Cálculo - Relación de Ejercicios (Integrales Indefinidas)

Calcula las siguientes integrales.

a) $\displaystyle\int \frac{2x}{3x^2+1}\,dx$

b) $\displaystyle\int x\sin(x^2)\,dx$

c) $\displaystyle\int e^{3\cos(x)}\sin(x)\,dx$

d) $\displaystyle\int x(4x^2+7)^9\,dx$

e) $\displaystyle\int \frac{x}{\sqrt{x+1}}\,dx$

f) $\displaystyle\int (2x+1)^{25}\,dx$

g) $\displaystyle\int x^3\sqrt[3]{x^4+1}\,dx$

h) $\displaystyle\int \frac{x+2}{x+1}\,dx$

i) $\displaystyle\int \frac{x+2}{x^2+4x}\,dx$

j) $\displaystyle\int \frac{xe^x}{(x+1)^2}\,dx$

k) $\displaystyle\int x^5e^{-x^3}\,dx$

l) $\displaystyle\int x^2e^x, dx$

$$\int \frac{2x}{3x^2+1}\,dx \to \begin{matrix} t = 3x^2+1 \\ dt = 6x\,dx \end{matrix} \to \frac{1}{3}\int \frac{dt}{t} = \frac{1}{3}\ln t + C = \frac{1}{3}\ln(3x^2+1) + C$$

$$\int x\,\mathrm{sen}(x^2)\,dx \to \begin{matrix} t = x^2 \\ dt = 2x\,dx \end{matrix} \to \frac{1}{2}\int \mathrm{sen}(t)\,dt = -\frac{1}{2}\cos(t) + C = -\frac{1}{2}\cos(x^2) + C$$

$$\int e^{3\cos(x)}\,\mathrm{sen}(x)\,dx \to \begin{matrix} t = 3\cos x \\ dt = -3\,\mathrm{sen}\,x \end{matrix} = -\frac{1}{3}\int e^t\,dt = -\frac{1}{3}e^t + C = -\frac{1}{3}e^{3\cos x} + C$$

$$\int x(4x^2+7)^9\,dx \to \begin{matrix} t = 4x^2+7 \\ dt = 8x\,dx \end{matrix} = \frac{1}{8}\int t^9\,dt = \frac{1}{80}t^{10} + C = \frac{1}{80}(4x^2+7)^{10} + C$$

$$\int \frac{x}{\sqrt{x+1}}\,dx \to \begin{matrix} t = x+1 \\ dt = dx \\ x = t-1 \end{matrix} = \int \frac{t-1}{\sqrt{t}}\,dt = \int \sqrt{t}\,dt - \int \frac{1}{\sqrt{t}}\,dt$$

$$= \frac{2}{3}\sqrt{t^3} - 2\sqrt{t} + C = 2\left(\frac{\sqrt{(x+1)^3}}{3} - \sqrt{x+1}\right) + C$$

$$\int (2x+1)^{25}\,dx \to \begin{matrix} t = 2x+1 \\ dt = 2\,dx \end{matrix} \to \frac{1}{2}\int t^{25}\,dt = \frac{1}{2}\,\frac{t^{26}}{26} + C = \frac{1}{52}(2x+1)^{26} + C$$

$$\int x^3\sqrt[3]{x^4+1}\,dx \to \begin{matrix} t = x^4+1 \\ dt = 4x^3\,dx \end{matrix} \to \frac{1}{4}\int \sqrt[3]{t}\,dt \to \frac{1}{4}\,\frac{3}{4}\,t^{\frac{4}{3}} = \frac{3}{16}t^{\frac{4}{3}} + C = \frac{3}{16}\sqrt[3]{(x^4+1)^4} + C$$

$$\int \frac{x+2}{x+1}\,dx \to \begin{matrix} t = x+1 \\ x+2 = t+1 \\ dt = dx \end{matrix} = \int \frac{t+1}{t}\,dt = \int dt + \int \frac{1}{t}\,dt = t + \ln t + C = x+1 + \ln(x+1) + C$$

$$\int \frac{x+2}{x^2+4x} \to \begin{matrix} t = x^2+4x \\ dt = 2x+4\,dx \end{matrix} = \frac{1}{2}\int \frac{1}{t}\,dt = \frac{1}{2}\ln t + C = \frac{1}{2}\ln(x^2+4x) + C$$

$$\int \frac{xe^x}{(x+1)^2}\,dx \rightarrow \begin{matrix} u = xe^x & du = (e^x + xe^x)\,dx \\ dv = \frac{1}{(x+1)^2}\,dx & v = -\frac{1}{x+1} \end{matrix} \rightarrow -\frac{xe^x}{x+1} + \int \frac{e^x + xe^x}{x+1}\,dx$$

$$= -\frac{xe^x}{x+1} + \int \frac{x+1}{x+1} e^x\,dx = -\frac{xe^x}{x+1} + \int e^x\,dx = -\frac{xe^x}{x+1} + e^x + C$$

$$\int x^5 e^{-x^3}\,dx \rightarrow \begin{matrix} t = x^3 \\ dt = 3x^2 \\ 3x^5 = t\,dt \end{matrix} \rightarrow \frac{1}{3}\int t e^{-t}\,dt = \frac{1}{3}(te^{-t} + e^{-t}) = \frac{1}{3}\frac{t+1}{e^t} + C$$

$$= \frac{1}{3}\frac{x^3+1}{e^{x^3}} + C$$

$$\int x^2 e^x\,dx \rightarrow \begin{matrix} u = x^2 & du = 2x\,dx \\ dv = e^x\,dx & v = e^x \end{matrix} = x^2e^x - \int 2xe^x\,dx = x^2e^x - 2\int xe^x\,dx$$

$$\rightarrow \begin{matrix} u = x & du = dx \\ dv = e^x dx & v = e^x \end{matrix} = x^2e^x - 2\left(xe^x - \int e^x\,dx\right) = x^2e^x - 2\,(xe^x - e^x + C)$$

$$x^2e^x - 2xe^x + 2e^x + C = e^x(x^2 - 2x + 2) + C$$