Team Reference of St. Petersburg Campus of Higher School of Economics.

SPb HSE: Abstract Economists
(Ermilov, Fedorov, Labutin)



// pollard begins

return b;

**if** (a > b)

return a-b;

return b-a;

11

12

13

14

15

18

19

21

31 32

33

34

}

const int max\_step = 4e5;

→ unsigned long long b){ if (!a) return 1;

while (a) swap(a, b%=a);

unsigned long long b){

int stage = 2, i = 0;

g = gcd(get(x, y), n);

if (i == max\_step)

if (i == stage) {

stage <<= 1;

g = gcd(get(x, y), n);

 $x = (x * (_int128)x + 1) \% n;$ 

while (g == 1) {

break;

y = x;

return g;

// pollard ends

unsigned long long gcd(unsigned long long a,

unsigned long long get (unsigned long long a,

unsigned long long pollard(unsigned long long n){

unsigned long long x = (rand() + 1) % n, y = 1,

## pragma

```
#pragma GCC optimize(''03, no-stack-protector'')
#pragma GCC target("sse, sse2, sse4, ssse3, popcnt, abm,
```

### Алгебра Pick

```
B + \Gamma / 2 - 1 = AREA,
```

где В — количество целочисленных точек внутри многоугольника, а  $\Gamma$  — количество целочисленных точек на границе многоугольника.

# Newton

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$\begin{aligned} & \mathbf{Catalan} \\ & C_n = \sum_{k=0}^{n-1} C_k C_{n-1-k} \\ & C_i = \frac{1}{n+1} {2n \choose n} \end{aligned}$$

## Кол-во графов

$$G_N := 2^{n(n-1)/2}$$

Количество связных помеченных графов

$$Conn_N = G_N - \frac{1}{N} \sum_{K=1}^{N-1} K\binom{N}{K} Conn_K G_{N-K}$$

Количество помеченных графов с К компонентами связности

$$D[N][K] = \sum_{S=1}^{N} {\binom{N-1}{S-1}} Conn_S D[N-S][K-1]$$

# Miller-Rabbin

```
a=a^t
FOR i = 1...s
    if a^2=1 && |a|!=1
        NOT PRIME
    a=a^2
return a==1 ? PRIME : NOT PRIME
```

# Интегрирование по формуле Симпсона $\int_a^b f(x)dx?$

$$h = \frac{b-a}{2n}$$

$$\int = (f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \dots + 4f(x_{2n-1}) + f(x_{2n})) \frac{h}{3}$$

$$O(n^4).$$

## Простые числа

 $x_i := a + ih, i = 0 \dots 2n$ 

1009,1013;10007,10009;100003,100019 1000003,1000033;10000019,10000079 100000007,100000037 10000000019,10000000033 100000000039,100000000061 10000000000031,10000000000067 10000000000000061,10000000000000069 

