Introduction to Parallel Computing
A.Y. 2023/2024
Homework 2: Parallelizing matrix operations using OpenMP
Results report

Alessandro Iepure, 228023
alessandro.iepure@studenti.unitn.it
`https://github.com/aleiepure/Parallel-assignment2`

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Abstract

This document analyzes the results and methodologies used while working on the assignment in the title. All work is original and done without input from outside sources except where explicitly noted.

1 Parallel matrix multiplication

The first task asks for the implementation of a parallel matrix by matrix multiplication algorithm in *C* using OpenMP. The goal of the task is to check the scalability of the code and evaluate its efficiency.

The most trivial method is called "row by column" and involves computing each element of the resulting matrix by taking the dot product of a row of the first matrix and the corresponding column of the second one. Other algorithms and techniques are more complex[4, 3] and exploit the hardware of the machine (registers, cache and memory alignment, etc.) or the content of the matrices to get better performance. Such methodologies are "row by row", "column by column", "Strassen's", and many others.

For the sake of simplicity, I choose to implement the "row by column" method. It was designed with dense matrices in mind but works fine with sparse matrices without exploiting their benefits, like reduced memory usage and data reutilization. I opted for this method as, in my assessment, the more complex ones surpass the scope of this assignment.

The algorithm was implemented sequentially and later I introduced OpenMP directives to obtain the parallel code.
Source code 1 shows the parallelized function with the directives used. The choice of which ones to use was made after reading the professor's material on the subject [6], OpenMP's 3.1 documentation [1, 2], and trying various combinations. In the end, I decided to use the following: `#pragma omp parallel for collapse(2) reduction(+:temp)`. In particular:

- `#pragma omp parallel`: creates a parallel region in which all code is distributed among the specified number of CPUs. The amount is controlled externally by the `OMP_NUM_THREADS` environment variable.
- `for`: `for`-loops are going to be distributed between available CPUs.
- `collapse(2)` applies to the previous `for` directive and specifies the number of nested loops to divide among the CPUs.
- `reduction(+:temp)` specifies the variable and the operation on which to perform the reduction.

Source code 1: Implemented algorithm

```
float **matMulPar(float **A, float **B, int a_rows, int a_columns, int b_rows, int b_columns) {
    double wt1, wt2;
    float temp;
    float **C = allocateMatrix(a_rows, b_columns);

    wt1 = omp_get_wtime();
```

```
#pragma omp parallel for collapse(2) reduction(+:temp)
    for (int i = 0; i < a_rows; i++) {
        for (int j = 0; j < b_columns; j++) {
            temp = 0;
            for (int k = 0; k < a_columns; k++) {
                temp += A[i][k] * B[k][j];
            }
            C[i][j] = temp;
        }
    }
    wt2 = omp_get_wtime();

    printf("%f\n", wt2 - wt1);
    return C;
}
```

Before the final run on the University's HPC cluster (GCC 4.8.5, OpenMP 3.1), the code was tested and debugged on my local machine, a laptop equipped with an Intel® Core™ i5-8300H at 2.30GHz and 16GB of RAM running Fedora 39 (GCC 13.2.1, OpenMP 4.5).
As an effort to simplify the compilation and execution of the various iterations of the code with different values, I wrote a *bash* script which later became the PBS job submission file. This script is responsible for creating the necessary directories on the file system, for compilation, and execution with the various values. The results are returned in the form of a series of `.CSV` files which are easier to work on with other programs.

Results analysis

The implemented algorithm was run with dense square matrices of sizes ranging from 200 to 2000 with up to 32 CPUs. The node responsible for the computation was `hpc-c11-node22.unitn.it` equipped with an Intel® Xeon® Gold 6252N CPU running at 2.30GHz and 256GB of RAM. The obtained execution times are reported in Table 1 alongside the FLOPS calculated using Equation 1 and expressed as GFLOPS.

$$\text{FLOPS} = \frac{2 \cdot \text{A rows} \cdot \text{A columns} \cdot \text{B columns}}{\text{execution time}} = \frac{2 \cdot \text{size}^3}{\text{execution time}} \tag{1}$$

