CÁLCULO (GRADO DE INGENIERÍA INFORMÁTICA)
Sexta sesión de prácticas

1. Calcular las siguientes integrales indefinidas utilizando el cambio de variable sugerido

$$\int \frac{dx}{x^2\sqrt{25-x^2}}, \quad x = 5\sin(t). \qquad \int \frac{dx}{x^2\sqrt{4+x^2}}, \quad x = 2\tan(t).$$

2. Calcular las siguientes integrales definidas (usar, cuando sea necesario, el mismo tipo de cambio de variable visto en el ejercicio anterior).

$$(a) \quad \int_1^{3/2} \frac{x^3-2x^2-4}{x^3+2x^2}dx, \qquad (d) \quad \int_0^{2\sqrt{3}} \frac{x^3}{\sqrt{16-x^2}}dx$$

$$(b) \quad \int_0^{\pi/2} \sin^2(x)\cos^2(x)dx, \qquad (e) \quad \int_{\pi/18}^{\pi/3} \sin(3x)\sin(6x)dx,$$

$$(c) \quad \int_0^{\pi} x^3\cos(x)dx, \qquad (f) \quad \int_0^{\pi/6} \frac{\cos^5(x)}{\sqrt{\sin(x)}}dx,$$

3. Calcular la derivada de las siguientes funciones en aquellos puntos en los que dicha derivada existe:

$$F(x) = \int_1^{\log(1+x^2)} e^t dt, \quad G(x) = \int_{x^2}^{x^3-3x} \arctan(t)dt$$

4. Calcular el área delimitada por las funciones $f(x) = 5x - x^2$ y $g(x) = x$ en el primer cuadrante (considerar la región que queda por encima de la recta $g(x) = x$).

1. Calcular las siguientes integrales indefinidas utilizando el cambio de variable sugerido

$$\int \frac{dx}{x^2\sqrt{25-x^2}}, \quad x = 5\sin(t).$$ $dx = 5\cos(t)\,dt \quad t = \arcsin\left(\frac{x}{5}\right)$

$$\int \frac{5\cos t\,dt}{25\operatorname{sen}^2 t\,\sqrt{25-25\operatorname{sen}^2 t}} \rightarrow \begin{matrix} 25 = 25\operatorname{sen}^2 t + 25\cos^2 t \\ 25 - 25\operatorname{sen}^2 t = 25\cos^2 t \end{matrix} \rightarrow \int \frac{5\cos t\,dt}{25\operatorname{sen}^2 t\sqrt{25\cos^2 t}} \rightarrow \int \frac{5\cos t\,dt}{25\operatorname{sen}^2 t\,5\cos t} = \frac{1}{25}\int \frac{dt}{\operatorname{sen}^2 t} = \frac{1}{25}(-\cot t) = \frac{-1}{25}\cot\left(\arcsin\left(\frac{x}{5}\right)\right) + C$$

$$\int \frac{dx}{x^2\sqrt{4+x^2}}, \quad x = 2\tan(t).$$ $t = \arctan\left(\frac{x}{2}\right) \quad dx = \frac{2}{\cos^2 t}\,dt$

$$\frac{4}{\cos^2 t} = \frac{4\operatorname{sen}^2 t}{\cos^2 t} + \frac{4\cos^2 t}{\cos^2 t} \rightarrow \frac{4}{\cos^2 t} = 4\operatorname{tg}^2 t + 1 \rightarrow \int \frac{1}{4\operatorname{tg}^2 t\sqrt{\frac{4}{\cos^2 t}}}\,\frac{2}{\cos^2 t}\,dt \rightarrow \int \frac{1}{4\operatorname{tg}^2 t\,\frac{2}{\cos t}}\,\frac{2}{\cos^2 t}\,dt \rightarrow \frac{1}{4}\int \frac{1}{\operatorname{tg}^2 t}\,\frac{1}{\cos t}\,dt \rightarrow \frac{1}{4}\int \frac{1}{\tan^2 t}\sec t \rightarrow \frac{1}{4}\int \cot^2 t\sec t$$

$$-\frac{1}{4}\frac{1}{\operatorname{sen} t} = -\frac{1}{4}\frac{1}{\operatorname{sen}\left(\arctan\left(\frac{x}{2}\right)\right)} + C$$

2. Calcular las siguientes integrales definidas (usar, cuando sea necesario, el mismo tipo de cambio de variable visto en el ejercicio anterior).

