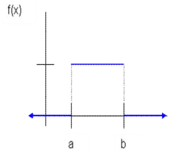
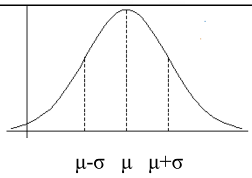
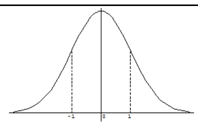
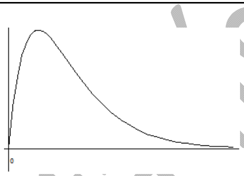
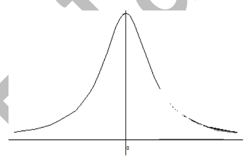
VARIABLEAS ALEATORIAS

V. Aleatoria	Discretas		Continuas
Espacio Muestral	$S_x=(x_1, x_2, x_3, \ldots\ldots\ldots x_n)$		$S_x = (-\infty ; \infty)$
Función de densidad	$f(x)=\begin{cases} P(X=x) & \text{si } x \in S_X \\ 0 & \text{si } x \notin S_X \end{cases}$ $P(X=x)>0 \quad \forall x \in S_X$ $\sum_{x\in S_X} P(X=x) = \sum_{x\in S_X} f(x) = 1$		$f(x)=\frac{\partial F(x)}{\partial x}=F'(x)$ $f(x)\geq 0, \ \forall x \in \Re$ $\int_{-\infty}^{\infty} f(x)dx=1$
Función de Distribución	$F(x)=P(X\leq x)=\sum_{x_i\leq x} P(X=x_i)$		$F(x)=P(X\leq x)=\int_{-\infty}^{x} f(t)dt \quad \forall x \in \Re$ $F(x)=\begin{cases} 0 & \text{si} \quad X\leq a \\ \int_a^x f(t)\ dt & \text{si} \quad a<X<b \\ 1 & \text{si} \quad X\geq b \end{cases}$
Media	$\mu=\alpha_1=E[X]=\sum_{\forall x_i \in S_X} x_i P(X=x_i)$		$\mu=\alpha_1=E[X]=\int_{-\infty}^{\infty} x f(x)dx$
Varianza	$\sigma^2=V(X)=\mu_2=E\left[(X-\mu)^2\right]=\sum_{\forall x_i\in S_X}(x_i-\mu)^2 P(X=x_i)=$ $=\alpha_2-\alpha_1^2=E\left[X^2\right]-\left(E[X]\right)^2=\left[\sum_{\forall x_i\in S_X} x_i^2 P(X=x_i)\right]-\mu^2$		$\sigma^2=V(X)=\mu_2=E\left[(X-\mu)^2\right]=\int_{-\infty}^{\infty}(x-\mu)^2 f(x)dx=$ $=\alpha_2-\alpha_1^2=E\left[X^2\right]-\left(E[X]\right)^2=\left[\int_{-\infty}^{\infty} x^2 f(x)dx\right]-\mu^2$