Table 1: Matrix multiplication - Run times and FLOPS

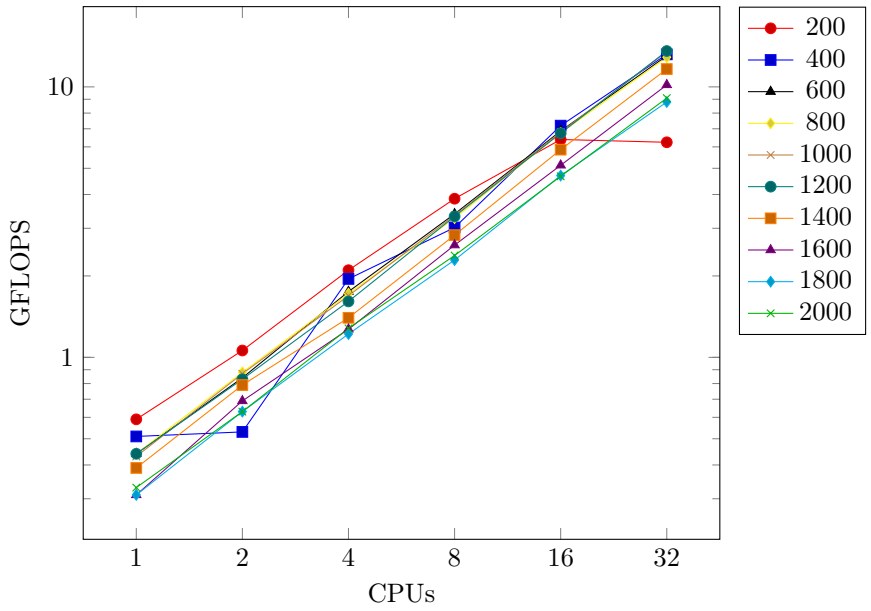
Size	Cores	Run Time [s]	GFLOPS	Size	Cores	Run Time [s]	GFLOPS
200	1	0.0272	0.59	1200	1	7.9264	0.44
	2	0.0150	1.06		2	4.1837	0.83
	4	0.0076	2.10		4	2.1500	1.61
	8	0.0041	3.86		8	1.0419	3.32
	16	0.0025	6.39		16	0.5124	6.74
	32	0.0026	6.24		32	0.2546	13.57
400	1	0.2509	0.51	1400	1	14.1042	0.39
	2	0.2405	0.53		2	6.9411	0.79
	4	0.0656	1.95		4	3.9240	1.40
	8	0.0424	3.02		8	1.9400	2.83
	16	0.0178	7.19		16	0.9362	5.86
	32	0.0097	13.19		32	0.4717	11.64
600	1	0.9862	0.44	1600	1	26.1967	0.31
	2	0.5122	0.84		2	11.8077	0.69
	4	0.2474	1.75		4	6.4396	1.27
	8	0.1273	3.39		8	3.1493	2.60
	16	0.0629	6.87		16	1.5955	5.13
	32	0.0331	13.03		32	0.8052	10.17
800	1	2.3327	0.44	1800	1	37.6769	0.31
	2	1.1591	0.88		2	18.6574	0.63
	4	0.5950	1.72		4	9.5689	1.22
	8	3.3135	3.27		8	5.1028	2.29

Table 1: Matrix multiplication - Run times and FLOPS

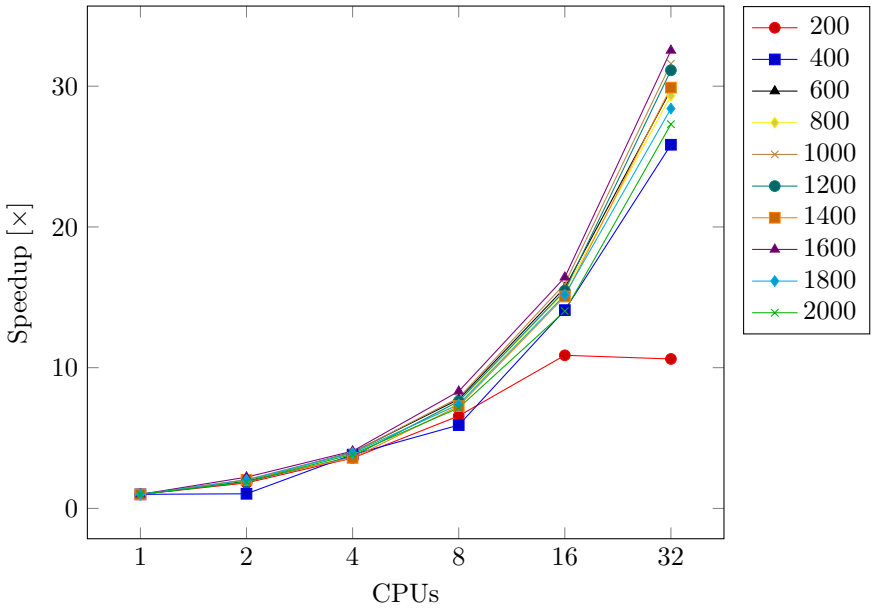
Size	Cores	Run Time [s]	GFLOPS	Size	Cores	Run Time [s]	GFLOPS
	16	0.1514	6.76		16	2.4839	4.70
	32	0.0769	12.87		32	1.3263	8.79
1000	1	4.6862	0.43	2000	1	48.0570	0.33
	2	2.2981	0.87		2	25.5150	0.63
	4	1.1806	1.69		4	12.4870	1.28
	8	0.5962	3.35		8	6.7303	2.38
	16	0.2936	6.81		16	3.4271	4.67
	32	0.1483	13.49		32	1.7600	9.09

