Chapter 1 Exercise Solutions Guide

Principles of Computer Systems

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1 Question 1: Manufacturing Cost and Yield

Key Formulas

1. **Cost per Die:**
$$\text{Cost per die} = \frac{\text{Cost per wafer}}{\text{Dies per wafer} \times \text{Yield}}$$

2. **Dies per Wafer:**
$$\text{Dies per wafer} \approx \frac{\text{Wafer area}}{\text{Die area}}$$

3. **Production Yield:**
$$\text{Yield} = \frac{1}{\left(1 + \frac{\text{Defects per area} \times \text{Die area}}{2}\right)^2}$$

1.1 Problem Data

- Silicon wafer diameter: $d = 25$ cm
- Wafer cost: $\text{Cost}_{\text{wafer}} = \$30$
- Die area: $A_{\text{die}} = 180 \text{ mm}^2$
- Defect rate: $D = 0.031 \text{ defects/cm}^2$

1.2 a) Calculate Yield

Step 1: Calculate Wafer Area

$$A_{\text{wafer}} = \pi r^2 = \pi \times (12.5)^2 = 490.87 \text{ cm}^2$$

Step 2: Calculate Number of Dies

$$\text{Dies per wafer} = \frac{49087 \text{ mm}^2}{180 \text{ mm}^2} = 272.71$$

Step 3: Calculate Die Area in cm^2

$$A_{\text{die}} = 180 \text{ mm}^2 = 1.80 \text{ cm}^2$$

Step 4: Calculate Yield

$$\text{Yield} = \frac{1}{\left(1 + \frac{0.031 \times 1.80}{2}\right)^2} = \frac{1}{(1 + 0.0279)^2} = \frac{1}{(1.0279)^2}$$

$$\boxed{\text{Yield} = 0.9465 \text{ or } 94.65\%}$$

1.3 b) Cost per Die

$$\text{Cost per die} = \frac{30}{272.71 \times 0.9465} = \frac{30}{258.12} = \boxed{\$0.116}$$

1.4 c) New Scenario with Changes

Changes:

- Dies per wafer: 20% increase $\rightarrow 272.71 \times 1.20 = 327.25$ dies
- Defect rate: 15% increase $\rightarrow 0.031 \times 1.15 = 0.0356$ defects/cm^2
- Die area: unchanged

Calculate New Yield:

$$\text{Yield}_{\text{new}} = \frac{1}{\left(1 + \frac{0.0356 \times 1.80}{2}\right)^2} = \frac{1}{(1 + 0.0320)^2}$$

$$\boxed{\text{Yield}_{\text{new}} = 0.9388 \text{ or } 93.88\%}$$

2 Question 2: Comparing Production Method Yields

2.1 Problem Data

- Old method yield: $\text{Yield}_{\text{old}} = 0.89$ (89%)
- New method yield: $\text{Yield}_{\text{new}} = 0.94$ (94%)
- Die area: $A_{\text{die}} = 180 \text{ mm}^2 = 1.80 \text{ cm}^2$

2.2 Calculate Defect Rate for Each Method

The defect rate can be calculated from the Yield formula:

$$\text{Yield} = \frac{1}{\left(1 + \frac{D \times A}{2}\right)^2}$$

Solving for D:

$$D = \frac{2}{A}\left(\frac{1}{\sqrt{\text{Yield}}} - 1\right)$$

Old Method Defect Rate:

$$D_{\text{old}} = \frac{2}{1.80}\left(\frac{1}{\sqrt{0.89}} - 1\right) = \frac{2}{1.80} \times (1.0604 - 1) = 0.0667 \text{ defects/cm}^2$$

New Method Defect Rate:

$$D_{\text{new}} = \frac{2}{1.80}\left(\frac{1}{\sqrt{0.94}} - 1\right) = \frac{2}{1.80} \times (1.0309 - 1) = 0.0349 \text{ defects/cm}^2$$

