

31 32

33

34

}

const int max_step = 4e5;

→ unsigned long long b){ if (!a) return 1;

while (a) swap(a, b%=a);

unsigned long long b){

int stage = 2, i = 0;

g = gcd(get(x, y), n);

if (i == max_step)

if (i == stage) {

stage <<= 1;

g = gcd(get(x, y), n);

 $x = (x * (_int128)x + 1) \% n;$

while (g == 1) {

break;

y = x;

return g;

// pollard ends

unsigned long long gcd(unsigned long long a,

unsigned long long get (unsigned long long a,

unsigned long long pollard(unsigned long long n){

unsigned long long x = (rand() + 1) % n, y = 1,

pragma

```
#pragma GCC optimize(''03, no-stack-protector'')
#pragma GCC target("sse, sse2, sse4, ssse3, popcnt, abm,
```

Алгебра Pick

```
B + \Gamma / 2 - 1 = AREA,
```

где В — количество целочисленных точек внутри многоугольника, а Γ — количество целочисленных точек на границе многоугольника.

Newton

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$\begin{aligned} & \mathbf{Catalan} \\ & C_n = \sum_{k=0}^{n-1} C_k C_{n-1-k} \\ & C_i = \frac{1}{n+1} {2n \choose n} \end{aligned}$$

Кол-во графов

$$G_N := 2^{n(n-1)/2}$$

Количество связных помеченных графов

$$Conn_N = G_N - \frac{1}{N} \sum_{K=1}^{N-1} K\binom{N}{K} Conn_K G_{N-K}$$

Количество помеченных графов с К компонентами связности

$$D[N][K] = \sum_{S=1}^{N} {\binom{N-1}{S-1}} Conn_S D[N-S][K-1]$$

Miller-Rabbin

```
a=a^t
FOR i = 1...s
    if a^2=1 && |a|!=1
        NOT PRIME
    a=a^2
return a==1 ? PRIME : NOT PRIME
```

Интегрирование по формуле Симпсона $\int_a^b f(x)dx?$

$$h = \frac{b-a}{2n}$$

$$\int = (f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \dots + 4f(x_{2n-1}) + f(x_{2n})) \frac{h}{3}$$

$$O(n^4).$$

Простые числа

 $x_i := a + ih, i = 0 \dots 2n$

1009,1013;10007,10009;100003,100019 1000003,1000033;10000019,10000079 100000007,100000037 10000000019,10000000033 100000000039,100000000061 10000000000031,10000000000067 10000000000000061,10000000000000069

