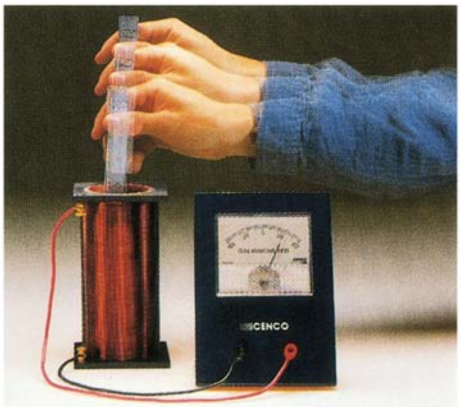
# Campos electromagnéticos dependientes del tiempo




$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_o}$$

$$\vec{\nabla} \mathbf{x} \vec{E} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \mathbf{x} \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{J}}$$

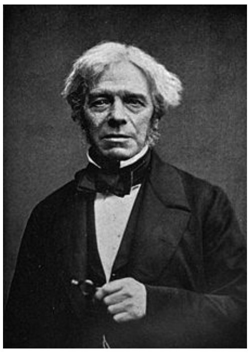
**Ecuaciones de Maxwell para campos estáticos**

**Electricidad**

**Magnetismo**

$\Longrightarrow$ **dos fenómenos independientes**

# Ley de Faraday-Henry

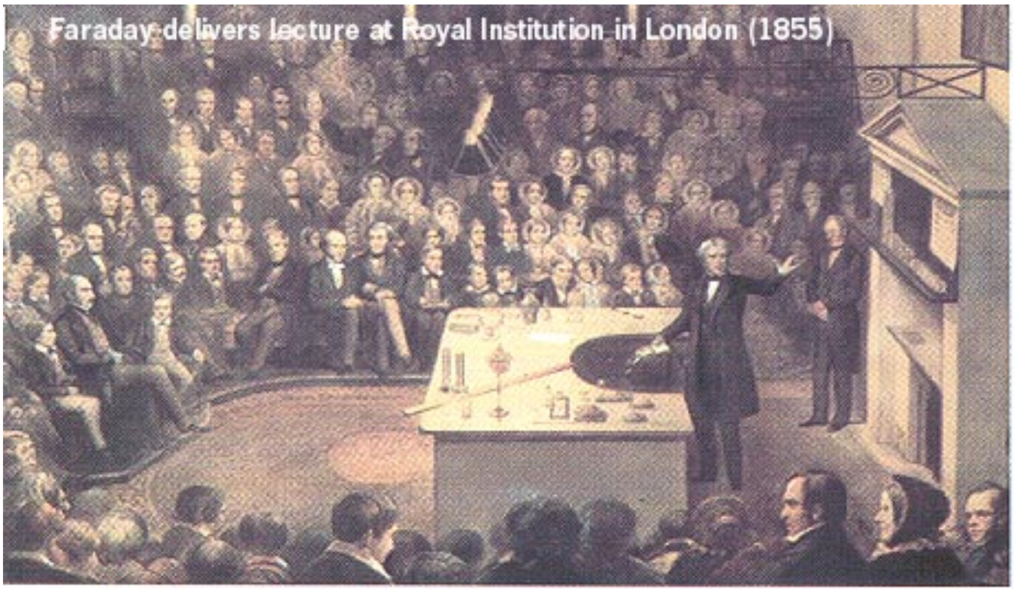


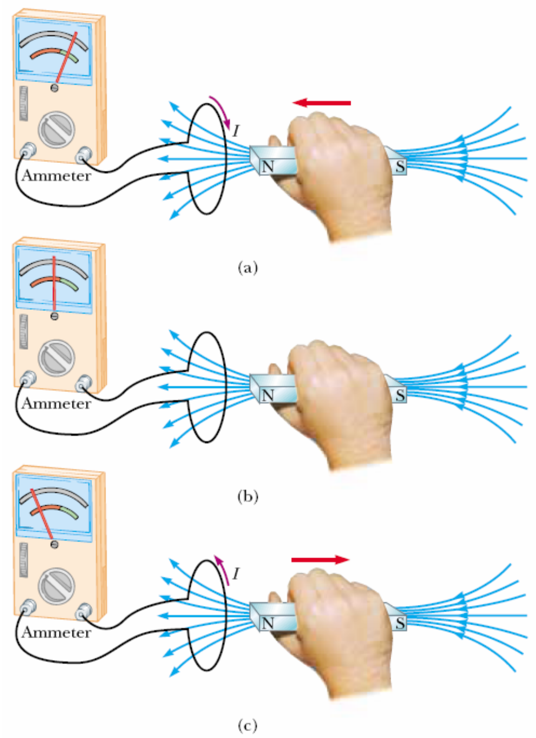
**Michael Faraday (1791-1867)**

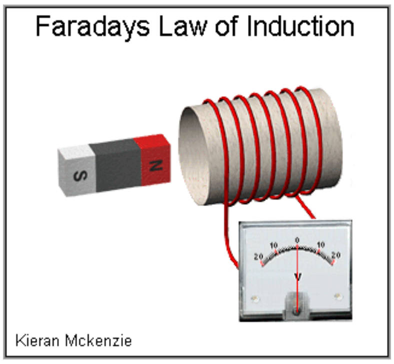
**Joseph Henry (1797-1878)**





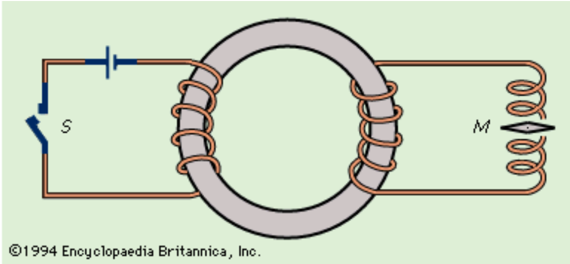



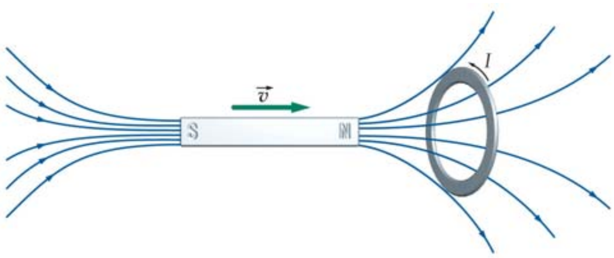




**Una corriente eléctrica puede producirse mediante un campo magnético variable**

## Experimento de Faraday

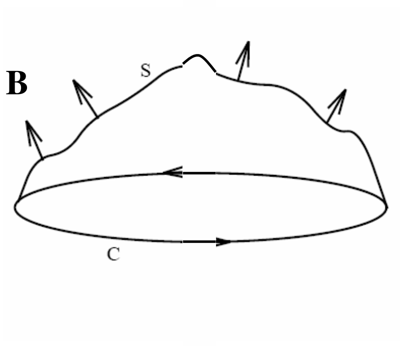



**Un campo magnético variable induce una fem en el circuito secundario**




$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

## Enunciado general de la ley de Faraday-Henry




$$\mathcal{E} = -\frac{d\phi_{\mathrm{m}}}{dt}$$

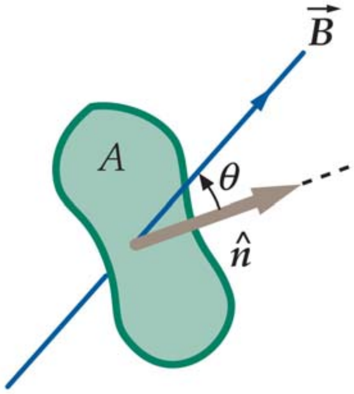
$$\phi_{\mathrm{m}} = \int_S \vec{B} \cdot \hat{n}\, dA = \int_S B_n\, dA$$

$$\boxed{\mathcal{E} = \oint_C \vec{E}_{\mathrm{nc}} \cdot d\vec{\ell} = -\frac{d}{dt}\int_S \vec{B} \cdot \hat{n}\, dA = -\frac{d\phi_{\mathrm{m}}}{dt}}$$