Defect Rate Reduction:

$$\text{Reduction} = \frac{D_{\text{old}} - D_{\text{new}}}{D_{\text{old}}} \times 100\% = \frac{0.0667 - 0.0349}{0.0667} \times 100\%$$

$$\boxed{\text{Defect Reduction} = 47.63\%}$$

3 Question 3: Processor Performance Comparison

3.1 Problem Data

Processor	Clock Rate	CPI
P1	2.6 GHz	1.5
P2	3.3 GHz	1.1
P3	4.2 GHz	1.9

3.2 a) Highest Instructions Per Second (IPS)

Formula:

$$\text{IPS} = \frac{\text{Clock Rate}}{\text{CPI}}$$

Calculations:

$$\text{IPS}_{P1} = \frac{2.6 \text{ GHz}}{1.5} = 1.733 \text{ billion IPS}$$
$$\text{IPS}_{P2} = \frac{3.3 \text{ GHz}}{1.1} = 3.000 \text{ billion IPS}$$
$$\text{IPS}_{P3} = \frac{4.2 \text{ GHz}}{1.9} = 2.211 \text{ billion IPS}$$
$$\boxed{\text{Answer: P2 has the highest IPS}}$$

3.3 b) Executing a Program in 11 Seconds

Formulas:

$$\text{Cycles} = \text{Clock Rate} \times \text{Time}$$
$$\text{Instructions} = \frac{\text{Cycles}}{\text{CPI}}$$

For P1:

$$\text{Cycles} = 2.6 \times 10^9 \text{ Hz} \times 11 \text{ s} = 28.6 \times 10^9 \text{ cycles}$$
$$\text{Instructions} = \frac{28.6 \times 10^9}{1.5} = 19.067 \times 10^9 \text{ instructions}$$

For P2:

$$\text{Cycles} = 3.3 \times 10^9 \times 11 = 36.3 \times 10^9 \text{ cycles}$$
$$\text{Instructions} = \frac{36.3 \times 10^9}{1.1} = 33.000 \times 10^9 \text{ instructions}$$

For P3:

$$\text{Cycles} = 4.2 \times 10^9 \times 11 = 46.2 \times 10^9 \text{ cycles}$$
$$\text{Instructions} = \frac{46.2 \times 10^9}{1.9} = 24.316 \times 10^9 \text{ instructions}$$

3.4 c) New Clock Rate for 15% Execution Time Reduction

Conditions:

- Execution time reduction: 15%
- CPI increase: 10%

Formula: If the new time must be 0.85× the old time:

$$\text{Time}_{\text{new}} = \frac{\text{IC} \times \text{CPI}_{\text{new}}}{\text{Clock Rate}_{\text{new}}} = 0.85 \times \text{Time}_{\text{old}}$$

$$\frac{\text{IC} \times \text{CPI} \times 1.1}{\text{Clock Rate}_{\text{new}}} = 0.85 \times \frac{\text{IC} \times \text{CPI}}{\text{Clock Rate}_{\text{old}}}$$

$$\text{Clock Rate}_{\text{new}} = \frac{1.1}{0.85} \times \text{Clock Rate}_{\text{old}} = 1.294 \times \text{Clock Rate}_{\text{old}}$$

Results:

$$\text{P1: } 2.6 \times 1.294 = \boxed{3.365 \text{ GHz}}$$

$$\text{P2: } 3.3 \times 1.294 = \boxed{4.271 \text{ GHz}}$$

$$\text{P3: } 4.2 \times 1.294 = \boxed{5.435 \text{ GHz}}$$

4 Question 4: ISA Implementation Comparison

4.1 Problem Data

Instruction Mix (for 1 million instructions):

Class	P1 (CPI)	P2 (CPI)
A (15%)	1	2
B (25%)	1	2
C (40%)	3	2
D (20%)	3	2

4.2 Calculate Average CPI

For P1:

$$\begin{aligned}\text{CPI}_{\text{avg}} &= 0.15 \times 1 + 0.25 \times 1 + 0.40 \times 3 + 0.20 \times 3 \\ &= 0.15 + 0.25 + 1.20 + 0.60 = \boxed{2.20}\end{aligned}$$