Числа для Фурье

```
• prime: 7340033 = 7 \cdot 2^{20} + 1; w : 5(w^{2^{20}} = 1)
```

```
• prime: 13631489 = 13 \cdot 2^{20} + 1; w: 3(w^{2^{20}} = 1)
                                                           1 // sk fast allocation begins
                                                               const int MAX_MEM = 5e8;
                                                               int mpos = 0;
                                                               char mem[MAX_MEM];
                                                               inline void * operator new ( size_t n ) {
                                                                 assert((mpos += n) <= MAX_MEM);</pre>
  • prime: 23068673 = 11 \cdot 2^{21} + 1; w: 38(w^{2^{21}} = 1)
                                                                 return (void *) (mem + mpos - n);
                                                               inline void operator delete ( void * ) noexcept {
                                                               → } // must have!
  • prime: 69206017 = 33 \cdot 2^{21} + 1; w: 45(w^{2^{21}} = 1)
                                                               // sk fast allocation ends
                                                           13
                                                               // sk fast read-write begins
  • prime: 81788929 = 39 \cdot 2^{21} + 1; w: 94(w^{2^{21}} = 1)
                                                               inline int readChar();
                                                               template <class T = int> inline T readInt();
                                                               template <class T> inline void writeInt( T x,
                                                               \rightarrow char end = 0);
  • prime: 104857601 = 25 \cdot 2^{22} + 1; w : 21(w^{2^{22}} = 1)
                                                               inline void writeChar( int x );
                                                           21
                                                               inline void writeWord( const char *s );
                                                               /** Read */
                                                               static const int buf_size = 2048;
  • prime: 113246209 = 27 \cdot 2^{22} + 1; w : 66(w^{2^{22}} = 1)
                                                               inline int getChar() {
                                                                 static char buf[buf_size];
                                                                 static int len = 0, pos = 0;
                                                                 if (pos == len)
                                                           31
  • prime: 138412033 = 33 \cdot 2^{22} + 1; w: 30(w^{2^{22}} = 1)
                                                                    pos = 0, len = fread(buf, 1, buf_size,
                                                           32
                                                                    \hookrightarrow stdin);
                                                                 if (pos == len)
                                                                    return -1;
                                                           34
                                                                 return buf[pos++];
                                                           35
  • prime: 167772161 = 5 \cdot 2^{25} + 1; w : 17(w^{2^{25}} = 1)
                                                               inline int readWord(char * buffer) {
                                                                 int c = getChar();
                                                                 while (c <= 32) {
                                                                    c = getChar();
  • prime: 469762049 = 7 \cdot 2^{26} + 1; w : 30(w^{2^{26}} = 1)
                                                                 int len = 0;
                                                                 while (c > 32) {
                                                                    *buffer = (char) c;
                                                                    c = getChar();
  • prime: 998244353 = 7.17.2^{23} + 1; w: 3^{7*17}.
                                                                    buffer++;
                                                                    len++;
                                                                 }
                                                                 return len;
  Erdős-Gallai theorem
A sequence of non-negative integers d_1 \geq \cdots \geq d_n can be
represented as the degree sequence of a finite simple graph
                                                               inline int readChar() {
on n vertices if and only if d_1 + \cdots + d_n \ d_1 + \cdots + d_n is even
                                                                 int c = getChar();
                                                                 while (c \ll 32)
                                                                    c = getChar();
  \sum_{i=1}^{k} d_i \le k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k) holds for every \frac{\partial^2}{\partial k}
                                                                 return c;
in 1 \le k \le n.
```

```
}
                                                            }
                                                            int x1, y1;
   template <class T>
                                                            int d = gcd (b%a, a, x1, y1);
61
                                                            x = y1 - (b / a) * x1;
   inline T readInt() {
      int s = 1, c = readChar();
                                                            y = x1;
      T x = 0;
                                                            return d;
                                                      12
     if (c == '-')
                                                      13
65
        s = -1, c = getChar();
66
                                                      14
      while ('^{0}' <= c && c <= '^{9}')
                                                          // extended euclid ends
        x = x * 10 + c - '0', c = getChar();
                                                       1 // kto begins
      return s == 1 ? x : -x;
                                                       2 //return pair(nmod,nr)
   }
70
                                                       3 //nr%mod1=r1, nr%mod2=r2
71
                                                          //nmod=mod1*mod2/gcd(mod1,mod2)
    /** Write */
72
                                                          //if input incosistent return mp(-1,-1)
73
                                                          pll kto (ll mod1, ll r1, ll mod2, ll r2)
   static int write_pos = 0;
                                                       7
                                                          {
    static char write_buf[buf_size];
                                                            11 d=_gcd(mod1,mod2);
                                                            if (r1\%d!=r2\%d)
   inline void writeChar( int x ) {
                                                              return mp(-1,-1);
      if (write_pos == buf_size)
                                                      10
                                                            ll rd=r1%d;
        fwrite(write_buf, 1, buf_size, stdout),
                                                      11
79
                                                            mod1/=d, mod2/=d, r1/=d, r2/=d;

    write_pos = 0;

                                                       12
      write_buf[write_pos++] = x;
80
                                                            if (mod1<mod2)
                                                       14
   }
81
                                                              swap(mod1,mod2), swap(r1,r2);
                                                       15
82
   template <class T>
                                                       16
83
                                                            11 k=(r2-r1)\%mod2;
    inline void writeInt( T x, char end ) {
                                                            if (k<0)
      if (x < 0)
                                                       18
                                                              k+=mod2;
        writeChar('-'), x = -x;
                                                       19
                                                            11 x, y;
      char s[24];
                                                            gcdex(mod1,mod2,x,y);
                                                      22
      int n = 0;
                                                            x\%=mod2;
      while (x \mid | !n)
                                                      23
                                                            if (x<0)
       s[n++] = 0 + x \% 10, x = 10;
                                                      24
      while (n--)
                                                              x += mod2;
                                                            k*=x, k\%=mod2;
                                                      26
        writeChar(s[n]);
                                                            return mp(mod1*mod2*d,(k*mod1+r1)*d+rd);
     if (end)
                                                      27
                                                          }
        writeChar(end);
                                                      28
95
                                                          // kto ends
   }
96
                                                          // FFT begins
   inline void writeWord( const char *s ) {
      while (*s)
99
                                                          const int MAX_LOG = 17;
        writeChar(*s++);
100
                                                          const int MAXN = (1 << MAX_LOG);</pre>
    }
101
                                                          int LOG = MAX_LOG;
102
                                                          int N = MAXN;
    struct Flusher {
103
      ~Flusher() {
104
                                                          typedef std::complex<double> cd;
        if (write_pos)
105
          fwrite(write_buf, 1, write_pos, stdout),
106
                                                          int rev[MAXN];
          \rightarrow write_pos = 0;
                                                          cd W[MAXN];
                                                       11
      }
                                                       12
    } flusher;
108
                                                          void precalc() {
109
                                                            const double pi = std::acos(-1);
                                                       14
110
                                                            for (int i = 0; i != N; ++i)
                                                       15
   // sk fast read-write ends
111
                                                              W[i] = cd(std::cos(2 * pi * i / N),
                                                       16
   // extended euclid begins
                                                               \rightarrow std::sin(2 * pi * i / N));
   int gcd (int a, int b, int & x, int & y) {
                                                            int last = 0;
                                                       18
     if (a == 0) {
                                                            for (int i = 1; i != N; ++i) {
        x = 0; y = 1;
                                                              if (i == (2 << last))
        return b;
                                                                 ++last;
                                                       21
```

```
vector<cd> r = inv_poly(a, n / 2);
22
       rev[i] = rev[i ^ (1 << last)] | (1 << (LOG <math>-80
                                                            r.resize(n);
                                                            vector<cd> q = mul_fft(a, r, n);
        \rightarrow 1 - last));
                                                      81
     }
                                                            for (int i = 0; i < n / 2; i++) {
                                                      82
   }
                                                              q[i] = -q[n / 2 + i];
                                                              q[n / 2 + i] = 0;
26
                                                      84
   void fft(vector<cd>& a) {
27
                                                      85
     for (int i = 0; i != N; ++i)
                                                            vector<cd> c = mul_fft(q, r, n);
28
                                                      86
       if (i < rev[i])</pre>
                                                            for (int i = n / 2; i < n; i++) {
          std::swap(a[i], a[rev[i]]);
                                                              r[i] = c[i - n / 2];
30
     for (int lvl = 0; lvl != LOG; ++lvl)
                                                            return r;
       for (int start = 0; start != N; start += (2 91
        for (int pos = 0; pos != (1 << lvl); ++pos)
                                                          // FFT ends
          ← {
                                                          // fast gauss begins
            cd x = a[start + pos];
            cd y = a[start + pos + (1 << lvl)];
                                                         using elem_t = int;
                                                          // a[i][rows[i][j].first]=rows[i][j].second;
            y *= W[pos << (LOG - 1 - lvl)];
                                                          \rightarrow b[i]=a[i][n]
                                                       5 bool gauss(vector<vector<pair<int, elem_t>>>
           a[start + pos] = x + y;
40

¬ rows, vector<elem_t> &res) {

            a[start + pos + (1 << lvl)] = x - y;
41
                                                            int n = rows.size();
          }
42
   }
43
                                                            res.resize(n + 1, 0);
44
                                                            vector<int> p(n + 1);
   void inv_fft(vector<cd>& a) {
45
                                                            iota(p.begin(), p.end(), 0);
                                                      10
     fft(a);
                                                            vector<int> toZero(n + 1, -1);
     std::reverse(a.begin() + 1, a.begin() + N);
                                                      11
47
                                                            vector<int> zro(n + 1);
                                                      12
48
                                                            vector<elem_t> a(n + 1);
                                                      13
     for (cd& elem: a)
49
       elem /= N;
                                                            // optional: sort rows
   }
51
52
                                                            sort(p.begin(), p.begin() + n, [&rows](int i,
                                                            → int j) { return rows[i].size() <</pre>
   vector<cd> mul_fft(vector<cd> a, vector<cd> b,
                                                            → rows[j].size(); });
    \rightarrow int n = N) {
                                                            vector<int> invP(n + 1);
     if (N != n) {
                                                      18
55
                                                            vector<vector<pair<int, elem_t>>> rs(n);
       N = n;
56
                                                            for (int i = 0; i < n; i++) {
                                                      20
       LOG = round(log2(N));
                                                              invP[p[i]] = i;
                                                      21
       precalc();
                                                              rs[i] = rows[p[i]];
                                                      22
                                                      23
     fft(a);
                                                            for (int i = 0; i < n; i++) {
                                                      24
     fft(b);
                                                              rows[i] = rs[i];
                                                      25
62
                                                              for (auto& el: rows[i]) {
     vector<cd> c(n);
63
                                                                if (el.first < n) {</pre>
                                                      27
     for (int i = 0; i < n; i++) {
                                                                   el.first = invP[el.first];
       c[i] = a[i] * b[i];
                                                      28
                                                      29
                                                              }
                                                            }
     inv_fft(c);
                                                      32
     return c;
69
                                                      33
70
                                                            for (int i = 0; i < n; i++) {
                                                      34
71
                                                              for (auto& el: rows[i]) {
72
                                                                a[el.first] = el.second;
   vector<cd> inv_poly(vector<cd>& a, int n = N) { 36
73
     if (n == 1) {
74
                                                              while (true) {
       vector<cd> res(1);
                                                                int k = -1;
       res[0] = cd(1) / a[0];
76
                                                                for (auto& el: rows[i]) {
       return res;
                                                      40
77
                                                                   if (!isZero(a[el.first]) &&
     }
                                                      41
78

    toZero[el.first] != -1 &&
```

```
(k == -1 || toZero[el.first] <</pre>
                                                      97 // fast gauss ends

    toZero[k])) {

              k = el.first;
43
                                                      1 // stable gauss begins
           }
                                                      2 // if at least one solution returns it in ans
         }
                                                         int gauss (vector < vector <double> > a,
          if (k == -1)

    vector < double > & ans) {

           break;
47
                                                           int n = (int) a.size();
                                                            int m = (int) a[0].size() - 1;
          int j = toZero[k];
         elem_t c = a[k];
                                                            vector<int> where (m, -1);
                                                            for (int col=0, row=0; col<m && row<n; ++col) {</pre>
         for (auto el: rows[j]) {
                                                              int sel = row;
            if (isZero(a[el.first]))
                                                              for (int i=row; i<n; ++i)</pre>
              rows[i].emplace_back(el.first, 0);
                                                                if (abs (a[i][col]) > abs (a[sel][col]))
           a[el.first] = sub(a[el.first], mult(c,
                                                      12

    el.second));
                                                      13
                                                              if (abs (a[sel][col]) < EPS)</pre>
         }
                                                                continue;
                                                      14
                                                              for (int i=col; i<=m; ++i)</pre>
         auto cond = [&a](const pair<int, elem_t>&
                                                                swap (a[sel][i], a[row][i]);
          → p) { return isZero(a[p.first]); };
                                                              where[col] = row;
         rows[i].erase(std::remove_if
          for (int i=0; i<n; ++i)</pre>
          → rows[i].end());
                                                                if (i != row) {
       }
                                                                  double c = a[i][col] / a[row][col];
                                                      21
                                                                  for (int j=col; j \le m; ++j)
       bool ok = false;
                                                                    a[i][j] -= a[row][j] * c;
       for (auto& el: rows[i]) {
                                                                }
         if (el.first < n && !isZero(a[el.first]))</pre>
                                                              ++row;
            toZero[el.first] = i;
           zro[i] = el.first;
66
              det = (det * a[el.first]) % MOD;
67
                                                      28
                                                            ans.assign (m, 0);
                                                            for (int i=0; i<m; ++i)</pre>
                                                      29
            elem_t c = divM(1, a[el.first]);
                                                              if (where[i] != -1)
           for (auto& el: rows[i]) {
                                                                ans[i] = a[where[i]][m] / a[where[i]][i];
              el.second = mult(a[el.first], c);
                                                            for (int i=0; i<n; ++i) {
              a[el.first] = 0;
                                                              double sum = 0;
                                                              for (int j=0; j<m; ++j)</pre>
                                                      34
                                                                sum += ans[j] * a[i][j];
           ok = true;
                                                              if (abs (sum - a[i][m]) > EPS)
           break;
                                                                return 0;
                                                      37
         }
                                                            }
                                                      38
                                                            for (int i=0; i<m; ++i)</pre>
       if (!ok) {
                                                              if (where[i] == -1)
                                                      41
           det = 0;
81
                                                                return INF;
                                                      42
         return false;
82
                                                            return 1;
                                                      44
     }
                                                          // stable gauss ends
     res[n] = sub(0, 1);
     for (int i = n - 1; i >= 0; i--) {
                                                          // simple geometry begins
       int k = zro[i];
       for (auto& el : rows[i])
                                                         struct Point {
         if (el.first != k)
                                                            double x, y;
           res[p[k]] = sub(res[p[k]],
                                                           Point operator+(const Point& p) const { return

→ mult(el.second, res[p[el.first]]));
                                                            \rightarrow \{x + p.x, y + p.y\}; \}
                                                           Point operator-(const Point& p) const { return
                                                            \hookrightarrow \{x - p.x, y - p.y\}; \}
     return true;
                                                           Point operator*(const double d) const { return
95
                                                            \rightarrow {x * d, y * d}; }
96
                                                            Point rotate() const { return {y, -x}; }
```

```
double operator*(const Point& p) const { return
                                                            Line 11(p1, p2), 12(q1, q2);
      \rightarrow x * p.x + y * p.y; }
                                                            if (11 || 12) return false;
     double operator^(const Point& p) const { return
                                                            x = 11 ^ 12;
      \rightarrow x * p.y - y * p.x; }
                                                            return on_segment(p1, p2, x, false);
     double dist() const { return sqrt(x * x + y * 59
      \rightarrow y); }
   };
                                                          // in case circles are not equal
12
                                                          inline bool intersect_circles(const Circle& c1,
13
   struct Line {
                                                          double a, b, c;
                                                            double d = (c2.c - c1.c).dist();
15
                                                            if (d > c1.r + c2.r + EPS \mid \mid d < fabs(c1.r -
     Line(const Point& p1, const Point& p2) {
16
                                                            \leftrightarrow c2.r) - EPS)
       a = p1.y - p2.y;
       b = p2.x - p1.x;
                                                              return false;
                                                            double cosa = (sqr(d) + sqr(c1.r) - sqr(c2.r))
       c = -a * p1.x - b * p1.y;
19
                                                            \rightarrow / (2 * c1.r * d);
       double d = sqrt(sqr(a) + sqr(b));
                                                            double 1 = c1.r * cosa, h = sqrt(sqr(c1.r) -
21
       a /= d, b /= d, c /= d;
                                                            \rightarrow sqr(1));
                                                            Point v = (c2.c - c1.c) * (1 / d), p = c1.c + v
23
     bool operator | | (const Line& 1) const { return
                                                            → * 1:
24
      \rightarrow fabs(a * 1.b - 1.a * b) < EPS; }
                                                            p1 = p + v.rotate() * h, p2 = p - v.rotate() *
     double dist(const Point& p) const { return
                                                             \hookrightarrow h;
      \rightarrow fabs(a * p.x + b * p.y + c); }
                                                            return true;
                                                      70
     Point operator (const Line& 1) const {
                                                      71
26
       return \{(1.c * b - c * 1.b) / (a * 1.b - 1.a_2)\}
        \rightarrow * b),
                                                          inline bool intersect_circle_and_line(const
            (1.c * a - c * 1.a) / (1.a * b - a *
                                                          → Circle& c, const Line& l, Point& p1, Point&
            \rightarrow 1.b)};
                                                          → p2) {
     }
                                                            double d = 1.dist(c.c);
     Point projection(const Point& p) const {
                                                            if (d > c.r + EPS)
                                                      75
30
       return p - Point{a, b} * (a * p.x + b * p.y \pi
                                                              return false;
31
                                                            Point p = 1.projection(c.c);

→ c);
                                                      77
     }
                                                            Point n{1.b, -1.a};
   };
                                                            double h = sqrt(sqr(c.r) - sqr(l.dist(c.c)));
33
                                                            p1 = p + n * h, p2 = p - n * h;
34
   struct Circle {
                                                            return true;
                                                      81
     Point c;
     double r:
37
     Circle(const Point& c, double r) : c(c), r(r) 84
                                                         // simple geometry ends
38
      ← {}
     Circle(const Point& a, const Point& b, const
                                                          // convex hull begins
      → Point& c) {
       Point p1 = (a + b) * 0.5, p2 = (a + c) * 0.5;
                                                          struct Point {
       Point q1 = p1 + (b - a).rotate(), q2 = p2 + 3
                                                            int x, y;
        \rightarrow (c - a).rotate();
                                                            Point operator-(const Point& p) const { return
       this->c = Line(p1, q1) ^ Line(p2, q2);
42
                                                            \rightarrow {x - p.x, y - p.y}; }
       r = (a - this->c).dist();
43
                                                            int64_t operator^(const Point& p) const {
     }
44
                                                            → return x * 111 * p.y - y * 111 * p.x; }
   };
45
                                                            int64_t dist() const { return x * 111 * x + y *
46
                                                            \rightarrow 111 * y; }
   inline bool on_segment(const Point& p1, const
47
                                                            bool operator<(const Point& p) const { return x
    → Point& p2, const Point& x, bool strictly) {
                                                            \rightarrow != p.x ? x < p.x : y < p.y; \}
     if (fabs((p1 - x) ^ (p2 - x)) > EPS)
48
                                                          };
       return false;
49
     return (p1 - x) * (p2 - x) < (strictly ? - EPS^{10}
50
                                                          // all point on convex hull are included

→ : EPS);

                                                          vector<Point> convex_hull(vector<Point> pt) {
   }
51
                                                            int n = pt.size();
                                                            Point p0 = *std::min_element(pt.begin(),
   // in case intersection is not a segment
                                                            \rightarrow pt.end());
   inline bool intersect_segments(const Point& p1,
                                                            std::sort(pt.begin(), pt.end(), [&p0](const

→ const Point& p2, const Point& q1, const

                                                            → Point& a, const Point& b) {
    \rightarrow Point& q2, Point& x) {
                                                              int64_t cp = (a - p0) ^ (b - p0);
```

```
return cp != 0 ? cp > 0 : (a - p0).dist() < 34
                                                                  add_line(v + v + 2, m, r, a, b, nw);
           (b - p0).dist();
                                                                  return;
     });
18
                                                        37
     int i = n - 1;
                                                                bool lef = f(nw, 1) < f(line[v], 1);
     for (; i > 0 && ((pt[i] - p0) ^ (pt[i - 1] -
                                                                bool mid = f(nw, m) < f(line[v], m);</pre>
21
      \rightarrow p0)) == 0; i--);
     std::reverse(pt.begin() + i, pt.end());
                                                                if (mid) swap(line[v], nw);
22
     vector<Point> ch;
                                                                if (1 == r - 1)
                                                        43
24
     for (auto& p : pt) {
                                                                  return;
                                                        44
       while (ch.size() > 1) {
                                                                if (lef != mid)
          auto& p1 = ch[(int) ch.size() - 1];
          auto& p2 = ch[(int) ch.size() - 2];
                                                                  add_line(v + v + 1, l, m, a, b, nw);
                                                        47
          int64_t cp = (p1 - p2) ^ (p - p1);
                                                                else
                                                        48
          if (cp >= 0) break;
                                                                  add_line(v + v + 2, m, r, a, b, nw);
                                                        49
          ch.pop_back();
                                                             }
       }
                                                        51
                                                              ftype get(int v, int 1, int r, int x) {
        ch.push_back(p);
                                                        52
                                                                if(1 == r - 1)
                                                        53
                                                        54
                                                                  return f(line[v], x);
     return ch;
                                                                int m = (1 + r) / 2;
                                                        55
36
                                                                if(x < m) {
37
                                                        56
                                                                  return min(f(line[v], x), get(v + v + 1, 1,
   // convex hull ends
                                                                  \rightarrow m, x));
                                                                } else {
                                                        58
   // convex hull trick begins
                                                                  return min(f(line[v], x), get(v + v + 2, m,
                                                                  \rightarrow r, x));
   typedef long long ftype;
                                                        60
   typedef complex<ftype> point;
                                                              }
                                                        61
   #define x real
   #define y imag
                                                           } cdt(maxn);
                                                        64
   ftype dot(point const& a, point const& b) {
                                                           // convex hull with stack
                                                        65
     return (conj(a) * b).x();
10
                                                           ftype cross(point a, point b) {
                                                        67
11
                                                             return (conj(a) * b).y();
                                                        68
   ftype f(point const& a, int x) {
                                                        69
     return dot(a, {compressed[x], 1});
                                                        70
      //return\ dot(a, \{x, 1\});
14
                                                           vector<point> hull, vecs;
                                                        72
16
                                                           void add_line(ftype k, ftype b) {
                                                        73
   int pos = 0;
17
                                                             point nw = \{k, b\};
                                                        74
18
                                                              while(!vecs.empty() && dot(vecs.back(), nw -
   // (x, y) \rightarrow (k, b) \rightarrow kb + x
                                                              \rightarrow hull.back()) < 0) {
   struct li_chao { // for min
20
                                                               hull.pop_back();
                                                        76
     vector<point> line;
21
                                                                vecs.pop_back();
                                                        77
                                                             }
                                                        78
     li_chao(int maxn) {
                                                              if(!hull.empty()) {
                                                        79
        line.resize(4 * maxn, \{0, inf\});
24
                                                                vecs.push_back(1i * (nw - hull.back()));
25
                                                             hull.push_back(nw);
     void add_line(int v, int 1, int r, int a, int _{83}
      \rightarrow b, point nw) {
        if (r <= a || b <= 1) return; // remove if n_{s5}
                                                           int get(ftype x) {
           [a, b) query
                                                             point query = \{x, 1\};
                                                             auto it = lower_bound(vecs.begin(), vecs.end(),
        int m = (1 + r) >> 1;
30
                                                              → query, [](point a, point b) {
31
                                                                return cross(a, b) > 0;
        if (!(a \le 1 \&\& r \le b)) \{ // remove if no
                                                              });
        \rightarrow [a, b) query
                                                             return dot(query, hull[it - vecs.begin()]);
          add_line(v + v + 1, l, m, a, b, nw);
```

```
}
                                                                a = par[upper[a]];
                                                              } else {
   // convex hull trick ends
                                                                back.push_back({t_in[upper[b]], t_in[b],

  false});
   // heavy-light begins
                                                                b = par[upper[b]];
                                                              }
                                                      62
   int sz[maxn];
                                                           }
                                                      63
                                                      64
   void dfs_sz(int v, int par = -1) {
                                                            front.push_back({min(t_in[a], t_in[b]),
     sz[v] = 1;
                                                            \rightarrow max(t_in[a], t_in[b]), t_in[a] > t_in[b]});
     for (int x : gr[v])
                                                            // for edge-weighted hld add:
       if (x != par) {
                                                            dfs_sz(x, v);
                                                            front.insert(front.end(), back.rbegin(),
         sz[v] += sz[x];
10

→ back.rend());
       }
11
     for (int i = 0; i < gr[v].size(); i++)</pre>
                                                           return front;
       if (gr[v][i] != par)
13
                                                         }
       if (sz[gr[v][i]] * 2 >= sz[v]) {
14
                                                      71
         swap(gr[v][i], gr[v][0]);
15
                                                         // heavy-light ends
         break;
       }
                                                         // max flow begins
17
   }
18
                                                         struct edge{
                                                           int from, to;
   int rev[maxn];
                                                           int c, f, num;
   int t_in[maxn];
21
   int upper[maxn];;
                                                           edge(int from, int to, int c, int
                                                            \rightarrow num):from(from), to(to), c(c), f(0),
   int par[maxn];
                                                            → num(num){}
   int dep[maxn];
                                                            edge(){}
                                                         };
   int T = 0;
   void dfs_build(int v, int uppr, int pr = -1) { 10
                                                         const int max_n = 600;
     rev[T] = v;
29
                                                         edge eds[150000];
     t_in[v] = T++;
                                                      12
30
     dep[v] = pr == -1 ? 0 : dep[pr] + 1;
                                                         int num = 0;
                                                      13
31
                                                         int it[max_n];
     par[v] = pr;
                                                      14
                                                      15
                                                         vector<int> gr[max_n];
     upper[v] = uppr;
                                                         int s, t;
                                                         vector<int> d(max_n);
                                                      17
     bool first = true;
                                                      18
     for (int x : gr[v])
                                                         bool bfs(int k) {
37
       if (x != pr) {
                                                      20
                                                           queue<int> q;
38
                                                           q.push(s);
         dfs_build(x, first ? upper[v] : x, v);
                                                      21
                                                           fill(d.begin(), d.end(), -1);
          first = false;
                                                      22
40
                                                           d[s] = 0;
41
                                                           while (!q.empty()) {
   }
                                                      24
42
                                                              int v = q.front();
                                                      25
                                                              q.pop();
   struct interval {
44
     int 1;
                                                              for (int x : gr[v]) {
45
                                                                int to = eds[x].to;
                                                      28
                                                                if (d[to] == -1 \&\& eds[x].c - eds[x].f >=
     bool inv; // should direction be reversed
   };
                                                                   (1 << k)){}
48
                                                                  d[to] = d[v] + 1;
                                                                  q.push(to);
   // node-weighted hld
                                                      31
                                                                }
   vector<interval> get_path(int a, int b) {
                                                      32
51
                                                              }
     vector<interval> front;
                                                      33
52
                                                           }
     vector<interval> back;
53
                                                           return (d[t] != -1);
     while (upper[a] != upper[b]) {
                                                      36
       if (dep[upper[a]] > dep[upper[b]]) {
         front.push_back({t_in[upper[a]], t_in[a], 38
                                                         int dfs(int v, int flow, int k) {

    true});
```

```
if (flow < (1 << k))
                                                          }
       return 0;
41
                                                          while (decomp(s, 1e9 + 5));
     if (v == t)
42
                                                      100
       return flow;
     for (; it[v] < gr[v].size(); it[v]++) {</pre>
                                                          // max flow ends
        int num = gr[v][it[v]];
                                                          // min-cost flow begins
       if (d[v] + 1 != d[num].to])
46
          continue;
47
                                                          long long ans = 0;
                                                       3
       int res = dfs(eds[num].to, min(flow,
                                                          int mx = 2 * n + 2;
        \rightarrow eds[num].c - eds[num].f), k);
       if (res){
49
                                                          memset(upd, 0, sizeof(upd));
          eds[num].f += res;
                                                          for (int i = 0; i < mx; i++)
          eds[num ^ 1].f -= res;
                                                            dist[i] = inf;
          return res;
52
                                                          dist[st] = 0;
53
                                                          queue<int> q;
     }
54
                                                          q.push(st);
                                                       11
     return 0;
                                                          upd[st] = 1;
                                                       12
56
                                                          while (!q.empty()){
57
                                                            int v = q.front();
   void add(int fr, int to, int c, int nm) {
                                                             q.pop();
                                                       15
     gr[fr].push_back(num);
                                                             if (upd[v]){
     eds[num++] = edge(fr, to, c, nm);
                                                       16
60
                                                               for (int x : gr[v]) {
     gr[to].push_back(num);
                                                       17
61
                                                                 edge &e = edges[x];
     eds[num++] = edge(to, fr, 0, nm); //corrected 18
62
                                                                 if (e.c - e.f > 0 && dist[v] != inf &&
   }
63
                                                                 \rightarrow dist[e.to] > dist[v] + e.w) {
64
                                                                   dist[e.to] = dist[v] + e.w;
   int ans = 0;
                                                                   if (!upd[e.to])
     for (int k = 30; k \ge 0; k--)
                                                                     q.push(e.to);
       while (bfs(k)) {
67
                                                                   upd[e.to] = true;
          memset(it, 0, sizeof(it));
                                                                   p[e.to] = x;
          while (int res = dfs(s, 1e9 + 500, k))
            ans += res;
       }
                                                              upd[v] = false;
                                                       27
                                                       28
                                                          }
                                                       29
   // decomposition
                                                       30
75
                                                          for (int i = 0; i < k; i++){
   int path_num = 0;
                                                       31
76
                                                            for (int i = 0; i < mx; i++)
                                                       32
   vector<int> paths[max_n];
                                                               d[i] = inf;
                                                      33
   int flows[max_n];
                                                            d[st] = 0;
                                                      34
                                                            memset(used, false, sizeof(used));
                                                      35
   int decomp(int v, int flow) {
                                                            set<pair<int, int> > s;
     if (flow < 1)
                                                             s.insert(make_pair(0, st));
                                                       37
       return 0;
                                                             for (int i = 0; i < mx; i++){
     if (v == t) {
                                                       38
83
                                                               int x;
       path_num++;
                                                       39
84
                                                               while (!s.empty() && used[(s.begin() ->
       flows[path_num - 1] = flow;

→ second)]){
       return flow;
                                                                 s.erase(s.begin());
     }
                                                       41
                                                               }
     for (int i = 0; i < gr[v].size(); i++) {</pre>
                                                               if (s.empty())
       int num = gr[v][i];
                                                                 break:
                                                       44
        int res = decomp(eds[num].to, min(flow,
90
                                                               x = s.begin() -> second;
                                                       45
        \rightarrow eds[num].f));
                                                               used[x] = true;
       if (res)
                 {
91
                                                               s.erase(s.begin());
         eds[num].f -= res;
                                                               for (int i = 0; i < gr[x].size(); i++){</pre>
                                                       48
         paths[path_num -
                                                                 edge &e = edges[gr[x][i]];
                                                       49
          → 1].push_back(eds[num].num);
                                                                 if (!used[e.to] && e.c - e.f > 0){
                                                       50
         return res;
                                                                   if (d[e.to] > d[x] + (e.c - e.f) * e.w +

    dist[x] - dist[e.to]){
     }
96
                                                                     d[e.to] = d[x] + (e.c - e.f) * e.w +
                                                       52
     return 0;
97

→ dist[x] - dist[e.to];
```

```
p[e.to] = gr[x][i];
                                                                     fre[b] = i;
              s.insert(make_pair(d[e.to], e.to));
                                                                     if (step == n - 1)
                                                                       base = b;
55
          }
                                                                   }
       }
                                                                 }
       dist[x] += d[x];
                                                                 if (base < 0) break;</pre>
59
                                                       50
                                                                 vi seq;
     int pos = t;
60
                                                      51
     while (pos != st){
                                                                 vb was(n, false);
       int id = p[pos];
                                                                 for (int x = base; x = es[fre[x] ^ 1].to)
                                                       53
62
       edges[id].f += 1;
                                                                 ← {
       edges[id ^ 1].f -= 1;
                                                                   if (!was[x]) {
                                                       54
       pos = edges[id].from;
                                                                     seq.pb(x);
                                                                     was[x] = true;
66
                                                      56
                                                                   } else {
   }
67
                                                      57
                                                                     seq.erase(
68