# Числа для Фурье

```
• prime: 7340033 = 7 \cdot 2^{20} + 1; w : 5(w^{2^{20}} = 1)
```

```
• prime: 13631489 = 13 \cdot 2^{20} + 1; w: 3(w^{2^{20}} = 1)
                                                           1 // sk fast allocation begins
                                                               const int MAX_MEM = 5e8;
                                                               int mpos = 0;
                                                               char mem[MAX_MEM];
                                                               inline void * operator new ( size_t n ) {
                                                                 assert((mpos += n) <= MAX_MEM);</pre>
  • prime: 23068673 = 11 \cdot 2^{21} + 1; w: 38(w^{2^{21}} = 1)
                                                                 return (void *) (mem + mpos - n);
                                                               inline void operator delete ( void * ) noexcept {
                                                               → } // must have!
  • prime: 69206017 = 33 \cdot 2^{21} + 1; w: 45(w^{2^{21}} = 1)
                                                               // sk fast allocation ends
                                                           13
                                                               // sk fast read-write begins
  • prime: 81788929 = 39 \cdot 2^{21} + 1; w: 94(w^{2^{21}} = 1)
                                                               inline int readChar();
                                                               template <class T = int> inline T readInt();
                                                               template <class T> inline void writeInt( T x,
                                                               \rightarrow char end = 0);
  • prime: 104857601 = 25 \cdot 2^{22} + 1; w : 21(w^{2^{22}} = 1)
                                                               inline void writeChar( int x );
                                                           21
                                                               inline void writeWord( const char *s );
                                                               /** Read */
                                                               static const int buf_size = 2048;
  • prime: 113246209 = 27 \cdot 2^{22} + 1; w : 66(w^{2^{22}} = 1)
                                                               inline int getChar() {
                                                                 static char buf[buf_size];
                                                                 static int len = 0, pos = 0;
                                                                 if (pos == len)
                                                           31
  • prime: 138412033 = 33 \cdot 2^{22} + 1; w: 30(w^{2^{22}} = 1)
                                                                    pos = 0, len = fread(buf, 1, buf_size,
                                                           32
                                                                    \hookrightarrow stdin);
                                                                 if (pos == len)
                                                                    return -1;
                                                           34
                                                                 return buf[pos++];
                                                           35
  • prime: 167772161 = 5 \cdot 2^{25} + 1; w : 17(w^{2^{25}} = 1)
                                                               inline int readWord(char * buffer) {
                                                                 int c = getChar();
                                                                 while (c <= 32) {
                                                                    c = getChar();
  • prime: 469762049 = 7 \cdot 2^{26} + 1; w : 30(w^{2^{26}} = 1)
                                                                 int len = 0;
                                                                 while (c > 32) {
                                                                    *buffer = (char) c;
                                                                    c = getChar();
  • prime: 998244353 = 7.17.2^{23} + 1; w: 3^{7*17}.
                                                                    buffer++;
                                                                    len++;
                                                                 }
                                                                 return len;
  Erdős-Gallai theorem
A sequence of non-negative integers d_1 \geq \cdots \geq d_n can be
represented as the degree sequence of a finite simple graph
                                                               inline int readChar() {
on n vertices if and only if d_1 + \cdots + d_n \ d_1 + \cdots + d_n is even
                                                                 int c = getChar();
                                                                 while (c \ll 32)
                                                                    c = getChar();
  \sum_{i=1}^{k} d_i \le k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k) holds for every \frac{\partial^2}{\partial k}
                                                                 return c;
in 1 \le k \le n.
```

```
}
                                                            }
                                                            int x1, y1;
   template <class T>
                                                            int d = gcd (b%a, a, x1, y1);
61
                                                            x = y1 - (b / a) * x1;
   inline T readInt() {
      int s = 1, c = readChar();
                                                            y = x1;
      T x = 0;
                                                            return d;
                                                       12
     if (c == '-')
                                                       13
65
        s = -1, c = getChar();
                                                       14
      while ('^{0}' <= c && c <= '^{9}')
                                                          // extended euclid ends
        x = x * 10 + c - '0', c = getChar();
      return s == 1 ? x : -x;
                                                       1 // kto begins
   }
70
                                                       2 //return pair(nmod,nr)
71
                                                          //nr\%mod1=r1, nr\%mod2=r2
    /** Write */
72
                                                          //nmod=mod1*mod2/gcd(mod1,mod2)
                                                          //if input incosistent return mp(-1,-1)
   static int write_pos = 0;
                                                       6 pll kto (ll mod1, ll r1, ll mod2, ll r2)
    static char write_buf[buf_size];
                                                       7
                                                            11 d=_gcd(mod1,mod2);
   inline void writeChar( int x ) {
                                                            if (r1\%d!=r2\%d)
      if (write_pos == buf_size)
                                                              return mp(-1,-1);
                                                       10
        fwrite(write_buf, 1, buf_size, stdout),
79
                                                            ll rd=r1%d;
                                                       11

    write_pos = 0;

                                                            mod1/=d, mod2/=d, r1/=d, r2/=d;
                                                       12
      write_buf[write_pos++] = x;
80
                                                       13
   }
81
                                                             if (mod1<mod2)
                                                       14
                                                               swap(mod1,mod2), swap(r1,r2);
                                                       15
   template <class T>
    inline void writeInt( T x, char end ) {
                                                            11 k=(r2-r1)\%mod2;
      if (x < 0)
                                                            if (k<0)
                                                       18
        writeChar('-'), x = -x;
                                                              k+=mod2;
                                                       19
      char s[24];
                                                       21
                                                            11 x, y;
      int n = 0;
                                                            gcdex(mod1,mod2,x,y);
                                                       22
      while (x \mid | !n)
                                                            x\%=mod2;
                                                       23
        s[n++] = '0' + x \% 10, x /= 10;
                                                            if (x<0)
                                                       24
      while (n--)
                                                              x += mod2;
        writeChar(s[n]);
                                                            k*=x, k\%=mod2;
                                                       26
     if (end)
                                                            return mp(mod1*mod2*d,(k*mod1+r1)*d+rd);
                                                       27
        writeChar(end);
95
                                                          }
   }
96
                                                          // kto ends
   inline void writeWord( const char *s ) {
                                                          // FFT begins
      while (*s)
        writeChar(*s++);
100
                                                          const int LOG = 19;
    }
101
                                                          const int N = (1 \ll LOG);
102
    struct Flusher {
103
                                                          typedef std::complex<double> cd;
      ~Flusher() {
104
        if (write_pos)
105
                                                          int rev[N];
          fwrite(write_buf, 1, write_pos, stdout),
106
                                                          cd W[N];
          \rightarrow write_pos = 0;
                                                       10
      }
                                                          void precalc() {
    } flusher;
108
                                                            const double pi = std::acos(-1);
109
                                                            for (int i = 0; i != N; ++i)
                                                       13
110
                                                              W[i] = cd(std::cos(2 * pi * i / N),
                                                       14
   // sk fast read-write ends
111
                                                               \rightarrow std::sin(2 * pi * i / N));
   // extended euclid begins
                                                            int last = 0;
                                                       16
   int gcd (int a, int b, int & x, int & y) {
                                                            for (int i = 1; i != N; ++i) {
                                                       17
     if (a == 0) {
                                                             if (i == (2 << last))
                                                       18
        x = 0; y = 1;
                                                                 ++last;
                                                       19
        return b;
                                                       20
```

```
rev[i] = rev[i ^ (1 << last)] | (1 << (LOG <math>_{-24}
                                                            for (int i = 0; i < n; i++) {
           1 - last));
                                                              rows[i] = rs[i];
     }
                                                              for (auto& el: rows[i]) {
22
   }
                                                                if (el.first < n) {</pre>
                                                      27
23
                                                                  el.first = invP[el.first];
   void fft(vector<cd>& a) {
25
     for (int i = 0; i != N; ++i)
                                                              }
26
                                                      30
       if (i < rev[i])</pre>
                                                            }
27
                                                      31
         std::swap(a[i], a[rev[i]]);
29
     for (int lvl = 0; lvl != LOG; ++lvl)
                                                            for (int i = 0; i < n; i++) {
                                                      34
       for (int start = 0; start != N; start += (2 35
                                                              for (auto& el: rows[i]) {
        a[el.first] = el.second;
         for (int pos = 0; pos != (1 << lv1); ++pos)
32
                                                              while (true) {
          ← {
           cd x = a[start + pos];
                                                                int k = -1;
33
           cd y = a[start + pos + (1 << lv1)];
                                                                for (auto& el: rows[i]) {
                                                                  if (!isZero(a[el.first]) &&
                                                      41
           y *= W[pos << (LOG - 1 - lvl)];

    toZero[el.first] != -1 &&
                                                                    (k == -1 || toZero[el.first] <</pre>
           a[start + pos] = x + y;

    toZero[k])) {

           a[start + pos + (1 << lvl)] = x - y;
                                                                    k = el.first;
39
                                                      43
         }
                                                                  }
40
                                                      44
                                                                }
   }
41
                                                                if (k == -1)
42
   void inv_fft(vector<cd>& a) {
                                                                  break;
43
                                                      47
     fft(a);
44
     std::reverse(a.begin() + 1, a.end());
                                                                int j = toZero[k];
                                                                elem_t c = a[k];
     for (cd& elem: a)
47
                                                      51
                                                                for (auto el: rows[j]) {
       elem /= N;
48
                                                                  if (isZero(a[el.first]))
                                                                    rows[i].emplace_back(el.first, 0);
                                                      54
   // FFT ends
                                                                  a[el.first] = sub(a[el.first], mult(c,
                                                      55

→ el.second));
   // fast gauss begins
                                                                }
                                                      57
   using elem_t = int;
                                                                auto cond = [&a](const pair<int, elem_t>&
   // a[i][rows[i][j].first]=rows[i][j].second;
                                                                → p) { return isZero(a[p.first]); };
    \rightarrow b[i]=a[i][n]
   bool gauss(vector<vector<pair<int, elem_t>>>
                                                                    rows[i].erase(std::remove_if(rows[i].begin(),
   → rows, vector<elem_t> &res) {
                                                                    rows[i].end(), cond), rows[i].end());
     int n = rows.size();
                                                              }
     res.resize(n + 1, 0);
                                                              bool ok = false;
                                                      62
     vector<int> p(n + 1);
                                                              for (auto& el: rows[i]) {
     iota(p.begin(), p.end(), 0);
10
                                                                if (el.first < n && !isZero(a[el.first])) {</pre>
     vector<int> toZero(n + 1, -1);
                                                                  toZero[el.first] = i;
     vector<int> zro(n + 1);
                                                                  zro[i] = el.first;
                                                      66
     vector<elem_t> a(n + 1);
13
                                                                    det = (det * a[el.first]) % MOD;
                                                      67
14
     // optional: sort rows
                                                                  elem_t c = divM(1, a[el.first]);
                                                                  for (auto& el: rows[i]) {
     sort(p.begin(), p.begin() + n, [&rows](int i,
                                                                    el.second = mult(a[el.first], c);
     → int j) { return rows[i].size() <</pre>
                                                                    a[el.first] = 0;
      → rows[j].size(); });
                                                                  }
     vector<int> invP(n + 1);
                                                      74
     vector<vector<pair<int, elem_t>>> rs(n);
19
                                                                  ok = true;
                                                      75
     for (int i = 0; i < n; i++) {
20
                                                                  break;
       invP[p[i]] = i;
21
                                                      77
       rs[i] = rows[p[i]];
22
                                                              }
                                                      78
     }
23
```

```
if (where [i] == -1)
        if (!ok) {
                                                                    return INF;
            det = 0;
                                                               return 1;
81
                                                         43
          return false;
                                                         44
        }
      }
                                                             // stable gauss ends
85
     res[n] = sub(0, 1);
                                                             // simple geometry begins
      for (int i = n - 1; i >= 0; i--) {
                                                             struct Point {
        int k = zro[i];
                                                          3
                                                               double x, y;
        for (auto& el : rows[i])
                                                               Point operator+(const Point& p) const { return
          if (el.first != k)
                                                                \hookrightarrow \{x + p.x, y + p.y\}; \}
            res[p[k]] = sub(res[p[k]],
                                                               Point operator-(const Point& p) const { return

→ mult(el.second, res[p[el.first]])); 6

                                                                \rightarrow \{x - p.x, y - p.y\}; \}
      }
92
                                                               Point operator*(const double d) const { return
93
                                                                \rightarrow {x * d, y * d}; }
     return true;
                                                               Point rotate() const { return {y, -x}; }
                                                               double operator*(const Point& p) const { return
                                                                \rightarrow x * p.x + y * p.y; }
   // fast gauss ends
                                                               double operator^(const Point& p) const { return
   // stable gauss begins
                                                                \rightarrow x * p.y - y * p.x; }
   // if at least one solution returns it in ans
                                                               double dist() const { return sqrt(x * x + y *
   int gauss (vector < vector <double> > a,
                                                                \rightarrow y); }
    _{\rightarrow} \quad \text{vector} {<} \underline{\text{double}}{>} \ \& \ \text{ans)} \ \{
                                                             };
                                                         12
      int n = (int) a.size();
                                                         13
      int m = (int) a[0].size() - 1;
                                                             struct Line {
                                                         14
                                                               double a, b, c;
                                                         15
      vector<int> where (m, -1);
                                                               Line(const Point& p1, const Point& p2) {
      for (int col=0, row=0; col<m && row<n; ++col) 1
                                                                 a = p1.y - p2.y;
        int sel = row;
                                                                 b = p2.x - p1.x;
        for (int i=row; i<n; ++i)</pre>
10
                                                                  c = - a * p1.x - b * p1.y;
          if (abs (a[i][col]) > abs (a[sel][col]))
            sel = i;
                                                                 double d = sqrt(sqr(a) + sqr(b));
12
        if (abs (a[sel][col]) < EPS)</pre>
13
                                                                  a /= d, b /= d, c /= d;
          continue:
                                                               }
        for (int i=col; i<=m; ++i)</pre>
                                                               bool operator | | (const Line& 1) const { return
          swap (a[sel][i], a[row][i]);
                                                                \rightarrow fabs(a * 1.b - 1.a * b) < EPS; }
        where[col] = row;
                                                               double dist(const Point& p) const { return
                                                                \rightarrow fabs(a * p.x + b * p.y + c); }
        for (int i=0; i<n; ++i)</pre>
                                                               Point operator^(const Line& 1) const {
                                                         26
          if (i != row) {
                                                                 return {(1.c * b - c * 1.b) / (a * 1.b - 1.a
20
            double c = a[i][col] / a[row][col];
21
                                                                  \rightarrow * b),
            for (int j=col; j<=m; ++j)</pre>
                                                                      (1.c * a - c * 1.a) / (1.a * b - a *
              a[i][j] -= a[row][j] * c;
                                                                       \rightarrow 1.b)};
          }
24
        ++row;
                                                               Point projection(const Point& p) const {
                                                                  return p - Point{a, b} * (a * p.x + b * p.y +
                                                         31
                                                                  \hookrightarrow c);
      ans.assign (m, 0);
                                                               }
28
      for (int i=0; i<m; ++i)</pre>
                                                             };
29
        if (where[i] != -1)
          ans[i] = a[where[i]][m] / a[where[i]][i]; _{35}
                                                             struct Circle {
      for (int i=0; i<n; ++i) {</pre>
                                                               Point c;
        double sum = 0;
                                                               double r;
        for (int j=0; j<m; ++j)</pre>
                                                               Circle(const Point& c, double r) : c(c), r(r)
          sum += ans[j] * a[i][j];
        if (abs (sum - a[i][m]) > EPS)
                                                               Circle(const Point& a, const Point& b, const
36
          return 0;
37
                                                                → Point& c) {
      }
                                                                 Point p1 = (a + b) * 0.5, p2 = (a + c) * 0.5;
                                                         40
                                                                 Point q1 = p1 + (b - a).rotate(), q2 = p2 +
39
                                                         41
      for (int i=0; i<m; ++i)</pre>
                                                                  \hookrightarrow (c - a).rotate();
40
```

```
this->c = Line(p1, q1) ^ Line(p2, q2);
