University of The Punjab (Gujranwala Campus)

"Computer Vision (Assignment)"

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Q:- Calculate the optimal threshold Using Otsu Method

Given:-

Gray scale image : {0, 0, 1, 1, 2, 2, 3, 3, 4, 4, 5

So the histogram is:

Pixel value (i)	Frequency p(i)
0	2
1	2
2	2
3	2
4	2
5	1

Calculation:

$N = 2+2+2+2+2+1 = 11$

(i) Compute the Mean Class Variance

$$\sigma_b^2(T) = P(w_0) \cdot P(w_1) \cdot (\mu_0 - \mu_1)^2$$

where

(1) $P(w_0) = \sum_{i=0}^{T=1} \frac{p(i)}{N}$ // (Probability of pixel belonging to background (class 0))

(2) $P(w_1) = \sum_{i=T}^{255} \frac{p(i)}{N}$ // (Probability of pixel belonging to foreground (class 1))

As, sum of all probabilites equals to 1, so we can calculate $P(w_1)$ as:

$$P(W_0) + P(W_1) = 1$$

$$P(W_1) = 1 - P(W_0)$$

(3) $\mu_0 = \sum_{i=0}^{T=1} \frac{i \cdot p(i)}{P(W_0)}$ (Mean pixel value of the background class (class 0))

(4) $\mu_1 = \sum_{i=T}^{255} \frac{i \cdot p(i)}{P(W_1)}$ (Mean pixel value of the foreground class (class 1))

Possible Thresholds:

$T = 0, T = 1, T = 2, T = 3, T = 4$

(ii) Compute Otsu's Between Class-Variance for each T:

• For T = 0 :

As Background : pixel values $\leq T$
Foreground : Pixel values $> T$

So,

Background = {0}
Foreground = {1, 2, 3, 4, 5}

$p(i) = \frac{\text{Frequency}}{N}$
$p(0) = 2/11$
$p(1) = 2/11$
$p(2) = p(3) = p(4) = 2/11$
$p(5) = 1/11$

$P(w_0) = 2/11$

$P(w_1) = 1 - 2/11 = 9/11$

$\mu_0 = \frac{(0 \times 2/11)}{2/11} = 0$

$\mu_1 = \frac{(1\times2)+(2\times2)+(3\times2)+(4\times2)+(5\times1)/11}{9/11}$

$= \frac{2+4+6+8+5}{9}$

$= 25/9 = 2.78$

$\sigma_b^2(T) = \frac{2}{11} \times \frac{9}{11} (0 - 2.78)^2 = 1.15$

• For T = 1 :

Background : {0, 1}
Foreground : {2, 3, 4, 5}

$P(w_0) = \frac{2+2}{11} = \frac{4}{11}$

$P(w_1) = 1 - \frac{4}{11} = \frac{11-4}{11} = \frac{7}{11}$

$\mu_0 = \frac{[(0\times2)+(1\times2)]/11}{4/11} = 0.5$

CamScanner

$$\mu_1 = \frac{[(2\times2)+(3\times2)+(4\times2)+(5\times1)]/11}{7/11} = \frac{23}{7} = 3.30$$

$$\sigma_b^2(T) = \frac{4}{11} \times \frac{7}{11} \times (0.5 - 3.30)^2 = 1.81$$

For $T = 2$:

Background $= \{0, 1, 2\}$

Foreground $= \{3, 4, 5\}$

$P(w_0) = 6/11$

$P(w_1) = 1 - 6/11 = 5/11$

$$\mu_0 = \frac{(0\times2)+(1\times2)+(2\times2)}{6} = 1$$

$$\mu_1 = \frac{(3\times2)+(4\times2)+(5\times1)}{5} = 3.8$$

$$\sigma_b^2(T) = \frac{6}{11} \times \frac{5}{11} \times (1 - 3.8)^2 = 1.94$$

For $T = 3$:

Background $= \{0, 1, 2, 3\}$

Foreground $= \{4, 5\}$

$P(w_0) = (2+2+2+2)/11 = 8/11$

$P(w_1) = 1 - 8/11 = 3/11$

$$\mu_0 = \frac{(0\times2)+(1\times2)+(2\times2)+(3\times2)}{8} = 1.5$$

$$\mu_1 = \frac{(4\times2)+(5\times1)}{3} = 4.33$$

$$\sigma_b^2(T) = \frac{8}{11} \times \frac{3}{11} \times (1.5 - 4.33)^2 = 1.59$$

For $T=4$:

Background : $\{0,1,2,3,4\}$
Foreground : $\{5\}$

$$P(w_0) = (2+2+2+2+2)/11 = 10/11$$
$$P(w_1) = 1 - \frac{10}{11} = 1/11$$

$$\mu_0 = \frac{(0\times2)+(1\times2)+(2\times2)+(3\times2)+(4\times2)}{10} = 2$$

$$\mu_0 = \frac{5\times1}{1} = 5$$

$$\delta_b^2(T) = P(w_0)\cdot P(w_1)\cdot(\mu_0-\mu_1)^2$$
$$= \frac{10}{11}\times\frac{1}{11}\times(2-5)^2$$
$$= 0.74$$

(3) Choosing Final Threshold Value

Comparing all ~~possible threshold~~ between-class variance value

$T=0 \longrightarrow 1.15$
$T=1 \longrightarrow 1.81$
$T=2 \longrightarrow 1.94$ (Max)
$T=3 \longrightarrow 1.59$
$T=4 \longrightarrow 0.74$

So, the optimal threhold is $\boxed{T=2}$