The parallel scalability of the algorithm was evaluated using the FLOPS as the performance metric. Plot 1, Plot 2, and Plot 3 allow us to see a visual representation of the performance, the speedup, and the efficiency, respectively.

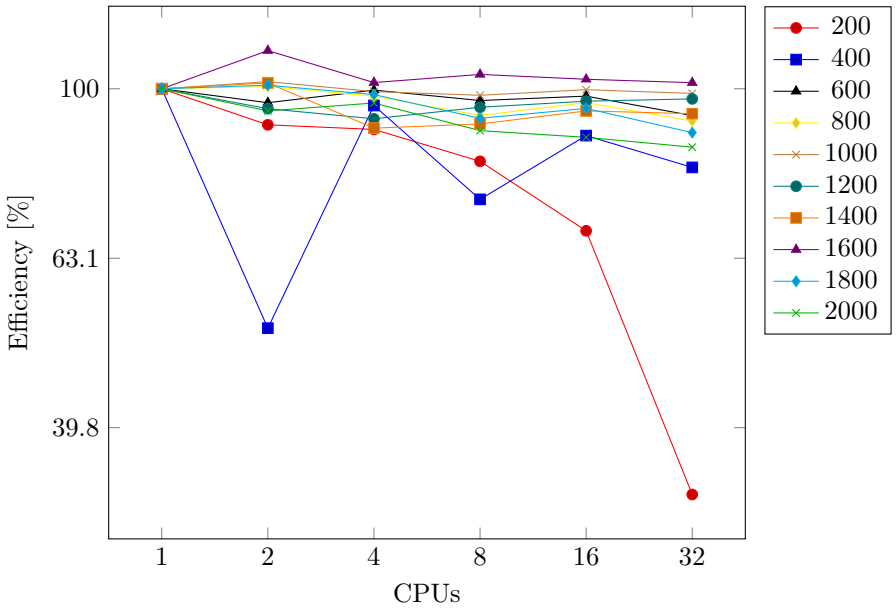
Plot 1: Parallel Matrix Multiplication - Performance graph by size of matrices



Plot 2: Parallel Matrix Multiplication - Speedup graph by size of matrices



Plot 3: Parallel Matrix Multiplication - Efficiency graph by size of matrices



It is easy to see that the execution has a better performance (Plot 1) the more CPUs it can utilize because of the higher number of operations it can execute per second. The same consideration is also valid for the speedup (Plot 2): the more CPUs, the faster the calculations are executed.
The spikes observable in the efficiency graph (plot 3) are because parallel systems are nonlinearly scalable. We must choose either a constant run time or a constant efficiency when scaling up the problem [5]. The general downward slope can be interrupted by undefined peaks caused by imbalance problems. In case in exam, it is particularly evident with 200×200 matrices (red line) and 400×400 matrices (blue line).

2 Parallel matrix transposition

The second task asks for the implementation of a matrix transposition algorithm in two different ways, written in *C* and parallelized with OpenMP. The end goal is to compare the scalability of the two versions and evaluate their efficiency.