                                                            int64_t operator^(const Point& p) const {
       r = (a - this -> c).dist();

→ return x * 111 * p.y - y * 111 * p.x; }

     }
                                                            int64_t dist() const { return x * 111 * x + y *
44
   };
                                                             → 111 * y; }
45
                                                            bool operator<(const Point& p) const { return x</pre>
   inline bool on_segment(const Point& p1, const
                                                             \rightarrow != p.x ? x < p.x : y < p.y; }
47
    → Point& p2, const Point& x, bool strictly) { 9
     if (fabs((p1 - x) ^ (p2 - x)) > EPS)
48
                                                          // all point on convex hull are included
       return false;
     return (p1 - x) * (p2 - x) < (strictly ? - EPS_{12})
                                                          vector<Point> convex_hull(vector<Point> pt) {

→ : EPS);

                                                            int n = pt.size();
                                                            Point p0 = *std::min_element(pt.begin(),
51
                                                       14

→ pt.end());
                                                            std::sort(pt.begin(), pt.end(), [&p0](const
   // in case intersection is not a segment
53
   inline bool intersect_segments(const Point& p1,
                                                             → Point& a, const Point& b) {

→ const Point& p2, const Point& q1, const

                                                              int64_t cp = (a - p0) ^ (b - p0);
                                                              return cp != 0 ? cp > 0 : (a - p0).dist() <
    → Point& q2, Point& x) {
    Line 11(p1, p2), 12(q1, q2);
                                                               \rightarrow (b - p0).dist();
     if (11 || 12) return false;
                                                            });
                                                       18
     x = 11 ^ 12;
     return on_segment(p1, p2, x, false);
                                                            int i = n - 1;
                                                            for (; i > 0 \&\& ((pt[i] - p0) ^ (pt[i - 1] -
                                                      21
59
                                                             \rightarrow p0)) == 0; i--);
60
   // in case circles are not equal
                                                            std::reverse(pt.begin() + i, pt.end());
   inline bool intersect_circles(const Circle& c1, 23

→ const Circle& c2, Point& p1, Point& p2) {
                                                            vector<Point> ch;
     double d = (c2.c - c1.c).dist();
                                                            for (auto& p : pt) {
     if (d > c1.r + c2.r + EPS || d < fabs(c1.r -
                                                              while (ch.size() > 1) {
      \hookrightarrow c2.r) - EPS)
                                                                auto& p1 = ch[(int) ch.size() - 1];
       return false;
                                                                auto& p2 = ch[(int) ch.size() - 2];
65
                                                      28
     double cosa = (sqr(d) + sqr(c1.r) - sqr(c2.r))_{29}
                                                                int64_t cp = (p1 - p2) ^ (p - p1);
                                                                if (cp >= 0) break;
      \rightarrow / (2 * c1.r * d);
     double l = c1.r * cosa, h = sqrt(sqr(c1.r) -
                                                                ch.pop_back();
      \rightarrow sqr(1));
     Point v = (c2.c - c1.c) * (1 / d), p = c1.c + 3v
                                                              ch.push_back(p);
     p1 = p + v.rotate() * h, p2 = p - v.rotate() *35
      \hookrightarrow h:
                                                            return ch;
     return true;
                                                      37
70
71
                                                          // convex hull ends
   inline bool intersect_circle_and_line(const
                                                          // convex hull trick begins
    \hookrightarrow Circle& c, const Line& l, Point& p1, Point& ^{1}
    → p2) {
                                                          typedef long long ftype;
     double d = 1.dist(c.c);
74
                                                          typedef complex<ftype> point;
     if (d > c.r + EPS)
75
                                                          #define x real
       return false;
                                                          #define y imag
     Point p = 1.projection(c.c);
     Point n{1.b, -1.a};
                                                          ftype dot(point const& a, point const& b) {
     double h = sqrt(sqr(c.r) - sqr(l.dist(c.c)));
                                                            return (conj(a) * b).x();
     p1 = p + n * h, p2 = p - n * h;
     return true;
81
82
                                                          ftype f(point const& a, int x) {
                                                       12
83
                                                            return dot(a, {compressed[x], 1});
                                                       13
   // simple geometry ends
                                                            //return\ dot(a, \{x, 1\});
                                                       14
   // convex hull begins
                                                      15
                                                          int pos = 0;
   struct Point {
     int x, y;
                                                         //(x, y) \rightarrow (k, b) \rightarrow kb + x
     Point operator-(const Point& p) const { return 9
                                                      20 struct li_chao { // for min
      \leftarrow \{x - p.x, y - p.y\}; \}
```

```
vector<point> line;
                                                               hull.pop_back();
21
                                                               vecs.pop_back();
     li_chao(int maxn) {
23
                                                       78
       line.resize(4 * maxn, {0, inf});
                                                             if(!hull.empty()) {
                                                               vecs.push_back(1i * (nw - hull.back()));
     void add_line(int v, int l, int r, int a, int 82
                                                             hull.push_back(nw);
27
      → b, point nw) {
       if (r <= a || b <= 1) return; // remove if no
        \rightarrow [a, b) query
                                                           int get(ftype x) {
                                                             point query = \{x, 1\};
       int m = (1 + r) >> 1;
                                                             auto it = lower_bound(vecs.begin(), vecs.end(),

    query, [](point a, point b) {
       if (!(a \le 1 \&\& r \le b)) \{ // remove if no ss
                                                               return cross(a, b) > 0;
32
        \rightarrow [a, b) query
                                                             });
          add_line(v + v + 1, l, m, a, b, nw);
                                                             return dot(query, hull[it - vecs.begin()]);
33
          add_line(v + v + 2, m, r, a, b, nw);
          return;
       }
                                                          // convex hull trick ends
                                                           // heavy-light begins
       bool lef = f(nw, 1) < f(line[v], 1);</pre>
       bool mid = f(nw, m) < f(line[v], m);</pre>
39
                                                           int sz[maxn];
40
       if (mid) swap(line[v], nw);
41
                                                           void dfs_sz(int v, int par = -1) {
                                                             sz[v] = 1;
       if (1 == r - 1)
43
                                                             for (int x : gr[v])
         return;
                                                               if (x != par) {
                                                                 dfs_sz(x, v);
        if (lef != mid)
46
                                                                 sz[v] += sz[x];
          add_line(v + v + 1, l, m, a, b, nw);
47
                                                       11
       else
48
                                                             for (int i = 0; i < gr[v].size(); i++)</pre>
                                                       12
          add_line(v + v + 2, m, r, a, b, nw);
                                                               if (gr[v][i] != par)
                                                       13
50
                                                               if (sz[gr[v][i]] * 2 >= sz[v]) {
                                                       14
51
                                                                 swap(gr[v][i], gr[v][0]);
                                                       15
     ftype get(int v, int 1, int r, int x) {
52
                                                                 break;
                                                       16
       if(1 == r - 1)
                                                               }
          return f(line[v], x);
                                                           }
       int m = (1 + r) / 2;
55
       if(x < m) {
                                                           int rev[maxn];
          return min(f(line[v], x), get(v + v + 1, 1^{20},
                                                           int t_in[maxn];
          \rightarrow m, x));
                                                           int upper[maxn];;
       } else {
                                                           int par[maxn];
          return min(f(line[v], x), get(v + v + 2, m_i^{23},
                                                           int dep[maxn];
          \rightarrow r, x));
60
                                                           int T = 0;
61
                                                       27
                                                           void dfs_build(int v, int uppr, int pr = -1) {
                                                       28
   } cdt(maxn);
                                                             rev[T] = v;
                                                       29
                                                             t_in[v] = T++;
                                                       30
   // convex hull with stack
                                                             dep[v] = pr == -1 ? 0 : dep[pr] + 1;
                                                       31
                                                             par[v] = pr;
   ftype cross(point a, point b) {
                                                             upper[v] = uppr;
     return (conj(a) * b).y();
                                                       34
69
                                                             bool first = true;
                                                       35
   vector<point> hull, vecs;
71
                                                             for (int x : gr[v])
                                                       37
                                                               if (x != pr) {
                                                       38
   void add_line(ftype k, ftype b) {
                                                                 dfs_build(x, first ? upper[v] : x, v);
                                                       39
     point nw = \{k, b\};
                                                                 first = false;
     while(!vecs.empty() && dot(vecs.back(), nw -
75
                                                               }
                                                       41
      \rightarrow hull.back()) < 0) {
                                                           }
                                                       42
```

```
int v = q.front();
                                                       25
43
   struct interval {
                                                               q.pop();
     int 1;
                                                               for (int x : gr[v]) {
45
                                                       27
                                                                 int to = eds[x].to;
     int r;
                                                       28
     bool inv; // should direction be reversed
                                                                 if (d[to] == -1 \&\& eds[x].c - eds[x].f >=
                                                                     (1 << k)){}
48
                                                                   d[to] = d[v] + 1;
49
                                                       30
   // node-weighted hld
                                                                    q.push(to);
50
                                                       31
   vector<interval> get_path(int a, int b) {
     vector<interval> front;
                                                               }
                                                       33
                                                             }
     vector<interval> back;
                                                       34
                                                       35
     while (upper[a] != upper[b]) {
                                                             return (d[t] != -1);
       if (dep[upper[a]] > dep[upper[b]]) {
                                                       37
56
          front.push_back({t_in[upper[a]], t_in[a], 38
57

    true});
                                                           int dfs(int v, int flow, int k) {
                                                             if (flow < (1 << k))
          a = par[upper[a]];
       } else {
                                                               return 0;
          back.push_back({t_in[upper[b]], t_in[b],
                                                             if (v == t)
          → false});
                                                               return flow;
          b = par[upper[b]];
                                                       44
                                                             for (; it[v] < gr[v].size(); it[v]++) {
       }
                                                               int num = gr[v][it[v]];
                                                       45
62
     }
                                                               if (d[v] + 1 != d[num].to])
63
                                                       46
                                                                 continue;
     front.push_back({min(t_in[a], t_in[b]),
                                                               int res = dfs(eds[num].to, min(flow,

    eds[num].c - eds[num].f), k);
      \rightarrow max(t_in[a], t_in[b]), t_in[a] > t_in[b]});
                                                               if (res){
     // for edge-weighted hld add:

→ "front.back().l++;"

                                                                 eds[num].f += res;
     front.insert(front.end(), back.rbegin(),
                                                                 eds[num ^ 1].f -= res;
                                                       51
      → back.rend());
                                                                 return res;
                                                       52
                                                               }
                                                       53
                                                             }
     return front;
                                                       55
                                                             return 0;
70
                                                       56
71
   // heavy-light ends
                                                       57
                                                           void add(int fr, int to, int c, int nm) {
   // max flow begins
                                                             gr[fr].push_back(num);
                                                       59
                                                             eds[num++] = edge(fr, to, c, nm);
                                                       60
   struct edge{
                                                             gr[to].push_back(num);
                                                       61
     int from, to;
                                                             eds[num++] = edge(to, fr, 0, nm); //corrected c
     int c, f, num;
                                                       63
     edge(int from, int to, int c, int
      \rightarrow num):from(from), to(to), c(c), f(0),
                                                           int ans = 0;
      _{\hookrightarrow} \quad \texttt{num(num)}\{\}
                                                             for (int k = 30; k >= 0; k--)
     edge(){}
                                                               while (bfs(k)) {
                                                       67
   };
                                                                 memset(it, 0, sizeof(it));
                                                       68
                                                                 while (int res = dfs(s, 1e9 + 500, k))
   const int max_n = 600;
                                                                    ans += res;
11
                                                       71
   edge eds[150000];
12
                                                       72
   int num = 0;
13
   int it[max_n];
                                                           // decomposition
                                                       74
   vector<int> gr[max_n];
                                                       75
   int s, t;
                                                           int path_num = 0;
                                                       76
   vector<int> d(max_n);
                                                           vector<int> paths[max_n];
                                                       77
                                                           int flows[max_n];
                                                       78
   bool bfs(int k) {
     queue<int> q;
20
                                                           int decomp(int v, int flow) {
     q.push(s);
21
                                                             if (flow < 1)
                                                       81
     fill(d.begin(), d.end(), -1);
22
                                                       82
                                                               return 0;
     d[s] = 0;
23
                                                             if (v == t) {
                                                       83
     while (!q.empty()) {
24
```

```
path_num++;
                                                              while (!s.empty() && used[(s.begin() ->
        flows[path_num - 1] = flow;

    second)]){
        return flow;
                                                                s.erase(s.begin());
86
                                                       41
      }
                                                              }
                                                       42
      for (int i = 0; i < gr[v].size(); i++) {</pre>
                                                              if (s.empty())
        int num = gr[v][i];
                                                                break;
                                                       44
        int res = decomp(eds[num].to, min(flow,
                                                              x = s.begin() -> second;
90
                                                       45
                                                              used[x] = true;

→ eds[num].f));
                                                       46
        if (res)
                                                              s.erase(s.begin());
                  {
          eds[num].f -= res;
                                                              for (int i = 0; i < gr[x].size(); i++){</pre>
          paths[path_num -
                                                                edge &e = edges[gr[x][i]];
          if (!used[e.to] && e.c - e.f > 0){
                                                                   if (d[e.to] > d[x] + (e.c - e.f) * e.w +
          return res;
                                                                   \rightarrow dist[x] - dist[e.to]){
95
      }
                                                                     d[e.to] = d[x] + (e.c - e.f) * e.w +
96

    dist[x] - dist[e.to];

97
      return 0;
                                                                    p[e.to] = gr[x][i];
                                                                     s.insert(make_pair(d[e.to], e.to));
                                                      54
   while (decomp(s, 1e9 + 5));
100
                                                      55
                                                                }
                                                       56
                                                              }
   // max flow ends
                                                       57
                                                              dist[x] += d[x];
                                                      58
   // min-cost flow begins
                                                      59
                                                            int pos = t;
   long long ans = 0;
                                                            while (pos != st){
                                                      61
   int mx = 2 * n + 2;
                                                              int id = p[pos];
                                                      62
                                                              edges[id].f += 1;
                                                      63
   memset(upd, 0, sizeof(upd));
                                                              edges[id ^ 1].f -= 1;
                                                      64
   for (int i = 0; i < mx; i++)
                                                              pos = edges[id].from;
                                                      65
     dist[i] = inf;
                                                      66
   dist[st] = 0;
                                                          }
                                                      67
   queue<int> q;
   q.push(st);
11
                                                          // min-cost flow ends
   upd[st] = 1;
12
                                                          // min circulation begins
   while (!q.empty()){
13
      int v = q.front();
                                                          class Solver2 { // Min-cost circulation
14
                                                       2
      q.pop();
                                                            struct Edge {
15
      if (upd[v]){
                                                              int to, ne, w, c;
        for (int x : gr[v]) {
                                                            };
17
          edge &e = edges[x];
                                                            vector<Edge> es;
          if (e.c - e.f > 0 && dist[v] != inf &&
                                                            vi firs;

    dist[e.to] > dist[v] + e.w) {

                                                            int curRes;
            dist[e.to] = dist[v] + e.w;
                                                            public:
20
            if (!upd[e.to])
                                                            Solver2(int n) : es(), firs(n, -1),
                                                       10
21
                                                          curRes(0) {}
              q.push(e.to);
            upd[e.to] = true;
                                                            // from, to, capacity (max.flow), cost
                                                      12
            p[e.to] = x;
                                                            int adde(int a, int b, int w, int c) {
                                                      13
          }
                                                       14
        }
                                                              e.to = b; e.ne = firs[a];
                                                       15
26
        upd[v] = false;
                                                              e.w = w; e.c = c;
27
                                                      16
                                                              es.pb(e);
28
