UDC 530.1

DARK ENERGY AND DARK MATTER AS A MANIFESTATION OF A SINGLE METRIC-LIKE DARK FIELD

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A theory has been constructed that describes dark energy and dark matter as a single dynamic dark field. The theory introduces 4 dimensionless fundamental physical constants and is free of such problems as the vacuum catastrophe and the problem of infinite choice of model. An inflationary model of the Universe has been constructed, according to which the Universe is eternal both in the future and in the past, and in the distant past before the Big Bang matter was compressed to maximum rigidity compatible with causality, when the speed of sound is equal to the speed of light. The values of the age of the Universe ≈ 13.6 billion years and the deceleration parameter $q_0 \approx -0.847$ are obtained. This value of q_0 is consistent with observations when the value obtained within the framework of the standard cosmological model ΛCDM diverges at the level of 1.9σ. A model of a galactic dark field halo has been constructed, which allows for regions in space where the gravitational force is directed away from the center. A galaxy rotation curve for a halo was obtained, depending on 7 parameters. In particular, depending on the parameter values, the rotation speed can increase in proportion to $\sqrt{r}$ or r. Equations for linear scalar perturbations of the dark field in the synchronous gauge are obtained. Based on the analysis of the anisotropy of the cosmic microwave background, approximate and assumed exact values of the introduced fundamental physical constants are obtained. The value of the Hubble constant $H_0 \approx 78.67$ (km/s)/Mpc is also obtained, thereby significantly reducing the Hubble tension.

Keywords: dark energy, dark matter, inflationary model of the Universe.

PACS: 04.20.Cv, 04.50.Kd, 11.10.-z
DOI: 00.000000/issn2226-8812.0000.0.0-00

Introduction

Based on observations of supernovae of Ia type in the late 1990s, it was concluded that the expansion of the Universe is accelerating over time. These observations were then supported by other sources. To explain the observations, the existence of an unknown type of energy, which was called “dark energy”, was postulated. There are two theoretical models of dark energy.

In the first model, which is currently the standard, dark energy is the cosmological constant introduced by Einstein in order for the equations to allow a spatially homogeneous static solution. The most important unsolved problem of the cosmological constant, also known as the vacuum catastrophe, is that most quantum field theories, based on the energy of the quantum vacuum, predict an enormous value of the cosmological constant – many orders of magnitude exceeding what is acceptable according to cosmological concepts.

In the second model, dark energy is the so-called quintessence – one or more dynamic scalar fields, the energy density of which can vary in space and time. The problem of the theory of quintessence is the unlimited choice of the kind of potential function. There is no particular reason to believe that potential should have any particular form [1].

This article presents the model of dark energy similar to quintessence, in which the dynamic field is described by a dimensionless (when all four coordinates have cm dimension) symmetric tensor of the

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second rank, similar to the metric tensor. We will call this field the "dark field". The theory introduces 4 dimensionless fundamental physical constants $\widetilde{\alpha}$, $\widetilde{\delta}$, $\widetilde{\varepsilon}$, $\widetilde{\Lambda}$ (of which a maximum of 3 are independent) and is devoid of the above problems.

In addition to the accelerating expansion of the Universe, the model is able to explain the abnormally high rotation speeds of the outer regions of galaxies, which makes it the theory of dark matter too. Nevertheless, an entity similar to dark matter is needed to explain the observed anisotropy of the cosmic microwave background. We will also call it "dark matter" with the remark that the main role in the formation of the galactic halo is played not by dark matter, but by the dark field.

1. Designations

Four-dimensional tensor indexes are denoted by the Latin letters i, k, l, ... and run through the values 0, 1, 2, 3. The metric signature $(+\,-\,-\,-)$ is accepted. The 4-volume element $d\Omega = dx^0dx^1dx^2dx^3$. The dot above the letter means differentiation by t.

2. The dark field model

In the modern standard cosmological model ΛCDM the Lagrangian is given by the linear combination of the scalar curvature R and the cosmological constant Λ. The scalar curvature is an odd value relative to the change of sign of the metric tensor, i.e. R changes sign when replacing $g_{ik} \to -g_{ik}$. The cosmological constant is obviously an even value. We will build the theory so that the parities of the dark field Lagrangian and scalar curvature coincide. To do this, we will describe the dark field with a real tensor of the second rankf_{ik} and define the Lagrangian term as the linear combination of all possible complete convolutions of $f_{ik;l}f_{mn;p}$.

The choice in favor of the symmetric dark field tensor was made based on the following intuitive considerations. The Friedman model describes a homogeneous and isotropic Universe [1]. Therefore, the solution of the dark field describing the Universe must also be homogeneous and isotropic, and with the same constant curvature as the metric. The solution will automatically have such properties if the dark field tensor is symmetric and its nonzero elements are $f_{00} = u\,(t)\,,\ f_{\alpha\beta} = v\,(t)\,g_{\alpha\beta}$; $\alpha = 1,2,3$; $\beta = 1,2,3$, where $u\,(t)$ and $v\,(t)$ are arbitrary functions of time.

In the case off the symmetric tensor f_{ik} only 5 independent complete convolutions of $f_{ik;l}f_{mn;p}$ can be constructed. Thus, the full action of the theory can be written in the following form [2]:

$$S = -\frac{c^3}{16\pi G}\int\left(R + \widetilde{\alpha} f_{ik;l}f^{ik;l} + \widetilde{\beta} f_{ik;l}f^{li;k} + \widetilde{\gamma} f_{;i}f^{;i} + \widetilde{\delta} h_i h^i + \widetilde{\varepsilon} f_{;i}h^i\right)\sqrt{-g}\,d\Omega + S_m, \tag{2.1}$$

where

$$f \equiv f_i^i, \ \ h_i \equiv f_{i;k}^k, \tag{2.2}$$

$\widetilde{\alpha}$, $\widetilde{\beta}$, $\widetilde{\gamma}$, $\widetilde{\delta}$, $\widetilde{\varepsilon}$ are dimensionless constants, S_m is the action for matter.

3. The dark field equation

Variation relative to f_{ik} leads to the equations

$$\delta\int f_{ik;l}f^{ik;l}\sqrt{-g}\,d\Omega = -\int 2\left(f^{ik;l}\right)_{;l}\delta f_{ik}\sqrt{-g}\,d\Omega, \tag{3.1}$$

$$\delta\int f_{ik;l}f^{li;k}\sqrt{-g}\,d\Omega = -\int \left(f^{li;k} + f^{lk;i}\right)_{;l}\delta f_{ik}\sqrt{-g}\,d\Omega, \tag{3.2}$$

$$\delta\int f_{;i}f^{;i}\sqrt{-g}\,d\Omega = -\int 2g^{ik}f_{;l}^{;l}\delta f_{ik}\sqrt{-g}\,d\Omega, \tag{3.3}$$

$$\delta\int h_i h^i\sqrt{-g}\,d\Omega = -\int \left(h^{i;k} + h^{k;i}\right)\delta f_{ik}\sqrt{-g}\,d\Omega, \tag{3.4}$$

$$\delta \int f_{;i} h^i \sqrt{-g} d\Omega = -\int \left(g^{ik} h^l{}_{;l} + f^{;i;k}\right) \delta f_{ik} \sqrt{-g} d\Omega. \tag{3.5}$$

Using them, we obtain the dark field equation from the variation of the action (2.1):

$$2\widetilde{\alpha}\left(f^{ik;l}\right)_{;l} + \widetilde{\beta}\left(f^{li;k} + f^{lk;i}\right)_{;l} + 2\widetilde{\gamma} g^{ik} f^{;l}_{;l} + \widetilde{\delta}\left(h^{i;k} + h^{k;i}\right) + \widetilde{\varepsilon}\left(g^{ik} h^l{}_{;l} + f^{;i;k}\right) = 0. \tag{3.6}$$

4. Einstein's equation with the dark field term

With a variation of the action relative to g^{pq} we need the formula

$$\int A_m{}^{kl} \delta\Gamma^m_{kl} \sqrt{-g} d\Omega = \int \frac{1}{2}\left(A_p{}^m{}_q + A_{pq}{}^m - A^m{}_{pq}\right)_{;m} \delta g^{pq} \sqrt{-g} d\Omega, \tag{4.1}$$

where A_{mkl} is an arbitrary tensor of the third rank. When f_{ik} is fixed, the equations follow from it:

$$\delta \int f_{ik;l} f^{ik;l} \sqrt{-g} d\Omega = \int \left(\widetilde{K}_{pq} - 2 f_{pi}\left(f^i{}_q{}^{;l}\right)_{;l}\right) \delta g^{pq} \sqrt{-g} d\Omega, \tag{4.2}$$

$$\delta \int f_{ik;l} f^{li;k} \sqrt{-g} d\Omega = \int \left(\widetilde{L}_{pq} - f_{pi}\left(f^{li}{}_{;q} + f^l{}_q{}^{;i}\right)_{;l}\right) \delta g^{pq} \sqrt{-g} d\Omega, \tag{4.3}$$

$$\delta \int f_{;i} f^{;i} \sqrt{-g} d\Omega = \int \left(\widetilde{M}_{pq} - 2 f_{pi}\left(\delta^i_q f^{;l}_{;l}\right)\right) \delta g^{pq} \sqrt{-g} d\Omega, \tag{4.4}$$

$$\delta \int h_i h^i \sqrt{-g} d\Omega = \int \left(\widetilde{N}_{pq} - f_{pi}\left(h^i{}_{;q} + h_q{}^{;i}\right)\right) \delta g^{pq} \sqrt{-g} d\Omega, \tag{4.5}$$

$$\delta \int f_{;i} h^i \sqrt{-g} d\Omega = \int \left(\widetilde{O}_{pq} - f_{pi}\left(\delta^i_q h^l{}_{;l} + f^{;i}_{;q}\right)\right) \delta g^{pq} \sqrt{-g} d\Omega, \tag{4.6}$$

where

$$\widetilde{K}_{pq} \equiv f_{kl;p} f^{kl}{}_{;q} + \widetilde{E}_{pq} + \widetilde{E}_{qp} - \frac{1}{2} g_{pq} f_{ik;l} f^{ik;l}, \tag{4.7}$$

$$\widetilde{E}_{pq} \equiv \left(f^m_i f^i{}_{p;q} - f_{pi} f^{im}{}_{;q}\right)_{;m}, \tag{4.8}$$

$$\widetilde{L}_{pq} \equiv \frac{1}{2}\left(f_{kp;l} f^{kl}{}_{;q} + f_{kq;l} f^{kl}{}_{;p}\right) + \widetilde{H}_{pq} + \widetilde{H}_{qp} - \frac{1}{2} g_{pq} f_{ik;l} f^{li;k}, \tag{4.9}$$

$$\widetilde{H}_{pq} \equiv \frac{1}{2}\left(f^m_i\left(f_{qp}{}^{;i} + f_q{}^i{}_{;p}\right) - f_{pi}\left(f_q{}^{m;i} + f_q{}^{i;m}\right)\right)_{;m}, \tag{4.10}$$

$$\widetilde{M}_{pq} \equiv f_{;p} f_{;q} - \frac{1}{2} g_{pq} f_{;i} f^{;i}, \tag{4.11}$$

$$\widetilde{N}_{pq} \equiv h_p h_q + \widetilde{G}_{pq} + \widetilde{G}_{qp} + \left(f^m_i h^i g_{pq} - h^m f_{pq}\right)_{;m} - \frac{1}{2} g_{pq} h_i h^i, \tag{4.12}$$

$$\widetilde{G}_{pq} \equiv \frac{1}{2}\left(f_{pi} h_q{}^{;i} - f_{qi} h^i{}_{;p}\right), \tag{4.13}$$

$$\widetilde{O}_{pq} \equiv \frac{1}{2}\left(f_{;p} h_q + f_{;q} h_p\right) + \frac{1}{2}\left(f^m_i f^{;i} g_{pq} - f^{;m} f_{pq}\right)_{;m} - \frac{1}{2} g_{pq} f_{;i} h^i. \tag{4.14}$$

From the variation of the action (2.1) relative to g^{pq} at the fixed f_{ik}, using (4.2)–(4.6) and the dark field equation (3.6), we obtain the Einstein equation [2]

$$R_{pq} - \frac{1}{2} g_{pq} R + \widetilde{T}_{pq} = \frac{8\pi G}{c^4} T_{pq}, \tag{4.15}$$

where $\widetilde{T}_{pq}$ is the energy-momentum tensor (EMT) of the dark field, equal to

$$\widetilde{T}_{pq} = \widetilde{\alpha}\widetilde{K}_{pq} + \widetilde{\beta}\widetilde{L}_{pq} + \widetilde{\gamma}\widetilde{M}_{pq} + \widetilde{\delta}\widetilde{N}_{pq} + \widetilde{\varepsilon}\widetilde{O}_{pq}. \tag{4.16}$$

5. The divergence of the EMT of the dark field and matter

Using the properties of the curvature tensor [2], from (4.7)–(4.10) we get

$$\left(\widetilde{E}^{pq}\right)_{;q} = \left(f_i^m \left(f^{ip;q}\right)_{;q} - f_i^p \left(f^{im;q}\right)_{;q}\right)_{;m} - f_i^m f^{iq;n} R^p{}_{nmq}, \tag{5.1}$$

$$\left(\widetilde{H}^{pq}\right)_{;q} = \frac{1}{2}\left(f_i^m \left(f^{qp;i} + f^{qi;p}\right) - f_i^p \left(f^{qm;i} + f^{qi;m}\right)\right)_{;q;m} + \frac{1}{2} f_i^q \left(f^{nm;i} + f^{ni;m}\right) R^p{}_{nmq}, \tag{5.2}$$

$$\left(\widetilde{E}^{qp}\right)_{;q} = -f_i^m f^{iq;n} R^p{}_{nmq}, \tag{5.3}$$

$$\left(\widetilde{H}^{qp}\right)_{;q} = \frac{1}{2} f_i^q \left(f^{nm;i} + f^{ni;m}\right) R^p{}_{nmq}, \tag{5.4}$$

$$\left(\widetilde{K}^{pq}\right)_{;q} = f_{kl}{}^{;p} \left(f^{kl;q}\right)_{;q} + 2 f_i^m f^{iq;n} R^p{}_{nmq} + \left(\widetilde{E}^{pq}\right)_{;q} + \left(\widetilde{E}^{qp}\right)_{;q}, \tag{5.5}$$

$$\left(\widetilde{L}^{pq}\right)_{;q} = \frac{1}{2} f_{kl}{}^{;p} \left(f^{qk;l} + f^{ql;k}\right)_{;q} - \frac{1}{2}\left(f_{i;q}^m \left(f^{qp;i} + f^{qi;p}\right) - f_{i;q}^p \left(f^{qm;i} + f^{qi;m}\right)\right)_{;m} -$$
$$- f_i^q \left(f^{nm;i} + f^{ni;m}\right) R^p{}_{nmq} + \left(\widetilde{H}^{pq}\right)_{;q} + \left(\widetilde{H}^{qp}\right)_{;q}. \tag{5.6}$$

The substitution (5.1) and (5.3) into (5.5) gives

$$\left(\widetilde{K}^{pq}\right)_{;q} = f_{kl}{}^{;p} \left(f^{kl;q}\right)_{;q} + \left(f_i^m \left(f^{ip;q}\right)_{;q} - f_i^p \left(f^{im;q}\right)_{;q}\right)_{;m}. \tag{5.7}$$

The substitution (5.2) and (5.4) into (5.6) gives

$$\left(\widetilde{L}^{pq}\right)_{;q} = \frac{1}{2} f_{kl}{}^{;p} \left(f^{qk;l} + f^{ql;k}\right)_{;q} + \frac{1}{2}\left(f_i^m \left(f^{qp;i} + f^{qi;p}\right)_{;q} - f_i^p \left(f^{qm;i} + f^{qi;m}\right)_{;q}\right)_{;m}. \tag{5.8}$$

From (5.7) and (5.8), taking into account the dark field equation (3.6), it follows

$$\left(\widetilde{\alpha}\widetilde{K}^{pq} + \widetilde{\beta}\widetilde{L}^{pq}\right)_{;q} = -\widetilde{\gamma}\widetilde{A}^p - \widetilde{\delta}\widetilde{B}^p - \widetilde{\varepsilon}\widetilde{C}^p, \tag{5.9}$$

where

$$\widetilde{A}^p \equiv f_{kl}{}^{;p} \left(g^{kl} f^{;q}_{;q}\right) + \left(f_i^m \left(g^{ip} f^{;q}_{;q}\right) - f_i^p \left(g^{im} f^{;q}_{;q}\right)\right)_{;m} = f^{;p} f^{;q}_{;q}, \tag{5.10}$$

$$\widetilde{B}^p \equiv \frac{1}{2} f_{kl}{}^{;p} \left(h^{k;l} + h^{l;k}\right) + \frac{1}{2}\left(f_i^m \left(h^{i;p} + h^{p;i}\right) - f_i^p \left(h^{i;m} + h^{m;i}\right)\right)_{;m} =$$
$$= f_{kl}{}^{;p} h^{k;l} + \frac{1}{2}\left(f_i^m \left(h^{i;p} + h^{p;i}\right) - f_i^p \left(h^{i;m} + h^{m;i}\right)\right)_{;m}, \tag{5.11}$$

$$\widetilde{C}^p \equiv \frac{1}{2} f_{kl}{}^{;p} \left(g^{kl} h^q{}_{;q} + f^{;k;l}\right) + \frac{1}{2}\left(f_i^m \left(g^{ip} h^q{}_{;q} + f^{;i;p}\right) - f_i^p \left(g^{im} h^q{}_{;q} + f^{;i;m}\right)\right)_{;m} =$$
$$= \frac{1}{2}\left(f^{;p} h^q{}_{;q} + f_{kl}{}^{;p} f^{;k;l}\right) + \frac{1}{2}\left(f_i^m f^{;i;p} - f_i^p f^{;i;m}\right)_{;m}. \tag{5.12}$$

It follows from (4.11) and (5.10):

$$\left(\widetilde{M}^{pq}\right)_{;q} - \widetilde{A}^p = 0. \tag{5.13}$$

It follows from (4.13) and (5.11):

$$\left(\widetilde{G}^{pq} + \widetilde{G}^{qp}\right)_{;q} - \widetilde{B}^p = -f_{kl}{}^{;p} h^{k;l} + f_{i;q}^p h^{q;i} + f_i^p h_q{}^{;i;q} - h_i h^{i;p} - f_{iq} h^{i;p;q}. \tag{5.14}$$

Using the properties of the curvature tensor [2], from (4.12) and (5.14) we get

$$\left(\widetilde{N}^{pq}\right)_{;q} - \widetilde{B}^p = 0. \tag{5.15}$$

Similarly, from (4.14) and (5.12) we get

$$\left(\widetilde{O}^{pq}\right)_{;q} - \widetilde{C}^p = 0. \tag{5.16}$$

It follows from (4.16), (5.9), (5.13), (5.15), (5.16) that the dark field EMT divergence is zero:

$$\widetilde{T}^{q}_{p;q} = 0. \tag{5.17}$$

By virtue of the collapsed Bianchi identities [2] and the Einstein equation (4.15), this leads to the similar differential equation for the matter EMT:

$$T^{q}_{p;q} = 0. \tag{5.18}$$

Thus, the same law is performed as in Einstein's original general theory of relativity.

