International Trade, Industrial Concentration, and Welfare

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Abstract

I develop an international trade model of hybrid competition motivated by the market structure of internationally integrated markets. In the model, consumers have CES preferences over two sets of varieties of a good that differ in their utility weights — major and minor varieties. Potential entrants have to pay an entry cost to get a blueprint and sell a variety in a market. Firms that receive blueprints of major varieties have market power, whereas firms that receive blueprints of minor varieties do not. I characterize the relationship between industrial concentration and competition and show that a decrease in trade costs increases domestic concentration of firms with market power. I then close the model and provide a welfare formula that adds three terms to the standard formula in gravity models: (i) a concentration term that captures the markup level and dispersion, (ii) a variety term, and (iii) an excess profits term due to firms' market power. Using novel data from Colombia that includes the actual identity of foreign sellers, I measure the welfare gains from a trade liberalization process that took place over the 2007-2017 period. Welfare calculations suggest an increase in welfare of between 0.12 and 0.43 log points for each log point decrease in tariffs.

Keywords: Gains from Trade, Welfare, Market Concentration, Competition, Market Power, Oligopoly, Market Structure

JEL codes: F10, F12, F13, L11, L13

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1 Introduction

In recent years, the interest in the structure of markets has increased, fueled by evidence showing a rise in market concentration in developed countries.[1] Such increase and evidence showing rising markups were often employed together to argue that overall competition in developed economies deteriorated.[2] However, there is plenty of evidence showing that a different type of competition, the one from abroad, increased and affected different margins over this period.[3] In fact, competition between domestic and foreign firms became a common feature of markets in developed countries.[4]

Has foreign competition modified domestic market structures? How does welfare depend on the structure of markets? Despite all these changes in the structure and composition of markets, standard trade models are usually silent about their competition and welfare effects. In this paper, I propose a trade model of hybrid competition both to stress the role of market structures in shaping competition and welfare gains from trade, and to focus on the different channels through which large and small firms can affect welfare.

Standard trade models with heterogeneous firms assume symmetric behavior between small and large firms. On the one hand, models with monopolistic competition and a continuum of firms gained popularity due to their analytic tractability and explanatory power at the cost of individual firms' sizes variation. On the other hand, models with granular firms assume that all firms have market power, even those with negligible size, at the cost of having to impose additional structure to firm entry. The model I propose maintains both the tractability of monopolistic competitive models and the role of large firms.

I construct an international trade model where consumers have CES preferences over two types of varieties that differ in their utility weights. One set of varieties is fixed and easily recognizable to consumers, which is captured by positive weights in the utility function. I call them *major* varieties. The other set of varieties are not individually recognized by consumers, but they value having access to many of them. I call them *minor* varieties. Origin-specific ex-ante homogeneous firms decide to enter into each market by comparing future expected profits and entry costs. Firms that choose to enter get a blueprint of a variety and produce. Those receiving major varieties have market power, and those receiving minor varieties do not. As a consequence, market power arises due to the type of varieties.

I motivate the model by providing facts that illustrate the market structure of integrated economies. Using firm-level Colombian data that includes both foreign and domestic firms selling in 121 Colombian manufacturing industries, I show that (i) firms with market shares higher than 1% are rare, (2) the size-rank linear relationship characteristic of Zipf's distributions such as Pareto does not hold for top firms, and (3) the probability of exit or entry of large firms is significantly lower than small firms, regardless of their origin.

I use the model to examine the interrelation between industrial concentration, competition and trade costs in international trade. I show that domestic concentration of large firms is negatively related to competition.[5]

[1]e.g. Gutierrez and Philippon (2017), OECD (2018), Bajgar et al. (2019).

[2]e.g. Philippon (2019), De Loecker et al. (2020), Autor et al. (2020a).

[3]e.g. Caliendo and Parro (2014), Acemoglu et al. (2016), Autor et al. (2020b).

[4]e.g Eaton et al. (2009), Caliendo et al. (2015).

[5]A recent paper by Amiti and Heise (2021) show that the increase in US domestic concentration observed in the last decades

Even though this is a feature of granular models in general, I stress the importance of only including firms suspected of market power. Attaching meaning to concentration measures over the entire set of firms does not have a structural interpretation under hybrid competition.[6] In addition, I show that a decrease in trade costs always decreases the CES industry price index, but more so if domestic firms are more concentrated. Intuitively, a reduction in trade costs shifts demand towards the more competitive, lower aggregate markup segment of the market, magnifying the impact of trade costs on the price index.

I solve the model in general equilibrium and derive an expression that extends the Arklokais et al. (2012) welfare formula, ACR henceforth, to include the different mechanisms through which small and large firms affect welfare. Importantly, the formula depends on information that can be directly observed, such as firms' market shares and the number of active firms in a market. Differently from papers that allow for variable markups, such as Arkolakis et al. (2018), the expression depends on the actual and not expected structure of markets. A key result of this approach is that changes in domestic concentration and the number of small firms are sufficient statistics to calculate the overall pro-competitive and variety gains. Moreover, the formula provides a term for the excess profits due to oligopolistic behavior, a channel present in national accounts and important in the context of the ongoing discussions about competition, but frequently dismissed due to free entry assumptions.

I provide a novel model-consistent concentration measure that considers markup asymmetries across large firms from all origins.[7] Changes in this measure capture changes in welfare *due to* changes in markups.[8] I show that this measure is closely related to the widely employed Herfindahl-Hirschman Index (HHI) but (i) it is not bounded above, and (ii) takes into account the importance of the set of firms for which it is calculated. For example, it would take a very low value in a highly concentrated but highly integrated domestic market. The HHI would treat markets with different levels of integration symmetrically.

I measure welfare changes in Colombia during a trade liberalization process that took place over the 2007-2017 period. Over that period of time, Colombia signed free trade agreements with countries representing about 42% of its 2007 imports and reduced average MFN tariffs by about 4 percentage points.[9] During this period, import penetration increased from 53% to 63% in manufacturing sectors. I find that welfare increased in about 0.7 to 2.6 log points when I employ my formula and the elasticity of substitution values used in ACR. Both the excess profits generated by oligopolistic behavior and the increase in the number of small varieties increase gains relative to the ACR term. On the contrary, domestic concentration increased, lowering the gains.

can be explained by an increase in foreign competition.

[6] In models with a continuum of firms and selection, changes in concentration capture changes in the shape of the productivity distribution when entry cutoffs change.

[7] This term resonates with the markup decomposition in Edmond et al. (2021) since it captures both the aggregate and misallocative impact of markups.

[8] In Impullitti et al. (2021), market concentration, as captured by the number of firms, increases in trade liberalization events producing a scale effect that benefit consumers due to a boost in innovation. My model lacks that mechanism and focuses on measurement.

[9] MFN stands for Most Favored Nation, the tariff applied by World Trade Organization members to each other when they do not have a preferential trade agreement in force.

The closest work in terms of welfare measurement in trade is Feenstra and Weinstein (2017). In that paper, they study the impact of globalization on US welfare by focusing on the pro-competitive and variety gains from trade. They do so by employing translog preferences and extensive product-specific concentration measures used both to deduce the number of firms acting in markets and capture crowing-out of variety space — a mechanism that is absent in CES preferences. They find that US welfare increased 1 percent over the 1992-2005 period, with each of the two mechanisms contributing by half.[10] In my model, I can distinguish these two channels plus the overall price effect as in ACR, and the excess profits generated by domestic oligopolistic firms in domestic and foreign markets. Moreover, I find that the pro-competitive term negatively impacted welfare in the case I study. The rationale behind this result is that large firms gained market share and therefore markups increased overall. This could have happened through reallocation of market shares across or within destinations for reasons accounted for but not identified in the model (e.g. changes in quality and thus in large firms weights). This is a key advantage of having a model-consistent concentration measure: It allows for market share reallocation across firms, regardless of their origin.

To the best of my knowledge, Parenti (2017) was the first to construct a trade model in which small and large firms compete.[11] My model differs from his along different dimensions. First, I assume firm heterogeneity within each group of firms and therefore I am able to both nest standard trade models with CES preferences and a continuum of heterogeneous firms (Melitz, 2003, Chaney, 2008, Melitz and Redding, 2015, to name a few relevant examples), and granular trade models (Eaton et al., 2012, Edmond et al., 2015, Gaubert and Itskhoki, 2021). Second, Parenti (2017) assumes that large firms can decide both prices and the number of products they produce, generating a richer setting along that dimension.[12] Finally, the focus of my paper is different. I focus on measuring welfare changes and characterizing the role of industrial concentration, whereas Parenti (2018) focuses on how the impact of trade liberalization can differ due to granularity in comparison to other models with homogeneous firms (e.g. Krugman, 1979).

In section 2, I establish empirical facts to support setting up a hybrid model. In section 3, I develop the model at the industry level. In section 4, I characterize the relationship between concentration, the industry price index and trade costs by means of analytical and numerical comparative statics. In section 5, I close the model in general equilibrium, derive the welfare formula and present a welfare-consistent concentration measure. In section 6, I study the Colombian trade liberalization event. In doing so, I argue that I have the right data to do so, characterize the trade liberalization event and how it relates to the different welfare terms, and calculate the welfare change over this period. Section 7 concludes.

[10]Feenstra (2018) studies welfare gains in the same setting by assuming QMOR preferences and a bounded Pareto, and finds similar results plus selection contributing with 25% gains.

[11]Shimomura and Thisse (2012) were the first to build a model where homogeneous large and small firms interact but focusing on a closed economy.

[12]There are other channels through which large firms can modify the impact of trade liberalization. For instance, Ludema and Yu (2016) focus on the quality upgrade mechanism: high productivity firms have a low pass-through due to their choice of high-quality products, especially in products with high quality scope.

2 Empirical Facts

In international trade theory, papers employing models with monopolistic competition and models with oligopoly have followed different paths and rarely addressed the same type of questions (cf. Head and Spencer, 2017). On the one hand, recent models using oligopoly have been mostly concerned with questions related to gains from trade from more efficient resource allocations (e.g. Edmond et al., 2015, Neary, 2016). On the other hand, models with monopolistic competition have mostly addressed firm selection and productivity (e.g. Melitz, 2003). In this section, I present facts to argue that constructing a hybrid model, where a subset of firms affects industry aggregates and the rest does not, is in line with empirical evidence and allows me to characterize the role of industrial concentration for welfare.

I use firm-level Colombian data of both domestic and foreign origins. Domestic information comes from the Annual Manufacturing Survey (AMS), which includes sales and four-digit ISIC Rev. 3 industry codes of all Colombian firms with more than 10 employees. Foreign information comes from Colombian customs and include the actual seller rather than the shipper or producer.[13]

The basic fact motivating having firms behave differently depending on their size comes from evidence showing that larger, more productive firms charge higher markups (De Loecker et al., 2016). Yet this fact alone does not justify having a hybrid theory since it can be obtained by assuming consumer preferences where more productive firms face a lower elasticity of demand, even with atomistic firms (e.g. quadratic preferences as in Melitz and Ottaviano, 2008). However, the industry-specific distribution of market shares that considers firms from all origins seem to suggest a strikingly high number of small firms, and a few large firms. In other words, the degree of granularity is apparently low, but not negligible, as stated by Fact 1. Therefore, assuming differential behavior across firms does not contradict the evidence.

Fact 1. ***Degree of Granularity***. *When we consider the actual firms from all origins selling in a market, only a small fraction of them has non-negligible market shares.*

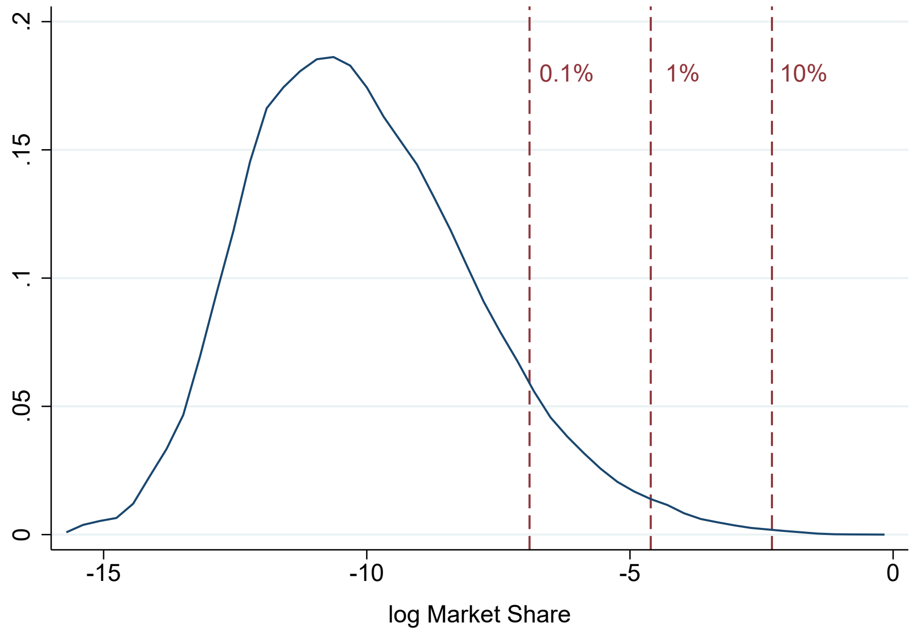
To show evidence supporting Fact 1, I present the distribution of industry-specific (log) market shares of firms from all origins selling in Colombian markets. In Figure 1, only a small fraction of firms have sizable market shares, i.e. are granular.

In Table 1 , I show the fraction of domestic and foreign firms that are granular using three different definitions of granularity — having market shares higher than 0.1%, 1% and 10%. Only about 2% of firms have market shares higher than 1%. Moreover, there are both domestic and foreign granular firms acting in domestic markets. When I use the granular definition of 0.1%, the fraction of foreign granular firms is larger; whereas under the more conservative definition of 10%, the faction of domestic and foreign firms are approximately the same. This shows that the vast number of firms are small in domestic markets and only a few have a size that may affect industry aggregates.[14]

[13] For more information about the data, please see the data section 6.1.

[14] One problem with this approach is that firms may be directly competing in narrower industry definition. Unfortunately, I don't observe domestic sales at a more disaggregated level. In any case, claims about market concentration concerns have

Figure 1: Distribution of Firms Market Shares in Colombian Domestic Markets



Market shares defined at 4-digit ISIC Rev. 3 industry level in 2007 (121 industries) taking into account both domestic and foreign firms.

Table 1: Share of Granular Firms

Granular definition	0.1%	1%	10%
Foreign firms - Non-granular	88.7%	94.2%	95%
Domestic firms - Non-granular	2.6%	4.3%	4.9%
Foreign firms - Granular	6.3%	0.8%	0.04%
Domestic firms - Granular	2.4%	0.7%	0.06%

Foreign and domestic firms selling in Colombia at 4-digit ISIC Rev. 3 industry codes in 2007 (121 industries).

Even though testing for oligopolistic behavior always relies on either demand or supply-side assumptions, a way of providing suggestive evidence for differential pricing at the top of the distribution is to check the relationship between the rank and size of firms.

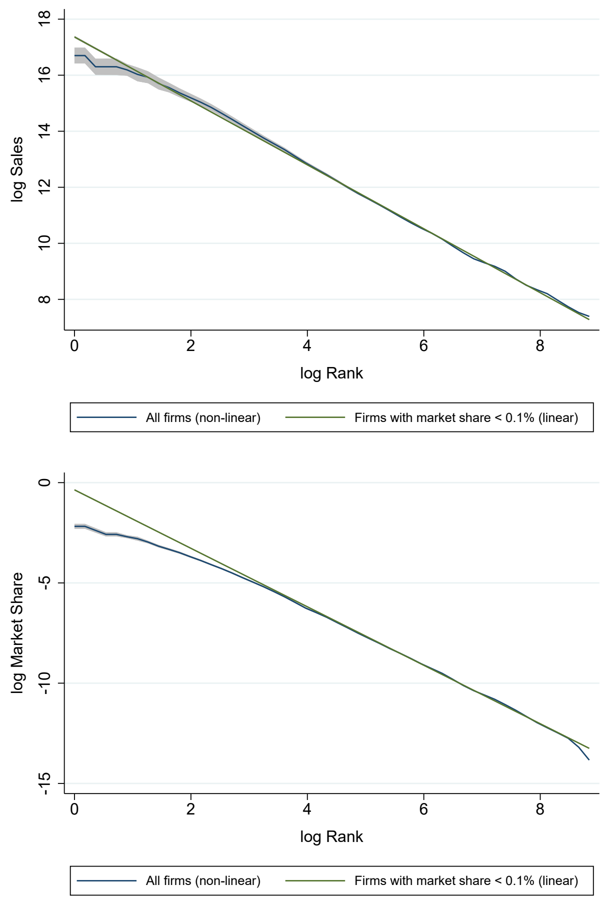
Fact 2. ***Size vs Rank Relationship at the Top***. *The relationship between the (log) size of firms and their (log) rank is linear when firms are not granular, and deviate for granular firms.*

In Figure 2 I show the relationship between the log industry-rank and log size of firms is well-fit with a linear function —it follows the Zipf's Law. A way to test if this relationship holds for the entire distribution is to fit a non-linear regression without considering firms with market shares higher than 1%. Both when we define size in terms of total sales and market shares, the predicted size for top firms is smaller than what is

usually be done at this or similar level of aggregation.

predicted by the linear relationship.[15].

Figure 2: Size vs Rank Relationship (logs)



Market shares and rank defined within 4-digit ISIC Rev. 3 industry level in 2007 (121 industries) taking into account both domestic and foreign firms.

Fact 2 has two implications. First, models in which the sales distribution is predicted to be Pareto distributed may overestimate the size of large firms. Second, granular firms may be charging higher prices than what these models predict, which is consistent with oligopolistic competition at the top.

Recent papers that assess the importance of granular firms in international trade and macroeconomics assume that they take their size into account when setting prices (e.g. Edmond et al., 2015; Gaubert and Itskhoki, 2021). The usual result of that assumption is that granular firms behave oligopolistically and charge markups above the constant CES-monopolistic competitive fixed mark-up. The issue in these papers is that firm entry is not analytically solvable and therefore the number of firms has to be solved numerically. Specifically, entry decisions need more structure to avoid multiple equilibria.[16] The following fact captures the idea that entry and exit of granular firms is significantly less frequent, and therefore imposing granularity

[15]Recent papers have been suggestive about firms' sales empirical distribution not fitting well the top part of theoretical sales distributions derived with Pareto productivity distributions, monopolistic competition and CES preferences (e.g. Head et al., 2014; Hottman et al., 2016)

[16]The usual assumption is ordering entry decisions by productivity draws (e.g. Eaton et al., 2012).

to the segment of firms with high churning may be an unnecessary assumption.

Fact 3. ***Granular Incumbency***. *The degree of churning of granular firms is lower than the one of non-granular firms.*

i Granular firms are less likely to exit than non-granular firms.

ii New firms are less likely to be granular.

In Table 2, I estimate different linear probability models relating entry and exit over the 2007-2010 ten-year period to a granular indicator to provide support for Fact 3. In the three first columns I use the granular definitions, 0.1%, 1% and 10%, to show that the probability of exiting is significantly lower for granular firms. Moreover, the higher the granular threshold, the lower the probability of exit, supporting the fact that the larger the firms the more likely are to remain. The last three columns show that non-granular firms are more likely to enter, providing support for granular incumbency.

Table 2: Granular Firm Exit and Entry Probability Differential

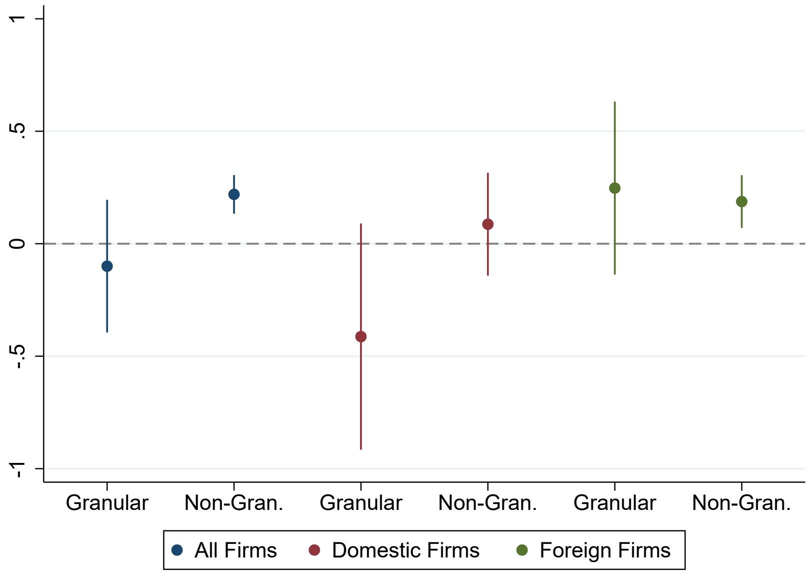
	Exit			Entry		
Granular Definition	0.1%	1%	10%	0.1%	1%	10%
Granular Firm	-0.295	-0.380	-0.452	-0.263	-0.382	-0.523
	(0.011)	(0.017)	(0.043)	(0.011)	(0.017)	(0.051)
Observations	139,639	129,667	127,666	177,445	166,216	164,281
R^2	0.057	0.035	0.025	0.042	0.024	0.015

OLS Regression. Each observation is a firm-country-industry. Granular indicator defined within 4-digit ISIC Rev. 3 industries (121 industries). Exit and entry defined by comparing 2007 with 2017. Non-granular firms are those with market shares lower than 0.1%. Industry Fixed effects. Standard errors in parenthesis clustered at the industry level.