                                                      17
29
                                                       18
                                                              firs[a] = sz(es) - 1;
   for (int i = 0; i < k; i++){
                                                              e.to = a; e.ne = firs[b];
                                                      20
31
      for (int i = 0; i < mx; i++)</pre>
                                                              e.w = 0; e.c = -c;
                                                      21
        d[i] = inf;
                                                      22
                                                              es.pb(e);
33
      d[st] = 0;
                                                              firs[b] = sz(es) - 1;
                                                      23
     memset(used, false, sizeof(used));
                                                              return sz(es) - 2;
                                                      24
35
      set<pair<int, int> > s;
36
                                                      25
                                                            // increase capacity of edge 'id' by 'w'
      s.insert(make_pair(0, st));
37
      for (int i = 0; i < mx; i++){
                                                            void ince(int id, int w) {
                                                      27
        int x;
                                                              es[id].w += w;
                                                      28
```

```
}
                                                           int prev = -1;
     int solve() {
                                                            memset(inA, 0, sizeof(inA));
       const int n = sz(firs);
                                                            memset(w, 0, sizeof(w));
31
                                                       11
                                                            for (int it = 0; it < curN; it++) {</pre>
                                                       12
       for (;;) {
                                                               int best = -1;
                                                       13
          vi d(n, 0), fre(n, -1);
                                                               for (int i = 0; i < n; i++)
                                                       14
                                                                 if (ex[i] && !inA[i]
         vi chd(n, -1);
                                                       15
35
                                                                     && (best == -1 \mid \mid w[i] > w[best]))
                                                       16
          int base = -1;
                                                                    best = i;
          for (int step = 0; step < n; step++) {</pre>
                                                               assert(best != -1);
            for (int i = 0; i < sz(es); i++) if</pre>
                                                               if (it == curN - 1) {
                                                       19
            \rightarrow (es[i].w > 0) {
                                                                 ans = min(ans, w[best]);
                                                                 for (int i = 0; i < n; i++){</pre>
              int b = es[i].to;
              int a = es[i ^ 1].to;
                                                                   g[i][prev] += g[best][i];
41
              if (d[b] <= d[a] + es[i].c) continue; 23
                                                                   g[prev][i] = g[i][prev];
              d[b] = d[a] + es[i].c;
              fre[b] = i;
                                                                 ex[best] = false;
              if (step == n - 1)
                                                               } else {
                base = b;
                                                                 inA[best] = true;
                                                      27
                                                                 for (int i = 0; i < n; i++)</pre>
           }
                                                       28
          }
                                                                   w[i] += g[best][i];
         if (base < 0) break;
                                                                   prev = best;
49
                                                       30
                                                               }
50
                                                       31
                                                            }
          vi seq;
51
                                                          }
          vb was(n, false);
         for (int x = base;; x = es[fre[x] ^ 1].to)_{34}
                                                          void solve(){
                                                            ans = INF;
            if (!was[x]) {
                                                            for (int i = n; i > 1; i--)
              seq.pb(x);
                                                               iterate(i);
                                                      37
55
              was[x] = true;
                                                            cout << ans << endl;</pre>
56
                                                      38
            } else {
                                                      39
              seq.erase(
                                                          // global cut ends
                  seq.begin(),
                                                          // bad hungarian begins
                                                       1
                  find(seq.begin(), seq.end(),
                                                          fill(par, par + 301, -1);
              ));
                                                          fill(par2, par2 + 301, -1);
              break;
           }
                                                          int ans = 0;
         }
                                                          for (int v = 0; v < n; v++){
         for (int i = 0; i < sz(seq); i++) {</pre>
                                                            memset(useda, false, sizeof(useda));
            int v = seq[i];
                                                            memset(usedb, false, sizeof(usedb));
            int eid = fre[v];
                                                            useda[v] = true;
                                                       10
            assert(es[eid].w > 0);
                                                            for (int i = 0; i < n; i++)</pre>
                                                       11
            es[eid].w--;
                                                              w[i] = make_pair(a[v][i] + row[v] + col[i],
            es[eid ^1].w++;
71
            curRes += es[eid].c;
72
                                                            memset(prev, 0, sizeof(prev));
                                                       13
                                                             int pos;
                                                       14
       }
                                                             while (true){
                                                       15
       return curRes;
75
                                                               pair<pair<int, int>, int> p =
                                                       16
     }
76

→ make_pair(make_pair(1e9, 1e9), 1e9);
   };
                                                               for (int i = 0; i < n; i++)
   // min circulation ends
                                                                 if (!usedb[i])
   // global cut begins
                                                                   p = min(p, make_pair(w[i], i));
   // g[i][j] = g[j][i] is sum of edges between i 20
                                                               for (int i = 0; i < n; i++)
                                                                 if (!useda[i])
      and j
   // ans is value of mincut
                                                                   row[i] += p.first.first;
                                                              for (int i = 0; i < n; i++)
   11 g[maxn] [maxn], w[maxn];
                                                      23
   ll ans;
                                                                 if (!usedb[i]){
                                                      24
bool ex[maxn], inA[maxn];
                                                                   col[i] -= p.first.first;
                                                      25
                                                                   w[i].first -= p.first.first;
                                                       26
                                                                 }
  void iterate(int curN){
                                                       27
```

```
ans += p.first.first;
                                                          int find_path(int root) {
       usedb[p.second] = true;
                                                             memset(used, 0, sizeof(used));
                                                             memset(p, -1, sizeof p);
       prev[p.second] = p.first.second; //us emopoŭe
30
                                                             for (int i = 0; i < N; i++)</pre>
        ⊶ в первую
        int x = par[p.second];
                                                               base[i] = i;
       if (x == -1){
                                                       39
                                                             used[root] = true;
         pos = p.second;
33
                                                       40
                                                             int qh = 0, qt = 0;
         break;
                                                       41
34
                                                             q[qt++] = root;
       useda[x] = true;
                                                             while (qh < qt) {
                                                       43
       for (int j = 0; j < n; j++)
                                                               int v = q[qh++];
                                                       44
          w[j] = min(w[j], \{a[x][j] + row[x] +
                                                               for (int to : gr[v]) {
                                                       45
                                                                 if (base[v] == base[to] || match[v] == to)
          \rightarrow col[j], x});

→ continue;

39
     }
                                                                 if (to == root || match[to] != -1 &&
40
     while (pos != -1){
                                                                 \rightarrow p[match[to]] != -1) {
41
       int nxt = par2[prev[pos]];
                                                                   int curbase = lca(v, to);
       par[pos] = prev[pos];
                                                                   memset(blossom, 0, sizeof(blossom));
       par2[prev[pos]] = pos;
                                                                   mark_path(v, curbase, to);
                                                       50
                                                                   mark_path(to, curbase, v);
       pos = nxt;
45
                                                       51
46
     }
                                                                   for (int i = 0; i < N; i++)
                                                                     if (blossom[base[i]]) {
47
                                                       53
   cout << ans << "\n";
                                                                       base[i] = curbase;
48
                                                       54
   for (int i = 0; i < n; i++)
                                                                       if (!used[i]) {
     cout << par[i] + 1 << "" << i + 1 << "\n";
                                                                         used[i] = true;
                                                                          q[qt++] = i;
                                                       57
   // bad hungarian ends
                                                                     }
   // Edmonds O(n^3) begins
                                                                 } else if (p[to] == -1) {
                                                                   p[to] = v;
                                                       61
   vector<int> gr[MAXN];
                                                                   if (match[to] == -1)
   int match[MAXN], p[MAXN], base[MAXN], q[MAXN];
                                                                     return to;
   bool used[MAXN], blossom[MAXN];
                                                                   to = match[to];
                                                       64
   int mark[MAXN];
                                                                   used[to] = true;
                                                       65
   int C = 1;
                                                                   q[qt++] = to;
   int lca(int a, int b) {
                                                               }
                                                       68
     C++;
10
                                                             }
                                                       69
     for (;;) {
11
                                                       70
       a = base[a];
                                                             return -1;
       mark[a] = C;
                                                       72
       if (match[a] == -1) break;
14
                                                       73
       a = p[match[a]];
15
                                                          memset(match, -1, sizeof match);
                                                       74
16
                                                             for (int i = 0; i < N; i++) {
                                                       75
17
                                                               if (match[i] == -1 \&\& !gr[i].empty()) {
                                                       76
     for (;;) {
18
                                                                 int v = find_path(i);
                                                       77
       b = base[b];
19
                                                                 while (v != -1) {
       if (mark[b] == C) return b;
                                                                   int pv = p[v], ppv = match[pv];
       b = p[match[b]];
21
                                                                   match[v] = pv; match[pv] = v;
                                                       80
22
                                                                   v = ppv;
                                                       81
   }
23
                                                                 }
                                                       83
   void mark_path(int v, int b, int children) {
                                                             }
     while (base[v] != b) {
26
       blossom[base[v]] = blossom[base[match[v]]] = 86
27
                                                          // Edmonds O(n^3) ends

    true;

       p[v] = children;
                                                          // string basis begins
       children = match[v];
                                                       2
29
                                                          vector<int> getZ(string s){
       v = p[match[v]];
30
     }
                                                            vector<int> z;
31
   }
                                                             z.resize(s.size(), 0);
32
                                                             int 1 = 0, r = 0;
33
```

```
for (int i = 1; i < s.size(); i++){
                                                               }
       if (i <= r)
                                                               while (i \le k) i += j - k;
          z[i] = min(r - i + 1, z[i - 1]);
                                                             }
       while (i + z[i] < s.size() \&\& s[z[i]] == s[i]
                                                             return s.substr (ans, n/2);
        \rightarrow + z[i]])
         z[i]++;
11
       if (i + z[i] - 1 > r){
                                                          // min cyclic shift ends
12
         r = i + z[i] - 1;
13
                                                          // suffix array O(n) begins
          1 = i;
       }
15
                                                           typedef vector<char> bits;
     }
16
                                                       4
     return z;
17
                                                           template<const int end>
18
                                                           void getBuckets(int *s, int *bkt, int n, int K) {
19
                                                             fill(bkt, bkt + K + 1, 0);
   vector<int> getP(string s){
20
                                                             forn(i, n) bkt[s[i] + !end]++;
     vector<int> p;
21
                                                             forn(i, K) bkt[i + 1] += bkt[i];
     p.resize(s.size(), 0);
                                                       10
     int k = 0;
23
                                                           void induceSAl(bits &t, int *SA, int *s, int
                                                       11
     for (int i = 1; i < s.size(); i++){</pre>
                                                           → *bkt, int n, int K) {
       while (k > 0 \&\& s[i] == s[k])
                                                       12
                                                             getBuckets<0>(s, bkt, n, K);
         k = p[k - 1];
                                                             forn(i, n) {
       if (s[i] == s[k])
27
                                                               int j = SA[i] - 1;
                                                       14
         k++;
28
                                                               if (j >= 0 \&\& !t[j])
                                                       15
       p[i] = k;
                                                                 SA[bkt[s[j]]++] = j;
                                                       16
     }
30
                                                             }
                                                       17
     return p;
31
                                                           }
                                                       18
   }
32
                                                           void induceSAs(bits &t, int *SA, int *s, int
                                                           → *bkt, int n, int K) {
   vector<int> getH(string s){
34
                                                             getBuckets<1>(s, bkt, n, K);
     vector<int> h;
35
                                                             for (int i = n - 1; i >= 0; i--) {
     h.resize(s.size() + 1, 0);
36
                                                               int j = SA[i] - 1;
     for (int i = 0; i < s.size(); i++)
                                                               if (j >= 0 \&\& t[j])
       h[i + 1] = ((h[i] * 111 * pow) + s[i] - 'a' ^{23}
                                                                 SA[--bkt[s[j]]] = j;
                                                       24
        \rightarrow 1) % mod;
                                                       25
     return h;
39
                                                           }
                                                       26
   }
41
                                                           void SA_IS(int *s, int *SA, int n, int K) { //
   int getHash(vector<int> &h, int 1, int r){
42
                                                           → require last symbol is 0
     int res = (h[r + 1] - h[1] * p[r - 1 + 1]) %
43
                                                           #define isLMS(i) (i \&\&\ t[i]\ \&\&\ !t[i-1])
      \rightarrow mod;
                                                             int i, j;
     if (res < 0)
44
                                                       31
                                                             bits t(n);
       res += mod;
45
                                                             t[n-1] = 1;
                                                       32
     return res;
                                                             for (i = n - 3; i >= 0; i--)
                                                       33
   }
47
                                                               t[i] = (s[i] < s[i+1] \mid | (s[i] = s[i+1]) \&\&
48
                                                                \rightarrow t[i+1]==1));
   // string basis ends
                                                             int bkt[K + 1];
                                                       35
   // min cyclic shift begins
                                                             getBuckets<1>(s, bkt, n, K);
                                                             fill(SA, SA + n, -1);
                                                       37
   string min_cyclic_shift (string s) {
                                                             forn(i, n)
                                                       38
     s += s;
                                                               if (isLMS(i))
                                                       39
     int n = (int) s.length();
                                                                 SA[--bkt[s[i]]] = i;
     int i=0, ans=0;
                                                             induceSAl(t, SA, s, bkt, n, K);
     while (i < n/2) {
                                                             induceSAs(t, SA, s, bkt, n, K);
                                                       42
       ans = i;
                                                             int n1 = 0;
                                                       43
       int j=i+1, k=i;
                                                             forn(i, n)
       while (j < n \&\& s[k] <= s[j]) {
                                                               if (isLMS(SA[i]))
                                                       45
         if (s[k] < s[j])
                                                                 SA[n1++] = SA[i];
11
                                                       46
           k = i;
                                                             fill(SA + n1, SA + n, -1);
12
                                                       47
                                                             int name = 0, prev = -1;
          else
            ++k;
                                                             forn(i, n1) {
                                                       49
          ++j;
                                                               int pos = SA[i];
                                                       50
```

```
bool diff = false;
                                                                for (p = 0, i = N - j; i < N; i++) y[p++] =
51
        for (int d = 0; d < n; d++)

   i:

          if (prev == -1 || s[pos+d] != s[prev+d] |t_{05}
                                                                for (int k = 0; k < N; ++k) if (SA[k] >= j)

    t[pos+d] != t[prev+d])

                                                                 \rightarrow y[p++] = SA[k] - j;
            diff = true, d = n;
                                                                Sort();
          else if (d > 0 && (isLMS(pos+d) ||
                                                                for (swap (x, y), p = 1, x[SA[0]] = 0, i = 1;
                                                        107
                                                                 \rightarrow i < N; ++i) x[SA [i]] = cmp (SA[i - 1],

    isLMS(prev+d)))

                                                                 \hookrightarrow SA[i], j) ? p - 1 : p++;
            d = n;
56
        if (diff)
                                                              }
          name++, prev = pos;
                                                            }
                                                        109
        SA[n1 + (pos >> 1)] = name - 1;
                                                        110
                                                            // common for all algorithms
                                                        111
      for (i = n - 1, j = n - 1; i >= n1; i--)
                                                            void kasaiLCP () {
                                                        112
61
                                                              for (int i = 0; i < N; i++) c[SA[i]] = i;</pre>
