

Presentation Week04: Recursion & Data types

RECURSION:

Using the function you are defining within itself.

For every recursive definition you NEED a base case. This means that at one point, the function must define some sort of behavior that does not use itself in the definition to end the recursive call. Without a base case, the function would call itself recursively indefinitely.

DEFINING MAP (INTRO TO RECURSION)

- In order to define map, we need to **recurse** through it's input list

```
map :: (a -> b) -> [a] -> [b]
map f [] = []
map f (x:xs) = f x : (map f xs)
```

```
Map f [] = []          ---->    BASE CASE
Map f(x:xs)          -----> Recursive portion
```

Example :

```
F:    add x = x + 1
```

```
x:xs = [1,2,3]
```

```
map f (x:xs) = add [1] : map add [2,3]
              =add [1] : add[2] : map add [3]
              =add [1] : add[2] : add[3] : map add [].    ---This is why a base case is needed, if there was
              no definition 'map f [] = [] '              the function would not stop calling itself.

              =add [1] : add[2] : add[3] : []
              =add [1] : add[2] : [4] : []
              =add [1] : [3] : [4] : []
              = [2] : [3] : [4] : []
              =[2,3,4]
```

** remember colon operator adds an element to the beginning of a list.

** Pattern matches to one of the definitions of map at every call of itself, until a base case is reached

Recursive definition of Factorial Function:

```
factorial :: (Eq a, Num a) => a -> a
factorial 0 = 1
factorial x = x * factorial(x-1)
```

Factorial definition pattern matches on the input, and gives output

RECURSIVE FUNCTIONS

Evaluation:

```
factorial 3
=
3 * factorial 2
=
3 * (2 * factorial 1)
=
3 * (2 * (1 * factorial 0))
=
3 * (2 * (1 * 1))
=
3 * (2 * 1)
=
3 * 2
=
6
```

Another Example:

```
pow :: (Eq a, Num a) => a -> a -> a
pow m 0 = 1
pow m n = m * pow m (n-1)
```

i.e. produces m^n output, recursively.

What happens if didn't have the base case? Well..

```
Pow 3 2 = 3 * pow 3 1
        = 3 * (3* pow 3 0)
        = 3 * (3* (3 * pow 3 -1) < -- never ends!
```

Recursive Types

-- Can define own types by the 'data' keyword.

RECURSIVE TYPES

- We can use recursion to define types with an infinite amount of values

```
data Nat = Zero | Succ Nat
```

- Nat is a new type, with constructors

```
Zero :: Nat
-- and
Succ :: Nat -> Nat
```

I.e. Nat rebuilds natural number (any positive whole number)

So now, 'Zero' and 'Succ Nat' are of type 'Nat'

$$\text{Zero} = 0$$

$$\text{Succ (Succ Zero)} = 2$$

*When defining out own data types must write: 'deriving Show' In order for ghci to print.

i.e.

```
data Nat = Zero | Succ Nat -- where successor is plus 1 of another Natural Number
    deriving Show --must write this for ghci to print, must turn into string
-- makes an instant of the show class
```

Zero	0
Succ Zero	1
Succ Succ Zero	2

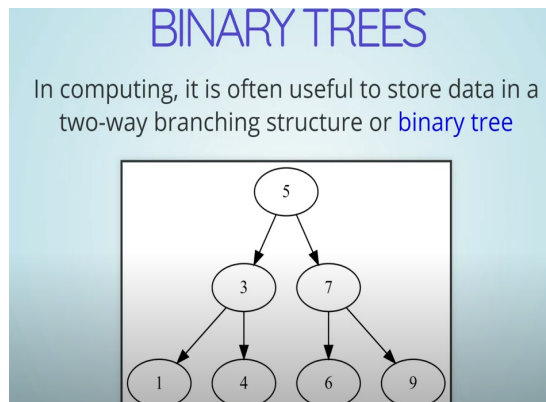
```
nat2int :: Num p => Nat -> p
nat2int Zero = 0
nat2int (Succ n) = 1 + (nat2int n)
```

Defining nat2int recursively, since nat is a recursive type with infinite amount of values.
I.e.

```
Nat2int (Succ Succ Zero) = 1 + nat2int (Succ Zero)
                        = 1 + 1 + nat2int Zero
                        = 1 + 1 + 0
                        = 2
```

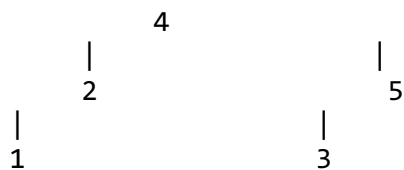
** now that we defined data type, we can use this data type and create functions.