6. Criterion of invariance of equations with respect to the transformation $f'_{ik}(x) = f_{ik}(x) + s(x)\, g_{ik}(x)$

The dark field equation (3.6) and the Einstein equation (4.15) are invariant with respect to the transformation

$$f'_{ik}(x) = f_{ik}(x) + C g_{ik}(x), \tag{6.1}$$

where C is an arbitrary dimensionless constant, as can be seen by simple calculations due to $g_{ik;l} = 0$. In the super-degenerate case, which is considered in section 14, the equations are invariant with respect to the more general transformation

$$f'_{ik}(x) = f_{ik}(x) + s(x)\, g_{ik}(x), \tag{6.2}$$

where $s(x)$ is an arbitrary dimensionless real scalar. We derive a criterion for the invariance of the equations with respect to this transformation. The substitution (6.2) into (3.6) and (4.7)–(4.14) gives

$$2\widetilde{\alpha}\left(f'^{\,ik;l}\right)_{;l} + \widetilde{\beta}\left(f'^{\,li;k} + f'^{\,lk;i}\right)_{;l} + 2\widetilde{\gamma} g^{ik} f'^{;l}_{;l} + \widetilde{\delta}\left(h'^{\,i;k} + h'^{\,k;i}\right) + \widetilde{\varepsilon}\left(g^{ik} h'^{\,l}_{\;;l} + f'^{\,;i;k}\right) =$$

$$= \left(2\widetilde{\alpha} + 8\widetilde{\gamma} + \widetilde{\varepsilon}\right) s^{;l}_{;l} g^{ik} + 2\left(\widetilde{\beta} + \widetilde{\delta} + 2\widetilde{\varepsilon}\right) s^{;i;k}, \tag{6.3}$$

$$\widetilde{K}'_{pq} = \widetilde{K}_{pq} + f_{;p} s_{;q} + f_{;q} s_{;p} + 4 s_{;p} s_{;q} - \frac{1}{2} g_{pq}\left(2 f_{;l} s^{;l} + 4 s_{;l} s^{;l}\right), \tag{6.4}$$

$$\widetilde{L}'_{pq} = \widetilde{L}_{pq} + h_p s_{;q} + h_q s_{;p} + s_{;p} s_{;q} + \left(f^m_i s^{;i} g_{pq} - s^{;m} f_{pq}\right)_{;m} - \frac{1}{2} g_{pq}\left(2 h_l s^{;l} + s_{;l} s^{;l}\right), \tag{6.5}$$

$$\widetilde{M}'_{pq} = \widetilde{M}_{pq} + 4\left(f_{;p} s_{;q} + f_{;q} s_{;p}\right) + 16 s_{;p} s_{;q} - \frac{1}{2} g_{pq}\left(8 f_{;l} s^{;l} + 16 s_{;l} s^{;l}\right), \tag{6.6}$$

$$\widetilde{N}'_{pq} = \widetilde{N}_{pq} + h_p s_{;q} + h_q s_{;p} + s_{;p} s_{;q} + \left(f^m_i s^{;i} g_{pq} - s^{;m} f_{pq}\right)_{;m} - \frac{1}{2} g_{pq}\left(2 h_l s^{;l} + s_{;l} s^{;l}\right), \tag{6.7}$$

$$\widetilde{O}'_{pq} = \widetilde{O}_{pq} + \frac{1}{2}\left(f_{;p} s_{;q} + f_{;q} s_{;p}\right) + 2\left(h_p s_{;q} + h_q s_{;p}\right) + 4 s_{;p} s_{;q} +$$

$$+ 2\left(f^m_i s^{;i} g_{pq} - s^{;m} f_{pq}\right)_{;m} - \frac{1}{2} g_{pq}\left(f_{;l} s^{;l} + 4 h_{;l} s^{;l} + 4 s_{;l} s^{;l}\right). \tag{6.8}$$

From (4.16) and (6.4)–(6.8) we get

$$\widetilde{T}'_{pq} = \widetilde{T}_{pq} + \left(2\widetilde{\alpha} + 8\widetilde{\gamma} + \widetilde{\varepsilon}\right)\left(\frac{1}{2}\left(f_{;p} s_{;q} + f_{;q} s_{;p}\right) + 2 s_{;p} s_{;q} - \frac{1}{2} g_{pq}\left(f_{;l} s^{;l} + 2 s_{;l} s^{;l}\right)\right) +$$

$$+ \left(\widetilde{\beta} + \widetilde{\delta} + 2\widetilde{\varepsilon}\right)\left(h_p s_{;q} + h_q s_{;p} + s_{;p} s_{;q} + \left(f^m_i s^{;i} g_{pq} - s^{;m} f_{pq}\right)_{;m} - \frac{1}{2} g_{pq}\left(2 h_l s^{;l} + s_{;l} s^{;l}\right)\right). \tag{6.9}$$

It follows from (6.3) and (6.9) that the dark field equation (3.6) and the Einstein equation (4.15) are invariant with respect to the transformation (6.2) if and only if

$$2\widetilde{\alpha} + 8\widetilde{\gamma} + \widetilde{\varepsilon} = 0, \;\; \widetilde{\beta} + \widetilde{\delta} + 2\widetilde{\varepsilon} = 0. \tag{6.10}$$

7. Secondary inflation

Next, we accept $c = 1$. We will describe the Universe in the Friedmann–Lemaitre–Robertson–Walker metric in spherical coordinates [1]

$$g_{00} = 1,\ \ g_{rr} = -a\left(t\right)^2 \left(1 - kr^2\right)^{-1},\ \ g_{\theta\theta} = -a\left(t\right)^2 r^2,\ \ g_{\varphi\varphi} = -a\left(t\right)^2 r^2 \sin^2\left(\theta\right), \tag{7.1}$$

where $a\left(t\right)$ is the scale factor; the parameter k takes three values depending on the sign of the curvature of the 3-space: $k = +1$ for spherical, $k = 0$ for planar, and $k = -1$ for hyperbolic space. Let's define the dark field tensor in the similar form:

$$f_{00} = u\left(t\right),\ \ f_{rr} = -v\left(t\right) a\left(t\right)^2 \left(1 - kr^2\right)^{-1},$$
$$f_{\theta\theta} = -v\left(t\right) a\left(t\right)^2 r^2,\ \ f_{\varphi\varphi} = -v\left(t\right) a\left(t\right)^2 r^2 \sin^2\left(\theta\right). \tag{7.2}$$

Equation (4.15) leads to the Friedman equation with the dark field term [1]

$$3\left(\dot{a}^2 + k\right) a^{-2} + \widetilde{T}_0^0 = 8\pi G\rho, \tag{7.3}$$

where ρ is the energy density of matter.

For further convenience, let's move from the set of the constants $\widetilde{\alpha}$, $\widetilde{\beta}$, $\widetilde{\gamma}$, $\widetilde{\delta}$, $\widetilde{\varepsilon}$ to the set $\widetilde{\alpha}$, $\widetilde{\delta}$, $\widetilde{\varepsilon}$, $\widetilde{\Lambda}$, $\widetilde{\Phi}$ via the reversible transformation

$$\widetilde{\beta} = \widetilde{\Phi} - \widetilde{\alpha},\ \ \widetilde{\gamma} = \widetilde{\Lambda} - \frac{7}{4}\widetilde{\Phi} - \widetilde{\delta} - \widetilde{\varepsilon}. \tag{7.4}$$

Using the DifferentialGeometry software package of the Maple computer mathematics system, write[1] (3.6) and (4.16), substituting (7.1), (7.2), (7.4) into them:

$$4\left(u - v\right)\left(\widetilde{\delta} + \frac{\widetilde{\varepsilon}}{2}\right) a\ddot{a} - 7a^2\left(\widetilde{\Phi} + \frac{4\widetilde{\delta}}{7} - \frac{4\widetilde{\Lambda}}{7} + \frac{2\widetilde{\varepsilon}}{7}\right)\ddot{v} - \left(\widetilde{\Phi} - \frac{4\widetilde{\Lambda}}{3}\right) a^2\ddot{u} - 4\Bigg(\left(u - v\right)\left(\widetilde{\Phi} + \widetilde{\alpha} + \widetilde{\delta} - \widetilde{\varepsilon}\right)\dot{a} +$$
$$+ \frac{3}{4} a\left(\left(\frac{25\widetilde{\Phi}}{3} - \frac{4\widetilde{\alpha}}{3} + \frac{16\widetilde{\delta}}{3} - 4\widetilde{\Lambda} + \frac{14\widetilde{\varepsilon}}{3}\right)\dot{v} + \dot{u}\left(\widetilde{\Phi} - \frac{4\widetilde{\Lambda}}{3}\right)\right)\Bigg)\dot{a} = 0, \tag{7.5}$$

$$4\left(\widetilde{\Phi} - \widetilde{\alpha} + \frac{3\widetilde{\varepsilon}}{2}\right)\left(u - v\right) a\ddot{a} - 21\left(\widetilde{\Phi} - \frac{4\widetilde{\alpha}}{21} + \frac{4\widetilde{\delta}}{7} - \frac{4\widetilde{\Lambda}}{7} + \frac{4\widetilde{\varepsilon}}{7}\right) a^2\ddot{v} -$$
$$-7a^2\left(\widetilde{\Phi} + \frac{4\widetilde{\delta}}{7} - \frac{4\widetilde{\Lambda}}{7} + \frac{2\widetilde{\varepsilon}}{7}\right)\ddot{u} + 12\Bigg(\left(\widetilde{\Phi} - \frac{\widetilde{\alpha}}{3} + \widetilde{\delta} + \widetilde{\varepsilon}\right)\left(u - v\right)\dot{a} -$$
$$-\frac{17}{12} a\left(\left(\widetilde{\Phi} + \frac{4\widetilde{\alpha}}{17} + \frac{8\widetilde{\delta}}{17} - \frac{12\widetilde{\Lambda}}{17} - \frac{2\widetilde{\varepsilon}}{17}\right)\dot{u} + \frac{63}{17}\left(\widetilde{\Phi} - \frac{4\widetilde{\alpha}}{21} + \frac{4\widetilde{\delta}}{7} - \frac{4\widetilde{\Lambda}}{7} + \frac{4\widetilde{\varepsilon}}{7}\right)\dot{v}\right)\Bigg)\dot{a} = 0, \tag{7.6}$$

$$\widetilde{T}_0^0 = \frac{3}{8a^2}\Bigg(4\left(u - v\right)^2\left(\widetilde{\Phi} + \widetilde{\alpha} + 3\widetilde{\delta}\right)\dot{a}^2 + 8\left(u - v\right) a\left(\left(\widetilde{\Phi} - \widetilde{\alpha} + \frac{3\widetilde{\varepsilon}}{2}\right)\dot{v} + \dot{u}\left(\widetilde{\delta} + \frac{\widetilde{\varepsilon}}{2}\right)\right)\dot{a} -$$
$$-a^2\left(\left(21\widetilde{\Phi} - 4\widetilde{\alpha} + 12\left(\widetilde{\delta} - \widetilde{\Lambda} + \widetilde{\varepsilon}\right)\right)\dot{v}^2 + 14\dot{u}\left(\widetilde{\Phi} + \frac{4\widetilde{\delta}}{7} - \frac{4\widetilde{\Lambda}}{7} + \frac{2\widetilde{\varepsilon}}{7}\right)\dot{v} + \dot{u}^2\left(\widetilde{\Phi} - \frac{4\widetilde{\Lambda}}{3}\right)\right)\Bigg), \tag{7.7}$$

$$\widetilde{T}_r^r = \widetilde{T}_\theta^\theta = \widetilde{T}_\varphi^\varphi = \frac{1}{8a^2}\Bigg(8a\left(u - v\right)^2\left(\widetilde{\Phi} + \widetilde{\alpha} + 3\widetilde{\delta}\right)\ddot{a} + 8\left(\widetilde{\Phi} - \widetilde{\alpha} + \frac{3\widetilde{\varepsilon}}{2}\right)\left(u - v\right) a^2\ddot{v} +$$
$$+8\left(u - v\right)\left(\widetilde{\delta} + \frac{\widetilde{\varepsilon}}{2}\right) a^2\ddot{u} + 4\left(u - v\right)^2\left(\widetilde{\Phi} + \widetilde{\alpha} + 3\widetilde{\delta}\right)\dot{a}^2 +$$

[1] File friedmann.mw from the repository https://github.com/alegorov/dark-field

$$+16a\left(u-v\right)\left(\widetilde{\Phi}+\widetilde{\alpha}+3\widetilde{\delta}\right)\left(\dot{u}-\dot{v}\right)\dot{a}+3a^2\left(\left(\frac{55\widetilde{\Phi}}{3}-\frac{4\widetilde{\alpha}}{3}+12\widetilde{\delta}-12\widetilde{\Lambda}+8\widetilde{\varepsilon}\right)\dot{v}^2+\right.$$
$$\left.+\frac{50}{3}\left(\widetilde{\Phi}-\frac{4\widetilde{\alpha}}{25}+\frac{8\widetilde{\delta}}{25}-\frac{12\widetilde{\Lambda}}{25}+\frac{2\widetilde{\varepsilon}}{5}\right)\dot{u}\dot{v}+\dot{u}^2\left(\widetilde{\Phi}+\frac{8\widetilde{\delta}}{3}-\frac{4\widetilde{\Lambda}}{3}+\frac{4\widetilde{\varepsilon}}{3}\right)\right)\Bigg). \tag{7.8}$$

The numerical solution of the equation system (7.3), (7.5)–(7.7) for various values of $\widetilde{\alpha}$, $\widetilde{\delta}$, $\widetilde{\varepsilon}$, $\widetilde{\Lambda}$, $\widetilde{\Phi}$ resulted in either to the expansion of the Universe without acceleration, or to an avalanche-like expansion. The following method was applied to solve this problem. We require that the scale factor for large values of time at the stage of secondary inflation increases exponentially in the same way as in the model with the cosmological constant:

$$a\left(t\right)=A\exp\left(\lambda t\right),\quad A>0,\quad \lambda>0. \tag{7.9}$$

In this case, (7.7) will be written as

$$\widetilde{T}_0^0=\frac{3}{8}\left(-21\widetilde{\Phi}+4\widetilde{\alpha}-12\widetilde{\delta}+12\widetilde{\Lambda}-12\widetilde{\varepsilon}\right)\dot{v}^2+$$
$$+\frac{3}{4}\left(\left(-7\widetilde{\Phi}-4\widetilde{\delta}+4\widetilde{\Lambda}-2\widetilde{\varepsilon}\right)\dot{u}+4\left(u-v\right)\left(\widetilde{\Phi}-\widetilde{\alpha}+\frac{3\widetilde{\varepsilon}}{2}\right)\lambda\right)\dot{v}+\frac{1}{8}\left(-3\widetilde{\Phi}+4\widetilde{\Lambda}\right)\dot{u}^2+$$
$$+3\left(u-v\right)\left(\widetilde{\delta}+\frac{\widetilde{\varepsilon}}{2}\right)\lambda\dot{u}+\frac{3}{2}\lambda^2\left(u-v\right)^2\left(\widetilde{\Phi}+\widetilde{\alpha}+3\widetilde{\delta}\right). \tag{7.10}$$

For large t, the right-hand side of (7.3) is zero [1], the first term of the left-hand side is $3\lambda^2$. Therefore, the value of $\widetilde{T}_0^0$ must be constant and negative. Note that according to (7.10), the value of $\widetilde{T}_0^0$ is constant if

$$u\left(t\right)=Ut+C,\quad v\left(t\right)=Ut+V+C, \tag{7.11}$$

where by virtue of $\widetilde{T}_0^0\neq 0$ at least one of the constants U and V is not zero. By substituting (7.9) and (7.11) into (7.5), (7.6), we get

$$\left(\widetilde{\varepsilon}+2\widetilde{\Phi}-\frac{2\widetilde{\alpha}}{7}+\frac{8\widetilde{\delta}}{7}-\frac{8\widetilde{\Lambda}}{7}\right)U+\frac{3}{7}\left(\widetilde{\varepsilon}-\frac{2\widetilde{\Phi}}{3}-\frac{2\widetilde{\alpha}}{3}\right)\lambda V=0, \tag{7.12}$$

$$\left(\widetilde{\varepsilon}+\frac{40\widetilde{\Phi}}{17}-\frac{4\widetilde{\alpha}}{17}+\frac{22\widetilde{\delta}}{17}-\frac{24\widetilde{\Lambda}}{17}\right)U+\frac{9}{17}\left(\widetilde{\varepsilon}+\frac{8\widetilde{\Phi}}{9}-\frac{4\widetilde{\alpha}}{9}+\frac{2\widetilde{\delta}}{3}\right)\lambda V=0. \tag{7.13}$$