For P2:

$$\begin{aligned}\text{CPI}_{\text{avg}} &= 0.15 \times 2 + 0.25 \times 2 + 0.40 \times 2 + 0.20 \times 2 \\ &= 0.30 + 0.50 + 0.80 + 0.40 = \boxed{2.00}\end{aligned}$$

4.3 Calculate Execution Time

$$\text{Execution Time} = \frac{\text{IC} \times \text{CPI}}{\text{Clock Rate}}$$

For P1:

$$\text{P1: } \frac{10^6 \times 2.20}{2.6 \times 10^9} = 0.8462 \text{ ms}$$

For P2:

$$\text{P2: } \frac{10^6 \times 2.00}{3.2 \times 10^9} = 0.6250 \text{ ms}$$

$$\boxed{\text{Answer: P2 is faster}}$$

5 Question 5: Compiler Comparison

5.1 Problem Data

- **Compiler A:**
  - Instruction count: $N_A = 1.3 \times 10^9$
  - Execution time: $T_A = 5.1$ s
- **Compiler B:**
  - Instruction count: $N_B = 1.6 \times 10^9$
  - Execution time: $T_B = 1.7$ s
- Clock cycle time: $\Delta t = 2 \text{ ns} = 2 \times 10^{-9}$ s

5.2 a) Average CPI

Formula:

$$\text{CPI} = \frac{T}{N \times \Delta t}$$

For Compiler A:

$$\text{CPI}_A = \frac{5.1}{1.3 \times 10^9 \times 2 \times 10^{-9}} = \frac{5.1}{2.6} = \boxed{1.9615}$$

For Compiler B:

$$\text{CPI}_B = \frac{1.7}{1.6 \times 10^9 \times 2 \times 10^{-9}} = \frac{1.7}{3.2} = \boxed{0.5312}$$

5.3 b) Clock Rate Ratio for Equal Execution Time

Equality Condition:

$$T_A = T_B$$
$$\frac{N_A \times \text{CPI}_A}{\text{Clock Rate}_A} = \frac{N_B \times \text{CPI}_B}{\text{Clock Rate}_B}$$
$$\frac{\text{Clock Rate}_A}{\text{Clock Rate}_B} = \frac{N_A \times \text{CPI}_A}{N_B \times \text{CPI}_B}$$

Calculation:

$$\frac{\text{Clock Rate}_A}{\text{Clock Rate}_B} = \frac{1.3 \times 10^9 \times 1.9615}{1.6 \times 10^9 \times 0.5312}$$
$$= \frac{2.550}{0.850} = \boxed{3.000}$$

$$\boxed{\text{Processor A must be 3 times faster than Processor B}}$$

5.4 c) Speedup with Compiler C

Compiler C:

- Instruction count: $N_C = 0.75 \times 10^9 = 7.5 \times 10^8$
- CPI: $\text{CPI}_C = 1.1$

Calculate Total Cycles:

$$\text{Total Cycles}_A = N_A \times \text{CPI}_A = 1.3 \times 10^9 \times 1.9615 = 2.550 \times 10^9$$
$$\text{Total Cycles}_B = N_B \times \text{CPI}_B = 1.6 \times 10^9 \times 0.5312 = 0.850 \times 10^9$$
$$\text{Total Cycles}_C = N_C \times \text{CPI}_C = 0.75 \times 10^9 \times 1.1 = 0.825 \times 10^9$$

Calculate Execution Time (on one processor):

$$\text{Clock Rate} = \frac{1}{2 \times 10^{-9}} = 0.5 \times 10^9 \text{ Hz}$$
$$\text{Time}_A = \frac{2.550 \times 10^9}{0.5 \times 10^9} = 5.1 \text{ s}$$
$$\text{Time}_B = \frac{0.850 \times 10^9}{0.5 \times 10^9} = 1.7 \text{ s}$$
$$\text{Time}_C = \frac{0.825 \times 10^9}{0.5 \times 10^9} = 1.65 \text{ s}$$