                                                      58
   // min-cost flow ends
                                                                         seq.begin(),
                                                                         find(seq.begin(), seq.end(),
   // min circulation begins
                                                       61
   class Solver2 { // Min-cost circulation
                                                                     ));
                                                       62
     struct Edge {
                                                                     break;
       int to, ne, w, c;
                                                                   }
                                                       64
                                                                 }
                                                       65
     vector<Edge> es;
                                                                 for (int i = 0; i < sz(seq); i++) {
     vi firs;
                                                                   int v = seq[i];
     int curRes;
                                                                   int eid = fre[v];
                                                       68
     public:
                                                                   assert(es[eid].w > 0);
     Solver2(int n) : es(), firs(n, -1),
10
                                                                   es[eid].w--;
   curRes(0) {}
11
                                                                   es[eid ^1].w++;
                                                       71
     // from, to, capacity (max.flow), cost
12
                                                                   curRes += es[eid].c;
                                                       72
     int adde(int a, int b, int w, int c) {
13
       Edge e;
14
                                                               }
       e.to = b; e.ne = firs[a];
                                                              return curRes;
                                                       75
       e.w = w; e.c = c;
16
                                                       76
       es.pb(e);
17
                                                          };
                                                      77
       firs[a] = sz(es) - 1;
                                                          // min circulation ends
       e.to = a; e.ne = firs[b];
                                                       1 // global cut begins
       e.w = 0; e.c = -c;
                                                       _2 // g[i][j] = g[j][i] is sum of edges between i
       es.pb(e);
                                                          \rightarrow and j
                                                          // ans is value of mincut
       firs[b] = sz(es) - 1;
       return sz(es) - 2;
                                                         11 g[maxn] [maxn], w[maxn];
24
                                                          ll ans;
25
     // increase capacity of edge 'id' by 'w'
                                                       bool ex[maxn], inA[maxn];
26
     void ince(int id, int w) {
                                                       7 int n;
27
       es[id].w += w;
                                                         void iterate(int curN){
28
     }
                                                          int prev = -1;
     int solve() {
                                                            memset(inA, 0, sizeof(inA));
                                                       10
30
                                                            memset(w, 0, sizeof(w));
       const int n = sz(firs);
                                                      11
31
                                                            for (int it = 0; it < curN; it++) {</pre>
                                                      12
32
       for (;;) {
                                                              int best = -1;
33
                                                      13
         vi d(n, 0), fre(n, -1);
                                                       14
                                                              for (int i = 0; i < n; i++)
         vi chd(n, -1);
                                                                 if (ex[i] && !inA[i]
                                                                     && (best == -1 \mid \mid w[i] > w[best]))
                                                       16
          int base = -1;
                                                                    best = i;
                                                       17
         for (int step = 0; step < n; step++) {</pre>
                                                              assert(best != -1);
                                                       18
            for (int i = 0; i < sz(es); i++) if
                                                              if (it == curN - 1) {
            \rightarrow (es[i].w > 0) {
                                                       20
                                                                 ans = min(ans, w[best]);
              int b = es[i].to;
                                                                 for (int i = 0; i < n; i++){
40
              int a = es[i ^ 1].to;
                                                                   g[i][prev] += g[best][i];
              if (d[b] <= d[a] + es[i].c) continue; 23
                                                                   g[prev][i] = g[i][prev];
42
                                                                 }
              d[b] = d[a] + es[i].c;
43
```

```
ex[best] = false;
                                                               int nxt = par2[prev[pos]];
       } else {
                                                               par[pos] = prev[pos];
          inA[best] = true;
                                                               par2[prev[pos]] = pos;
27
                                                       44
         for (int i = 0; i < n; i++)</pre>
                                                               pos = nxt;
                                                       45
            w[i] += g[best][i];
                                                          }
            prev = best;
30
                                                       47
                                                          cout << ans << ''\n'';
                                                       48
31
     }
                                                          for (int i = 0; i < n; i++)</pre>
32
   }
                                                            cout << par[i] + 1 << "" << i + 1 << "\n";</pre>
33
   void solve(){
34
                                                       51
     ans = INF;
                                                          // bad hungarian ends
35
     for (int i = n; i > 1; i--)
                                                          // Edmonds O(n^3) begins
       iterate(i);
     cout << ans << endl;</pre>
38
                                                          vector<int> gr[MAXN];
39
                                                          int match[MAXN], p[MAXN], base[MAXN], q[MAXN];
   // global cut ends
                                                          bool used[MAXN], blossom[MAXN];
   // bad hungarian begins
                                                          int mark[MAXN];
                                                          int C = 1;
   fill(par, par + 301, -1);
  fill(par2, par2 + 301, -1);
                                                          int lca(int a, int b) {
                                                            C++;
                                                       10
   int ans = 0;
                                                            for (;;) {
                                                       11
   for (int v = 0; v < n; v++){
                                                       12
                                                               a = base[a];
     memset(useda, false, sizeof(useda));
                                                               mark[a] = C;
                                                       13
     memset(usedb, false, sizeof(usedb));
                                                              if (match[a] == -1) break;
                                                       14
     useda[v] = true;
10
                                                               a = p[match[a]];
     for (int i = 0; i < n; i++)
11
       w[i] = make_pair(a[v][i] + row[v] + col[i], 17
        \rightarrow v);
                                                            for (;;) {
     memset(prev, 0, sizeof(prev));
                                                               b = base[b];
13
     int pos;
                                                               if (mark[b] == C) return b;
     while (true){
                                                               b = p[match[b]];
                                                       21
       pair<pair<int, int>, int> p =
16
                                                       22

→ make_pair(make_pair(1e9, 1e9), 1e9);
       for (int i = 0; i < n; i++)
17
          if (!usedb[i])
                                                          void mark_path(int v, int b, int children) {
                                                       25
           p = min(p, make_pair(w[i], i));
                                                            while (base[v] != b) {
                                                       26
       for (int i = 0; i < n; i++)
                                                               blossom[base[v]] = blossom[base[match[v]]] =
                                                       27
          if (!useda[i])

    true:

            row[i] += p.first.first;
                                                               p[v] = children;
                                                       28
       for (int i = 0; i < n; i++)
                                                               children = match[v];
23
                                                       29
         if (!usedb[i]){
                                                               v = p[match[v]];
24
            col[i] -= p.first.first;
                                                            }
25
                                                       31
            w[i].first -= p.first.first;
                                                          }
                                                       32
         }
       ans += p.first.first;
                                                          int find_path(int root) {
                                                       34
       usedb[p.second] = true;
                                                            memset(used, 0, sizeof(used));
                                                       35
       prev[p.second] = p.first.second; //us emopoŭ<sub>86</sub>
                                                            memset(p, -1, sizeof p);
        ⊶ в первую
                                                            for (int i = 0; i < N; i++)
       int x = par[p.second];
                                                               base[i] = i;
31
       if (x == -1){
                                                       39
         pos = p.second;
                                                            used[root] = true;
                                                       40
          break;
                                                            int qh = 0, qt = 0;
                                                       41
                                                            q[qt++] = root;
                                                       42
       useda[x] = true;
                                                            while (qh < qt) {
                                                       43
       for (int j = 0; j < n; j++)
                                                              int v = q[qh++];
                                                       44
         w[j] = min(w[j], \{a[x][j] + row[x] +
                                                               for (int to : gr[v]) {
38
                                                      45
          \rightarrow col[j], x});
                                                                 if (base[v] == base[to] || match[v] == to)
39

→ continue;

                                                                 if (to == root || match[to] != -1 &&
40
                                                       47
     while (pos != -1){
                                                                 \rightarrow p[match[to]] != -1) {
41
```

```
int curbase = lca(v, to);
                                                            p.resize(s.size(), 0);
            memset(blossom, 0, sizeof(blossom));
                                                            int k = 0;
                                                            for (int i = 1; i < s.size(); i++){</pre>
            mark_path(v, curbase, to);
                                                      24
            mark_path(to, curbase, v);
                                                              while (k > 0 \&\& s[i] == s[k])
                                                       25
            for (int i = 0; i < N; i++)
                                                                k = p[k - 1];
              if (blossom[base[i]]) {
                                                               if (s[i] == s[k])
                                                      27
                base[i] = curbase;
                                                                k++;
                                                      28
54
                if (!used[i]) {
                                                              p[i] = k;
                                                      29
                                                            }
                  used[i] = true;
                  q[qt++] = i;
                                                            return p;
                                                      31
                }
                                                      32
              }
                                                      33
          } else if (p[to] == -1) {
                                                          vector<int> getH(string s){
                                                      34
            p[to] = v;
                                                            vector<int> h;
                                                      35
61
            if (match[to] == -1)
                                                            h.resize(s.size() + 1, 0);
                                                      36
62
              return to;
                                                            for (int i = 0; i < s.size(); i++)
63
                                                      37
                                                              h[i + 1] = ((h[i] * 111 * pow) + s[i] - 'a' +
            to = match[to];
            used[to] = true;
                                                               \rightarrow 1) % mod;
            q[qt++] = to;
                                                            return h;
                                                       39
          }
                                                          }
                                                       40
       }
                                                       41
     }
                                                          int getHash(vector<int> &h, int 1, int r){
                                                       42
69
                                                            int res = (h[r + 1] - h[1] * p[r - 1 + 1]) %
70
     return -1;
                                                             \rightarrow mod;
71
                                                            if (res < 0)
72
                                                              res += mod;
73
   memset(match, -1, sizeof match);
                                                            return res;
74
     for (int i = 0; i < N; i++) {
                                                          }
       if (match[i] == -1 && !gr[i].empty()) {
76
                                                      48
          int v = find_path(i);
                                                          // string basis ends
77
          while (v != -1) {
                                                          // min cyclic shift begins
            int pv = p[v], ppv = match[pv];
                                                       1
            match[v] = pv; match[pv] = v;
                                                       2
                                                          string min_cyclic_shift (string s) {
            v = ppv;
                                                            s += s;
          }
                                                            int n = (int) s.length();
       }
                                                            int i=0, ans=0;
     }
                                                            while (i < n/2) {
85
                                                               ans = i;
   // Edmonds O(n^3) ends
                                                              int j=i+1, k=i;
   // string basis begins
                                                              while (j < n \&\& s[k] <= s[j]) {
                                                       10
                                                                 if (s[k] < s[j])
                                                       11
   vector<int> getZ(string s){
                                                                  k = i;
                                                       12
     vector<int> z;
                                                                 else
                                                       13
     z.resize(s.size(), 0);
                                                                   ++k;
                                                       14
     int 1 = 0, r = 0;
                                                                 ++j;
     for (int i = 1; i < s.size(); i++){
                                                               }
       if (i <= r)
                                                               while (i \le k) i += j - k;
                                                       17
          z[i] = min(r - i + 1, z[i - 1]);
                                                            }
       while (i + z[i] < s.size() \&\& s[z[i]] == s[i_{19}]
10
                                                            return s.substr (ans, n/2);
        \rightarrow + z[i]])
         z[i]++;
       if (i + z[i] - 1 > r){
                                                          // min cyclic shift ends
         r = i + z[i] - 1;
13
         l = i;
                                                          // suffix array O(n) begins
       }
15
     }
                                                          typedef vector<char> bits;
16
     return z;
17
                                                          template<const int end>
18
                                                       5
                                                          void getBuckets(int *s, int *bkt, int n, int K) {
19
   vector<int> getP(string s){
                                                            fill(bkt, bkt + K + 1, 0);
20
     vector<int> p;
                                                            forn(i, n) bkt[s[i] + !end]++;
21
```

```
forn(i, K) bkt[i + 1] += bkt[i];
                                                              int *s1 = SA + n - n1;
   }
                                                              if (name < n1)
10
   void induceSAl(bits &t, int *SA, int *s, int
                                                                SA_IS(s1, SA, n1, name - 1);
11
                                                        66
    \rightarrow *bkt, int n, int K) {
                                                        67
                                                              else
     getBuckets<0>(s, bkt, n, K);
                                                                forn(i, n1)
     forn(i, n) {
                                                                  SA[s1[i]] = i;
13
        int j = SA[i] - 1;
                                                              getBuckets<1>(s, bkt, n, K);
                                                        70
14
        if (j \ge 0 \&\& !t[j])
                                                              for (i = 1, j = 0; i < n; i++)
                                                        71
15
          SA[bkt[s[j]]++] = j;
                                                                if (isLMS(i))
                                                                  s1[j++] = i;
17
                                                        73
   }
                                                              forn(i, n1)
18
                                                        74
                                                                SA[i] = s1[SA[i]];
   void induceSAs(bits &t, int *SA, int *s, int
    → *bkt, int n, int K) {
                                                              fill(SA + n1, SA + n, -1);
                                                              for (i = n1 - 1; i >= 0; i--) {
     getBuckets<1>(s, bkt, n, K);
                                                        77
20
     for (int i = n - 1; i >= 0; i--) {
                                                                j = SA[i], SA[i] = -1;
21
                                                        78
        int j = SA[i] - 1;
                                                                SA[--bkt[s[j]]] = j;
22
                                                        79
        if (j >= 0 && t[j])
                                                             }
          SA[--bkt[s[j]]] = j;
                                                              induceSAl(t, SA, s, bkt, n, K);
                                                        81
24
     }
                                                              induceSAs(t, SA, s, bkt, n, K);
25
                                                        82
   }
26
                                                        83
27
                                                           // suffix array O(n) ends
   void SA_IS(int *s, int *SA, int n, int K) { //
28
    → require last symbol is 0
                                                           // suffix array O(n log n) begins
   #define isLMS(i) (i \&\&\ t[i]\ \&\&\ !t[i-1])
                                                           string str;
     int i, j;
                                                           int N, m, SA [MAX_N], LCP [MAX_N];
     bits t(n);
                                                           int x [MAX_N], y [MAX_N], w [MAX_N], c [MAX_N];
31
     t[n-1] = 1;
     for (i = n - 3; i \ge 0; i--)
                                                           inline bool cmp (const int a, const int b, const
        t[i] = (s[i] < s[i+1] \mid | (s[i] = s[i+1] \&\&
                                                            \rightarrow int 1) { return (y [a] == y [b] && y [a + 1]
34
        \rightarrow t[i+1]==1));
                                                               == y [b + 1]); }
     int bkt[K + 1];
35
                                                        92
     getBuckets<1>(s, bkt, n, K);
                                                           void Sort () {
     fill(SA, SA + n, -1);
                                                              for (int i = 0; i < m; ++i) w[i] = 0;
                                                        94
37
                                                             for (int i = 0; i < N; ++i) ++w[x[y[i]]];
     forn(i, n)
                                                        95
                                                             for (int i = 0; i < m - 1; ++i) w[i + 1] +=
        if (isLMS(i))
          SA[--bkt[s[i]]] = i;
                                                              induceSAl(t, SA, s, bkt, n, K);
                                                             for (int i = N - 1; i \ge 0; --i)
                                                        97
41
     induceSAs(t, SA, s, bkt, n, K);
                                                              \hookrightarrow SA[--w[x[y[i]]] = y[i];
42
     int n1 = 0;
                                                        98
43
     forn(i, n)
        if (isLMS(SA[i]))
                                                           void DA () {
45
                                                       100
          SA[n1++] = SA[i];
                                                             for (int i = 0; i < N; ++i) x[i] = str[i], y[i]
                                                       101
     fill(SA + n1, SA + n, -1);
                                                              \hookrightarrow = i;
     int name = 0, prev = -1;
                                                              Sort ();
     forn(i, n1) {
                                                              for (int i, j = 1, p = 1; p < N; j <<= 1, m =
49
                                                       103
        int pos = SA[i];
                                                              \rightarrow p) {
50
                                                                for (p = 0, i = N - j; i < N; i++) y[p++] =
        bool diff = false;
51
        for (int d = 0; d < n; d++)
          if (prev == -1 || s[pos+d] != s[prev+d] | t_{05}
                                                                for (int k = 0; k < N; ++k) if (SA[k] >= j)
                                                                \hookrightarrow y[p++] = SA[k] - j;
          \rightarrow t[pos+d] != t[prev+d])
            diff = true, d = n;
                                                                Sort();
          else if (d > 0 && (isLMS(pos+d) ||
                                                                for (swap (x, y), p = 1, x[SA[0]] = 0, i = 1;
                                                       107
55
                                                                \rightarrow i < N; ++i) x[SA [i]] = cmp (SA[i - 1],

    isLMS(prev+d)))