(a) $\displaystyle\int_1^{3/2} \frac{x^3-2x^2-4}{x^3+2x^2}dx$ →

$$\begin{array}{l|l} x^3-2x^2-4 & x^3+2x^2 \\ -x^3-2x^2 & 1 \\ -4x^2-4 & \end{array}$$

→ $\displaystyle\int_1^{3/2} 1 + \frac{-4x^2-4}{x^3+2x^2}$ →

$\frac{x^3-2x^2-4}{x^3+2x^2} = \frac{A}{x}+\frac{B}{x+2}+\frac{C}{x^2}$ → $x^3-2x^2-4 = A(x+2)x^2 + Bx^3 + C(x+2)x$

$x=0$ → $-4=2C$ → $C=-2$

$x=-2$ → $-20=-8B$ → $B=\frac{5}{2}$

$x=1$ → $-8 = 3A+B+3C = -\frac{9}{6} = -\frac{3}{2}$

→ $\displaystyle\int_1^{3/2} 1\,dx + \int_1^{3/2} \frac{-3/2}{x} + \frac{5/2}{x+2} + \frac{-2}{x^2}\,dx = \left[x - \frac{3}{2}\ln|x| + \frac{5}{2}\ln|x+2| + \frac{2}{x}\right]_1^{3/2}$ → $-0{,}5319$

(b) $\displaystyle\int_0^{\pi/2} \sin^2(x)\cos^2(x)dx$ → $\displaystyle\int_0^{\pi/2} sen^2 x\,(1-sen^2 x)\,dx$

→ $\displaystyle\int_0^{\pi/2} sen^2 x - sen^4 x\, dx$ → $\displaystyle\int_0^{\pi/2} sen^2 x\,dx - \int_0^{\pi/2} sen^4 x\,dx = \int_0^{\pi/2} \frac{1}{2} sen\,x\,dx - \int_0^{\pi/2} \frac{3}{4} sen^3 x\,dx$

$\left[\frac{1}{2} sen\,x\cos x - \frac{3}{4} sen^3 x \cos x\right]_0^{\pi/2}$ → $\frac{1}{16}\pi$

(c) $\displaystyle\int_0^{\pi} x^3\cos(x)dx$ $\quad u=x^3 \quad du=3x^2 \quad dv=\cos x\,dx \quad v=sen\,x$

$x^3 sen\,x - \int 3x^2 sen\,x\,dx \quad u=3x^2 \quad du=6x \quad dv=sen\,x\,dx \quad v=-\cos x$

$x^3 sen\,x - 3x^2\cos x - \int 6x\cos x\,dx \quad u=6x \quad du=6 \quad dv=\cos x\,dx \quad v=sen\,x$

$x^3 sen\,x - 3x^2\cos x - 6x\,sen\,x + \int 6\,sen\,x\,dx = \left[x^3 sen\,x - 3x^2\cos x - 6x\,sen\,x - 6\cos x + C\right]_0^{\pi}$

→ $-17{,}6088$

(d) $\displaystyle\int_0^{2\sqrt{3}} \frac{x^3}{\sqrt{16-x^2}}dx$ $\quad x=4\,sen\,t \quad dx=4\cos t\,dt \quad t=arcsen\left(\frac{x}{4}\right)$

→ $\displaystyle\int \frac{64\,sen^3 t}{\sqrt{16-16\,sen^2 t}}\,4\cos t\,dt$ → $\displaystyle\int \frac{64\,sen^3 t}{\sqrt{16\cos^2 t}}\,4\cos t\,dt$ → $\displaystyle\int 64\,sen^3 t\,dt + 64\int sen^3 t\,dt$