        if (SA[i] >= 0)
                                                        113
62
          SA[j--] = SA[i];
                                                              for (int i = 0, j, k = 0; i < N; LCP [c[i++]] =
63
                                                        114
      int *s1 = SA + n - n1;
64
                                                                if (c [i] > 0) for (k ? k-- : 0, j = SA[c[i]]
      if (name < n1)
        SA_IS(s1, SA, n1, name - 1);
                                                                 \rightarrow -1]; str[i + k] == str[j + k]; k++);
                                                                else k = 0;
      else
                                                        116
        forn(i, n1)
                                                        117
          SA[s1[i]] = i;
                                                        118
      getBuckets<1>(s, bkt, n, K);
                                                            void suffixArray () { // require last symbol is
                                                        119
70
      for (i = 1, j = 0; i < n; i++)
                                                            \hookrightarrow char(0)
71
        if (isLMS(i))
                                                              m = 256;
72
          s1[j++] = i;
                                                              N = str.size();
                                                        121
      forn(i, n1)
                                                              DA ();
74
                                                        122
        SA[i] = s1[SA[i]];
                                                              kasaiLCP ();
                                                        123
                                                            }
      fill(SA + n1, SA + n, -1);
                                                        124
      for (i = n1 - 1; i \ge 0; i--) {
                                                            // suffix array O(n log n) ends
                                                        125
77
        j = SA[i], SA[i] = -1;
                                                            // bad suffix automaton begins
        SA[--bkt[s[j]]] = j;
79
                                                            struct node{
      induceSAl(t, SA, s, bkt, n, K);
81
                                                              map<char, int> go;
      induceSAs(t, SA, s, bkt, n, K);
                                                              int len, suff;
   }
                                                              long long sum_in;
    // suffix array O(n) ends
                                                              node(){}
                                                            };
    // suffix array O(n log n) begins
86
   string str;
                                                            node v[max_n * 4];
    int N, m, SA [MAX_N], LCP [MAX_N];
    int x [MAX_N], y [MAX_N], w [MAX_N], c [MAX_N]; ^{\scriptscriptstyle 11}
                                                            int add_node(int max_len){
                                                              //v[number].sum_in = 0;
    inline bool cmp (const int a, const int b, const^{13}
                                                              v[number].len = max_len;
    \rightarrow int 1) { return (y [a] == y [b] && y [a + 1]^{14}
                                                              v[number].suff = -1;
    \rightarrow == y [b + 1]); }
                                                              number++;
                                                        16
92
                                                              return number - 1;
                                                        17
   void Sort () {
                                                            }
                                                        18
      for (int i = 0; i < m; ++i) w[i] = 0;
                                                        19
      for (int i = 0; i < N; ++i) ++w[x[y[i]]];
                                                            int last = add_node(0);
                                                        20
      for (int i = 0; i < m - 1; ++i) w[i + 1] +=
                                                        21

    w[i];
                                                            void add_char(char c) {
      for (int i = N - 1; i >= 0; --i)
                                                              int cur = last;
      \Rightarrow SA[--w[x[y[i]]]] = y[i];
                                                              int new_node = add_node(v[cur].len + 1);
                                                        24
   }
98
                                                              last = new_node;
                                                        25
                                                              while (cur != -1){
   void DA () {
100
                                                                if (v[cur].go.count(c) == 0){
      for (int i = 0; i < N; ++i) x[i] = str[i], y[i]
101
                                                                  v[cur].go[c] = new_node;
      //v[new\_node].sum\_in += v[cur].sum\_in;
      Sort ();
                                                                  cur = v[cur].suff;
      for (int i, j = 1, p = 1; p < N; j <<= 1, m = ^{30}
103
                                                                  if (cur == -1)
      → p) {
                                                                     v[new_node].suff = 0;
                                                        32
```

```
}else{
          int a = v[cur].go[c];
                                                          int go(int v, char c);
          if (v[a].len == v[cur].len + 1){
                                                       40
            v[new\_node].suff = a;
                                                          int get_link(int v) {
                                                       41
          }else{
                                                            if (t[v].link == -1)
                                                       42
            int b = add_node(v[cur].len + 1);
                                                               if (v == 0 || t[v].p == 0)
                                                       43
            v[b].go = v[a].go;
                                                                 t[v].link = 0;
                                                       44
            v[b].suff = v[a].suff;
                                                               else
            v[new_node].suff = b;
                                                                 t[v].link = go(get_link(t[v].p), t[v].pch);
            while (cur != -1 && v[cur].go.count(c) !#
                                                             return t[v].link;
            \rightarrow 0 && v[cur].go[c] == a){
                                                          }
                                                       48
              v[cur].go[c] = b;
              //v[a].sum_in -= v[cur].sum_in;
                                                          int go(int v, char c) {
              //v[b].sum_in += v[cur].sum_in;
                                                            if (t[v].go[c] == -1)
                                                      51
45
              cur = v[cur].suff;
                                                               if (t[v].next[c] != -1)
                                                       52
46
                                                                 t[v].go[c] = t[v].next[c];
47
                                                       53
            v[a].suff = b;
         }
                                                                 t[v].go[c] = v == 0 ? 0 : go(get_link(v),
                                                       55
49
                                                                 return;
50
                                                            return t[v].go[c];
51
52
     }
                                                       57
53
                                                       58
                                                          // aho-corasick ends
54
   // bad suffix automaton ends
                                                          // pollard begins
   // aho-corasick begins
                                                          const int max_step = 4e5;
                                                       3
   struct vertex {
     int next[K];
                                                          unsigned long long gcd(unsigned long long a,
     bool leaf;

    unsigned long long b){
                                                            if (!a) return 1;
     int p;
     char pch;
                                                            while (a) swap(a, b\%=a);
     int link;
                                                            return b;
                                                          }
     int go[K];
   };
10
                                                       10
                                                          unsigned long long get (unsigned long long a,
11
   vertex t[NMAX+1];

    unsigned long long b){
   int sz;
                                                            if (a > b)
                                                       12
                                                               return a-b;
                                                       13
   void init() {
                                                             else
                                                       14
     t[0].p = t[0].link = -1;
                                                       15
                                                               return b-a;
     memset(t[0].next, 255, sizeof t[0].next);
                                                       16
17
     memset(t[0].go, 255, sizeof t[0].go);
                                                       17
18
                                                          unsigned long long pollard(unsigned long long n){
     sz = 1;
19
                                                       18
                                                            unsigned long long x = (rand() + 1) \% n, y = 1,
   }

    g;

21
   void add_string(const string & s) {
                                                             int stage = 2, i = 0;
     int v = 0;
                                                             g = gcd(get(x, y), n);
     for (size_t i=0; i<s.length(); ++i) {</pre>
                                                            while (g == 1) {
24
       char c = s[i] - 'a';
                                                               if (i == max_step)
25
       if (t[v].next[c] == -1) {
                                                                 break;
26
                                                       24
         memset(t[sz].next, 255, sizeof t[sz].next)2;
                                                               if (i == stage) {
         memset(t[sz].go, 255, sizeof t[sz].go);
                                                                 y = x;
         t[sz].link = -1;
                                                                 stage <<= 1;
          t[sz].p = v;
                                                       28
          t[sz].pch = c;
                                                              x = (x * (__int128)x + 1) \% n;
                                                       29
          t[v].next[c] = sz++;
                                                       30
                                                               g = gcd(get(x, y), n);
                                                      31
33
        v = t[v].next[c];
                                                            }
34
                                                      32
                                                            return g;
35
                                                       33
     t[v].leaf = true;
                                                          }
                                                      34
36
   }
37
                                                       35
```

```
// pollard ends
                                                            phi -= phi/nn;
                                                            num++;
   // linear sieve begins
                                                      18
                                                          if (!((num == 1 && n % 2 != 0) || n == 4 || n ==
   const int N = 1000000;
                                                             2 || (num == 2 && n % 2 == 0 && n % 4 != 0)))
   int pr[N + 1], sz = 0;
                                                            cout << "-1\n";
   /* minimal prime, mobius function, euler function
                                                            continue;
    → */
   int lp[N + 1], mu[N + 1], phi[N + 1];
                                                          vector<long long> v;
                                                          long long pp = phi;
   lp[1] = mu[1] = phi[1] = 1;
                                                          for (long long x:primes){
   for (int i = 2; i <= N; ++i) {
                                                            if (x*x>pp)
     if (lp[i] == 0)
                                                              break;
                                                      27
       lp[i] = pr[sz++] = i;
                                                            if (pp \% x == 0){
12
     for (int j = 0; j < sz && pr[j] <= lp[i] && i **
                                                              while (pp \% x == 0)
13
      \rightarrow pr[j] <= N; ++j)
                                                               pp /= x;
       lp[i * pr[j]] = pr[j];
                                                              v.push_back(x);
15
     mu[i] = lp[i] == lp[i / lp[i]] ? 0 : -1 * mu[i_{33}]
                                                         }
16
     → / lp[i]];
                                                          if (pp != 1){
     phi[i] = phi[i / lp[i]] * (lp[i] == lp[i /
                                                            v.push_back(pp);
                                                      35
      → lp[i]] ? lp[i] : lp[i] - 1);
                                                      36
                                                          while (true){
18
19
                                                            long long a = primes[rand()%5000]%n;
   // linear sieve ends
                                                            if (gcd(a, n) != 1)
                                                      39
                                                              continue;
                                                      40
   // discrete log in sqrt(p) begins
                                                            bool bb = false;
                                                            for (long long x:v)
                                                      42
   int k = sqrt((double)p) + 2;
                                                              if (pow(a, phi/x) == 1){
                                                      43
                                                                bb = true;
                                                      44
   for (int i = k; i >= 1; i--)
                                                                break;
     mp[bin(b, (i * 111 * k) \% (p-1), p)] = i;
                                                              }
                                                            if (!bb){
                                                      47
   bool answered = false;
                                                              cout << a << ''\n'';
                                                      48
   int ans = INT32_MAX;
                                                              break;
   for (int i = 0; i <= k; i++){
10
                                                      50
     int sum = (n * 111 * bin(b, i, p)) % p;
                                                      51
     if (mp.count(sum) != 0){
       int an = mp[sum] * 111 * k - i;
                                                         // prime roots mod n ends
       if (an < p)
14
         ans = min(an, ans);
                                                      1
                                                          // simplex begins
15
     }
                                                      2
                                                         const double EPS = 1e-9;
   }
17
                                                         typedef vector<double> vdbl;
   // discrete log in sqrt(p) ends
                                                          // n variables, m inequalities
   // prime roots mod n begins
                                                          // Ax <= b, c*x -> max, x >= 0
                                                          double simplex( int n, int m, const vector<vdbl>
   int num = 0;
                                                          \leftrightarrow &a0, const vdbl &b, const vdbl &c, vdbl &x )
   long long phi = n, nn = n;
   for (long long x:primes){
                                                          ← {
     if (x*x>nn)
                                                            // Ax + Ez = b, A[m]*x -> max
       break;
                                                            // x = 0, z = b, x \ge 0, z \ge 0
                                                      11
     if (nn \% x == 0){
                                                            vector\langle vdbl \rangle a(m + 2, vdbl(n + m + 2));
                                                      12
       while (nn \% x == 0)
                                                            vector<int> p(m);
                                                      13
         nn /= x;
                                                            forn(i, n)
                                                      14
                                                              a[m + 1][i] = c[i];
       phi -= phi/x;
11
                                                      15
                                                            forn(i, m) {
       num++;
12
                                                      16
     }
                                                              forn(j, n)
13
                                                      17
   }
                                                                a[i][j] = a0[i][j];
14
                                                      18
   if (nn != 1){
                                                              a[i][n + i] = 1;
                                                      19
```

```
a[i][m + n] = -1;
                                                            swap(a[m], a[m + 1]);
       a[i][m + n + 1] = b[i];
                                                            if (!iterate(n + m))
       p[i] = n + i;
                                                              return INFINITY;
22
                                                      79
                                                            x = vdbl(n, 0);
                                                            forn(i, m)
     // basis: enter "j", leave "ind+n"
                                                              if (p[i] < n)
                                                      82
     auto pivot = [&]( int j, int ind ) {
                                                                x[p[i]] = a[i][n + m + 1];
26
                                                      83
                                                            return -a[m][n + m + 1];
       double coef = a[ind][j];
                                                      84
27
       assert(fabs(coef) > EPS);
                                                          }
                                                      85
       forn(col, n + m + 2)
29
          a[ind][col] /= coef;
                                                          // simplex usage:
       forn(row, m + 2)
                                                          vdbl x(n);
          if (row != ind && fabs(a[row][j]) > EPS) \{89
                                                          double result = simplex(n, m, a, b, c, x);
            coef = a[row][j];
                                                          if (isinf(result))
33
                                                            puts(''Unbounded'');
            forn(col, n + m + 2)
                                                      91
              a[row][col] -= a[ind][col] * coef;
                                                          else if (isnan(result))
                                                      92
            a[row][j] = 0; // reduce precision erroms
                                                            puts(''No solution'');
          }
                                                          else {
       p[ind] = j;
                                                            printf("%.9f :", result);
                                                      95
     };
                                                            forn(i, n)
                                                              printf(" %.9f", x[i]);
                                                      97
     // the Simplex itself
                                                            puts('"');
                                                      98
41
     auto iterate = [&]( int nn ) {
42
                                                      99
       for (int run = 1; run; ) {
43
                                                      100
          run = 0;
                                                          // simplex ends
          forn(j, nn) {
45
                                                         // sum over subsets begins
            if (a[m][j] > EPS) { // strictly positive
                                                          // fast subset convolution O(n \ 2^n)
                                                         for(int i = 0; i < (1 << N); ++i)
              double mi = INFINITY, t;
                                                            F[i] = A[i];
              int ind = -1;
                                                          for(int i = 0; i < N; ++i) for(int mask = 0; mask
              forn(i, m)
                                                          \rightarrow < (1<<N); ++mask){
                if (a[i][j] > EPS && (t = a[i][n + m])
                                                            if(mask & (1<<i))
                \rightarrow + 1] / a[i][j]) < mi - EPS)
                                                              F[mask] += F[mask^(1<< i)];
                  mi = t, ind = i;
                                                       7
              if (ind == -1)
                                                          // sum over subsets ends
                return false;
              pivot(j, ind);
                                                          // algebra begins
56
          }
57
                                                          Pick
       }
                                                                / 2 - 1,
                                                          B + Γ
       return true;
                                                          где В - количество целочисленных точек внутри
                                                              многоугольника, |\mathsf{a}| \Gamma | - | \mathsf{количество}|
                                                              целочисленных точек на границе
     int mi = min_element(b.begin(), b.end()) -
      → b.begin();
                                                              многоугольника.
     if (b[mi] < -EPS) {
63
       a[m][n + m] = -1;
                                                          Newton
64
                                                          x_{i+1}=x_{i-f}(x_i)/f'(x_i)
       pivot(n + m, mi);
       assert(iterate(n + m + 1));
        if (a[m][m + n + 1] > EPS) // optimal value _{10}
                                                          Catalan
           is positive
                                                          C_n = \sum_{k=0}^{n-1} C_{k} C_{n-1-k}
         return NAN;
                                                          C_i = \frac{1}{n+1}  binom 2n  n
       forn(i, m)
                                                      13
          if (p[i] == m + n) {
                                                          G_N:=2^{n(n-1)/2}
                                                      14
            int j = 0;
                                                          Количество связных помеченных графов
            while (find(p.begin(), p.end(), j) !=
                                                          Conn_N = G_N - \frac 1 N \sum
            \rightarrow p.end() || fabs(a[i][j]) < EPS)
                                                              \lambdalimits_{K=1}^{N-1} K \bigcup binom N K Conn_K
              j++, assert(j < m + n);