Transposing a matrix involves flipping it over its main diagonal, exchanging rows and columns, resulting in a new matrix where the rows of the original one become the columns of the transposed matrix and vice versa. The first algorithm does verbatim what described above, while the other divides the original matrix into blocks of fixed sizes, transposes them using the same principle, and finally transposes each block's content.

Both algorithms were written and tested sequentially and parallelized later by introducing the OpenMP directives visible in source code 2. The choice of which ones to use was made after reading the professor's material on the subject [6], OpenMP's 3.1 documentation [1, 2], and trying various combinations[1].

Source code 2: Implemented algorithm

```
float **matTpar(float **M, int rows, int cols) {
    double wt1, wt2;
    float **T = allocateMatrix(cols, rows);

    wt1 = omp_get_wtime();
#pragma omp parallel for collapse(2)
    for (int i = 0; i < rows; i++)
        for (int j = 0; j < cols; j++)
            T[j][i] = M[i][j];
    wt2 = omp_get_wtime();

    printf("%f, ", wt2 - wt1);
    return T;
}
```

[1] For an explanation of each part of the directive, refer to Task 1, Parallel matrix multiplication.

```
float **matBlockTpar(float **M, int rows, int cols, int b_rows, int b_cols) {
    double wt1, wt2;
    float **T = allocateMatrix(cols, rows);

    wt1 = omp_get_wtime();
#pragma omp parallel for collapse(4)
    for (int i = 0; i < rows; i += b_rows) {
        for (int j = 0; j < cols; j += b_cols) {
            for (int x = 0; x < b_rows; x++) {
                for (int y = 0; y < b_cols; y++) {
                    T[j + y][i + x] = M[i + x][j + y];
                }
            }
        }
    }
    wt2 = omp_get_wtime();

    printf("%f\n", wt2 - wt1);
    return T;
}
```

Compilation and execution are achieved, both locally (see previous task for the architecture) and on the HPC cluster, via the same PBS script used for task 1, integrated with the new commands. The outputs of the runs are returned in `CSV` files for easier analysis with external programs.

Results analysis

The implemented code was executed with square matrices of sizes between 20000 and 50000, each with square blocks of 1000, 2000, 5000, and 10000. All combinations were run with up to 32 CPUs on `hpc-c11-node22.unitn.it`[2] Execution times are reported in Table 2 and Table 3 alongside the achieved bandwidth, calculated using Equation 2. Due to the way I wrote the *C* code, the algorithm without the blocks runs four times per matrix size per number of CPUs. The bandwidth reported in Table 2 refers to the average of the runtimes.

$$\text{Bandwidth} = \frac{\text{Rows} \cdot \text{Columns} \cdot \text{size\_of}(\textit{float})}{\text{execution time}} = \frac{\text{size}^2 \cdot 4}{\text{execution time}} \quad [\text{B/s}] \tag{2}$$

Table 2: Matrix transposition - run times and bandwidth

Size	Cores	Run 1 [s]	Run 2 [s]	Run 3 [s]	Run 4 [s]	Average [s]	Bandwidth [GB/s]
20000	1	6.5606	6.6204	6.6265	6.5600	6.5919	60.68
	2	3.4106	3.3133	3.3296	3.3078	3.3404	119.75
	4	2.0776	1.8602	2.0055	2.0716	2.0037	199.63
	8	0.9174	0.9229	0.9655	0.9833	0.9473	422.26
	16	0.5285	0.5373	0.5404	0.5687	0.5437	735.70
	32	0.3506	0.3423	0.3365	0.3830	0.3532	1132.77
30000	1	15.6335	15.5900	15.5903	16.2771	15.7727	57.06
	2	8.4734	8.1278	8.2218	8.2349	8.2645	108.90
	4	4.3465	4.8519	4.9496	4.8396	4.7469	189.60
	8	2.2628	2.4801	2.6046	2.2887	2.4091	373.59
	16	1.2341	1.8231	1.2558	1.2322	1.3863	649.21
	32	0.8240	0.8335	1.1785	0.7841	0.9050	994.44
40000	1	27.9708	28.9593	27.9213	27.9582	28.2024	56.73
	2	14.9170	14.8816	14.8110	14.9570	14.8919	107.44
	4	8.4412	8.3528	8.6755	8.3326	8.4505	189.34
	8	4.3593	4.4382	4.2855	4.1860	4.3172	370.16
	16	3.2731	3.0261	2.2626	2.5232	2.7712	577.36
	32	3.1133	2.7508	2.5513	1.9700	2.5964	616.24

[2]refer to task 1 for its architecture.