The equations (7.12) and (7.13) can be written as the product of a square matrix and the column $\left(U,\ \lambda V\right)^T$ is equal to zero. Since the column is non-zero, the determinant of the specified matrix must be zero:

$$\frac{\widetilde{\delta}^2}{4}+\left(\widetilde{\Phi}-\frac{\widetilde{\Lambda}}{4}+\frac{\widetilde{\varepsilon}}{4}\right)\widetilde{\delta}+\widetilde{\Phi}^2+\left(\widetilde{\varepsilon}-\frac{7\widetilde{\Lambda}}{6}\right)\frac{\widetilde{\Phi}}{2}-\frac{\widetilde{\Lambda}\widetilde{\alpha}}{12}+\frac{\widetilde{\varepsilon}^2}{16}=0. \tag{7.14}$$

First, consider the case off

$$\widetilde{\Lambda}\neq 0, \tag{7.15}$$

and the case off $\widetilde{\Lambda}=0$ is considered in the sections 13 and 14. Then, according to (7.14), the constant $\widetilde{\alpha}$ can be expressed through the rest:

$$\widetilde{\alpha}=\left(4\widetilde{\Lambda}\right)^{-1}\left(12\widetilde{\delta}^2+\left(48\widetilde{\Phi}-12\widetilde{\Lambda}+12\widetilde{\varepsilon}\right)\widetilde{\delta}+48\widetilde{\Phi}^2+\left(24\widetilde{\varepsilon}-28\widetilde{\Lambda}\right)\widetilde{\Phi}+3\widetilde{\varepsilon}^2\right). \tag{7.16}$$

By substituting (7.16) into (7.12) and (7.13), we get:

$$\left(\left(\widetilde{\Phi}+\frac{\widetilde{\delta}}{2}-\frac{2\widetilde{\Lambda}}{3}+\frac{\widetilde{\varepsilon}}{4}\right)U+\left(\widetilde{\Phi}+\frac{\widetilde{\delta}}{2}+\frac{\widetilde{\varepsilon}}{4}\right)\lambda V\right)\left(\frac{\widetilde{\varepsilon}}{4}+\widetilde{\Phi}+\frac{\widetilde{\delta}}{2}-\frac{\widetilde{\Lambda}}{2}\right)=0, \tag{7.17}$$

$$\left(\left(\widetilde{\Phi}+\frac{\widetilde{\delta}}{2}-\frac{2\widetilde{\Lambda}}{3}+\frac{\widetilde{\varepsilon}}{4}\right)U+\left(\widetilde{\Phi}+\frac{\widetilde{\delta}}{2}+\frac{\widetilde{\varepsilon}}{4}\right)\lambda V\right)\left(\frac{\widetilde{\varepsilon}}{4}+\widetilde{\Phi}+\frac{\widetilde{\delta}}{2}-\frac{3\widetilde{\Lambda}}{4}\right)=0. \tag{7.18}$$

By virtue of (7.15), the pair of the equations (7.17) and (7.18) is equivalent to the one equation

$$\left(\widetilde{\Phi}+\frac{\widetilde{\delta}}{2}-\frac{2\widetilde{\Lambda}}{3}+\frac{\widetilde{\varepsilon}}{4}\right)U+\left(\widetilde{\Phi}+\frac{\widetilde{\delta}}{2}+\frac{\widetilde{\varepsilon}}{4}\right)\lambda V=0. \tag{7.19}$$

Since by virtue of (7.15) both coefficients in (7.19) before U and V cannot be equal to zero, the constants U and V must be expressed as

$$U=-\left(\widetilde{\Phi}+\frac{\widetilde{\delta}}{2}+\frac{\widetilde{\varepsilon}}{4}\right)\lambda W,\ \ V=\left(\widetilde{\Phi}+\frac{\widetilde{\delta}}{2}-\frac{2\widetilde{\Lambda}}{3}+\frac{\widetilde{\varepsilon}}{4}\right)W,\ \ W\neq 0. \tag{7.20}$$

The substitution (7.11), (7.16), (7.20) into (7.10) gives

$$\widetilde{T}_0^0=-\frac{1}{16}\widetilde{\Phi}\left(12\widetilde{\Phi}+6\widetilde{\delta}-8\widetilde{\Lambda}+3\widetilde{\varepsilon}\right)^2\lambda^2W^2. \tag{7.21}$$

Hence, from $\widetilde{T}_0^0<0$ follows $\widetilde{\Phi}>0$. Considering that according to (7.4) $\widetilde{\Phi}=\widetilde{\alpha}+\widetilde{\beta}$, by making in the action (2.1) the replacement $f_{ik}\to\left(\widetilde{\alpha}+\widetilde{\beta}\right)^{-\frac{1}{2}}f_{ik}$, lead the constant $\widetilde{\Phi}$ to

$$\widetilde{\Phi}=1. \tag{7.22}$$

By virtue of $\widetilde{T}_0^0\neq 0$ from (7.21) and (7.22) follows:

$$12+6\widetilde{\delta}-8\widetilde{\Lambda}+3\widetilde{\varepsilon}\neq 0. \tag{7.23}$$

8. Primary inflation

Let's assume that before the Big Bang, there was an eternal in the past stage of primary inflation, in which the Universe expanded exponentially:

$$a\left(t\right)=A\exp\left(\lambda t\right),\ \ A>0,\ \ \lambda>0. \tag{8.1}$$

Also set the functions $u\left(t\right)$ and $v\left(t\right)$ in the exponential form:

$$u\left(t\right)=U\exp\left(\eta t\right)+C,\ \ v\left(t\right)=V\exp\left(\eta t\right)+C, \tag{8.2}$$

where at least one of the constants U and V is not equal to zero. The substitution (7.16), (7.22), (8.1), (8.2) into (7.5), (7.6) leads to the equations

$$\begin{gathered}\left(3\left(-\frac{7}{4}-\widetilde{\delta}-\frac{\widetilde{\varepsilon}}{2}+\widetilde{\Lambda}\right)V+U\left(-\frac{3}{4}+\widetilde{\Lambda}\right)\right)\widetilde{\Lambda}\eta^2-9\left(U-V\right)\left(\widetilde{\delta}+\frac{\widetilde{\varepsilon}}{2}+2\right)\left(\widetilde{\delta}-\widetilde{\Lambda}+\frac{\widetilde{\varepsilon}}{2}+2\right)\lambda^2+\\ +3\left(\left(3\widetilde{\Lambda}^2+\left(-7\widetilde{\delta}-\frac{7\widetilde{\varepsilon}}{2}-\frac{53}{4}\right)\widetilde{\Lambda}+3\left(\widetilde{\delta}+\frac{\widetilde{\varepsilon}}{2}+2\right)^2\right)V+U\widetilde{\Lambda}\left(-\frac{3}{4}+\widetilde{\Lambda}\right)\right)\eta\lambda=0,\end{gathered} \tag{8.3}$$

$$\begin{gathered}6\left(\widetilde{\delta}-\frac{3\widetilde{\Lambda}}{2}+\frac{\widetilde{\varepsilon}}{2}+2\right)\left(U-V\right)\left(\widetilde{\delta}+\frac{\widetilde{\varepsilon}}{2}+2\right)\lambda^2+\\ +3\left(\left(-3\widetilde{\Lambda}^2+\left(6\widetilde{\delta}+3\widetilde{\varepsilon}+\frac{49}{4}\right)\widetilde{\Lambda}-3\left(\widetilde{\delta}+\frac{\widetilde{\varepsilon}}{2}+2\right)^2\right)V+\right.\\ \left.+\left(-\widetilde{\Lambda}^2+\left(-\frac{\widetilde{\delta}}{3}-\frac{\widetilde{\varepsilon}}{6}-\frac{11}{12}\right)\widetilde{\Lambda}+\left(\widetilde{\delta}+\frac{\widetilde{\varepsilon}}{2}+2\right)^2\right)U\right)\eta\lambda+\end{gathered}$$

$$+\eta^2\left(\left(-3\widetilde{\Lambda}^2+\left(6\widetilde{\delta}+3\widetilde{\varepsilon}+\frac{49}{4}\right)\widetilde{\Lambda}-3\left(\widetilde{\delta}+\frac{\widetilde{\varepsilon}}{2}+2\right)^2\right)V+U\widetilde{\Lambda}\left(\widetilde{\delta}-\widetilde{\Lambda}+\frac{\widetilde{\varepsilon}}{2}+\frac{7}{4}\right)\right)=0. \tag{8.4}$$

They can be written as the product of a square matrix and the column $(U,\ V)^T$ is equal to zero. Since the column is non-zero, the determinant of the specified matrix must be zero:

$$\widetilde{\Lambda}\left(12+6\widetilde{\delta}-8\widetilde{\Lambda}+3\widetilde{\varepsilon}\right)^2\eta^2\left(\eta+3\lambda\right)^2=0. \tag{8.5}$$

By virtue of (7.15) and (7.23), this is equivalent to the equation

$$\eta^2\left(\eta+3\lambda\right)^2=0. \tag{8.6}$$

The equation $\eta=0$ leads to the solution of the form $f_{ik}=const\cdot g_{ik}$, in which $\widetilde{T}_{pq}=0$, therefore this case is excluded. Hence, from (8.6) we definitely find

$$\eta=-3\lambda. \tag{8.7}$$

The equations (8.3) and (8.4), after substituting (8.7) into them, become equivalent to the one equation

$$\left(2\widetilde{\delta}+\widetilde{\varepsilon}+4\right)\left(\left(\widetilde{\delta}-\widetilde{\Lambda}+\frac{\widetilde{\varepsilon}}{2}+2\right)U+2\left(\widetilde{\delta}-\frac{3\widetilde{\Lambda}}{2}+\frac{\widetilde{\varepsilon}}{2}+2\right)V\right)=0. \tag{8.8}$$

In the case off $2\widetilde{\delta}+\widetilde{\varepsilon}+4\neq 0$, we get from it

$$\left(\widetilde{\delta}-\widetilde{\Lambda}+\frac{\widetilde{\varepsilon}}{2}+2\right)U+2\left(\widetilde{\delta}-\frac{3\widetilde{\Lambda}}{2}+\frac{\widetilde{\varepsilon}}{2}+2\right)V=0. \tag{8.9}$$

Since by virtue of (7.15) both coefficients in (8.9) before U and V cannot be equal to zero, the constants U and V must be expressed as

$$U=2\left(\widetilde{\delta}-\frac{3\widetilde{\Lambda}}{2}+\frac{\widetilde{\varepsilon}}{2}+2\right)W,\ \ V=-\left(\widetilde{\delta}-\widetilde{\Lambda}+\frac{\widetilde{\varepsilon}}{2}+2\right)W,\ \ W\neq 0. \tag{8.10}$$

The substitution (7.16), (7.22), (8.1), (8.2), (8.7), (8.10) into (7.7), (7.8) leads to the equations

$$\widetilde{T}^0_0=\frac{45}{32}\left(12+6\widetilde{\delta}-8\widetilde{\Lambda}+3\widetilde{\varepsilon}\right)^2W^2\lambda^2\exp\left(-6\lambda t\right), \tag{8.11}$$

$$\widetilde{T}^r_r=\widetilde{T}^\theta_\theta=\widetilde{T}^\varphi_\varphi=-\widetilde{T}^0_0. \tag{8.12}$$

Taking into account (7.23), (8.1), (8.11), at $t\to-\infty$ the first term of the left side of the Friedman equation (7.3) becomes negligible compared to the second, which leads to

$$\rho\sim\exp\left(-6\lambda t\right)\sim a^{-6}. \tag{8.13}$$

By virtue of (5.18), this is possible only if the equation of state of matter has the form $p=\rho$ [1], which is consistent with (8.12). This means that in the distant past, before the Big Bang, matter was compressed to maximum rigidity compatible with causality, when the speed of sound is equal to the speed of light.

In the case off $2\widetilde{\delta}+\widetilde{\varepsilon}+4=0$, we assign

$$\widetilde{\varepsilon}=-2\widetilde{\delta}-4. \tag{8.14}$$

The substitution (7.16), (7.22), (8.1), (8.2), (8.7), (8.14) into (7.7), (7.8) leads to the equations

$$\widetilde{T}^0_0=\frac{9}{2}\left(\left(\widetilde{\Lambda}+\frac{5}{4}\right)U^2+V\left(6\widetilde{\Lambda}-\frac{5}{2}\right)U+9\left(\widetilde{\Lambda}+\frac{5}{36}\right)V^2\right)\lambda^2\exp\left(-6\lambda t\right) \tag{8.15}$$

and (8.12). In this case, the constants U and V can take any values, including (8.10), leading to a positive value of (8.15). Thus, we come back to (8.13).

9. Solutions in the early stages of the Universe's evolution

Let's assume that in the early stages of the Universe's evolution, $a(t)$, $u(t)$, and $v(t)$ are functions of the form

$$a(t) = \widetilde{a}_0 t^{\mu}, \;\; u(t) = u_0 t^{\chi} + C, \;\; v(t) = v_0 t^{\chi} + C, \tag{9.1}$$

where $\widetilde{a}_0 > 0$ and at least one of the constants u_0 and v_0 is not zero. The substitution (7.16), (7.22), (9.1) into (7.5), (7.6) leads to the equations

$$\left(\widetilde{\delta} - \widetilde{\Lambda} + \frac{\widetilde{\varepsilon}}{2} + 2\right)\left(\widetilde{\delta} + \frac{\widetilde{\varepsilon}}{2} + 2\right)(u_0 - v_0)\mu^2 + \left(-\frac{\chi}{3}(u_0 + 3v_0)\widetilde{\Lambda}^2 + \right.$$
$$\left. + \left(\frac{1}{3}\left(\left(\frac{53}{4} + 7\widetilde{\delta} + \frac{7\widetilde{\varepsilon}}{2}\right)\chi - \widetilde{\delta} - \frac{\widetilde{\varepsilon}}{2}\right)v_0 + \frac{1}{4}\left(\chi + \frac{4\widetilde{\delta}}{3} + \frac{2\widetilde{\varepsilon}}{3}\right)u_0\right)\widetilde{\Lambda} - \chi v_0\left(\widetilde{\delta} + \frac{\widetilde{\varepsilon}}{2} + 2\right)^2\right)\mu -$$
$$-\frac{\chi}{9}(-1 + \chi)\widetilde{\Lambda}\left((u_0 + 3v_0)\widetilde{\Lambda} + 3\left(-\frac{7}{4} - \widetilde{\delta} - \frac{\widetilde{\varepsilon}}{2}\right)v_0 - \frac{3}{4}u_0\right) = 0, \tag{9.2}$$

$$\left(\widetilde{\delta} - \frac{3\widetilde{\Lambda}}{2} + \frac{\widetilde{\varepsilon}}{2} + 2\right)\left(\widetilde{\delta} + \frac{\widetilde{\varepsilon}}{2} + 2\right)(u_0 - v_0)\mu^2 + \left(\left(\left(-\frac{3}{2}\widetilde{\Lambda}^2 + \left(3\widetilde{\delta} + \frac{3\widetilde{\varepsilon}}{2} + \frac{49}{8}\right)\widetilde{\Lambda} - \right.\right.\right.$$
$$\left.\left. -\frac{3}{2}\left(\widetilde{\delta} + \frac{\widetilde{\varepsilon}}{2} + 2\right)^2\right)v_0 + \frac{1}{2}\left(-\widetilde{\Lambda}^2 + \left(-\frac{\widetilde{\delta}}{3} - \frac{\widetilde{\varepsilon}}{6} - \frac{11}{12}\right)\widetilde{\Lambda} + \left(\widetilde{\delta} + \frac{\widetilde{\varepsilon}}{2} + 2\right)^2\right)u_0\right)\chi -$$
$$\left. -\frac{1}{2}\left(\left(-\widetilde{\delta} - \frac{\widetilde{\varepsilon}}{2} - \frac{8}{3}\right)\widetilde{\Lambda} + \left(\widetilde{\delta} + \frac{\widetilde{\varepsilon}}{2} + 2\right)^2\right)(u_0 - v_0)\right)\mu +$$
$$+\frac{1}{6}\left(\left(-3\widetilde{\Lambda}^2 + \left(6\widetilde{\delta} + 3\widetilde{\varepsilon} + \frac{49}{4}\right)\widetilde{\Lambda} - 3\left(\widetilde{\delta} + \frac{\widetilde{\varepsilon}}{2} + 2\right)^2\right)v_0 + u_0\widetilde{\Lambda}\left(\widetilde{\delta} - \widetilde{\Lambda} + \frac{\widetilde{\varepsilon}}{2} + \frac{7}{4}\right)\right)\chi(-1 + \chi) = 0. \tag{9.3}$$

They can be written as the product of a square matrix and the column $(u_0, \;\; v_0)^T$ is equal to zero. Since the column is non-zero, the determinant of the specified matrix must be zero:

$$\widetilde{\Lambda}\left(12 + 6\widetilde{\delta} - 8\widetilde{\Lambda} + 3\widetilde{\varepsilon}\right)^2 \chi(\chi + 3\mu - 1)\left(\chi^2 + 3\mu\chi - \chi - 8\mu\right) = 0. \tag{9.4}$$

By virtue of (7.15) and (7.23), this is equivalent to the equation

$$\chi(\chi + 3\mu - 1)\left(\chi^2 + 3\mu\chi - \chi - 8\mu\right) = 0. \tag{9.5}$$

The equation $\chi = 0$ leads to the solution of the form $f_{ik} = const \cdot g_{ik}$, in which $\widetilde{T}_{pq} = 0$, therefore this case is excluded.