**En una bobina con N vueltas, todas de la misma área**

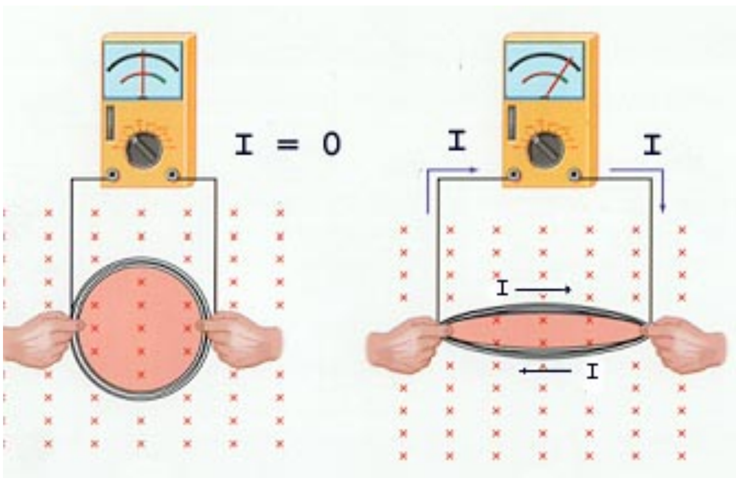
$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

$\Phi_B(t)$ ?

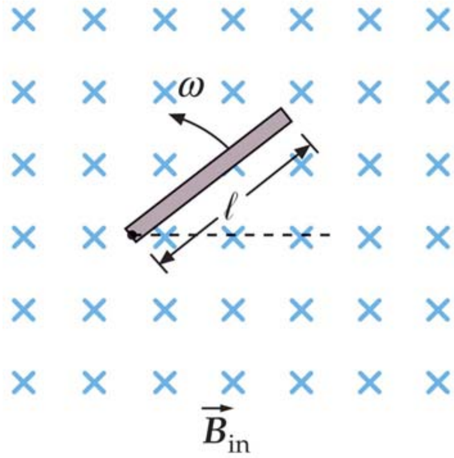


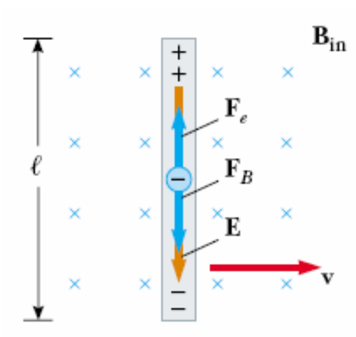

$$\mathcal{E} = -\frac{d}{dt}(BA\cos\theta)$$

- B(t)
- A(t)
- θ(t)



# fem de movimiento



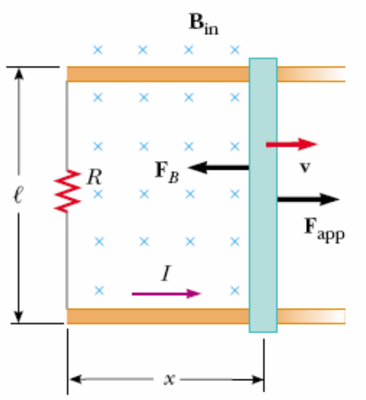




$$qE = qvB$$

$$E = vB$$

$$\Delta V = E\ell = B\ell v$$

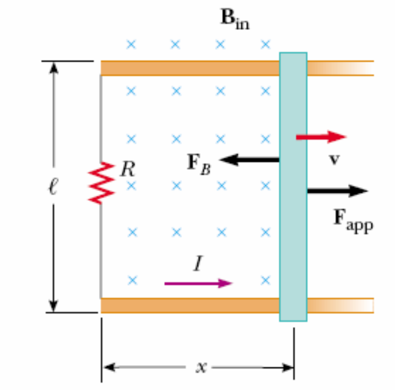
**Si se invierte v, se invierte la polaridad de V**

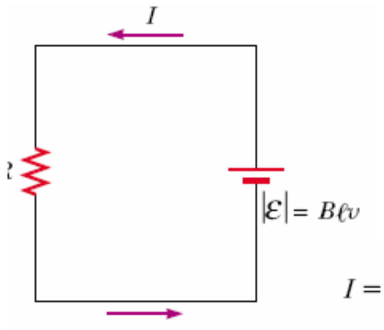



$$\Phi_B = B\ell x$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(B\ell x) = -B\ell\frac{dx}{dt}$$

$$\mathcal{E} = -B\ell v$$




$$I = \frac{|\mathcal{E}|}{R} = \frac{B\ell v}{R}$$

**Consideración energética: si v = cte** $\longrightarrow$ $F_{app} = I\ell B$

**La potencia entregada por la fuerza aplicada es:**

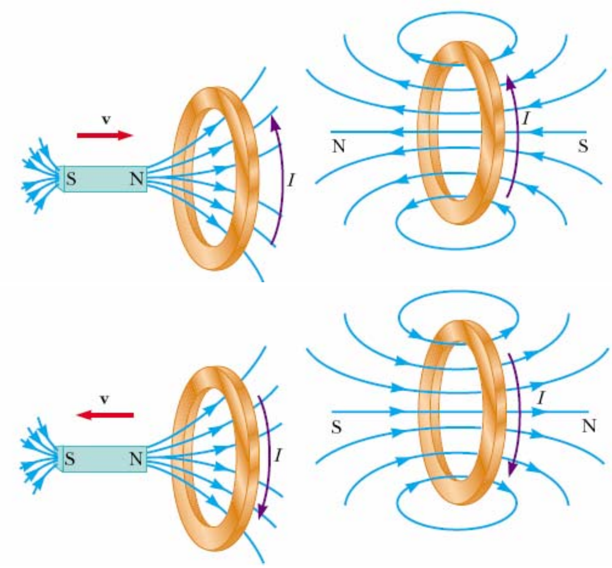
$$\mathcal{P} = F_{app}v = (I\ell B)\,v = \frac{B^2\,\ell^2\,v^2}{R} = \frac{\mathcal{E}^2}{R}$$

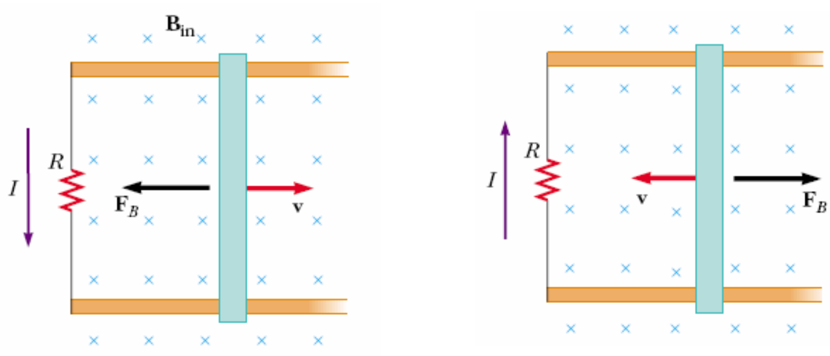
**Energía Mecánica** → **Energía Eléctrica** → **Energía Térmica**

# Ley de Lenz

**Heinrich Lenz (1804-1865)**

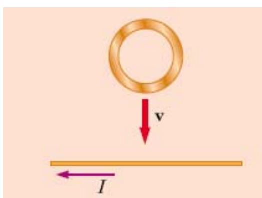
La polaridad de una fem inducida es tal que tiende a producir una corriente eléctrica que creara un flujo magnético que se opone al cambio de $\Phi_B$ a través del lazo.



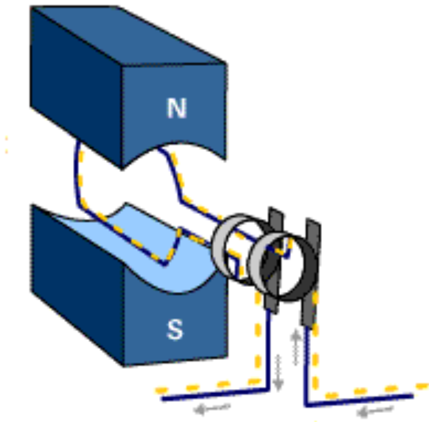




Qué sucede si el sentido de I es contrario al obtenido utilizando la ley de Lenz en los dos casos anteriores ?