Calculate Speedup:

$$\text{Speedup}_{C \text{ vs } A} = \frac{\text{Time}_A}{\text{Time}_C} = \frac{5.1}{1.65} = \boxed{3.0909 \text{ or } 209.09\%}$$
$$\text{Speedup}_{C \text{ vs } B} = \frac{\text{Time}_B}{\text{Time}_C} = \frac{1.7}{1.65} = \boxed{1.0303 \text{ or } 3.03\%}$$

6 Question 6: Power Consumption Analysis

6.1 Problem Data

Characteristic	Pentium 4	Core i5
Clock Rate	3.6 GHz	3.4 GHz
Voltage	1.25 V	0.9 V
Static Power	15 W	30 W
Dynamic Power	80 W	40 W

6.2 a) Calculate Capacitive Load (C)

Dynamic Power Formula:

$$P_{\text{dynamic}} = C \times V^2 \times f$$

$$C = \frac{P_{\text{dynamic}}}{V^2 \times f}$$

For Pentium 4:

$$C_{P4} = \frac{80}{(1.25)^2 \times 3.6 \times 10^9} = \frac{80}{1.5625 \times 3.6 \times 10^9}$$

$$C_{P4} = \frac{80}{5.625 \times 10^9} = \boxed{1.422 \times 10^{-8} \text{ F} = 14.22 \text{ nF}}$$

For Core i5:

$$C_{i5} = \frac{40}{(0.9)^2 \times 3.4 \times 10^9} = \frac{40}{0.81 \times 3.4 \times 10^9}$$

$$C_{i5} = \frac{40}{2.754 \times 10^9} = \boxed{1.452 \times 10^{-8} \text{ F} = 14.52 \text{ nF}}$$

6.3 b) Total Power and Static Power Ratio

For Pentium 4:

$$P_{\text{total}} = P_{\text{static}} + P_{\text{dynamic}} = 15 + 80 = \boxed{95 \text{ W}}$$

$$\frac{P_{\text{static}}}{P_{\text{total}}} = \frac{15}{95} = \boxed{0.1579 \text{ or } 15.79\%}$$

For Core i5:

$$P_{\text{total}} = 30 + 40 = \boxed{70 \text{ W}}$$

$$\frac{P_{\text{static}}}{P_{\text{total}}} = \frac{30}{70} = \boxed{0.4286 \text{ or } 42.86\%}$$

6.4 c) Voltage Reduction for 15% Total Power Decrease

Condition:

$$P_{\text{total,new}} = 0.85 \times P_{\text{total,old}}$$

$$P_{\text{static}} + P_{\text{dynamic,new}} = 0.85 \times (P_{\text{static}} + P_{\text{dynamic,old}})$$

$$P_{\text{dynamic,new}} = 0.85 \times P_{\text{total,old}} - P_{\text{static}}$$

For Pentium 4:

$$P_{\text{dynamic,new}} = 0.85 \times 95 - 15 = 80.75 - 15 = 65.75 \text{ W}$$

From $P_{\text{dynamic}} = C \times V^2 \times f$:

$$V_{\text{new}} = \sqrt{\frac{P_{\text{dynamic,new}}}{C \times f}} = \sqrt{\frac{65.75}{14.222 \times 10^{-9} \times 3.6 \times 10^9}}$$

$$V_{\text{new}} = \sqrt{\frac{65.75}{51.2}} = \sqrt{1.2842} = \boxed{1.1332 \text{ V}}$$

Voltage Reduction:

$$\frac{V_{\text{new}} - V_{\text{old}}}{V_{\text{old}}} = \frac{1.1332 - 1.25}{1.25} = \frac{-0.1168}{1.25} = \boxed{-9.34\%}$$

