Team Reference of St. Petersburg Campus of Higher School of Economics.
SPb HSE: Abstract Economists
(Ermilov, Fedorov, Labutin)

```cpp
// pollard begins

const int max_step = 4e5;

unsigned long long gcd(unsigned long long a, unsigned long long b){
  if (!a) return 1;
  while (a) swap(a, b%=a);
  return b;
}

unsigned long long get(unsigned long long a, unsigned long long b){
  if (a > b)
    return a-b;
  else
    return b-a;
}

unsigned long long pollard(unsigned long long n){
  unsigned long long x = (rand() + 1) % n, y = 1, g;
  int stage = 2, i = 0;
  g = gcd(get(x, y), n);
  while (g == 1) {
    if (i == max_step)
      break;
    if (i == stage) {
      y = x;
      stage <<= 1;
    }
    x = (x * (__int128)x + 1) % n;
    i++;
    g = gcd(get(x, y), n);
  }
  return g;
}

// pollard ends
```

pragma

```
#pragma GCC optimize(”O3, no-stack-protector”)
#pragma GCC target(”sse, sse2, sse4, ssse3, popcnt, abm,
```

Алгебра Pick

В + Г / 2 − 1 = AREA,
где В — количество целочисленных точек внутри многоугольника, а Г — количество целочисленных точек на границе многоугольника.

Newton

$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$

Catalan

$C_n = \sum_{k=0}^{n-1} C_k C_{n-1-k}$
$C_i = \frac{1}{n+1}\binom{2n}{n}$

Кол-во графов

$G_N := 2^{n(n-1)/2}$
Количество связных помеченных графов
$Conn_N = G_N - \frac{1}{N}\sum_{K=1}^{N-1} K\binom{N}{K} Conn_K G_{N-K}$

Количество помеченных графов с K компонентами связности
$D[N][K] = \sum_{S=1}^{N} \binom{N-1}{S-1} Conn_S D[N-S][K-1]$

Miller-Rabbin

```
a=a^t
FOR i = 1...s
    if a^2=1 && |a|!=1
        NOT PRIME
    a=a^2
return a==1 ? PRIME : NOT PRIME
```

Интегрирование по формуле Симпсона

$\int_a^b f(x)dx$?
$x_i := a + ih, i = 0 \dots 2n$
$h = \frac{b-a}{2n}$

$\int = (f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \dots + 4f(x_{2n-1}) + f(x_{2n})))\frac{h}{3}$
$O(n^4)$.

Простые числа

1009,1013;10007,10009;100003,100019
1000003,1000033;10000019,10000079
100000007,100000037
10000000019,10000000033
1000000000039,1000000000061
100000000000031,100000000000067
10000000000000061,10000000000000069
100000000000000003,100000000000000009

Числа для Фурье

- prime: $7340033 = 7 \cdot 2^{20} + 1; w : 5(w^{2^{20}} = 1)$

- prime: $13631489 = 13 \cdot 2^{20} + 1; w : 3(w^{2^{20}} = 1)$
- prime: $23068673 = 11 \cdot 2^{21} + 1; w : 38(w^{2^{21}} = 1)$
- prime: $69206017 = 33 \cdot 2^{21} + 1; w : 45(w^{2^{21}} = 1)$
- prime: $81788929 = 39 \cdot 2^{21} + 1; w : 94(w^{2^{21}} = 1)$
- prime: $104857601 = 25 \cdot 2^{22} + 1; w : 21(w^{2^{22}} = 1)$
- prime: $113246209 = 27 \cdot 2^{22} + 1; w : 66(w^{2^{22}} = 1)$
- prime: $138412033 = 33 \cdot 2^{22} + 1; w : 30(w^{2^{22}} = 1)$
- prime: $167772161 = 5 \cdot 2^{25} + 1; w : 17(w^{2^{25}} = 1)$
- prime: $469762049 = 7 \cdot 2^{26} + 1; w : 30(w^{2^{26}} = 1)$
- prime: $998244353 = 7 \cdot 17 \cdot 2^{23} + 1; w : 3^{7*17}$.

Erdős–Gallai theorem

A sequence of non-negative integers $d_1 \geq \cdots \geq d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only if $d_1 + \cdots + d_n$ $d_1 + \cdots + d_n$ is even and

$\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k)$ holds for every k in $1 \leq k \leq n$.

```cpp
// sk fast allocation begins
const int MAX_MEM = 5e8;
int mpos = 0;
char mem[MAX_MEM];
inline void * operator new ( size_t n ) {
  assert((mpos += n) <= MAX_MEM);
  return (void *)(mem + mpos - n);
}
inline void operator delete ( void * ) noexcept { } // must have!

// sk fast allocation ends



// sk fast read-write begins

inline int readChar();
template <class T = int> inline T readInt();
template <class T> inline void writeInt( T x, char end = 0 );
inline void writeChar( int x );
inline void writeWord( const char *s );

/** Read */

static const int buf_size = 2048;

inline int getChar() {
  static char buf[buf_size];
  static int len = 0, pos = 0;
  if (pos == len)
    pos = 0, len = fread(buf, 1, buf_size, stdin);
  if (pos == len)
    return -1;
  return buf[pos++];
}

inline int readWord(char * buffer) {
  int c = getChar();
  while (c <= 32) {
    c = getChar();
  }

  int len = 0;
  while (c > 32) {
    *buffer = (char) c;
    c = getChar();
    buffer++;
    len++;
  }
  return len;
}

inline int readChar() {
  int c = getChar();
  while (c <= 32)
    c = getChar();
  return c;
```

```
}

template <class T>
inline T readInt() {
  int s = 1, c = readChar();
  T x = 0;
  if (c == '-')
    s = -1, c = getChar();
  while ('0' <= c && c <= '9')
    x = x * 10 + c - '0', c = getChar();
  return s == 1 ? x : -x;
}

/** Write */

static int write_pos = 0;
static char write_buf[buf_size];

inline void writeChar( int x ) {
  if (write_pos == buf_size)
    fwrite(write_buf, 1, buf_size, stdout),
    ↪  write_pos = 0;
  write_buf[write_pos++] = x;
}

template <class T>
inline void writeInt( T x, char end ) {
  if (x < 0)
    writeChar('-'), x = -x;

  char s[24];
  int n = 0;
  while (x || !n)
    s[n++] = '0' + x % 10, x /= 10;
  while (n--)
    writeChar(s[n]);
  if (end)
    writeChar(end);
}

inline void writeWord( const char *s ) {
  while (*s)
    writeChar(*s++);
}

struct Flusher {
  ~Flusher() {
    if (write_pos)
      fwrite(write_buf, 1, write_pos, stdout),
      ↪  write_pos = 0;
  }
} flusher;


// sk fast read-write ends
```

```
// extended euclid begins

int gcd (int a, int b, int & x, int & y) {
  if (a == 0) {
    x = 0; y = 1;
    return b;
  }
  int x1, y1;
  int d = gcd (b%a, a, x1, y1);
  x = y1 - (b / a) * x1;
  y = x1;
  return d;
}

// extended euclid ends
```

```
// kto begins
//return pair(nmod,nr)
//nr%mod1=r1, nr%mod2=r2
//nmod=mod1*mod2/gcd(mod1,mod2)
//if input incosistent return mp(-1,-1)
pll kto (ll mod1, ll r1, ll mod2, ll r2)
{
  ll d=__gcd(mod1,mod2);
  if (r1%d!=r2%d)
    return mp(-1,-1);
  ll rd=r1%d;
  mod1/=d, mod2/=d, r1/=d, r2/=d;

  if (mod1<mod2)
    swap(mod1,mod2), swap(r1,r2);

  ll k=(r2-r1)%mod2;
  if (k<0)
    k+=mod2;

  ll x, y;
  gcdex(mod1,mod2,x,y);
  x%=mod2;
  if (x<0)
    x+=mod2;
  k*=x, k%=mod2;
  return mp(mod1*mod2*d,(k*mod1+r1)*d+rd);
}
// kto ends
```

```
// FFT begins

const int LOG = 19;
const int N = (1 << LOG);

typedef std::complex<double> cd;

int rev[N];
cd W[N];

void precalc() {
  const double pi = std::acos(-1);
  for (int i = 0; i != N; ++i)
    W[i] = cd(std::cos(2 * pi * i / N),
    ↪  std::sin(2 * pi * i / N));

  int last = 0;
  for (int i = 1; i != N; ++i) {
    if (i == (2 << last))
      ++last;