Binary Tree



Binary trees contain Nodes
nodes with no branches at all, called leaves
Each node has two branches, child nodes; HENCE BINARY

Sorted Trees : Every node/leaf on left side of Node is LESS THAN node.
: Every node /leaf on right side of Node is GREATER THAN node



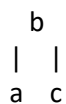
[1,2,3,4,5]

```
-- d
-- / \
-- b f
-- / \ / \
-- a c e g
```

Tree1 = Node 'b' (Leaf 'a') (Leaf 'c')

Node 'd' (Tree1) (Tree t2)

Node 'd' (Node b (Leaf a) (Leaf c)) (Tree t2)



Lists:

- Groups together values
- Lists are created by putting values inside square brackets separated by commas [1,2,3,4,5]

List comprehension

LIST COMPREHENSION

In Haskell, a similar **comprehension** notation can be used to construct new **lists** from old lists

```
[x^2 | x <- [1..5]]
```

The list [1, 4, 9, 16, 25] of all numbers x^2 such that x is an element of the list [1..5]

- Can specify new lists from old lists

`X<-[1..5]` called a GENERATOR ! List comprehensions can have multiple generators

- Comprehensions can have **multiple generators**

```
[(x,y) | x <- [1,2,3], y <- [4,5]]  
== [(1,4),(1,5),(2,4),(2,5),(3,4),(3,5)]
```

- Try switching the position of the generators around, what happens?

i.e. `[(x,y) | y<-[4,5], x<-[1,2,3]]`
`== [(1,4),(2,4),(3,4),(1,5),(2,5),(3,5)]`. <<--- changed the order

Guards

Restrict values produced by generator.

i.e.

List = `[x | x< [1..10], even x]`

What will be produced? ->> [2,4,6,8,10]

Type Declaration

A new name for an existing type

i.e. Data declarations CREATE NEW types, will type declarations RENAME EXISTING types

i.e. `type Pos = (Int , Int)`

`String = [Char]` -> list of char ; type alias or type synonym

- Main use for them would be to make code more readable.

```
origin :: Pos
origin = (0,0)
```

Could've easily had it as:

```
origin :: (Int, Int)
origin :: (0,0)
```

Increases readability!!

Type Parameters:

`type Pair a = (a, a)` -- can now utilize polymorphism

Nested Types:

`type Trans = Pos -> Pos`

We created the type "pos" and nested it creating another type "Trans"

When creating data types: i.e. `:t Circle` (w/o the Float) would return signature `Float -> Shape` , treats it as if it were a function definition. This type must be completed with a float to be of type 'Shape'

Data Parameters

- Data declarations can have parameters (can involve polymorphic variables)
i.e. `data Maybe a = Nothing | Just a`

*This will take in a value of any type 'a' and return either Nothing OR Just 'a'; i.e. can create multiple varieties of this type.

DERIVING

Deriving (Show, Eq) will automatically create instances of the type class mentioned i.e.

```
data Maybe1 a = Nothing1 | Just1 a
  deriving (Show, Eq)
```

```
Deriving in the prim class derives
instance [safe] Eq a => Eq (Maybe1 a)
  -- Defined at /Users/alessandraguerinoni/Desktop/1JC3/Haskell Tutorial Projects/Week04Tut/src/Lib.hs:284:21
```

Type Constructors vs Data Constructors:

Constructor:

Constructor can mean:

Type constructor

Data constructor (or value constructor)

```
Data Bool = True | False
```

A **type constructor** is the thing on the left hand side of the equals sign.

The **data constructor(s)** are the things on the right hand side of the equals sign.

You use type constructors where a type is expected & you use data constructors where a value is expected.

Data constructors

```
data Colour = Red | Green | Blue
```

Here, we have three data constructors.

Colour is a type, and Red, Green and Blue are constructors that contain a value of type Colour.

A data constructor either contains a value like a variable would, or takes other values as its argument and creates a new value.

```
data SBTTree = Leaf String
             | Branch String SBTTree SBTTree
```

type SBTTree that contains two data constructors.

there are two functions (namely Leaf and Branch) that will construct values of the SBTTree type.

Type constructors

BTree is a type constructor.

```
data BTree a = Leaf a
             | Branch a (BTree a) (BTree a)
```

type variable 'a' as a parameter to the type constructor.

In this declaration, BTree has become a function. It takes a type as its argument and it returns a new type.

If we pass in, say, Bool as an argument to BTree, it returns the type BTree Bool,

If you want to, you can view BTree as a function with the kind

`BTree :: * -> *` -- these are called "Kinds"

BTree is from a concrete type -> concrete type.

Concrete type (examples include Int, [Char] and Maybe Bool) which is a type that can be assigned to a value

Wrapping up

A **data constructor** is a "function" that takes 0 or more values and gives you back a new value.

A **type constructor** is a "function" that takes 0 or more types and gives you back a new type.

```
data Maybe a = Nothing
             | Just a
```

Here, Maybe is a type constructor that returns a concrete type.

Just is a data constructor that returns a value.

Nothing is a data constructor that contains a value.

```
Maybe :: * -> *
```

In other words, Maybe takes a concrete type and returns a concrete type.