For Core i5:

$$P_{\text{dynamic,new}} = 0.85 \times 70 - 30 = 59.5 - 30 = 29.5 \text{ W}$$

$$V_{\text{new}} = \sqrt{\frac{29.5}{14.524 \times 10^{-9} \times 3.4 \times 10^9}} = \sqrt{\frac{29.5}{49.376}} = \boxed{0.7729 \text{ V}}$$

Voltage Reduction:

$$\frac{0.7729 - 0.9}{0.9} = \boxed{-14.12\%}$$

7 Question 7: Multiprocessor System Analysis

7.1 Problem Data

- Arithmetic instructions: $N_{\text{arith}} = 2.56 \times 10^9$, CPI = 1
- Load/Store instructions: $N_{\text{LS}} = 1.28 \times 10^9$, CPI = 12
- Branch instructions: $N_{\text{branch}} = 256 \times 10^6$, CPI = 4
- Clock rate: $f = 3$ GHz

7.2 a) Execution Time and Speedup

Execution Time Formula for p processors:

$$T(p) = \frac{\left(\frac{N_{\text{arith}}}{p} \times \text{CPI}_{\text{arith}} + \frac{N_{\text{LS}}}{p} \times \text{CPI}_{\text{LS}} + N_{\text{branch}} \times \text{CPI}_{\text{branch}}\right)}{f}$$

For p = 1:

$$\text{Cycles} = 2.56 \times 10^9 \times 1 + 1.28 \times 10^9 \times 12 + 256 \times 10^6 \times 4$$
$$= 2.56 \times 10^9 + 15.36 \times 10^9 + 1.024 \times 10^9 = 18.944 \times 10^9$$
$$T(1) = \frac{18.944 \times 10^9}{3 \times 10^9} = \boxed{6.315 \text{ s}}$$

For p = 2:

$$\text{Cycles} = 1.28 \times 10^9 \times 1 + 0.64 \times 10^9 \times 12 + 256 \times 10^6 \times 4$$
$$= 1.28 \times 10^9 + 7.68 \times 10^9 + 1.024 \times 10^9 = 9.984 \times 10^9$$
$$T(2) = \frac{9.984 \times 10^9}{3 \times 10^9} = \boxed{3.328 \text{ s}}$$

For p = 4:

$$\text{Cycles} = 0.64 \times 10^9 \times 1 + 0.32 \times 10^9 \times 12 + 256 \times 10^6 \times 4$$
$$= 0.64 \times 10^9 + 3.84 \times 10^9 + 1.024 \times 10^9 = 5.504 \times 10^9$$
$$T(4) = \frac{5.504 \times 10^9}{3 \times 10^9} = \boxed{1.835 \text{ s}}$$

For p = 8:

$$\text{Cycles} = 0.32 \times 10^9 \times 1 + 0.16 \times 10^9 \times 12 + 256 \times 10^6 \times 4$$
$$= 0.32 \times 10^9 + 1.92 \times 10^9 + 1.024 \times 10^9 = 3.264 \times 10^9$$
$$T(8) = \frac{3.264 \times 10^9}{3 \times 10^9} = \boxed{1.088 \text{ s}}$$

Processors	Execution Time (s)	Speedup
1	6.315	1.000
2	3.328	1.897
4	1.835	3.442
8	1.088	5.804

Calculate Speedup:

$$\text{Speedup}(p) = \frac{T(1)}{T(p)}$$

7.3 b) New CPI for Load/Store

Goal: Single-core processor execution time = 4-processor system execution time

$$T_{\text{single}} = T(4) = 1.835 \text{ s}$$

$$1.835 = \frac{2.56 \times 10^9 \times 1 + 1.28 \times 10^9 \times \text{CPI}_{\text{LS,new}} + 256 \times 10^6 \times 4}{3 \times 10^9}$$

$$1.835 \times 3 \times 10^9 = 2.56 \times 10^9 + 1.28 \times 10^9 \times \text{CPI}_{\text{LS,new}} + 1.024 \times 10^9$$