These estimations implicitly follow a supply-side approach for modelling granularity as it has been standard in the literature. Specifically, firms are the ones identified as granular and therefore their identity is important when supporting the fact. However, granularity can also be defined following a demand-side approach. In concrete, consumer preferences may assign positive weights to a subset of varieties within an industry, regardless of which firms produce them. In this case, the number of granular varieties within an industry is the relevant indicator instead of the identity of granular firms.

In Figure 3, I plot the average midpoint growth rate of the number of granular and non-granular firms by origin. The growth of the number of granular firms is insignificant in all cases: when I include all firms, or when I distinguish by origin —domestic and foreign. The total number of non-granular firms did grow over this period, mainly explained by the growth in the number of foreign non-granular firms. This result provides support to the fact that the number of granular firms may stay constant on average, regardless of the identity of firms.

Figure 3: Average Number of Granular and Non-Granular Firm Growth by Origin



Each dot is the average 2007-2017 number of firms mid-point growth across industries defined at the 4-digit ISIC Rev. 3 level (86 industries for all firms, 37 industries for domestic firms, and 55 industries for foreign firms due to availability of granular firms —10% definition.

In this section, I provided different facts to support having an hybrid approach when modelling firms in international trade. The coexistence of a large pool of small firms with a few granular firms suggest that assuming they behave in an identical manner may not be suitable for answering welfare-related questions that rely on different mechanisms such as changes in markups and firm entry. On the one hand, the monopolistic competitive model assumes that all firms behave as small. Given the existence of granular firms, such assumption may conflict with profit-maximizing behavior in firms. On the other hand, assuming that the long fringe of small firms charge variable markups introduces the "integer problem" and as a consequence the model needs to be solved numerically by imposing structure to the entry decision (Neary, 2010). In the next section, I introduce a hybrid model that will help me characterize the role of industrial concentration and its relation with welfare through variable markups, without losing the analytical entry of small firms.

3 Model

In this section, I develop an industry-level hybrid model focusing on the role of firm concentration and its relationship with the standard international trade model with heterogeneous firms.

3.1 Environment

Consumers have defined preferences over a fixed set of discrete varieties and a continuum of varieties. I call the first type of varieties "major varieties" and the ones included in the continuum "minor varieties". The elasticity of substitution across all varieties is $\sigma > 1$, which means that they cannot perfectly substitute among them. As a result, consumers have love of variety.

Firms decide to pay an entry cost to produce any of the varieties in the economy based on the expected profits of owning a variety blueprint. They assign a probability sufficiently close to zero to get a blueprint of a major variety and therefore do not consider the potential extraordinary profits of getting to produce these varieties. However, firms produce major varieties in the long term given that such probability is not exactly zero. I call firms producing major varieties "large" or "granular" firms, and firms producing minor varieties "small" firms. The identity of the firm is not relevant in the model because consumers have preferences over varieties, not firms.

I assume there are two countries with similar characteristics, each of them with its own set of firms deciding to enter into each market. Domestic firms selling in the foreign country, and foreign firms selling in the domestic country face an ad-valorem iceberg trade cost τ.

3.2 Consumer Preferences

Consumers have CES preferences with elasticity of substitution $\sigma > 1$ over a set of two different type of varieties:

$$U = \left[\int_{\omega \in \Omega^m} q(\omega)^{\frac{\sigma}{\sigma-1}} d\omega + \sum_{\omega \in \Omega^M} q(\omega)^{\frac{\sigma}{\sigma-1}} \right]^{\frac{\sigma-1}{\sigma}}$$

I call varieties included in the set Ω^m *minor varieties* and varieties included in the set Ω^M *major varieties*. The main difference between the two is that consumers assign a positive weight to major varieties an a $d\omega$ weight to minor varieties.[17] This means that each individual major variety is valued by consumers, but not each minor varieties. In this last case, consumers value having access to many of them. As an example, we can think of soft drink brands. There are a few brands that are easily recognized by the average consumer, and plenty of others that fill the market but are not individually recognized. Consumers value having many available options.

For now, I assume that consumers spend an exogenous amount E on the industry. The industry price index has the standard CES expression over the two set of varieties:

$$P = \left[P_m^{1-\sigma} + P_M^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{1}$$

where $P_m = \left[\int_{\omega \in \Omega^m} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}$, and $P_M = \left[\sum_{\omega \in \Omega^M} p(\omega)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$. The inverse demand function is the same regardless of their type, $p = q^{-\frac{1}{\sigma}} Q^{\frac{1-\sigma}{\sigma}} E$, where Q is the quantity index (i.e. U).

[17] I assume a unit weight for simplicity in the exposition, but the more general case allows for variety-specific weights —e.g. quality.

3.3 Firm Entry

An exogenous number of ex-ante identical firms have to pay a market-specific entry cost to get a blueprint from $\Omega := \cup_{j=m,M}\Omega^j$. Getting a major variety is improbable but not impossible ($Pr(\omega \in \Omega^M) = 0$), i.e. they are part of the event space but have zero probability measure. On the contrary, firms get a minor variety *almost surely* ($Pr(\omega \in \Omega^m) = 1$).

Firms draw unit costs from a distribution that depends on the variety type. Firms that got major varieties draw a unit cost from $g^M(c)$, where $c \in \mathcal{A}^M := [c_L^M, c_H^M]$ with $c_L^M > 0$ and $c_H^M < +\infty$, whereas firms that got minor varieties draw a unit cost from $g^m(c)$, where $c \in \mathcal{A}^m := [c_L^m, c_H^m]$ with $c_L^m > 0$ and $c_H^m < +\infty$. In principle, there are no conditions on the relationship between these two cost distributions and may even be the same.

In this setting, firms will not consider major varieties when deciding whether enter. Therefore, I focus on minor varieties and foreign firms deciding to enter into the domestic economy. Given that I assume that the set $\mathcal{A}^m$ is a linear continuum, the entry cost K determines a threshold above which firms will not enter:

$$c^s_{fd,*} = \frac{P_d}{\tau w_f}\left[\frac{\tilde{\sigma}E_d}{(1-\beta)K}\right]^{\frac{1}{\sigma-1}} \tag{2}$$

where P_d is the domestic price index, E_d is the domestic expenditure, τ is an ad-valorem trade cost, and β is the survival probability.[18] Note that the cutoff $c^s_{fd,*}$ has a superscript s. I index ex-ante variety variables by m and M, and ex-post variables by s and l, i.e. small and large. The intuition is that firms getting major varieties are large ex-post, and firms getting minor varieties are small ex-post. The equilibrium cutoff $c^s_{fd,*}$ is an ex-post object and therefore it is indexed by s.

3.4 Pricing

All firms maximize profits by choosing quantities.[19] The first-order condition of a foreign firm selling variety i in the domestic economy is as follows: [20]

$$p_{fd,i} - c_{fd,i}\tau w_f = \frac{1}{\sigma}p_{fd,i} + \frac{\sigma-1}{\sigma}s_{fd,i}p_{fd,i} \tag{3}$$

The key object that differentiates small and large firms is $s_{fd,i}$, which is the market share of i when all varieties sold in that market are taken into account — large and small, domestic and foreign.

$$\begin{cases} s(\omega) \in (0,1) & \text{if } \omega \in \Omega^M \\ s(\omega) = 0 & \text{if } \omega \in \Omega^m \end{cases} \tag{4}$$

Firms getting major varieties have a positive market share, whereas firms getting minor varieties have

[18] $\tilde{\sigma} \equiv (\sigma-1)^{\sigma-1}\sigma^{-\sigma}$
[19] Assuming price competition delivers similar qualitative predictions.
[20] Derivation in Appendix A.1.

zero market shares. Market shares are equilibrium objects and therefore depend on all the fundamentals of the economy, including the unit cost c associated with variety ω. A lower unit cost implies a higher market share *ceteris paribus* only in the case of large firms.

We can return to equation 3. On the left hand side we observe the standard marginal gain of increasing the quantity produced since it is the difference between the market price $p_{fd,i}$ and the effective unit cost. The first term on the right hand side captures the marginal cost of increasing the quantity produced since doing so generates a movement along the demand curve that decreases the price. Large firms have positive market shares and therefore recognize that by increasing the quantity produced, they also increase the quantity index and thus reduce the industry price index. This increases competition and pushes prices further down, which is captured by the last term. Therefore, that large firms produce less than if they were miopic about their size, i.e. in a monopolistically competitive setting. In fact, the more productive they are the lower their quantity produced is relative to monopolistic competition, because the marginal cost of increasing production increases with the market share.

The optimality condition 3 delivers the following firm-specific optimal markup:[21]

$$\mu_{fd,i} = \tilde{\mu} \times (1 - s_{fd,i})^{-1} \tag{5}$$

where $\tilde{\mu} \equiv \frac{\sigma}{\sigma-1}$ is the markup that the firm would charge under monopolistic competition. Therefore, the term $(1 - s_{fd,i})^{-1}$ is the excess markup charged by firms with positive market shares.

The coexistence of major and minor varieties implies that competition is hybrid. The subset of firms producing minor varieties price as in a monopolistic competitive setting, whereas the subset of firms producing major varieties price as in a oligopolistic competitive setting.

3.5 Industry Equilibrium

Both industries, domestic and foreign, are populated with four firm types: domestic-large (d, l), domestic-small (d, s), foreign-large (f, l), foreign-small (f, s). Below I define the domestic industry equilibrium by taking wages and expenditure as given.

Therefore, given the distribution of productivities of major and minor varieties trade costs τ_{fd} and τ_{df}, the survival probability β, the entry cost K, and the distribution of small firms' productivity, G^s, the domestic-side equilibrium conditions are defined as follows:[22]

[21] The associated elasticity of demand is $-\nu^l_{f,i} = (s^l_{f,i} + (1 - s^l_{f,i})/\sigma)^{-1}$.

[22] The foreign-side equilibrium conditions are defined analogously.

$$
\begin{aligned}
s^l_{r,i} &= (p^l_{r,i})^{1-\sigma}(P)^{\sigma-1} && (6)\\
p^l_{r,i} &= \tilde{\mu}(1-s^l_{r,i})^{-1}\tau_r c^l_{r,i} w_r && (7)\\
p^s_r(c) &= \tilde{\mu}\tau_r c^s_r w_r && (8)\\
c^s_{r,*} &= \frac{P}{\tau_r w_r}\left[\frac{\tilde{\sigma}E}{(1-\beta)K}\right]^{\frac{1}{\sigma-1}} && (9)\\
P^l_r &= \Big[\sum_i^{N^l_r}(p^l_{r,i})^{1-\sigma}\Big]^{\frac{1}{1-\sigma}} && (10)\\
P^s_r &= \Big[\int_{c^s_L}^{c^s_{r,*}}(p^s_{r,i})^{1-\sigma}dG^s(c^s)\Big]^{\frac{1}{1-\sigma}} && (11)\\
P &= [(P^l_d)^{1-\sigma}+(P^s_d)^{1-\sigma}+(P^l_f)^{1-\sigma}+(P^s_f)^{1-\sigma}]^{\frac{1}{1-\sigma}} && (12)
\end{aligned}
$$

for $r \in (f,d)$ and $i = 1...N^l_r$.

Firms' market shares $s^l_{r,i}$ are defined relative to the entire market. We need to define the following equilibrium market shares that are useful in subsequent derivations.

Definition 1. *Given firm types $\tilde{r} \in \{(d,l),(d,s),(f,l),(f,s)\}$ and the domestic industry equilibrium defined in equations 6-12, the market share of firm i within its type is defined as $z_{\tilde{r},i} \equiv \frac{(p_{\tilde{r},i})^{1-\sigma}}{(P_{\tilde{r}})^{1-\sigma}}$.*

Definition 2. *Given the industry equilibrium defined in equations 6-12, aggregate equilibrium market shares are defined as:*

(i) Share of foreign firms (import penetration): $\lambda_f \equiv \frac{(P^l_f)^{1-\sigma}+(P^s_f)^{1-\sigma}}{P^{1-\sigma}}$

(ii) Share of large firms: $h^l \equiv \frac{(P^l_f)^{1-\sigma}+(P^l_d)^{1-\sigma}}{P^{1-\sigma}}$

(iii) Share of r firms within large firms: $\lambda^l_r \equiv \frac{(P^l_r)^{1-\sigma}}{(P^l_d)^{1-\sigma}+(P^l_f)^{1-\sigma}}$

(iv) Share of large firms within r firms: $h^l_r \equiv \frac{(P^l_r)^{1-\sigma}}{(P^l_r)^{1-\sigma}+(P^s_r)^{1-\sigma}}$

where $r \in (f,d)$.

Given definitions 1 and 2, foreign firm i's overall market share can be written either as $s^l_{f,i} = \lambda_f h^l_f z^l_{f,i}$ or $s^l_{f,i} = h^l \lambda^l_f z^l_{f,i}$.

4 Comparative Statics

4.1 Theoretical Results

In this section, I derive theoretical results to characterize the relationship between industrial concentration, the CES industry price index, and trade costs. In order to do so, I define an increase in competition as follows:

Definition 3. *Any shock that decreases the CES industry price index P is a shock that increases competition.*

In this model, a decrease in P causes both downward pressure on large firms' markups and exit of less-productive small firms. These are two features present in many oligopolistic and monopolistic competitive models that are generally interpreted as characteristics of more competitive environments. Therefore, I use P to capture changes in the state of competition.

4.1.1 Relative Market Shares

In the standard model with a continuum of monopolistically competitive firms, the role of industrial concentration is limited to reflecting the interaction between underlying productivity distribution and the entry cutoff. With granular firms this is different. Changes in trade costs generate changes in the country-specific distribution of market shares due to firm-specific heterogeneous pass-throughs. Moreover, changes in trade costs do not need to directly affect firms to modify the distribution of shares since changes in competition also affect large domestic firms' markups. Therefore, industrial concentration not only reflects the underlying productivity distribution but also the state of competition in the industry.

Before formalizing the previous discussion, let's first note the following:

$$d\log s^l_{r,i} = (1-\sigma)d\log(p^l_{r,i}/P) \tag{13}$$

which directly follows from equation 6. This means that a change in the ratio of any exogenous consumer price determinant to the price index is a sufficient statistic for a change in firm-specific overall market share. The reason is that it captures both the direct impact of such effect and the overall change in competition, which aggregates all markup and entry responses, including firm's own.

The previous discussion implies that the effective impact of trade liberalization on individual foreign firms' market shares has to be measured by τ/P in the case of foreign firms, and by $1/P$ in the case of domestic firms (given that there is no direct effect of trade costs on their prices). The following proposition uses this idea to establish the relative response of market shares to trade liberalization.

Proposition 1. ***Relative Market Shares Response to Trade Costs****. A decrease in effective trade costs, τ/P, that increases competition:*

(i) decreases the market share of the relatively more productive large foreign firms,

$$\frac{d\log z^l_{f,i}/z^l_{f,j}}{d\log \tau/P} > 0 \tag{14}$$

where $c^l_{f,j} > c^l_{f,i}$; and

(ii) increases the market share of the relatively more productive large domestic firms,

$$\frac{d\log z^l_{d,i'}/z^l_{d,j'}}{d\log 1/P} > 0 \tag{15}$$

where $c^l_{d,j'} > c^l_{d,i'}$.

Proof: See Appendix A.2.

In order to explore the result in Proposition 1, I define the markup pass-through as follows:

$$\psi^l_{r,i} \equiv -\frac{\partial \log \mu^l_{r,i}}{\partial \log p^l_{r,i}} = (\sigma-1)\frac{s^l_{r,i}}{1-s^l_{r,i}} \tag{16}$$

where $r \in (f,d)$.[23] Note that this elasticity is increasing in firm i market share, which indicates that larger firms react more strongly to changes in either trade costs or competition.[24] For instance, a decrease in trade costs leads to higher markup increases by relatively more productive foreign firms and thus lowers their share relative to their less productive foreign competitors.[25] Domestic firms will face more competition once trade costs go down, and as a result their markups will decrease. The relatively more productive firms will do so at a greater extent and therefore will gain market share.

4.1.2 Industry Price Index

In this section I examine how trade costs affect the CES industry price index in the hybrid model. To do so, I introduce more structure to the model by assuming that the productivity distribution G^m is bounded Pareto, with shape parameter k and bounds c_L and c_H.[26] Let's first define two small and large-specific objects:

$$\Psi^l_r \equiv \frac{\partial \log P^l_r}{\partial \log P} = \sum_{i=1}^{N^l_r} z^l_{r,i}\frac{\psi^l_{r,i}}{1+\psi^l_{r,i}} \tag{17}$$

$$\Lambda^s_r \equiv -\frac{\partial \log P^s_r}{\partial \log P} = \frac{k-(\sigma-1)}{\sigma-1}\frac{(c^s_{r,*})^{k-(\sigma-1)}}{(c^s_{r,*})^{k-(\sigma-1)}-(c^s_L)^{k-(\sigma-1)}} \tag{18}$$

The object Ψ^l_r is the weighted average of large firms' equilibrium responses to a change in competition. In fact, each firm-specific term $\frac{\psi^l_{r,i}}{1+\psi^l_{r,i}}$ is the firm-specific equilibrium markup response to changes in

[23] I follow Amiti et al. (2019) in defining a term $\psi^l_{r,i}$ as the negative of the markup elasticity.

[24] This mechanism is not present in small firms because $\frac{\partial \log \mu^l_{r,i}}{\partial \log p^l_{r,i}} = 0$.

[25] The underlying mechanism can be understood by examining equation 3: Even though the decline in trade costs increases the marginal gain of increasing production, a relatively more productive firm i, given its relatively larger size, acknowledges that it need not to increase production as much as less productive firm j to equate those gains to the marginal costs of increasing production. As a result, firm i increases production less than firm j and the decline in $p^l_{r,i}$ is lower than the decline in $p^l_{f,j}$, inducing i's markup to increase more as a consequence.

[26] Note that the unbounded Pareto distribution is a special case with $c_L = 0$.

determinants of its own prices (e.g. trade costs in the case of foreign firms).

In contrast to large firms, small firms do not respond individually to changes in competition, and only do so at the industry level through the extensive margin. This is captured by Λ_r^s, that shows that when competition decreases, more firms enter decreasing P_r^s. The terms Ψ_r^l and Λ_r^l have the opposite sign showing that small and large firms react in opposite directions to changes in competition.[27]

In the following proposition I show how changes in trade costs affect the industry price index.

Proposition 2. ***Industry Price Index Elasticity.***

(a) The elasticity of the price index with respect to trade costs can be decomposed into a (i) price term (19), (ii) a relative large firms concentration term (20), (iii) relative small firms entry term (21), and (iv) a cross-size term (22):

$$\begin{aligned}
\frac{d\log P}{d\log \tau} &= s_f && (19)\\
&+ (h^l)^2\frac{\lambda_f^l(1-\lambda_f^l)}{H}(\Psi_d^l - \Psi_f^l) && (20)\\
&+ (1-h^l)^2\frac{\lambda_f^s(1-\lambda_f^s)}{H}(\Lambda_f^s - \Lambda_d^s) && (21)\\
&+ \frac{(1-h^l)h^l}{H}\Big[\lambda_f^s(1-\lambda_f^l)[\Psi_d^l + \Lambda_f^s] - (1-\lambda_f^s)\lambda_f^l[\Psi_f^l + \Lambda_d^s]\Big] && (22)
\end{aligned}$$

where $H \equiv 1 - h^l\Psi^l + (1-h^l)\Lambda^s > 0$ is the overall industry equilibrium response.

(b) The elasticity of the price index with respect to trade costs takes values between 0 *and* 1.

Proof: See Appendix A.3

There are two special cases that are worth highlighting in part (a). The first one is when there are only small firms ($N_f^l = N_d^l = 0$). In that case, this expression only retains the price effect and the term 21, which captures the gains from trade due to product variety. In a symmetric setting, this term is positive as long as there are more small domestic firms than small foreign firms in the industry, all else equal. In the special case where the Pareto distribution is unbounded ($c_L^s = 0$), this term vanishes showing that there are no gains from trade due to product variety in the standard monopolistic model, as argued in Feenstra (2018).

The second special case is when there are no small firms ($c_L^s = c_H^s$). In this case, the effect only come from the pro-competitive term, 20, which captures whether markups decrease or increase depending on the relative pass-through between domestic and foreign large firms. All else equal, higher concentration implies $\Psi_d^l - \Psi_f^l >$ and thus the effect is magnified.