PRINCIPALES DISTRIBUCIONES DISCRETAS

Nombre	**Notación**	**Espacio Muestral**	**Función de densidad**	**Media**	**Varianza**
Uniforme	$U(K)$	$S=(x_1, x_2, \ldots, x_K)$	$f(x)=1/K$	$E[X]=\frac{1}{K}\sum_{\forall x_i\in S_X} x_i$	$\sigma^2=V(X)=\left[\frac{1}{K}\sum_{\forall x_i\in S_X} x_i^2\right]-\mu^2$
		$S=(1,2,\ldots,K)$		$E[X]=\frac{K+1}{2}$	$V[X]=\frac{K^2-1}{12}$
Binaria	$B(p)$	S= (0,1)	$f(x)=p^x(1-p)^{1-x}$	$E[X]=p$	$V[X]=p(1-p)=pq$
Binomial	$b(n,p)$	S=(0,1,2,…,n)	$f(x)=\binom{n}{x}p^x(1-p)^{n-x}$	$E[X]=np$	$V[X]=npq$
Hipergeométrica	$h(n,a,b)$ $H(N,n,a)$	S=(0,1,2,…,n) $\forall$ n≤a N = a + b	$f(x)=\frac{\binom{a}{x}\binom{b}{n-x}}{\binom{a+b}{n}} \begin{cases} x\leq a \\ n-x\leq b \\ n\leq a+b \end{cases}$	$E[X]=n\frac{a}{N}$	$V[X]=\frac{N-n}{N-1}n\frac{a}{N}\frac{b}{N}$
Poissón	$X\in P(\lambda)$; $\lambda=np$	S=(0,1,2,…..)	$f(x)=e^{-\lambda}\frac{\lambda^x}{x!}$	$E[X]=\lambda$	$V[X]=\lambda$
Geométrica	$X\in G(p)$	$S=\{1,2,3,\ldots\ldots\}$	$f(x)=(1-p)^{x-1}p$	$E[X]=\frac{1}{p}$	$V[X]=\frac{q}{p^2}$
Binomial Negativa	$X\in bn(k;p)$	$S=(k,k+1,k+2,\ldots.)$	$f(x)=\binom{x-1}{k-1}p^{k-1}(1-p)^{x-k}p$	$E[X]=\frac{k}{p}$	$V[X]=\frac{kq}{p^2}$

PRINCIPALES DISTRIBUCIONES CONTINUAS

Nombre	Notación	Gráfica	Función de densidad	Media	Varianza
Uniforme o Rectangular	$X \in U(a,b)$ $X \in R(a,b)$		$f(x)=\begin{cases} 0 & \text{si } X \notin (a,b) \\ \frac{1}{b-a} & \text{si } X \in (a,b) \end{cases}$	$\mu = \frac{a+b}{2}$	$\sigma^2 = \frac{(b-a)^2}{12}$
Normal	$X \in N(\mu;\sigma^2)$		$f(x)=\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$ $\forall\ X \in \Re$	μ	σ^2
Normal tipificada o Estandar	$Z \in N(0;1)$		$f(z)=\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$	0	1
Gamma	$X \in \gamma(a,p)$ $a,p>0$		$f(x)=\begin{cases} \frac{a^p}{\Gamma(p)} x^{p-1} a^{-ax} & \text{si } \begin{cases} X>0 \\ a,p>0 \end{cases} \\ 0 & \text{en el resto} \end{cases}$ siendo: $\Gamma(p)=\int_0^{+\infty} x^{p-1}e^{-x}\,dx$	$E[X]=\frac{p}{a}$	$V[X]=\frac{p}{a^2}$
Exponencial	$X \in \gamma(a,1) \equiv Exp(a)\ a>0$		$f(x)=\begin{cases} a\,e^{-ax} & ; \quad X \geq 0, a>0 \\ 0 & ; \quad \text{resto} \end{cases}$	$E[X]=\frac{1}{a}$	$V[X]=\frac{1}{a^2}$
Chi-Cuadrado	$X \in \gamma\left(\frac{1}{2},\frac{n}{2}\right) \equiv \chi^2_{(n)}$ $n \equiv$ "Grados de Libertad"		$f(x)=\begin{cases} \frac{\left(\frac{1}{2}\right)^{n/2}}{\Gamma(n/2)} x^{n/2-1} e^{-x/2} & \text{si } X>0 \\ 0 & \text{en el resto} \end{cases}$	n	2n
t- Student	$X \in t_{(n)}$ $n \equiv$ "Grados de Libertad"		$f(x)=\frac{\Gamma\left(\frac{n+1}{2}\right)}{\sqrt{n\pi}\,\Gamma\left(\frac{n}{2}\right)}\left(1+\frac{x^2}{n}\right)^{-\left(\frac{n+1}{2}\right)} \quad \forall X \in \Re$	0	$\frac{n}{n-2} \quad \forall n>2$
F- Snedecor	$X \in F_{(m,n)} \equiv F_{(n,d)} \equiv F_{(n_1,n_2)}$ m :"Grados de Libertad del Numerador" n : "Grados de Libertad del Denominador"		$f(x)=\frac{\left(\frac{m}{n}\right)^2 \Gamma\left(\frac{m+n}{2}\right)}{\Gamma\left(\frac{m}{2}\right)\Gamma\left(\frac{n}{2}\right)} x^{\frac{m}{2}-1}\left(1+\frac{m}{n}x\right)^{-\left(\frac{m+n}{2}\right)}$ $\forall\ X>0$	$\frac{n}{n-2} \quad \forall n>2$	$\frac{2n^2(m+n-2)}{m(n-2)^2(n-4)}$ $\forall n>4$