As the result of (5.18) the energy density of matter ρ is proportional to a^{-4} at the stage of radiation dominance and proportional to a^{-3} at the stage of matter dominance [1]. Therefore, under the assumption (9.1) the Friedman equation (7.3) will be written as

$$3\mu^2 t^{-2} + 3k\widetilde{a}_0^{-2} t^{-2\mu} + \widetilde{T}_0^0 = const \cdot t^{-\xi\mu}, \tag{9.6}$$

where $\xi = 4$ in the case off radiation dominance or $\xi = 3$ in the case off matter dominance. From the substitution (9.1) into (7.7) we find

$$\widetilde{T}_0^0 \sim t^{-2+2\chi}. \tag{9.7}$$

The assumption that the second term of the left part of (9.6) has the smallest degree of t among all the terms of the left part, leads to the contradiction $-2 \geq -2\mu$, $-2\mu = -\xi\mu$. If the first term has the smallest degree, then we have

$$\chi > 0, \;\; -2 = -\xi\mu, \tag{9.8}$$

and if the third one, then

$$\chi < 0, \quad -2 + 2\chi = -\xi\mu. \tag{9.9}$$

Under the condition (9.9), the equation (9.5) has no solution, and under the condition (9.8) it has the unique solution

$$\chi = (2\xi)^{-1}\left(\xi - 6 + \sqrt{\xi^2 + 52\xi + 36}\right). \tag{9.10}$$

As a result, we uniquely define

$$\mu = \frac{1}{2}, \quad \chi = -\frac{1}{4} + \frac{\sqrt{65}}{4} \approx 1.7656 \tag{9.11}$$

in the case off radiation dominance and

$$\mu = \frac{2}{3}, \quad \chi = -\frac{1}{2} + \frac{\sqrt{201}}{6} \approx 1.8629 \tag{9.12}$$

in the case off matter dominance.

10. Solution of the dark field equation in the Friedmann–Lemaitre–Robertson–Walker metric

The equation system (7.5), (7.6) for (7.16), (7.22), (7.23) admits the solution of the form

$$u(t) = \frac{24}{12 + 6\widetilde{\delta} - 8\widetilde{\Lambda} + 3\widetilde{\varepsilon}} \int_0^t \left(7\left(2 + \widetilde{\delta} + \frac{\widetilde{\varepsilon}}{2} - \frac{8\widetilde{\Lambda}}{7}\right) a(\tau)^{-9}\dot{a}(\tau)\, n(\tau) - \left(2 + \widetilde{\delta} + \frac{\widetilde{\varepsilon}}{2} - \widetilde{\Lambda}\right) w(\tau)\right) d\tau + C, \tag{10.1}$$

$$v(t) = u(t) + 4a(t)^{-8}\, n(t), \tag{10.2}$$

where

$$n(t) = \int_0^t a(\tau)^8\, w(\tau)\, d\tau \tag{10.3}$$

and the function $w(t)$ satisfies the equation

$$\dot{w} = 5a^{-1}\dot{a}w - 40a^{-10}\dot{a}^2 n. \tag{10.4}$$

Let's assume that in the early stages of the Universe's evolution, $a(t)$ and $w(t)$ are power functions of the form

$$a(t) = \widetilde{a}_0 t^{\mu}, \quad w(t) = w_0 t^{\chi - 1}, \quad \widetilde{a}_0 > 0, \quad w_0 \neq 0, \tag{10.5}$$

on condition (9.8). Then, by virtue of (10.1)–(10.3), the functions $u(t)$ and $v(t)$ have the form (9.1). The substitution (10.5) into (10.4) leads to the equation

$$\chi^2 + 3\mu\chi - \chi - 8\mu = 0, \tag{10.6}$$

which is equivalent to the equation (9.5) under the condition (9.8) and, accordingly, has the solutions (9.10)–(9.12). Thus, we have proved that the solution (10.1) satisfies the initial conditions (9.1)–(9.12).

Let ε be a time point close to zero. Then the equations (7.5), (7.6) together with the initial conditions

$$u(\varepsilon) = u_0\varepsilon^{\chi} + C, \quad v(\varepsilon) = v_0\varepsilon^{\chi} + C, \quad \dot{u}(\varepsilon) = u_0\chi\varepsilon^{\chi-1}, \quad \dot{v}(\varepsilon) = v_0\chi\varepsilon^{\chi-1} \tag{10.7}$$

form the Cauchy problem. From the uniqueness of the solution of this problem, it follows that the system (10.1)–(10.4) is the only possible solution under the conditions (9.1)–(9.12) in the early stages of the expansion of the Universe.

The substitution (7.16), (7.22), (10.1)–(10.3) into (7.7) and (7.8) yields

$$\widetilde{T}_0^0 = -6w^2 + 240a^{-18}\dot{a}^2 n^2, \tag{10.8}$$

$$\widetilde{T}_r^r = \widetilde{T}_\theta^\theta = \widetilde{T}_\varphi^\varphi = a^{-18}\left(160a\ddot{a} - 2480\dot{a}^2\right) n^2 + \left(480a^{-9}\dot{a}w - 32a^{-8}\dot{w}\right) n - 26w^2. \tag{10.9}$$

11. The deceleration parameter and the age of the Universe

Let t_0 be the present moment of time, $H_0 \equiv \dot{a}(t_0)\, a(t_0)^{-1}$ be the Hubble constant, $a_0 \equiv a(t_0)$. Then, due to the fact that the energy density of matter ρ is proportional to a^{-4} at the stage of radiation dominance and proportional to a^{-3} at the stage of matter dominance, ρ can be written as the following combination [1]:

$$\rho = \frac{3H_0^2}{8\pi G}\left(\Omega_m\left(\frac{a_0}{a}\right)^3 + \Omega_r\left(\frac{a_0}{a}\right)^4\right). \tag{11.1}$$

Hence, the Friedman equation (7.3) is written as

$$3\dot{a}^2 a^{-2} + \widetilde{T}_0^0 = 3H_0^2\left(\Omega_k\left(\frac{a_0}{a}\right)^2 + \Omega_m\left(\frac{a_0}{a}\right)^3 + \Omega_r\left(\frac{a_0}{a}\right)^4\right), \tag{11.2}$$

where

$$\Omega_k = -k a_0^{-2} H_0^{-2}. \tag{11.3}$$

After making the substitutions $t \to H_0^{-1}t$, $t_0 \to H_0^{-1}t_0$, $a \to a_0 a$, $w \to H_0 w$ we pass to the dimensionless t, t_0, a, w. In this case, we get

$$a(t_0) = \dot{a}(t_0) = 1. \tag{11.4}$$

Taking into account (10.8), the equation (11.2) takes the form

$$\dot{a}^2 a^{-2} - 2w^2 + 80a^{-18}\dot{a}^2 n^2 = \Omega_k a^{-2} + \Omega_m a^{-3} + \Omega_r a^{-4}. \tag{11.5}$$

Consider the expansion of the Universe at the stage of matter dominance, so accept

$$\Omega_r = 0. \tag{11.6}$$

Then (11.5) can be written as

$$\dot{a} = \sqrt{\frac{\Omega_k + \Omega_m a^{-1} + 2w^2 a^2}{1 + 80a^{-16}n^2}}. \tag{11.7}$$

From (10.3) we get

$$\dot{n} = a^8 w. \tag{11.8}$$

At the early stage, the solution has the form (10.5), where μ, χ are defined by the equations (9.12), and by virtue of $\dot{a}^2 a^{-2} \simeq \Omega_m a^{-3}$ we have

$$\widetilde{a}_0 = 3^{\frac{2}{3}} 2^{-\frac{2}{3}} \Omega_m^{\frac{1}{3}}. \tag{11.9}$$

Hence, we can write the initial conditions for a and w at the time close to zero ε in the form

$$a(\varepsilon) = \widetilde{a}_0 \varepsilon^{\mu}, \quad w(\varepsilon) = w_0 \varepsilon^{\chi-1}. \tag{11.10}$$

From (10.3) and (10.5) we find the initial condition for n:

$$n(\varepsilon) = \int_0^{\varepsilon} \left(\widetilde{a}_0 \tau^{\mu}\right)^8 w_0 \tau^{\chi-1} d\tau. \tag{11.11}$$

The equations (10.4), (11.7), (11.8) with the initial conditions (11.10), (11.11) form the Cauchy problem, the solution of which describes the expansion of the Universe at the stage of matter dominance. It is noteworthy that this problem does not depend on the introduced fundamental physical constants $\widetilde{\alpha}$, $\widetilde{\delta}$, $\widetilde{\varepsilon}$, $\widetilde{\Lambda}$. For the specified parameters Ω_k, Ω_m the values of t_0 and w_0 are determined from the requirements of (11.4).

According to the standard cosmological model, we accept that space is flat: $\Omega_k = 0$. In calculations we set $\varepsilon = 0.001$. Take $\Omega_m = \Omega_b + \Omega_c = 0.2302$ according to (16.1). With these parameters the calculations result is the $t_0 \approx 1.093$ and $w_0 \approx 0.9493$. The deceleration parameter $q_0 \approx -0.847$ is also obtained, which is consistent with the result of [3], in which the observed value is $q_0 = -1.08 \pm 0.29$, when the value obtained within the framework of the standard cosmological model ΛCDM differs from observations at the level of 1.9σ [3]. Also, according to calculations, the age of the Universe is $t_0 H_0^{-1} \approx 13.6$ billion years and the redshifts $3 \leq z \leq 14$ correspond to the following age values:

z	3	4	5	6	7	8	9	10	11	12	13	14
billion y.	2.16	1.54	1.18	0.93	0.76	0.64	0.55	0.47	0.42	0.37	0.33	0.3

12. Galactic dark field halo

To construct a galactic halo model, we consider the stationary centrally symmetric dark field in Minkowski space in spherical coordinates:

$$f_{00} = X(r), \quad f_{0r} = f_{r0} = S(r), \quad f_{rr} = -Y(r), \quad f_{\theta\theta} = -Z(r) r^2, \quad f_{\varphi\varphi} = -Z(r) r^2 \sin^2(\theta). \tag{12.1}$$

Using the DifferentialGeometry software package of the Maple computer mathematics system, write[2] (3.6) and (4.16) by substituting the Minkowski metric and (7.4), (7.22), (12.1) into them:

$$\begin{gathered}
-\left(\widetilde{\alpha}-\widetilde{\delta}+\widetilde{\Lambda}-\widetilde{\varepsilon}-\frac{7}{4}\right) r^2 X'' + \left(\widetilde{\delta}-\widetilde{\Lambda}+\frac{\widetilde{\varepsilon}}{2}+\frac{7}{4}\right) r^2 Y'' + \\
+2\left(\widetilde{\delta}-\widetilde{\Lambda}+\widetilde{\varepsilon}+\frac{7}{4}\right) r^2 Z'' - 2\left(\widetilde{\alpha}-\widetilde{\delta}+\widetilde{\Lambda}-\widetilde{\varepsilon}-\frac{7}{4}\right) r X' + \\
+4r\left(\widetilde{\delta}-\widetilde{\Lambda}+\frac{5\widetilde{\varepsilon}}{4}+\frac{7}{4}\right) Z' + 2r\left(\widetilde{\delta}-\widetilde{\Lambda}+\frac{7}{4}\right) Y' - \widetilde{\varepsilon}(-Z+Y) = 0,
\end{gathered} \tag{12.2}$$

$$\begin{gathered}
2\left(\widetilde{\delta}-\widetilde{\Lambda}+\frac{\widetilde{\varepsilon}}{2}+\frac{7}{4}\right) r^2 X'' + 4\left(\widetilde{\delta}-\widetilde{\Lambda}+\frac{\widetilde{\varepsilon}}{2}+\frac{7}{4}\right) r^2 Z'' - 2\left(\widetilde{\Lambda}-\frac{3}{4}\right) r^2 Y'' - \\
-4r\left(\widetilde{\alpha}-3\widetilde{\delta}+2\widetilde{\Lambda}-\frac{5\widetilde{\varepsilon}}{2}-\frac{9}{2}\right) Z' + 4r\left(\widetilde{\delta}-\widetilde{\Lambda}+\widetilde{\varepsilon}+\frac{7}{4}\right) X' + \\
+\left(-4\widetilde{\Lambda}+3\right) r Y' + 4(-Z+Y)\left(\widetilde{\alpha}+\widetilde{\delta}-\frac{\widetilde{\varepsilon}}{2}+1\right) = 0,
\end{gathered} \tag{12.3}$$

$$\begin{gathered}
-\left(\widetilde{\alpha}-2\widetilde{\delta}+2\widetilde{\Lambda}-2\widetilde{\varepsilon}-\frac{7}{2}\right) r^2 Z'' + \left(\widetilde{\delta}-\widetilde{\Lambda}+\widetilde{\varepsilon}+\frac{7}{4}\right) r^2 X'' + \left(\widetilde{\delta}-\widetilde{\Lambda}+\frac{\widetilde{\varepsilon}}{2}+\frac{7}{4}\right) r^2 Y'' + \\
+r\left(\widetilde{\alpha}+\widetilde{\delta}-2\widetilde{\Lambda}-\frac{\widetilde{\varepsilon}}{2}+\frac{5}{2}\right) Y' - 2\left(\widetilde{\alpha}-2\widetilde{\delta}+2\widetilde{\Lambda}-2\widetilde{\varepsilon}-\frac{7}{2}\right) r Z' + \\
+2r\left(\widetilde{\delta}-\widetilde{\Lambda}+\frac{3\widetilde{\varepsilon}}{4}+\frac{7}{4}\right) X' - 2\left(\widetilde{\delta}+\frac{\widetilde{\varepsilon}}{2}+1\right)(-Z+Y) = 0,
\end{gathered} \tag{12.4}$$

$$\left(S'' r^2 + 2S' r - 2S\right)\left(\widetilde{\alpha}+\widetilde{\delta}+1\right) = 0, \tag{12.5}$$

$$\begin{gathered}
\widetilde{T}_0^0 = \frac{1}{2r^2}\Bigg(4r^2 S\left(\widetilde{\alpha}+\widetilde{\delta}-1\right) S'' - 2r^2 (X-Y)\left(\widetilde{\alpha}-\frac{\widetilde{\varepsilon}}{2}-1\right) X'' + \\
+2\left(\widetilde{\delta}+\frac{\widetilde{\varepsilon}}{2}\right) r^2 (X-Y) Y'' + 2r^2 \widetilde{\varepsilon}(X-Y) Z'' - 2r^2\left(\widetilde{\delta}-\frac{\widetilde{\Lambda}}{2}+\frac{\widetilde{\varepsilon}}{2}+\frac{3}{8}\right)(Y')^2 + \\
+2\left(r\left(\widetilde{\alpha}+\widetilde{\Lambda}-\frac{\widetilde{\varepsilon}}{2}-\frac{11}{4}\right) X' - 2r\left(\widetilde{\delta}-\widetilde{\Lambda}+\widetilde{\varepsilon}+\frac{7}{4}\right) Z' + \left(4\widetilde{\delta}+\widetilde{\varepsilon}\right) X - 4Y\widetilde{\delta} - Z\widetilde{\varepsilon}\right) r Y' - \\
-r^2\left(\widetilde{\alpha}+\widetilde{\delta}-\widetilde{\Lambda}-\frac{1}{4}\right)(X')^2 - 4r\Bigg(r\left(\widetilde{\delta}-\widetilde{\Lambda}+\frac{\widetilde{\varepsilon}}{2}+\frac{7}{4}\right) Z' + \left(\widetilde{\alpha}-\frac{\widetilde{\varepsilon}}{2}-1\right) X + \left(-\widetilde{\alpha}-\widetilde{\delta}+1\right) Y + \\
+Z\left(\widetilde{\delta}+\frac{\widetilde{\varepsilon}}{2}\right)\Bigg) X' + 3\left(\widetilde{\alpha}+\widetilde{\delta}-\frac{5}{3}\right) r^2 (S')^2 + 8Sr\left(\widetilde{\alpha}+\frac{5\widetilde{\delta}}{2}-1\right) S' +
\end{gathered}$$