```

```
    rev[i] = rev[i ^ (1 << last)] | (1 << (LOG - 1 - last));
  }
}

void fft(vector<cd>& a) {
  for (int i = 0; i != N; ++i)
    if (i < rev[i])
      std::swap(a[i], a[rev[i]]);

  for (int lvl = 0; lvl != LOG; ++lvl)
    for (int start = 0; start != N; start += (2 << lvl))
      for (int pos = 0; pos != (1 << lvl); ++pos) {
        cd x = a[start + pos];
        cd y = a[start + pos + (1 << lvl)];

        y *= W[pos << (LOG - 1 - lvl)];

        a[start + pos] = x + y;
        a[start + pos + (1 << lvl)] = x - y;
      }
}

void inv_fft(vector<cd>& a) {
  fft(a);
  std::reverse(a.begin() + 1, a.end());

  for (cd& elem: a)
    elem /= N;
}

// FFT ends

// fast gauss begins

using elem_t = int;
// a[i][rows[i][j].first]=rows[i][j].second;  b[i]=a[i][n]
bool gauss(vector<vector<pair<int, elem_t>>> rows, vector<elem_t> &res) {
  int n = rows.size();

  res.resize(n + 1, 0);
  vector<int> p(n + 1);
  iota(p.begin(), p.end(), 0);
  vector<int> toZero(n + 1, -1);
  vector<int> zro(n + 1);
  vector<elem_t> a(n + 1);

  // optional: sort rows

  sort(p.begin(), p.begin() + n, [&rows](int i, int j) { return rows[i].size() < rows[j].size(); });
  vector<int> invP(n + 1);
  vector<vector<pair<int, elem_t>>> rs(n);
  for (int i = 0; i < n; i++) {
    invP[p[i]] = i;
    rs[i] = rows[p[i]];
  }
  for (int i = 0; i < n; i++) {
    rows[i] = rs[i];
    for (auto& el: rows[i]) {
      if (el.first < n) {
        el.first = invP[el.first];
      }
    }
  }


  for (int i = 0; i < n; i++) {
    for (auto& el: rows[i]) {
      a[el.first] = el.second;
    }
    while (true) {
      int k = -1;
      for (auto& el: rows[i]) {
        if (!isZero(a[el.first]) && toZero[el.first] != -1 &&
          (k == -1 || toZero[el.first] < toZero[k])) {
          k = el.first;
        }
      }
      if (k == -1)
        break;

      int j = toZero[k];
      elem_t c = a[k];

      for (auto el: rows[j]) {
        if (isZero(a[el.first]))
          rows[i].emplace_back(el.first, 0);
        a[el.first] = sub(a[el.first], mult(c, el.second));
      }

      auto cond = [&a](const pair<int, elem_t>& p) { return isZero(a[p.first]); };

      rows[i].erase(std::remove_if(rows[i].begin(), rows[i].end(), cond), rows[i].end());
    }

    bool ok = false;
    for (auto& el: rows[i]) {
      if (el.first < n && !isZero(a[el.first])) {
        toZero[el.first] = i;
        zro[i] = el.first;
//        det = (det * a[el.first]) % MOD;

        elem_t c = divM(1, a[el.first]);
        for (auto& el: rows[i]) {
          el.second = mult(a[el.first], c);
          a[el.first] = 0;
        }

        ok = true;
        break;
      }
    }
```

```
    if (!ok) {
//      det = 0;
      return false;
    }
  }

  res[n] = sub(0, 1);
  for (int i = n - 1; i >= 0; i--) {
    int k = zro[i];
    for (auto& el : rows[i])
      if (el.first != k)
        res[p[k]] = sub(res[p[k]],
          ↪ mult(el.second, res[p[el.first]]));
  }

  return true;
}

// fast gauss ends

// stable gauss begins
// if at least one solution returns it in ans
int gauss (vector < vector<double> > a,
↪ vector<double> & ans) {
  int n = (int) a.size();
  int m = (int) a[0].size() - 1;

  vector<int> where (m, -1);
  for (int col=0, row=0; col<m && row<n; ++col) {
    int sel = row;
    for (int i=row; i<n; ++i)
      if (abs (a[i][col]) > abs (a[sel][col]))
        sel = i;
    if (abs (a[sel][col]) < EPS)
      continue;
    for (int i=col; i<=m; ++i)
      swap (a[sel][i], a[row][i]);
    where[col] = row;

    for (int i=0; i<n; ++i)
      if (i != row) {
        double c = a[i][col] / a[row][col];
        for (int j=col; j<=m; ++j)
          a[i][j] -= a[row][j] * c;
      }
    ++row;
  }

  ans.assign (m, 0);
  for (int i=0; i<m; ++i)
    if (where[i] != -1)
      ans[i] = a[where[i]][m] / a[where[i]][i];
  for (int i=0; i<n; ++i) {
    double sum = 0;
    for (int j=0; j<m; ++j)
      sum += ans[j] * a[i][j];
    if (abs (sum - a[i][m]) > EPS)
      return 0;
  }

  for (int i=0; i<m; ++i)
    if (where[i] == -1)
      return INF;
  return 1;
}

// stable gauss ends
```

```
// simple geometry begins

struct Point {
  double x, y;
  Point operator+(const Point& p) const { return
  ↪ {x + p.x, y + p.y}; }
  Point operator-(const Point& p) const { return
  ↪ {x - p.x, y - p.y}; }
  Point operator*(const double d) const { return
  ↪ {x * d, y * d}; }
  Point rotate() const { return {y, -x}; }
  double operator*(const Point& p) const { return
  ↪ x * p.x + y * p.y; }
  double operator^(const Point& p) const { return
  ↪ x * p.y - y * p.x; }
  double dist() const { return sqrt(x * x + y *
  ↪ y); }
};

struct Line {
  double a, b, c;
  Line(const Point& p1, const Point& p2) {
    a = p1.y - p2.y;
    b = p2.x - p1.x;
    c = - a * p1.x - b * p1.y;

    double d = sqrt(sqr(a) + sqr(b));
    a /= d, b /= d, c /= d;
  }
  bool operator||(const Line& l) const { return
  ↪ fabs(a * l.b - l.a * b) < EPS; }
  double dist(const Point& p) const { return
  ↪ fabs(a * p.x + b * p.y + c); }
  Point operator^(const Line& l) const {
    return {(l.c * b - c * l.b) / (a * l.b - l.a
    ↪ * b),
        (l.c * a - c * l.a) / (l.a * b - a *
        ↪ l.b)};
  }
  Point projection(const Point& p) const {
    return p - Point{a, b} * (a * p.x + b * p.y +
    ↪ c);
  }
};

struct Circle {
  Point c;
  double r;
  Circle(const Point& c, double r) : c(c), r(r)
  ↪ {}
  Circle(const Point& a, const Point& b, const
  ↪ Point& c) {
    Point p1 = (a + b) * 0.5, p2 = (a + c) * 0.5;
    Point q1 = p1 + (b - a).rotate(), q2 = p2 +
    ↪ (c - a).rotate();
```

```
    this->c = Line(p1, q1) ^ Line(p2, q2);
    r = (a - this->c).dist();
  }
};

inline bool on_segment(const Point& p1, const Point& p2, const Point& x, bool strictly) {
  if (fabs((p1 - x) ^ (p2 - x)) > EPS)
    return false;
  return (p1 - x) * (p2 - x) < (strictly ? - EPS : EPS);
}

// in case intersection is not a segment
inline bool intersect_segments(const Point& p1, const Point& p2, const Point& q1, const Point& q2, Point& x) {
  Line l1(p1, p2), l2(q1, q2);
  if (l1 || l2) return false;
  x = l1 ^ l2;
  return on_segment(p1, p2, x, false);
}

// in case circles are not equal
inline bool intersect_circles(const Circle& c1, const Circle& c2, Point& p1, Point& p2) {
  double d = (c2.c - c1.c).dist();
  if (d > c1.r + c2.r + EPS || d < fabs(c1.r - c2.r) - EPS)
    return false;
  double cosa = (sqr(d) + sqr(c1.r) - sqr(c2.r)) / (2 * c1.r * d);
  double l = c1.r * cosa, h = sqrt(sqr(c1.r) - sqr(l));
  Point v = (c2.c - c1.c) * (1 / d), p = c1.c + v * l;
  p1 = p + v.rotate() * h, p2 = p - v.rotate() * h;
  return true;
}

inline bool intersect_circle_and_line(const Circle& c, const Line& l, Point& p1, Point& p2) {
  double d = l.dist(c.c);
  if (d > c.r + EPS)
    return false;
  Point p = l.projection(c.c);
  Point n{l.b, -l.a};
  double h = sqrt(sqr(c.r) - sqr(l.dist(c.c)));
  p1 = p + n * h, p2 = p - n * h;
  return true;
}

// simple geometry ends

// convex hull begins

struct Point {
  int x, y;
  Point operator-(const Point& p) const { return {x - p.x, y - p.y}; }
  int64_t operator^(const Point& p) const { return x * 1ll * p.y - y * 1ll * p.x; }
  int64_t dist() const { return x * 1ll * x + y * 1ll * y; }
  bool operator<(const Point& p) const { return x != p.x ? x < p.x : y < p.y; }
};

// all point on convex hull are included
vector<Point> convex_hull(vector<Point> pt) {
  int n = pt.size();
  Point p0 = *std::min_element(pt.begin(), pt.end());
  std::sort(pt.begin(), pt.end(), [&p0](const Point& a, const Point& b) {
    int64_t cp = (a - p0) ^ (b - p0);
    return cp != 0 ? cp > 0 : (a - p0).dist() < (b - p0).dist();
  });

  int i = n - 1;
  for (; i > 0 && ((pt[i] - p0) ^ (pt[i - 1] - p0)) == 0; i--);
  std::reverse(pt.begin() + i, pt.end());

  vector<Point> ch;
  for (auto& p : pt) {
    while (ch.size() > 1) {
      auto& p1 = ch[(int) ch.size() - 1];
      auto& p2 = ch[(int) ch.size() - 2];
      int64_t cp = (p1 - p2) ^ (p - p1);
      if (cp >= 0) break;
      ch.pop_back();
    }
    ch.push_back(p);
  }

  return ch;
}

// convex hull ends

// convex hull trick begins

typedef long long ftype;
typedef complex<ftype> point;
#define x real
#define y imag

ftype dot(point const& a, point const& b) {
  return (conj(a) * b).x();
}

ftype f(point const& a, int x) {
  return dot(a, {compressed[x], 1});
  //return dot(a, {x, 1});
}

int pos = 0;

// (x, y) -> (k, b) -> kb + x
struct li_chao { // for min