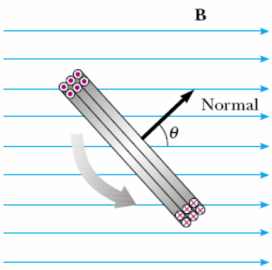
Determinar el sentido de la corriente inducida en la espira circular.

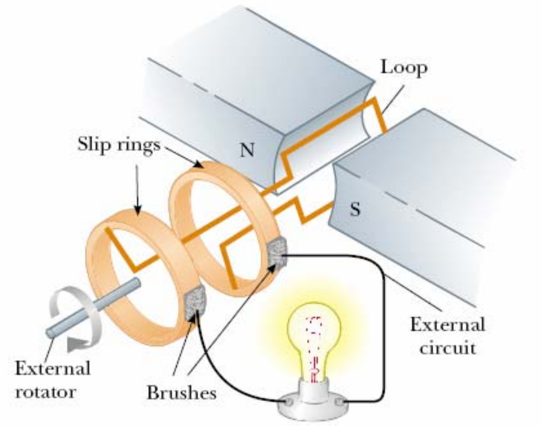


# Aplicaciones de la ley de Faraday




## Generador eléctrico

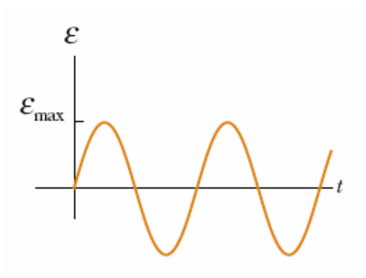


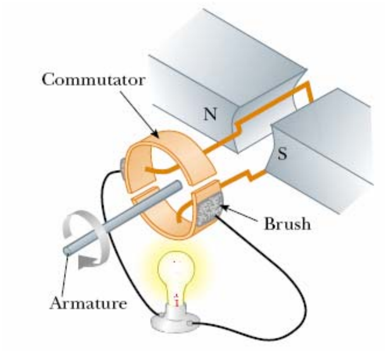



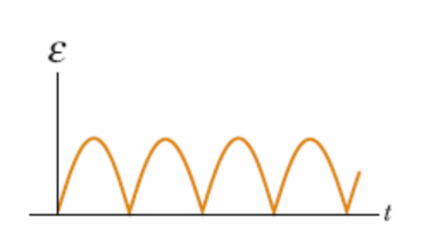

$$\Phi_B = BA\cos\theta = BA\cos\omega t$$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -NAB\frac{d}{dt}(\cos\omega t) = NAB\omega\sin\omega t$$