            d = n;
                                                                    SA[i], j) ? p - 1 : p++;
56
        if (diff)
                                                             }
57
          name++, prev = pos;
                                                           }
                                                       109
        SA[n1 + (pos >> 1)] = name - 1;
59
                                                       110
                                                           // common for all algorithms
                                                       111
     for (i = n - 1, j = n - 1; i >= n1; i--)
                                                           void kasaiLCP () {
61
                                                       112
        if (SA[i] >= 0)
                                                       113
                                                              for (int i = 0; i < N; i++) c[SA[i]] = i;
62
          SA[j--] = SA[i];
63
```

```
for (int i = 0, j, k = 0; i < N; LCP [c[i++]] 46
                                                                       cur = v[cur].suff;
      \rightarrow k)
                                                                     }
        if (c [i] > 0) for (k ? k-- : 0, j = SA[c[i]_{48}]
                                                                     v[a].suff = b;
115
         \rightarrow -1]; str[i + k] == str[j + k]; k++);
                                                                   }
        else k = 0;
                                                                  return;
116
    }
117
118
   void suffixArray () { // require last symbol is 53
119
    \hookrightarrow char(0)
      m = 256;
                                                            // bad suffix automaton ends
120
      N = str.size();
121
                                                            // aho-corasick begins
      DA ();
122
                                                         2
      kasaiLCP ();
123
                                                            struct vertex {
124
                                                              int next[K];
    // suffix array O(n log n) ends
125
                                                              bool leaf;
    // bad suffix automaton begins
                                                              int p;
                                                              char pch;
                                                              int link;
    struct node{
      map<char, int> go;
                                                              int go[K];
      int len, suff;
                                                            };
      long long sum_in;
      node(){}
                                                            vertex t[NMAX+1];
                                                        12
   };
                                                            int sz;
                                                        13
                                                        14
                                                            void init() {
   node v[max_n * 4];
                                                              t[0].p = t[0].link = -1;
11
                                                        16
    int add_node(int max_len){
                                                              memset(t[0].next, 255, sizeof t[0].next);
12
                                                        17
      //v[number].sum_in = 0;
                                                              memset(t[0].go, 255, sizeof t[0].go);
                                                        18
      v[number].len = max_len;
                                                              sz = 1;
14
                                                        19
      v[number].suff = -1;
                                                        20
15
      number++;
                                                        21
                                                            void add_string(const string & s) {
      return number - 1;
                                                        22
    }
                                                              int v = 0;
                                                        23
18
                                                              for (size_t i=0; i<s.length(); ++i) {</pre>
                                                        24
19
                                                                char c = s[i] - 'a';
    int last = add_node(0);
20
                                                        25
                                                                if (t[v].next[c] == -1) {
21
    void add_char(char c) {
                                                                   memset(t[sz].next, 255, sizeof t[sz].next);
22
                                                                  memset(t[sz].go, 255, sizeof t[sz].go);
      int cur = last;
                                                        28
      int new_node = add_node(v[cur].len + 1);
                                                                   t[sz].link = -1;
      last = new_node;
                                                                   t[sz].p = v;
      while (\operatorname{cur} != -1){
                                                        31
                                                                   t[sz].pch = c;
26
        if (v[cur].go.count(c) == 0){
                                                                   t[v].next[c] = sz++;
27
                                                        32
          v[cur].go[c] = new_node;
                                                                }
28
          //v[new\_node].sum\_in += v[cur].sum\_in;
                                                                v = t[v].next[c];
          cur = v[cur].suff;
                                                              }
                                                        35
          if (cur == -1)
                                                              t[v].leaf = true;
                                                            }
            v[new_node].suff = 0;
                                                        37
        }else{
                                                        38
          int a = v[cur].go[c];
                                                            int go(int v, char c);
34
                                                        39
          if (v[a].len == v[cur].len + 1){
                                                        40
35
            v[new_node].suff = a;
                                                            int get_link(int v) {
          }else{
                                                              if (t[v].link == -1)
            int b = add_node(v[cur].len + 1);
                                                                if (v == 0 || t[v].p == 0)
                                                        43
            v[b].go = v[a].go;
                                                                   t[v].link = 0;
                                                        44
            v[b].suff = v[a].suff;
                                                        45
                                                                else
            v[new_node].suff = b;
                                                                   t[v].link = go(get_link(t[v].p), t[v].pch);
            while (cur != -1 && v[cur].go.count(c) !#
                                                              return t[v].link;
42
             \rightarrow 0 && v[cur].go[c] == a){
                                                            }
                                                        48
              v[cur].go[c] = b;
               //v[a].sum_in -= v[cur].sum_in;
                                                           int go(int v, char c) {
44
               //v[b].sum_in += v[cur].sum_in;
                                                              if (t[v].go[c] == -1)
                                                       51
45
```

```
if (t[v].next[c] != -1)
                                                             lp[i] = pr[sz++] = i;
52
         t[v].go[c] = t[v].next[c];
                                                            for (int j = 0; j < sz && pr[j] <= lp[i] && i *

    pr[j] <= N; ++j)
</pre>
54
         t[v].go[c] = v == 0 ? 0 : go(get_link(v), 14)
                                                             lp[i * pr[j]] = pr[j];

→ c);
     return t[v].go[c];
                                                           mu[i] = lp[i] == lp[i / lp[i]] ? 0 : -1 * mu[i]
56
                                                            → / lp[i]];
57
                                                           phi[i] = phi[i / lp[i]] * (lp[i] == lp[i /
58
                                                      17
                                                            → lp[i]] ? lp[i] : lp[i] - 1);
   // aho-corasick ends
                                                      18
   // pollard begins
                                                      19
                                                         // linear sieve ends
   const int max_step = 4e5;
                                                         // discrete log in sqrt(p) begins
   unsigned long long gcd(unsigned long long a,
                                                         int k = sqrt((double)p) + 2;

    unsigned long long b){
     if (!a) return 1;
     while (a) swap(a, b\%=a);
                                                         for (int i = k; i >= 1; i--)
     return b;
                                                           mp[bin(b, (i * 111 * k) \% (p-1), p)] = i;
                                                         bool answered = false;
10
                                                         int ans = INT32_MAX;
   unsigned long long get(unsigned long long a,
11
                                                         for (int i = 0; i <= k; i++){

    unsigned long long b){
     if (a > b)
                                                           int sum = (n * 111 * bin(b, i, p)) \% p;
                                                      11
12
       return a-b;
                                                           if (mp.count(sum) != 0){
                                                      12
13
                                                             int an = mp[sum] * 111 * k - i;
     else
14
                                                      13
                                                              if (an < p)
       return b-a;
15
   }
                                                                ans = min(an, ans);
16
17
   unsigned long long pollard(unsigned long long n){
     unsigned long long x = (rand() + 1) \% n, y = 1_{1}
                                                         // discrete log in sqrt(p) ends

→ g;
     int stage = 2, i = 0;
20
                                                         // prime roots mod n begins
     g = gcd(get(x, y), n);
21
     while (g == 1) {
                                                      2
                                                         int num = 0;
       if (i == max_step)
                                                      3
23
                                                         long long phi = n, nn = n;
         break;
                                                         for (long long x:primes){
       if (i == stage) {
                                                           if (x*x>nn)
         y = x;
26
                                                             break;
         stage <<= 1;
27
       }
                                                           if (nn \% x == 0){
28
                                                              while (nn \% x == 0)
       x = (x * (\_int128)x + 1) \% n;
                                                               nn /= x;
                                                      10
30
                                                              phi -= phi/x;
                                                      11
       g = gcd(get(x, y), n);
31
                                                              num++;
32
                                                           }
                                                      13
     return g;
33
                                                         }
   }
                                                      14
34
                                                         if (nn != 1){
                                                      15
35
                                                           phi -= phi/nn;
   // pollard ends
                                                           num++;
                                                      17
   // linear sieve begins
                                                      18
                                                         if (!((num == 1 && n % 2 != 0) || n == 4 || n ==
   const int N = 1000000;
                                                          \rightarrow 2 | | (num == 2 && n % 2 == 0 && n % 4 != 0)))
   int pr[N + 1], sz = 0;
                                                           cout << "-1\n";
   /* minimal prime, mobius function, euler function
                                                           continue;
   int lp[N + 1], mu[N + 1], phi[N + 1];
                                                         vector<long long> v;
                                                      23
                                                         long long pp = phi;
                                                     24
   lp[1] = mu[1] = phi[1] = 1;
                                                         for (long long x:primes){
   for (int i = 2; i <= N; ++i) {
                                                           if (x*x>pp)
                                                     26
     if (lp[i] == 0)
                                                             break;
11
                                                      27
```

```
if (pp \% x == 0){
                                                                   forn(col, n + m + 2)
       while (pp % x == 0)
                                                                     a[row][col] -= a[ind][col] * coef;
         pp /= x;
                                                                   a[row][j] = 0; // reduce precision error
30
                                                       36
       v.push_back(x);
31
                                                       37
                                                              p[ind] = j;
32
   }
                                                            };
33
                                                       39
   if (pp != 1){
34
                                                       40
                                                            // the Simplex itself
     v.push_back(pp);
35
                                                       41
                                                            auto iterate = [&]( int nn ) {
36
   while (true){
                                                               for (int run = 1; run; ) {
37
                                                       43
     long long a = primes[rand()%5000]%n;
                                                                 run = 0;
                                                       44
     if (gcd(a, n) != 1)
                                                                 forn(j, nn) {
                                                                   if (a[m][j] > EPS) { // strictly positive
       continue;
     bool bb = false;
                                                                     run = 1;
41
                                                       47
                                                                     double mi = INFINITY, t;
     for (long long x:v)
42
                                                       48
       if (pow(a, phi/x) == 1){
                                                                     int ind = -1;
43
         bb = true;
                                                                     forn(i, m)
          break;
                                                                       if (a[i][j] > EPS \&\& (t = a[i][n + m])
45
                                                       51
       }
                                                                        \rightarrow + 1] / a[i][j]) < mi - EPS)
46
                                                                         mi = t, ind = i;
     if (!bb){
                                                       52
       cout << a << "\n";
                                                                     if (ind == -1)
       break;
                                                                       return false;
                                                       54
49
     }
                                                                     pivot(j, ind);
50
                                                       55
   }
51
                                                                 }
52
                                                               }
   // prime roots mod n ends
                                                       58
                                                       59
                                                              return true;
   // simplex begins
                                                       60
                                                       61
   const double EPS = 1e-9;
                                                            int mi = min_element(b.begin(), b.end()) -
                                                       62
                                                             → b.begin();
   typedef vector<double> vdbl;
                                                            if (b[mi] < -EPS) {
                                                               a[m][n + m] = -1;
                                                       64
   // n variables, m inequalities
                                                              pivot(n + m, mi);
                                                       65
   // Ax <= b, c*x -> max, x >= 0
                                                               assert(iterate(n + m + 1));
   double simplex( int n, int m, const vector<vdbl>
67
                                                               if (a[m][m + n + 1] > EPS) // optimal value
    → &a0, const vdbl &b, const vdbl &c, vdbl &x)
                                                               → is positive
    ← {
                                                                return NAN;
     // Ax + Ez = b, A[m]*x -> max
10
                                                               forn(i, m)
     // x = 0, z = b, x >= 0, z >= 0
11
                                                                 if (p[i] == m + n) {
     vector < vdbl > a(m + 2, vdbl(n + m + 2));
                                                                   int j = 0;
                                                       71
     vector<int> p(m);
13
                                                                   while (find(p.begin(), p.end(), j) !=
                                                       72
     forn(i, n)
14
                                                                   \rightarrow p.end() || fabs(a[i][j]) < EPS)
       a[m + 1][i] = c[i];
15
                                                                     j++, assert(j < m + n);
     forn(i, m) {
                                                                   pivot(j, i);
                                                       74
       forn(j, n)
17
                                                       75
          a[i][j] = a0[i][j];
                                                            }
                                                       76
       a[i][n + i] = 1;
                                                            swap(a[m], a[m + 1]);
                                                       77
       a[i][m + n] = -1;
                                                            if (!iterate(n + m))
                                                      78
       a[i][m + n + 1] = b[i];
21
                                                              return INFINITY;
                                                       79
       p[i] = n + i;
22
                                                            x = vdbl(n, 0);
23
                                                            forn(i, m)
                                                       81
                                                               if (p[i] < n)
                                                       82
     // basis: enter "j", leave "ind+n"
                                                                 x[p[i]] = a[i][n + m + 1];
                                                       83
     auto pivot = [&]( int j, int ind ) {
                                                            return -a[m][n + m + 1];
       double coef = a[ind][j];
                                                          }
                                                       85
       assert(fabs(coef) > EPS);
       forn(col, n + m + 2)
29
                                                          // simplex usage:
          a[ind][col] /= coef;
30
                                                          vdbl x(n);
       forn(row, m + 2)
                                                          double result = simplex(n, m, a, b, c, x);
          if (row != ind && fabs(a[row][j]) > EPS) \{_{90}
32
                                                          if (isinf(result))
            coef = a[row][j];
```

```
puts(''Unbounded'');
                                                                                                                        x_i := a+ih, i=0 \setminus ldots 2n
        else if (isnan(result))
                                                                                                                        h = \sqrt{frac \{b-a\} \{2n\}}
            puts(''No solution'');
       else {
                                                                                                                        \int int = (f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) +
            printf("%.9f :", result);
                                                                                                                         forn(i, n)
                                                                                                                          \rightarrow f(x<sub>{2n}))) \frac h 3</sub>
                printf(" %.9f", x[i]);
                                                                                                                        Погрешность имеет порядок уменьшения как O(n^4).
            puts('"');
                                                                                                                       // algebra ends
100
       // simplex ends
101
       // sum over subsets begins
                                                                                                                       // wavelet tree begins
       // fast subset convolution O(n 2^n)
      for(int i = 0; i < (1<<N); ++i)
                                                                                                                       struct wavelet_tree{
            F[i] = A[i];
                                                                                                                            int lo, hi;
      for(int i = 0; i < N; ++i) for(int mask = 0; mask
                                                                                                                            wavelet_tree *1, *r;
         \rightarrow < (1<<N); ++mask){
                                                                                                                            vi b;
            if(mask & (1<<i))</pre>
                 F[mask] += F[mask^(1<<i)];
                                                                                                                             //nos are in range [x,y]
        }
                                                                                                                            //array indices are [from, to)
       // sum over subsets ends
                                                                                                                            wavelet_tree(int *from, int *to, int x, int y){
                                                                                                                 10
                                                                                                                                 lo = x, hi = y;
                                                                                                                 11
        // algebra begins
                                                                                                                                 if(lo == hi or from >= to) return;
                                                                                                                 12
                                                                                                                 13
       Pick
                                                                                                                                 int mid = (lo+hi)/2;
                                                                                                                 14
        B + \Gamma / 2 - 1,
                                                                                                                                 auto f = [mid](int x){
        где В - количество целочисленных точек внутри
                                                                                                                                    return x <= mid;
                 многоугольника, |a|\Gamma| количество
                                                                                                                                 };
                                                                                                                 17
                целочисленных точек на границе
                                                                                                                                 b.reserve(to-from+1);
                                                                                                                 18