```

```
  vector<point> line;

  li_chao(int maxn) {
    line.resize(4 * maxn, {0, inf});
  }

  void add_line(int v, int l, int r, int a, int b, point nw) {
    if (r <= a || b <= l) return; // remove if no [a, b) query

    int m = (l + r) >> 1;

    if (!(a <= l && r <= b)) { // remove if no [a, b) query
      add_line(v + v + 1, l, m, a, b, nw);
      add_line(v + v + 2, m, r, a, b, nw);
      return;
    }

    bool lef = f(nw, l) < f(line[v], l);
    bool mid = f(nw, m) < f(line[v], m);

    if (mid) swap(line[v], nw);

    if (l == r - 1)
      return;

    if (lef != mid)
      add_line(v + v + 1, l, m, a, b, nw);
    else
      add_line(v + v + 2, m, r, a, b, nw);
  }

  ftype get(int v, int l, int r, int x) {
    if(l == r - 1)
      return f(line[v], x);
    int m = (l + r) / 2;
    if(x < m) {
      return min(f(line[v], x), get(v + v + 1, l, m, x));
    } else {
      return min(f(line[v], x), get(v + v + 2, m, r, x));
    }
  }

} cdt(maxn);

// convex hull with stack

ftype cross(point a, point b) {
  return (conj(a) * b).y();
}

vector<point> hull, vecs;

void add_line(ftype k, ftype b) {
  point nw = {k, b};
  while(!vecs.empty() && dot(vecs.back(), nw - hull.back()) < 0) {
    hull.pop_back();
    vecs.pop_back();
  }
  if(!hull.empty()) {
    vecs.push_back(1i * (nw - hull.back()));
  }
  hull.push_back(nw);
}

int get(ftype x) {
  point query = {x, 1};
  auto it = lower_bound(vecs.begin(), vecs.end(), query, [](point a, point b) {
    return cross(a, b) > 0;
  });
  return dot(query, hull[it - vecs.begin()]);
}

// convex hull trick ends
```

```
// heavy-light begins

int sz[maxn];

void dfs_sz(int v, int par = -1) {
  sz[v] = 1;
  for (int x : gr[v])
    if (x != par) {
      dfs_sz(x, v);
      sz[v] += sz[x];
    }
  for (int i = 0; i < gr[v].size(); i++)
    if (gr[v][i] != par)
    if (sz[gr[v][i]] * 2 >= sz[v]) {
      swap(gr[v][i], gr[v][0]);
      break;
    }
}

int rev[maxn];
int t_in[maxn];
int upper[maxn];;
int par[maxn];
int dep[maxn];

int T = 0;

void dfs_build(int v, int uppr, int pr = -1) {
  rev[T] = v;
  t_in[v] = T++;
  dep[v] = pr == -1 ? 0 : dep[pr] + 1;
  par[v] = pr;
  upper[v] = uppr;

  bool first = true;

  for (int x : gr[v])
    if (x != pr) {
      dfs_build(x, first ? upper[v] : x, v);
      first = false;
    }
}
```

```

struct interval {
  int l;
  int r;
  bool inv; // should direction be reversed
};

// node-weighted hld
vector<interval> get_path(int a, int b) {
  vector<interval> front;
  vector<interval> back;

  while (upper[a] != upper[b]) {
    if (dep[upper[a]] > dep[upper[b]]) {
      front.push_back({t_in[upper[a]], t_in[a], true});
      a = par[upper[a]];
    } else {
      back.push_back({t_in[upper[b]], t_in[b], false});
      b = par[upper[b]];
    }
  }

  front.push_back({min(t_in[a], t_in[b]), max(t_in[a], t_in[b]), t_in[a] > t_in[b]});
  // for edge-weighted hld add: "front.back().l++;"
  front.insert(front.end(), back.rbegin(), back.rend());

  return front;
}

// heavy-light ends

// max flow begins

struct edge{
  int from, to;
  int c, f, num;
  edge(int from, int to, int c, int num):from(from), to(to), c(c), f(0), num(num){}
  edge(){}
};

const int max_n =  600;

edge eds[150000];
int num = 0;
int it[max_n];
vector<int> gr[max_n];
int s, t;
vector<int> d(max_n);

bool bfs(int k) {
  queue<int> q;
  q.push(s);
  fill(d.begin(), d.end(), -1);
  d[s] = 0;
  while (!q.empty()) {
    int v = q.front();
    q.pop();
    for (int x : gr[v]) {
      int to = eds[x].to;
      if (d[to] == -1 && eds[x].c - eds[x].f >= (1 << k)){
        d[to] = d[v] + 1;
        q.push(to);
      }
    }
  }

  return (d[t] != -1);
}

int dfs(int v, int flow, int k) {
  if (flow < (1 << k))
    return 0;
  if (v == t)
    return flow;
  for (; it[v] < gr[v].size(); it[v]++) {
    int num = gr[v][it[v]];
    if (d[v] + 1 != d[num].to])
      continue;
    int res = dfs(eds[num].to, min(flow, eds[num].c - eds[num].f), k);
    if (res){
      eds[num].f += res;
      eds[num ^ 1].f -= res;
      return res;
    }
  }
  return 0;
}

void add(int fr, int to, int c, int nm) {
  gr[fr].push_back(num);
  eds[num++] = edge(fr, to, c, nm);
  gr[to].push_back(num);
  eds[num++] = edge(to, fr, 0, nm); //corrected c
}

int ans = 0;
  for (int k = 30; k >= 0; k--)
    while (bfs(k)) {
      memset(it, 0, sizeof(it));
      while (int res = dfs(s, 1e9 + 500, k))
        ans += res;
    }


// decomposition

int path_num = 0;
vector<int> paths[max_n];
int flows[max_n];

int decomp(int v, int flow) {
  if (flow < 1)
    return 0;
  if (v == t) {
```

```
    path_num++;
    flows[path_num - 1] = flow;
    return flow;
  }
  for (int i = 0; i < gr[v].size(); i++) {
    int num = gr[v][i];
    int res = decomp(eds[num].to, min(flow,
    ↪  eds[num].f));
    if (res)  {
      eds[num].f -= res;
      paths[path_num -
      ↪  1].push_back(eds[num].num);
      return res;
    }
  }
  return 0;
}

while (decomp(s, 1e9 + 5));

// max flow ends
```

```
// min-cost flow begins

long long ans = 0;
int mx = 2 * n + 2;

memset(upd, 0, sizeof(upd));
for (int i = 0; i < mx; i++)
  dist[i] = inf;
dist[st] = 0;
queue<int> q;
q.push(st);
upd[st] = 1;
while (!q.empty()){
  int v = q.front();
  q.pop();
  if (upd[v]){
    for (int x : gr[v])  {
      edge &e = edges[x];
      if (e.c - e.f > 0 && dist[v] != inf &&
      ↪  dist[e.to] > dist[v] + e.w) {
        dist[e.to] = dist[v] + e.w;
        if (!upd[e.to])
          q.push(e.to);
        upd[e.to] = true;
        p[e.to] = x;
      }
    }
    upd[v] = false;
  }
}

for (int i = 0; i < k; i++){
  for (int i = 0; i < mx; i++)
    d[i] = inf;
  d[st] = 0;
  memset(used, false, sizeof(used));
  set<pair<int, int> > s;
  s.insert(make_pair(0, st));
  for (int i = 0; i < mx; i++){
    int x;
    while (!s.empty() && used[(s.begin() ->
    ↪  second)]){
      s.erase(s.begin());
    }
    if (s.empty())
      break;
    x = s.begin() -> second;
    used[x] = true;
    s.erase(s.begin());
    for (int i = 0; i < gr[x].size(); i++){
      edge &e = edges[gr[x][i]];
      if (!used[e.to] && e.c - e.f > 0){
        if (d[e.to] > d[x] + (e.c - e.f) * e.w +
        ↪  dist[x] - dist[e.to]){
          d[e.to] = d[x] + (e.c - e.f) * e.w +
          ↪  dist[x] - dist[e.to];
          p[e.to] = gr[x][i];
          s.insert(make_pair(d[e.to], e.to));
        }
      }
    }
    dist[x] += d[x];
  }
  int pos = t;
  while (pos != st){
    int id = p[pos];
    edges[id].f += 1;
    edges[id ^ 1].f -= 1;
    pos = edges[id].from;
  }
}

// min-cost flow ends
```

```
// min circulation begins
class Solver2 { // Min-cost circulation
  struct Edge {
    int to, ne, w, c;
  };
  vector<Edge> es;
  vi firs;
  int curRes;
  public:
  Solver2(int n) : es(), firs(n, -1),
curRes(0) {}
  // from, to, capacity (max.flow), cost
  int adde(int a, int b, int w, int c) {
    Edge e;
    e.to = b; e.ne = firs[a];
    e.w = w; e.c = c;
    es.pb(e);
    firs[a] = sz(es) - 1;

    e.to = a; e.ne = firs[b];
    e.w = 0; e.c = -c;
    es.pb(e);
    firs[b] = sz(es) - 1;
    return sz(es) - 2;
  }
  // increase capacity of edge 'id' by 'w'
  void ince(int id, int w) {
    es[id].w += w;
```

```
  }
  int solve() {
    const int n = sz(firs);

    for (;;) {
      vi d(n, 0), fre(n, -1);
      vi chd(n, -1);

      int base = -1;
      for (int step = 0; step < n; step++) {
        for (int i = 0; i < sz(es); i++) if
        ↪  (es[i].w > 0) {
          int b = es[i].to;
          int a = es[i ^ 1].to;
          if (d[b] <= d[a] + es[i].c) continue;
          d[b] = d[a] + es[i].c;
          fre[b] = i;
          if (step == n - 1)
            base = b;
        }
      }
      if (base < 0) break;

      vi seq;
      vb was(n, false);
      for (int x = base;; x = es[fre[x] ^ 1].to)
      ↪  {
        if (!was[x]) {
          seq.pb(x);
          was[x] = true;
        } else {
          seq.erase(
              seq.begin(),
              find(seq.begin(), seq.end(),
              x
          ));
          break;
        }
      }
      for (int i = 0; i < sz(seq); i++) {
        int v = seq[i];
        int eid = fre[v];
        assert(es[eid].w > 0);
        es[eid].w--;
        es[eid ^ 1].w++;
        curRes += es[eid].c;
      }
    }
    return curRes;
  }
};
// slow min circulation ends
```

```
// global cut begins
// g[i][j] = g[j][i] is sum of edges between i
↪  and j
// ans is value of mincut
ll g[maxn][maxn], w[maxn];
ll ans;
bool ex[maxn], inA[maxn];
