Demostración PCA

$$\min_{w} \mathbb{E}_x\left\{\| x_n - \hat{x}_n \|_2^2 = \mathbb{E}_x\left\{\langle x_n - z_n w^T, x_n - z_n w^T \rangle\right\}\right.$$

$$= \mathbb{E}_x\left\{ x_n x_n^T - 2x_n (z_n w^T) + z_n w^T (z_n w^T)^T \right\}; \quad x_n \in \mathbb{R}^{1\times p}, \; w \in \mathbb{R}^p$$

$$= \mathbb{E}_x\left\{ x_n x_n^T - 2x_n w w^T x_n^T + x_n w w^T w w^T x_n^T \right\}; \quad w^T w = 1$$

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$$\min_{w} - \mathbb{E}_x\left[\| x_n - \hat{x}_n \|_2^2 \right] = \min_{w} - \mathbb{E}_x\left\{ x_n w w^T x_n^T \right\}$$

$$\text{s.t} \quad w^T w = 1, \quad z_n = x_n w, \quad z_n \in \mathbb{R}$$

$$\min_{w} - \mathbb{E}_x\left\{ z_n z_n^T \right\} = - \mathbb{E}_x\left\{ z_n^T z_n \right\}$$

$$= - \mathbb{E}_x\left\{ w^T x_n^T x_n w \right\} = - w^T \mathbb{E}\left\{ x_n^T x_n \right\} w; \quad \mathbb{E}_x\left\{ x_n^T x_n \right\}$$

$$\min_{w} - w^T \Sigma_x w = \max_{w} w^T \Sigma_x w \qquad R_{xx} \in \mathbb{R}^{p\times p}$$

$$\text{s.t. } w^T w = 1, \quad z_n = x_n w \qquad \text{s.t } w^T w = 1$$

$$\mathcal{L}(w, \lambda) = w^T \Sigma_x w - \lambda (w^T w - 1); \quad \frac{dt}{dw} = 2\Sigma_x w - 2\lambda w^T$$

$$\Sigma_x w = \lambda w \rightarrow e : \varphi$$

$W = 0$

$$\boxed{\begin{aligned} & w^T \Sigma_\lambda w = \lambda w^T w = \lambda \\ & \mathbb{E}\left\{ w^T x_n^T x_n w \right\} = \mathbb{E}\left\{ z_n^T z_n \right\} : \sigma_z^2 = \lambda \end{aligned}}$$