In the case where there are both large and small firms, each of the previous terms is qualified by how much more productive large firms are. This is captured by h_l: the more productive large firms are relative to small firms, the higher will be their aggregate market share, even after taking into account their higher

[27]The function Λ_r^s is proportional to the hazard function o_r^s of the bounded Pareto distribution of export sales, as shown by Melitz and Redding (2015) —$o_r^s = (\sigma-1)\Lambda_r^s$.

markups. In addition, the terms 20 and 21 are not sufficient to capture the total effect since there are across-size effects, as captured by term 22. For example, a decrease in trade costs increases foreign entry by more when Λ_f^s is high, and therefore amplifies the impact on prices by further decreasing domestic markups.

In part (b) of Proposition 2 I establish that the price index elasticity is always positive and bounded above. To see this, we can write the price index as follows:

$$\frac{d\log P}{d\log\tau} = s_f\frac{H_f}{H} \tag{23}$$

where $H_f \equiv 1 - h_f^l\Psi_f^l + (1-h_f^l)\Lambda_f^s > 0$.[28] Given that $\frac{H_f}{H} \in [0,1]$, the elasticity is always positive and depends on the ratio of foreign to overall equilibrium responses.

4.1.3 Domestic Concentration

Given the evidence of an increase in domestic concentration in developed economies, it is useful to study the predictions of this model in a setting where foreign competition increases. In light of the model, such increase can be caused by any factor decreasing the relative price of imports such as transportation costs or an increase in foreign firms' productivities.

I analyze the relationship between competition and the domestic concentration of large firms by means of the Herfindahl-Hirschman Index (HHI). This measure is relevant because it is related to both the excess profits large firms make due to oligopolsitic competition, and the consumer welfare loss due to markup dispersion. I explore this in the general equilibrium section of the paper.

Proposition 3. ***Domestic Concentration and Competition***. *(a) The elasticity of domestic concentration as captured by the HHI with respect to the CES industry price index is:*

$$\frac{d\log HHI_d}{d\log P} \quad = \quad -2(\sigma-1)\sum_{i=1}^{N_d}\left[\gamma_{d,i}^l - z_{d,i}^l\right]\frac{\psi_{d,i}^l}{1+\psi_{d,i}^l} \tag{24}$$

where $\gamma_{d,i}^l \equiv \frac{(z_{d,i}^l)^2}{\sum_{i=1}^{N_d^l}(z_{d,i}^l)^2}$ are HHI-specific weights.

(b) Any shock that increases competition increases domestic concentration at the industry level.

Proof: See Appendix A.4.

The impact of P on HHI_d depends on the reallocation of market shares due to changes in markups. This is captured by the discrete weights that each function, P and HHI, assigns to individual firms' responses, $z_{d,i}^l$ and $\gamma_{d,i}^l$ respectively. Its sign is always well-defined because HHI is an increasing convex function in the unit interval, which implies assigning higher weights to relatively larger firms. This means that the difference

[28] Note that $\Psi_f^l \in (0,1)$ and therefore $1 - h_f^l\Psi_f^l > 0$.

in weights, $\gamma^l_{d,i} - z^l_{d,i}$, is higher for the higher markup equilibrium responses. As a result, the impact of P on HHI_d is negative.[29]

4.2 Numerical Results

In this section I provide a numerical exercise to illustrate the previous results.

4.2.1 Large Firms' Productivities

So far, I have not impose any kind of restriction on the distribution of productivities of domestic and foreign large firms. In this section, I interpret the distribution of productivities of large firms as a particular draw $(\{c^l_{d,i}\}|^{N^l_d}_{i=1}, \{c^l_{f,i}\}|^{N^l_f}_{i=1})$ from an unbounded Pareto distribution G^M with shape parameter k and scale parameter $1/c^m_L$. This means that the productivity distribution of all firms, large and small, can be understood as a compound of two distributions: the one for the small firms and the distribution that generated the observed draws of large firms' productivities.[30]

4.2.2 Parameters

In Table 3, I list the parameter values required to conduct a numerical exercise.

Table 3: Parameter Values.

Parameter	Definition	Value	Source/Explanation
k	Pareto shape parameter	4.3	GI2021
σ	Elasticity of substitution	4.5	Average GI2021-MR2015
c^m_H	Upper bound of the unit cost distribution	1	Normalization
c^m_L	Lower bound of the unit cost distribution	0.125	Implies average large firm productivity to be 8x relative to small firms
N^l_f, N^l_d	Number of domestic and foreign large firms	4	Commonly used value to calculate concentration ratios (HS2017)
N	Number of potential entrants	1000	Normalization
$\tilde{E} \equiv \frac{E}{K(1-\beta)}$	Entry shifter	10000	Guarantees an internal solution

GI2021: Gaubert and Itskhoki (2021), MR2015: Melitz and Redding (2015), HS2017: Head and Spencer (2017).

The two parameters that govern the curvature of the distribution of sales are σ and k. I set $\sigma = 4.5$, which is the average between $\sigma = 4$, the value used in Melitz and Redding (2015), which features a monopolistic competitive model and bounded Pareto, and $\sigma = 5$, the one used in Gaubert and Itskhoki (2018), which features a pure oligopolistic model. In the case of k, values used in the literature do not differ much and are between 4.25 and 4.5 in general. I choose 4.3 as in Gaubert and Itskhoki (2021).

I set $c^m_L = 0.125$, which determines the relative productivity between small and large, given the normalization $c^m_H = 1$. This value implies that large firms are assumed to be approximately 8 times more productive

[29] The relationship between HHI and P can also be analyzed when there are only small firms. If we assume an unbounded Pareto distribution, the effect is $\frac{d \log HHI_d}{d \log P} = -\frac{k}{\sigma - 1}$, which is also negative. Note, however, that in this case the effect depends exclusively on the underlying productivity distribution and cost cutoff expression.

[30] This implies that the overall productivity distribution is unbounded Pareto with scale parameter $1/c^m_H$. This is helpful to make the model potentially comparable to a model with a continuum or firms over the entire cost support $(0, c^m_H)$.

than small firms on average.[31]

I assume that there are four large domestic and foreign firms producing major varieties in the domestic market, N_d^l and N_f^l, given that it is a value traditionally used to measure the degree of oligopoly tightness (cf. Head and Spencer, 2017). Moreover, it is a widely used value to calculate concentration ratios.[32]

The rest of parameters/exogenous variables only affect entry directly. The number of potential entrants N captures the degree of contestability in the market given that it determines how many small firms could enter, imposing potential competition on large firms. Consumer expenditure E, the entry cost K and the discount factor β only modify the cost cutoff. I set $N = 1000$ as a normalization and construct an entry shifter $\tilde{E} \equiv \frac{E}{K(1-\beta)}$. I assume it takes a value that guarantees an internal solution ($\tilde{E} = 10000$).

4.2.3 Quantification

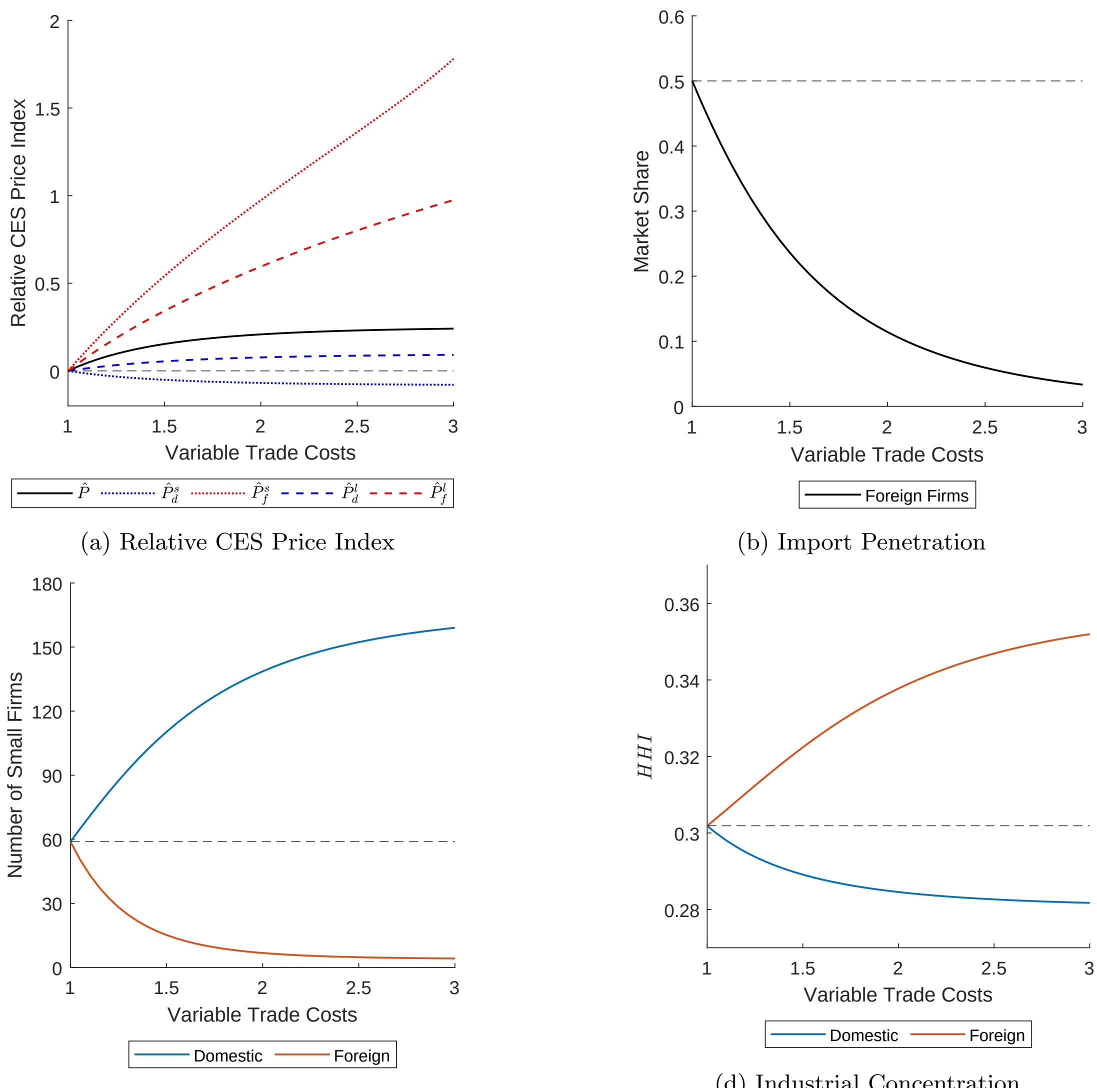
In Figure 4 I show the industry solution of the model at different variable trade costs.[33]

[31]This value is derived from $\frac{\int_0^{c_L^m} c dG^M(c)}{\int_{c_L^m}^{c_H^m} c dG^m(c)} = \frac{(\frac{c_L^m}{c_H^m})(c_H^m)^{k+1} - (c_L^m)^{k+1}}{(c_H^m)^{k+1} - (c^s m_L)^{k+1}}$, which using the chosen parameters is equal to 8.

[32]The US Census uses 4, 8, 20, and 50 top firms to calculate the share of top firms.

[33]Details in Appendix B and Graziano (2020)

Figure 4: Hybrid Industry Model Solution at Different Variable Trade Costs.



(a) Relative CES Price Index

(b) Import Penetration

(c) Number of Small Firms

(d) Industrial Concentration of Large Firms

Equilibrium solution at each level of trade costs using parameters in Table 3 and procedure in Appendix B.

In Panel 4a I plot the price index and each component relative to free trade ($\hat{P} = \log P_{\tau_f=\bar{\tau_f}} - \log P_{\tau_f=1}$). The increase in trade costs causes more domestic entry, which lowers P_d^s, but also causes large domestic firms to increase their markups, which increases P_d^l. Foreign price indices increase as expected, with the large firm price index increasing less due to markup reductions. The overall price index increases, but substitution towards domestic varieties implies that at high trade costs it is not as affected by them. This substitution can be seen in Panel 4b, where the share of foreign firms decreases to less than 5% at $\tau_f = 3$.

Panels 4c and 4d show what happens with small and large firms when trade costs increase. Given the imposed symmetry between the two countries, when $\tau_f = 1$ there are the same number of foreign and domestic firms selling in the industry, and the distribution of foreign and domestic market shares is the same in the case of large firms. When trade costs increase, foreign firms exit and there is more entry of domestic firms. In the case of concentration, an increase in trade costs decreases concentration of large domestic firms, as captured by the HHI, because large firms charge higher markups an therefore absorb less demand. The opposite happens with large foreign firms. This illustrates the result in Proposition 3.

5 General Equilibrium

In this section, I close the model by providing the market clearing conditions and derive a welfare expression that extends the ACR formula to include exporters' profits and large firms' concentration capturing the markup level and dispersion. In doing so, I provide the expression for a model-consistent concentration expression.

5.1 Market Clearing Conditions

I assume that there is a constant number of exogenous firms selling major varieties which derive profits by exploiting the fact that consumers assign non-negligible weights to them. I use wages of the domestic country as the numeraire ($w_d = 1$).

The equilibrium conditions needed to close the model are the goods market clearing conditions,

$$Y_d = X_{dd} + X_{df} \tag{25}$$

$$Y_f = X_{ff} + X_{fd} \tag{26}$$

labor market clearing conditions,

$$Y_d = L_d + \Pi_{dd} + \Pi_{df} \tag{27}$$

$$Y_f = w_f L_f + \Pi_{ff} + \Pi_{fd} \tag{28}$$

and the trade balance,

$$X_{df} = X_{fd} \tag{29}$$

where Y_d is aggregate income of country d, X_{df} is exports from d to f, and Π_{df} are aggregate profits of d firms in f.

Note that the main difference with models with monopolistic competition and exogenous entry *a la* Chaney (2008) is that the income expression needs to account for the excess profits generated by large firms at home and abroad not as a fraction of total income but as independent equilibrium objects.

5.2 Excess Profits

Large firms charge markups that are above the standard constant CES-MC markup, which implies that demand will be reallocated towards smaller firms. This is the same mechanism as in Edmond et al. (2015), with the difference that the set of small firms selling minor varieties imposes extra competitive pressure on them.

Proposition 4. ***Excess Profits***. *Aggregate profits of firms from d in market f are equal to the monopolistic competitive profits times a factor capturing excess profits due to oligopolistic pricing in large firms.*

$$\Pi_{df} \quad = \quad \Pi_{df}^{MC} \times \left[1 + (\sigma - 1)\lambda_{df}(h_{df}^{l})^2 HHI_{df}\right] \tag{30}$$

where $\Pi_{df}^{MC} \equiv \frac{\lambda_{fd} E_d}{\sigma}$ *is profits under monopolistic competition.*

Proof: See Appendix A.5

Note that expression A.5 is analogous to the firm-specific markup expression. Both of them have a factor increasing the value they would have under monopolistic competition. In this case, it depends on a weighted average of concentration of large firms across markets and the importance of these firms in the importing country.

This expression captures three different mechanisms that increase profits above the MC level. First, the higher the penetration in foreign markets λ_{fd}, the higher the profits large firms make. Therefore, markets that are highly integrated generate higher profits for their members, *ceteris paribus*. Second, the higher is the importance of large firms within trade flows h_{fd}^{l}, the more excess profits there will be due to oligopolistic pricing. Finally, the more concentrated are large firms as captured exactly by the Herfindahl-Hirschman Index HHI_{fd} in foreign markets, the higher their profits. Importantly, this concentration measure may come from the fact that consumers value some varieties more than others and as such firms are able to derive excess profits even by lowering demand by charging higher markups.

Finally, the excess profit expression can be measured with observed data. As suggested before, the main assumption it requires is determining the subset of granular firms.

5.3 Welfare

This model does not fall within the ACR and Arkolakis et al. (2018) family of gravity models because aggregate profits are not a constant share of total revenue. Regardless of it, I can extend that formula to capture the change in domestic profits coming from changes in excess profits, the dispersion of markups in the domestic market taking into account *all* firms selling major varieties in the market, and the change in the number of small firms, capturing changes in available varieties.

5.3.1 Gains From Trade Decomposition

Welfare gains in international trade models have different sources. In CES-MC models with heterogeneous firms, gains are usually come from selection of more efficient firms and the expansion in the set of available varieties. When preferences are not CES or firms have market power, trade can induce lower markups — the so-called pro-competitive effect.

In the hybrid model, direct gains can come from the expansion in the set of varieties supplied by small firms, a reduction in markups of large firms in the domestic market, and higher profits of domestic firms at home and abroad. Moreover, trade can induce general equilibrium gains as in models covered by ACR. The augmented ACR formula including the additional terms is described in Proposition 5.

Proposition 5. ***Gains from Trade Decomposition.*** *The total change in aggregate welfare of country d due to changes in underlying conditions is:*

$$d\log W_d = -\underbrace{\frac{d\log\lambda_{dd}}{\sigma-1}}_{\textit{Gross Price Effect}} - \tag{31}$$

$$- \underbrace{d\log\left[1-\sum_{r\in f,d}\lambda_{dr}(h^l_{dr})^2 HHI_{dr}\right]}_{\textit{Excess Profits Effect}} - \tag{32}$$

$$- \underbrace{d\mathcal{C}_d}_{\textit{Pro-Competitive Effect}} + \tag{33}$$

$$+ \underbrace{d\mathcal{V}_d}_{\textit{Variety Effect}} + \tag{34}$$

$$+ \underbrace{\left[d\mathcal{C}_d - \frac{d\mathcal{C}_{dd}}{\lambda_{dd}}\right] - \left[d\mathcal{V}_d - d\mathcal{V}_{dd}\right]}_{\textit{Terms of Trade Adjustments}} \tag{35}$$

where $d\mathcal{C}_d$ is a function of the equilibrium change in granular market shares, and $d\mathcal{V}_d$ is a function of the equilibrium change in the number of firms producing minor varieties.

Proof: See Appendix A.6.

The first row is the ACR term as it would appear in an Armington model and captures changes in aggregate domestic prices by means of absorption.[34] The higher the share of domestic varieties in total expenditure, the higher the pressure on wages and therefore the higher prices. The second line captures the change in excess profits of domestic firms at home and abroad. Given that I assume that profits are distributed equally among consumers, an increase in profits translate into higher welfare due to higher purchasing power. The third line captures markup dispersion through a concentration factor $\mathcal{C}_d$ that takes into account *all* large firms selling major varieties in domestic markets. Note that this term partially offset the excess profit terms because a higher dispersion implies that markups in varieties with high demand are higher. The fourth line takes into account the number of available minor varieties from *all* origins that

[34] Varieties in the Armington models are country-specific, whereas in the HM model are firm-specific.

consumers can access. The last term adjusts for the effect of markups and varieties on wages. I study these last three terms in the next sections.

5.3.2 Variety Gains

In CES-MC trade models with Pareto productivity distribution, gains from trade beyond that of comparative advantages come from firm selection (c.f. Feenstra, 2018). Moreover, when productivities are assumed to be distributed by a bounded Pareto, variety gains are restored. I take a different approach which is to be agnostic about the underlying productivity distribution and assume that the number of potential entrants is fixed. In that case, the variety term is determined by the number of *observed* firms selling minor varieties.

Proposition 6. ***Gains from Variety****. Assuming that there is a fixed number of potential entrants N^e, the change in welfare due to change in the available minor varieties is captured by the following function $\mathcal{V}$:*

$$d\mathcal{V}_d = \sum_{r\in f,d} \frac{\lambda_{rd}(1-h^l_{rd})}{\sigma-1} d\log N^s_{rd} \tag{36}$$

Proof: See Appendix A.7.

Equation 36 is a weighted average of the change in the number of firms selling minor varieties. Therefore, an increase in overall entry tend to increase welfare provided they do not lose importance relative to large firms — the overall change in the number of firms is not a sufficient statistic for the variety effect due to the assumption of a fixed number of major varieties.

Assuming the number of potential entrants is fixed has the following two benefits. First, it allows me to be agnostic with respect to the underlying productivity distribution. In other words, the rate at which firms enter captures the shape of such distribution. Second, it allows me to measure variety gains term by the number of firms that I identify as small. Given the completeness of the data, I argue that the trade-off of making the aforementioned assumption is justified.

In CES-MC models wit Pareto productivity distribution, the exponent of the ACR term also depends on the shape parameter. The assumption I employ implies that such exponent has the same expression that in the Armington model. In other words, the ACR term only depends on the consumers side because size-specific supply-side adjustments are captured by the pro-competitive and variety term.

5.3.3 Concentration and Pro-Competitive Gains

In recent work, change in industrial concentration has been taken as indicative of changes in firms' market power and thus of lower consumer welfare (e.g. Gutierrez and Philippon, 2017). The measures that are usually employed with this goal are the HHI and the share of top firms. Moreover, they are usually calculated by using domestic firms alone, frequently taking exports also into account (c.f. Grullon et al., 2019). In this model, those measures are not valid statistics for welfare change due to market power for two reasons. First, measures like the HHI do not properly weight the importance of different firms for consumers. Second, consumers have access to varieties produced by firms from different origins and thus considering only domestic firms is insufficient.