int n;
void iterate(int curN){
 int prev = -1;
  memset(inA, 0, sizeof(inA));
  memset(w, 0, sizeof(w));
  for (int it = 0; it < curN; it++) {
    int best = -1;
    for (int i = 0; i < n; i++)
      if (ex[i] && !inA[i]
          && (best == -1 || w[i] > w[best]))
        best = i;
    assert(best != -1);
    if (it == curN - 1) {
      ans = min(ans, w[best]);
      for (int i = 0; i < n; i++){
        g[i][prev] += g[best][i];
        g[prev][i] = g[i][prev];
      }
      ex[best] = false;
    } else {
      inA[best] = true;
      for (int i = 0; i < n; i++)
        w[i] += g[best][i];
        prev = best;
    }
  }
}
void solve(){
  ans = INF;
  for (int i = n; i > 1; i--)
    iterate(i);
  cout << ans << endl;
}
// global cut ends
```

```
// bad hungarian begins

fill(par, par + 301, -1);
fill(par2, par2 + 301, -1);

int ans = 0;
for (int v = 0; v < n; v++){
  memset(useda, false, sizeof(useda));
  memset(usedb, false, sizeof(usedb));
  useda[v] = true;
  for (int i = 0; i < n; i++)
    w[i] = make_pair(a[v][i] + row[v] + col[i],
    ↪  v);
  memset(prev, 0, sizeof(prev));
  int pos;
  while (true){
    pair<pair<int, int>, int> p =
    ↪  make_pair(make_pair(1e9, 1e9), 1e9);
    for (int i = 0; i < n; i++)
      if (!usedb[i])
        p = min(p, make_pair(w[i], i));
    for (int i = 0; i < n; i++)
      if (!useda[i])
        row[i] += p.first.first;
    for (int i = 0; i < n; i++)
      if (!usedb[i]){
        col[i] -= p.first.first;
        w[i].first -= p.first.first;
      }
```

```
    ans += p.first.first;
    usedb[p.second] = true;
    prev[p.second] = p.first.second; //из второй в первую
    int x = par[p.second];
    if (x == -1){
      pos = p.second;
      break;
    }
    useda[x] = true;
    for (int j = 0; j < n; j++)
      w[j] = min(w[j], {a[x][j] + row[x] + col[j], x});

  }
  while (pos != -1){
    int nxt = par2[prev[pos]];
    par[pos] = prev[pos];
    par2[prev[pos]] = pos;
    pos = nxt;
  }
}
cout << ans << "\n";
for (int i = 0; i < n; i++)
  cout << par[i] + 1 << " " << i + 1 << "\n";

// bad hungarian ends
```

```
// Edmonds O(n^3) begins

vector<int> gr[MAXN];
int match[MAXN], p[MAXN], base[MAXN], q[MAXN];
bool used[MAXN], blossom[MAXN];
int mark[MAXN];
int C = 1;

int lca(int a, int b) {
  C++;
  for (;;) {
    a = base[a];
    mark[a] = C;
    if (match[a] == -1) break;
    a = p[match[a]];
  }

  for (;;) {
    b = base[b];
    if (mark[b] == C) return b;
    b = p[match[b]];
  }
}

void mark_path(int v, int b, int children) {
  while (base[v] != b) {
    blossom[base[v]] = blossom[base[match[v]]] = true;
    p[v] = children;
    children = match[v];
    v = p[match[v]];
  }
}
```

```
int find_path(int root) {
  memset(used, 0, sizeof(used));
  memset(p, -1, sizeof p);
  for (int i = 0; i < N; i++)
    base[i] = i;

  used[root] = true;
  int qh = 0, qt = 0;
  q[qt++] = root;
  while (qh < qt) {
    int v = q[qh++];
    for (int to : gr[v]) {
      if (base[v] == base[to] || match[v] == to)
        continue;
      if (to == root || match[to] != -1 &&
          p[match[to]] != -1) {
        int curbase = lca(v, to);
        memset(blossom, 0, sizeof(blossom));
        mark_path(v, curbase, to);
        mark_path(to, curbase, v);
        for (int i = 0; i < N; i++)
          if (blossom[base[i]]) {
            base[i] = curbase;
            if (!used[i]) {
              used[i] = true;
              q[qt++] = i;
            }
          }
      } else if (p[to] == -1) {
        p[to] = v;
        if (match[to] == -1)
          return to;
        to = match[to];
        used[to] = true;
        q[qt++] = to;
      }
    }
  }

  return -1;
}

memset(match, -1, sizeof match);
  for (int i = 0; i < N; i++) {
    if (match[i] == -1 && !gr[i].empty()) {
      int v = find_path(i);
      while (v != -1) {
        int pv = p[v], ppv = match[pv];
        match[v] = pv; match[pv] = v;
        v = ppv;
      }
    }
  }

// Edmonds O(n^3) ends
```

```
// string basis begins

vector<int> getZ(string s){
  vector<int> z;
  z.resize(s.size(), 0);
  int l = 0, r = 0;
```

```
  for (int i = 1; i < s.size(); i++){
    if (i <= r)
      z[i] = min(r - i + 1, z[i - l]);
    while (i + z[i] < s.size() && s[z[i]] == s[i
    ↪  + z[i]])
      z[i]++;
    if (i + z[i] - 1 > r){
      r = i + z[i] - 1;
      l = i;
    }
  }
  return z;
}

vector<int> getP(string s){
  vector<int> p;
  p.resize(s.size(), 0);
  int k = 0;
  for (int i = 1; i < s.size(); i++){
    while (k > 0 && s[i] == s[k])
      k = p[k - 1];
    if (s[i] == s[k])
      k++;
    p[i] = k;
  }
  return p;
}

vector<int> getH(string s){
  vector<int> h;
  h.resize(s.size() + 1, 0);
  for (int i = 0; i < s.size(); i++)
    h[i + 1] = ((h[i] * 1ll * pow) + s[i] - 'a' +
    ↪  1) % mod;
  return h;
}

int getHash(vector<int> &h, int l, int r){
  int res = (h[r + 1] - h[l] * p[r - l + 1]) %
  ↪  mod;
  if (res < 0)
    res += mod;
  return res;
}

// string basis ends
```

```
// min cyclic shift begins

string min_cyclic_shift (string s) {
  s += s;
  int n = (int) s.length();
  int i=0, ans=0;
  while (i < n/2) {
    ans = i;
    int j=i+1, k=i;
    while (j < n && s[k] <= s[j]) {
      if (s[k] < s[j])
        k = i;
      else
        ++k;
      ++j;
    }
    while (i <= k)  i += j - k;
  }
  return s.substr (ans, n/2);
}

// min cyclic shift ends
```

```
// suffix array O(n) begins

typedef vector<char> bits;

template<const int end>
void getBuckets(int *s, int *bkt, int n, int K) {
  fill(bkt, bkt + K + 1, 0);
  forn(i, n) bkt[s[i] + !end]++;
  forn(i, K) bkt[i + 1] += bkt[i];
}
void induceSAl(bits &t, int *SA, int *s, int
↪  *bkt, int n, int K) {
  getBuckets<0>(s, bkt, n, K);
  forn(i, n) {
    int j = SA[i] - 1;
    if (j >= 0 && !t[j])
      SA[bkt[s[j]]++] = j;
  }
}
void induceSAs(bits &t, int *SA, int *s, int
↪  *bkt, int n, int K) {
  getBuckets<1>(s, bkt, n, K);
  for (int i = n - 1; i >= 0; i--) {
    int j = SA[i] - 1;
    if (j >= 0 && t[j])
      SA[--bkt[s[j]]] = j;
  }
}

void SA_IS(int *s, int *SA, int n, int K) { //
↪  require last symbol is 0
#define isLMS(i) (i && t[i] && !t[i-1])
  int i, j;
  bits t(n);
  t[n-1] = 1;
  for (i = n - 3; i >= 0; i--)
    t[i] = (s[i]<s[i+1] || (s[i]==s[i+1] &&
    ↪  t[i+1]==1));
  int bkt[K + 1];
  getBuckets<1>(s, bkt, n, K);
  fill(SA, SA + n, -1);
  forn(i, n)
    if (isLMS(i))
      SA[--bkt[s[i]]] = i;
  induceSAl(t, SA, s, bkt, n, K);
  induceSAs(t, SA, s, bkt, n, K);
  int n1 = 0;
  forn(i, n)
    if (isLMS(SA[i]))
      SA[n1++] = SA[i];
  fill(SA + n1, SA + n, -1);
  int name = 0, prev = -1;
  forn(i, n1) {
    int pos = SA[i];
```

```
    bool diff = false;
    for (int d = 0; d < n; d++)
      if (prev == -1 || s[pos+d] != s[prev+d] ||
      ↪  t[pos+d] != t[prev+d])
        diff = true, d = n;
      else if (d > 0 && (isLMS(pos+d) ||
      ↪  isLMS(prev+d)))
        d = n;
    if (diff)
      name++, prev = pos;
    SA[n1 + (pos >> 1)] = name - 1;
  }
  for (i = n - 1, j = n - 1; i >= n1; i--)
    if (SA[i] >= 0)
      SA[j--] = SA[i];
  int *s1 = SA + n - n1;
  if (name < n1)
    SA_IS(s1, SA, n1, name - 1);
  else
    forn(i, n1)
      SA[s1[i]] = i;
  getBuckets<1>(s, bkt, n, K);
  for (i = 1, j = 0; i < n; i++)
    if (isLMS(i))
      s1[j++] = i;
  forn(i, n1)
    SA[i] = s1[SA[i]];
  fill(SA + n1, SA + n, -1);
  for (i = n1 - 1; i >= 0; i--) {
    j = SA[i], SA[i] = -1;
    SA[--bkt[s[j]]] = j;
  }
  induceSAl(t, SA, s, bkt, n, K);
  induceSAs(t, SA, s, bkt, n, K);
}
// suffix array O(n) ends

// suffix array O(n log n) begins
string str;
int N, m, SA [MAX_N], LCP [MAX_N];
int x [MAX_N], y [MAX_N], w [MAX_N], c [MAX_N];

inline bool cmp (const int a, const int b, const
↪  int l) { return (y [a] == y [b] && y [a + l]
↪  == y [b + l]); }

void Sort () {
  for (int i = 0; i < m; ++i) w[i] = 0;
  for (int i = 0; i < N; ++i) ++w[x[y[i]]];
  for (int i = 0; i < m - 1; ++i) w[i + 1] +=
  ↪  w[i];
  for (int i = N - 1; i >= 0; --i)
  ↪  SA[--w[x[y[i]]]] = y[i];
}

void DA () {
  for (int i = 0; i < N; ++i) x[i] = str[i], y[i]
  ↪  = i;
  Sort ();
  for (int i, j = 1, p = 1; p < N; j <<= 1, m =
  ↪  p) {
    for (p = 0, i = N - j; i < N; i++) y[p++] =
    ↪  i;
    for (int k = 0; k < N; ++k) if (SA[k] >= j)
    ↪  y[p++] = SA[k] - j;
    Sort();
    for (swap (x, y), p = 1, x[SA[0]] = 0, i = 1;
    ↪  i < N; ++i) x[SA [i]] = cmp (SA[i - 1],
    ↪  SA[i], j) ? p - 1 : p++;
  }
}

// common for all algorithms
void kasaiLCP () {