Table 2: Matrix transposition - run times and bandwidth

Size	Cores	Run 1 [s]	Run 2 [s]	Run 3 [s]	Run 4 [s]	Average [s]	Bandwidth [GB/s]
50000	1	45.9321	44.8595	45.1216	47.1102	45.7558	54.64
	2	23.8948	25.4520	25.4178	25.2524	25.0042	99.98
	4	13.8669	13.7983	13.6129	13.5577	13.7090	182.36
	8	8.6068	7.5582	8.0404	8.4306	8.1590	306.41
	16	4.1942	5.3581	5.3738	4.5863	4.8781	512.49
	32	3.3619	4.7703	3.2806	3.4283	3.7103	673.80

Table 3: Matrix transposition in blocks - run times and bandwidth

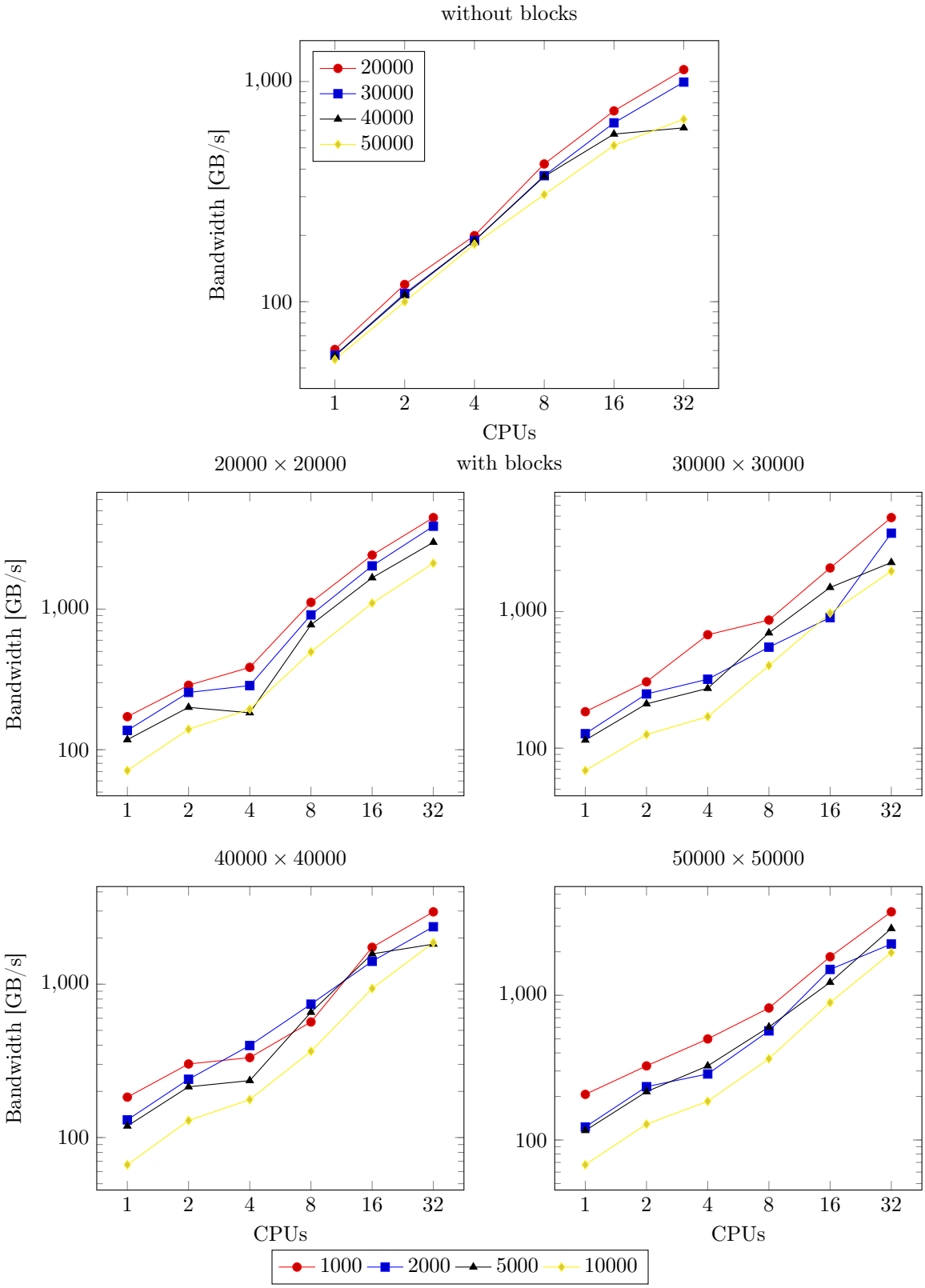
Size	Block size	Cores	Run time [s]	Bandwidth [GB/s]
20000	1000	1	2.3274	171.87
		2	1.3910	287.56
		4	1.0382	385.29
		8	0.3582	1116.72
		16	0.1652	2421.84
		32	0.0894	4473.82
	2000	1	2.9142	137.26
		2	1.5654	255.53
		4	1.3991	285.90
		8	0.4399	909.24
		16	0.1971	2029.22
		32	0.1029	3886.55
	5000	1	3.3928	117.90
		2	1.9972	200.28
		4	2.1841	183.14
		8	0.5174	773.12
		16	0.2396	1669.54
		32	0.1340	2985.25
	10000	1	5.6197	71.18
		2	2.8618	139.77
		4	2.0601	194.16
		8	0.8063	496.08
		16	0.3628	1102.46
		32	0.1888	2118.68
30000	1000	1	4.8689	184.85
		2	2.9412	306.00
		4	1.3300	676.71
		8	1.0382	866.91
		16	0.4314	2086.13
		32	0.1845	4877.28
	2000	1	7.0540	127.59
		2	3.6089	249.38
		4	2.8164	319.56
		8	1.6397	548.88
		16	0.9963	903.34
		32	0.2399	3750.95
	5000	1	7.8568	114.55
		2	4.2641	211.06
		4	3.2850	273.97
		8	1.2891	698.14
		16	0.5993	1501.79
		32	0.3935	2287.10
	10000	1	13.1201	68.60