                многоугольника.
                                                                                                                                 b.pb(0);
                                                                                                                                 //b[i] = no of elements from first "i"
       Newton
                                                                                                                                  \hookrightarrow elements that go to left node
        x_{i+1}=x_{i-f}(x_{i})/f'(x_{i})
                                                                                                                                 for(auto it = from; it != to; it++)
                                                                                                                 21
                                                                                                                                     b.pb(b.back() + f(*it));
       Catalan
       C_n = \sum_{k=0}^{n-1} C_{k} C_{n-1-k}
                                                                                                                                 //see how lambda function is used here
                                                                                                                                 auto pivot = stable_partition(from, to, f);
       C_i = \frac{1}{n+1}  binom 2n  n 
                                                                                                                                 1 = new wavelet_tree(from, pivot, lo, mid);
       G_N:=2^{n(n-1)/2}
                                                                                                                 27
                                                                                                                                 r = new wavelet_tree(pivot, to, mid+1, hi);
        Количество связных помеченных графов
                                                                                                                 28
        Conn_N = G_N - \frac 1 N \sum
                                                                                                                 29
                                                                                                                             //kth smallest element in [1, r]
         \downarrow \limits_{K=1}^{N-1} K \binom N K Conn_K
                                                                                                                             int kth(int 1, int r, int k){
                                                                                                                 31
         \hookrightarrow G_{N-K}
                                                                                                                                 if(l > r) return 0;
                                                                                                                 32
                                                                                                                                 if(lo == hi) return lo;
        Количество помеченных графов с К компонентами
                                                                                                                                 //how many nos are there in left node from
         → СВЯЗНОСТИ
                                                                                                                                  \hookrightarrow [l, r]
        D[N][K] = \sqrt{\sum_{s=1}^{N} N \cdot \sum_{s=1}^{N} N \cdot 
                                                                                                                                 int inleft = b[r] - b[1-1];
         \hookrightarrow Conn_S D[N-S][K-1]
                                                                                                                                 int lb = b[1-1]; //amt of nos from first
                                                                                                                                  \rightarrow (l-1) nos that go in left
       Miller-Rabbin
21
                                                                                                                                 int rb = b[r]; //amt of nos from first (r)
                                                                                                                 37
        a=a^t
22
                                                                                                                                  \hookrightarrow nos that go in left
       FOR i = 1...s
                                                                                                                                 //so [lb+1, rb] represents nos from [l, r]
            if a^2=1 && |a|!=1
                                                                                                                                  → that go to left
                NOT PRIME
                                                                                                                                 if(k <= inleft) return this->l->kth(lb+1, rb
                                                                                                                 39
            a=a^2
                                                                                                                                  \rightarrow , k);
       return a==1 ? PRIME : NOT PRIME
                                                                                                                                 //(l-1-lb) is amt of nos from first (l-1) nos
29
                                                                                                                                  → that go to right
        Интегрирование по формуле Симпсона
                                                                                                                                 //(r-rb) is amt of nos from first (r) nos
                                                                                                                 42
        \int \int a^b f(x) dx - ?
                                                                                                                                  → that go to right
31
```

```
//so [l-lb, r-rb] represents nos from [l, r]_{52}
                                                                for (int i = 0; i < m + m; ++i) t_[i] = 0;
        → that go to right
                                                                for (int i = 0; i < m; ++i)
       return this->r->kth(l-lb, r-rb, k-inleft);
                                                                  if (p[i])
44
     }
                                                                    for (int j = 0; j < m; ++j)
45
   };
                                                                      t_{i} = (t_{i} + j) + p[i] * q[j]) %

→ MOD;

47
   // wavelet tree ends
                                                                for (int i = m + m - 1; i >= m; --i)
                                                        57
                                                                  if (t_[i])
                                                        58
   // berlecamp-massey begins
                                                                    for (int j = m - 1; ~j; --j)
                                                                       t_{i} = [i - j - 1] = (t_{i} - j - 1] + t_{i}
   const int SZ = MAXN;
                                                                       → * h[j]) % MOD;
                                                                for (int i = 0; i < m; ++i) p[i] = t_[i];
                                                        61
   11 qp(ll a, ll b) {
                                                        62
     11 x = 1;
                                                        63
     a \%= MOD;
                                                              inline 11 calc(11 K) {
                                                        64
     while (b) {
                                                                for (int i = m; ~i; --i)
                                                        65
        if (b & 1) x = x * a \% MOD;
                                                                  s[i] = t[i] = 0;
       a = a * a % MOD;
                                                                s[0] = 1;
                                                        67
        b >>= 1;
11
                                                                if (m != 1) t[1] = 1; else t[0] = h[0];
                                                        68
     }
12
                                                                while (K) {
                                                        69
     return x;
13
                                                                  if (K & 1) mull(s, t);
                                                                  mull(t, t);
                                                        71
15
                                                                  K >>= 1:
                                                        72
   namespace linear_seq {
                                                                }
                                                        73
     inline vector<int> BM(vector<int> x) {
                                                                11 su = 0;
        vector<int> ls, cur;
                                                                for (int i = 0; i < m; ++i) su = (su + s[i] *
                                                        75
        int lf, ld;
19
                                                                \rightarrow a[i]) % MOD;
        for (int i = 0; i < int(x.size()); ++i) {</pre>
20
                                                                return (su % MOD + MOD) % MOD;
          11 t = 0;
21
          for (int j = 0; j < int(cur.size()); ++j)</pre>
            t = (t + x[i - j - 1] * (11) cur[j]) %
                                                              inline int work(vector<int> x, ll n) {
            \rightarrow MOD;
                                                                if (n < int(x.size())) return x[n];</pre>
          if ((t - x[i]) \% MOD == 0) continue;
                                                                vector < int > v = BM(x);
                                                        81
          if (!cur.size()) {
                                                                m = v.size();
                                                        82
            cur.resize(i + 1);
26
                                                                if (!m) return 0;
                                                        83
            lf = i;
27
                                                                for (int i = 0; i < m; ++i) h[i] = v[i], a[i]
            ld = (t - x[i]) \% MOD;
                                                                \rightarrow = x[i];
            continue;
                                                                return calc(n);
                                                        85
          11 k = -(x[i] - t) * qp(1d, MOD - 2) % MOD;
          vector<int> c(i - lf - 1);
                                                              //b = a0/(1-p)
          c.pb(k);
                                                              inline void calc_generating_function(const
          for (int j = 0; j < int(ls.size()); ++j)</pre>
34

    vector<int>& b, vector<int>& p,

            c.pb(-ls[j] * k \% MOD);
35

    vector<int>& a0) {

          if (c.size() < cur.size())</pre>
                                                                p = BM(b);
                                                        90

    c.resize(cur.size());

                                                                a0.resize(p.size());
          for (int j = 0; j < int(cur.size()); ++j)</pre>
                                                                for (int i = 0; i < a0.size(); ++i) {</pre>
            c[j] = (c[j] + cur[j]) \% MOD;
                                                                  a0[i] = b[i];
          if (i - lf + (int) ls.size() >= (int)
                                                                  for (int j = 0; j < i; ++j) {
                                                        94

    cur.size())

                                                                    a0[i] += MOD - (p[j] * 111 * b[i - j -
            ls = cur, lf = i, ld = (t - x[i]) \% MOD;
40
                                                                     \hookrightarrow 1]) % MOD;
41
          cur = c;
                                                                    if (a0[i] > MOD) {
                                                        96
        }
                                                        97
                                                                      a0[i] -= MOD;
        for (int i = 0; i < int(cur.size()); ++i)</pre>
                                                                    }
                                                        98
          cur[i] = (cur[i] % MOD + MOD) % MOD;
                                                                  }
       return cur;
                                                                }
     }
                                                       101
47
                                                       102
     int m:
48
     11 a[SZ], h[SZ], t_[SZ], s[SZ], t[SZ];
49
                                                           // berlecamp-massey ends
                                                       104
     inline void mull(ll* p, ll* q) {
                                                        1 // AND-FFT begins
51
```

```
void fast_fourier(vector<int>& a) { // AND-FFT. 31
                                                            int get ( int x ) {
     for (int k = 1; k < SZ(a); k *= 2)
                                                              return jump[x] == x ? x : (jump[x] = get
     for (int start = 0; start < (1 << K); start +=</pre>
                                                               \rightarrow (jump[x]));
      \rightarrow 2 * k) {
                                                            }
       for (int off = 0; off < k; ++off) {</pre>
                                                          };
       int a_val = a[start + off];
                                                      35
       int b_val = a[start + k + off];
                                                      36
                                                          struct Treap {
       a[start + off] = b_val;
                                                            int value, add;
10
                                                      38
       a[start + k + off] = add(a_val, b_val);
                                                            int source, target, height;
11
                                                      39
                                                            int min_value, min_path;
13
   }
                                                            Treap *left, *right;
14
                                                       42
15
                                                            Treap ( int _source, int _target, int _value )
   void inverse_fast_fourier(vector<int>& a) {
     for (int k = 1; k < SZ(a); k *= 2)
                                                            \rightarrow : value (_value), add (0), source
     for (int start = 0; start < (1 << K); start +=</pre>
                                                             \rightarrow 2 * k) {
                                                            height = rand ();
       for (int off = 0; off < k; ++off) {</pre>
                                                            min_value = value, min_path = 0;
       int a_val = a[start + off];
                                                            left = right = 0;
       int b_val = a[start + k + off];
                                                       48
21
22
       a[start + off] = sub(b_val, a_val);
                                                            Treap& operator += ( int sub ) {
       a[start + k + off] = a\_val;
                                                            add += sub;
                                                            return *this;
                                                      52
25
                                                      53
   }
27
                                                            void push () {
                                                      55
   // AND-FFT ends
                                                            if (!add)
                                                      56
                                                      57
                                                              return;
   // 2-chinese begins
                                                            if (left) {
                                                              left->add += add;
                                                      59
   template <typename Info>
                                                      60
   class DSU {
                                                            if (right) {
                                                       61
     public:
                                                              right->add += add;
     DSU ( int n ) : jump (new int[n]), rank (new

    int [n]), info (new Info [n]) {

                                                            value += add;
                                                      64
       for (int i = 0; i < n; i++) {
                                                            min_value += add;
       jump[i] = i;
                                                            add = 0;
       rank[i] = 0;
10
11
                                                            void recalc () {
     Info& operator [] ( int x ) {
12
                                                            min_value = value;
       return info[get (x)];
                                                            min_path = 0;
                                                      71
14
                                                            if (left && left->min_value + left->add <
     void merge (int a, int b, const Info &comment

    min_value) {
                                                              min_value = left->min_value + left->add;
       a = get (a);
                                                              min_path = -1;
                                                      74
       b = get (b);
17
                                                            }
                                                       75
       if (rank[a] <= rank[b]) {</pre>
18
                                                            if (right && right->min_value + right->add <</pre>
       jump[a] = b;
                                                             \rightarrow min_value) {
       rank[b] += rank[a] == rank[b];
                                                              min_value = right->min_value + right->add;
                                                      77
       info[b] = comment;
                                                              min_path = +1;
                                                       78
       } else {
                                                            }
                                                       79
       jump[b] = a;
                                                            }
       info[a] = comment;
                                                      81
25
     }
26
                                                          Treap* treap_merge ( Treap *x, Treap *y ) {
     private:
27
                                                            if (!x)
                                                      84
     int *jump, *rank;
                                                            return y;
                                                      85
     Info *info;
```

```
if (!y)
                                                              for (int i = 0; i < m; i++) {
                                                        143
      return x;
                                                                int a, b, c;
                                                                assert (scanf (''\d\d\d'', &a, &b, &c) == 3);
      if (x->height < y->height) {
                                                        145
                                                                g[b] = treap_merge (g[b], new Treap (b, a,
      x->push();
      x->right = treap_merge (x->right, y);
                                                              }
      x->recalc ();
                                                        147
                                                              DSU <pair <int, Treap*> > dsu (n + 1);
      return x;
                                                        148
      } else {
                                                              for (int i = 0; i < n; i++) {
                                                        149
                                                                dsu[i] = make_pair (i, g[i]);
      y->push ();
      y->left = treap_merge (x, y->left);
                                                              }
                                                        151
      y->recalc ();
                                                        152
                                                              int ans = 0, k = n;
      return y;
                                                        153
                                                              int jump[2 * n], jump_from[2 * n], parent[2 *
      }
                                                              \rightarrow n], c[n];
99
                                                              vector <int> children[2 * n];
100
    Treap* treap_getmin ( Treap *x, int &source, int 6
                                                              memset (c, 0, sizeof (c[0]) * n);
101
    memset (parent, -1, sizeof (parent[0]) * 2 *
      assert (x);
                                                               \rightarrow n);
102
      x->push();
                                                              vector <int> finish;
103
                                                        158
      if (x->min_path == 0) {
                                                              for (int i = 0; i < n; i++) {
                                                        159
                                                                if (dsu[i].first == 0)
      // memory leak, sorry
                                                        160
      source = x->source;
                                                                continue;
                                                        161
106
                                                                int u = i;
      target = x->target;
107
                                                        162
      value = x->value + x->add;
                                                                c[u] = 1;
                                                        163
      return treap_merge (x->left, x->right);
                                                                while (true) {
                                                        164
      } else if (x-\min_{path} == -1) {
                                                                int source, target, value;
110
                                                        165
      x->left = treap_getmin (x->left, source,
                                                                dsu[u].second = treap_getmin (dsu[u].second,
                                                        166
111

    target, value);

→ source, target, value);
      value += x->add;
                                                                if (dsu[target] == dsu[u])
                                                        167
112
      x->recalc ();
                                                                  continue;
113
                                                        168
      return x;
                                                                treap_add (dsu[u].second, -value);
                                                        169
114
      } else if (x->min_path == +1) {
                                                                ans += value;
      x->right = treap_getmin (x->right, source,
                                                                jump_from[dsu[u].first] = source;
116
                                                        171

    target, value);

                                                                jump[dsu[u].first] = target;
                                                        172
      value += x->add;
                                                                if (dsu[target].first == 0)
117
                                                        173
      x->recalc ();
                                                        174
                                                                  break;
      return x;
                                                                if (!c[target]) {
                                                        175
119
      } else
                                                                  c[target] = 1;
120
                                                        176
      assert (0);
                                                                  u = target;
121
                                                        177
   }
                                                                  continue;
122
                                                                }
123
                                                        179
   Treap* treap_add ( Treap *x, int add ) {
                                                                assert (k < 2 * n);
124
                                                        180
      if (!x)
                                                                int node = k++, t = target;
125
                                                        181
                                                                parent[dsu[u].first] = node;
      return 0;
126
      return \&((*x) += add);
                                                                children[node].push_back (dsu[u].first);
127
                                                        183