  for (int i = 0; i < N; i++) c[SA[i]] = i;
  for (int i = 0, j, k = 0; i < N; LCP [c[i++]] =
  ↪  k)
    if (c [i] > 0) for (k ? k-- : 0, j = SA[c[i]
    ↪  - 1]; str[i + k] == str[j + k]; k++);
    else k = 0;
}

void suffixArray () { // require last symbol is
↪  char(0)
  m = 256;
  N = str.size();
  DA ();
  kasaiLCP ();
}
// suffix array O(n log n) ends

// bad suffix automaton begins

struct node{
  map<char, int> go;
  int len, suff;
  long long sum_in;
  node(){}
};

node v[max_n * 4];

int add_node(int max_len){
  //v[number].sum_in = 0;
  v[number].len = max_len;
  v[number].suff = -1;
  number++;
  return number - 1;
}

int last = add_node(0);

void add_char(char c) {
  int cur = last;
  int new_node = add_node(v[cur].len + 1);
  last = new_node;
  while (cur != -1){
    if (v[cur].go.count(c) == 0){
      v[cur].go[c] = new_node;
      //v[new_node].sum_in += v[cur].sum_in;
      cur = v[cur].suff;
      if (cur == -1)
        v[new_node].suff = 0;
```

```
    }else{
      int a = v[cur].go[c];
      if (v[a].len == v[cur].len + 1){
        v[new_node].suff = a;
      }else{
        int b = add_node(v[cur].len + 1);
        v[b].go = v[a].go;
        v[b].suff = v[a].suff;
        v[new_node].suff = b;
        while (cur != -1 && v[cur].go.count(c) != 0 && v[cur].go[c] == a){
          v[cur].go[c] = b;
          //v[a].sum_in -= v[cur].sum_in;
          //v[b].sum_in += v[cur].sum_in;
          cur = v[cur].suff;
        }
        v[a].suff = b;
      }
      return;
    }
  }
}

// bad suffix automaton ends
```

```
// aho-corasick begins

struct vertex {
  int next[K];
  bool leaf;
  int p;
  char pch;
  int link;
  int go[K];
};

vertex t[NMAX+1];
int sz;

void init() {
  t[0].p = t[0].link = -1;
  memset(t[0].next, 255, sizeof t[0].next);
  memset(t[0].go, 255, sizeof t[0].go);
  sz = 1;
}

void add_string(const string & s) {
  int v = 0;
  for (size_t i=0; i<s.length(); ++i) {
    char c = s[i]-'a';
    if (t[v].next[c] == -1) {
      memset(t[sz].next, 255, sizeof t[sz].next);
      memset(t[sz].go, 255, sizeof t[sz].go);
      t[sz].link = -1;
      t[sz].p = v;
      t[sz].pch = c;
      t[v].next[c] = sz++;
    }
    v = t[v].next[c];
  }
  t[v].leaf = true;
}

int go(int v, char c);

int get_link(int v) {
  if (t[v].link == -1)
    if (v == 0 || t[v].p == 0)
      t[v].link = 0;
    else
      t[v].link = go(get_link(t[v].p), t[v].pch);
  return t[v].link;
}

int go(int v, char c) {
  if (t[v].go[c] == -1)
    if (t[v].next[c] != -1)
      t[v].go[c] = t[v].next[c];
    else
      t[v].go[c] = v == 0 ? 0 : go(get_link(v), c);
  return t[v].go[c];
}

// aho-corasick ends
```

```
// pollard begins

const int max_step = 4e5;

unsigned long long gcd(unsigned long long a, unsigned long long b){
  if (!a) return 1;
  while (a) swap(a, b%=a);
  return b;
}

unsigned long long get(unsigned long long a, unsigned long long b){
  if (a > b)
    return a-b;
  else
    return b-a;
}

unsigned long long pollard(unsigned long long n){
  unsigned long long x = (rand() + 1) % n, y = 1, g;
  int stage = 2, i = 0;
  g = gcd(get(x, y), n);
  while (g == 1) {
    if (i == max_step)
      break;
    if (i == stage) {
      y = x;
      stage <<= 1;
    }
    x = (x * (__int128)x + 1) % n;
    i++;
    g = gcd(get(x, y), n);
  }
  return g;
}
```

```
// pollard ends

// linear sieve begins

const int N = 1000000;

int pr[N + 1], sz = 0;
/* minimal prime, mobius function, euler function */
int lp[N + 1], mu[N + 1], phi[N + 1];

lp[1] = mu[1] = phi[1] = 1;
for (int i = 2; i <= N; ++i) {
  if (lp[i] == 0)
    lp[i] = pr[sz++] = i;
  for (int j = 0; j < sz && pr[j] <= lp[i] && i * pr[j] <= N; ++j)
    lp[i * pr[j]] = pr[j];

  mu[i] = lp[i] == lp[i / lp[i]] ? 0 : -1 * mu[i / lp[i]];
  phi[i] = phi[i / lp[i]] * (lp[i] == lp[i / lp[i]] ? lp[i] : lp[i] - 1);
}

// linear sieve ends

// discrete log in sqrt(p) begins

int k = sqrt((double)p) + 2;

for (int i = k; i >= 1; i--)
  mp[bin(b, (i * 1ll * k) % (p-1), p)] = i;

bool answered = false;
int ans = INT32_MAX;
for (int i = 0; i <= k; i++){
  int sum = (n * 1ll * bin(b, i, p)) % p;
  if (mp.count(sum) != 0){
    int an = mp[sum] * 1ll * k - i;
    if (an < p)
      ans = min(an, ans);
  }
}

// discrete log in sqrt(p) ends

// prime roots mod n begins

int num = 0;
long long phi = n, nn = n;
for (long long x:primes){
  if (x*x>nn)
    break;
  if (nn % x == 0){
    while (nn % x == 0)
      nn /= x;
    phi -= phi/x;
    num++;
  }
}
if (nn != 1){
  phi -= phi/nn;
  num++;
}
if (!((num == 1 && n % 2 != 0) || n == 4 || n == 2 || (num == 2 && n % 2 == 0 && n % 4 != 0))) {
  cout << ''-1\n'';
  continue;
}
vector<long long> v;
long long pp = phi;
for (long long x:primes){
  if (x*x>pp)
    break;
  if (pp % x == 0){
    while (pp % x == 0)
      pp /= x;
    v.push_back(x);
  }
}
if (pp != 1){
  v.push_back(pp);
}
while (true){
  long long a = primes[rand()%5000]%n;
  if (gcd(a, n) != 1)
    continue;
  bool bb = false;
  for (long long x:v)
    if (pow(a, phi/x) == 1){
      bb = true;
      break;
    }
  if (!bb){
    cout << a << ''\n'';
    break;
  }
}

// prime roots mod n ends

// simplex begins

const double EPS = 1e-9;

typedef vector<double> vdbl;

// n variables, m inequalities
// Ax <= b, c*x -> max, x >= 0
double simplex( int n, int m, const vector<vdbl> &a0, const vdbl &b, const vdbl &c, vdbl &x ) {
  // Ax + Ez = b, A[m]*x -> max
  // x = 0, z = b, x >= 0, z >= 0
  vector<vdbl> a(m + 2, vdbl(n + m + 2));
  vector<int> p(m);
  forn(i, n)
    a[m + 1][i] = c[i];
  forn(i, m) {
    forn(j, n)
      a[i][j] = a0[i][j];
    a[i][n + i] = 1;
```

```
    a[i][m + n] = -1;
    a[i][m + n + 1] = b[i];
    p[i] = n + i;
  }

  // basis: enter "j", leave "ind+n"
  auto pivot = [&]( int j, int ind ) {
    double coef = a[ind][j];
    assert(fabs(coef) > EPS);
    forn(col, n + m + 2)
      a[ind][col] /= coef;
    forn(row, m + 2)
      if (row != ind && fabs(a[row][j]) > EPS) {
        coef = a[row][j];
        forn(col, n + m + 2)
          a[row][col] -= a[ind][col] * coef;
        a[row][j] = 0; // reduce precision error
      }
    p[ind] = j;
  };

  // the Simplex itself
  auto iterate = [&]( int nn ) {
    for (int run = 1; run; ) {
      run = 0;
      forn(j, nn) {
        if (a[m][j] > EPS) { // strictly positive
          run = 1;
          double mi = INFINITY, t;
          int ind = -1;
          forn(i, m)
            if (a[i][j] > EPS && (t = a[i][n + m
            ↪  + 1] / a[i][j]) < mi - EPS)
              mi = t, ind = i;
          if (ind == -1)
            return false;
          pivot(j, ind);
        }
      }
    }
    return true;
  };

  int mi = min_element(b.begin(), b.end()) -
  ↪  b.begin();
  if (b[mi] < -EPS) {
    a[m][n + m] = -1;
    pivot(n + m, mi);
    assert(iterate(n + m + 1));
    if (a[m][m + n + 1] > EPS) // optimal value
    ↪  is positive
      return NAN;
    forn(i, m)
      if (p[i] == m + n) {
        int j = 0;
        while (find(p.begin(), p.end(), j) !=
        ↪  p.end() || fabs(a[i][j]) < EPS)
          j++, assert(j < m + n);
        pivot(j, i);
      }
  }
  swap(a[m], a[m + 1]);
  if (!iterate(n + m))
    return INFINITY;
  x = vdbl(n, 0);
  forn(i, m)
    if (p[i] < n)
      x[p[i]] = a[i][n + m + 1];
  return -a[m][n + m + 1];
}

// simplex usage:
vdbl x(n);
double result = simplex(n, m, a, b, c, x);
if (isinf(result))
  puts("Unbounded");
else if (isnan(result))
  puts("No solution");
else {
  printf("%.9f :", result);
  forn(i, n)
    printf(" %.9f", x[i]);
  puts("");
}

// simplex ends

// sum over subsets begins
// fast subset convolution O(n 2^n)
for(int i = 0; i < (1<<N); ++i)
  F[i] = A[i];
for(int i = 0; i < N; ++i) for(int mask = 0; mask
↪  < (1<<N); ++mask){
  if(mask & (1<<i))
    F[mask] += F[mask^(1<<i)];
}
// sum over subsets ends

// algebra begins

Pick
B + Г / 2 - 1,
где B - количество целочисленных точек внутри
↪  многоугольника, а Г - количество
↪  целочисленных точек на границе
↪  многоугольника.

Newton
x_{i+1}=x_i-f(x_i)/f'(x_i)

Catalan
C_n = \sum \limits_{k=0}^{n-1} C_{k} C_{n-1-k}
C_i = \frac 1 {n + 1} \binom {2n} {n}

G_N:=2^{n(n-1)/2}
Количество связных помеченных графов
Conn_N = G_N - \frac 1 N \sum
↪  \limits_{K=1}^{N-1} K \binom N K Conn_K
↪  G_{N-K}

Количество помеченных графов с K компонентами
↪  связности
```

```
D[N][K]=\sum \limits_{S=1}^N \binom {N-1} {S-1} Conn_S D[N-S][K-1]

Miller-Rabbin
a=a^t
FOR i = 1...s
  if a^2=1 && |a|!=1
    NOT PRIME