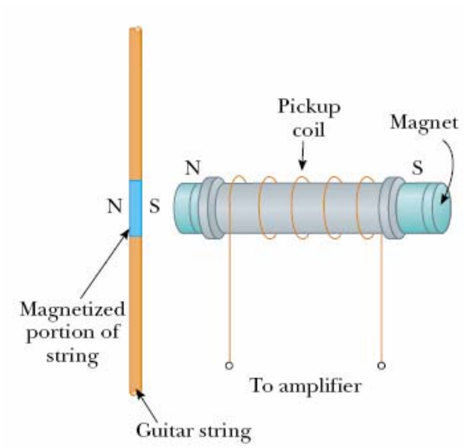
$$\mathcal{E}_{\max} = NAB\omega$$



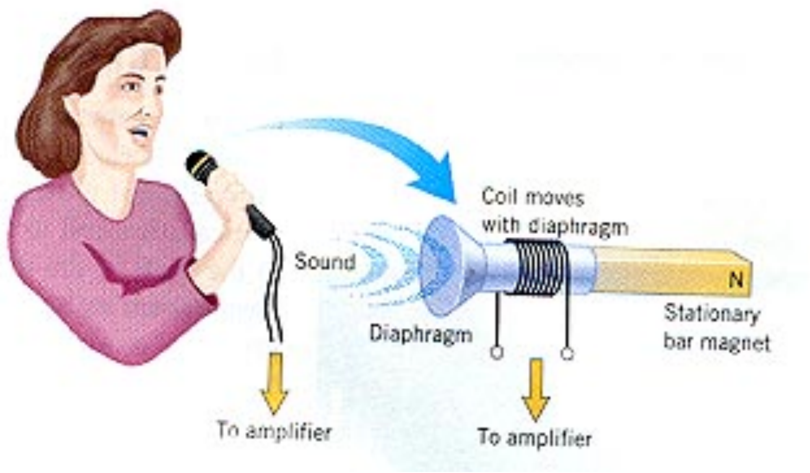



## Guitarra eléctrica

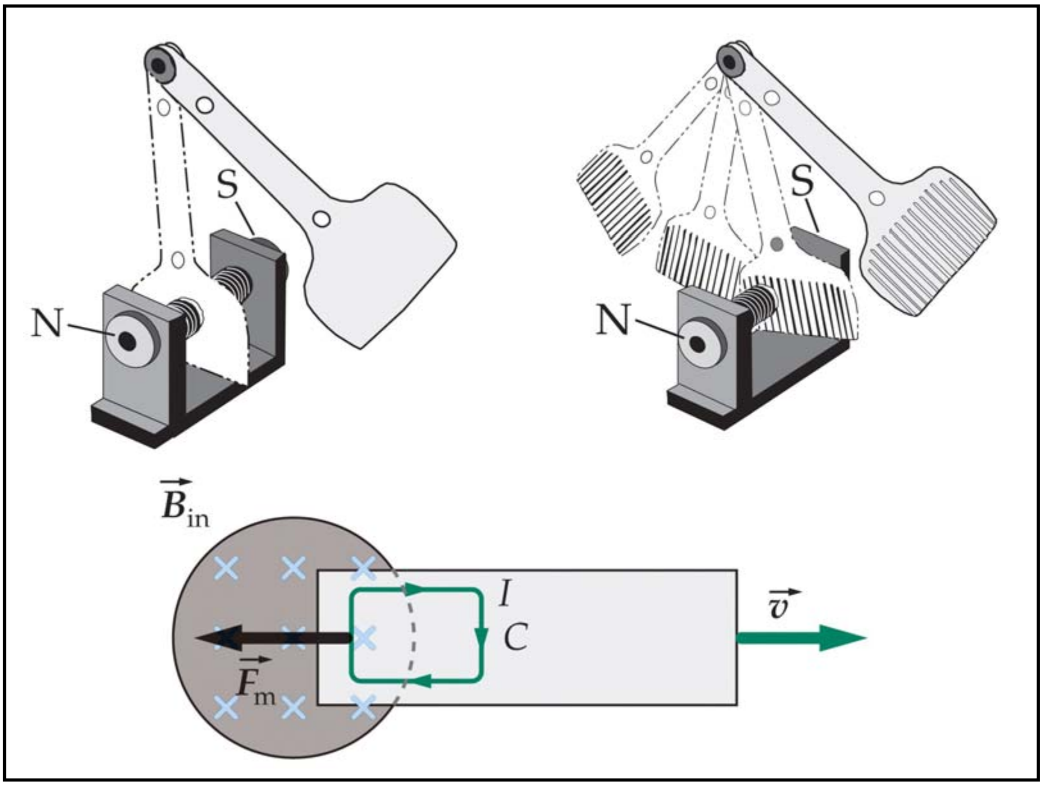



**Micrófono de bobina móvil**

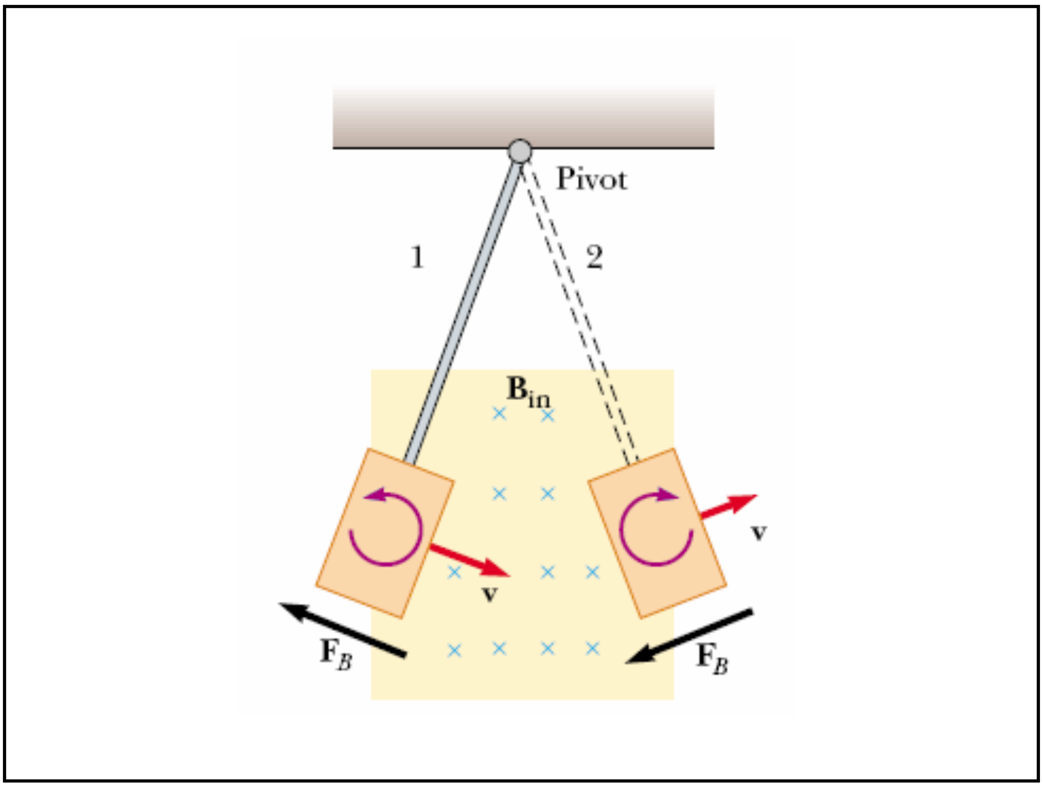



# Corrientes de Foucault

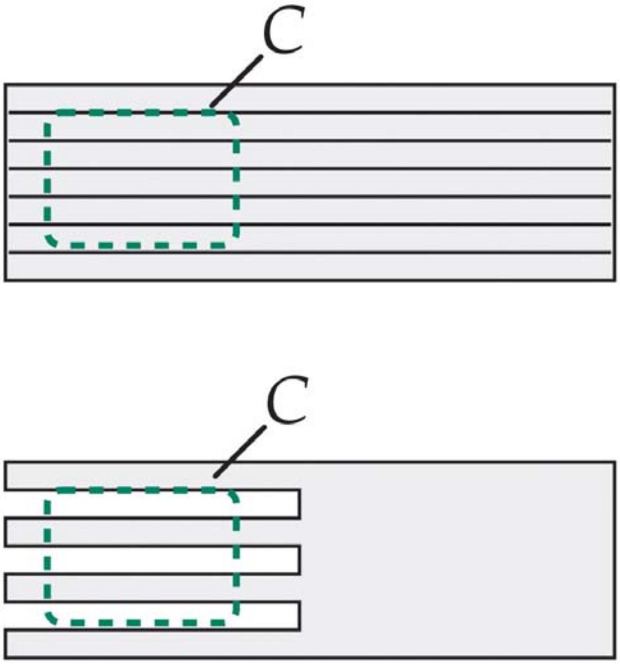
**Jean Leon Foucault (1819-1868 )**



S
N
S
N
$\vec{B}_{in}$
$I$
$C$
$\vec{v}$
$\vec{F}_m$

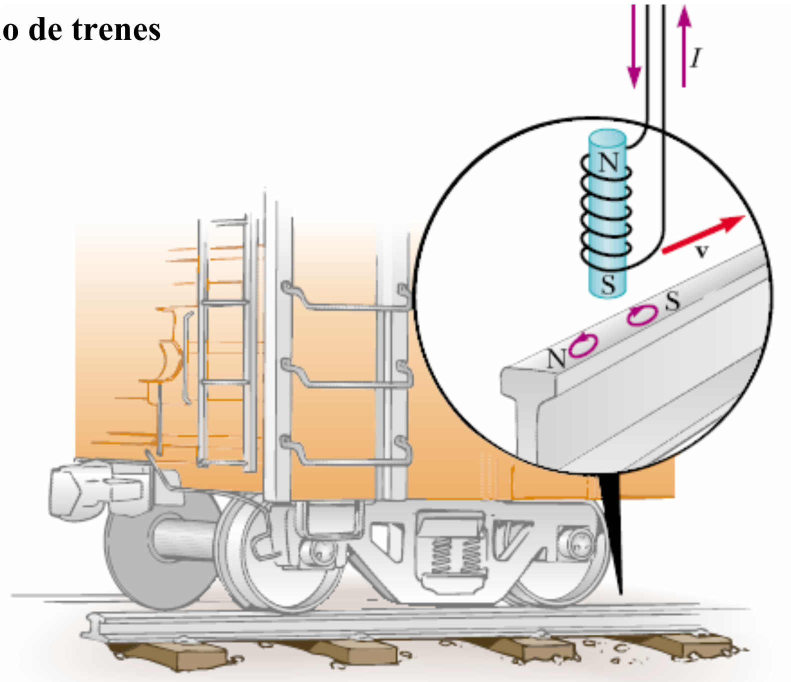



Pivot
1
2
$B_{in}$
v
v
$F_B$
$F_B$

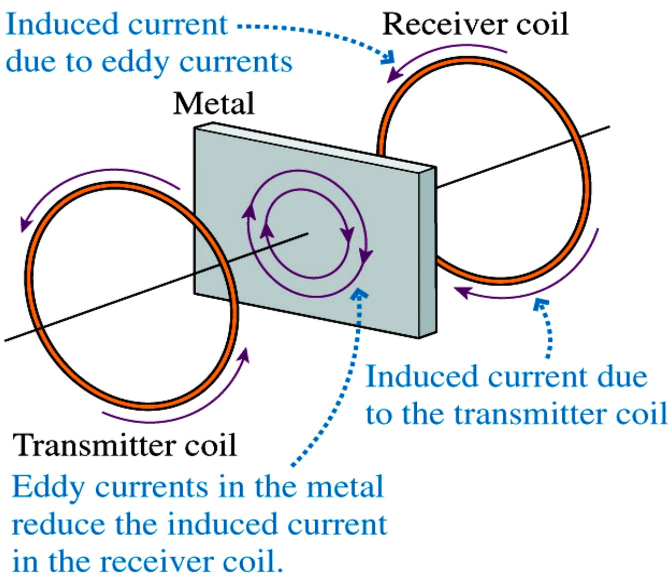


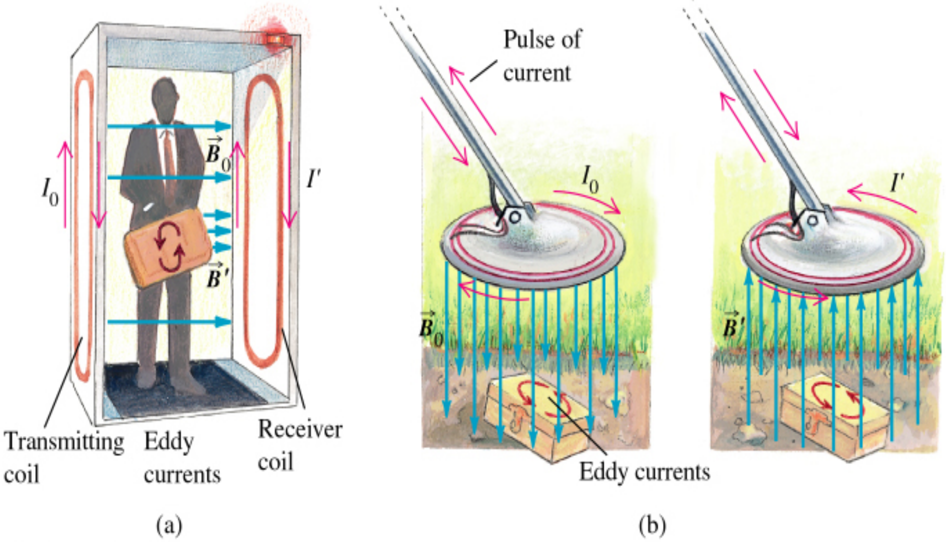

Son útiles para amortiguar oscilaciones mecánicas

**Freno de trenes**



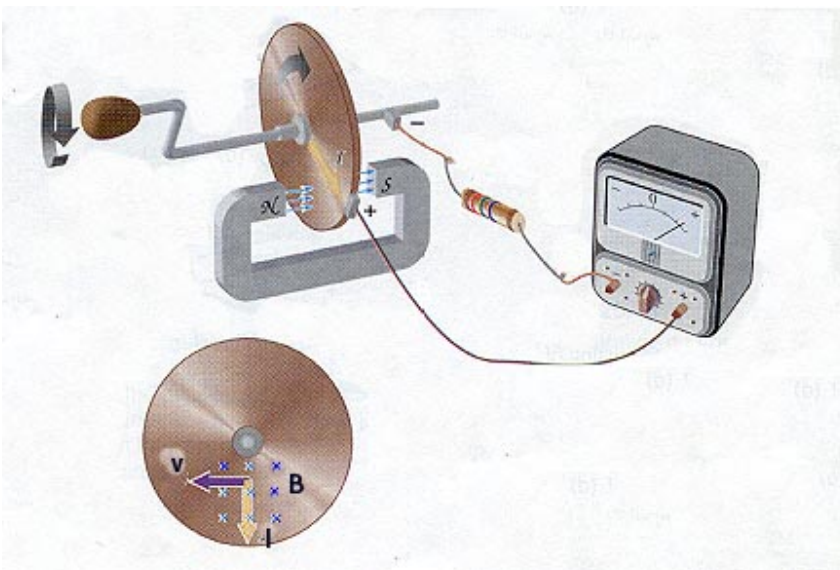

**Detector de metales**

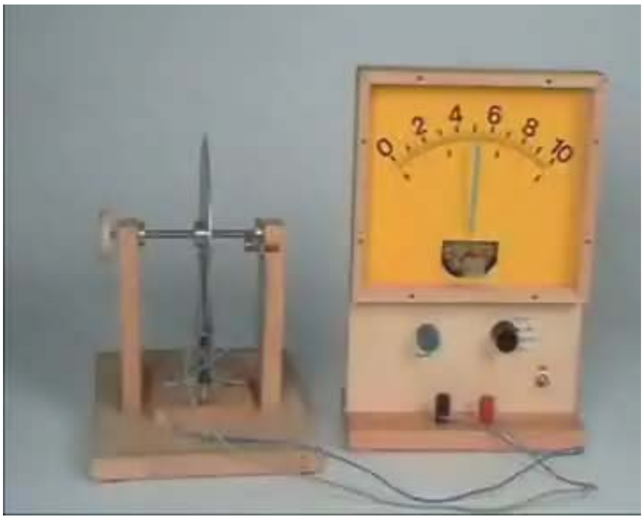




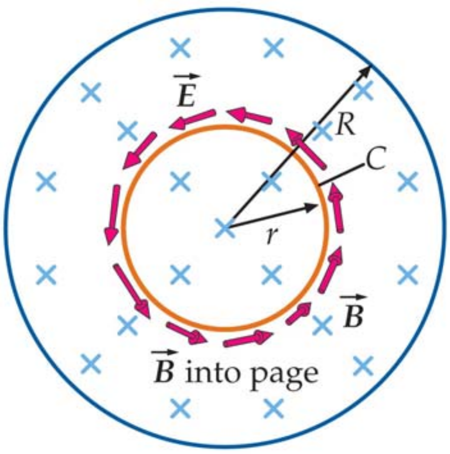


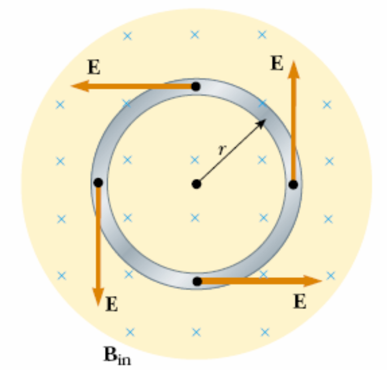

## El primer generador






# fem inducidas y campos eléctricos






$$\mathscr{E} = -\frac{d\phi_m}{dt}$$

$$\phi_m = \int_S \vec{B} \cdot \hat{n}\, dA = \int_S B_n\, dA$$