[2] File halo.mw from the repository https://github.com/alegorov/dark-field

$$+2\left(\widetilde{\alpha}-2\widetilde{\delta}+2\widetilde{\Lambda}-2\widetilde{\varepsilon}-\frac{7}{2}\right)r^2\left(Z'\right)^2-4r\left(\left(\widetilde{\alpha}-\widetilde{\delta}-1\right)Y+\left(\widetilde{\delta}-\widetilde{\varepsilon}\right)X-Z\left(\widetilde{\alpha}-\widetilde{\varepsilon}-1\right)\right)Z'+$$

$$+4\widetilde{\delta}\left(-Z+Y\right)X+2\left(\widetilde{\alpha}+1\right)Y^2-4Z\left(\widetilde{\alpha}+\widetilde{\delta}+1\right)Y+2\left(1+\widetilde{\alpha}+2\widetilde{\delta}\right)Z^2-2S^2\left(\widetilde{\alpha}-2\widetilde{\delta}+1\right)\Bigg), \quad (12.6)$$

$$\widetilde{T}_r^r=\frac{1}{2r^2}\Bigg(-2\left(\widetilde{\alpha}-2\widetilde{\delta}+2\widetilde{\Lambda}-2\widetilde{\varepsilon}-\frac{7}{2}\right)r^2\left(Z'\right)^2+4r\Bigg(r\left(\widetilde{\delta}-\widetilde{\Lambda}+\widetilde{\varepsilon}+\frac{7}{4}\right)X'+$$

$$+r\left(\widetilde{\delta}-\widetilde{\Lambda}+\frac{\widetilde{\varepsilon}}{2}+\frac{7}{4}\right)Y'+\left(-Z+Y\right)\left(\widetilde{\alpha}-\widetilde{\varepsilon}-1\right)\Bigg)Z'-r^2\left(\widetilde{\alpha}-\widetilde{\delta}+\widetilde{\Lambda}-\widetilde{\varepsilon}-\frac{7}{4}\right)\left(X'\right)^2+$$

$$+2\left(r\left(\widetilde{\delta}-\widetilde{\Lambda}+\frac{\widetilde{\varepsilon}}{2}+\frac{7}{4}\right)Y'-\widetilde{\varepsilon}\left(-Z+Y\right)\right)rX'-\left(\widetilde{\Lambda}-\frac{3}{4}\right)r^2\left(Y'\right)^2-4\left(-Z+Y\right)\left(\widetilde{\delta}+\frac{\widetilde{\varepsilon}}{2}\right)rY'+$$

$$+r^2\left(\widetilde{\alpha}+\widetilde{\delta}+1\right)\left(S'\right)^2+4SS'\widetilde{\delta}r+2\left(S+Y-Z\right)\left(S-Y+Z\right)\left(1+\widetilde{\alpha}+2\widetilde{\delta}\right)\Bigg), \quad (12.7)$$

$$\widetilde{T}_\theta^\theta=\widetilde{T}_\varphi^\varphi=\frac{1}{2r}\Bigg(2r\left(-Z+Y\right)\left(\widetilde{\alpha}-\widetilde{\varepsilon}-1\right)Z''-2\left(-Z+Y\right)\left(\widetilde{\delta}+\frac{\widetilde{\varepsilon}}{2}\right)rY''-r\widetilde{\varepsilon}\left(-Z+Y\right)X''+$$

$$+2SS''\widetilde{\delta}r-2r\left(\widetilde{\delta}-\frac{\widetilde{\Lambda}}{2}+\frac{\widetilde{\varepsilon}}{2}+\frac{3}{8}\right)\left(Y'\right)^2+2\Bigg(\left(\widetilde{\alpha}-\widetilde{\delta}+2\widetilde{\Lambda}-\frac{3\widetilde{\varepsilon}}{2}-\frac{9}{2}\right)rZ'-r\left(\widetilde{\delta}-\widetilde{\Lambda}+\widetilde{\varepsilon}+\frac{7}{4}\right)X'-$$

$$-2\left(-Z+Y\right)\left(1+\widetilde{\alpha}+2\widetilde{\delta}\right)\Bigg)Y'-4r\left(\widetilde{\delta}-\widetilde{\Lambda}+\frac{\widetilde{\varepsilon}}{2}+\frac{5}{4}\right)\left(Z'\right)^2+4\Bigg(-r\left(\widetilde{\delta}-\widetilde{\Lambda}+\frac{3\widetilde{\varepsilon}}{4}+\frac{7}{4}\right)X'+\left(-Z+Y\right)\times$$

$$\times\left(1+\widetilde{\alpha}+2\widetilde{\delta}\right)\Bigg)Z'+r\left(\widetilde{\alpha}-\widetilde{\delta}+\widetilde{\Lambda}-\widetilde{\varepsilon}-\frac{7}{4}\right)\left(X'\right)^2-S'\left(r\left(\widetilde{\alpha}-\widetilde{\delta}+1\right)S'-4S\left(1+\widetilde{\alpha}+2\widetilde{\delta}\right)\right)\Bigg), \quad (12.8)$$

$$\widetilde{T}_r^0=-\widetilde{T}_0^r=\frac{1}{2r^2}\Bigg(r^2\left(X-Y\right)\left(\widetilde{\alpha}+\widetilde{\delta}+1\right)S''+2\left(\widetilde{\delta}+\frac{\widetilde{\varepsilon}}{2}+1\right)r^2SY''-$$

$$-2r^2\left(\widetilde{\alpha}-\frac{\widetilde{\varepsilon}}{2}\right)SX''+2\widetilde{\varepsilon}Sr^2Z''+2r\left(X-Y\right)\left(\widetilde{\alpha}+\widetilde{\delta}+1\right)S'-$$

$$-4\left(-r\left(\widetilde{\alpha}-\widetilde{\delta}-1\right)Z'-r\left(\widetilde{\delta}+1\right)Y'+rX'\widetilde{\alpha}+\frac{1}{2}\left(X+Y-2Z\right)\left(\widetilde{\alpha}+\widetilde{\delta}+1\right)\right)S\Bigg). \quad (12.9)$$

Since $S\left(r\right)$ does not appear in the equations (12.2)–(12.4), according to (12.5) in the case off $\widetilde{\alpha}+\widetilde{\delta}+1=0$ the function $S\left(r\right)$ becomes arbitrary, which leads to uncertainty of the EMT of the dark field. Therefore, the inequality must be satisfied

$$\widetilde{\alpha}+\widetilde{\delta}+1\neq 0. \quad (12.10)$$

When this inequality is satisfied, the general solution of (12.5) is written as

$$S\left(r\right)=S_1 r+S_2 r^{-2}. \quad (12.11)$$

In the case off $\widetilde{\alpha}=0$ the equation system (12.2)–(12.4) admits the solution of the form

$$X\left(r\right)=-3Y\left(r\right)-rY'\left(r\right),\quad Z\left(r\right)=Y\left(r\right)+2^{-1}rY'\left(r\right). \quad (12.12)$$

Due to the arbitrariness of $Y\left(r\right)$ in this solution, the EMT of the dark field also becomes indeterminate, which leads to the inequality

$$\widetilde{\alpha}\neq 0. \quad (12.13)$$

It can be shown that for the stability of the Universe the stronger requirement $\widetilde{\alpha}>0$ must be performed, the proof of which is beyond the scope of this article. The arguments (12.10)–(12.13) are also valid in the degenerate and super-degenerate cases discussed in the sections (13) and (14).

The general solution of the equation system (12.2)–(12.4) will be sought in the form of a linear combination of power functions. To find out which degrees to use, consider functions of the form

$$X(r) = X_0 r^x, \;\; Y(r) = Y_0 r^x, \;\; Z(r) = Z_0 r^x, \tag{12.14}$$

where at least one of the constants X_0, Y_0, Z_0 is not equal to zero. The substitution (12.14) into (12.2)–(12.4) can be written as the product of a square matrix and the column $(X_0, \;\; Y_0, \;\; Z_0)^T$ is equal to zero. Since the column is non-zero, the determinant of the specified matrix must be zero:

$$\widetilde{\alpha}\left(-3\widetilde{\delta}^2+\left(3\widetilde{\Lambda}-3\widetilde{\varepsilon}-\frac{33}{4}\right)\widetilde{\delta}-\frac{3\widetilde{\varepsilon}^2}{4}-3\widetilde{\varepsilon}+(\widetilde{\alpha}+3)\widetilde{\Lambda}-\frac{3\widetilde{\alpha}}{4}-\frac{21}{4}\right)x^2(x+1)^2(x-2)(x+3)=0, \tag{12.15}$$

which, taking into account (7.16) and (7.22), is equivalent to the equation

$$\widetilde{\alpha}\left(12+6\widetilde{\delta}-8\widetilde{\Lambda}+3\widetilde{\varepsilon}\right)^2\widetilde{\Lambda}^{-1}x^2(x+1)^2(x-2)(x+3)=0. \tag{12.16}$$

Hence, by virtue of (7.23) and (12.13), the desired degree x is equal to one of the values 0, -1, 2, -3. Now we can write the solution of the equation system (12.2)–(12.4) in the general form:

$$X(r)=\left(2-2\widetilde{\alpha}+2\widetilde{\delta}+3\widetilde{\varepsilon}\right)X_1r^{-1}-5\Big(7+4\widetilde{\delta}^2+\left(4\widetilde{\varepsilon}-4\widetilde{\alpha}-4\widetilde{\Lambda}+11\right)\widetilde{\delta}+\widetilde{\varepsilon}^2+$$

$$+(4-2\widetilde{\alpha})\widetilde{\varepsilon}+\left(4\widetilde{\Lambda}-7\right)\widetilde{\alpha}-4\widetilde{\Lambda}\Big)L_2r^2+X_0+C, \tag{12.17}$$

$$Y(r)=-\widetilde{\varepsilon}X_1r^{-1}+2\left(2+2\widetilde{\delta}+\widetilde{\varepsilon}\right)L_1r^{-1}+\Big(7+12\widetilde{\alpha}^2+\left(36\widetilde{\Lambda}-40\widetilde{\delta}-38\widetilde{\varepsilon}-67\right)\widetilde{\alpha}+$$

$$+4\widetilde{\delta}^2+\left(11-4\widetilde{\Lambda}+4\widetilde{\varepsilon}\right)\widetilde{\delta}+\widetilde{\varepsilon}^2-4\widetilde{\Lambda}+4\widetilde{\varepsilon}\Big)L_2r^2+2L_3r^{-3}+C, \tag{12.18}$$

$$Z(r)=-\widetilde{\varepsilon}X_1r^{-1}+\left(2+2\widetilde{\alpha}+2\widetilde{\delta}-\widetilde{\varepsilon}\right)L_1r^{-1}+2\Big(\widetilde{\varepsilon}^2+\left(4-3\widetilde{\alpha}+4\widetilde{\delta}\right)\widetilde{\varepsilon}+2\widetilde{\alpha}^2+$$

$$+\left(3-4\widetilde{\Lambda}\right)\widetilde{\alpha}+\left(\widetilde{\delta}+1\right)\left(7+4\widetilde{\delta}-4\widetilde{\Lambda}\right)\Big)L_2r^2-L_3r^{-3}+C. \tag{12.19}$$

The generality of the solution is proved by the equality of the number of different constants to the total order of differential equations. According to (12.11), (12.17)–(12.19) if $L_2 \neq 0$ or $S_1 \neq 0$, then the solution of the equation system (12.2)–(12.5) diverges at $r \to \infty$. Therefore, in such cases, the dark field is stationary only in the inner region of space, and accordingly in the outer region it changes with time. The substitution (12.11), (12.17)–(12.19) into (12.7)–(12.9) gives

$$\widetilde{T}_r^0 = \widetilde{T}_0^r = 0, \tag{12.20}$$

$$\widetilde{T}_\theta^\theta = \widetilde{T}_\varphi^\varphi = \left(1+\frac{r}{2}\frac{\partial}{\partial r}\right)\widetilde{T}_r^r. \tag{12.21}$$

The equation (12.21) can also be obtained from (5.17) and (12.20).

In the case off a weak gravitational field, the metric can be represented as the sum of the Minkowski metric and a small perturbation, and then linearize the Einstein equation by perturbation. From the linearity of the obtained equations, it follows that the gravitational potential Φ can be represented as the sum of the potential Φ_m induced by matter (including black holes) and the potential Φ_f induced by dark field. Each component of Φ_m and Φ_f can be considered separately, independently of the other. We are only interested in the potential of Φ_f, so for simplicity, without losing generality, we consider a galaxy consisting only of dark field halo.

Consider the static centrally symmetric metric tensor in spherical coordinates, which we define as

$$g_{00}=1+A(r), \;\; g_{rr}=-(1+B(r))^{-1}, \;\; g_{\theta\theta}=-r^2, \;\; g_{\varphi\varphi}=-r^2\sin^2(\theta), \tag{12.22}$$

where $A(r)$ and $B(r)$ are small perturbations of elements of the Minkowski metric. The function $A(r)$ is related to the gravitational potential $\Phi(r)$ by the equation $A(r) = 2c^{-2}\Phi(r)$ [2], from where we get the expression for the square of the rotation speed:

$$V_h^2(r) = 2^{-1}c^2 r A'(r). \tag{12.23}$$

The linearization of the Einstein tensor in (4.15) by $A(r)$ and $B(r)$ gives the system of the differential equations

$$r^{-1}B' + r^{-2}B = \widetilde{T}_0^0, \quad r^{-1}A' + r^{-2}B = \widetilde{T}_r^r, \tag{12.24}$$

$$\left(1 + \frac{r}{2}\frac{\partial}{\partial r}\right)\left(r^{-1}A' + r^{-2}B\right) = \widetilde{T}_\theta^\theta = \widetilde{T}_\varphi^\varphi, \tag{12.25}$$

which is consistent with (12.20), (12.21). The rotation speeds of galaxies are much less than the speed of light. Therefore, we will assume that the EMT elements of the dark field in the equation system (12.24), (12.25) are small values, which is consistent with the smallness of A and B. To calculate the first approximation of $\widetilde{T}_0^0$, $\widetilde{T}_r^r$, $\widetilde{T}_\theta^\theta$, $\widetilde{T}_\varphi^\varphi$ it is sufficient to use the unperturbed Minkowski metric. Thus, the substitution (12.6), (12.7), (12.11), (12.17)–(12.19) into (12.24) is correct. From this substitution, taking into account (7.16), (7.22), we find

$$A'(r) = Q_3 r^3 + Q_1 r + Q_0 + R_g r^{-2} + Q_{-3} r^{-3} + Q_{-5} r^{-5} - 36 L_3^2 r^{-7}, \tag{12.26}$$

$$\begin{aligned} Q_3 = 54\left(12\widetilde{\Lambda}\right)^{-4}\left(12 + 6\widetilde{\delta} - 8\widetilde{\Lambda} + 3\widetilde{\varepsilon}\right)^4 \Bigl(48\left(132\widetilde{\delta}^2 + 737\widetilde{\delta} + 998\right)\widetilde{\Lambda}^2 - \\ -132\left(67 + 24\widetilde{\delta}\right)\left(4 + 2\widetilde{\delta} + \widetilde{\varepsilon}\right)^2\widetilde{\Lambda} + 396\left(4 + 2\widetilde{\delta} + \widetilde{\varepsilon}\right)^4\Bigr) L_2^2, \end{aligned} \tag{12.27}$$

$$\begin{aligned} Q_1 = 4S_1^2 + 10\Bigl(\left(7 - 4\widetilde{\Lambda}\right)\widetilde{\alpha}^2 + \left(12\widetilde{\delta}^2 + \left(13 - 12\widetilde{\Lambda} + 12\widetilde{\varepsilon}\right)\widetilde{\delta} + 3\widetilde{\varepsilon}^2 + 8\widetilde{\Lambda} - 4\widetilde{\varepsilon} - 14\right)\widetilde{\alpha} + \\ +4\widetilde{\delta}^2 + \left(11 + 4\widetilde{\varepsilon} - 4\widetilde{\Lambda}\right)\widetilde{\delta} + \widetilde{\varepsilon}^2 - 4\widetilde{\Lambda} + 4\widetilde{\varepsilon} + 7\Bigr) X_0 L_2, \end{aligned} \tag{12.28}$$

$$\begin{aligned} Q_0 = 8\left(4\widetilde{\Lambda}\right)^{-4}\left(12 + 6\widetilde{\delta} - 8\widetilde{\Lambda} + 3\widetilde{\varepsilon}\right)^2 \Bigl(16\Bigl(72\left(2X_1 - L_1\right)\widetilde{\delta}^3 + \left(36\left(2X_1 - L_1\right)\widetilde{\varepsilon} + 32\left(22X_1 - 23L_1\right)\right)\widetilde{\delta}^2 + \\ + \left(\left(208X_1 - 275L_1\right)\widetilde{\varepsilon} + 256\left(4X_1 - 9L1\right)\right)\widetilde{\delta} + 24\left(4X_1 - 19L_1\right)\widetilde{\varepsilon} + 32\left(12X_1 - 71L_1\right)\Bigr)\widetilde{\Lambda}^3 - \\ -4\left(4 + 2\widetilde{\delta} + \widetilde{\varepsilon}\right)^2\Bigl(36\left(11X_1 - 4L_1\right)\widetilde{\delta}^2 + 2\left(36\left(2X_1 - L1\right)\widetilde{\varepsilon} + 652X_1 - 575L_1\right)\widetilde{\delta} + \\ + \left(208X_1 - 275L_1\right)\widetilde{\varepsilon} + 16\left(61X_1 - 121L_1\right)\Bigr)\widetilde{\Lambda}^2 + 6\left(4 + 2\widetilde{\delta} + \widetilde{\varepsilon}\right)^4 \times \\ \times\left(12\left(5X_1 - L_1\right)\widetilde{\delta} + 6\left(2X_1 - L_1\right)\widetilde{\varepsilon} + 100X_1 - 69L_1\right)\widetilde{\Lambda} - 27\left(4 + 2\widetilde{\delta} + \widetilde{\varepsilon}\right)^6 X_1\Bigr) L_2, \end{aligned} \tag{12.29}$$

$$\begin{aligned} Q_{-3} = 4\Bigl(\left(1 - \widetilde{\alpha} + \widetilde{\delta} + 2\widetilde{\varepsilon}\right)X_1 - \left(2 + 2\widetilde{\delta} + \widetilde{\varepsilon}\right)L_1\Bigr) \times \\ \times\Bigl(\left(1 - \widetilde{\alpha}\right)\left(1 - \widetilde{\alpha} + \widetilde{\delta} + 2\widetilde{\varepsilon}\right)X_1 + \left(2\widetilde{\alpha}\widetilde{\delta} + \widetilde{\varepsilon}\widetilde{\alpha} + 2\widetilde{\varepsilon}\right)L_1\Bigr), \end{aligned} \tag{12.30}$$

$$Q_{-5} = 4\Bigl(\left(\widetilde{\alpha} - 1\right)\left(1 - \widetilde{\alpha} + \widetilde{\delta} + 2\widetilde{\varepsilon}\right)X_1 - \left(\widetilde{\alpha}\left(2\widetilde{\delta} + \widetilde{\varepsilon} - 6\right) + 6\widetilde{\delta} + 11\widetilde{\varepsilon} + 6\right)L_1\Bigr) L_3 + 2\left(3\widetilde{\alpha} - 1\right)S_2^2, \tag{12.31}$$

where R_g is the gravitational radius of the black hole in the center of the galaxy. Since we are interested in the gravitational potential induced only by the dark field, let's accept

$$R_g = 0. \tag{12.32}$$

By setting the different values of the 7 parameters X_0, X_1, L_1, L_2, L_3, S_1, S_2 in (12.26)–12.31, we can obtain the rotation curve $V_h(r)$ of various shapes. There are parameter values, for example, at $L_3 \neq 0$, that create regions in space where the gravitational force is directed away from the center.