  a=a^2
return a==1 ? PRIME : NOT PRIME


Интегрирование по формуле Симпсона
\int_a^b f(x)dx - ?
x_i := a+ih, i=0\ldots 2n
h = \frac {b-a} {2n}

\int = (f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \ldots + 4f(x_{2n-1}) + f(x_{2n}))) \frac h 3
Погрешность имеет порядок уменьшения как O(n^4).

// algebra ends
```

```
// wavelet tree begins

struct wavelet_tree{
  int lo, hi;
  wavelet_tree *l, *r;
  vi b;

  //nos are in range [x,y]
  //array indices are [from, to)
  wavelet_tree(int *from, int *to, int x, int y){
    lo = x, hi = y;
    if(lo == hi or from >= to) return;

    int mid = (lo+hi)/2;
    auto f = [mid](int x){
      return x <= mid;
    };
    b.reserve(to-from+1);
    b.pb(0);
    //b[i] = no of elements from first "i" elements that go to left node
    for(auto it = from; it != to; it++)
      b.pb(b.back() + f(*it));

    //see how lambda function is used here
    auto pivot = stable_partition(from, to, f);
    l = new wavelet_tree(from, pivot, lo, mid);
    r = new wavelet_tree(pivot, to, mid+1, hi);
  }

  //kth smallest element in [l, r]
  int kth(int l, int r, int k){
    if(l > r) return 0;
    if(lo == hi) return lo;
    //how many nos are there in left node from [l, r]
    int inleft = b[r] - b[l-1];
    int lb = b[l-1]; //amt of nos from first (l-1) nos that go in left
    int rb = b[r]; //amt of nos from first (r) nos that go in left
    //so [lb+1, rb] represents nos from [l, r] that go to left
    if(k <= inleft) return this->l->kth(lb+1, rb , k);

    //(l-1-lb) is amt of nos from first (l-1) nos that go to right
    //(r-rb) is amt of nos from first (r) nos that go to right
    //so [l-lb, r-rb] represents nos from [l, r] that go to right
    return this->r->kth(l-lb, r-rb, k-inleft);
  }
};

// wavelet tree ends
```

```
// berlecamp-massey begins

const int SZ = MAXN;

ll qp(ll a, ll b) {
  ll x = 1;
  a %= MOD;
  while (b) {
    if (b & 1) x = x * a % MOD;
    a = a * a % MOD;
    b >>= 1;
  }
  return x;
}

namespace linear_seq {
  inline vector<int> BM(vector<int> x) {
    vector<int> ls, cur;
    int lf, ld;
    for (int i = 0; i < int(x.size()); ++i) {
      ll t = 0;
      for (int j = 0; j < int(cur.size()); ++j)
        t = (t + x[i - j - 1] * (ll) cur[j]) % MOD;
      if ((t - x[i]) % MOD == 0) continue;
      if (!cur.size()) {
        cur.resize(i + 1);
        lf = i;
        ld = (t - x[i]) % MOD;
        continue;
      }
      ll k = -(x[i] - t) * qp(ld, MOD - 2) % MOD;
      vector<int> c(i - lf - 1);
      c.pb(k);
      for (int j = 0; j < int(ls.size()); ++j)
        c.pb(-ls[j] * k % MOD);
      if (c.size() < cur.size())
        c.resize(cur.size());
      for (int j = 0; j < int(cur.size()); ++j)
        c[j] = (c[j] + cur[j]) % MOD;
```

```
      if (i - lf + (int) ls.size() >= (int) cur.size())
        ls = cur, lf = i, ld = (t - x[i]) % MOD;
      cur = c;
    }
    for (int i = 0; i < int(cur.size()); ++i)
      cur[i] = (cur[i] % MOD + MOD) % MOD;
    return cur;
  }

  int m;
  ll a[SZ], h[SZ], t_[SZ], s[SZ], t[SZ];

  inline void mull(ll* p, ll* q) {
    for (int i = 0; i < m + m; ++i) t_[i] = 0;
    for (int i = 0; i < m; ++i)
      if (p[i])
        for (int j = 0; j < m; ++j)
          t_[i + j] = (t_[i + j] + p[i] * q[j]) % MOD;
    for (int i = m + m - 1; i >= m; --i)
      if (t_[i])
        for (int j = m - 1; ~j; --j)
          t_[i - j - 1] = (t_[i - j - 1] + t_[i] * h[j]) % MOD;
    for (int i = 0; i < m; ++i) p[i] = t_[i];
  }

  inline ll calc(ll K) {
    for (int i = m; ~i; --i)
      s[i] = t[i] = 0;
    s[0] = 1;
    if (m != 1) t[1] = 1; else t[0] = h[0];
    while (K) {
      if (K & 1) mull(s, t);
      mull(t, t);
      K >>= 1;
    }
    ll su = 0;
    for (int i = 0; i < m; ++i) su = (su + s[i] * a[i]) % MOD;
    return (su % MOD + MOD) % MOD;
  }

  inline int work(vector<int> x, ll n) {
    if (n < int(x.size())) return x[n];
    vector<int> v = BM(x);
    m = v.size();
    if (!m) return 0;
    for (int i = 0; i < m; ++i) h[i] = v[i], a[i] = x[i];
    return calc(n);
  }

  //b=a0/(1-p)
  inline void calc_generating_function(const vector<int>& b, vector<int>& p,
  vector<int>& a0) {
    p = BM(b);
    a0.resize(p.size());
    for (int i = 0; i < a0.size(); ++i) {
      a0[i] = b[i];
      for (int j = 0; j < i; ++j) {
        a0[i] += MOD - (p[j] * 1ll * b[i - j - 1]) % MOD;
        if (a0[i] > MOD) {
          a0[i] -= MOD;
        }
      }
    }
  }
}

// berlecamp-massey ends
```

```
// AND-FFT begins

void fast_fourier(vector<int>& a) { // AND-FFT.
  for (int k = 1; k < SZ(a); k *= 2)
  for (int start = 0; start < (1 << K); start += 2 * k) {
    for (int off = 0; off < k; ++off) {
    int a_val = a[start + off];
    int b_val = a[start + k + off];

    a[start + off] = b_val;
    a[start + k + off] = add(a_val, b_val);
    }
  }
}

void inverse_fast_fourier(vector<int>& a) {
  for (int k = 1; k < SZ(a); k *= 2)
  for (int start = 0; start < (1 << K); start += 2 * k) {
    for (int off = 0; off < k; ++off) {
    int a_val = a[start + off];
    int b_val = a[start + k + off];

    a[start + off] = sub(b_val, a_val);
    a[start + k + off] = a_val;
    }
  }
}

// AND-FFT ends
```

```
// 2-chinese begins

template <typename Info>
class DSU {
  public:
  DSU ( int n ) : jump (new int[n]), rank (new int [n]), info (new Info [n]) {
    for (int i = 0; i < n; i++) {
    jump[i] = i;
    rank[i] = 0;
    }
  }
  Info& operator [] ( int x ) {
    return info[get (x)];
  }
```

```
  void merge ( int a, int b, const Info &comment
  ) {
    a = get (a);
    b = get (b);
    if (rank[a] <= rank[b]) {
    jump[a] = b;
    rank[b] += rank[a] == rank[b];
    info[b] = comment;
    } else {
    jump[b] = a;
    info[a] = comment;
    }
  }
  private:
  int *jump, *rank;
  Info *info;

  int get ( int x ) {
    return jump[x] == x ? x : (jump[x] = get
    (jump[x]));
  }
};


struct Treap {
  int value, add;
  int source, target, height;
  int min_value, min_path;

  Treap *left, *right;

  Treap ( int _source, int _target, int _value )
  : value (_value), add (0), source
  (_source), target (_target) {
  height = rand ();
  min_value = value, min_path = 0;
  left = right = 0;
  }

  Treap& operator += ( int sub ) {
  add += sub;
  return *this;
  }

  void push () {
  if (!add)
    return;
  if (left) {
    left->add += add;
  }
  if (right) {
    right->add += add;
  }
  value += add;
  min_value += add;
  add = 0;
  }

  void recalc () {
  min_value = value;
  min_path = 0;
  if (left && left->min_value + left->add <
  min_value) {
    min_value = left->min_value + left->add;
    min_path = -1;
  }
  if (right && right->min_value + right->add <
  min_value) {
    min_value = right->min_value + right->add;
    min_path = +1;
  }
  }
};

Treap* treap_merge ( Treap *x, Treap *y ) {
  if (!x)
  return y;
  if (!y)
  return x;
  if (x->height < y->height) {
  x->push ();
  x->right = treap_merge (x->right, y);
  x->recalc ();
  return x;
  } else {
  y->push ();
  y->left = treap_merge (x, y->left);
  y->recalc ();
  return y;
  }
}

Treap* treap_getmin ( Treap *x, int &source, int
&target, int &value ) {
  assert (x);
  x->push ();
  if (x->min_path == 0) {
  // memory leak, sorry
  source = x->source;
  target = x->target;
  value = x->value + x->add;
  return treap_merge (x->left, x->right);
  } else if (x->min_path == -1) {
  x->left = treap_getmin (x->left, source,
  target, value);
  value += x->add;
  x->recalc ();
  return x;
  } else if (x->min_path == +1) {
  x->right = treap_getmin (x->right, source,
  target, value);
  value += x->add;
  x->recalc ();
  return x;
  } else
  assert (0);
}

Treap* treap_add ( Treap *x, int add ) {
  if (!x)
  return 0;
  return &((*x) += add);
```

```
}


int main () {
  int n, m;
  while (scanf (''%d%d'', &n, &m) == 2) {
  Treap * g[n + 1];
  for (int i = 0; i <= n; i++)
    g[i] = 0;
  for (int i = 1; i <= n; i++) {
    int a;
    assert (scanf (''%d'', &a) == 1);
    g[i] = treap_merge (g[i], new Treap (i, 0, a));
  }
  n++;
  for (int i = 0; i < m; i++) {
    int a, b, c;
    assert (scanf (''%d%d%d'', &a, &b, &c) == 3);
    g[b] = treap_merge (g[b], new Treap (b, a, c));
  }
  DSU <pair <int, Treap*> > dsu (n + 1);
  for (int i = 0; i < n; i++) {
    dsu[i] = make_pair (i, g[i]);
  }

  int ans = 0, k = n;
  int jump[2 * n], jump_from[2 * n], parent[2 * n], c[n];
  vector <int> children[2 * n];
  memset (c, 0, sizeof (c[0]) * n);
  memset (parent, -1, sizeof (parent[0]) * 2 * n);