$$5.505 \times 10^9 = 3.584 \times 10^9 + 1.28 \times 10^9 \times \text{CPI}_{\text{LS,new}}$$

$$1.921 \times 10^9 = 1.28 \times 10^9 \times \text{CPI}_{\text{LS,new}}$$

$$\boxed{\text{CPI}_{\text{LS,new}} = 1.50}$$

8 Question 8: Benchmark Analysis

8.1 Problem Data

- Instruction count: $N = 2.389 \times 10^{12}$
- Execution time: $T = 875$ s
- Reference time: $T_{\text{ref}} = 9650$ s
- Clock cycle time: $\Delta t = 0.6$ ns

8.2 a) SPECratio

$$\text{SPECratio} = \frac{T_{\text{ref}}}{T} = \frac{9650}{875} = \boxed{11.03}$$

8.3 b) Calculate CPI

Formula:

$$\text{CPI} = \frac{T}{N \times \Delta t}$$

$$\text{CPI} = \frac{875}{2.389 \times 10^{12} \times 0.6 \times 10^{-9}} = \frac{875}{1.433 \times 10^{3}} = \boxed{0.6104}$$

8.4 c) 10% Increase in Instruction Count

$$N_{\text{new}} = N \times 1.10 = 2.389 \times 10^{12} \times 1.10 = 2.628 \times 10^{12}$$

$$T_{\text{new}} = N_{\text{new}} \times \text{CPI} \times \Delta t = 2.628 \times 10^{12} \times 0.6104 \times 0.6 \times 10^{-9}$$

$$T_{\text{new}} = \boxed{962.50 \text{ s}}$$

$$\text{Increase} = \boxed{10\%}$$

8.5 d) 5% CPI Increase and 10% Instruction Increase

$$\text{CPI}_{\text{new}} = 0.6104 \times 1.05 = 0.6410$$

$$N_{\text{new}} = 2.628 \times 10^{12}$$

$$T_{\text{new}} = 2.628 \times 10^{12} \times 0.6410 \times 0.6 \times 10^{-9} = \boxed{1010.62 \text{ s}}$$

$$\text{Increase} = \boxed{15.5\%}$$

$$\text{SPECratio}_{\text{new}} = \frac{9650}{1010.62} = \boxed{9.55}$$

8.6 e) New Processor CPI

Conditions:

- Clock rate: $f_{\text{new}} = 4$ GHz
- 15% reduction in instructions: $N_{\text{new}} = 2.389 \times 10^{12} \times 0.85 = 2.031 \times 10^{12}$
- Execution time: $T_{\text{new}} = 750$ s

$$\Delta t_{\text{new}} = \frac{1}{4 \times 10^9} = 0.25 \times 10^{-9} \text{ s}$$

$$\text{CPI}_{\text{new}} = \frac{T_{\text{new}}}{N_{\text{new}} \times \Delta t_{\text{new}}} = \frac{750}{2.031 \times 10^{12} \times 0.25 \times 10^{-9}}$$

$$\text{CPI}_{\text{new}} = \frac{750}{507.75} = \boxed{1.4771}$$

8.7 f) Clock Rate for 10% Time Reduction

Goal: Execution time at 90% of previous time

$$T_{\text{target}} = 750 \times 0.9 = 675 \text{ s}$$

Formula:

$$\text{Clock Rate} = \frac{N \times \text{CPI}}{T_{\text{target}}}$$

$$f_{\text{new}} = \frac{2.031 \times 10^{12} \times 1.4771}{675} = \boxed{4.44 \text{ GHz}}$$

8.8 g) Clock Rate for 20% Time Reduction and 15% CPI Reduction

$$T_{\text{target}} = 750 \times 0.8 = 600 \text{ s}$$

$$\text{CPI}_{\text{new}} = 1.4771 \times 0.85 = 1.2555$$

$$f_{\text{new}} = \frac{2.031 \times 10^{12} \times 1.2555}{600} = \boxed{4.25 \text{ GHz}}$$