$$\mathscr{E} = \oint_C \vec{E}_{nc} \cdot d\vec{\ell} = -\frac{d}{dt}\int_S \vec{B} \cdot \hat{n}\, dA = -\frac{d\phi_m}{dt}$$

**El campo eléctrico inducido no es conservativo**

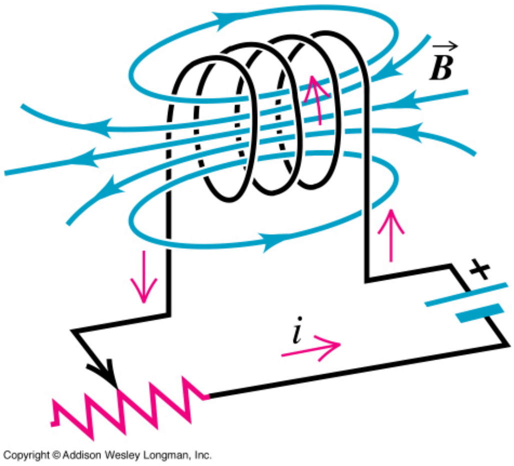
$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B} \cdot d\vec{A}$$

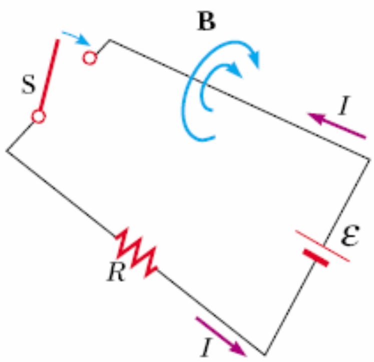
$$\left.\begin{array}{l} \oint_L \vec{E} \cdot d\vec{l} = \int_S \nabla \wedge \vec{E} \cdot d\vec{s} \\ \dfrac{d}{dt}\int_S \vec{B} \cdot d\vec{s} = \int_S \dfrac{\partial \vec{B}}{\partial t} \cdot d\vec{s} \end{array}\right\} \rightarrow \nabla \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

**La ley de Faraday-Henry en forma diferencial**

# Autoinducción




**S se cierra** $\Longrightarrow$ I (t)

$\Downarrow$

B(t)

**Variación temporal de $\Phi_B$ a través del área del circuito**

**fem autoinducida**

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

B α I

$\Phi_B$ α I

**Definimos:**

$$\Phi_B = L\ I$$

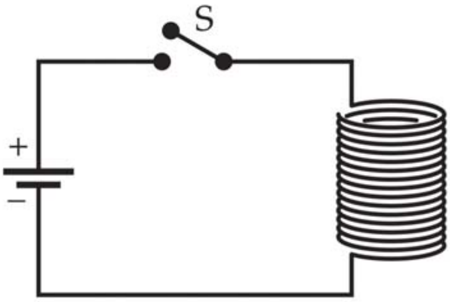
**L: coeficiente de autoinductancia o inductancia** $\Longrightarrow$