As an example, let us consider one of two classes of solutions without a singularity at the zero point of the rotation curve at

$$X_0 = 0,\ \ L_3 = 0,\ \ S_2 = 0,\ \ X_1 = 4\widetilde{\Lambda}\left(2 + 2\widetilde{\delta} + \widetilde{\varepsilon}\right)\left(12 + 6\widetilde{\delta} - 8\widetilde{\Lambda} + 3\widetilde{\varepsilon}\right)^{-1} M_1,\ \ L_1 = -\left(4 + 2\widetilde{\delta} + \widetilde{\varepsilon}\right) M_1, \tag{12.33}$$

which is consistent with (7.23). In this case, according to (7.16), (7.22), (12.23), (12.26)–12.32, the square of the rotation speed is expressed as follows:

$$V_h^2\left(r\right) = c^2\left(W_4 L_2^2 r^4 + 2S_1^2 r^2 + W_1 M_1 L_2 r\right), \tag{12.34}$$

$$W_4 = 27\left(12\widetilde{\Lambda}\right)^{-4}\left(12 + 6\widetilde{\delta} - 8\widetilde{\Lambda} + 3\widetilde{\varepsilon}\right)^4\left(48\left(132\widetilde{\delta}^2 + 737\widetilde{\delta} + 998\right)\widetilde{\Lambda}^2 - \right.$$

$$\left. -132\left(67 + 24\widetilde{\delta}\right)\left(4 + 2\widetilde{\delta} + \widetilde{\varepsilon}\right)^2\widetilde{\Lambda} + 396\left(4 + 2\widetilde{\delta} + \widetilde{\varepsilon}\right)^4\right), \tag{12.35}$$

$$W_1 = 2\left(4\widetilde{\Lambda}\right)^{-3}\left(12 + 6\widetilde{\delta} - 8\widetilde{\Lambda} + 3\widetilde{\varepsilon}\right)^2\left(4 + 2\widetilde{\delta} + \widetilde{\varepsilon}\right)\left(3\widetilde{\varepsilon}^2 + 6\left(4 + 2\widetilde{\delta} - \widetilde{\Lambda}\right)\widetilde{\varepsilon} + \right.$$

$$\left. +4\left(2 + \widetilde{\delta}\right)\left(6 + 3\widetilde{\delta} - 4\widetilde{\Lambda}\right)\right)\left(228\widetilde{\delta}\left(4 + \widetilde{\delta} - \widetilde{\Lambda} + \widetilde{\varepsilon}\right) + 57\widetilde{\varepsilon}^2 + 8\left(114 - 68\widetilde{\Lambda} + 57\widetilde{\varepsilon}\right)\right). \tag{12.36}$$

In the case off $S_1 = 0$, $W_1 M_1 L_2 > 0$ and $\left|W_4 L_2\right| \ll \left|W_1 M_1\right| R_s^{-3}$, where R_s is the radius of the halo, the rotation speed increases proportionally to $\sqrt{r}$, which approximately corresponds to observations of many galaxies [4]. If the second term in parentheses dominates in (12.34), then $V_h \sim r$, which is observed, for example, at the galaxy NGC 2976 [4].

13. The degenerate case

Secondary inflation

Consider the case off

$$\widetilde{\Lambda} = 0. \tag{13.1}$$

By substituting (13.1) into (7.14), we get

$$\left(4\widetilde{\Phi} + 2\widetilde{\delta} + \widetilde{\varepsilon}\right)^2 = 0. \tag{13.2}$$

Hence, we accept

$$\widetilde{\varepsilon} = -2\left(2\widetilde{\Phi} + \widetilde{\delta}\right). \tag{13.3}$$

The equations (7.12) and (7.13), after substituting (13.1), (13.3) into them, become equivalent to the one equation

$$\left(7\widetilde{\Phi} + \widetilde{\alpha} + 3\widetilde{\delta}\right)\left(U + \lambda V\right) = 0. \tag{13.4}$$

First, consider the case off

$$7\widetilde{\Phi} + \widetilde{\alpha} + 3\widetilde{\delta} \neq 0, \tag{13.5}$$

and the case off $7\widetilde{\Phi} + \widetilde{\alpha} + 3\widetilde{\delta} = 0$ is considered in the section (14). According to (13.4) and (13.5), we accept

$$U = -\lambda V. \tag{13.6}$$

The substitution (7.11), (13.1), (13.3), (13.6) into (7.10) gives

$$\widetilde{T}_0^0 = -9\widetilde{\Phi}\lambda^2 V^2. \tag{13.7}$$

Hence, from $\widetilde{T}_0^0 < 0$ the equation (7.22) follows, by virtue of which (13.5) is written as

$$7 + \widetilde{\alpha} + 3\widetilde{\delta} \neq 0. \tag{13.8}$$

Primary inflation

The substitution (7.22), (8.1), (8.2), (13.1), (13.3) into (7.5), (7.6) leads to the equations

$$4\left(7+\widetilde{\alpha}+3\widetilde{\delta}\right)(U-V)\lambda^2+3\eta\left(V\left(-\frac{31}{3}-\frac{4\widetilde{\alpha}}{3}-4\widetilde{\delta}\right)+U\right)\lambda+\eta^2(U-V)=0, \tag{13.9}$$

$$-8\left(7+\widetilde{\alpha}+3\widetilde{\delta}\right)(U-V)\lambda^2-4\eta\left(\left(-3\widetilde{\alpha}-9\widetilde{\delta}-\frac{81}{4}\right)V+U\left(\widetilde{\alpha}+3\widetilde{\delta}+\frac{25}{4}\right)\right)\lambda+$$
$$+\left(\left(27+4\alpha+12\widetilde{\delta}\right)V+U\right)\eta^2=0. \tag{13.10}$$

They can be written as the product of a square matrix and the column $(U,\ V)^T$ is equal to zero. Since the column is non-zero, the determinant of the specified matrix must be zero:

$$\left(7+\widetilde{\alpha}+3\widetilde{\delta}\right)\eta^2(\eta+3\lambda)^2=0. \tag{13.11}$$

By virtue of (13.8), this is equivalent to (8.6). The equations (13.9) and (13.10) after the substitution (8.7) into them become equivalent to the one equation

$$\left(7+\widetilde{\alpha}+3\widetilde{\delta}\right)(U+2V)=0. \tag{13.12}$$

According to (13.8) and (13.12), we accept

$$U=-2V. \tag{13.13}$$

The substitution (7.22), (8.1), (8.2), (8.7), (13.1), (13.3), (13.13) into (7.7), (7.8) leads to the equations

$$\widetilde{T}_0^0=\frac{405}{8}V^2\lambda^2\exp(-6\lambda t) \tag{13.14}$$

and (8.12).

Solutions in the early stages of the Universe's evolution

The substitution (7.22), (9.1), (13.1), (13.3) into (7.5), (7.6) leads to the equations

$$\left(7+\widetilde{\alpha}+3\widetilde{\delta}\right)(u_0-v_0)\mu^2+\left(\left(2-\left(\widetilde{\alpha}+3\widetilde{\delta}+\frac{31}{4}\right)\chi\right)v_0+\frac{3}{4}u_0\chi-2u_0\right)\mu+\frac{\chi}{4}(\chi-1)(u_0-v_0)=0, \tag{13.15}$$

$$\left(\left(5-3\left(\frac{27}{4}+\widetilde{\alpha}+3\widetilde{\delta}\right)\chi+3\widetilde{\delta}+\widetilde{\alpha}\right)v_0+\left(\left(\widetilde{\alpha}+3\widetilde{\delta}+\frac{25}{4}\right)\chi-5-\widetilde{\alpha}-3\widetilde{\delta}\right)u_0\right)\frac{\mu}{2}+$$
$$+\left(7+\widetilde{\alpha}+3\widetilde{\delta}\right)(u_0-v_0)\mu^2-\frac{\chi}{8}\left(\left(4\widetilde{\alpha}+12\widetilde{\delta}+27\right)v_0+u_0\right)(\chi-1)=0. \tag{13.16}$$

They can be written as the product of a square matrix and the column $(u_0,\ v_0)^T$ is equal to zero. Since the column is non-zero, the determinant of the specified matrix must be zero:

$$\left(7+\widetilde{\alpha}+3\widetilde{\delta}\right)\chi(\chi+3\mu-1)\left(\chi^2+3\mu\chi-\chi-8\mu\right)=0. \tag{13.17}$$

By virtue of (13.8), this is equivalent to (9.5).

Solution of the dark field equation in the Friedmann–Lemaitre–Robertson–Walker metric

The equation system (7.5), (7.6) for (7.22), (13.1), (13.3) admits the solution of the form

$$u(t)=4\int_0^t\left(7a(\tau)^{-9}\dot{a}(\tau)n(\tau)-w(\tau)\right)d\tau+C \tag{13.18}$$

and (10.2), where $n(\tau)$ and $w(\tau)$ are defined by the equations (10.3), (10.4). For (10.5), by virtue of (13.18), (10.2), (10.3), the functions $u(t)$ and $v(t)$ have the form (9.1). By substituting (7.22), (10.2), (10.3), (13.1), (13.3), (13.18) into (7.7) and (7.8), we get (10.8) and (10.9).

Galactic dark field halo

The substitution (7.22), (13.1), (13.3) into (12.15) gives

$$\widetilde{\alpha}\left(7+\widetilde{\alpha}+3\widetilde{\delta}\right)x^2\left(x+1\right)^2\left(x-2\right)\left(x+3\right)=0. \tag{13.19}$$

Hence, by virtue of (12.13) and (13.8), the desired degree x is equal to one of the values 0, -1, 2, -3. Now we can write the solution of the equation system (12.2)–(12.4) for (7.22), (13.1), (13.3) in the general form:

$$X\left(r\right)=\left(5+\widetilde{\alpha}+2\widetilde{\delta}\right)X_1r^{-1}-5L_2r^2+X_0+C, \tag{13.20}$$

$$Y\left(r\right)=-\left(2+\widetilde{\delta}\right)X_1r^{-1}+2L_1r^{-1}+\left(1+12\widetilde{\alpha}\right)L_2r^2+2L_3r^{-3}+C, \tag{13.21}$$

$$Z\left(r\right)=-\left(2+\widetilde{\delta}\right)X_1r^{-1}-\left(3+2\widetilde{\delta}+\widetilde{\alpha}\right)L_1r^{-1}+2\left(1+2\widetilde{\alpha}\right)L_2r^2-L_3r^{-3}+C. \tag{13.22}$$

The substitution (7.22), (12.11), (13.1), (13.3), (13.20)–(13.22) into (12.7)–(12.9) yields (12.20) and (12.21). The substitution (7.22), (12.6), (12.7), (12.11), (13.1), (13.3), (13.20)–(13.22) into (12.24) yields (12.26), where

$$Q_3=6\left(44\widetilde{\alpha}^2-121\widetilde{\alpha}-9\right)L_2^2, \tag{13.23}$$

$$Q_1=4S_1^2+10\left(1+7\widetilde{\alpha}\right)X_0L_2, \tag{13.24}$$

$$Q_0=2\left(2\left(3\widetilde{\alpha}^2+16\widetilde{\alpha}+1\right)\left(7+\widetilde{\alpha}+3\widetilde{\delta}\right)X_1+\left(45\widetilde{\alpha}^2+114\widetilde{\alpha}\widetilde{\delta}+212\widetilde{\alpha}-18\widetilde{\delta}-49\right)L_1\right)L_2, \tag{13.25}$$

$$Q_{-3}=\left(\left(7+\widetilde{\alpha}+3\widetilde{\delta}\right)X_1-2L_1\right)\left(\left(1-\widetilde{\alpha}\right)\left(7+\widetilde{\alpha}+3\widetilde{\delta}\right)X_1+4\left(2+\widetilde{\alpha}+\widetilde{\delta}\right)L_1\right), \tag{13.26}$$

$$Q_{-5}=2\left(\left(\widetilde{\alpha}-1\right)\left(7+\widetilde{\alpha}+3\widetilde{\delta}\right)X_1-2\left(19+5\widetilde{\alpha}+8\widetilde{\delta}\right)L_1\right)L_3+2\left(3\widetilde{\alpha}-1\right)S_2^2. \tag{13.27}$$

As an example, let us consider one of two classes of solutions without a singularity at the zero point of the rotation curve at

$$X_0=0,\ \ L_3=0,\ \ S_2=0,\ \ X_1=2\left(7+\widetilde{\alpha}+3\widetilde{\delta}\right)^{-1}L_1, \tag{13.28}$$

which is consistent with (13.8). In this case, according to (12.23), (12.26), 12.32, (13.23)–(13.27), the square of the rotation speed is expressed as follows:

$$V_h^2\left(r\right)=c^2\left(W_4L_2^2r^4+2S_1^2r^2+W_1L_1L_2r\right), \tag{13.29}$$

$$W_4=3\left(44\widetilde{\alpha}^2-121\widetilde{\alpha}-9\right),\ \ W_1=3\left(5+\widetilde{\alpha}+2\widetilde{\delta}\right)\left(19\widetilde{\alpha}-3\right). \tag{13.30}$$

In the case off $S_1=0$, $W_1L_1L_2>0$ and $|W_4L_2|\ll|W_1L_1|\,R_s^{-3}$, where R_s is the radius of the halo, the rotation speed increases proportionally to $\sqrt{r}$. If the second term in parentheses dominates in (13.29), then $V_h\sim r$.

14. The super-degenerate case

Secondary inflation

Consider the case off

$$\widetilde{\Lambda}=0,\ \ \widetilde{\varepsilon}=-2\left(2\widetilde{\Phi}+\widetilde{\delta}\right),\ \ \widetilde{\alpha}=-7\widetilde{\Phi}-3\widetilde{\delta}, \tag{14.1}$$

in which, taking into account (7.4), the equation (6.10) is performed, as the result of which the dark field equation (3.6) and the Einstein equation (4.15) are invariant with respect to the transformation (6.2). We define the function $v\left(t\right)$ as

$$v\left(t\right)=u\left(t\right)+b\left(t\right). \tag{14.2}$$

The equations (7.5) and (7.6), after substituting (14.1), (14.2) into them, become equivalent to the one equation

$$\widetilde{\Phi}\left(a^2\ddot{b}+3\dot{a}a\dot{b}+8\ddot{a}ba-8\dot{a}^2b\right)=0. \tag{14.3}$$

The substitution (14.1), (14.2) into (7.7), (7.8) gives

$$\widetilde{T}_0^0=-\frac{3\widetilde{\Phi}}{8a^2}\left(24\dot{a}^2b^2+16\dot{a}a\dot{b}b+a^2\dot{b}^2\right), \tag{14.4}$$

$$\widetilde{T}_r^r=\widetilde{T}_\theta^\theta=\widetilde{T}_\varphi^\varphi=-\frac{2\widetilde{\Phi}}{a^2}\left(\ddot{b}ba^2+\frac{13}{16}a^2\dot{b}^2+3\ddot{a}b^2a+6\dot{a}a\dot{b}b+\frac{3}{2}\dot{a}^2b^2\right). \tag{14.5}$$

In the case off (7.9), the equation (14.4) is written as

$$\widetilde{T}_0^0=-\frac{3}{8}\widetilde{\Phi}\left(24\lambda^2b^2+16\lambda b\dot{b}+\dot{b}^2\right). \tag{14.6}$$

Note that according to (14.6), the value off $\widetilde{T}_0^0$ is constant if

$$b\left(t\right)=V. \tag{14.7}$$

With (7.9) and (14.7), the equation (14.3) is performed. By substituting (14.7) into (14.6), we get

$$\widetilde{T}_0^0=-9\widetilde{\Phi}\lambda^2V^2. \tag{14.8}$$

Hence, from $\widetilde{T}_0^0<0$ it follows (7.22).

Primary inflation

Search the function $b\left(t\right)$ in the exponential form

$$b\left(t\right)=V\exp\left(\eta t\right),\ \ V\neq 0. \tag{14.9}$$

By substituting (7.22), (8.1), (14.9) into (14.3), we get

$$\eta\left(\eta+3\lambda\right)=0. \tag{14.10}$$

The case off $\eta=0$ corresponds to the secondary inflation discussed in the previous subsection, hence from (14.10) we arrive at (8.7). The substitution (7.22), (8.1), (8.7), (14.9) into (14.4), (14.5) leads to the equations

$$\widetilde{T}_0^0=\frac{45}{8}V^2\lambda^2\exp\left(-6\lambda t\right) \tag{14.11}$$

and (8.12).

Solutions in the early stages of the Universe's evolution

Let's define $a\left(t\right)$ and $b\left(t\right)$ as the power functions

$$a\left(t\right)=\widetilde{a}_0t^\mu,\ \ b\left(t\right)=b_0t^\chi,\ \ \widetilde{a}_0>0,\ \ b_0\neq 0. \tag{14.12}$$

The substitution (7.22), (14.12) into (14.3) yields the equation

$$\chi^2+3\mu\chi-\chi-8\mu=0, \tag{14.13}$$

which is equivalent to (9.5) at (9.8).

Solution of the dark field equation in the Friedmann–Lemaitre–Robertson–Walker metric

The equation (14.3) for (7.22) admits the solution of the form

$$b(t) = 4a(t)^{-8} n(t), \tag{14.14}$$

where the function $n(t)$ is defined by the equations (10.3), (10.4). For (10.5), by virtue of (10.3), (14.14), the function $b(t)$ has the form (14.12). By substituting (7.22), (10.3), (14.14) into (14.4) and (14.5), we get (10.8) and (10.9).