  vector <int> finish;
  for (int i = 0; i < n; i++) {
    if (dsu[i].first == 0)
    continue;
    int u = i;
    c[u] = 1;
    while (true) {
    int source, target, value;
    dsu[u].second = treap_getmin (dsu[u].second, source, target, value);
    if (dsu[target] == dsu[u])
      continue;
    treap_add (dsu[u].second, -value);
    ans += value;
    jump_from[dsu[u].first] = source;
    jump[dsu[u].first] = target;
    if (dsu[target].first == 0)
      break;
    if (!c[target]) {
      c[target] = 1;
      u = target;
      continue;
    }
    assert (k < 2 * n);
    int node = k++, t = target;
    parent[dsu[u].first] = node;
    children[node].push_back (dsu[u].first);
    dsu[u].first = node;
    Treap *v = dsu[u].second;
    while (dsu[t].first != node) {
      int next = jump[dsu[t].first];
      parent[dsu[t].first] = node;
      children[node].push_back (dsu[t].first);
      v = treap_merge (v, dsu[t].second);
      dsu.merge (u, t, make_pair (node, v));
      t = next;
    }
    }
    u = i;
    while (dsu[u].first) {
    int next = jump[dsu[u].first];
    finish.push_back (dsu[u].first);
    dsu.merge (u, 0, make_pair (0, (Treap *)0));
    u = next;
    }
  }
  bool ok[k];
  int res[n];
  memset (ok, 0, sizeof (ok[0]) * k);
  memset (res, -1, sizeof (res[0]) * n);
  function <void (int, int)> add_edge = [&ok, &parent, &res, &n] ( int a, int b ) {
    assert (0 <= a && a < n);
    assert (0 <= b && b < n);
    assert (res[a] == -1);
    res[a] = b;
    while (a != -1 && !ok[a]) {
    ok[a] = true;
    a = parent[a];
    }
  };
  function <void (int)> reach = [&ok, &reach, &children, &jump, &jump_from, &add_edge](
    int u ) {
    if (!ok[u])
    add_edge (jump_from[u], jump[u]);
    for (auto x : children[u])
    reach (x);
  };
  for (auto x : finish)
     reach (x);
  printf (''%d\n'', ans);
  for (int i = 1; i < n; i++)
    printf (''%d%c'', res[i] ? res[i] : -1, ''\n ''[i < n - 1]);
  }
  return 0;
}

// 2-chinese ends

// general max weighet match begins

#define DIST(e)  (lab[e.u]+lab[e.v]-g[e.u][e.v].w*2)
using namespace std;
typedef long long ll;
const int N = 1023, INF = 1e9;
struct Edge {
```

```
  int u, v, w;
} g[N][N];
int n, m, n_x, lab[N], match[N], slack[N], st[N], pa[N], flower_from[N][N], S[N], vis[N];
vector < int> flower[N];
deque < int> q;
void update_slack(int u, int x) {
  if (!slack[x] || DIST(g[u][x]) < DIST(g[slack[x]][x])) slack[x] = u;
}
void set_slack(int x) {
  slack[x] = 0;
  for (int u = 1; u <= n; ++u)
    if (g[u][x].w>0 && st[u] != x && S[st[u]] == 0) update_slack(u, x);
}
void q_push(int x) {
  if (x <= n) return q.push_back(x);
  for (int i = 0; i < flower[x].size(); i++)
    q_push(flower[x][i]);
}
void set_st(int x, int b) {
  st[x] = b;
  if (x <= n) return;
  for (int i = 0; i < flower[x].size(); ++i)
    set_st(flower[x][i], b);
}
int get_pr(int b, int xr) {
  int pr = find(flower[b].begin(), flower[b].end(), xr)-flower[b].begin();
  if (pr % 2 == 1) {
    reverse(flower[b].begin() +1, flower[b].end());
    return (int) flower[b].size()-pr;
  }
  else return pr;
}
void set_match(int u, int v) {
  match[u] = g[u][v].v;
  if (u <= n) return;
  Edge e = g[u][v];
  int xr = flower_from[u][e.u], pr = get_pr(u, xr);
  for (int i = 0; i < pr; ++i)
    set_match(flower[u][i], flower[u][i^1]);
  set_match(xr, v);
  rotate(flower[u].begin(), flower[u].begin() +pr, flower[u].end());
}
void augment(int u, int v) {
  int xnv = st[match[u]];
  set_match(u, v);
  if (!xnv) return;
  set_match(xnv, st[pa[xnv]]);
  augment(st[pa[xnv]], xnv);
}
int get_lca(int u, int v) {
  static int t = 0;
  for (++t; u || v; swap(u, v)) {
    if (u == 0) continue;
    if (vis[u] == t) return u;
    vis[u] = t;
    u = st[match[u]];
    if (u) u = st[pa[u]];
  }
  return 0;
}
void add_blossom(int u, int lca, int v) {
  int b = n+1;
  while (b <= n_x && st[b]) ++b;
  if (b>n_x) ++n_x;
  lab[b] = 0, S[b] = 0;
  match[b] = match[lca];
  flower[b].clear();
  flower[b].push_back(lca);
  for (int x = u, y; x != lca; x = st[pa[y]])
    flower[b].push_back(x), flower[b].push_back(y = st[match[x]]), q_push(y);
  reverse(flower[b].begin() +1, flower[b].end());
  for (int x = v, y; x != lca; x = st[pa[y]])
    flower[b].push_back(x), flower[b].push_back(y = st[match[x]]), q_push(y);
  set_st(b, b);
  for (int x = 1; x <= n_x; ++x) g[b][x].w = g[x][b].w = 0;
  for (int x = 1; x <= n; ++x) flower_from[b][x] = 0;
  for (int i = 0; i < flower[b].size(); ++i) {
    int xs = flower[b][i];
    for (int x = 1; x <= n_x; ++x)
      if (g[b][x].w == 0 || DIST(g[xs][x]) < DIST(g[b][x]))
        g[b][x] = g[xs][x], g[x][b] = g[x][xs];
    for (int x = 1; x <= n; ++x)
      if (flower_from[xs][x]) flower_from[b][x] = xs;
  }
  set_slack(b);
}
void expand_blossom(int b)  // S[b]  ==  1 {
  for (int i = 0; i < flower[b].size(); ++i)
    set_st(flower[b][i], flower[b][i]);
  int xr = flower_from[b][g[b][pa[b]].u], pr = get_pr(b, xr);
  for (int i = 0; i < pr; i += 2) {
    int xs = flower[b][i], xns = flower[b][i+1];
    pa[xs] = g[xns][xs].u;
    S[xs] = 1, S[xns] = 0;
    slack[xs] = 0, set_slack(xns);
    q_push(xns);
  }
  S[xr] = 1, pa[xr] = pa[b];
  for (int i = pr+1; i < flower[b].size(); ++i) {
    int xs = flower[b][i];
    S[xs] = -1, set_slack(xs);
  }
  st[b] = 0;
}
bool on_found_Edge(const Edge &e) {
  int u = st[e.u], v = st[e.v];
  if (S[v] == -1) {
    pa[v] = e.u, S[v] = 1;
```

```
    int nu = st[match[v]];
    slack[v] = slack[nu] = 0;
    S[nu] = 0, q_push(nu);
  }
  else if (S[v] == 0) {
    int lca = get_lca(u, v);
    if (!lca) return augment(u, v), augment(v, u), 1;
    else add_blossom(u, lca, v);
  }
  return 0;
}
bool matching() {
  fill(S, S+n_x+1, -1), fill(slack, slack+n_x+1, 0);
  q.clear();
  for (int x = 1; x <= n_x; ++x)
    if (st[x] == x && !match[x]) pa[x] = 0, S[x] = 0, q_push(x);
  if (q.empty()) return 0;
  for (;;) {
    while (q.size()) {
      int u = q.front();
      q.pop_front();
      if (S[st[u]] == 1) continue;
      for (int v = 1; v <= n; ++v)
        if (g[u][v].w>0 && st[u] != st[v]) {
          if (DIST(g[u][v]) == 0) {
            if (on_found_Edge(g[u][v])) return 1;
          }
          else update_slack(u, st[v]);
        }
    }
    int d = INF;
    for (int b = n+1; b <= n_x; ++b)
      if (st[b] == b && S[b] == 1) d = min(d, lab[b]/2);
    for (int x = 1; x <= n_x; ++x)
      if (st[x] == x && slack[x]) {
        if (S[x] == -1) d = min(d, DIST(g[slack[x]][x]));
        else if (S[x] == 0) d = min(d, DIST(g[slack[x]][x])/2);
      }
    for (int u = 1; u <= n; ++u) {
      if (S[st[u]] == 0) {
        if (lab[u] <= d) return 0;
        lab[u]- = d;
      }
      else if (S[st[u]] == 1) lab[u] += d;
    }
    for (int b = n+1; b <= n_x; ++b)
      if (st[b] == b) {
        if (S[st[b]] == 0) lab[b] += d*2;
        else if (S[st[b]] == 1) lab[b]- = d*2;
      }
    q.clear();
    for (int x = 1; x <= n_x; ++x)
      if (st[x] == x && slack[x] && st[slack[x]] != x && DIST(g[slack[x]][x]) == 0)
        if (on_found_Edge(g[slack[x]][x])) return 1;
    for (int b = n+1; b <= n_x; ++b)
      if (st[b] == b && S[b] == 1 && lab[b] == 0) expand_blossom(b);
  }
  return 0;
}
pair < ll, int> weight_blossom() {
  fill(match, match+n+1, 0);
  n_x = n;
  int n_matches = 0;
  ll tot_weight = 0;
  for (int u = 0; u <= n; ++u) st[u] = u, flower[u].clear();
  int w_max = 0;
  for (int u = 1; u <= n; ++u)
    for (int v = 1; v <= n; ++v) {
      flower_from[u][v] = (u == v?u:0);
      w_max = max(w_max, g[u][v].w);
    }
  for (int u = 1; u <= n; ++u) lab[u] = w_max;
  while (matching()) ++n_matches;
  for (int u = 1; u <= n; ++u)
    if (match[u] && match[u] < u)
      tot_weight += g[u][match[u]].w;
  return make_pair(tot_weight, n_matches);
}
int main() {
  cin>>n>>m;
  for (int u = 1; u <= n; ++u)
    for (int v = 1; v <= n; ++v)
      g[u][v] = Edge {u, v, 0};
  for (int i = 0, u, v, w; i < m; ++i) {
    cin>>u>>v>>w;
    g[u][v].w = g[v][u].w = w;
  }
  cout << weight_blossom().first << '\n';
  for (int u = 1; u <= n; ++u) cout << match[u] << ' ';
}

// general max weighet match ends
```

```
// fast hashtable begins

#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
gp_hash_table<int, int> table;

const int RANDOM = chrono ::high_resolution_clock ::now().time_since_epoch().count();
struct chash {
  int operator()(int x) { return hash<int>{}(x ^ RANDOM); }
};
gp_hash_table<key, int, chash> table;

// fast hashtable ends
```