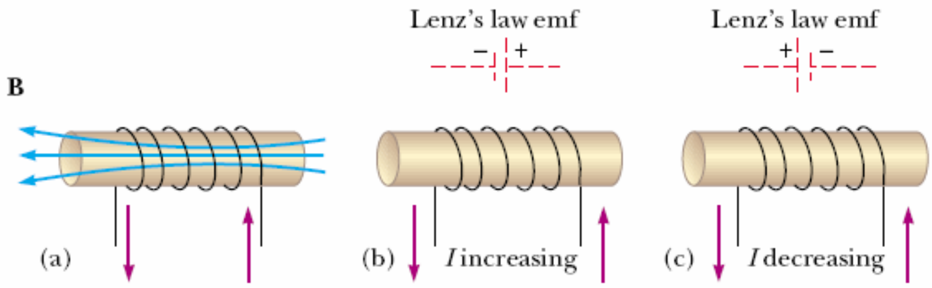
$$\varepsilon_L = -L\frac{dI}{dt}$$

$L = \Phi_B \ / \ I = -\mathcal{E}_L \ / \ (dI/dt)$


**[ L] = T m² / A = Wb / A = V s / A = H (Henry)**

**La inductancia de un circuito depende de su geometría**




$$L = \frac{N\Phi_B}{I}$$

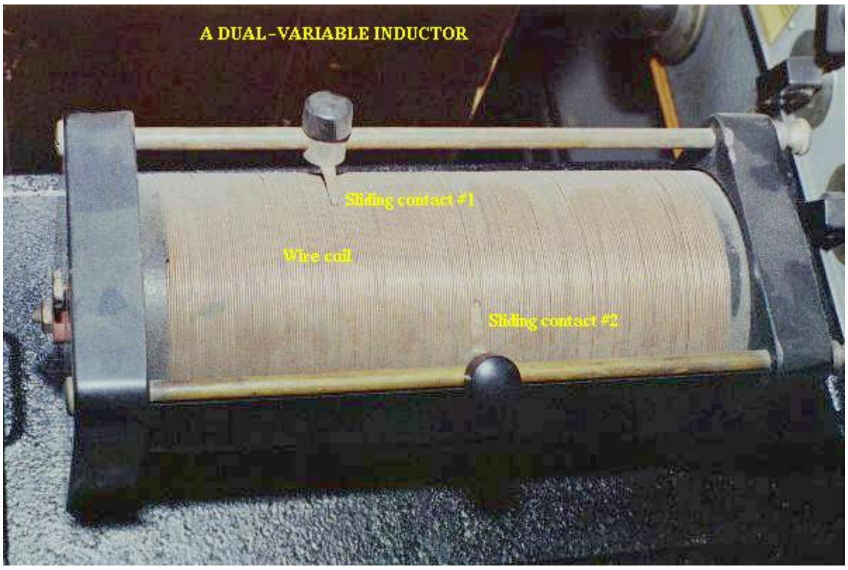
$$B = \mu_0 n I = \mu_0 \frac{N}{\ell} I \qquad n = N/\ell$$

$$\Phi_B = BA = \mu_0 \frac{NA}{\ell} I$$

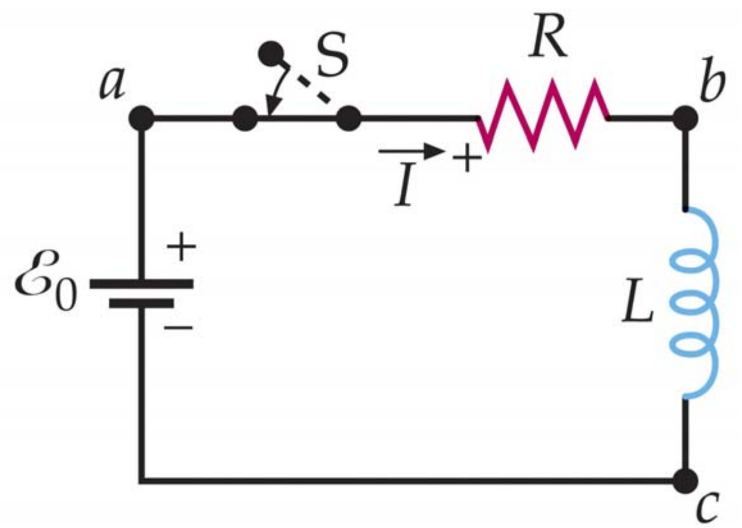
$$L = \frac{N\Phi_B}{I} = \frac{\mu_0 N^2 A}{\ell}$$

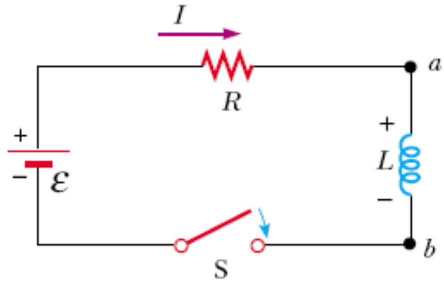
$$L = \mu_0 \frac{(n\ell)^2}{\ell} A = \mu_0 n^2 A\ell = \mu_0 n^2 V$$

$$V = A\ell$$


A DUAL-VARIABLE INDUCTOR
Sliding contact #1
Wire coil
Sliding contact #2

# Circuito *RL*


a
S
R
b
I
+
$\mathcal{E}_0$
+
−
L
c




$$\mathcal{E} - IR - L\frac{dI}{dt} = 0$$

$$x = (\mathcal{E}/R) - I, \quad dx = -dI.$$

$$x + \frac{L}{R}\frac{dx}{dt} = 0 \qquad \frac{dx}{x} = -\frac{R}{L}\,dt \qquad \int_{x_0}^{x}\frac{dx}{x} = -\frac{R}{L}\int_0^t dt$$

$$\ln\frac{x}{x_0} = -\frac{R}{L}t \quad \Longrightarrow \quad x = x_0 e^{-Rt/L}$$

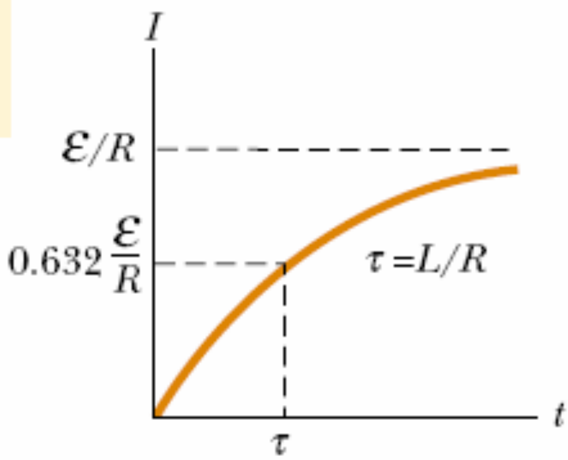
**$x_0 = x\,(t=0) = \mathcal{E} / R$**

$$\frac{\mathcal{E}}{R} - I = \frac{\mathcal{E}}{R}e^{-Rt/L} \qquad I = \frac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right)$$

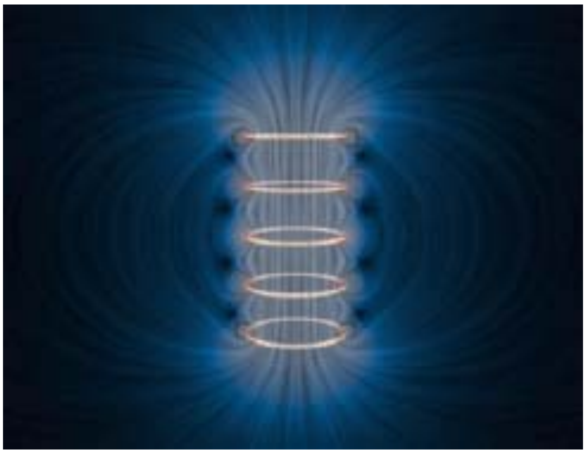
$$I = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)$$

$$\tau = \frac{L}{R}$$

**Constante de tiempo del circuito RL**



# Energía del Campo Magnético



La fem inducida evita que la batería establezca una corriente: la batería efectúa trabajo contra el inductor