                                                                dsu[u].first = node;
128
                                                        184
                                                                Treap *v = dsu[u].second;
129
                                                        185
                                                                while (dsu[t].first != node) {
130
                                                        186
    int main () {
                                                                  int next = jump[dsu[t].first];
131
                                                        187
                                                                  parent[dsu[t].first] = node;
      int n, m;
                                                        188
132
      while (scanf (''%d%d'', &n, &m) == 2) {
                                                                  children[node].push_back (dsu[t].first);
133
      Treap * g[n + 1];
                                                                  v = treap_merge (v, dsu[t].second);
                                                        190
134
      for (int i = 0; i <= n; i++)
                                                                  dsu.merge (u, t, make_pair (node, v));
135
                                                        191
        g[i] = 0;
                                                                  t = next:
136
                                                        192
                                                                }
      for (int i = 1; i <= n; i++) {
137
                                                        193
        int a;
                                                                }
                                                        194
138
        assert (scanf (''%d'', &a) == 1);
                                                                u = i;
139
                                                        195
                                                                while (dsu[u].first) {
        g[i] = treap_merge (g[i], new Treap (i, 0, 196
140
           a));
                                                                int next = jump[dsu[u].first];
      }
                                                                finish.push_back (dsu[u].first);
                                                        198
141
                                                                dsu.merge (u, 0, make_pair (0, (Treap *)0));
      n++:
142
                                                        199
```

```
void q_push(int x) {
        u = next:
200
        }
                                                            if (x <= n) return q.push_back(x);</pre>
      }
                                                            for (int i = 0; i < flower[x].size(); i++)</pre>
202
                                                       23
      bool ok[k];
                                                                q_push(flower[x][i]);
      int res[n];
      memset (ok, 0, sizeof (ok[0]) * k);
                                                          void set_st(int x, int b) {
                                                       25
      memset (res, -1, sizeof (res[0]) * n);
                                                            st[x] = b;
206
                                                       26
      function <void (int, int)> add_edge = [&ok,
                                                            if (x <= n) return;</pre>
                                                       27
207
      for (int i = 0; i < flower[x].size(); ++i)</pre>
        assert (0 \leq a && a \leq n);

    set_st(flower[x][i], b);

        assert (0 <= b \&\& b < n);
                                                          }
                                                       29
        assert (res[a] == -1);
                                                          int get_pr(int b, int xr) {
                                                       30
        res[a] = b;
                                                            int pr = find(flower[b].begin(),
211
        while (a != -1 \&\& !ok[a]) {

→ flower[b].end(), xr)-flower[b].begin();
212
        ok[a] = true;
                                                            if (pr % 2 == 1) {
                                                       32
213
                                                              reverse(flower[b].begin() +1,
        a = parent[a];
214
        }

    flower[b].end());

                                                              return (int) flower[b].size()-pr;
216
      function <void (int)> reach = [&ok, &reach,
                                                            }
217
      else return pr;
       → int u ) {
        if (!ok[u])
                                                          void set_match(int u, int v) {
                                                       38
218
                                                            match[u] = g[u][v].v;
        add_edge (jump_from[u], jump[u]);
219
                                                       39
                                                            if (u <= n) return;</pre>
        for (auto x : children[u])
220
        reach (x);
                                                            Edge e = g[u][v];
221
                                                       41
                                                            int xr = flower_from[u][e.u], pr = get_pr(u,
      };
222
                                                       42
      for (auto x : finish)
                                                             \hookrightarrow xr);
                                                            for (int i = 0; i < pr; ++i)</pre>
         reach (x);
      printf (''%d\n'', ans);

    set_match(flower[u][i], flower[u][i^1]);

225
      for (int i = 1; i < n; i++)
                                                            set_match(xr, v);
226
                                                       44
        printf (''%d%c'', res[i] ? res[i] : -1, ''\n ''[i
                                                            rotate(flower[u].begin(), flower[u].begin()
227
         \rightarrow < n - 1]);
                                                             → +pr, flower[u].end());
      }
                                                          }
228
                                                          void augment(int u, int v) {
      return 0;
                                                       47
229
                                                            int xnv = st[match[u]];
230
                                                       48
                                                            set_match(u, v);
    // 2-chinese ends
                                                            if (!xnv) return;
                                                       50
232
                                                            set_match(xnv, st[pa[xnv]]);
                                                      51
    // general max weighet match begins
                                                            augment(st[pa[xnv]], xnv);
                                                       52
    #define DIST(e)
                                                          int get_lca(int u, int v) {
                                                       54
    \rightarrow (lab[e.u]+lab[e.v]-g[e.u][e.v].w*2)
                                                            static int t = 0;
                                                       55
   using namespace std;
                                                            for (++t; u || v; swap(u, v)) {
   typedef long long 11;
                                                              if (u == 0) continue;
                                                       57
   const int N = 1023, INF = 1e9;
                                                              if (vis[u] == t) return u;
                                                       58
   struct Edge {
                                                              vis[u] = t;
                                                       59
     int u, v, w;
                                                              u = st[match[u]];
   } g[N][N];
                                                              if (u) u = st[pa[u]];
    int n, m, n_x, lab[N], match[N], slack[N], st[N],
                                                            }

→ pa[N], flower_from[N][N], S[N], vis[N];

                                                            return 0;
                                                       63
   vector < int> flower[N];
11
                                                          }
                                                       64
   deque < int> q;
                                                          void add_blossom(int u, int lca, int v) {
                                                       65
   void update_slack(int u, int x) {
                                                            int b = n+1;
      if (!slack[x] || DIST(g[u][x]) <</pre>
14
                                                            while (b \leq n_x && st[b]) ++b;
                                                       67
      \rightarrow DIST(g[slack[x]][x])) slack[x] = u;
                                                            if (b>n_x) ++n_x;
   }
15
                                                            lab[b] = 0, S[b] = 0;
   void set_slack(int x) {
16
                                                            match[b] = match[lca];
      slack[x] = 0;
17
                                                            flower[b].clear();
      for (int u = 1; u <= n; ++u)
18
                                                            flower[b].push_back(lca);
        if (g[u][x].w>0 \&\& st[u] != x \&\& S[st[u]] ==_{73}
                                                            for (int x = u, y; x != lca; x = st[pa[y]])

→ 0) update_slack(u, x);
   }
20
```

```
flower[b].push_back(x), flower[b].push_back(y)
                                                               q.clear();
         \Rightarrow = st[match[x]]), q_push(y);
                                                               for (int x = 1; x \le n_x; ++x)
                                                                 if (st[x] == x && !match[x]) pa[x] = 0, S[x]
      reverse(flower[b].begin() +1, flower[b].end())24
      for (int x = v, y; x != lca; x = st[pa[y]])
                                                                  \rightarrow = 0, q_push(x);
        flower[b].push_back(x), flower[b].push_back(y)
                                                               if (q.empty()) return 0;
         \rightarrow = st[match[x]]), q_push(y);
                                                               for (;;) {
      set_st(b, b);
                                                                 while (q.size()) {
                                                        131
      for (int x = 1; x \le n_x; ++x) g[b][x].w =
                                                                   int u = q.front();
                                                        132
      \rightarrow g[x][b].w = 0;
                                                                   q.pop_front();
      for (int x = 1; x \le n; ++x) flower_from[b][x]<sub>34</sub>
                                                                   if (S[st[u]] == 1) continue;
                                                                   for (int v = 1; v \le n; ++v)
      \rightarrow = 0;
                                                                     if (g[u][v].w>0 \&\& st[u] != st[v]) {
      for (int i = 0; i < flower[b].size(); ++i) { 136</pre>
        int xs = flower[b][i];
                                                                        if (DIST(g[u][v]) == 0) {
        for (int x = 1; x \le n_x; ++x)
                                                                          if (on_found_Edge(g[u][v])) return 1;
83
                                                        138
          if (g[b][x].w == 0 \mid \mid DIST(g[xs][x]) <
                                                        139
           \rightarrow DIST(g[b][x]))
                                                                        else update_slack(u, st[v]);
            g[b][x] = g[xs][x], g[x][b] = g[x][xs];_{141}
        for (int x = 1; x \le n; ++x)
           if (flower_from[xs][x]) flower_from[b][x] 145
                                                                 int d = INF;
                                                                 for (int b = n+1; b \le n_x; ++b)
              xs;
                                                                   if (st[b] == b \&\& S[b] == 1) d = min(d,
      }
      set_slack(b);
                                                                       lab[b]/2);
89
   }
                                                                 for (int x = 1; x \le n_x; ++x)
90
                                                        146
    void expand_blossom(int b) // S[b] == 1 {
                                                                   if (st[x] == x \&\& slack[x]) {
91
      for (int i = 0; i < flower[b].size(); ++i)</pre>
                                                                     if (S[x] == -1) d = min(d,
        set_st(flower[b][i], flower[b][i]);
                                                                      → DIST(g[slack[x]][x]));
      int xr = flower_from[b][g[b][pa[b]].u], pr = {}_{149}
                                                                     else if (S[x] == 0) d = min(d,

    get_pr(b, xr);

                                                                      \rightarrow DIST(g[slack[x]][x])/2);
      for (int i = 0; i < pr; i += 2) {
        int xs = flower[b][i], xns = flower[b][i+1];51
                                                                 for (int u = 1; u <= n; ++u) {
        pa[xs] = g[xns][xs].u;
                                                                   if (S[st[u]] == 0) {
        S[xs] = 1, S[xns] = 0;
                                                                     if (lab[u] <= d) return 0;</pre>
        slack[xs] = 0, set_slack(xns);
                                                                     lab[u] - = d;
                                                        154
                                                                   }
        q_push(xns);
                                                        155
100
                                                                   else if (S[st[u]] == 1) lab[u] += d;
                                                        156
101
                                                                 }
      S[xr] = 1, pa[xr] = pa[b];
      for (int i = pr+1; i < flower[b].size(); ++i) 15{
                                                                 for (int b = n+1; b \le n_x; ++b)
103
        int xs = flower[b][i];
                                                                   if (st[b] == b) {
104
        S[xs] = -1, set_slack(xs);
                                                                     if (S[st[b]] == 0) lab[b] += d*2;
105
                                                         160
      }
                                                                     else if (S[st[b]] == 1) lab[b] - = d*2;
                                                                   }
      st[b] = 0;
                                                        162
107
                                                                 q.clear();
    }
                                                        163
108
    bool on_found_Edge(const Edge &e) {
                                                                 for (int x = 1; x \le n_x; ++x)
                                                        164
      int u = st[e.u], v = st[e.v];
                                                                   if (st[x] == x \&\& slack[x] \&\& st[slack[x]]
      if (S[v] == -1) {
                                                                       != x \&\& DIST(g[slack[x]][x]) == 0)
111
        pa[v] = e.u, S[v] = 1;
                                                                     if (on_found_Edge(g[slack[x]][x])) return
112
                                                        166
        int nu = st[match[v]];

→ 1;
113
        slack[v] = slack[nu] = 0;
                                                                 for (int b = n+1; b \le n_x; ++b)
114
                                                                   if (st[b] == b \&\& S[b] == 1 \&\& lab[b] == 0)
        S[nu] = 0, q_push(nu);
115
                                                        168
                                                                       expand_blossom(b);
116
      else if (S[v] == 0) {
                                                               }
        int lca = get_lca(u, v);
                                                               return 0;
                                                        170
118
        if (!lca) return augment(u, v), augment(v,
                                                        171
119
                                                            pair < 11, int> weight_blossom() {
         \rightarrow u). 1:
                                                        172
        else add_blossom(u, lca, v);
                                                               fill(match, match+n+1, 0);
120
      }
                                                        174
                                                               n x = n:
121
                                                               int n_matches = 0;
      return 0;
122
                                                        175
    }
                                                        176
                                                               11 tot_weight = 0;
123
                                                               for (int u = 0; u \le n; ++u) st[u] = u,
    bool matching() {
124
      fill(S, S+n_x+1, -1), fill(slack, slack+n_x+1,

    flower[u].clear();

125
      \rightarrow 0);
                                                               int w_max = 0;
```

```
for (int u = 1; u \le n; ++u)
                                                              double x, y;
179
        for (int v = 1; v <= n; ++v) {
                                                              bool operator< (const point & p) const {</pre>
180
                                                                 return x < p.x - EPS \mid \mid abs (x - p.x) < EPS
          flower_from[u][v] = (u == v?u:0);
181
                                                        32
          w_max = max(w_max, g[u][v].w);
                                                                 \rightarrow && y < p.y - EPS;
182
                                                              }
      for (int u = 1; u <= n; ++u) lab[u] = w_max;
                                                            };
                                                        34
184
      while (matching()) ++n_matches;
185
                                                        35
      for (int u = 1; u \le n; ++u)
                                                            map<point,int> ids;
186
        if (match[u] && match[u] < u)</pre>
                                                            vector<point> p;
          tot_weight += g[u][match[u]].w;
                                                            vector < vector<int> > g;
                                                        38
188
      return make_pair(tot_weight, n_matches);
189
                                                        39
    }
                                                            int get_point_id (point pt) {
190
    int main() {
                                                              if (!ids.count(pt)) {
                                                        41
191
                                                                 ids[pt] = (int)p.size();
      cin>>n>>m;
                                                        42
192
      for (int u = 1; u \le n; ++u)
                                                                 p.push_back (pt);
193
                                                        43
        for (int v = 1; v \le n; ++v)
                                                                 g.resize (g.size() + 1);
194
                                                        44
          g[u][v] = Edge \{u, v, 0\};
      for (int i = 0, u, v, w; i < m; ++i) {
                                                              return ids[p];
                                                        46
196
        cin>>u>>v>>w;
                                                            }
                                                        47
        g[u][v].w = g[v][u].w = w;
                                                            void intersect (pair<point, point> a,
                                                             → pair<point,point> b, vector<point> & res) {
      cout << weight_blossom().first << '\n';</pre>
200
      for (int u = 1; u <= n; ++u) cout << match[u] 50</pre>
                                                               ... стандартная процедура, пересекает два
201
      отрезка а и b и закидывает результат в res
202
                                                                  если отрезки перекрываются, то закидывает
    // general max weighted match ends
204
                                                                   те концы, которые попали внутрь первого
    // planar begins
                                                                   отрезка ...
                                                            }
    // обход граней
                                                        53
    int n; // число вершин
                                                            int main() {
                                                        54
   vector < vector<int> > g; // граф
                                                              // входные данные
                                                        55
   vector < vector<char> > used (n);
                                                              int m;
    for (int i=0; i<n; ++i)</pre>
                                                              vector < pair<point,point> > a (m);
                                                        57
      used[i].resize (g[i].size());
                                                               ... чтение ...
    for (int i=0; i<n; ++i)</pre>
      for (size_t j=0; j<g[i].size(); ++j)</pre>
10
                                                               // построение графа
                                                        60
        if (!used[i][j]) {
11
                                                              for (int i=0; i<m; ++i) {
                                                        61
          used[i][j] = true;
                                                                 vector<point> cur;
12
                                                        62
          int v = g[i][j], pv = i;
                                                        63
                                                                 for (int j=0; j<m; ++j)
          vector<int> facet;
                                                                   intersect (a[i], a[j], cur);
                                                        64
          for (;;) {
                                                                 sort (cur.begin(), cur.end());
                                                        65
            facet.push_back (v);
16
                                                                 for (size_t j=0; j+1<cur.size(); ++j) {</pre>
            vector<int>::iterator it = find
                                                                   int x = get_id (cur[j]), y = get_id
17

    (g[v].begin(), g[v].end(), pv);
                                                                   \hookrightarrow (cur[j+1]);
             //vector<int>::iterator it =
                                                                   if (x != y) {
             \rightarrow lower_bound(g[v].begin(), g[v].end()<sub>69</sub>
                                                                     g[x].push_back (y);
             \rightarrow pv, cmp_ang(v));
                                                                     g[y].push_back (x);
             // cmp_ang(v) -- true, если меньше
                                                        71
             \hookrightarrow полярный угол относ v
                                                                 }
                                                        72
             if (++it == g[v].end()) it =
20
                                                              }
             \rightarrow g[v].begin();
                                                              int n = (int) g.size();
                                                        74
             if (used[v][it-g[v].begin()]) break;
                                                               // сортировка по углу и удаление кратных рёбер
                                                        75
            used[v][it-g[v].begin()] = true;
                                                              for (int i=0; i<n; ++i) {</pre>
                                                        76
            pv = v, v = *it;
                                                                 sort (g[i].begin(), g[i].end(), cmp_ang (i));
          }
                                                                 g[i].erase (unique (g[i].begin(),
                                                        78
               вывод facet - текущей грани ...

    g[i].end()), g[i].end());
25
                                                              }
26
                                                        79
                                                            }
27
    // построение планарного графа
                                                            // planar ends
    struct point {
```

```
// fast hashtable begins
   #include <ext/pb_ds/assoc_container.hpp>
   using namespace __gnu_pbds;
   gp_hash_table<int, int> table;
   const int RANDOM = chrono ::
   \hookrightarrow high_resolution_clock ::
   → now().time_since_epoch().count();
   struct chash {
     int operator()(int x) { return hash<int>{}(x ^

    RANDOM); }

   gp_hash_table<key, int, chash> table;
11
   // fast hashtable ends
     xmodmap -e 'keycode 94='
   setxkbmap us
```