            pivot(j, i);
                                                      17
75
                                                          Количество помеченных графов с К компонентами
     }
76
                                                              СВЯЗНОСТИ
```

```
D[N][K] = \sum_{s=1}^{N} \sum_{s=1}^{N} 
                                                              int lb = b[1-1]; //amt of nos from first
                                                               \hookrightarrow (l-1) nos that go in left
    \hookrightarrow Conn_S D[N-S][K-1]
                                                              int rb = b[r]; //amt of nos from first (r)
20
                                                               \rightarrow nos that go in left
   Miller-Rabbin
21
                                                              //so [lb+1, rb] represents nos from [l, r]
   a=a^t
22
                                                               → that go to left
   FOR i = 1...s
                                                              if(k <= inleft) return this->l->kth(lb+1, rb
     if a^2=1 && |a|!=1
                                                      39
                                                               \rightarrow , k);
       NOT PRIME
     a=a^2
                                                              //(l-1-lb) is amt of nos from first (l-1) nos
   return a==1 ? PRIME : NOT PRIME
                                                       41
                                                               → that go to right
                                                              //(r-rb) is amt of nos from first (r) nos
                                                               Интегрирование по формуле Симпсона
30
                                                              //so [l-lb, r-rb] represents nos from [l, r]
   \int int_a^b f(x) dx - ?
                                                       43
31
                                                               \hookrightarrow that go to right
   x_i := a+ih, i=0 ldots 2n
                                                              return this->r->kth(l-lb, r-rb, k-inleft);
                                                       44
   h = \{ frac \{b-a\} \{2n\} \}
33
                                                          };
   int = (f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) +
    2f(x_4) + 1dots + 4f(x_{2n-1}) +
                                                          // wavelet tree ends
    \rightarrow f(x<sub>{2n}))) \frac h 3</sub>
   Погрешность имеет порядок уменьшения как O(n^4).
                                                          // berlecamp-massey begins
   // algebra ends
                                                          const int SZ = MAXN;
   // wavelet tree begins
                                                          11 qp(11 a, 11 b) {
   struct wavelet_tree{
                                                            11 x = 1;
                                                            a \%= MOD;
     int lo, hi;
     wavelet_tree *1, *r;
                                                            while (b) {
     vi b;
                                                              if (b \& 1) x = x * a \% MOD;
                                                              a = a * a \% MOD;
                                                       10
     //nos are in range [x,y]
                                                              b >>= 1;
                                                            }
     //array indices are [from, to)
                                                      12
     wavelet_tree(int *from, int *to, int x, int y).{
                                                            return x;
10
       lo = x, hi = y;
11
       if(lo == hi or from >= to) return;
                                                      15
                                                          namespace linear_seq {
       int mid = (lo+hi)/2;
                                                            inline vector<int> BM(vector<int> x) {
                                                      17
       auto f = [mid](int x){
                                                              vector<int> ls, cur;
                                                       18
         return x <= mid;
                                                       19
                                                              int lf, ld;
       };
                                                      20
                                                              for (int i = 0; i < int(x.size()); ++i) {</pre>
       b.reserve(to-from+1);
                                                                11 t = 0:
                                                      21
       b.pb(0);
                                                                for (int j = 0; j < int(cur.size()); ++j)</pre>
19
                                                      22
       //b[i] = no of elements from first "i"
                                                                   t = (t + x[i - j - 1] * (11) cur[j]) %
        → elements that go to left node
                                                                if ((t - x[i]) \% MOD == 0) continue;
       for(auto it = from; it != to; it++)
21
                                                      24
          b.pb(b.back() + f(*it));
                                                                if (!cur.size()) {
                                                       25
                                                                   cur.resize(i + 1);
       //see how lambda function is used here
                                                                   lf = i;
24
       auto pivot = stable_partition(from, to, f); 28
                                                                   ld = (t - x[i]) \% MOD;
25
       1 = new wavelet_tree(from, pivot, lo, mid); 29
                                                                   continue;
       r = new wavelet_tree(pivot, to, mid+1, hi); 30
                                                                }
                                                                11 k = -(x[i] - t) * qp(1d, MOD - 2) % MOD;
28
                                                                vector<int> c(i - lf - 1);
                                                      32
     //kth smallest element in [l, r]
                                                                c.pb(k);
                                                      33
     int kth(int 1, int r, int k){
                                                                for (int j = 0; j < int(ls.size()); ++j)</pre>
                                                                   c.pb(-ls[j] * k % MOD);
       if(1 > r) return 0;
                                                      35
32
       if(lo == hi) return lo;
                                                                if (c.size() < cur.size())</pre>
33
       //how many nos are there in left node from

    c.resize(cur.size());

                                                                for (int j = 0; j < int(cur.size()); ++j)</pre>
        \hookrightarrow [l, r]
       int inleft = b[r] - b[1-1];
                                                                   c[j] = (c[j] + cur[j]) \% MOD;
                                                      38
```

```
if (i - lf + (int) ls.size() >= (int)
                                                                 a0[i] = b[i];

    cur.size())

                                                                 for (int j = 0; j < i; ++j) {
                                                                   a0[i] += MOD - (p[j] * 111 * b[i - j -
            ls = cur, lf = i, ld = (t - x[i]) % MOD;95
40
                                                                    \rightarrow 1]) % MOD;
          cur = c;
                                                                   if (a0[i] > MOD) {
       for (int i = 0; i < int(cur.size()); ++i)</pre>
                                                                     a0[i] -= MOD;
                                                       97
          cur[i] = (cur[i] % MOD + MOD) % MOD;
                                                       98
44
       return cur;
45
                                                               }
                                                             }
47
                                                      101
     int m;
                                                      102
     11 a[SZ], h[SZ], t_[SZ], s[SZ], t[SZ];
                                                      103
                                                          // berlecamp-massey ends
     inline void mull(ll* p, ll* q) {
51
       for (int i = 0; i < m + m; ++i) t_[i] = 0;
52
                                                          // AND-FFT begins
       for (int i = 0; i < m; ++i)
53
          if (p[i])
                                                          void fast_fourier(vector<int>& a) { // AND-FFT.
            for (int j = 0; j < m; ++j)
                                                             for (int k = 1; k < SZ(a); k *= 2)
              t_{i} = (t_{i} + j) + p[i] * q[j])
                                                             for (int start = 0; start < (1 << K); start +=</pre>
              \hookrightarrow MOD;
                                                             \rightarrow 2 * k) {
       for (int i = m + m - 1; i \ge m; --i)
                                                               for (int off = 0; off < k; ++off) {</pre>
          if (t_[i])
                                                               int a_val = a[start + off];
            for (int j = m - 1; ~j; --j)
                                                               int b_val = a[start + k + off];
              t_{i} - j - 1 = (t_{i} - j - 1) + t_{i}
              → * h[j]) % MOD;
                                                               a[start + off] = b_val;
       for (int i = 0; i < m; ++i) p[i] = t_[i];</pre>
                                                               a[start + k + off] = add(a_val, b_val);
                                                               }
                                                             }
                                                       13
     inline 11 calc(11 K) {
64
                                                          }
                                                       14
       for (int i = m; ~i; --i)
65
                                                       15
          s[i] = t[i] = 0;
66
                                                          void inverse_fast_fourier(vector<int>& a) {
                                                       16
       s[0] = 1;
                                                             for (int k = 1; k < SZ(a); k *= 2)
                                                       17
       if (m != 1) t[1] = 1; else t[0] = h[0];
                                                             for (int start = 0; start < (1 << K); start +=</pre>
       while (K) {
                                                             \rightarrow 2 * k) {
          if (K & 1) mull(s, t);
                                                               for (int off = 0; off < k; ++off) {</pre>
                                                       19
         mull(t, t);
                                                               int a_val = a[start + off];
                                                       20
         K >>= 1;
                                                               int b_val = a[start + k + off];
                                                       21
       }
       11 su = 0;
                                                               a[start + off] = sub(b_val, a_val);
       for (int i = 0; i < m; ++i) su = (su + s[i])
                                                               a[start + k + off] = a_val;
        → a[i]) % MOD;
                                                               }
       return (su % MOD + MOD) % MOD;
     }
                                                          }
     inline int work(vector<int> x, ll n) {
79
                                                          // AND-FFT ends
       if (n < int(x.size())) return x[n];</pre>
80
       vector < int > v = BM(x);
                                                          // 2-chinese begins
       m = v.size();
       if (!m) return 0;
                                                          template <typename Info>
       for (int i = 0; i < m; ++i) h[i] = v[i], a[i]
                                                          class DSU {
        \rightarrow = x[i];
                                                             public:
       return calc(n);
85
                                                            DSU ( int n ) : jump (new int[n]), rank (new
86
                                                             → int [n]), info (new Info [n]) {
87
                                                               for (int i = 0; i < n; i++) {
     //b=a0/(1-p)
                                                               jump[i] = i;
     inline void calc_generating_function(const
                                                               rank[i] = 0;

  vector<int>& b, vector<int>& p,

                                                       10
      → vector<int>& a0) {
                                                            }
                                                       11
       p = BM(b);
                                                            Info& operator [] ( int x ) {
                                                       12
       a0.resize(p.size());
91
                                                               return info[get (x)];
                                                       13
       for (int i = 0; i < a0.size(); ++i) {</pre>
                                                             }
                                                       14
```

```
void merge (int a, int b, const Info &comment,2
                                                           if (left && left->min_value + left->add <
      → ) {

    min_value) {
                                                             min_value = left->min_value + left->add;
       a = get(a);
16
                                                      73
       b = get (b);
                                                             min_path = -1;
                                                      74
       if (rank[a] <= rank[b]) {</pre>
                                                      75
       jump[a] = b;
                                                           if (right && right->min_value + right->add <</pre>
                                                      76
       rank[b] += rank[a] == rank[b];

    min_value) {
20
                                                             min_value = right->min_value + right->add;
       info[b] = comment;
                                                      77
21
       } else {
                                                             min_path = +1;
                                                      78
       jump[b] = a;
                                                           }
                                                      79
       info[a] = comment;
                                                           }
                                                      80
                                                         };
                                                      81
     }
                                                         Treap* treap_merge ( Treap *x, Treap *y ) {
     private:
                                                     83
27
     int *jump, *rank;
                                                           if (!x)
                                                     84
28
     Info *info;
                                                           return y;
29
                                                      85
                                                           if (!y)
     int get ( int x ) {
                                                           return x;
                                                      87
       return jump[x] == x ? x : (jump[x] = get
                                                           if (x->height < y->height) {
          (jump[x]));
                                                           x->push();
     }
                                                           x->right = treap_merge (x->right, y);
   };
                                                           x->recalc ();
                                                      91
34
                                                           return x;
35
                                                      92
                                                           } else {
   struct Treap {
                                                           y->push ();
37
                                                           y->left = treap_merge (x, y->left);
     int value, add;
                                                     95
                                                           y->recalc ();
     int source, target, height;
                                                     96
     int min_value, min_path;
                                                     97
                                                           return y;
41
     Treap *left, *right;
                                                         }
                                                     99
42
                                                     100
43
     Treap ( int _source, int _target, int _value )01
                                                         Treap* treap_getmin ( Treap *x, int &source, int

⇒ : value (_value), add (0), source

                                                         assert (x);
                                                     102
     height = rand ();
                                                           x->push();
                                                     103
                                                           if (x->min_path == 0) {
     min_value = value, min_path = 0;
     left = right = 0;
                                                           // memory leak, sorry
                                                     105
                                                           source = x->source;
                                                     106
                                                           target = x->target;
                                                     107
     Treap& operator += ( int sub ) {
                                                           value = x->value + x->add;
     add += sub;
                                                           return treap_merge (x->left, x->right);
51
                                                     109
     return *this;
                                                           } else if (x->min_path == -1) {
                                                     110
     }
                                                           x->left = treap_getmin (x->left, source,
                                                     111

    target, value);

     void push () {
                                                           value += x->add;
55
                                                     112
     if (!add)
                                                           x->recalc ();
56
                                                     113
       return;
                                                           return x;
57
                                                     114
     if (left) {
                                                           } else if (x->min_path == +1) {
                                                     115
       left->add += add;
                                                           x->right = treap_getmin (x->right, source,
                                                     116

    target, value);

     if (right) {
                                                           value += x->add;
       right->add += add;
                                                           x->recalc ();
                                                     118
62
                                                           return x;
                                                     119
63
     value += add;
                                                           } else
                                                     120
     min_value += add;
                                                           assert (0);
                                                     121
     add = 0;
                                                     122
                                                     123
                                                         Treap* treap_add ( Treap *x, int add ) {
                                                     124
     void recalc () {
                                                           if (!x)
                                                     125
     min_value = value;
                                                     126
                                                           return 0;
70
     min_path = 0;
                                                           return \&((*x) += add);
                                                     127
71
```

```
}
                                                               dsu[u].first = node;
128
                                                      184
                                                               Treap *v = dsu[u].second;
129
                                                               while (dsu[t].first != node) {
130
                                                      186
    int main () {
                                                                 int next = jump[dsu[t].first];
131
                                                      187
      int n, m;
                                                                 parent[dsu[t].first] = node;
      while (scanf (''%d%d'', &n, &m) == 2) {
                                                                 children[node].push_back (dsu[t].first);
133
                                                      189
      Treap * g[n + 1];
                                                                 v = treap_merge (v, dsu[t].second);
134
                                                      190
      for (int i = 0; i <= n; i++)
                                                                 dsu.merge (u, t, make_pair (node, v));
135
                                                      191
        g[i] = 0;
                                                                 t = next;
136
                                                               }
      for (int i = 1; i <= n; i++) {
137
                                                      193
                                                               }
        int a;
138
                                                      194
        assert (scanf (''%d'', &a) == 1);
                                                               u = i;
                                                      195
        g[i] = treap_merge (g[i], new Treap (i, 0,
                                                               while (dsu[u].first) {
140
                                                               int next = jump[dsu[u].first];
         → a));
                                                      197
      }
                                                               finish.push_back (dsu[u].first);
141
                                                      198
      n++;
                                                               dsu.merge (u, 0, make_pair (0, (Treap *)0));
142
                                                      199
      for (int i = 0; i < m; i++) {
                                                               u = next;
        int a, b, c;
                                                               }
144
        assert (scanf (''%d%d%d'', &a, &b, &c) == 3); 202
                                                            }
145
        g[b] = treap_merge (g[b], new Treap (b, a, 203
                                                            bool ok[k];
146
                                                            int res[n];
                                                            memset (ok, 0, sizeof (ok[0]) * k);
                                                      205
147
      DSU <pair <int, Treap*> > dsu (n + 1);
                                                            memset (res, -1, sizeof (res[0]) * n);
148
                                                      206
                                                            function <void (int, int)> add_edge = [&ok,
      for (int i = 0; i < n; i++) {
149
        dsu[i] = make_pair (i, g[i]);
                                                             150
                                                               assert (0 \leq a && a \leq n);
151
                                                      208
                                                               assert (0 \leq b && b \leq n);