CONVERGENCIAS ENTRE DISTRIBUCIONES

<table>
<tr><td colspan="3">$X \in h(n,a,b) \rightarrow X^* \in b\left(n; p = a/N\right)$ Si: $n \leq 0.05N$ siendo $N = a + b$</td></tr>
<tr><td colspan="3">$X \in b(n,p) \rightarrow X^* \in P(\lambda)$ Si: $\begin{cases} n \geq 50 \\ p \leq 0.1 \end{cases}$ o bien $\lambda \leq 18$</td></tr>
<tr><td colspan="3">$X \in b(n,p) \rightarrow X^* \in N(np; npq) \Rightarrow Z^* = \dfrac{X^* - np}{\sqrt{npq}} \rightarrow N(0,1)$ Si: $\begin{cases} n \geq 30 \\ 0.1 < p < 0.9 \end{cases}$ ó $\begin{cases} n \geq 30 \\ \min.(p,q) \geq 0.1 \end{cases}$</td></tr>
<tr><td colspan="3">$X \in P(\lambda) \rightarrow X^* \in N(\lambda; \lambda) \Rightarrow Z^* = \dfrac{X^* - \lambda}{\sqrt{\lambda}} \rightarrow N(0,1)$ Si: $\lambda \geq 18$</td></tr>
<tr><td>$X \in \chi^2_{(n)} \xrightarrow{n>30} X^* \in N(n; 2n)$
$Z^* = \dfrac{X^* - n}{\sqrt{2n}} \rightarrow N(0;1)$</td><td colspan="2">$X \in \chi^2_{(n)} \Rightarrow X^* = \sqrt{2X} - \sqrt{2n-1} \xrightarrow{n>30} X^* \in N(0;1)$</td></tr>
<tr><td>$X \in t_{(n)} \xrightarrow{n \to \infty} X^* \in N(0\,;1)$</td><td>$X \in F_{(n_1, n_2)} \xrightarrow{n_2 \to \infty} X^* \in \chi^2_{(n_1)}$</td><td>$X^* \in \chi^2_{(n_1)} \xrightarrow[n_1>30]{} X^{**} \in N(\mu = n_1; \sigma^2 = 2n_1)$</td></tr>
</table>

CORRECCIÓN DE CONTINUIDAD DE YATES	
$X \equiv$ Discreta	$Y \equiv$ Continua
$X = a$	$a-0.5 \leq Y \leq a+0.5$
$a < X < b$	$a+0.5 \leq Y \leq b-0.5$
$a \leq X \leq b$	$a-0.5 \leq Y \leq b+0.5$
$a \leq X < b$	$a-0.5 \leq Y \leq b-0.5$
$a < X \leq b$	$a+0.5 \leq Y \leq b+0.5$

DESIGUALDAD DE TCHEBYCHEFF

Dada una v.a. $X \in D(\mu, \sigma^2)$, para cualquier constante k > 0 se cumple:

$$P\left(|X - \mu| \leq k\sigma\right) \geq 1 - \frac{1}{k^2} \Leftrightarrow P(\mu - k\sigma \leq X \leq \mu + k\sigma) \geq 1 - \frac{1}{k^2}$$

O bien: $P\left(|X - \mu| > k\sigma\right) < \dfrac{1}{k^2}$