In the following proposition I derive the welfare-consistent concentration measure in the hybrid model with CES preferences (HM-CES).

Proposition 7. *Welfare-Consistent HM-CES Concentration Factor*. *The change in welfare due to change in the distribution of markups of N^l granular firms can be measured by the change in a concentration factor $\mathcal{C}$, which has the following expression:*

$$\mathcal{C} = -\sum_{i}^{N^l}[s_i + \log(1 - s_i)] \tag{37}$$

Proof: See Appendix A.8

The expression of the concentration factor $\mathcal{C}$ is rooted in the markup expression but also takes into account the change in the importance of large firms. This can be seen by applying the formula to the general current setting and collapsing the first term:

$$\mathcal{C}_d = -\sum_{r\in f,d}\lambda_{rd}h^l_{rd} - \sum_{r\in f,d}\sum_{i}^{N^l_{rd}}\log(1 - \lambda_{rd}h^l_{rd}z^l_{rd,i}) \tag{38}$$

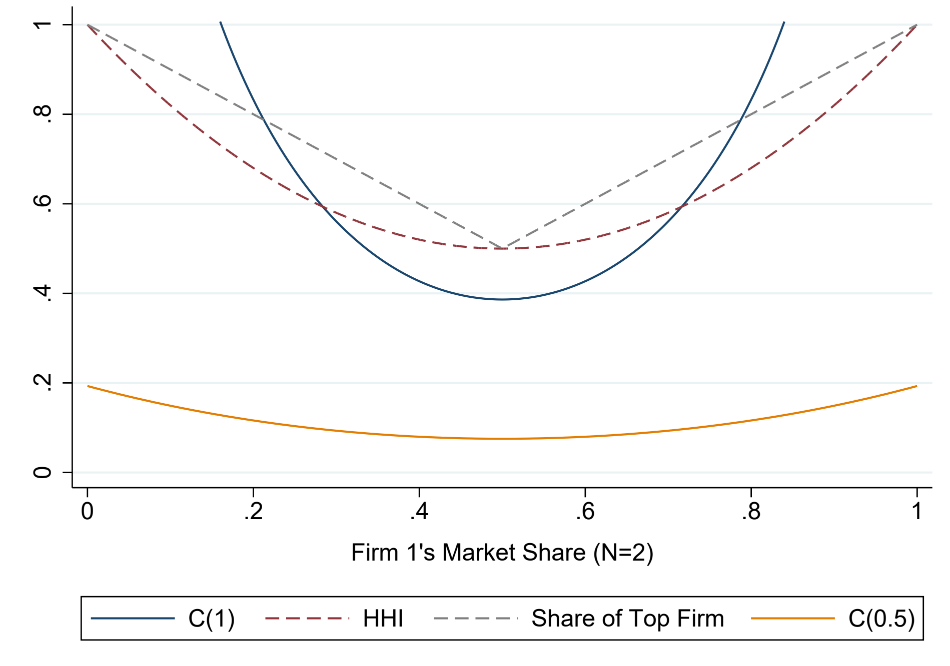
When we write this equation in changes, the first term measures the direct impact of a change in overall markups by means of the weighted average across origins of the importance of large firms. The second term captures the impact of the change in the dispersion of markups by considering firms from all origins.

The HM-CES concentration factor is closely related to the HHI. It has two main mensurable differences. First, it is not contain within the $(0, 1)$ interval because changes in welfare may be higher or lower than what changes in HHI imply, depending on the importance of large firms. The second reason is that even when the market share of large firms is one (i.e. there are no small firms), the function $\mathcal{C}$ weights individual firms differently.

In order to illustrate these two differences, let's consider a setting with two large firms and parametrize $\mathcal{C}(x)$ with x, which is the importance of large firms (i.e. $\lambda_{rd}h^l_{rd}$). In Figure 5 I plot the value of HHI and the share of the top firm along with $\mathcal{C}(x)$ under two parameter values, $x = \{0.5, 1\}$., as a function of one of the firms' market share s (the second one is by definition $1 - s$).

First, note that when $x = 1$, the function $\mathcal{C}$ assigns higher weights to larger firms and therefore suggests that an increase in the importance of top large firms is more detrimental to consumers than what changes in HHI and the share of top firms would imply. Second, note that when the importance of large firms diminishes to $x = 0.5$, the impact of changes in concentration on consumers are actually overstated by the HHI. In this sense, using changes in HHI or the share of the top firm as measures of the impact of changes in concentration on welfare is insufficient.

Figure 5: Concentration Factor $\mathcal{C}(x)$ and HHI



5.3.4 Terms-of-Trade Adjustments

The last line of proposition 5 includes two terms measuring terms-of-trade adjustments due to general equilibrium effects. Concretely, it that takes into account how changes in aggregate relative markups between domestic and foreign firms and changes between the relative domestic and foreign small firms fringes affect domestic wages. The price term includes those effects and therefore we need to make this adjustment to avoid double-counting them with the pro-competitive and variety term. Hence, to distinguish between different effects, I define the net price effect $\mathcal{P}_d$:

$$d\mathcal{P}_d = \frac{d \log \lambda_{dd}}{\sigma - 1} - \left[d\mathcal{C}_d - \frac{d\mathcal{C}_{dd}}{\lambda_{dd}} \right] - \left[d\mathcal{V}_{dd} - d\mathcal{V}_d \right] \tag{39}$$

5.3.5 Sufficient Statistics

Based on the previous discussions, equation 40 distinguishes among all the channels through which trade can provide welfare gains:

$$d \log W_d = - \underbrace{d\mathcal{P}_d}_{\text{Net Price Effect}} - \underbrace{d\mathcal{C}_d}_{\text{Pro-Competitive Effect}} + \underbrace{d\mathcal{V}_d}_{\text{Variety Effect}} + \underbrace{d\mathcal{H}_d}_{\text{Excess Profits}} \tag{40}$$

where $\mathcal{H}_d \equiv -d\log\left[1-\sum_{r\in f,d}\lambda_{dr}(h^l_{dr})^2 HHI_{dr}\right]$.

This expression can be further collapsed to a smaller set of sufficient statistics that depend only on domestic firms' information. The expression is as follows:

$$d\log W_d = -\frac{d\log\lambda_{dd}}{\sigma-1} - \frac{d\mathcal{C}_{dd}}{\lambda_{dd}} + d\mathcal{V}_{dd} + d\mathcal{H}_d \tag{41}$$

Note that this is the ACR-Armington plus three terms. First, the concentration factor only for domestic large firms. Surprisingly, an increase in domestic concentration decreases welfare in this model. The intuition is that an increase in domestic concentration captures more competition and therefore lower labor demand and thus relative lower wages. Moreover, the impact is higher in sectors with higher import penetration. Given the convex relation between markups and market shares, a given change in $\mathcal{C}$ is indicative of a more detrimental shock when the domestic share is lower.

The second extra term is the change in the number of domestic firms active in domestic markets. The intuition in this case is the opposite. An increase in the number of domestic firms captures an increase in labor demand and thus wages.

Finally, the last extra term is the change in excess profits and also requires domestic exporter-level data.

6 Measuring Changes in Welfare

In this section, I apply the welfare formula to Colombia for the 2007-2017 period. During that period, Colombia undertook a trade liberalization process based on two strategies. First, it decreased its Most-Favored Nation (MFN) tariffs to all WTO members by an average of 4 percentage points. Second, it signed free trade agreements with many countries, including two of its main commercial partners, the US and the EU.

In order to apply the formulas to measure changes in welfare, I extend the model to a setting with multiple countries and sectors.[35] Doing so allows me to use a rich set of data that includes firm information of Colombian firms selling in domestic and foreign markets, and information about the actual foreign firms selling in domestic Colombian markets.

6.1 Data

I use data from different sources to closely measure each term in the welfare formula at the industry-origin level. The first source of information is the Annual Manufacturing Survey (AMS) conducted by DANE (National Administrative Department of Statistics, by its acronym in Spanish). This survey retrieves multiple information from all manufacturing firms with more than 10 employees. I use sales information

[35] Derivations in Appendix C.

to construct domestic market shares within each ISIC Rev.3 four digits industry code (121 manufacturing industries).

The second source of information is exports and imports from Colombian customs at ten digits of the Harmonized System (HS10), firm and country level. First, I use export data to construct country-specific market shares at the ISIC3 level after converting HS10 to ISIC3. Second, I use import data to construct foreign firms' market shares in Colombian industries.

The import DANE data has a specific feature that is relevant for this setting. Along with the information of the exporting country, it includes information about the actual seller and its country. Therefore, the exporting firm may be located in the US but ship the good from China, where other firm or a subsidiary may produce it. Hence, market shares are constructed taking into account firms that are *selling* the product, not *producing* it. The implicit assumption is that the seller is the one that price goods.[36]

I use information about Colombian penetration in foreign markets from The International Trade Production Database for Estimation (ITDP-E). This data is prepared by Borchert et al. (2020) of bilateral trade flows at the ISIC Rev.3 four digits industry code including domestic sales. Therefore, I employ this data to measure the importance of Colombian firms in each ISIC industry and importing country.

Finally, the formula is not parameter free since it includes the elasticity of substitution across varieties. I use their US estimations in Broda and Weinstein (2006) and take the median for each ISIC sector.

In Table 4, I summarize all the data sources.

Table 4: Data Sources.

Object	Concept	Data
λ_{dd}	Absorpion	AMS, Imports DANE
λ_{df}	Share of domestic firms in foreign markets	ITDP-E in Borchert et al. (2020)
λ_{fd}	Share of foreign firms in domestic market	AMS, Imports DANE
h^l_{dd}	Share of domestic large firms in domestic market	AMS
h^l_{df}	Share of domestic large firms in foreign markets	Exports DANE
h^l_{fd}	Share of foreign large firms in domestic markets	Imports DANE
HHI_{df}	Domestic large firms' HHI in foreign markets	Exports DANE
HHI_{dd}	Domestic large firms' HHI in domestic market	AMS
$\mathcal{C}_d$	HM-CES Concentration Factor	AMS, Imports DANE
N^s_{dd}	Number of domestic small firms in domestic market	AMS
N^s_{fd}	Number of foreign small firms in domestic market	Imports DANE
σ	Elasticity of substitution	Broda and Weinstein (2006)

[36]The seller information was provided in a messy string variable that contains variations in names of the same firm. I realized an extensive country-specific cleaning process to identify names of the same firm with consisted of (i) removing punctuation marks and different cases, (ii) removing firms' types (e.g. "limited" in English, "sociedad de responsabilidad limitada" in Spanish, "Gesellschaft mit beschränkter Haftung" in German, etc.), (iii) using different clustering techniques sequentially (e.g. Nearest Neighbor method from openrefine.org), and (iv) visual inspection. I reduced the number of foreign firms in 54% in total after no further name similarities were found.

6.2 Descriptive Statistics

In Table 5, I characterize how the domestic and export market structure changed over the 2007-2017 period.

Table 5: Descriptive Statistics.

	2007	2017
Domestic Market Structure		
Import Penetration	0.602	0.698
All Firms		
HHI	0.063	0.062
Share of top firm	0.143	0.144
Share of top 4 firms	0.355	0.331
Number of Firms	1154	1466
Domestic Firms		
HHI	0.128	0.160
Share of top firm	0.191	0.236
Share of top 4 firms	0.580	0.573
Number of Firms	67	80
Foreign Firms		
HHI	0.067	0.068
Share of top firms	0.158	0.160
Share of top 4 firms	0.355	0.342
Number of Firms	1115	1427
Exporter Market Structure		
Foreign Penetration - All Countries	0.020	0.015
Exporters		
HHI	0.712	0.725
Share of top firm	0.779	0.790
Share of top 4 firms	0.951	0.958
Number of Firms	11	9
Foreign Penetration - Top 10 Countries	0.038	0.033
Exporters		
HHI	0.522	0.529
Share of top firm	0.625	0.632
Share of top 4 firms	0.880	0.894
Number of Firms	28	24

Domestic market structure statistics are averages across 4-digit ISIC Rev. 3 industries. Export market structure statistics are averages across 4-digit ISIC Rev. 3 industries and importing countries. Top 10 countries are Venezuela, USA, Ecuador, Peru, China, Dominican Republic, Mexico, Italy, Spain, ans Switzerland (2007).

The trade liberalization process can be illustrated by the increase on average import penetration of 10 percentage points (pp). This is larger that the increase in Chinese penetration into the US after its WTO accession in the 2000-2010 period, which was about 5 pp (c.f. Handley and Limao, 2017).

Industrial concentration of domestic firms increased over this period as illustrated by the HHI and the share of the top firm, where both of them increased more than 3 pp. This is comparable to the increase in domestic concentration in the US over the period when calculated over NAICS industry codes (c.f. Gutierrez and Philippon, 2017; Grullon et al., 2019).

When we include foreign firms into the domestic market structure calculations, concentration did not increase and remained the same. This is shown in the top panel of Table 5. Note that the average number of firms in domestic markets actually increased, plausibly providing domestic consumers with more varieties.

Colombian exporters did not increase their average penetration in foreign markets. Over this period of time, Colombian penetration decreased by 0.5 pp both when we consider all markets and top 10 destinations. In addition, their export structure remained mostly the same, barely showing more concentrated export sales.

6.3 Welfare Changes in Trade Liberalization

In this section I calculate welfare changes by dividing firms in granular and non-granular exporters.

6.3.1 Granular Determinations

I assume three levels of granularity as when I presented facts: firms with market shares higher than 0.1%, 1%, and 10%. In Table 6 I show statistics about the share of each of them in domestic markets and foreign markets.

Most of domestic sales are done by granular firms, regardless of their origin. When we use the 1% definition, granular firms market shares are 64% in total, distributed evenly between domestic and foreign firms in 2007. Foreign non-granular firms explain most of the remaining total sales. Note that over this period, the share of foreign granular firms increased, suggesting that a higher fraction of profits may have shifted overseas.

The share of Colombian firms in foreign markets is usually very low. On average, granular Colombian firms have a 1.6% market share when we consider all destinations, and 2.8% when we only consider the top 10 destinations. The market share of granular Colombian firms abroad decreased over this period, which may indicate lower national income due to lower profits.

6.3.2 Trade Liberalization

Over this period of time, Colombia went through a trade liberalization process by decreasing MFN tariffs and signing free trade agreements with many of its main partners. In this section, I characterize the relationship between changes in tariffs and the welfare components.

Table 6: Average Market Shares by Type of Firms and Market

			2007	2017
Granularity >0.1%				
	Domestic Markets			
		Foreign firms - Non-granular	0.100	0.118
		Domestic firms - Non-granular	0.010	0.011
		Foreign firms - Granular	0.501	0.580
		Domestic firms - Granular	0.388	0.290
	Export Markets			
		Exporting firms - Non-granular	0.001	0.001
		Exporting firms - Granular	0.019	0.015
	Top 10 Export Markets			
		Exporting firms - Non-granular	0.002	0.002
		Exporting firms - Granular	0.036	0.031
Granularity >1%				
	Domestic Markets			
		Foreign firms - Non-granular	0.281	0.332
		Domestic firms - Non-granular	0.076	0.070
		Foreign firms - Granular	0.321	0.366
		Domestic firms - Granular	0.322	0.231
	Export Markets			
		Exporting firms - Non-granular	0.004	0.003
		Exporting firms - Granular	0.016	0.012
	Top 10 Export Markets			
		Exporting firms - Non-granular	0.010	0.007
		Exporting firms - Granular	0.028	0.026
Granularity >10%				
	Domestic Markets			
		Foreign firms - Non-granular	0.506	0.595
		Domestic firms - Non-granular	0.293	0.227
		Foreign firms - Granular	0.096	0.103
		Domestic firms - Granular	0.105	0.074
	Export Markets			
		Exporting firms - Non-granular	0.013	0.009
		Exporting firms - Granular	0.008	0.006
	Top 10 Export Markets			
		Exporting firms - Non-granular	0.028	0.018
		Exporting firms - Granular	0.010	0.014

Domestic market shares are averages across 4-digit ISIC Rev. 3 industries. Export market shares statistics are averages across 4-digit ISIC Rev. 3 industries and importing countries. Top 10 countries are Venezuela, USA, Ecuador, Peru, China, Dominican Republic, Mexico, Italy, Spain, ans Switzerland (2007)

Table 7: Industry Average Change in (log) Ad-valorem Tariffs Applied by and to Colombia. 2017-2007.

		Mean	S.D.	p. 25	p. 50	p. 75	N
Tariffs Applied by Colombia							
Industry Average		-0.0601	0.0242	-0.0748	-0.0617	-0.0446	116
Industry-Exporter Average		-0.0615	0.0484	-0.0897	-0.0587	-0.0312	4349
	FTA countries	-0.0836	0.0472	-0.105	-0.0824	-0.0568	2099
	RoW	-0.0409	0.0396	-0.0618	-0.0426	-0.00982	2250
Tariffs Applied to Colombia							
Industry-Importer Average		-0.00872	0.0358	-0.00371	0	0	3463
	FTA countries	-0.00937	0.0278	0	0	0	1506
	RoW	-0.00823	0.0410	-0.00965	0	0	1957

Industries defined at the 4-digit ISIC Rev. 3 classification. Data from TRAINS. FTA countries: Chile, Canada, USA, Venezuela, Costa Rica, El Salvador, Honduras, Guatemala, South Korea, and the 28 EU members (including the UK).

In Table 7 I present the average change in log ad-valorem tariffs across ISIC industries. Colombian applied tariffs decreased by 6 pp. over this 10-year period. Tariffs decrease further for countries that signed agreements with Colombia over this period of time, at an average of 8 pp. In addition, Colombia undertook an unilateral liberalization by decreasing their MFN tariffs to non-FTA countries members of the WTO by an average of 4 pp.

Tariffs applied to Colombian goods also decreased, but only by 0.9 pp. Countries that signed an agreement with Colombia lowered tariffs marginally more, although the difference against non-FTA countries is insignificant. These agreements probably decreased non-trade barriers given that many of these countries were already treating Colombian goods with preferentially.[37]

How does the different welfare factors relate with the decrease of Colombian tariffs and of its partners? To see if changes in these factors correlate with changes in tariffs in the way the theory would predict, I estimate the following equation.

$$\Delta \log W_k^c = \beta \Delta \log \tau_k + \delta_K + u_k \tag{42}$$

where $\log W_k^c$ is the welfare component c (gross price term, concentration, variety and excess profits) in industry k, $\Delta \log \tau_k$ is the ad-valorem change in tariffs by Colombia, δ_K is a 2-digit industry fixed effect, and u_k is an error term. I estimate this equation by a robust regression method that downweights outliers, given the low number of observations. I show the results in table 8.

[37]For example, the US and the EU applied GSP tariffs to Colombian, which are often zero.

Table 8: Welfare Terms and Tariff Change Relationship. 2017-2007.

	Gross Price		Concentration		Variety		Excess Profits	
	No FE	Ind FE	No FE	Ind FE	No FE	Ind FE	No FE	Ind FE
$\Delta \log \tau$	0.376	0.209	0.120	0.097	-0.459	-0.386	0.020	-0.009
	(0.519)	(0.615)	(0.044)	(0.052)	(0.291)	(0.343)	(0.010)	(0.011)
Observations	90	88	116	115	116	116	100	98
R^2	0.006	0.598	0.063	0.264	0.021	0.252	0.036	0.986

OLS robust regressions the 4-digit ISIC Rev. 3 industry level. Ind FE columns include 2-digit broad industry fixed effects. Granularity defined at the 1 % level. Standard errors in parenthesis. Observations differ due to either (i) weights assigned by robust procedure or (ii) absence of observed activity.

In columns 1 and 2 of Table 8 I regress the industry-specific gross price term on the change on average ad-valorem tariffs with and without fixed effects. The coefficients are positive as we would expect, but the estimates are noisy. In columns 3 and 4, I estimate the relationship with the concentration factor when we define granularity at the 1% level. A decrease in tariffs decreases overall concentration as measured by using the HM-CES model-consistent measure. This can be capturing the existence of more granular firms and therefore more competition among them.[38] The impact of the change in tariffs on the variety term is negative as expected although noisy. Finally, decreasing tariffs seem to have decreased domestic excess profit.

Tariffs vary among partners and therefore we can use country variation to analyze the correlation of the change in tariffs with country-specific welfare terms. To do so, I estimate the following equation.

$$\Delta \log W_{jk}^{c} = \beta \Delta \log \tau_{jk} + \delta_k^1 + \delta_j^2 + v_k \tag{43}$$

In this case, I include industry and country fixed effects. Results in Table 9.

[38] Note that the model assumes that the number of major varieties is fixed but that does not imply that there is always a firm producing them. In the empirical calculations, a granular firm is defined as granular if in at least one period has market share greater than the threshold.