Table 3: Matrix transposition in blocks - run times and bandwidth

Size	Block size	Cores	Run time [s]	Bandwidth [GB/s]
40000		2	7.1527	125.83
		4	5.2946	169.98
		8	2.2368	402.35
		16	0.9249	9733.03
		32	0.4554	1976.34
	1000	1	8.7225	183.43
		2	5.3019	301.78
		4	4.8171	332.15
		8	2.8220	566.97
		16	0.9196	1739.83
		32	0.5399	2963.67
	2000	1	12.2732	130.37
		2	6.6707	239.85
		4	4.0177	398.23
		8	2.1598	740.81
		16	1.1351	1409.55
		32	0.6754	2368.82
	5000	1	13.4611	118.68
		2	7.4703	214.18
		4	6.8054	235.11
		8	2.4402	655.69
		16	1.0151	1576.22
		32	0.8766	1825.21
	10000	1	24.0736	66.46
		2	12.3900	129.14
		4	9.0544	176.71
		8	4.3846	364.91
		16	1.7084	936.55
		32	0.8562	1868.69
50000	1000	1	12.0920	206.75
		2	7.6869	325.23
		4	5.0051	499.49
		8	3.0580	817.54
		16	1.3538	1846.62
		32	0.6621	3775.99
	2000	1	20.3203	123.03
		2	10.6947	233.76
		4	8.7424	285.96
		8	4.4028	567.81
		16	1.6551	1510.47
		32	1.1047	2263.04
	5000	1	21.5054	116.25
		2	11.5888	215.73
		4	7.6854	325.29
		8	4.1386	604.07
		16	2.0332	1229.56
		32	0.8638	2894.14
	10000	1	37.0605	67.46
		2	19.4318	128.66
		4	13.5048	185.12
		8	6.8553	364.68
		16	2.8058	891.01
		32	1.2871	1942.34