                                                      209
      int ans = 0, k = n;
                                                               assert (res[a] == -1);
      int jump[2 * n], jump_from[2 * n], parent[2 * 211
                                                               res[a] = b;
154
                                                               while (a != -1 \&\& !ok[a]) {
      \rightarrow n], c[n];
      vector <int> children[2 * n];
                                                               ok[a] = true;
155
      memset (c, 0, sizeof(c[0]) * n);
                                                               a = parent[a];
      memset (parent, -1, sizeof (parent[0]) * 2 *
                                                               }
                                                      215
157
      \rightarrow n);
                                                            };
                                                      216
      vector <int> finish;
                                                            function <void (int)> reach = [&ok, &reach,
158
                                                      217
      for (int i = 0; i < n; i++) {
                                                             if (dsu[i].first == 0)
                                                                int u ) {
160
        continue;
                                                               if (!ok[u])
161
                                                      218
        int u = i;
                                                               add_edge (jump_from[u], jump[u]);
162
                                                      219
        c[u] = 1;
                                                               for (auto x : children[u])
                                                              reach (x);
        while (true) {
                                                      221
164
        int source, target, value;
                                                            };
165
        dsu[u].second = treap_getmin (dsu[u].second 223
                                                            for (auto x: finish)
        → source, target, value);
                                                                reach (x);
        if (dsu[target] == dsu[u])
                                                            printf (''%d\n'', ans);
                                                      225
167
          continue;
                                                            for (int i = 1; i < n; i++)
                                                      226
168
                                                               printf (''%d%c'', res[i] ? res[i] : -1, ''\n ''[i
        treap_add (dsu[u].second, -value);
169
        ans += value;
                                                                 < n - 1]);
170
                                                            }
        jump_from[dsu[u].first] = source;
171
                                                      228
        jump[dsu[u].first] = target;
                                                            return 0;
                                                      229
                                                          }
        if (dsu[target].first == 0)
                                                      230
          break;
                                                      231
174
        if (!c[target]) {
                                                      232
                                                          // 2-chinese ends
175
          c[target] = 1;
176
                                                          // general max weighet match begins
                                                       1
          u = target;
177
          continue;
178
                                                          #define DIST(e)
179
                                                              (lab[e.u]+lab[e.v]-g[e.u][e.v].w*2)
        assert (k < 2 * n);
                                                          using namespace std;
        int node = k++, t = target;
                                                          typedef long long ll;
        parent[dsu[u].first] = node;
182
                                                          const int N = 1023, INF = 1e9;
        children[node].push_back (dsu[u].first);
183
                                                          struct Edge {
```

```
vis[u] = t;
     int u, v, w;
   } g[N][N];
                                                                u = st[match[u]];
   int n, m, n_x, lab[N], match[N], slack[N], st[N]6,
                                                                if (u) u = st[pa[u]];
    \  \, \hookrightarrow \  \, \text{pa[N], flower\_from[N][N], S[N], vis[N];}
   vector < int> flower[N];
                                                              return 0;
   deque < int> q;
                                                           }
                                                        64
   void update_slack(int u, int x) {
                                                           void add_blossom(int u, int lca, int v) {
                                                        65
     if (!slack[x] \mid\mid DIST(g[u][x]) <
                                                              int b = n+1;
                                                              while (b \le n_x && st[b]) ++b;
        DIST(g[slack[x]][x])) slack[x] = u;
   }
                                                              if (b>n_x) ++n_x;
15
   void set_slack(int x) {
                                                              lab[b] = 0, S[b] = 0;
                                                              match[b] = match[lca];
     slack[x] = 0;
     for (int u = 1; u \le n; ++u)
                                                              flower[b].clear();
        if (g[u][x].w>0 \&\& st[u] != x \&\& S[st[u]] ==_{72}
                                                              flower[b].push_back(lca);
19
        \rightarrow 0) update_slack(u, x);
                                                              for (int x = u, y; x != lca; x = st[pa[y]])
                                                        73
   }
                                                                flower[b].push_back(x), flower[b].push_back(y
20
                                                        74
   void q_push(int x) {
                                                                \rightarrow = st[match[x]]), q_push(y);
     if (x <= n) return q.push_back(x);</pre>
                                                              reverse(flower[b].begin() +1, flower[b].end());
                                                        75
     for (int i = 0; i < flower[x].size(); i++)</pre>
                                                              for (int x = v, y; x != lca; x = st[pa[y]])
         q_push(flower[x][i]);
                                                                flower[b].push_back(x), flower[b].push_back(y
                                                        77
   }
24
                                                                \Rightarrow = st[match[x]]), q_push(y);
   void set_st(int x, int b) {
                                                              set_st(b, b);
                                                        78
25
                                                              for (int x = 1; x \le n_x; ++x) g[b][x].w =
     st[x] = b;
26
     if (x <= n) return;</pre>
                                                              \rightarrow g[x][b].w = 0;
     for (int i = 0; i < flower[x].size(); ++i)</pre>
                                                              for (int x = 1; x <= n; ++x) flower_from[b][x]</pre>
         set_st(flower[x][i], b);
                                                              \rightarrow = 0;
                                                              for (int i = 0; i < flower[b].size(); ++i) {</pre>
   }
29
   int get_pr(int b, int xr) {
                                                                int xs = flower[b][i];
     int pr = find(flower[b].begin(),
                                                                for (int x = 1; x \le n_x; ++x)
                                                        83
31

→ flower[b].end(), xr)-flower[b].begin();

                                                                  if (g[b][x].w == 0 \mid \mid DIST(g[xs][x]) <
                                                        84
     if (pr % 2 == 1) {
                                                                   → DIST(g[b][x]))
32
       reverse(flower[b].begin() +1,
                                                                    g[b][x] = g[xs][x], g[x][b] = g[x][xs];
        → flower[b].end());
                                                                for (int x = 1; x \le n; ++x)
       return (int) flower[b].size()-pr;
                                                                  if (flower_from[xs][x]) flower_from[b][x] =
34
                                                        87
     }
                                                                      xs;
                                                              }
     else return pr;
                                                              set_slack(b);
37
                                                        89
   void set_match(int u, int v) {
38
                                                        90
     match[u] = g[u][v].v;
                                                           void expand_blossom(int b) // S[b] == 1 {
                                                        91
39
     if (u <= n) return;</pre>
                                                              for (int i = 0; i < flower[b].size(); ++i)</pre>
                                                                set_st(flower[b][i], flower[b][i]);
     Edge e = g[u][v];
41
     int xr = flower_from[u][e.u], pr = get_pr(u,
                                                              int xr = flower_from[b][g[b][pa[b]].u], pr =

    xr);

    get_pr(b, xr);

     for (int i = 0; i < pr; ++i)</pre>
                                                              for (int i = 0; i < pr; i += 2) {

    set_match(flower[u][i], flower[u][i^1]);

                                                                int xs = flower[b][i], xns = flower[b][i+1];
                                                                pa[xs] = g[xns][xs].u;
     set_match(xr, v);
44
                                                        97
     rotate(flower[u].begin(), flower[u].begin()
                                                                S[xs] = 1, S[xns] = 0;
        +pr, flower[u].end());
                                                                slack[xs] = 0, set_slack(xns);
                                                                q_push(xns);
46
                                                       100
   void augment(int u, int v) {
                                                              }
                                                       101
47
     int xnv = st[match[u]];
                                                              S[xr] = 1, pa[xr] = pa[b];
     set_match(u, v);
                                                              for (int i = pr+1; i < flower[b].size(); ++i) {</pre>
                                                       103
                                                                int xs = flower[b][i];
     if (!xnv) return;
50
                                                       104
     set_match(xnv, st[pa[xnv]]);
                                                                S[xs] = -1, set_slack(xs);
51
                                                       105
     augment(st[pa[xnv]], xnv);
                                                              }
52
                                                       106
   }
                                                              st[b] = 0;
53
                                                       107
   int get_lca(int u, int v) {
54
                                                       108
     static int t = 0;
                                                           bool on_found_Edge(const Edge &e) {
                                                       109
     for (++t; u || v; swap(u, v)) {
                                                              int u = st[e.u], v = st[e.v];
                                                       110
        if (u == 0) continue;
                                                       111
                                                              if (S[v] == -1) {
57
        if (vis[u] == t) return u;
                                                                pa[v] = e.u, S[v] = 1;
                                                       112
```

```
int nu = st[match[v]];
                                                                     if (on_found_Edge(g[slack[x]][x])) return
113
        slack[v] = slack[nu] = 0;
                                                                      S[nu] = 0, q_push(nu);
                                                                 for (int b = n+1; b \le n_x; ++b)
115
                                                        167
      }
                                                                   if (st[b] == b \&\& S[b] == 1 \&\& lab[b] == 0)
116
      else if (S[v] == 0) {
                                                                       expand_blossom(b);
        int lca = get_lca(u, v);
                                                               }
118
        if (!lca) return augment(u, v), augment(v,
                                                               return 0;
119
                                                        170
                                                        171
         \rightarrow u), 1;
                                                            pair < 11, int> weight_blossom() {
        else add_blossom(u, lca, v);
120
      }
                                                               fill(match, match+n+1, 0);
                                                        173
121
      return 0:
                                                               n x = n:
122
                                                        174
    }
                                                               int n_matches = 0;
                                                        175
123
    bool matching() {
                                                               11 tot_weight = 0;
124
                                                               for (int u = 0; u \le n; ++u) st[u] = u,
      fill(S, S+n_x+1, -1), fill(slack, slack+n_x+1,77)
125
      → 0);

    flower[u].clear();

      q.clear();
                                                               int w_max = 0;
126
      for (int x = 1; x \le n_x; ++x)
                                                               for (int u = 1; u <= n; ++u)
        if (st[x] == x \&\& !match[x]) pa[x] = 0, S[x]_{80}
                                                                 for (int v = 1; v <= n; ++v) {
128
         \rightarrow = 0, q_push(x);
                                                                   flower_from[u][v] = (u == v?u:0);
                                                                   w_max = max(w_max, g[u][v].w);
      if (q.empty()) return 0;
129
                                                        182
                                                                 }
      for (;;) {
        while (q.size()) {
                                                               for (int u = 1; u \le n; ++u) lab[u] = w_max;
131
                                                        184
                                                               while (matching()) ++n_matches;
          int u = q.front();
132
                                                        185
          q.pop_front();
                                                               for (int u = 1; u <= n; ++u)
133
           if (S[st[u]] == 1) continue;
                                                                 if (match[u] && match[u] < u)</pre>
134
                                                        187
          for (int v = 1; v \le n; ++v)
                                                                   tot_weight += g[u][match[u]].w;
135
                                                        188
             if (g[u][v].w>0 \&\& st[u] != st[v]) {
                                                               return make_pair(tot_weight, n_matches);
                                                        189
                                                            }
               if (DIST(g[u][v]) == 0) {
                                                        190
                 if (on_found_Edge(g[u][v])) return 19;
                                                            int main() {
138
                                                               cin>>n>>m;
139
                                                        192
                                                               for (int u = 1; u <= n; ++u)
               else update_slack(u, st[v]);
140
                                                        193
                                                                 for (int v = 1; v \le n; ++v)
        }
                                                                   g[u][v] = Edge \{u, v, 0\};
                                                        195
142
        int d = INF;
                                                               for (int i = 0, u, v, w; i < m; ++i) {
143
                                                        196
        for (int b = n+1; b \le n_x; ++b)
                                                                 cin>>u>>v>>w;
144
                                                        197
           if (st[b] == b && S[b] == 1) d = min(d,
                                                                 g[u][v].w = g[v][u].w = w;
           \rightarrow lab[b]/2);
                                                        199
        for (int x = 1; x \le n_x; ++x)
                                                               cout << weight_blossom().first << '\n';</pre>
146
                                                        200
                                                               for (int u = 1; u <= n; ++u) cout << match[u]</pre>
          if (st[x] == x \&\& slack[x]) {
                                                        201
147
            if (S[x] == -1) d = min(d,
                                                               → DIST(g[slack[x]][x]));
                                                        202
             else if (S[x] == 0) d = min(d,
149
                                                        203
                                                            // general max weighted match ends
             \rightarrow DIST(g[slack[x]][x])/2);
                                                        204
          }
                                                            // planar begins
        for (int u = 1; u <= n; ++u) {
151
          if (S[st[u]] == 0) {
152
                                                            // обход граней
             if (lab[u] <= d) return 0;</pre>
                                                            int n; // число вершин
            lab[u] - = d;
                                                            vector < vector<int> > g; // zpap
          }
                                                            vector < vector < char> > used (n);
          else if (S[st[u]] == 1) lab[u] += d;
                                                            for (int i=0; i<n; ++i)
                                                               used[i].resize (g[i].size());
        for (int b = n+1; b \le n_x; ++b)
158
                                                            for (int i=0; i<n; ++i)</pre>
          if (st[b] == b) {
159
                                                               for (size_t j=0; j<g[i].size(); ++j)</pre>
            if (S[st[b]] == 0) lab[b] += d*2;
160
                                                                 if (!used[i][j]) {
                                                         11
             else if (S[st[b]] == 1) lab[b] - = d*2;
161
                                                         12
                                                                   used[i][j] = true;
          }
162
                                                                   int v = g[i][j], pv = i;
                                                         13
        q.clear();
                                                                   vector<int> facet;
        for (int x = 1; x \le n_x; ++x)
                                                                   for (;;) {
          if (st[x] == x \&\& slack[x] \&\& st[slack[x]]^{15}
                                                                     facet.push_back (v);
           \rightarrow != x && DIST(g[slack[x]][x]) == 0)
                                                                     vector<int>::iterator it = find
                                                         17
                                                                      \rightarrow (g[v].begin(), g[v].end(), pv);
```

```
//vector<int>::iterator it =
                                                                if (x != y) {
            \rightarrow lower_bound(g[v].begin(), g[v].end()_{69}
                                                                  g[x].push_back (y);
            \rightarrow pv, cmp_ang(v));
                                                                  g[y].push_back(x);
            // cmp_ang(v) -- true, если меньше
                                                              }
                полярный угол относ v
                                                      72
            if (++it == g[v].end()) it =
                                                            }
                                                      73
                                                            int n = (int) g.size();

    g[v].begin();

                                                      74
            if (used[v][it-g[v].begin()]) break;
                                                            // сортировка по углу и удаление кратных рёбер
                                                      75
21
                                                            for (int i=0; i<n; ++i) {
            used[v][it-g[v].begin()] = true;
                                                              sort (g[i].begin(), g[i].end(), cmp_ang (i));
            pv = v, v = *it;
                                                              g[i].erase (unique (g[i].begin(),

    g[i].end()), g[i].end());
              вывод facet - текущей грани ...
26
                                                      80
   // построение планарного графа
                                                          // planar ends
   struct point {
     double x, y;
     bool operator< (const point & p) const {</pre>
       return x < p.x - EPS \mid \mid abs (x - p.x) < EPS
        \rightarrow && y < p.y - EPS;
     }
33
   };
34
35
   map<point,int> ids;
36
   vector<point> p;
37
   vector < vector<int> > g;
   int get_point_id (point pt) {
40
     if (!ids.count(pt)) {
41
       ids[pt] = (int)p.size();
42
       p.push_back (pt);
                                                          // fast hashtable begins
       g.resize (g.size() + 1);
44
45
                                                          #include <ext/pb_ds/assoc_container.hpp>
     return ids[p];
46
                                                          using namespace __gnu_pbds;
   }
47
                                                          gp_hash_table<int, int> table;
48
   void intersect (pair<point, point> a,
49
                                                          const int RANDOM = chrono ::
    → pair<point,point> b, vector<point> & res) {
                                                           → high_resolution_clock ::
      ... стандартная процедура, пересекает два
                                                          → now().time_since_epoch().count();
                                                          struct chash {
          отрезка а и b и закидывает результат в res
                                                            int operator()(int x) { return hash<int>{}(x ^

¬ RANDOM); }

      ... если отрезки перекрываются, то закидывает
                                                          };
          те концы, которые попали внутрь первого
                                                          gp_hash_table<key, int, chash> table;
          отрезка ...
52
                                                          // fast hashtable ends
   int main() {
     // входные данные
     int m;
56
     vector < pair<point,point> > a (m);
57
      ... чтение ...
     // построение графа
     for (int i=0; i<m; ++i) {
       vector<point> cur;
62
       for (int j=0; j<m; ++j)</pre>
63
          intersect (a[i], a[j], cur);
       sort (cur.begin(), cur.end());
       for (size_t j=0; j+1<cur.size(); ++j) {</pre>
          int x = get_id (cur[j]), y = get_id
          \rightarrow (cur[j+1]);
```