Table 9: Country-Specific Welfare Terms and Tariff Change Relationship. 2017-2007.

	Applied by Colombia		Applied to Colombia
	Concentration	Variety	Excess Profits
$\Delta \log \tau$	-0.007	-1.357	-0.028
	(0.007)	(0.488)	(0.015)
Observations	3,366	3,206	2,607
R^2	0.073	0.359	0.128

OLS regressions the country and 4-digit ISIC Rev. 3 industry level. Estimations include country and industry fixed effects. Granularity defined at the 1 % level. Robust standard errors in parenthesis.

In the first column, I estimate the relationship between the change in country-industry specific tariffs and the concentration factor. Results are insignificant. In the second column, I estimate the relationship with the variety term, and the result is negative and significant as we would expect. A decrease in tariffs is associated with an increase in the number of varieties imported from that origin.

In the last column, I estimate the impact of the change in tariffs applied to Colombia on the Colombian excess profits factor of granular firms. The result is negative as expected, showing an association between facing lower tariffs abroad and getting higher profits.

In conclusion, the relationship between changes in tariffs and the different welfare components seems to hold overall. It is likely that the tariff aggregation at the industry level defined by 4-digit ISIC codes does not help in getting precise estimates. In addition, the trade liberalization undertaken by Colombia is not limited to tariffs alone since modern agreements are deep and cover multiple areas that may further reduce trade costs. In the next section I measure the change in welfare, which should account for all observed and unobserved changes in trade conditions.

6.3.3 Welfare Gains

In this section, I calculate each term of the welfare formula to measure the change in underlying trade conditions that affected aggregate welfare in Colombia. I consider three different levels of granularity, 0.1%, 1% and 10%, and three different elasticities of substitution.

In ACR, they estimate welfare gains between 0.7 to 1.4 percent in the US relative to autarky. In this setting, I compare a before and after scenario in Colombia that increased aggregate import penetration in manufacturing sectors from 53% to 63%. Using the same elasticity values as in ACR, this represents aggregate welfare gains of between 2.6 to 5.7 percent.

Table 10: Gains from Trade Decomposition by Granular Assumption.

	0.1%			1%			10%		
	$\sigma = 5$	$\sigma = 10$	σ f/BW	$\sigma = 5$	$\sigma = 10$	σ f/BW	$\sigma = 5$	$\sigma = 10$	σ f/BW
Total Welfare Change	1.823	0.390	8.62	2.57	0.722	6.16	8.81	3.73	-0.031
Net Price Effect	1.41	0.089	9.54	1.50	0.123	8.28	7.76	3.19	12.13
Gross Price Effect (ACR-Armington)	2.56	1.14	11.69	2.56	1.14	11.69	2.56	1.14	11.69
TOT adjustment									
Relative Concentration	-0.965	-0.965	-0.965	-0.975	-0.975	-0.975	-0.467	-0.467	-0.467
Relative Variety	-0.192	-0.085	-1.190	-0.092	-0.041	-2.437	5.667	2.519	0.908
Excess Profit Effect	0.239	0.239	0.239	0.247	0.247	0.247	0.182	0.182	0.182
Pro-Competitive Effect	-0.0313	-0.0313	-0.0313	-0.0282	-0.0282	-0.0282	-0.0497	-0.0497	-0.0497
Variety Effect	0.209	0.0929	-1.13	0.857	0.381	-2.34	0.919	0.408	-12.3

σ from BW are the median estimation within 4-digit ISIC Rev. 3 industry codes in Broda and Weinstein (2006).

I consider a multi-sector and multi-country economy and as such the proper formula takes into account the weight of each sector. Table 10 shows the results in this setting. The gross price term ranges from 1.1 to 2.7 when I use the fixed elasticities. When I employ variable elasticities from Broda and Weinstein (2006), the gross price term is substantially higher, capturing that sectors with high weight or large changes in import penetration are more inelastic.

Colombian granular firms increased aggregate excess profits over this period regardless of the level of granularity I assume. Focusing on the 1% assumption, excess profits contribute with 0.2 percent to welfare.

The increase in the foreign number of firms explains the gains due to more varieties within industries when I consider constant σ. Gains range from 0.4 to 0.9 in this case.

The pro-competitive term shows that overall markup dispersion increased over this period causing a small decrease in welfare of about 0.03 percent.

The net price effect considers the gross price term plus changes in wages due to relative foreign and domestic concentration and entry. These effects diminish the range of the price effect from 0.1 to 1.5 percent. Overall welfare gains ranged from 0.7 to 2.6 with constant σ and were higher at 6.2 percent with variable σ.

In Table 11 I replicate the preferred columns using the more compact sufficient statistic approach. The increase in domestic concentration decreased welfare by 1 percent. When I assume $\sigma = 5$, the increase in excess profits and domestic varieties almost perfectly offset concentration and thus the total welfare change is the same as in ACR. When I assume $\sigma = 10$, welfare gains are lower than in ACR-Armington.

Table 11: Gains from Trade. Sufficient Statistics.

	Welfare Change		Welfare Elasticity	
	$\sigma = 5$	$\sigma = 10$	$\sigma = 5$	$\sigma = 10$
Total Welfare Change	2.571	0.722	0.428	0.120
ACR (+)	2.56	1.14	0.426	0.190
Domestic Concentration (-)	1.00	1.00	0.167	0.167
Domestic Varieties (+)	0.765	0.340	0.127	0.0566
Excess Profit Effect (+)	0.247	0.247	0.0411	0.0411

Granular assumption: 1%.

We can use information from Table 7 to approximate an aggregate elasticity of welfare to tariffs. Needlessly to say, many other things changed over this period and therefore we cannot interpret the value neither as a constant nor as a direct relationship. However, it is useful to have the obtained results as a reference. In the right panel of Table 11 we can see that a 1% decrease in tariffs is related to an increase in welfare of between 0.12 to 0.43 percent.

6.4 Comparisons

Using different modelling strategies, other papers provide alternative calculations that depend on sufficient statistics. First, gravity models covered in ACR have different underlying structures that would yield alternative calculations. Second, ACDR extend such models to include also preferences that would yield variable markups. Third, Feenstra and Weinstein (2017) use domestic and foreign concentration information to construct the statistics that capture welfare changes under translog preferences. In this section, I compare my results with the ones that these models would yield in the Colombian setting.

6.4.1 ACR and ACDR

Gravity models presented in ACR only require the change in absorption except when there are heterogeneous firms and no free entry. In this case, we also require the change domestic firms, similarly to what I showed in the hybrid model. This is not surprising given CES preferences and use of the continuum for small firms.

In the upper panel of Table 12, I present the different ACR calculations. The first one is the one under Armington, which uses $1-\sigma$ as the trade elasticity. This term is the one that shows up in the welfare formula I derived, basically because firms heterogeneity is captured through the entry term. As shown above, the

HM-CES and ACR-Armington calculations are almost identical.

In the second row I present the calculation for a monopolistic competitive without free entry, which is the closest to the HM-CES. In this case, ACR shows substantially lower gains, even if I do not consider the excess profits and pro-competitive effect, which are not included in ACR by definition.

Table 12: Hybrid Competition and ACR/ACDR Welfare Calculations.

	Gains from Trade
HM - CES	2.57
ACR	
Armington	2.56
Pareto w/o free entry	1.09
ACDR	
Homothetic demand	2.05
Non-homothetic demand	1.93

$\sigma = 5$, Pareto shape parameter $k = 5$, $\eta = .06$, a parameter in ACDR that captures both the degree of non-homotheticity and mark-up dispersion. Granular assumption: 1%.

I calculate ACDR gains with and without homothetic preferences in the lower panel using their baseline parameter values.[39] In both cases, their gains are lower.

6.4.2 FW

Feenstra and Weinstein (2017) measure US welfare gains arising from pro-competitive and variety changes over a period of increasing globalization. To do so, they employ translog preferences and derive sufficient statistics that require the number of firms selling in the US for each industry and country of origin. Given that they do not have that precise information, they use the inverse of HHI to capture the number of firms. I can calculate their precise expressions with the Colombian data, so I present calculations with both the exact data and their approximations.

[39] In the non-homothetic case I use $\eta = .06$, which in their case captures both the degree of non-homotheticity and mark-up dispersion.

Table 13: Hybrid Competition and FW Pro-Competitive and Variety Calculations.

	HM - CES	FW - translog	
		Approx.	Exact
Pro-Competitive + Variety	0.829	2.54	0.846
Pro-competitive ($-\mathcal{C}$)	-0.0282	2.73	2.38
Variety ($\mathcal{V}$)	0.857	-0.199	-1.54
—*New Varieties*		1.62	0.283
—*Crowding-out (HHI)*		-1.82	-1.82

Average $1/\gamma = 8.06$ from Feenstra and Weinstein (2017) used for translog calculations (γ is an industry-specific common parameter across goods in the compensated elasticity of the demand). Granular assumption: 1%.

In the first column of Table 13, I replicate the HM-CES results taking into account only the pro-competitive and variety gains. In the second column, I use the Colombian data but their approximate expressions to calculate each term. In both cases, FW gains are higher. Moreover, the translog welfare formula also allows to differentiate two terms within the variety term: new varieties and crowing-out of the parameter space — the latter captures the decreasing importance of new varieties. New varieties increase gains, but crowding goes in the opposite direction.

I can calculate the exact FW expressions thanks to the richness of the Colombian data. In column 3, I show that gains are substantially lower when I do so. Even though it is not possible to sign the bias of the approximation, in this case, using the not-exact FW formulas more than triples welfare gains. The exact calculations give the same gains as in the HM-CES model.[40]

In conclusion, the HM-CES model delivers higher gains than ACR, ACDR and FW in this case. In the first two cases, they do not account for ex-post pro-competitive and profit gains, whereas in the last one, they do not account for price and profit gains.

7 Conclusion

In this paper, I constructed a trade model of hybrid competition where large and small firms differ due to the type of varieties they produce. Firms that produce major varieties, those easily recognized by consumers, have market power, whereas the rest do not. I motivated this approach by presenting facts that account for markets that accommodate both a few granular firms and a high number of small firms.

I theoretically showed that concentration is negatively related to competition at the industry level when

[40]One potential way in which the approximation may be biasing results is that they approximate N as $1/HHI \approx N$. The problem is that $1/HHI = N/(1 + CV^2)$, where CV is the coefficient of variation of firms' sales. If the relative dispersion decreased over this period, the change in the number of firms is overstated.

we take into account only large firms. In addition, I showed that relative domestic to foreign concentration regulates the impact of trade costs on the industry price index, magnifying (attenuating) it if domestic (foreign) are relatively more concentrated.

I closed the model in general equilibrium an derived a welfare expression that extends the well-know ACR welfare formula. This formula adds a model-specific concentration factor and the number of small firms to account for pro-competitive and variety terms. Moreover, it adds an excess profits term to account for oligopolsitic profits.

Using a multi-country, multi-sector version of the model, I measured the welfare gains in Colombia over the 2007-2017 period. Over those ten years, Colombia undertook a trade liberalization process that increased import penetration in about ten percentage points. I found that welfare increased in about 0.7 to 2.6 percentage points in my preferred specification. Each point tariffs decreased were associated with an increase of 0.12 to 0.43 in welfare, without considering the tariff revenue loss.

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A Proofs and Analytic Derivations

A.1 Markups and Elasticity of Demand

Firms maximize the their profits by choosing quantities taking into account their effect on aggregates.

Firms' i in r problem:

$$\max_{q^l_{r,i}}(p^l_{r,i}/\tau - c^l_{f,i}w_r)q^l_{f,i}$$

subject to $p^l_{r,i} = (q^l_{r,i})^{-\frac{1}{\sigma}} Q^{\frac{1-\sigma}{\sigma}} E$.

First order condition (FOC):

$$(p^l_{r,i})'_q q^l_{r,i} + p^l_{r,i} - c^l_{r,i}w_r\tau = 0$$

where $(p^l_{r,i})'_q = -\frac{1}{\sigma}\frac{p^l_{r,i}}{q^l_{r,i}} - \frac{\sigma-1}{\sigma}\frac{p^l_{r,i}}{Q}Q'_q$ and $Q'_q = \frac{Q}{Q^{\frac{\sigma-1}{\sigma}}}(q^l_{r,i})^{-\frac{1}{\sigma}}$.

Therefore, the FOC is:

$$-\frac{1}{\sigma}p^l_{r,i} - \frac{\sigma-1}{\sigma}\frac{p^l_{r,i}}{Q^{\frac{\sigma-1}{\sigma}}}(q^l_{r,i})^{\frac{\sigma-1}{\sigma}} + p^l_{r,i} = c^l_{r,i}w_r\tau$$

Given that $s^l_{r,i} = \frac{p^l_{r,i}q^l_{r,i}}{PQ} = \frac{(q^l_{r,i})^{\frac{\sigma-1}{\sigma}}}{Q^{\frac{\sigma-1}{\sigma}}}$, we can write the markup as a function of the market share:

$$p^l_{r,i}[1 - \frac{1}{\sigma} - \frac{\sigma-1}{\sigma}s^l_{r,i}] = c^l_{r,i}w_r\tau$$
$$\frac{p^l_{r,i}}{c^l_{r,i}w_r\tau} = \frac{\sigma}{(\sigma-1)(1-s^l_{r,i})}$$

The firm-specific elasticity of demand $-\nu^l$ can be derived by using the Lerner Index:

$$\frac{1}{-\nu^l_{r,i}} = \frac{p^l_{r,i} - c^l_{r,i}w_r\tau}{p^l_{r,i}}$$
$$-\nu^l_{r,i} = \frac{1}{s^l_{r,i} + (1-s^l_{r,i})/\sigma}$$

where it can be seen that $-\nu^l_{r,i}$ is decreasing in $s^l_{r,i}$ and therefore large firms face a more inelastic demands.

A.2 Proposition 1. Relative Market Shares Response to Trade Liberalization.

The first point of the proposition implies we need to prove the following:

$$\frac{d\log z^l_{f,i}/z^l_{f,j}}{d\log \tau/P} > 0$$

where $c^l_{f,j} > c^l_{f,i}$. Note that by proving for τ it can be extended to any change in the relative price of imports.

The market shares within large foreign firms are: $z^l_{f,i} = (p^l_{f,i})^{1-\sigma}/(P^l_f)^{1-\sigma}$, therefore $d\log z^l_{f,i} = d\log(p^l_{f,i})^{1-\sigma} - d\log(P^l_f)^{1-\sigma}$. Given that, we only need to derive $d\log(p^l_{f,i})^{1-\sigma}$ since $d\log z^l_{f,i} - d\log z^l_{f,j} = d\log(p^l_{f,i})^{1-\sigma} - d\log(p^l_{f,j})^{1-\sigma}$.

$$\begin{aligned} d\log p^l_{f,i} &= d\log[\tilde{\mu}(1-s^l_{f,i})^{-1}c^l_{f,i}w_f\tau] \\ &= \frac{s^l_{f,i}}{1-s^l_{f,i}}d\log s^l_{f,i} + d\log\tau \end{aligned}$$

where I assumed fixed $c^l_{f,i}$ and w_f. Note that $d\log s^l_{f,i} = (1-\sigma)d\log p^l_{f,i} - (1-\sigma)d\log P$. Therefore:

$$
\begin{aligned}
d\log p^l_{f,i} &= \frac{s^l_{f,i}}{1-s^l_{f,i}}(1-\sigma)[d\log p^l_{f,i} - d\log P] + d\log\tau \\
&= -\psi^l_{f,i}[d\log p^l_{f,i} - d\log P] + d\log\tau \\
&= \frac{\psi^l_{f,i}}{1+\psi^l_{f,i}} d\log P + \frac{1}{1+\psi^l_{f,i}} d\log\tau
\end{aligned}
$$

where I used the definition $\psi^l_{f,i} \equiv -\frac{\partial \log \mu^l_{f,i}}{\partial \log p^l_{f,i}} = (\sigma-1)\frac{s^l_{r,i}}{1-s^l_{r,i}}$.

Subtract the price of the two large foreign firms:

$$
\begin{aligned}
d\log p^l_{f,i} - d\log p^l_{f,j} &= \frac{\psi^l_{f,i}}{1+\psi^l_{f,i}} d\log P + \frac{1}{1+\psi^l_{f,i}} d\log\tau - \frac{\psi^l_{j,i}}{1+\psi^l_{j,i}} d\log P - \\
&- \frac{1}{1+\psi^l_{f,j}} d\log\tau \\
&= \frac{\psi^l_{f,i}}{1+\psi^l_{f,i}} d\log P - \frac{\psi^l_{f,i}}{1+\psi^l_{f,i}} d\log\tau - \frac{\psi^l_{j,i}}{1+\psi^l_{j,i}} d\log P + \\
&+ \frac{\psi^l_{f,i}}{1+\psi^l_{f,j}} d\log\tau \\
&= -\frac{\psi^l_{f,i}}{1+\psi^l_{f,i}} d\log \tau/P + \frac{\psi^l_{f,j}}{1+\psi^l_{f,j}} d\log\tau/P
\end{aligned}
$$

where in the second line I used $\frac{1}{1+\psi^l_{f,i}} - 1 = -\frac{\psi^l_{f,i}}{1+\psi^l_{f,i}}$. Finally:

$$
\begin{aligned}
d\log z^l_{f,i} - d\log z^l_{f,j} &= (\sigma-1)[\frac{\psi^l_{f,i}}{1+\psi^l_{f,i}} d\log\tau/P - \frac{\psi^l_{f,j}}{1+\psi^l_{f,j}} d\log\tau/P] \\
&= \frac{\sigma-1}{(1+\psi^l_{f,i})(1+\psi^l_{f,j})}\left[\psi^l_{f,i} - \psi^l_{f,j}\right] d\log\tau/P
\end{aligned}
$$

Given that $\frac{\sigma-1}{(1+\psi^l_{f,i})(1+\psi^l_{f,j})} > 0$, we need $\psi^l_{f,i} - \psi^l_{f,j} > 0$ which follows from the fact that $z^l_{f,i} > z^l_{f,j}$.

The second point holds symmetrically by comparing two domestic firms and noting that $\tau = 1$. The decrease in τ decreases P and then increases the ratio $z^l_{d,i'}/z^l_{d,j'}$, where $c^l_{d,j'} > c^l_{d,i'}$.

A.3 Proposition 2. Industry Price Index Elasticity.

Totally differentiating the price index I get:

$$
d\log P = h^l d\log P^l + (1-h^l) d\log P^s
$$

where $h^l = \frac{(P^l)^{1-\sigma}}{(P^l)^{1-\sigma} + (P^s)^{1-\sigma}}$. Hence, I can derive the impact on each subset of firms and then add them up.

Large Firms. Rewriting the price index of domestic and foreign large firms directly as a function of the individual firms' prices we get:

$$
\begin{aligned}
d\log P^l &= \lambda^l_f d\log P^l_f + (1-\lambda^l_f) d\log P^l_d \\
&= \lambda^l_f \sum_{k=i}^{N^l_f} z^l_{f,k} d\log p^l_{f,k} + (1-\lambda^l_f) \sum_{k=i}^{N^l_d} z^l_{d,k} d\log p^l_{d,k}
\end{aligned}
$$

where $\lambda^l_f = \frac{(P^l_f)^{1-\sigma}}{(P^l_f)^{1-\sigma} + (P^l_d)^{1-\sigma}}$.

We already derived $d\log p^l_{f,k} = \frac{\psi^l_{f,k}}{1+\psi^l_{f,k}} d\log P - \frac{\psi^l_{f,k}}{1+\psi^l_{f,k}} d\log\tau$ when proving Proposition 1, thus:

$$
\begin{aligned}
d\log P^l &= \lambda_f^l \sum_{k=i}^{N_f^l} z_{f,k}^l [\frac{\psi_{f,k}^l}{1+\psi_{f,k}^l} d\log P + \\
&+ \frac{1}{1+\psi_{f,k}^l} d\log \tau] + (1-\lambda_f^l) \sum_{k=i}^{N_d^l} z_{d,k}^l \frac{\psi_{d,k}^l}{1+\psi_{d,k}^l} d\log P \\
&= \lambda_f^l \Psi_f^l \log P + s_f^l (1-\Psi_f^l) d\log\tau + (1-\lambda_f^l)\Psi_d^l d\log P \\
&= \Psi^l \log P + \lambda_f^l (1-\Psi_f^l) d\log\tau
\end{aligned}
$$

where I used the definition $\Psi_f^l \equiv \sum_{k=i}^{N_f^l} z_{f,k}^l \frac{\psi_{f,k}^l}{1+\psi_{f,k}^l}$, the fact that $1-\Psi_f^l = \sum_{k=i}^{N_f^l} z_{f,k}^l \frac{1}{1+\psi_{f,k}^l}$, and I defined $\frac{\partial \log P^l}{\partial \log P} \equiv \lambda_f^l \Psi_f^l + (1-\lambda_f^l)\Psi_d^l \equiv \Psi$.