Energía suministrada por la batería $\Longrightarrow$ Calor Joule + Energía almacenada en el inductor

$$\mathcal{E} - IR - L\frac{dI}{dt} = 0$$

$$I\mathcal{E} = I^2R + LI\frac{dI}{dt}$$

U: energía almacenada en el inductor

$$\frac{dU}{dt} = LI\frac{dI}{dt}$$

$$U = \int dU = \int_0^I LI\,dI = L\int_0^I I\,dI$$

$$\boxed{U = \tfrac{1}{2}LI^2}$$

$$U = \tfrac{1}{2}C(\Delta V)^2.$$

Análogo eléctrico

Para calcular la densidad de energía almacenada en el campo magnético, consideramos un solenoide:

$$L = \mu_0 n^2 A\ell$$

El campo magnético dentro del solenoide es $B = \mu_0 n I$

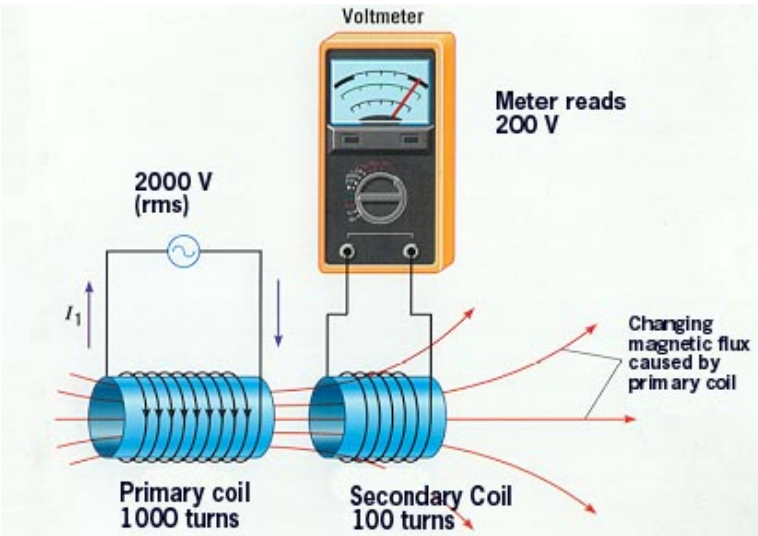
$$U = \frac{1}{2} L I^2 = \frac{1}{2} \mu_0 n^2 A\ell \left(\frac{B}{\mu_0 n}\right)^2 = \frac{B^2}{2\mu_0} A\ell$$

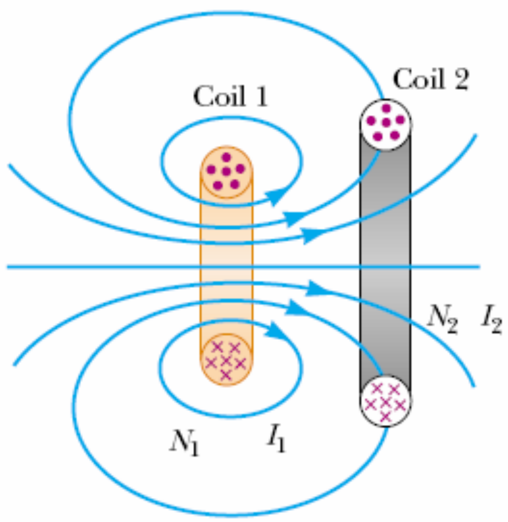
$A\ell$ volumen del solenoide donde se encuentra confinado B

$$u_B = \frac{U}{A\ell} = \frac{B^2}{2\mu_0}$$

Es válida para cualquier región del espacio donde haya B

# Inductancia mutua






$\Phi_{12}$

**Flujo magnético a través de la bobina 2 producido por la bobina 1.**

**Definimos inductancia mutua de la bobina 2 respecto a la 1**

$$M_{12} \equiv \frac{N_2 \Phi_{12}}{I_1}$$

**M12 depende de la geometría de ambos circuitos y de la orientación de uno respecto del otro.**

**La fem inducida en la bobina 2 es:**

$$\mathcal{E}_2 = -N_2 \frac{d\Phi_{12}}{dt} = -N_2 \frac{d}{dt}\left(\frac{M_{12} I_1}{N_2}\right) = -M_{12} \frac{dI_1}{dt}$$

**De la misma manera:**

$$\mathcal{E}_1 = -M_{21} \frac{dI_2}{dt}$$

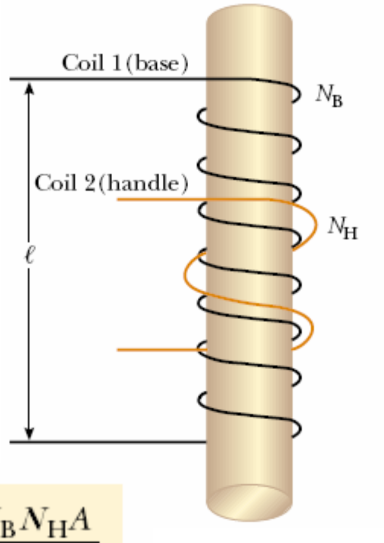
**Se puede demostrar que** $M_{12} = M_{21} = M$

$$\mathcal{E}_2 = -M \frac{dI_1}{dt} \qquad \mathcal{E}_1 = -M \frac{dI_2}{dt}$$

**[M] = Hy**

**Ejemplo:**

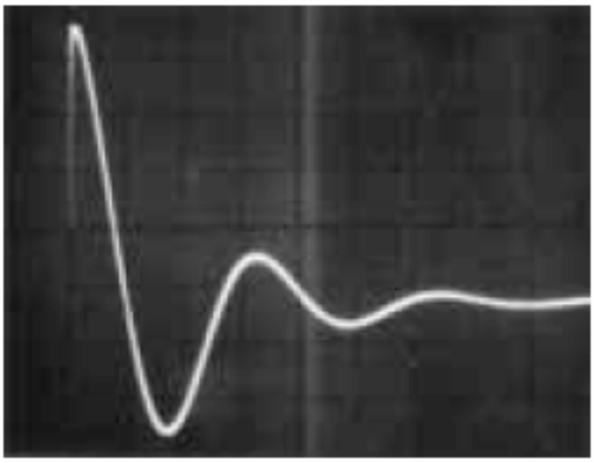
$$B = \frac{\mu_0 N_B I}{\ell}$$

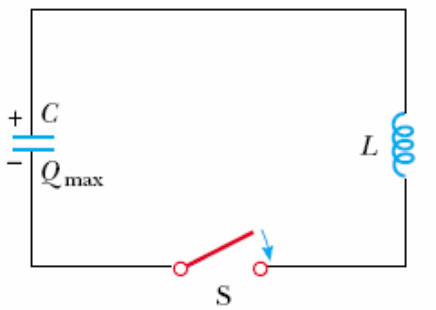



$$M = \frac{N_H \Phi_{BH}}{I} = \frac{N_H B A}{I} = \boxed{\mu_0 \frac{N_B N_H A}{\ell}}$$