Galactic dark field halo

The general solution of the equation system (12.2)–(12.4) for (7.22), (14.1) will be sought as a linear combination of power functions. To find out which degrees to use, consider functions of the form

$$Y(r) = X(r) + Y_0 r^x, \;\; Z(r) = X(r) + Z_0 r^x, \tag{14.15}$$

where at least one of the constants Y_0, Z_0 is not equal to zero. The substitution (7.22), (14.1), (14.15) into (12.2) and (12.4) can be written as the product of a square matrix and the column $(Y_0, \;\; Z_0)^T$ is equal to zero. Since the column is non-zero, the determinant of the specified matrix must be zero:

$$\widetilde{\alpha} x (x+1)(x-2)(x+3) = 0. \tag{14.16}$$

Hence, by virtue of (12.13), the desired degree x is equal to one of the values 0, -1, 2, -3. Now we can write the solution of the equation system (12.2)–(12.4) for (7.22), (14.1) in the general form:

$$Y(r) = X(r) + 2L_1 r^{-1} + 2\left(13 + 6\widetilde{\delta}\right) L_2 r^2 + 2L_3 r^{-3} - X_0, \tag{14.17}$$

$$Z(r) = X(r) + \left(4 + \widetilde{\delta}\right) L_1 r^{-1} + \left(7 + 4\widetilde{\delta}\right) L_2 r^2 - L_3 r^{-3} - X_0. \tag{14.18}$$

The substitution (7.22), (12.11), (14.1), (14.17), (14.18) into (12.7)–(12.9) yields (12.20) and (12.21). By substituting (7.22), (12.6), (12.7), (12.11), (14.1), (14.17), (14.18) into (12.24), we get (12.26), where

$$Q_3 = 2\left(132\widetilde{\delta}^2 + 737\widetilde{\delta} + 998\right) L_2^2, \tag{14.19}$$

$$Q_1 = 4S_1^2 + \left(160 + 70\widetilde{\delta}\right) X_0 L_2, \tag{14.20}$$

$$Q_0 = -2\left(14 + 3\widetilde{\delta}\right)\left(16 + 7\widetilde{\delta}\right) L_1 L_2, \tag{14.21}$$

$$Q_{-3} = 8\left(5 + 2\widetilde{\delta}\right) L_1^2, \tag{14.22}$$

$$Q_{-5} = \left(64 + 28\widetilde{\delta}\right) L_1 L_3 - \left(44 + 18\widetilde{\delta}\right) S_2^2. \tag{14.23}$$

It follows from (12.23), (12.26), (14.21), (14.22) that an increase in rotation speed proportional to $\sqrt{r}$ is possible only if

$$\widetilde{\delta} = -\frac{5}{2}. \tag{14.24}$$

In this case, as an example, let us consider the class of solutions without a singularity at the zero point of the rotation curve at

$$X_0 = 0, \;\; L_3 = 0, \;\; S_2 = 0. \tag{14.25}$$

According to (12.23), (12.26), 12.32, (14.19)–(14.25), the square of the rotation speed is expressed as (13.29), where

$$W_4 = -\frac{39}{2}, \;\; W_1 = \frac{39}{4}. \tag{14.26}$$

15. Linear scalar perturbations of the dark field in the synchronous gauge

In order to comply with the article [5], the metric signature $(-\ +\ +\ +)$ is accepted in this section, the Latin letters i, j, k, ... denote three-dimensional spatial indexes, the speed of light is set equal to $c = 1$. Also, the conformal time τ is used as the zero coordinate, which is determined by the equation

$$d\tau = dt/a, \tag{15.1}$$

and the dot above the letter means differentiation by τ.

According to (15.1), when passing from t to τ, the equations (10.4) and (11.8) can be written as

$$\dot{W} = 6BW - 40B^2N, \ \ \dot{N} = W - 8BN, \tag{15.2}$$

where

$$B \equiv \dot{a}/a, \ \ W \equiv aw, \ \ N \equiv a^{-8}n. \tag{15.3}$$

Consider the perturbed Friedmann–Lemaitre–Robertson–Walker metric in the Cartesian coordinates x^1, x^2, x^3 for the flat Universe [5]:

$$g_{00} = -a\left(\tau\right)^2, \tag{15.4}$$

$$g_{ij} = a\left(\tau\right)^2\left(\delta_{ij} + \int d^3k e^{i\vec{k}\cdot\vec{x}}\left\{\hat{k}_i\hat{k}_j h\left(\vec{k},\tau\right) + \left(\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij}\right)6\eta\left(\vec{k},\tau\right)\right\}\right), \tag{15.5}$$

where $h\left(\vec{k},\tau\right)$ and $\eta\left(\vec{k},\tau\right)$ are the Fourier images of the small scalar perturbations of the metric tensor in the synchronous gauge, $\vec{k}$ is the wave vector, $\hat{k}_j \equiv k^j/k$. Let's define the perturbed dark field tensor in the similar form:

$$f_{00} = -a\left(\tau\right)^2\left(u\left(\tau\right) + \int d^3k e^{i\vec{k}\cdot\vec{x}}\widetilde{x}\left(\vec{k},\tau\right)\right), \tag{15.6}$$

$$f_{0j} = f_{j0} = a\left(\tau\right)^2\int d^3k e^{i\vec{k}\cdot\vec{x}}\hat{k}_j i\widetilde{m}\left(\vec{k},\tau\right), \tag{15.7}$$

$$f_{ij} = a\left(\tau\right)^2\left(v\left(\tau\right)\delta_{ij} + \int d^3k e^{i\vec{k}\cdot\vec{x}}\left\{\hat{k}_i\hat{k}_j\widetilde{y}\left(\vec{k},\tau\right) + \left(\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij}\right)6\widetilde{z}\left(\vec{k},\tau\right)\right\}\right), \tag{15.8}$$

where $\widetilde{x}\left(\vec{k},\tau\right)$, $\widetilde{y}\left(\vec{k},\tau\right)$, $\widetilde{z}\left(\vec{k},\tau\right)$, $\widetilde{m}\left(\vec{k},\tau\right)$ are the Fourier images of the small scalar perturbations of the dark field tensor. Let's write the perturbed EMT of the dark field as

$$\widetilde{T}^{\mu}_{\nu} = \widetilde{\overline{T}}^{\mu}_{\nu} + \int d^3k e^{i\vec{k}\cdot\vec{x}}\delta\widetilde{T}^{\mu}_{\nu}\left(\vec{k},\tau\right), \tag{15.9}$$

where $\delta\widetilde{T}^{\mu}_{\nu}\left(\vec{k},\tau\right)$ is the Fourier image of the small perturbation of the dark field EMT, $\widetilde{\overline{T}}^{\mu}_{\nu}$ is the unperturbed EMT of the dark field, the elements of which, according to (10.8), (10.9), (15.1)–(15.3), taking into account the metric signature change, are equal to

$$\widetilde{\overline{T}}^{0}_{0} = a^{-2}\left(6W^2 - 240B^2N^2\right), \tag{15.10}$$

$$\widetilde{\overline{T}}^{1}_{1} = \widetilde{\overline{T}}^{2}_{2} = \widetilde{\overline{T}}^{3}_{3} = a^{-2}\left(\left(1200B^2 - 160\dot{B}\right)N^2 + \left(26W - 320BN\right)W\right). \tag{15.11}$$

Using the DifferentialGeometry software package of the Maple computer mathematics system, linearize[3] (3.6) and (4.16) by small perturbations, substituting (7.4), (7.22), (15.4)–(15.8) into them:

$$\widetilde{XX}\left(k, \widetilde{x}, \widetilde{y}, \widetilde{z}, \widetilde{m}, h, \eta, u, v, a, \dot{\widetilde{x}}, \dot{\widetilde{y}}, \dot{\widetilde{m}}, \dot{h}, \dot{u}, \dot{v}, \dot{a}, \ddot{\widetilde{x}}, \ddot{\widetilde{y}}, \ddot{h}, \ddot{v}, \ddot{a}\right) = 0, \tag{15.12}$$

[3] File scalar-perturbations.mw from the repository https://github.com/alegorov/dark-field

$$\widetilde{MM}\left(k,\widetilde{x},\widetilde{y},\widetilde{z},\widetilde{m},h,\eta,u,v,a,\dot{\widetilde{x}},\dot{\widetilde{y}},\dot{\widetilde{z}},\dot{\widetilde{m}},\dot{h},\dot{\eta},\dot{v},\dot{a},\ddot{\widetilde{m}},\ddot{a}\right)=0, \tag{15.13}$$

$$\widetilde{YY}\left(k,\widetilde{x},\widetilde{y},\widetilde{z},\widetilde{m},h,\eta,u,v,a,\dot{\widetilde{y}},\dot{\widetilde{z}},\dot{\widetilde{m}},\dot{h},\dot{\eta},\dot{u},\dot{v},\dot{a},\ddot{\widetilde{y}},\ddot{\widetilde{z}},\ddot{h},\ddot{\eta},\ddot{v},\ddot{a}\right)=0, \tag{15.14}$$

$$\widetilde{ZZ}\left(k,\widetilde{x},\widetilde{y},\widetilde{z},\widetilde{m},h,\eta,u,v,a,\dot{\widetilde{x}},\dot{\widetilde{y}},\dot{\widetilde{z}},\dot{\widetilde{m}},\dot{h},\dot{\eta},\dot{u},\dot{v},\dot{a},\ddot{\widetilde{x}},\ddot{\widetilde{y}},\ddot{\widetilde{z}},\ddot{h},\ddot{\eta},\ddot{v},\ddot{a}\right)=0, \tag{15.15}$$

$$\delta\widetilde{T}_0^0=\widetilde{X}\left(k,\widetilde{x},\widetilde{y},\widetilde{z},\widetilde{m},h,\eta,u,v,a,\dot{\widetilde{x}},\dot{\widetilde{y}},\dot{\widetilde{m}},\dot{h},\dot{u},\dot{v},\dot{a}\right), \tag{15.16}$$

$$\delta\widetilde{T}_j^0=-\delta\widetilde{T}_0^j=k^j\widetilde{M}\left(k,\widetilde{x},\widetilde{y},\widetilde{z},\widetilde{m},h,\eta,u,v,a,\dot{\widetilde{x}},\dot{\widetilde{y}},\dot{\widetilde{z}},\dot{\widetilde{m}},\dot{h},\dot{\eta},\dot{u},\dot{v},\dot{a},\ddot{\widetilde{m}},\ddot{u},\ddot{v},\ddot{a}\right), \tag{15.17}$$

$$\begin{aligned}\delta\widetilde{T}_j^i=\;&k^ik^j\widetilde{Y}\left(k,\widetilde{y},\widetilde{z},\widetilde{m},h,\eta,u,v,a,\dot{\widetilde{y}},\dot{\widetilde{z}},\dot{\widetilde{m}},\dot{h},\dot{\eta},\dot{u},\dot{v},\dot{a},\ddot{\widetilde{y}},\ddot{\widetilde{z}},\ddot{h},\ddot{\eta},\ddot{u},\ddot{v},\ddot{a}\right)+\\&+\delta_{ij}\widetilde{Z}\left(k,\widetilde{x},\widetilde{y},\widetilde{z},\widetilde{m},h,\eta,u,v,a,\dot{\widetilde{x}},\dot{\widetilde{y}},\dot{\widetilde{z}},\dot{\widetilde{m}},\dot{h},\dot{\eta},\dot{u},\dot{v},\dot{a},\ddot{\widetilde{x}},\ddot{\widetilde{y}},\ddot{\widetilde{z}},\ddot{h},\ddot{\eta},\ddot{u},\ddot{v},\ddot{a}\right),\end{aligned} \tag{15.18}$$

where the expressions for $\widetilde{XX}$, $\widetilde{MM}$, $\widetilde{YY}$, $\widetilde{ZZ}$, $\widetilde{X}$, $\widetilde{M}$, $\widetilde{Y}$, $\widetilde{Z}$ are not given in this article due to their bulkiness, but in the file according to the last above-mentioned footnote. To exclude $\ddot{h}\left(\overrightarrow{k},\tau\right)$ and $\ddot{\eta}\left(\overrightarrow{k},\tau\right)$ from (15.12)–(15.17), move from $\widetilde{x}\left(\overrightarrow{k},\tau\right)$, $\widetilde{y}\left(\overrightarrow{k},\tau\right)$, $\widetilde{z}\left(\overrightarrow{k},\tau\right)$ to $x\left(\overrightarrow{k},\tau\right)$, $y\left(\overrightarrow{k},\tau\right)$, $z\left(\overrightarrow{k},\tau\right)$ using the following substitutions:

$$\begin{aligned}\widetilde{x}=\;&\frac{1}{2\left(12+6\widetilde{\delta}-8\widetilde{\Lambda}+3\widetilde{\varepsilon}\right)^2}\left(-84\left(\frac{\widetilde{\varepsilon}}{2}+2-\frac{4\widetilde{\Lambda}}{3}+\widetilde{\delta}\right)\left(\frac{\widetilde{\varepsilon}}{2}+2-\frac{8\widetilde{\Lambda}}{7}+\widetilde{\delta}\right)(u-v)\,h+\right.\\&\left.+\left(-192\widetilde{\Lambda}^2+\widetilde{\Lambda}\left(384\widetilde{\delta}+192\widetilde{\varepsilon}+784\right)-192\left(\frac{\widetilde{\varepsilon}}{2}+2+\widetilde{\delta}\right)^2\right)x-192\widetilde{\Lambda}\left(\widetilde{\delta}-\widetilde{\Lambda}+\frac{\widetilde{\varepsilon}}{2}+\frac{7}{4}\right)z\right),\end{aligned} \tag{15.19}$$

$$\begin{aligned}\widetilde{y}=\;&\frac{6}{\left(12+6\widetilde{\delta}-8\widetilde{\Lambda}+3\widetilde{\varepsilon}\right)^2}\left(3\left(\frac{\widetilde{\varepsilon}}{2}+2-\frac{4\widetilde{\Lambda}}{3}+\widetilde{\delta}\right)\left(\left(\widetilde{\delta}-\frac{8\widetilde{\Lambda}}{3}+\frac{\widetilde{\varepsilon}}{2}+2\right)u+\right.\right.\\&\left.\left.+\left(\frac{\widetilde{\varepsilon}}{2}+2+\widetilde{\delta}\right)v\right)h-16\widetilde{\Lambda}\left(\left(\widetilde{\delta}-\widetilde{\Lambda}+\frac{\widetilde{\varepsilon}}{2}+\frac{7}{4}\right)x+\left(\widetilde{\Lambda}-\frac{3}{4}\right)z\right)\right),\end{aligned} \tag{15.20}$$

$$\begin{aligned}\widetilde{z}=\;&\frac{12}{\widetilde{\alpha}\widetilde{\Lambda}\left(12+6\widetilde{\delta}-8\widetilde{\Lambda}+3\widetilde{\varepsilon}\right)^2}\left(\frac{9}{32}\left(4+2\widetilde{\delta}-\frac{8\widetilde{\Lambda}}{3}+\widetilde{\varepsilon}\right)^2\times\right.\\&\times\left(\left(\widetilde{\Lambda}\left(-4\widetilde{\delta}-\frac{32}{3}\right)+\left(4+2\widetilde{\delta}+\widetilde{\varepsilon}\right)^2\right)u+\left(\widetilde{\Lambda}\left(-4\widetilde{\delta}-8\right)+\left(4+2\widetilde{\delta}+\widetilde{\varepsilon}\right)^2\right)v\right)\eta+\\&+\frac{\widetilde{\Lambda}}{8}\left(4+2\widetilde{\delta}-\frac{8\widetilde{\Lambda}}{3}+\widetilde{\varepsilon}\right)\left(\widetilde{\Lambda}\left(-4\widetilde{\delta}-8\right)+\left(\widetilde{\varepsilon}+2\widetilde{\delta}+\frac{7}{2}\right)\left(4+2\widetilde{\delta}+\widetilde{\varepsilon}\right)\right)(u-v)\,h+\\&+\left(\widetilde{\Lambda}\left(-4\widetilde{\delta}-\frac{28}{3}\right)+\left(4+2\widetilde{\delta}+\widetilde{\varepsilon}\right)^2\right)\times\\&\left.\times\left(\frac{3}{32}\left(4+2\widetilde{\delta}-\frac{8\widetilde{\Lambda}}{3}+\widetilde{\varepsilon}\right)^2y+\widetilde{\Lambda}\left(\left(\widetilde{\delta}-\widetilde{\Lambda}+\frac{\widetilde{\varepsilon}}{2}+\frac{7}{4}\right)x+\left(\widetilde{\Lambda}-\frac{3}{4}\right)z\right)\right)\right)\end{aligned} \tag{15.21}$$

in the non-degenerate case;

$$\widetilde{x}=-\frac{7}{6}(u-v)\,h+\frac{6z-2\left(27+4\widetilde{\alpha}+12\widetilde{\delta}\right)x}{3\left(7+\widetilde{\alpha}+3\widetilde{\delta}\right)}, \tag{15.22}$$

$$\widetilde{y}=\frac{1}{2}(u+v)\,h+\frac{2(x+3z)}{7+\widetilde{\alpha}+3\widetilde{\delta}}, \tag{15.23}$$

$$\widetilde{z}=\frac{1}{12\widetilde{\alpha}}\left(6\left(\left(\widetilde{\alpha}-1\right)u+\left(\widetilde{\alpha}+1\right)v\right)\eta-\left(u-v\right)h+2\widetilde{\alpha}\left(y-\frac{2\left(x+3z\right)}{7+\widetilde{\alpha}+3\widetilde{\delta}}\right)\right) \tag{15.24}$$

in the degenerate case;

$$\widetilde{x}=\frac{1}{2\widetilde{\alpha}}\left(7\widetilde{\delta}+16\right)\left(u-v\right)h+z, \tag{15.25}$$

$$\widetilde{y}=-\frac{1}{2\widetilde{\alpha}}\left(\left(\left(3\widetilde{\delta}+8\right)u+3\left(\widetilde{\delta}+2\right)v\right)h+16\left(7+3\widetilde{\delta}\right)x\right)+3z, \tag{15.26}$$

$$\widetilde{z}=-\frac{1}{2\widetilde{\alpha}}\left(\left(3\widetilde{\delta}+8\right)u+3\left(\widetilde{\delta}+2\right)v\right)\eta+\frac{1}{6}y-\frac{4}{3}x-\frac{1}{2}z \tag{15.27}$$

in the super-degenerate case, in which the expressions $\widetilde{XX}$, $\widetilde{YY}$, $\widetilde{ZZ}$ are linearly dependent:

$$\widetilde{ZZ}+\frac{1}{3}\widetilde{XX}+\frac{1}{12}\widetilde{YY}=0, \tag{15.28}$$

and when substituting (15.25)–(15.27), the function $z\left(\overrightarrow{k},\tau\right)$ and its derivatives are completely reduced in all the equations (15.12)–(15.18).