Small Firms. We can analogously write the change in small firms' price index as follows:

$$
d\log P^s = \lambda_f^s d\log P_f^s + (1-\lambda_f^s) d\log P_d^s
$$

where $\lambda_f^s = \frac{(P_f^s)^{1-\sigma}}{(P_f^s)^{1-\sigma}+(P_d^s)^{1-\sigma}}$.

The foreign price index for small firms is as follows:

$$
\begin{aligned}
(P_f^s)^{1-\sigma} &= N \int_{c_L^s}^{c_{f,*}^s} p(c)^{1-\sigma} dG^s(j) \\
&= kN \frac{\tilde{\mu}^{1-\sigma} w_f^{1-\sigma} \tau^{1-\sigma}}{(c_H^s)^k - (c_L^s)^k} \int_{c_L^s}^{c_{f,*}^s} (c^s)^{k-\sigma} d(c^s) \\
&= kN \frac{\tilde{\mu}^{1-\sigma} w_f^{1-\sigma} \tau^{1-\sigma}}{(c_H^s)^k - (c_L^s)^k} \left[\frac{(c^s)^{k-(\sigma-1)}}{k-(\sigma-1)}\right]\Bigg|_{c_L^s}^{c_{f,*}^s} \\
&= kN \frac{\tilde{\mu}^{1-\sigma} w_f^{1-\sigma} \tau^{1-\sigma}}{(c_H^s)^k - (c_L^s)^k} \left[\frac{(c_{f,*}^s)^{k-(\sigma-1)} - (c_L^s)^{k-(\sigma-1)}}{k-(\sigma-1)}\right]
\end{aligned}
$$

where I need that $k-(\sigma-1) > 0$ to have a well-defined Pareto distribution of sales.

Differentiating this expression yields:

$$
d\log (P_f^s)^{1-\sigma} = (1-\sigma) d\log\tau + \lambda_f^s d\log c_{f,*}^s
$$

where $\lambda_f = (k-(\sigma-1)) \frac{(c_{f,*}^s)^{k-(\sigma-1)}}{(c_{f,*}^s)^{k-(\sigma-1)} - (c_L^s)^{k-(\sigma-1)}}$ is the hazard function of foreign sales distribution under bounded Pareto. Since $c_{f,*}^s = \frac{P}{w_f}\left[\frac{\tilde{\sigma}E}{(1-\beta)K}\right]^{\frac{1}{\sigma-1}} \tau^{-\frac{\sigma}{\sigma-1}}$ we have:

$$
\begin{aligned}
d\log (P_f^s)^{1-\sigma} &= (1-\sigma) d\log\tau + \lambda_f^s d\log \left[\frac{P}{w_f}\left[\frac{\tilde{\sigma}E}{(1-\beta)K}\right]^{\frac{1}{\sigma-1}} \tau^{-\frac{\sigma}{\sigma-1}}\right] \\
&= (1-\sigma) d\log\tau + \lambda_f^s d\log P - \lambda_f^s \frac{\sigma}{\sigma-1} d\log\tau \\
&= \lambda_f^s d\log P - (\sigma-1)\left[1 + \frac{\lambda_f^s}{\sigma-1}\frac{\sigma}{\sigma-1}\right] d\log\tau
\end{aligned}
$$

where I assumed exogenous w_f and E. The small domestic firms price index is analogous but without the direct tariff impact. Therefore, both effects are:

$$
d\log P_f^s = -\Lambda_f^s d\log P + \left[1 + \Lambda_f^s \frac{\sigma}{\sigma-1}\right] d\log\tau
$$
$$
d\log P_d^s = -\Lambda_d^s d\log P
$$

where I defined $\Lambda_f^s \equiv \frac{\lambda_f^s}{\sigma-1}$ as in the text.

Therefore, the total impact of small firms is:

$$
\begin{aligned}
d\log P^s &= \lambda_f^s\Big[-\Lambda_f^s d\log P + \Big[1+\Lambda_f^s\frac{\sigma}{\sigma-1}\Big]d\log\tau\Big] + (1-\lambda_f^s)\Big[-\Lambda_d^s d\log P\Big] \\
&= -\Lambda^s d\log P + s_f^s\Big[1+\Lambda_f^s\frac{\sigma}{\sigma-1}\Big]d\log\tau
\end{aligned}
$$

where $\Lambda^s \equiv \lambda_f^s\Lambda_f^s + (1-\lambda_f^s)\Lambda_d^s$.

Total Impact. To derive the total impact of τ on P we put together previous derivations:

$$
\begin{aligned}
d\log P &= h^l\Big[\Psi^l d\log P + \lambda_f^l(1-\Psi_f^l)d\log\tau\Big] + \\
&+ (1-h^l)\Big[-\Lambda^s d\log P + \lambda_f^s(1+\Lambda_f^s\frac{\sigma}{\sigma-1})d\log\tau\Big] \\
&= \Big[h^l\Psi^l - (1-h^l)\Lambda^s\Big]d\log P + \\
&+ \Big[h^l\lambda_f^l(1-\Psi_f^l) + (1-h^l)\lambda_f^s(1+\Lambda_f^s\frac{\sigma}{\sigma-1})\Big]d\log\tau
\end{aligned}
$$

Defining $H \equiv 1 - h^l\Psi^l + (1-h^l)\Lambda^s$ yields:

$$
\frac{d\log P}{d\log\tau} = \frac{h^l s_f^l(1-\Psi_f^l) + (1-h^l)\lambda_f^s(1+\Lambda_f^s)}{H}
$$

Decomposition. We can write the pride index elasticity as follows:

$$
\frac{d\log P}{d\log\tau} = \Theta^l + \Theta^s
$$

where $\Theta^l \equiv \frac{h^l\lambda_f^l(1-\Psi_f^l)}{H}$ and $\Theta^s \equiv \frac{(1-h^l)\lambda_f^s(1+\Lambda_f^s)}{H}$. Then, we can work on each term of the elasticity:

$$
\begin{aligned}
\Theta^l &= \frac{h^l\lambda_f^l}{H} - \frac{h^l\lambda_f^l\Psi_f^l}{H} \\
&= h^l\lambda_f^l + \frac{h^l\lambda_f^l}{H}(1-H) - \frac{h^l\lambda_f^l\Psi_f^l}{H} \\
&= h^l\lambda_f^l + \frac{h^l\lambda_f^l}{H}(h^l\Psi^l - h^l\Psi_f^l + h^l\Psi_f^l - (1-h^l)\Lambda^s) - \frac{h^l\lambda_f^l\Psi_f^l}{H} \\
&= h^l\lambda_f^l + (h^l)^2\frac{\lambda_f^l(1-\lambda_f^l)}{H}(\Psi_d^l - \Psi_f^l) - \lambda_f^l\frac{h^l(1-h^l)}{H}(\Psi_f^l + \Lambda^s) \\
\Theta^s &= \frac{(1-h^l)\lambda_f^s(1+\Lambda_f^s)}{H} \\
&= (1-h^l)\lambda_f^s + \frac{(1-h^l)\lambda_f^s}{H}(1-H) + \frac{(1-h^l)\lambda_f^s\Lambda_f^s}{H} \\
&= (1-h^l)\lambda_f^s + \frac{(1-h^l)^2\lambda_f^s}{H}(\Lambda_f^s - \Lambda^s) - \frac{(1-h^l)\lambda_f^s}{H}(1-h^l)\Lambda_f^s + \\
&+ \frac{(1-h^l)\lambda_f^s}{H}h^l\Psi^l + \frac{(1-h^l)\lambda_f^s\Lambda_f^s}{H} \\
&= (1-h^l)\lambda_f^s + \frac{(1-h^l)^2\lambda_f^s}{H}(\Lambda_f^s - \lambda_f^s\Lambda_f^s - (1-\lambda_f^s)\Lambda_d^s) + \\
&+ \frac{(1-h^l)s_f^s}{H}(\Lambda_f^s - (1-h^l)\Lambda_f^s + h^l\Psi^l) \\
&= (1-h^l)\lambda_f^s + (1-h^l)^2\frac{\lambda_f^s(1-\lambda_f^s)}{H}(\Lambda_f^s - \Lambda_d^s) + \\
&+ \lambda_f^s\frac{(1-h^l)h^l}{H}(\Lambda_f^s + \Psi^l)
\end{aligned}
$$

Adding both terms yields the final result:

$$
\begin{aligned}
\frac{d\log P}{d\log\tau} &= s_f + (h^l)^2\frac{\lambda_f^l(1-\lambda_f^l)}{H}(\Psi_d^l-\Psi_f^l) + (1-h^l)^2\frac{\lambda_f^s(1-\lambda_f^s)}{H}(\Lambda_f^s-\Lambda_d^s) + \\
&+ \frac{(1-h^l)h^l}{H}\Big[\lambda_f^s(1-\lambda_f^l)[\Psi_d^l+\Lambda_f^s] - (1-\lambda_f^s)\lambda_f^l[\Psi_f^l+\Lambda_d^s]\Big]
\end{aligned}
$$

Sign. This result follows directly from equation 44. We can further reduced it by noting that $h^l\lambda_f^l = \lambda_f h_f^l$ and hence $h^l\lambda_f^l + (1-h^l)\lambda_f^s = \lambda_f$:

$$
\frac{d\log P}{d\log\tau} = s_f\frac{H_f}{H}
$$

where $H_f \equiv 1 - h_f^l\Psi_f^l + (1-h_f^l)\Lambda_f^s$.

Given that $1-h^l\Psi^l$ and $1-h_f^l\Psi_f^l$ are both positive because both h^l and Ψ^l are between zero and one, this expression is always negative.

In terms of the upper bound, note that we can be write the elasticity as follows:

$$
\frac{d\log P}{d\log\tau} = \frac{\lambda_f H_f}{\lambda_f H_f + (1-\lambda_f)H_d}
$$

where $H_r \equiv 1 - h_r^l\Psi_r^l + (1-h_r^l)\Lambda_r^s \in [0,1]$ and therefore the elasticity can only takes values between 0 and 1 (in the last case when $\lambda_f = 1$).

A.4 Proposition 3. Domestic Concentration and Competition.

In this proof I derive the expression for the impact of price index changes on any increasing and convex function $\mathcal{F}$ with respect to market shares $\mathcal{F}$. Particularly, I focus on homogeneous concentration functions $\mathcal{F}_h = \sum_{i=1}^{N_d^l}(z_{d,i}^l)^t$, with $t > 1$. The proposition result follows when $t = 2$ ($\mathcal{F}_h = HHI$). Moreover, I also include small firms to characterize the effect under bounded Pareto distribution.

Decomposition. In order to prove part (a) of this proposition, I start by the general definition of $\mathcal{F}$:

$$
\mathcal{F}^l(\{z_{d,i}\}_{i=1}^N) = \sum_{i=1}^N m(z_{d,i};W_d)
$$

where $m(z_{d,i};W_d)$ is a function of internal market shares $z_{d,i}$ and can contain other factors, which I summarize in W_d. The theoretical version of this measure can consider the continuum of small firms. Hence, the full concentration measure is:

$$
\mathcal{F}(\{z_{d,i}\}_{i=1}^N) = \sum_{i=1}^{N_d} m[h_d^l z_{d,i}^l] + \int_{c_L^s}^{c_{d,*}^s} m[(1-h_d^l)z^s(j)]dG^s(j)
$$

Large Firms.

$$
\begin{aligned}
d\log\Big[\sum_{i=1}^N m[h_d^l z_{d,i}^l]\Big] &= \sum_{i=1}^N \gamma_{d,i}^l \iota_i^l[d\log z_{d,i}^l + d\log h_d^l] \\
&= \sum_{i=1}^{N_d} \gamma_{d,i}^l \iota_i^m d\log z_{d,i}^l + \iota^{m,l} d\log h_d^l
\end{aligned}
$$

where $\gamma_{d,i}^l = \frac{m[h_d^l z_{d,i}]}{\sum_{k=1}^{N_d} m[h_d^l z_{d,k}]}$ are $\mathcal{F}$-specific weights and $\iota_i^m = \frac{m_i' h_d^l z_{d,i}}{m_i}$ is the elasticity of m with respect to the domestic market share of i, where I define $\iota^{m,l} = \sum_{i=1}^{N_d}\gamma_{d,i}^l\iota_i^m$ to be the weighted elasticity of changes in the large firm aggregate market share.

The first term captures reallocation within large firms:

$$\begin{aligned}\sum_{i=1}^{N_d}\gamma^l_{d,i}\iota^m_i d\log z^l_{d,i} &= \sum_{i=1}^{N_d}\gamma^l_{d,i}\iota^m_i\Big[d\log(p^l_{d,i})^{1-\sigma}-d\log(P^l_d)^{1-\sigma}\Big]\\ &= (1-\sigma)\sum_{i=1}^{N_d}\gamma^l_{d,i}\iota^m_i d\log p^l_{d,i}-\iota^{m,l}(1-\sigma)\sum_{i=1}^{N_d}z^l_{d,i}d\log p^l_{d,i}\\ &= (1-\sigma)\iota^{m,l}\sum_{i=1}^{N_d}\Big[\tilde\gamma^l_{d,i}-z^l_{d,i}\Big]d\log p^l_{d,i}\end{aligned}$$

where $\tilde\gamma^l_{d,i}\equiv\gamma^l_{d,i}\frac{\iota^m_i}{\iota^{m,l}}\in(0,1)$.

Assuming that the concentration function is homogeneous of degree t simplifies this expression due to the following:

$$\iota^m_i=\frac{m'_i h^l_d z_{d,i}}{m_i}=\frac{(h^l_d z_{d,i})^{t-1}h^l_d z_{d,i}}{(h^l_d z_{d,i})^t}=t$$

and implies $\iota^{m,l}=t$ and $\tilde\gamma^l_{d,i}=\gamma^l_{d,i}=\frac{z^t_{d,i}}{\sum_{k=1}^{N_d}z^t_{d,i}}$. As a result, the term for large firms is $t(1-\sigma)\sum_{i=1}^{N_d}\Big[\gamma^l_{d,i}-z^l_{d,i}\Big]d\log p^l_{d,i}$ in the case of $\mathcal{F}_h$.

Small Firms. Small firms are atomistic so the effect of competition on concentration acts through changes in the productivity distribution of firms that enter.

$$\int_{c^s_L}^{c^s_{d,*}}m[(1-h^l_d)z(j)]dG^s(j)=$$

$$=\int_{c^s_L}^{c^s_{d,*}}m[(1-h^l_d)z(j)]d\Big[\frac{(c^s)^k-(c^s_L)^k}{(c^s_H)^k-(c^s_L)^k}\Big]$$

$$=\frac{k}{(c^s_H)^k-(c^s_L)^k}\int_{c^s_L}^{c^s_{d,*}}m[(1-h^l_d)\frac{p(c)^{1-\sigma}}{(P^s_d)^{1-\sigma}}](c^s)^{k-1}d(c^s)$$

At this point I assume that m is homogeneous of degree t, which means that $m[\frac{p(c)^{1-\sigma}}{P_d^{1-\sigma}}]=\Big[\frac{p(c)^{1-\sigma}}{P_d^{1-\sigma}}\Big]^t$.

$$\int_{c^s_L}^{c^s_{d,*}}m[(1-h^l_d)z(j)]dG^s(j)=$$

$$=\frac{k}{(c^s_H)^k-(c^s_L)^k}\int_{c^s_L}^{c^s_{d,*}}m[\frac{p(c)^{1-\sigma}}{P_d^{1-\sigma}}](c^s)^{k-1}d(c^s)$$

$$=\frac{k}{(c^s_H)^k-(c^s_L)^k}\frac{(1-h^l_d)^t}{(P^s_d)^{t(1-\sigma)}}\int_{c^s_L}^{c^s_{d,*}}p(c)^{t(1-\sigma)}(c^s)^{k-1}d(c^s)$$

$$=\frac{k(1-h^l_d)^t}{(c^s_H)^k-(c^s_L)^k}\frac{\tilde\mu^{t(1-\sigma)}T_d^{t(1-\sigma)}}{(P^s_d)^{t(1-\sigma)}}\int_{c^s_L}^{c^s_{d,*}}(c^s)^{t(1-\sigma)+k-1}d(c^s)$$

$$=\frac{k(1-h^l_d)^t}{(c^s_H)^k-(c^s_L)^k}\frac{\tilde\mu^{t(1-\sigma)}T_d^{t(1-\sigma)}}{(P^s_d)^{t(1-\sigma)}}\Big[\frac{(c^s_{d,*})^{k-t(\sigma-1)}-(c^s_L)^{k-t(\sigma-1)}}{k-t(\sigma-1)}\Big]$$

where I need that $k-t(\sigma-1)>0$ to have a well-defined Pareto distribution of sales to the power of t.

Taking logs and differentiating this expression yields:

$$\begin{aligned}d\log\int_{c^s_L}^{c^s_{d,*}}m[(1-h^l_d)z(j)]dG^s(j) &= td\log(1-h^l_d)+\lambda^s_{t,d}d\log P+\\ &+ t(\sigma-1)d\log P^s_d\end{aligned}$$

where $\lambda^s_{t,d}\equiv[k-t(\sigma-1)]\frac{c_{d,*}^{k-t(\sigma-1)}}{c_{d,*}^{k-t(\sigma-1)}-c_L^{k-t(\sigma-1)}}$ and I used that $d\log c^s_{d,*}=d\log P$.

Given that $(1-\sigma)d\log P^s_d=\lambda^s_d d\log c^s_{d,*}$ we get:

$$d\log\int_{c^s_L}^{c^s_{d,*}}m[(1-h^l_d)z(j)]dG^s(j)=td\log(1-h^l_d)+\big[\lambda^s_{t,d}-t\lambda^s_d\big]d\log P \tag{44}$$

Total Impact. Adding up both derivations in the case of homogeneous concentration functions yields:

$$\begin{aligned} d\log \mathcal{F}_h &= t(1-\sigma)\sum_{i=1}^{N_d}\left[\gamma^l_{d,i} - z_{d,i}\right]\frac{\psi^l_{d,i}}{1+\psi^l_{d,i}} d\log P + \\ &+ \left[\lambda^s_{t,d} - t\lambda^s_d\right] d\log P + t\frac{1-2h^l_d}{1-h^l_d} d\log h^l_d \end{aligned}$$

where I used $d\log p^l_{d,i} = \frac{\psi^l_{d,i}}{1+\psi^l_{d,i}} d\log P$.

The change in $\log h^l_d$ captures the reallocation of market share between large and small firms and is as follows:

$$\begin{aligned} d\log h^l_d &= (1-\sigma)\left[\sum_{i=1}^{N_d} z_{d,i} d\log p_{d,i} - d\log P_d\right] \\ &= (1-\sigma)\Psi^l_d d\log P - (1-\sigma)h^l_d d\log P^l_d - (1-\sigma)(1-h^l_d) d\log P^s_d\Big] \\ &= (1-h^l_d)(1-\sigma)\left[\Psi^l_d + \Lambda^s_d\right] d\log P \end{aligned}$$

Replacing this last derivation into the main expression and rearranging yields the result:

$$\begin{aligned} \frac{d\log \mathcal{F}_h}{d\log P} &= t(1-\sigma)\sum_{i=1}^{N_d}\left[\gamma^l_{d,i} - z_{d,i}\right]\frac{\psi^l_{d,i}}{1+\psi^l_{d,i}} + \left[\lambda^s_{t,d} - t\lambda^s_d\right] + \\ &+ t(1-2h^l_d)(1-\sigma)\left[\Psi^l_d + \Lambda^s_d\right] \end{aligned}$$

Setting $t=2$ yields $\frac{d\log HHI}{d\log P}$ in text.

Sign. I follow the same approach than in part (a) where I use any concentration measure homogeneous of degree t and note that the HHI is a special case when $t=2$.

Sign of the Large Firms Effect. I need to prove that $\sum_{i=1}^{N_d}\left[\gamma^l_{d,i} - z_{d,i}\right]\frac{\psi^l_{d,i}}{1+\psi^l_{d,i}} > 0$ which means that the sign of the large firms effect is negative (since it is multiplied by $(1-\sigma)$).

(1) First, I need to show that there exists a firm $i*$ above which $\gamma_i - z_i > 0$. For any i, we can write it as follows:

$$\begin{aligned} \gamma_i - z_i &= \frac{z_i^t}{\sum_{j=1}^N z_j^t} - z_i \\ &= \frac{\omega_i z_i}{\sum_{j=1}^N \omega_j z_j} - z_i \end{aligned}$$

where $\omega_i = \frac{z_i^{t-1}}{\sum_{j=1}^N z_j^{t-1}}$ are weights that put more weight on larger firms given that $t>1$. Define $\bar{\omega} = 1/N$ as the particular case for which all shares are equally weighted. Given that $\sum_{j=1}^N z_j = 1$, we can write it as:

$$\gamma_i - z_i = \frac{\omega_i z_i}{\sum_{j=1}^N \omega_j z_j} - \frac{\bar{\omega} z_i}{\sum_{j=1}^N \bar{\omega} z_j}$$

This expression shows that it is the difference of the contribution of observation i between using ω_i and $\bar{\omega}$ weights.