 $\mathbf{D}\mathbf{M}$ 

xmodmap -e 'keycode 94=' setxkbmap us

Кол-во корневых деревьев:

$$t(G) = \frac{1}{n}\lambda_2 \dots \lambda_n \ (\lambda_1 = 0)$$

Кол-во эйлеровых циклов: 
$$e(D) = t^-(D, x) \cdot \prod_{y \in D} (outdeg(y) - 1)!$$

## Наличие совершенного паросочетания:

Т – матрица с нулями на диагонали. Если есть ребро

$$(i,j)$$
, to  $a_{i,j} := x_{i,j}$ ,  $a_{j,i} = -x_{i,j}$ 

 $\det(T) = 0 \Leftrightarrow$  нет совершенного паросочетания.

Fast subset convolution 
$$(f*g)(S) := \sum_{T \subseteq S} f(T)g(S \setminus T)$$

$$\hat{f}(X) := \sum f(S)$$

$$\hat{f}(X) := \sum_{S \subseteq X} \hat{f}(S)$$

$$f(S) = \sum_{X \subseteq S} (-1)^{|S \setminus X|} \hat{f}(X)$$

$$\hat{f}_0(X) := f(X)$$

$$\hat{f}_{0}(X) := f(X) 
\hat{f}_{j}(X) = \begin{cases} \hat{f}_{j-1}(X) & \text{if } j \notin X \\ \hat{f}_{j-1}(X \setminus j) + \hat{f}_{j-1}(X) & \text{if } j \in X \end{cases} 
\hat{f}_{n}(X) == \hat{f}(X)$$

$$\hat{f}_n(X) = \hat{f}(X)$$

$$f_0(S) := \hat{f}(S)$$

$$f_0(S) := f(S) f_j(S) = \begin{cases} f_{j-1}(S) & \text{if } j \notin S \\ -f_{j-1}(S \setminus j) + f_{j-1}(S) & \text{if } j \in S \end{cases} f_n(S) == f(S)$$

$$f_n(S) = f(S)$$

$$\begin{split} \hat{f}(k,X) &:= \sum_{S \subseteq, |S| = k} f(S) \\ f(S) &== \hat{f}(|S|,S) \end{split}$$

$$f(S) == \hat{f}(|S|, S)$$

$$(\hat{f} \otimes \hat{g})(k,X) := \sum_{j=0}^{k} \hat{f}(j,X)\hat{g}(k-j,X)$$
$$(f * g)(S) = \sum_{X \subseteq S} (-1)^{|S \setminus X|} (\hat{f} \otimes \hat{g})(|S|,X)$$

$$(f * g)(S) = \sum_{X \in \mathcal{C}} (-1)^{|S \setminus X|} (\hat{f} \otimes \hat{g})(|S|, X)$$

calculate using  $f_i$ !