→ $64\,\frac{2}{3}\,sen^2 t\cos t$ → $\frac{128}{3}\,sen^2 t\cos t$ → $\left[\frac{128}{3}\,sen^2\left(arcsen\left(\frac{x}{4}\right)\right)\cos\left(arcsen\left(\frac{x}{2}\right)\right) + C\right]_0^{2\sqrt{3}}$

→ $\frac{40}{3}$

(e) $\displaystyle\int_{\pi/18}^{\pi/3} \sin(3x)\sin(6x)dx$, $\quad 3x=t \quad 6x=2t \quad dx=\frac{1}{3}dt$

→ $\displaystyle\int sen\,t\,sen\,2t\,\frac{1}{3}dt \qquad 2\,sen\,t\cos t$

→ $\displaystyle\frac{2}{3}\int sen^2 t\cos t\,dt \quad u=sen\,t \quad t=arcsen\,u \quad du=\cos t$

→ $\displaystyle\frac{2}{3}\int u\cos t\,\frac{du}{\cos t}$ → $\displaystyle\frac{2}{3}\int u^2 du$ → $\frac{2}{3}\,\frac{u^3}{3}$ → $\frac{2}{3}\,\frac{sen^3 t}{3}$ → $\left[\frac{2\,sen^3 3x}{9} + C\right]_{\pi/18}^{\pi/3}$

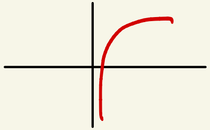
→ $-\frac{1}{36}$

(f) $\displaystyle\int_0^{\pi/6} \frac{\cos^5(x)}{\sqrt{\sin(x)}}dx$, → $\displaystyle\int \cos x\,\frac{\cos^4 x}{\sqrt{sen\,x}}\,dx$ → $\displaystyle\int \cos x\,\frac{(sen^2 x - 1)^2}{\sqrt{sen\,x}}\,x$

$\displaystyle\int \frac{(t^2-1)^2}{\sqrt{t}}\,dt$ → $\displaystyle\int t^{3/2} + t^2 + \frac{1}{\sqrt{t}}\,dt$ → $\frac{2t^{5/2}}{9} + \frac{t^3}{3} + 2\sqrt{t}$ → $\left[\frac{2\,sen^{9/2} x}{9} + \frac{sen^3 x}{3} + 2\sqrt{sen\,x} + C\right]_0^{\pi/6}$

3. Calcular la derivada de las siguientes funciones en aquellos puntos en los que dicha derivada existe:

$$F(x) = \int_1^{\log(1+x^2)} e^t dt,$$ $\rightarrow \log(1+x^2) \ \exists \ \forall x \in \mathbb{R} \quad (1+x^2 > 0)$

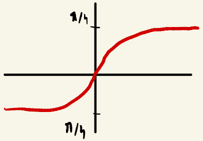


$\lim_{x \to 0} \log x = -\infty$

e^t cont $\forall x \in \mathbb{R}$

$F'(x) = f(x) = e^t \rightarrow e^{\log(x^2+1)} - e$

$$G(x) = \int_{x^2}^{x^3-3x} \arctan(t)dt$$



$f(x) = \text{arctg}\, x$

$f : \mathbb{R} \rightarrow \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

$F'(x) = f(x) = \left[\text{arctg}\, x\right]_{x^2}^{x^3-3x}$

$F'(x) = \text{arctg}(x^3-3x) - \text{arctg}(x^2) \ \exists \ \forall x \in \mathbb{R}$

4. Calcular el área delimitada por las funciones $f(x) = 5x - x^2$ y $g(x) = x$ en el primer cuadrante (considerar la región que queda por encima de la recta $g(x) = x$).

Ptos de corte

$$5x - x^2 = x \rightarrow 4x - x^2 = 0 \rightarrow x(4-x) = 0 \rightarrow x=0 \quad x=4$$

$$\int_0^4 4x - x^2 \, dx = \left[2x^2 - \frac{x^3}{3}\right]_0^4 \rightarrow \left(2 \cdot 16 - \frac{64}{3}\right) - 0 = \frac{32}{3} u^2 = A$$