I claim there is a i^* such that:

(i) $\gamma_i - z_i \geq 0$ if $i^* \geq i$

(ii) $\gamma_i - z_i < 0$ if $i^* < i$

To prove claim (i), assume that $i \geq i^*$ and $\gamma_i - z_i < 0$:

$$\frac{\omega_i z_i}{\sum_{j=1}^N \omega_j z_j} < \frac{\bar{\omega} z_i}{\sum_{j=1}^N \bar{\omega} z_j}$$

but given that ω_i is increasing in z_i, then the contribution of $i > i^*$ has to be higher for these weights. Thus, $\gamma_i - z_i > 0$ for $i > i^*$.

To prove claim (ii), we can follow the same logic assuming that $l < i^*$ and $\gamma_l - z_l \geq 0$:

$$\frac{\omega_l z_l}{\sum_{j=1}^N \omega_j z_j} \geq \frac{\bar{\omega} z_l}{\sum_{j=1}^N \bar{\omega} z_j}$$

but given that ω_l is increasing in z_l, then the contribution of $i < i^*$ has to be lower for these weights. Thus, $\gamma_l - z_l \leq 0$ for $i < i^*$.

(2) Define $X_i = \frac{\psi^l_{d,i}}{1+\psi^l_{d,i}}$ and $Z_i = \gamma^l_{d,i} - z^l_{d,i}$. Define two sets of firms: A for firms such as $i^* \geq i$ and B for firm such as $i^* > i$. Since X_i is increasing in z_i then $X^B_i > X^A_j$ for any $i \in A$ and $j \in B$. Lets assume that the expression is negative:

$$Z^A \sum_{i\in A} Z^A_i X_i + Z^B \sum_{i \in B} Z^B_i X_i < 0$$

where $Z^A = \sum_{i\in A} Z_i$, $Z^B = \sum_{i\in B} Z_i$, and $Z^A_i = Z_i/Z^A$ and $Z^B_i = Z_i/Z^B$. Note that $Z^A + Z^B = 0$ and thus:

$$\begin{aligned}
-Z^B \sum_{i\in A} Z^A_i X_i + Z^B \sum_{i \in B} Z^B_i X_i &< 0 \\
Z^B \sum_{i\in B} Z^B_i X_i &< Z^B \sum_{i \in A} Z^A_i X_i \\
\sum_{i\in B} Z^B_i X_i &< \sum_{i \in A} Z^A_i X_i
\end{aligned}$$

The left hand side is a weighted average of all X_i in B and the right hand side is a weighted average of all X_i in A. Since we assumed that $X^B_i > X^A_j$ for any $i \in A$ and $j \in B$ we arrived to a contradiction. Therefore:

$$Z^A \sum_{i\in A} Z^A_i X_i + Z^B \sum_{i \in B} Z^B_i X_i > 0$$

which proves that $(1-\sigma)t \sum_{i=1}^{N_d} \left[\gamma^l_{d,i} - z_{d,i}\right] \frac{\psi^l_{d,i}}{1+\psi^l_{d,i}} < 0$.

Sign of the Small Firms Effect. Assume the sign is positive:

$$\begin{aligned}
\lambda^s_{t,d} - t\lambda^s_d &> 0 \\
t\lambda^s_d\left(\frac{\lambda^s_{t,d}}{t\lambda^s_d} - 1\right) &> 0 \\
\frac{\lambda^s_{t,d}}{t\lambda^s_d} - 1 &> 0 \\
\frac{\frac{(c^s_{d,*})^{k-t(\sigma-1)}}{(c^s_{d,*})^{k-t(\sigma-1)} - (c^s_L)^{k-t(\sigma-1)}}}{\frac{(c^s_{d,*})^{k-(\sigma-1)}}{(c^s_{d,*})^{k-(\sigma-1)} - (c^s_L)^{k-(\sigma-1)}}} \frac{k-t(\sigma-1)}{tk - t(\sigma-1)} &> 1 \\
\frac{1 - v^{k-(\sigma-1)}}{1 - v^{k-t(\sigma-1)}} \frac{k-t(\sigma-1)}{tk - t(\sigma-1)} &> 1
\end{aligned}$$

where $v \equiv \frac{c^s_L}{c^s_{d,*}} \in (0,1)$. This means we can define the LHS as the function $\mathcal{G}(t;\sigma,k,v)$ and given that $t \in (1,\infty)$, check the limit of $\mathcal{G}$ at both boundaries:

$$\begin{aligned}
\lim_{t\to\infty} \mathcal{G}(t;\sigma,k,v) &= \lim_{t\to\infty} \left[\frac{1 - v^{k-(\sigma-1)}}{1 - v^{k-t(\sigma-1)}} \frac{k-t(\sigma-1)}{tk - t(\sigma-1)}\right] \\
&= \lim_{t\to\infty} \left[\frac{1 - v^{k-(\sigma-1)}}{1 - v^{k-t(\sigma-1)}}\right] \lim_{t\to\infty} \left[\frac{\frac{k}{t} - (\sigma-1)}{k - (\sigma-1)}\right] \\
&= \frac{1 - v^{k-(\sigma-1)}}{k - (\sigma-1)} \frac{\lim_{t\to\infty} \frac{k}{t} - (\sigma-1)}{\lim_{t\to\infty}(1 - v^{k-t(\sigma-1)})} \\
&= 0
\end{aligned}$$

where the last result follows from $\lim_{t\to\infty} \frac{k}{t} = 0$ and $\lim_{t\to\infty} v^{k-t(\sigma-1)} = \infty$. This means that as t increases the impact of P on small firms concentration is negative because the inequality 45 is a contradiction.

When $t \to 1^+$, we have:

$$\lim_{t \to 1^+} \mathcal{G}(t;\sigma,k,v) \quad = \quad 1$$

Therefore, if $\frac{d\mathcal{G}}{dt} < 0$ for all $t \in (1,\infty)$, the inequality 45 is contradiction for all t in its support:

$$\frac{d\mathcal{G}}{dt} \quad = \quad \frac{1-v^{k-(\sigma-1)}}{k-(\sigma-1)} \frac{d\left[\frac{\frac{k}{t}-(\sigma-1)}{(1-v^{k-t(\sigma-1)})}\right]}{dt} < 0$$

where the sign follows from $\frac{1-v^{k-(\sigma-1)}}{k-(\sigma-1)} > 0$ and $\frac{d\left[\frac{\frac{k}{t}-(\sigma-1)}{(1-v^{k-t(\sigma-1)})}\right]}{dt} < 0$. Therefore, 45 is a contradiction for all the support and hence the sign of the small firms effect is negative. Note that this includes $t = 2$, the HHI particular case.

Sign of the Cross-Effect. The sign of the cross-size effect depends on the relative market share between small domestic and large firms. If we assume that large domestic firms have more than half of the market ($h_d^l > \frac{1}{2}$), then this term is negative because both Ψ_d^l and Λ_d^s are positive.

Overall sign. The large and small firms' effects are negative. Given that the cross-size effect is positive if $h_d^l < \frac{1}{2}$, then having $h_d^l \geq \frac{1}{2}$ is sufficient to have a negative overall effect.

A.5 Proposition 4. Excess Profits.

Profits. I derive the expression for aggregate profits of firms in f at d to show that it has the expression in the proposition.

Large Firms Since there is no free entry, firms make profits. Aggregate profits are the sum of individual firms profits. Let's see the case of f firms selling in d:

$$\begin{aligned}
\Pi_{df}^l &= \sum_{i=1}^{N_{df}} \pi_{df,i} \\
&= \sum_{i=1}^{N_{df}} (p_{df,i} - c_{df,i} w_f) q_{df,i} \\
&= \sum_{i=1}^{N_{df}} (1 - c_{df,i} w_f / p_{df,i}) q_{df,i} p_{df,i} \\
&= \sum_{i=1}^{N_{df}} \left[1 - \frac{c_{df,i} w_f}{\tilde{\mu}(1-s_{df,i})^{-1} w_f c_{df,i}}\right] r_{df,i} \\
&= \sum_{i=1}^{N_{df}} \left[1 - \frac{(\sigma-1)(1-s_{df,i})}{\sigma}\right] r_{df,i} \\
&= \frac{\lambda_{df} h_{df}^l E_d}{\sigma} \sum_{i=1}^{N_{df}} \left[\sigma - (\sigma-1)(1-s_{df,i})\right] z_{df,i} \\
&= \frac{\lambda_{df} h_{df}^l E_d}{\sigma} \sum_{i=1}^{N_{df}} \left[1 + (\sigma-1) s_{df,i}\right] z_{df,i} \\
&= \frac{\lambda_{df} h_{df}^l E_d}{\sigma} \left[1 + \sum_{i=1}^{N_{df}} (\sigma-1) \lambda_{df} h_{df}^l (z_{df,i})^2\right] \\
&= \frac{\lambda_{df} h_{df}^l E_d}{\sigma} \left[1 + (\sigma-1) \lambda_{df} h_{df}^l HHI_{df}\right]
\end{aligned}$$

Small Firms Small firms make profits, but since the don't have market share their specific distribution of productivities don't matter. The derivation is the same but assuming that $z_{df} = 0$.

$$\Pi_{df}^s \quad = \quad \frac{\lambda_{df}(1-h_{df}^l) E_d}{\sigma}$$

Total Profits Therefore, the sum of small and large firms profits in a market is:

$$\begin{aligned}\Pi_{df} &= \Pi^s_{df} + \Pi^s_{df} \\ &= \frac{\lambda_{df} h^l_{df} E_d}{\sigma}\Big[1 + (\sigma-1)\lambda_{df} h^l_{df} HHI_{df}\Big] + \frac{\lambda_{df}(1-h^l_{df})E_d}{\sigma} \\ &= \frac{\lambda_{df} E_d}{\sigma}\Big[1 + (\sigma-1)\lambda_{df}(h^l_{df})^2 HHI_{df}\Big]\end{aligned}$$

A.6 Proposition 5. Gains from Trade Decomposition.

Changes in Welfare Given the CES structure, we have:

$$d\log W_d = d\log Y_d - d\log P_d$$

Income Term We took the domestic wage as a numeraire, so we start from both income equations:

$$\begin{aligned}Y_d &= L_d + \frac{\lambda_{dd} E_d H_{dd}}{\sigma} + \frac{\lambda_{df} E_f H_{df}}{\sigma} \\ Y_f &= w_f L_f + \frac{\lambda_{ff} E_f H_{ff}}{\sigma} + \frac{\lambda_{fd} E_d H_{fd}}{\sigma}\end{aligned}$$

where $H_{df} \equiv 1 + (\sigma-1)\lambda_{df}(h^l_{df})^2 HHI_{df}$. Note that both only depend on observed market shares and concentration indices. Using the goods market clearing condition $Y_d = E_d$ and the trade balance $\lambda_{df} E_f = \lambda_{fd} E_d$, we can work on the first equation:

$$\begin{aligned}Y_d &= L_d + \frac{\lambda_{dd} Y_d H_{dd}}{\sigma} + \frac{\lambda_{fd} Y_d H_{df}}{\sigma} \\ Y_d\Big(1 - \frac{\lambda_{dd} H_{dd}}{\sigma} - \frac{\lambda_{fd} H_{df}}{\sigma}\Big) &= L_d \\ Y_d &= \sigma L_d\left[\frac{1}{\sigma - \lambda_{dd} H_{dd} - \lambda_{fd} H_{df}}\right] \\ Y_d &= \sigma L_d\left[\frac{1}{\sigma - \lambda_{dd}\Big[1 + (\sigma-1)\lambda_{dd}(h^l_{dd})^2 HHI_{dd}\Big] - \lambda_{fd}\Big[1 + (\sigma-1)\lambda_{df}(h^l_{df})^2 HHI_{df}\Big]}\right] \\ Y_d &= \sigma L_d\left[\frac{1}{(\sigma-1) - \Big[(\sigma-1)(\lambda_{dd})^2(h^l_{dd})^2 HHI_{dd}\Big] - \Big[(\sigma-1)(1-\lambda_{dd})\lambda_{df}(h^l_{df})^2 HHI_{df}\Big]}\right] \\ Y_d &= \frac{\sigma}{\sigma-1} L_d\left[\frac{1}{1 - (\lambda_{dd})^2(h^l_{dd})^2 HHI_{dd} - (1-\lambda_{dd})\lambda_{df}(h^l_{df})^2 HHI_{df}}\right]\end{aligned}$$

The previous expression implies that concentration increases welfare through higher income:

$$d\log Y_d = -d\log\Big[1 - (\lambda_{dd})^2(h^l_{dd})^2 HHI_{dd} - (1-\lambda_{dd})\lambda_{df}(h^l_{df})^2 HHI_{df}\Big]$$

Price Index Term I assume a bounded Pareto distribution for small firms in these derivations, without loss of generality.

Foreign Firms Let's first derive it fully for foreign firms:

$$\begin{aligned}(1-\sigma)d\log P_{fd} &= h^l_{fd}(1-\sigma)d\log P^l_{fd} + (1-h^l_{fd})(1-\sigma)d\log P^s_{fd} \\ &= h^l_{fd}\Big[d\log \sum_i^{N^l_{fd}} c^{1-\sigma}_{fd,i}(1-s_{fd,i})^{\sigma-1}\Big] + (1-h^l_{fd})\Big[d\log \int_{c_L}^{c^*_{fd}} c^{1-\sigma} dG(c)\Big] + (1-\sigma)d\log w_f\end{aligned}$$

Large firms:

$$\begin{aligned}d\log \sum_i^{N^l_{fd}} c^{1-\sigma}_{fd,i}(1-s_{fd,i})^{\sigma-1} &= \sum_i^{N_{fd}} z_{fd,i}\, d\log(1-s_{fd,i})^{\sigma-1} \\ &= (1-\sigma)\sum_i^{N_{dd}} \frac{z_{fd,i}}{1-s_{fd,i}} ds_{fd,i}\end{aligned}$$

Small firms:

$$\begin{aligned}
d\log\int_{c_L}^{c^*_{fd}} c^{1-\sigma}dG(c) &= d\log\Big[\frac{k}{k-\sigma+1}\frac{(c^*)^{k-\sigma+1}-c_L^{k-\sigma+1}}{c_H^{k-\sigma+1}-c_L^{k-\sigma+1}}\Big] \\
&= (k-\sigma+1)\frac{(c^*)^{k-\sigma+1}}{(c^*)^{k-\sigma+1}-c_L^{k-\sigma+1}}d\log c^* \\
&= \gamma(c^*)d\log c^*
\end{aligned}$$

where $\gamma(c^*)$ is the hazard function as defined by Melitz and Redding (2015).
Putting both together:

$$\begin{aligned}
(1-\sigma)d\log P_{fd} &= h^l_{fd}(1-\sigma)\Big[\sum_i^{N_{fd}}\frac{z_{fd,i}}{1-s_{fd,i}}ds_{fd,i}\Big]+(1-h^l_{fd})\gamma(c^*_{fd})d\log c^*_{fd}+(1-\sigma)d\log w_f \\
d\log P_{fd} &= h^l_{fd}\Big[\sum_i^{N_{fd}}\frac{z_{fd,i}}{1-s_{fd,i}}ds_{fd,i}\Big]-(1-h^l_{fd})\widetilde{\gamma}(c^*_{fd})d\log c^*_{fd}+d\log w_f
\end{aligned}$$

where $\widetilde{\gamma}(c^*_{fd})\equiv\gamma(c^*_{fd})/(\sigma-1)$.

Domestic Firms Domestic firms are analogous, so:

$$d\log P_{dd} = h^l_{dd}\Big[\sum_i^{N_{dd}}\frac{z_{dd,i}}{1-s_{dd,i}}ds_{dd,i}\Big]-(1-h^l_{dd})\widetilde{\gamma}(c^*_{dd})d\log c^*_{dd}+d\log w_d$$

All

$$\begin{aligned}
d\log P_d &= \sum_{r\in f,d}\sum_i^{N_{rd}}\frac{s_{rd,i}}{1-s_{rd,i}}ds_{rd,i}- \\
&- \sum_{r\in f,d}\lambda_{rd}(1-h^l_{rd})\widetilde{\gamma}(c^*_{rd})d\log c^*_{rd} \\
&+ \sum_{r\in f,d}\lambda_{rd}d\log w_r
\end{aligned}$$

We can define $d\mathcal{V}\equiv\sum_{r\in f,d}\lambda_{rd}(1-h^l_{rd})\widetilde{\gamma}(c^*_{rd})d\log c^*_{rd}$ and $d\mathcal{C}\equiv\sum_{r\in f,d}\sum_i^{N_{rd}}\frac{s_{rd,i}}{1-s_{rd,i}}ds_{rd,i}$ and get:

$$d\log P_d = d\mathcal{C}-d\mathcal{V}+\sum_{r\in f,d}\lambda_{rd}d\log w_r$$

The first term captures the change in the dispersion and level of markups, the second one captures the change in variety, and the last one is the price effect.

Regrouping Terms I follow ACR in how to eliminate $\lambda_{fd}d\log w_f$. Let's first calculate the change in the overall share of f in d:

$$\begin{aligned}
d\log\lambda_{fd} &= d\log P_{fd}^{1-\sigma} \\
&= h^l_{fd}d\log(P^l_{fd})^{1-\sigma}+(1-h^l_{fd})d\log(P^s_{fd})^{1-\sigma}+(1-\sigma)d\log w_f \\
&= \frac{1-\sigma}{\lambda_{fd}}\sum_i^{N_{fd}}\frac{s_{fd,i}}{1-s_{fd,i}}ds_{fd,i}+(1-h^l_{fd})\gamma(c^*_{df})d\log c^*_{fd}+(1-\sigma)d\log w_f
\end{aligned}$$

So the difference is:

$$\begin{aligned}
d\log\lambda_{fd}-d\log\lambda_{dd} &= (1-\sigma)[d\log w_f-d\log w_d]+ \\
&+ \frac{1-\sigma}{\lambda_{fd}}\sum_i^{N_{fd}}\frac{s_{fd,i}}{1-s_{fd,i}}ds_{fd,i}-\frac{1-\sigma}{\lambda_{dd}}\sum_i^{N_{dd}}\frac{s_{dd,i}}{1-s_{dd,i}}ds_{dd,i} \\
&+ \gamma(c^*_{df})d\log c^*_{fd}-\gamma(c^*_{dd})d\log c^*_{dd}
\end{aligned}$$

And we can derive for the change in wages, taking into account that $d\log w_d=0$ (numeraire):

$$
\begin{aligned}
d\log w_f &= \frac{d\log\lambda_{dd} - d\log\lambda_{fd}}{\sigma-1} + \\
&+ \frac{1}{\lambda_{dd}}\sum_i^{N_{dd}} \frac{s_{dd,i}}{1-s_{dd,i}} ds_{dd,i} - \frac{1}{\lambda_{fd}}\sum_i^{N_{fd}} \frac{s_{fd,i}}{1-s_{fd,i}} ds_{fd,i} \\
&+ \widetilde{\gamma}(c^*_{df}) d\log c^*_{fd} - \widetilde{\gamma}(c^*_{dd}) d\log c^*_{dd}
\end{aligned}
$$

Analogously for domestic firms:

$$
\begin{aligned}
d\log w_d &= \frac{d\log\lambda_{dd} - d\log\lambda_{dd}}{\sigma-1} + \\
&+ \frac{1}{\lambda_{dd}}\sum_i^{N_{dd}} \frac{s_{dd,i}}{1-s_{dd,i}} ds_{dd,i} - \frac{1}{\lambda_{dd}}\sum_i^{N_{dd}} \frac{s_{dd,i}}{1-s_{dd,i}} ds_{dd,i} \\
&+ \widetilde{\gamma}(c^*_{dd}) d\log c^*_{dd} - \widetilde{\gamma}(c^*_{d}) d\log c^*_{dd}
\end{aligned}
$$

Putting all together:

$$
\begin{aligned}
d\log P_d &= d\mathcal{C} + d\mathcal{V} + \lambda_{fd} d\log w_f + \lambda_{dd} d\log w_d \\
&= d\mathcal{C} + d\mathcal{V} + \\
&+ \sum_{r\in(d,f)} \lambda_{rd} \frac{d\log\lambda_{dd} - d\log\lambda_{rd}}{\sigma-1} + \\
&+ \sum_{r\in(d,f)} \lambda_{rd} \Big[\frac{1}{\lambda_{dd}}\sum_i^{N_{dd}} \frac{s_{dd,i}}{1-s_{dd,i}} ds_{dd,i} - \frac{1}{\lambda_{rd}}\sum_i^{N_{rd}} \frac{s_{rd,i}}{1-s_{rd,i}} ds_{rd,i}\Big] + \\
&+ \sum_{r\in(d,f)} \lambda_{rd} \Big[\widetilde{\gamma}(c^*_{rd}) d\log c^*_{rd} - \widetilde{\gamma}(c^*_{dd}) d\log c^*_{dd}\Big] \\
&= d\mathcal{C} + d\mathcal{V} + \frac{d\log\lambda_{dd}}{\sigma-1} \\
&+ \frac{1}{\lambda_{dd}}\sum_i^{N_{dd}} \frac{s_{dd,i}}{1-s_{dd,i}} ds_{dd,i} - \sum_{r\in(d,f)} \lambda_{rd} \Big[\frac{1}{\lambda_{rd}}\sum_i^{N_{rd}} \frac{s_{rd,i}}{1-s_{rd,i}} ds_{rd,i}\Big] + \\
&+ \sum_{r\in(d,f)} \lambda_{rd} \Big[\widetilde{\gamma}(c^*_{rd}) d\log c^*_{rd}\Big] - \widetilde{\gamma}(c^*_{dd}) d\log c^*_{dd}
\end{aligned}
$$

Expression The final expression is then:

$$
\begin{aligned}
d\log W_d &= -d\log\Big[1 - (\lambda_{dd})^2 (h^l_{dd})^2 HHI_{dd} - (1-\lambda_{dd})\lambda_{df} (h^l_{df})^2 HHI_{df}\Big] - \\
&- \frac{d\log\lambda_{dd}}{\sigma-1} - \\
&- d\mathcal{C}_d + \lambda_{fd}\Big(\frac{d\mathcal{C}_{fd}}{\lambda_{fd}} - \frac{d\mathcal{C}_{dd}}{\lambda_{dd}}\Big) - \\
&+ d\mathcal{V}_d + \lambda_{fd}(d\mathcal{V}_{dd} - d\mathcal{V}_{fd})
\end{aligned}
$$

A.7 Proposition 6. Gains from Variety.