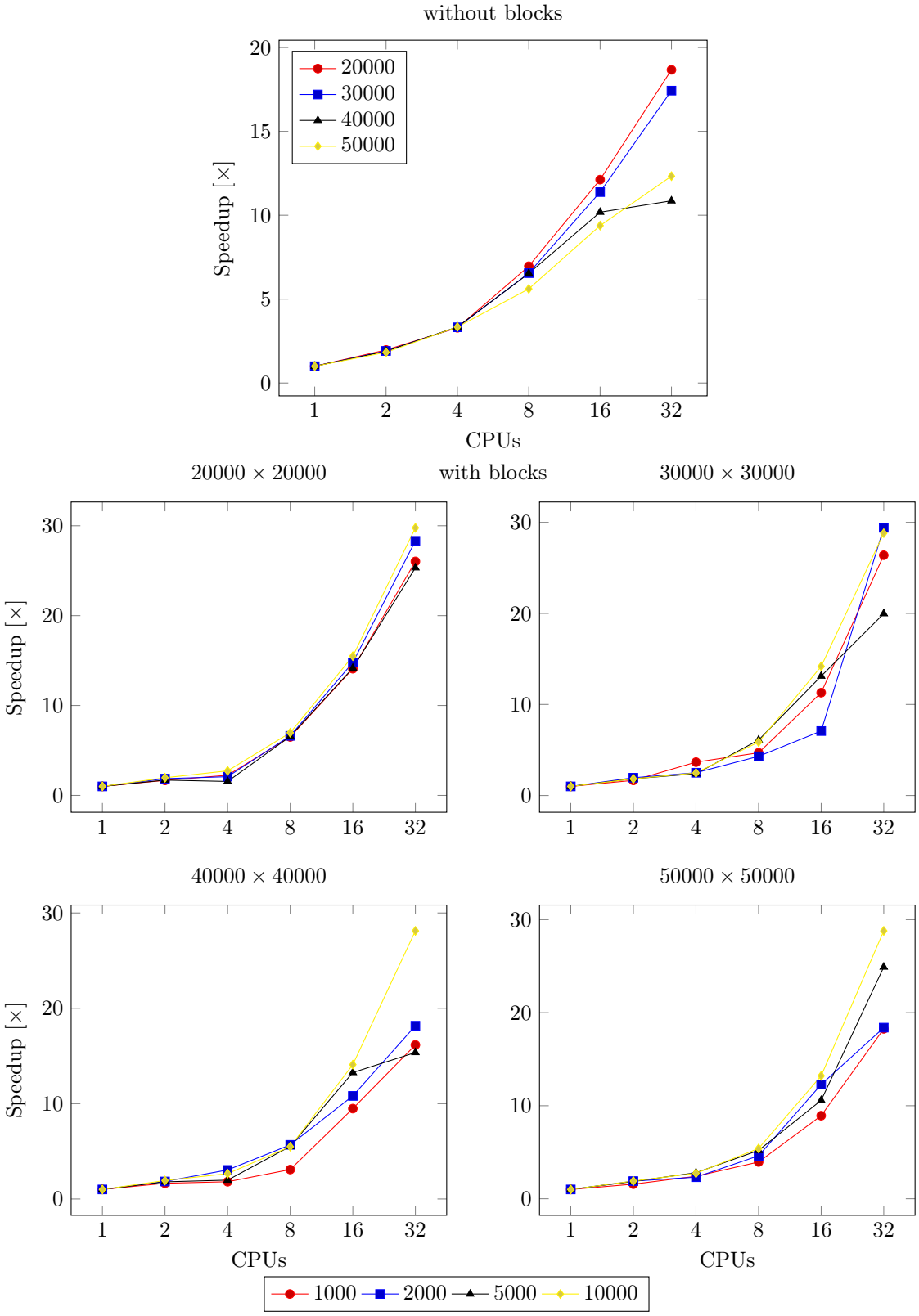
The parallel scalability of the algorithm was evaluated using the bandwidth as the performance metric. Plot 4, Plot 5, and Plot 6 visualize the performance, the speedup, and the efficiency respectively.