**Usualmente M se determina experimentalmente.**

# Oscilaciones eléctricas
# Circuitos LC y RLC





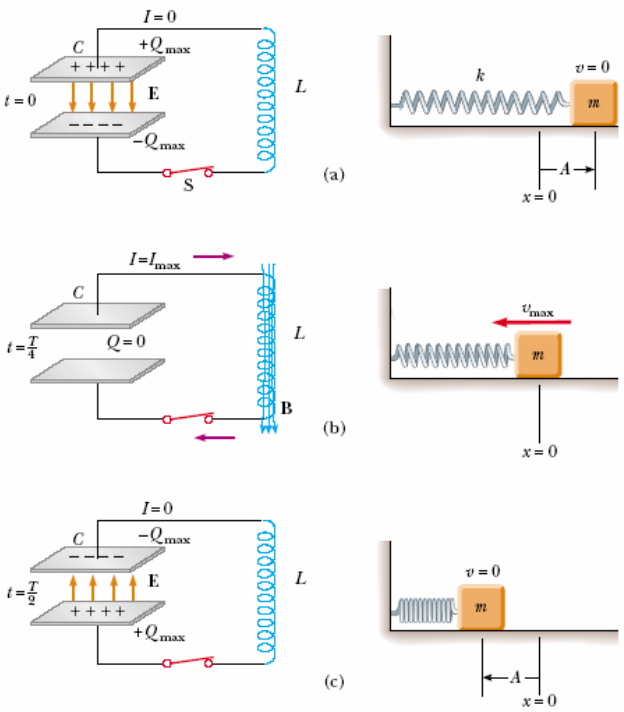

$$U = U_C + U_L = \frac{Q^2}{2C} + \frac{1}{2}LI^2$$

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{Q^2}{2C} + \frac{1}{2}LI^2\right) = \frac{Q}{C}\frac{dQ}{dt} + LI\frac{dI}{dt} = 0$$

$$I = dQ/dt. \longrightarrow \frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0$$

$$\frac{d^2Q}{dt^2} = -\frac{1}{LC}Q$$

**Ecuación diferencial del oscilador armónico**




$$\frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x$$

$$\omega = \sqrt{k/m}.$$

$$x = A\cos(\omega t + \phi)$$

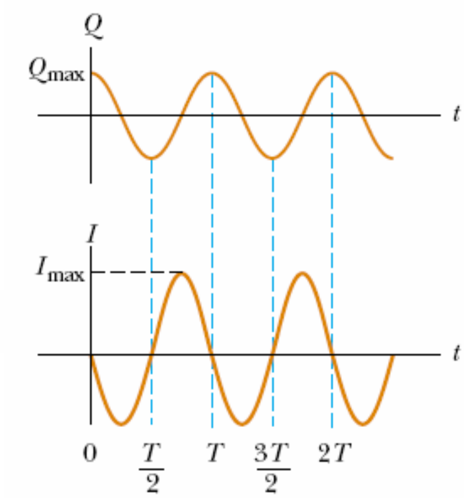
$$Q = Q_{\max}\cos(\omega t + \phi)$$

$$\boxed{\omega = \frac{1}{\sqrt{LC}}}$$

$$Q = Q_{\max}\cos(\omega t + \phi) \qquad I = \frac{dQ}{dt} = -\omega Q_{\max}\sin(\omega t + \phi)$$

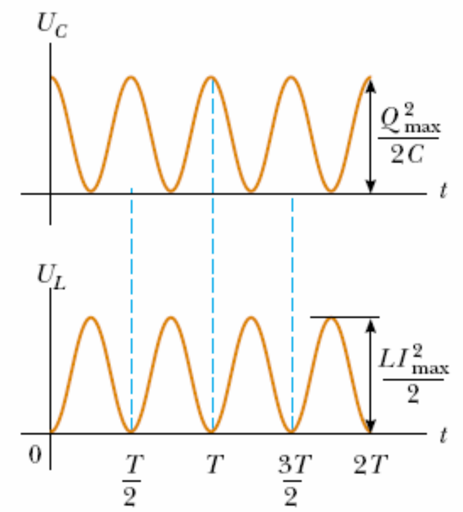
$I = 0$ at $t = 0$ $\qquad 0 = -\omega Q_{\max}\sin\phi$

$$Q = Q_{\max}\cos\omega t$$




$$I = -\omega Q_{\max}\sin\omega t = -I_{\max}\sin\omega t$$

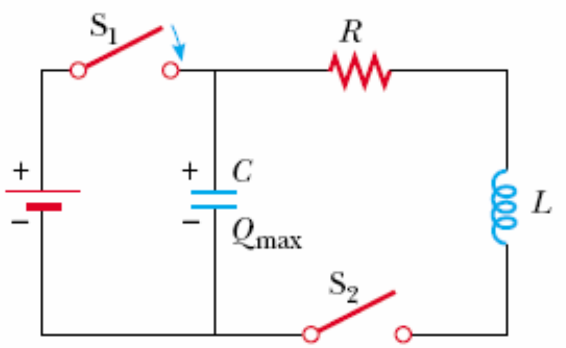
$$U = U_C + U_L = \frac{Q_{max}^2}{2C}\cos^2 \omega t + \tfrac{1}{2} L I_{max}^2 \sin^2 \omega t$$




$$\frac{Q_{max}^2}{2C} = \frac{L I_{max}^2}{2}$$

$$U = \frac{Q_{max}^2}{2C}(\cos^2 \omega t + \sin^2 \omega t) = \frac{Q_{max}^2}{2C}$$

**Que ocurre si agrego R??**



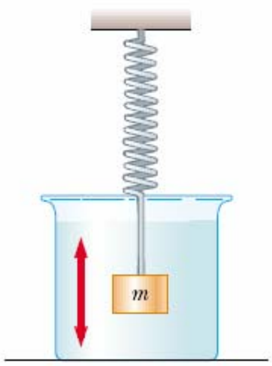

$$\frac{dU}{dt} = -I^2 R$$

$$LI\frac{dI}{dt} + \frac{Q}{C}\frac{dQ}{dt} = -I^2 R$$

$$LI\frac{d^2Q}{dt^2} + I^2 R + \frac{Q}{C} I = 0$$

$$L\frac{d^2Q}{dt^2} + IR + \frac{Q}{C} = 0$$

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0$$



$$L\,\frac{d^2Q}{dt^2} + R\,\frac{dQ}{dt} + \frac{Q}{C} = 0 \;\leftrightarrow\; m\,\frac{d^2x}{dt^2} + b\,\frac{dx}{dt} + kx = 0$$

$$\begin{array}{ccc} Q & \leftrightarrow & x \\ I & \leftrightarrow & v_x \\ \Delta V & \leftrightarrow & F_x \\ R & \leftrightarrow & b \\ C & \leftrightarrow & 1/k \\ L & \leftrightarrow & m \end{array}$$

$$m\,\frac{d^2x}{dt^2} + b\,\frac{dx}{dt} + kx = 0$$

$$\boxed{Q = Q_{\max}\, e^{-Rt/2L} \cos \omega_d t}$$

**donde**

$$\omega_d = \left[\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right]^{1/2}$$

$$I = \frac{dQ}{dt} \;\leftrightarrow\; v_x = \frac{dx}{dt}$$

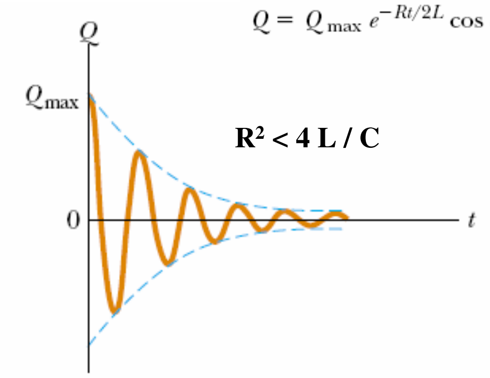
$$\frac{dI}{dt} = \frac{d^2Q}{dt^2} \;\leftrightarrow\; a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$$

$$U_L = \tfrac{1}{2}LI^2 \;\leftrightarrow\; K = \tfrac{1}{2}mv^2$$

$$U_C = \tfrac{1}{2}\frac{Q^2}{C} \;\leftrightarrow\; U = \tfrac{1}{2}kx^2$$

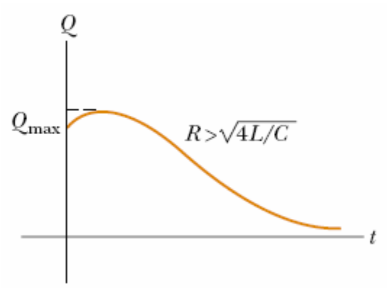
$$I^2R \;\leftrightarrow\; bv^2$$

$$Q = Q_{\max}\, e^{-Rt/2L} \cos \omega_d t \qquad \omega_d = \left[\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right]^{1/2}$$




**El sistema esta subamortiguado**

**$R^2 = 4\,L / C$ amortiguamiento crítico**




**$R^2 > 4\,L / C$ sobreamortiguado**