The number of actual firms is a function of entrants and the productivity distribution.

$$
N = N^e G(c^*) = \frac{(c^*)^{k-\sigma-1} - c_L^{k-\sigma-1}}{c_H^{k-\sigma-1} - c_L^{k-\sigma-1}}
$$

where I used the bounded Pareto for illustration again.

Assuming a fixed number of entrants imply that changes in the number of firms capture changes in the cutoff:

$$
d\log N = \gamma(c^*) d\log c^* = (\sigma-1)\widetilde{\gamma}(c^*) d\log c^*
$$

Therefore:

$$d\mathcal{V}_d = \sum_{r\in f,d} \frac{\lambda_{rd}(1-h^l_{rd})}{\sigma-1} d\log N^s_{rd}$$

A.8 Proposition 7. Welfare-Consistent HM-CES Concentration Factor.

The aggregate change in markup term has the following functional form:

$$d\mathcal{C} \equiv \sum_{r\in f,d} \sum_{i}^{N_{rd}} \frac{s_{rd,i}}{1-s_{rd,i}} ds_{rd,i}$$

This expression depends both on the markups' functional form and the importance of each granular firm. So the question we can ask is: what is the implicit function $\mathcal{C}$ this expression defines?

The starting point is to totally differentiate $\mathcal{C}$:

$$\begin{aligned} d\mathcal{C} &= \sum_{i}^{N^l} \frac{\partial \mathcal{C}}{\partial s_i} ds_i \\ &= \sum_{i}^{N^l} \frac{s_i}{1-s_i} ds_i \end{aligned}$$

where $N^l \equiv \cup_{r=d,f} N^l_r$. Then, $\frac{\partial \mathcal{C}}{\partial s_i} = \frac{s_i}{1-s_i}$.

Therefore, we can take the anti-derivative:

$$\begin{aligned} \int d\mathcal{C} &= \int \sum_{i}^{N^l} \frac{s_i}{1-s_i} ds_i \\ &= \sum_{i}^{N^l} \int \frac{s_i}{1-s_i} ds_i \\ &= \sum_{i}^{N^l} \int \frac{s_i}{1-s_i} ds_i \\ &= \sum_{i}^{N^l} [-s_i - \log(1-s_i)] + C \end{aligned}$$

where C is a constant of integration. Since expressions are in changes, I assume $C = 0$.

Hence, $\mathcal{C} = \sum_i^{N^l} \Big[-s_i - \log(1-s_i) + C \Big]$ is a function that satisfies that $d\mathcal{C} = \sum_i^{N^l} \frac{s_i}{1-s_i} ds_i$.

B Numerical Solution

The model is characterized by the set of equilibrium conditions defined in equations 6-12. In order to solve it numerically, I nest the equilibrium condition related to large firms into the conditions related to small firms. Specifically, I propose a cutoff for domestic and foreign firms and calculate the price index for small firms. Then I solve the oligopoly game played by large firms, conditional on the price index of small firms' varieties. With both indices, I construct the overall price index and the resulting entry cutoffs. I compare the latest with the initially proposed cutoffs and if the distance is outside the established tolerance, I iterate.

Formally, let's define the set of parameters $\Theta \equiv \{\sigma, k, K, \beta, c_L, c_H\}$, and the set of exogenous variables

$\Xi \equiv \{E_d, N^e, N^l_d, N^l_f, \{c^l_{d,i}\}|_{i=1}^{N^l_d}, \{c^l_{f,i}\}|_{i=1}^{N^l_f}\}$, where $\{N^l_d, N^l_f\} \in \mathbb{Z}^+$, and a policy variable τ.[41]

Endogenous variables are $S \equiv \{c^s_{d,*}, c^s_{f,*}, P^s, \{z^l_{d,i}\}|_{i=1}^{N^l_d}, \{z^l_{f,i}\}|_{i=1}^{N^l_f}, s^l\}$.

Therefore, I conduct the following steps:

1. I propose an initial set of cutoffs $c^0 \equiv \{[c^s_{d,*}]^0, [c^s_{f,*}]^0\}$ and calculate $[P^s]^0 \equiv P^s(c^0; \tau, \Xi, \Theta)$.
2. I solve for large firm's internal market shares: $[z^l_{r,i}]^0 = z^l_{r,i}([P^s]^0; \tau, \Xi, \Theta)]$, for $i = 1 ... N^l_r$, $r \in (d,f)$, and their overall share $[s^l]^0 = s^l([P^s]^0; \tau, \Xi, \Theta)]$ conditional on the small firms' price index $[P^s]^0$.

[41] Note that by choosing N^l_d and c_{Ld}, and conditioning on c_H, the number of domestic potential entrants is determined and does not need to be added to the set of exogenous variables. The same holds for the number of potential foreign entrants.

3. I construct the large firm's price index:

   $[P^l]^0 \equiv P^l([\{z^l_{d,i}\}]^0|_{i=1}^{N^l_d}, [\{z^l_{f,i}\}]^0|_{i=1}^{N^l_f}\}, [s^l]^0; \tau, \Xi, \Theta)$.
4. I construct the overall price index $P^0 = \left[([P^s]^0)^{1-\sigma} + ([P^l]^0)^{1-\sigma}\right]^{1/(1-\sigma)}$.
5. Derive the new cutoffs $c^1 \equiv \{[c^s_{d,*}(P^0; \tau, \Xi, \Theta)], [c^s_{f,*}(P^0; \Xi, \Theta)]\}$
6. If $|c^1 - c^0| < \epsilon$, then the problem is solved, otherwise I use c^1 to iterate the process.

This iterative process delivers a solution $S(\tau_0, T_{f0})$ conditional on the value of the trade policy variable τ_0. Given that each solution depends on the specific draw of large firms productivities, I solve the model U times in each case and average the result. Therefore, I use for each endogenous variable $S^y \in S$ the following: $S^y(\tau_0, T_{f0}) = \frac{\sum_{u=1}^{U} S^y_u(\tau_0, T_{f0})}{U}$, where $U = 1000$.

In the numerical exercise, I vary either τ and T_{f0}, depending the case, such that I get a set of solutions for the R values of these variables, τ: $\bar{S} \equiv \{S_1(\tau_1), ..., S_R(\tau_R)\}$, or w_f: $\bar{S} \equiv \{S_1(T_{f1}), ..., S_R(\tau_{fR})\}$.

C Multi-Country and Multi-Industry Model

In order to bring the model to the data, let's extend it to account for multiple sectors indexed by x. I assume fixed importer-specific expenditure shares α_{jx} (i.e. Cobb-Douglas).[42]

Total change in welfare is:

$$\begin{aligned} d\log W_i &= d\log Y_i - d\log P_i \\ &= d\log Y_i - \sum_{\Omega^X} \alpha_{jx} d\log P_{ix} dx \end{aligned}$$

Equilibrium Conditions We need the following conditions to hold:

1. Domestic and foreign good market clearing (GMC)
2. Domestic and foreign labor market clearing (LMC)
3. Trade Balance (TB)

They determine: wages, expenditure and price indices in each country. I include conditions for the domestic economy d, and I normalize its wages $w_d = 1$.

Good market clearing conditions:

$$Y_i = \sum_j \sum_x X_{ijx}$$

Labor market clearing conditions:

$$E_i = Y_i \equiv w_i L_i + \sum_j \sum_x \Pi_{ijx}$$

Trade Balance:

$$\sum_i \sum_x X_{ijx} = \sum_i \sum_x X_{jix}$$

Profits

Large Firms I assume firms act in different sectors, so domestic firms derive profits as before:

[42] I could assume a continuum of sectors as in Gaubert and Itskhoki (2021), given that in this case it is rational for large firms to not consider their effect on aggregates. Given that I bring it to the data, I assume a discrete number of sectors for exposition.

$$
\begin{aligned}
\Pi^l_{ijx} &= \sum_{i=1}^{N_{ijx}} \pi_{ij,k} \\
&= \sum_{k=1}^{N_{ijx}} (p_{ijx,k} - c_{ijx,k} w_i) q_{ijx,k} \\
&= \sum_{k=1}^{N_{ijx}} (1 - c_{ijx,k} w_i / p_{ijx,k}) q_{ijx,k} p_{ijx,k} \\
&= \sum_{k=1}^{N_{ijx}} \Big[1 - \frac{c_{ijx,k} w_i}{\tilde{\mu}(1 - s_{ijx,k})^{-1} w_i c_{ijx,k}}\Big] r_{ijx,k} \\
&= \sum_{k=1}^{N_{ijx}} \Big[1 - \frac{(\sigma - 1)(1 - s_{ijx,k})}{\sigma}\Big] r_{ijx,k} \\
&= \frac{\lambda_{ijx} h^l_{ijx} \alpha_{jx} E_j}{\sigma} \sum_{k=1}^{N_{ijx}} \Big[\sigma - (\sigma - 1)(1 - s_{ijx,k})\Big] z_{ijx,k} \\
&= \frac{\lambda_{ijx} h^l_{ijx} \alpha_{jx} E_j}{\sigma} \Big[1 + (\sigma - 1) \lambda_{ijx} h^l_{ijx} HHI_{ijx}\Big]
\end{aligned}
$$

Note that $\lambda_{ijx} \equiv \frac{P^{1-\sigma}_{ijx}}{P^{1-\sigma}_{jx}}$ is the share of j's imports in sector x from i, which is a different object.

Small Firms Doing a similar derivation, we get the following:

$$
\Pi^s_{ijx} = \frac{\lambda_{ijx}(1 - h^l_{ijx}) \alpha_{jx} E_j}{\sigma} \Big[1 + (\sigma - 1) \lambda_{ijx} h^l_{ijx} HHI_{ijx}\Big]
$$

Total Profits For a given sector-partner, we have that profits are:

$$
\Pi_{ijx} = \frac{\lambda_{ijx} \alpha_{jx} E_j}{\sigma} \Big[1 + (\sigma - 1) \lambda_{ijx} (h^l_{ijx})^2 HHI_{ijx}\Big]
$$

Now to get total profits, we need to sum across all sectors and importers:

$$
\begin{aligned}
\Pi_i &= \sum_j \sum_x \frac{\lambda_{ijx} \alpha_{jx} E_j}{\sigma} \Big[1 + (\sigma - 1) \lambda_{ijx} (h^l_{ijx})^2 HHI_{ijx}\Big] \\
&= \frac{\sum_j \sum_x \lambda_{ijx} \alpha_{jx} E_j}{\sum_j \sum_x \lambda_{ijx} \alpha_{jx} E_j} \sum_j \sum_x \frac{\lambda_{ijx} \alpha_{jx} E_j}{\sigma} \Big[1 + (\sigma - 1) \lambda_{ijx} (h^l_{ijx})^2 HHI_{ijx}\Big] \\
&= Y_i \sum_j \sum_x \frac{a_{ijx}}{\sigma} \Big[1 + (\sigma - 1) \lambda_{ijx} (h^l_{ijx})^2 HHI_{ijx}\Big] \\
&= \frac{Y_i}{\sigma} \Big[1 + (\sigma - 1) \sum_j \sum_x a_{ijx} \lambda_{ijx} (h^l_{ijx})^2 HHI_{ijx}\Big]
\end{aligned}
$$

where I used that $Y_i = \sum_j \sum_x \lambda_{ijx} \alpha_{jx} E_j$ (labor market condition) and defined $a_{ijx} \equiv \frac{\lambda_{ijx} \alpha_{jx} E_j}{\sum_j \sum_x \lambda_{ijx} \alpha_{jx} E_j}$, which is the contribution of x exports to j to total income (since it contains domestic sales as well).

Welfare

Income The income equation is:

$$
\begin{aligned}
Y_i &= w_i L_i + \Pi_i \\
&= w_i L_i + \frac{Y_i}{\sigma} \Big[1 + (\sigma - 1) \sum_j \sum_x a_{ijx} \lambda_{ijx} (h^l_{ijx})^2 HHI_{ijx}\Big]
\end{aligned}
$$

From where we can get the expression for income:

$$
\begin{aligned}
Y_i &= \frac{\sigma w_i L_i}{\sigma - \left[1 + (\sigma-1)\sum_j \sum_x a_{ijx}\lambda_{ijx}(h^l_{ijx})^2 HHI_{ijx}\right]} \\
&= \frac{\sigma w_i L_i}{(\sigma-1)(1 - \sum_j \sum_x a_{ijx}\lambda_{ijx}(h^l_{ijx})^2 HHI_{ijx}\big]} \\
&= \frac{\sigma}{\sigma-1}\frac{w_i L_i}{1 - \sum_j \sum_x a_{ijx}\lambda_{ijx}(h^l_{ijx})^2 HHI_{ijx}}
\end{aligned}
$$

That this equation is similar to the markup one, and shows there is an "excess return" to the labor income (i.e. profits). The higher is the concentration, the higher is this return.

Therefore, the change in income can be measure as follows for the domestic economy (considering that the numeraire is w_i):

$$
d\log Y_i = -d\log\left[1 - \sum_j \sum_x a_{ijx}\lambda_{ijx}(h^l_{ijx})^2 HHI_{ijx}\right]
$$

Price Index The total change in the price index in i is:

$$
\begin{aligned}
d\log P_i &= \sum_x \alpha_{ix} d\log P_{ix} \\
&= \sum_x \alpha_{ix} \sum_j \lambda_{jix} d\log P_{jix}
\end{aligned}
$$

The price index of an exporter-importer-sector is:

$$
d\log P_{jix} = h^l_{jix}\Big[\sum_k^{N_{jix}} \frac{z_{jix,k}}{1 - s_{jix,k}} ds_{jix,k}\Big] - (1 - h^l_{jix})\widetilde{\gamma}(c^*_{jix}) d\log c^*_{jix} + d\log w_j
$$

So we replace as follows:

$$
\begin{aligned}
d\log P_i &= \sum_x \alpha_{ix} d\log P_{ix} \\
&= \sum_x \alpha_{ix} \sum_j \lambda_{jix}[h^l_{jix}\Big[\sum_k^{N_{jix}} \frac{z_{jix,k}}{1 - s_{jix,k}} ds_{jix,k}\Big] - (1 - h^l_{jix})\widetilde{\gamma}(c^*_{jix}) d\log c^*_{jix} + d\log w_j] \\
&= \sum_x \alpha_{ix} \sum_j \Big[\sum_k^{N_{jix}} \frac{s_{jix,k}}{1 - s_{jix,k}} ds_{jix,k}\Big] - \lambda_{jix}(1 - h^l_{jix})\widetilde{\gamma}(c^*_{jix}) d\log c^*_{jix} + \lambda_{jix} d\log w_j] \\
&= \sum_x \alpha_{ix} d\mathcal{C}_{ix} - \sum_x \alpha_{ix} d\mathcal{V}_{ix} + \sum_x \sum_j \alpha_{ix}\lambda_{jix} d\log w_j
\end{aligned}
$$

We take the difference as before, but considering a specific sector:

$$
\begin{aligned}
d\log \lambda_{jix} - d\log \lambda_{iix} &= (1-\sigma)[d\log w_j - d\log w_i] + \\
&+ \frac{1-\sigma}{\lambda_{jix}} \sum_i^{N_{jix}} \frac{s_{jix,k}}{1 - s_{jix,k}} ds_{jix,k} - \frac{1-\sigma}{\lambda_{iix}} \sum_i^{N_{iix}} \frac{s_{iix,k}}{1 - s_{iix,k}} ds_{iix,k} \\
&+ \gamma(c^*_{jix}) d\log c^*_{jix} - \gamma(c^*_{iix}) d\log c^*_{iix}
\end{aligned}
$$

And we can derive for the change in wages, and again taking into account that $d\log w_i = 0$ (numeraire):

$$
\begin{aligned}
d\log w_j &= \frac{d\log \lambda_{iix} - d\log \lambda_{jix}}{\sigma - 1} + \\
&+ \frac{1}{\lambda_{iix}} \sum_i^{N_{iix}} \frac{s_{iix,k}}{1 - s_{iix,k}} ds_{iix,k} - \frac{1}{\lambda_{jix}} \sum_i^{N_{jix}} \frac{s_{jix,k}}{1 - s_{jix,k}} ds_{jix,k} \\
&+ \widetilde{\gamma}(c^*_{jix}) d\log c^*_{jix} - \widetilde{\gamma}(c^*_{iix}) d\log c^*_{iix}
\end{aligned}
$$

And replace back:

$$
\begin{aligned}
d\log P_i &= \sum_x \alpha_{ix} d\mathcal{C}_{ix} - \sum_x \alpha_{ix} d\mathcal{V}_{ix} + \\
&+ \sum_x \alpha_{ix} \sum_j \lambda_{jix} \Big[\frac{d\log\lambda_{iix} - d\log\lambda_{jix}}{\sigma - 1} + \\
&+ \frac{1}{\lambda_{iix}} \sum_i^{N_{iix}} \frac{s_{iix,k}}{1 - s_{iix,k}} ds_{iix,k} - \frac{1}{\lambda_{jix}} \sum_i^{N_{jix}} \frac{s_{jix,k}}{1 - s_{jix,k}} ds_{jix,k} \\
&+ \widetilde{\gamma}(c^*_{jix}) d\log c^*_{jix} - \widetilde{\gamma}(c^*_{iix}) d\log c^*_{iix} \Big] \\
&= \sum_x \alpha_{ix} d\mathcal{C}_{ix} - \sum_x \alpha_{ix} d\mathcal{V}_{ix} + \sum_x \alpha_{ix} \frac{d\log\lambda_{iix}}{\sigma - 1} \\
&+ \sum_x \alpha_{ix} \left[\frac{d\mathcal{C}_{iix}}{\lambda_{iix}} - \sum_j \lambda_{jix} \Big[\frac{d\mathcal{C}_{jix}}{\lambda_{jix}} \Big] \right] \\
&+ \sum_x \alpha_{ix} \left[\sum_j \lambda_{jix} d\mathcal{V}_{jix} - d\mathcal{V}_{iix} \right]
\end{aligned}
$$

Expression Using the fact that the variety term can be captured by the number of firms as before, we get the following ACR-type welfare formula:

$$
\begin{aligned}
d\log W_i &= \underbrace{-d\log\left[1 - \sum_j \sum_x a_{ijx} \lambda_{ijx} (h^l_{ijx})^2 HHI_{ijx}\right]}_{\text{Excess Profits Effect}} - \underbrace{\sum_x \alpha_{ix} d\mathcal{C}_{ix}}_{\text{Pro-Competitive Effect}} + \underbrace{\sum_x \alpha_{ix} d\mathcal{V}_{ix}}_{\text{Variety Effect}} - \\
&- \underbrace{\sum_x \alpha_{ix} \frac{d\log\lambda_{iix}}{\sigma - 1}}_{\text{Price Effect (ACR)}} + \underbrace{\sum_x \alpha_{ix} \left[\frac{d\mathcal{C}_{iix}}{\lambda_{iix}} - \sum_j \lambda_{jix} \Big[\frac{d\mathcal{C}_{jix}}{\lambda_{jix}} \Big] \right] + \sum_x \alpha_{ix} \left[\sum_j \lambda_{jix} d\mathcal{V}_{jix} - d\mathcal{V}_{iix} \right]}_{\text{Relative Variety Adj.}}
\end{aligned}
$$