Plot 4: Parallel Matrix Transposition - Performance graph by size of matrices

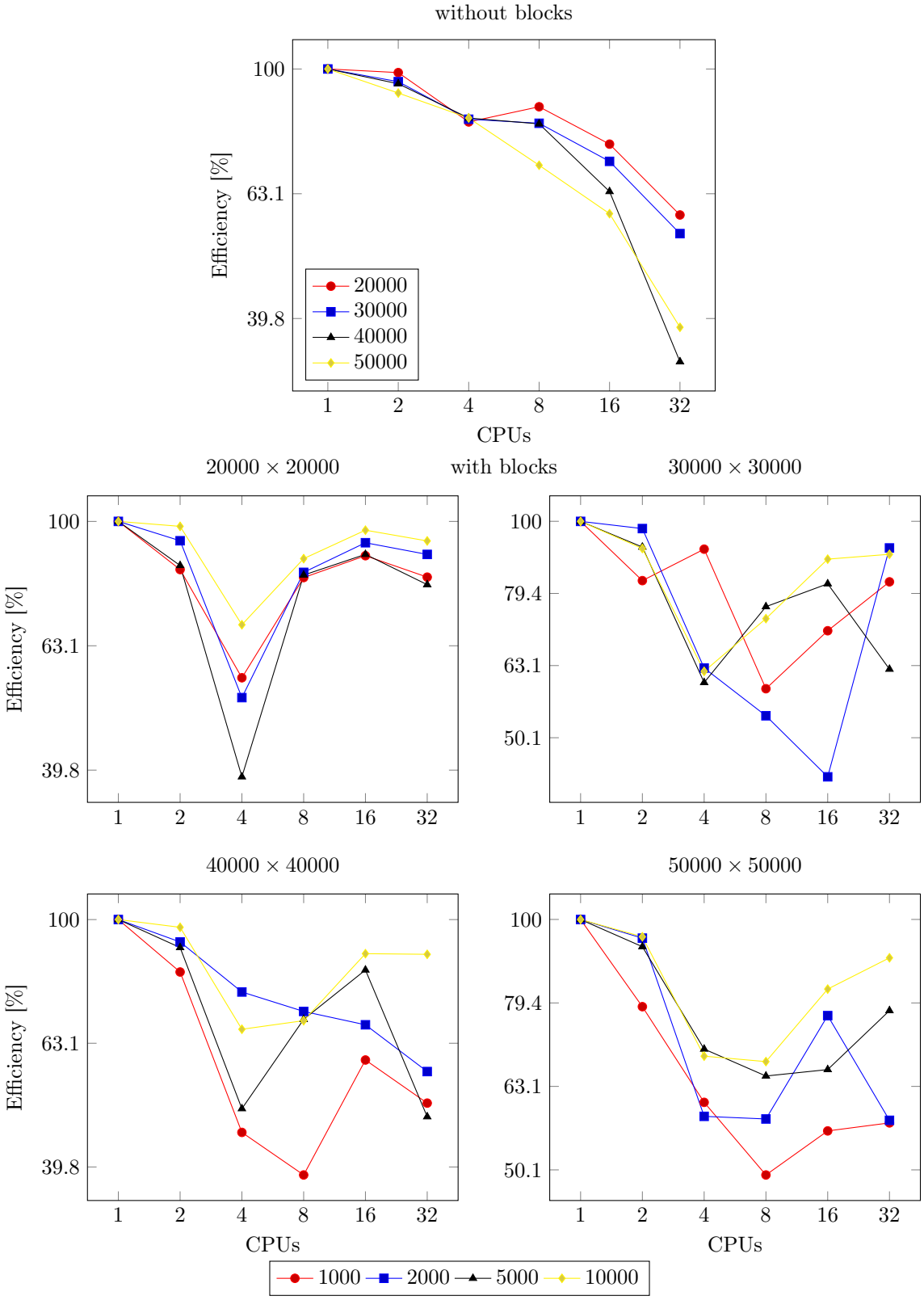


As expected, the algorithms achieve a better performance the more CPUs they have access to. The same can be said about the speedup: the higher the number of CPUs, the faster the calculations are completed. Unfortunately, the efficiency is all over the place because the algorithms are not linearly scalable [5]. Implementing the parallelization differently or using another algorithm altogether could be some of the possible fixes for this undefined yet mathematically correct issue.

Plot 5: Parallel Matrix Transposition - Speedup graph by size of matrices



Plot 6: Parallel Matrix Transposition - Efficiency graph by size of matrices



Conclusions

The analyzed problems highlight the advantages of parallel processing over sequentially run algorithms in terms of speed and achievable performance. This improvement does not strictly follow a linear pattern with efficiency challenges due to non-linear scalability. The challenges can be addressed by using different methods or more sophisticated algorithms that are optimized for the given hardware.
This report provides only insights into simple academic problems that could represent real-world applications even if they are overly simplified.

References

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