$\mathbf{D}M$

Кол-во корневых деревьев:

$$t(G) = \frac{1}{n}\lambda_2 \dots \lambda_n \ (\lambda_1 = 0)$$

$$to n$$
 Кол-во эйлеровых циклов: $e(D) = t^-(D, x) \cdot \prod_{y \in D} (outdeg(y) - 1)!$

Пентагональная теорема Эйлера:
$$\prod_{n=1}^{\infty}(1-x^n)=1+\sum_{k=1}^{\infty}(-1)^k\left(x^{k(3k-1)/2}+x^{k(3k+1)/2}\right)$$

Наличие совершенного паросочетания:

Т – матрица с нулями на диагонали. Если есть ребро (i,j), to $a_{i,j} := x_{i,j}, a_{j,i} = -x_{i,j}$ $\det(T) = 0 \Leftrightarrow$ нет совершенного паросочетания.

Fast subset convolution
$$(f * g)(S) := \sum_{T \subseteq S} f(T)g(S \setminus T)$$

$$\hat{f}(X) := \sum_{S \subseteq X} f(S)$$

$$f(S) = \sum_{X \subseteq S} (-1)^{|S \setminus X|} \hat{f}(X)$$

$$\hat{f}_0(X) := f(X)$$

$$\hat{f}_j(X) = \begin{cases} \hat{f}_{j-1}(X) & \text{if } j \notin X \\ \hat{f}_{j-1}(X \setminus j) + \hat{f}_{j-1}(X) & \text{if } j \in X \end{cases}$$

$$\hat{f}_n(X) == \hat{f}(X)$$

$$f_0(S) := \hat{f}(S)$$

$$f_j(S) = \begin{cases} f_{j-1}(S) & \text{if } j \notin S \\ f_{j-1}(S) & \text{if } j \notin S \end{cases}$$

$$f_0(S) := \hat{f}(S)$$

$$f_j(S) = \begin{cases} f_{j-1}(S) & \text{if } j \notin S \\ -f_{j-1}(S \setminus j) + f_{j-1}(S) & \text{if } j \in S \end{cases}$$

$$f_n(S) := f(S)$$

$$\begin{split} \hat{f}(k,X) &:= \sum_{S \subseteq, |S| = k} f(S) \\ f(S) &== \hat{f}(|S|,S) \end{split}$$

$$\begin{split} &(\hat{f}\otimes\hat{g})(k,X):=\sum_{j=0}^k\hat{f}(j,X)\hat{g}(k-j,X)\\ &(f*g)(S)=\sum_{X\subseteq S}(-1)^{|S\backslash X|}(\hat{f}\otimes\hat{g})(|S|,X)\\ &\text{calculate using }f_j! \end{split}$$