The substitutions (15.19)–(15.27) lead to the following equations:

$$\begin{aligned}\ddot{x}&=B\left(p_1\dot{x}+p_2\dot{z}\right)+p_3k\dot{\widetilde{m}}+\left(p_4\dot{B}+p_5B^2+p_6k^2\right)x+p_7k^2y+\left(p_8\dot{B}+p_9B^2+p_{10}k^2\right)z+\\&+p_{11}kB\widetilde{m}+\left(p_{12}BN+p_{13}W\right)\dot{h}+\left(\left(p_{14}B^2+p_{15}k^2\right)N+p_{16}BW\right)h+p_{17}k^2N\eta,\end{aligned} \tag{15.29}$$

$$\begin{aligned}\ddot{y}&=p_{18}B\dot{y}+p_{19}k\dot{\widetilde{m}}+p_{20}k^2x+\left(p_{21}\dot{B}+p_{22}B^2+p_{23}k^2\right)y+p_{24}k^2z+p_{25}kB\widetilde{m}+\left(p_{26}BN+p_{27}W\right)\times\\&\times\left(\dot{h}+6\dot{\eta}\right)+N\left(\left(p_{28}\dot{B}+p_{29}B^2+p_{30}k^2\right)h+\left(p_{31}\dot{B}+p_{32}B^2+p_{33}k^2\right)\eta\right),\end{aligned} \tag{15.30}$$

$$\begin{aligned}\ddot{z}&=B\left(p_{34}\dot{x}+p_{35}\dot{z}\right)+p_{36}k\dot{\widetilde{m}}+\left(p_{37}\dot{B}+p_{38}B^2+p_{39}k^2\right)x+p_{40}k^2y+\left(p_{41}\dot{B}+p_{42}B^2+p_{43}k^2\right)z+\\&+p_{44}kB\widetilde{m}+\left(p_{45}BN+p_{46}W\right)\dot{h}+\left(\left(p_{47}B^2+p_{48}k^2\right)N+p_{49}BW\right)h+p_{50}k^2N\eta,\end{aligned} \tag{15.31}$$

$$\begin{aligned}\ddot{\widetilde{m}}&=k\left(p_{51}\dot{x}+p_{52}\dot{y}+p_{53}\dot{z}\right)+B\left(p_{54}\dot{\widetilde{m}}+k\left(p_{55}x+p_{56}y+p_{57}z\right)\right)+\left(p_{58}\dot{B}+p_{59}B^2+p_{60}k^2\right)\widetilde{m}+\\&+kN\left(p_{61}\dot{h}+p_{62}\dot{\eta}\right)+k\left(\left(p_{63}BN+p_{64}W\right)h+\left(p_{65}BN+p_{66}W\right)\eta\right),\end{aligned} \tag{15.32}$$

$$\begin{aligned}a^2\widetilde{X}&=W\left(p_{67}\dot{x}+p_{68}\dot{z}\right)+p_{69}kN\dot{\widetilde{m}}+\left(\left(p_{70}B^2+p_{71}k^2\right)N+p_{72}BW\right)x+p_{73}k^2Ny+\\&+\left(\left(p_{74}B^2+p_{75}k^2\right)N+p_{76}BW\right)z+\left(p_{77}BN+p_{78}W\right)k\widetilde{m}+p_{79}BN^2\dot{h}+\\&+\left(\left(p_{80}B^2+p_{81}k^2\right)N^2+p_{82}W^2\right)h+p_{83}k^2N^2\eta,\end{aligned} \tag{15.33}$$

$$\begin{aligned}ia^2\widetilde{M}&=N\left(p_{84}\dot{x}+p_{85}\dot{y}+p_{86}\dot{z}\right)+\left(p_{87}BN+p_{88}W\right)x+\left(p_{89}BN+p_{90}W\right)y+\left(p_{91}BN+p_{92}W\right)z+\\&+p_{93}kN\widetilde{m}+N^2\left(p_{94}\dot{h}+p_{95}\dot{\eta}\right)+N\left(\left(p_{96}BN+p_{97}W\right)h+\left(p_{98}BN+p_{99}W\right)\eta\right)\end{aligned} \tag{15.34}$$

in the non-degenerate case, where the values of the constants p_1–p_{99} are given in the file the-non-degenerate-case.mw from the repository https://github.com/alegorov/dark-field;

$$\begin{aligned}\ddot{x}&=p_1B\left(\dot{x}+\dot{z}\right)+p_2k\dot{\widetilde{m}}+\left(p_3\dot{B}+p_4B^2+p_5k^2\right)x+k^2\left(p_6y+p_7z\right)+\\&+p_8kB\widetilde{m}+\left(p_9BN+p_{10}W\right)\dot{h}+\left(p_{11}k^2N+p_{12}BW\right)h+p_{13}k^2N\eta,\end{aligned} \tag{15.35}$$

$$\begin{aligned}\ddot{y}&=p_{14}B\dot{y}+p_{15}k\dot{\widetilde{m}}+p_{16}k^2x+\left(p_{17}\dot{B}+p_{18}B^2+p_{19}k^2\right)y+p_{20}k^2z+p_{21}kB\widetilde{m}+\left(p_{22}BN+p_{23}W\right)\times\\&\times\left(\dot{h}+6\dot{\eta}\right)+N\left(\left(p_{24}\dot{B}+p_{25}B^2+p_{26}k^2\right)h+\left(p_{27}\dot{B}+p_{28}B^2+p_{29}k^2\right)\eta\right),\end{aligned} \tag{15.36}$$

$$\begin{aligned}\ddot{z}&=B\left(p_{30}\dot{x}+p_{31}\dot{z}\right)+p_{32}k\dot{\widetilde{m}}+\left(p_{33}\dot{B}+p_{34}B^2+p_{35}k^2\right)x+k^2\left(p_{36}y+p_{37}z\right)+\\&+p_{38}kB\widetilde{m}+\left(p_{39}BN+p_{40}W\right)\dot{h}+\left(\left(p_{41}B^2+p_{42}k^2\right)N+p_{43}BW\right)h+p_{44}k^2N\eta,\end{aligned} \tag{15.37}$$

$$\ddot{\widetilde{m}} = k\left(p_{45}\dot{x} + p_{46}\dot{y} + p_{47}\dot{z}\right) + B\left(p_{48}\dot{\widetilde{m}} + k\left(p_{49}x + p_{50}y + p_{51}z\right)\right) + \left(p_{52}\dot{B} + p_{53}B^2 + p_{54}k^2\right)\widetilde{m} +$$
$$+ kN\left(p_{55}\dot{h} + p_{56}\dot{\eta}\right) + k\left(\left(p_{57}BN + p_{58}W\right)h + \left(p_{59}BN + p_{60}W\right)\eta\right), \tag{15.38}$$

$$a^2\widetilde{X} = p_{61}W\dot{x} + p_{62}kN\dot{\widetilde{m}} + \left(\left(p_{63}B^2 + p_{64}k^2\right)N + p_{65}BW\right)x + k^2N\left(p_{66}y + p_{67}z\right) +$$
$$+ \left(p_{68}BN + p_{69}W\right)k\widetilde{m} + p_{70}BN^2\dot{h} + \left(\left(p_{71}B^2 + p_{72}k^2\right)N^2 + p_{73}W^2\right)h + p_{74}k^2N^2\eta, \tag{15.39}$$

$$ia^2\widetilde{M} = N\left(p_{75}\dot{x} + p_{76}\dot{y} + p_{77}\dot{z}\right) + \left(p_{78}BN + p_{79}W\right)x + \left(p_{80}BN + p_{81}W\right)y + p_{82}kN\widetilde{m} +$$
$$+ N^2\left(p_{83}\dot{h} + p_{84}\dot{\eta}\right) + N\left(\left(p_{85}BN + p_{86}W\right)h + \left(p_{87}BN + p_{88}W\right)\eta\right) \tag{15.40}$$

in the degenerate case, where the values of the constants p_1–p_{88} are given in the file the-degenerate-case.mw from the repository https://github.com/alegorov/dark-field;

$$\ddot{x} = p_1B\dot{x} + p_2k\dot{\widetilde{m}} + \left(p_3\dot{B} + p_4B^2 + p_5k^2\right)x + p_6k^2y + p_7kB\widetilde{m} +$$
$$+ \left(p_8BN + p_9W\right)\dot{h} + k^2N\left(p_{10}h + p_{11}\eta\right), \tag{15.41}$$

$$\ddot{y} = p_{12}B\dot{y} + p_{13}k\dot{\widetilde{m}} + p_{14}k^2x + \left(p_{15}\dot{B} + p_{16}B^2 + p_{17}k^2\right)y + p_{18}kB\widetilde{m} + \left(p_{19}BN + p_{20}W\right)\times$$
$$\times\left(\dot{h} + 6\dot{\eta}\right) + N\left(\left(p_{21}\dot{B} + p_{22}B^2 + p_{23}k^2\right)h + \left(p_{24}\dot{B} + p_{25}B^2 + p_{26}k^2\right)\eta\right), \tag{15.42}$$

$$\ddot{\widetilde{m}} = k\left(p_{27}\dot{x} + p_{28}\dot{y}\right) + B\left(p_{29}\dot{\widetilde{m}} + k\left(p_{30}x + p_{31}y\right)\right) + \left(p_{32}\dot{B} + p_{33}B^2 + p_{34}k^2\right)\widetilde{m} +$$
$$+ kN\left(p_{35}\dot{h} + p_{36}\dot{\eta}\right) + k\left(\left(p_{37}BN + p_{38}W\right)h + \left(p_{39}BN + p_{40}W\right)\eta\right), \tag{15.43}$$

$$a^2\widetilde{X} = p_{41}W\dot{x} + p_{42}kN\dot{\widetilde{m}} + \left(\left(p_{43}B^2 + p_{44}k^2\right)N + p_{45}BW\right)x + p_{46}k^2Ny + \left(p_{47}BN + p_{48}W\right)k\widetilde{m} +$$
$$+ p_{49}BN^2\dot{h} + \left(\left(p_{50}B^2 + p_{51}k^2\right)N^2 + p_{52}W^2\right)h + p_{53}k^2N^2\eta, \tag{15.44}$$

$$ia^2\widetilde{M} = N\left(p_{54}\dot{x} + p_{55}\dot{y}\right) + \left(p_{56}BN + p_{57}W\right)x + \left(p_{58}BN + p_{59}W\right)y + p_{60}kN\widetilde{m} +$$
$$+ N^2\left(p_{61}\dot{h} + p_{62}\dot{\eta}\right) + N\left(\left(p_{63}BN + p_{64}W\right)h + \left(p_{65}BN + p_{66}W\right)\eta\right) \tag{15.45}$$

in the super-degenerate case, where the values of the constants p_1–p_{66} are given in the file the-super-degenerate-case.mw from the repository https://github.com/alegorov/dark-field.

It follows from (9.1), (9.11), (15.1) that at the stage of radiation dominance, the following is performed:

$$t \sim \tau^2, \;\; a \sim \tau. \tag{15.46}$$

Hence, according to (10.5), we get

$$w \sim \tau^{2\chi - 2}, \tag{15.47}$$

where the constant χ is defined by the second equation of (9.11). From (9.11), (10.3), (15.1), (15.3), (15.46), (15.47) we arrive at the approximate equations

$$B \simeq \tau^{-1}, \;\; W \simeq W_0\tau^{\sigma}, \;\; N \simeq W_0\left(\sigma + 9\right)^{-1}\tau^{\sigma+1}, \tag{15.48}$$

where

$$\sigma \equiv 2\chi - 1 = \left(\sqrt{65} - 3\right)/2 \approx 2.5312, \tag{15.49}$$

W_0 is the dimensionless coefficient determined from the condition that at the current time $a = 1\,\mathrm{Mpc}$ and $\dot{a} = H_0 \cdot \mathrm{Mpc}^2$. At the stage of radiation dominance, we have [5]

$$h \simeq C\left(k\tau\right)^2, \;\; \eta \simeq 2C. \tag{15.50}$$

By substituting (15.48), (15.50) into (15.29)–(15.45), we get

$$x \sim CW_0\tau^{\sigma+3}, \;\; y \sim CW_0\tau^{\sigma+1}, \;\; z \sim CW_0\tau^{\sigma+3}, \;\; \widetilde{m} \sim CW_0\tau^{\sigma+2}, \tag{15.51}$$

$$a^2\widetilde{X} \sim CW_0^2\tau^{2\sigma+2}, \;\; ia^2\widetilde{M} \sim CW_0^2\tau^{2\sigma+1}, \tag{15.52}$$

which, taking into account (15.16) and (15.17) gives

$$a^2\delta\widetilde{T}_0^0 \sim CW_0^2\tau^{2\sigma+2}, \;\; a^2\delta\widetilde{T}_j^0 \sim -ik^jCW_0^2\tau^{2\sigma+1}. \tag{15.53}$$

Also, at the stage of radiation dominance, we have [5]

$$\delta T_0^0 \sim \rho C\left(k\tau\right)^2, \;\; \delta T_j^0 \sim ik^j\rho Ck^2\tau^3, \tag{15.54}$$

where, by virtue of (7.3), (11.1), (15.1), the approximation is performed:

$$8\pi Ga^2\rho \simeq 3\tau^{-2}. \tag{15.55}$$

It follows from (15.54), (15.55):

$$8\pi Ga^2\delta T_0^0 \sim Ck^2, \;\; 8\pi Ga^2\delta T_j^0 \sim ik^jCk^2\tau. \tag{15.56}$$

Numerical calculations show that

$$W_0 \sim 10^{-17}. \tag{15.57}$$

In the CAMB program for analysis of the anisotropy of the cosmic microwave background, the initial conditions for differential equations are given at

$$10^{-3} \le \tau \le 10^{-1}, \;\; 10^{-6} \le k \le 1, \;\; 10^{-7} \le k\tau \le 10^{-3}. \tag{15.58}$$

With such values of τ and k, by virtue of (15.53), (15.56), (15.57), the following is performed:

$$\left|\delta\widetilde{T}_\nu^0\right| \ll \left|8\pi G\delta T_\nu^0\right|. \tag{15.59}$$

This makes it possible for simplicity to set initial conditions for small dark field perturbations in the trivial form practically with no loss of calculation accuracy:

$$x = y = z = \widetilde{m} = \dot{x} = \dot{y} = \dot{z} = \dot{\widetilde{m}} = 0. \tag{15.60}$$

16. Analysis of the anisotropy of the cosmic microwave background

Changes have been made to the CAMB program[4], according to the previous section. The angular anisotropy spectras TT, TE, EE of the cosmic microwave background in the multipole range $2 \le l \le 2500$ were taken as the analyzed data, which were calculated in the original CAMB program using the parameters obtained from the observational data from the Planck space observatory in [6]. In the modified program, the parameters were selected to minimize the differences between the resulting spectras and the analyzed ones. As a result, the best match of the spectras was with an accuracy of 0.5% in the degenerate case with the following parameters in the notation of [6]:

$$\Omega_bh^2 \approx 0.02238, \;\; \Omega_ch^2 \approx 0.12011, \;\; H_0 \approx 78.67\,(\mathrm{km/s})/\mathrm{Mpc}, \tag{16.1}$$

$$\tau \approx 0.0534, \;\; n_s \approx 0.966, \;\; 10^9A_s \approx 2.097, \tag{16.2}$$

$$\widetilde{\alpha} \approx 1, \;\; \widetilde{\delta} \approx -3. \tag{16.3}$$

There is reason to believe that the equations (16.3) are performed exactly:

$$\widetilde{\alpha} = 1, \;\; \widetilde{\delta} = -3, \tag{16.4}$$

since in this case, by virtue of (13.26), the coefficient Q_{-3} in (12.26) is zero:

$$Q_{-3} = 0, \tag{16.5}$$

[4]The repository https://github.com/alegorov/CAMBdf

as a result, according to (13.23)–(13.27), a class of solutions without a singularity at the zero point of the rotation curve arises with the weaker requirement for the parameters of the halo:

$$L_3 = 0, \;\; S_2 = 0, \tag{16.6}$$

this probably favors the formation of galaxies. Also note that for (16.4), the equation (13.23) takes the form

$$Q_3 = -516 L_2^2. \tag{16.7}$$

This means that at a nonzero parameter L_2, due to the negativity of the coefficient Q_3 at sufficiently large distances, the gravitational force is directed away from the center, which fits well into the overall picture of the expansion of the Universe.

From (7.4), (7.22), (13.1), (13.3), (16.4) we obtain the assumed exact values of all the initial constants of the theory:

$$\widetilde{\alpha} = 1, \;\; \widetilde{\beta} = 0, \;\; \widetilde{\gamma} = -\frac{3}{4}, \;\; \widetilde{\delta} = -3, \;\; \widetilde{\varepsilon} = 2. \tag{16.8}$$

The question of the existence of fundamental causes of such values is the subject of further research.

A recent measurement of the Hubble constant without reference to any cosmological model was made from observations of the James Webb Space Telescope in [7], in which the value $H_0 = 74.7 \pm 3.1$ (km/s)/Mpc was obtained. The H_0 estimate in (16.1) differs from this measured value at the level of 1.3σ when the value $H_0 \approx 67.66$ (km/s)/Mpc obtained within the framework of the standard cosmological model ΛCDM in [6] diverges at the level of 2.3σ. Thus, the proposed theory significantly reduces the Hubble tension. Perhaps, with a direct analysis of the observational data from the Planck space observatory, this tension will disappear completely.

Conclusion

We have considered an important problem in astrophysics and cosmology related to the presence of dark matter and dark energy. Indeed, in the most popular cosmological model, which has cold dark matter and the Λ-term (ΛCDM), there is currently no satisfactory understanding of the nature of dark matter and dark energy. The proposed theory solves this problem to some extent, and the cosmological values derived from it are in good agreement with observations.

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Please cite this article in English as:
Egorov A. A. Dark energy and dark matter as a manifestation of a single metric-like dark field. *Space, Time and Fundamental Interactions*, 0000, no. 0, pp. 1–28.