DM

Кол-во корневых деревьев:
$t(G) = \frac{1}{n}\lambda_2 \dots \lambda_n \ (\lambda_1 = 0)$
Кол-во эйлеровых циклов:
$e(D) = t^-(D, x) \cdot \prod_{y \in D} (outdeg(y) - 1)!$
Наличие совершенного паросочетания:
T – матрица с нулями на диагонали. Если есть ребро (i, j), то $a_{i,j} := x_{i,j}$, $a_{j,i} = -x_{i,j}$
$\det(T) = 0 \Leftrightarrow$ нет совершенного паросочетания.

Fast subset convolution
$(f * g)(S) := \sum_{T \subseteq S} f(T)g(S \setminus T)$
$\hat{f}(X) := \sum_{S \subseteq X} f(S)$
$f(S) = \sum_{X \subseteq S} (-1)^{|S \setminus X|} \hat{f}(X)$

$\hat{f}_0(X) := f(X)$
$\hat{f}_j(X) = \begin{cases} \hat{f}_{j-1}(X) & if \quad j \notin X \\ \hat{f}_{j-1}(X \setminus j) + \hat{f}_{j-1}(X) & if \quad j \in X \end{cases}$
$\hat{f}_n(X) == \hat{f}(X)$

$f_0(S) := \hat{f}(S)$
$f_j(S) = \begin{cases} f_{j-1}(S) & if \quad j \notin S \\ -f_{j-1}(S \setminus j) + f_{j-1}(S) & if \quad j \in S \end{cases}$
$f_n(S) == f(S)$

$\hat{f}(k, X) := \sum_{S \subseteq, |S| = k} f(S)$
$f(S) == \hat{f}(|S|, S)$

$(\hat{f} \otimes \hat{g})(k, X) := \sum_{j=0}^{k} \hat{f}(j, X)\hat{g}(k - j, X)$
$(f * g)(S) = \sum_{X \subseteq S} (-1)^{|S \setminus X|} (\hat{f} \otimes \hat{g})(|S|, X)$
calculate using f_j!

xmodmap -e 'keycode 94='
setxkbmap us