Logistics Project

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A company needs to organize how to transport 50 tons of garbage material from point 1 to point 8. The manager identifies a network of streets that connects 1 to 8. Given the type of material transported, only a limited amount of material is allowed to be routed on each street. The table below reports the set of links available and the associated capacity. The company wants to select the minimum number of links to route the material. Nevertheless, there are some links that are mandatory to be selected in the route, i.e., link (3,4) and link (5,6): these two links have to be used at their maximum capacity.

Arcs	Capacity	Arcs	Capacity
(1,2)	15	(5,2)	8
(1,3)	25	(5,4)	7
(1,4)	10	(5,3)	6
(1,5)	17	(5,6)	15
(1,6)	13	(5,7)	4
(1,7)	21	(6,2)	10
(2,3)	10	(6,4)	6
(2,4)	5	(6,5)	8
(2,5)	9	(6,3)	10
(2,6)	11	(6,7)	8
(2,7)	13	(7,2)	11
(3,2)	10	(7,4)	7
(3,4)	20	(7,5)	5
(3,5)	5	(7,6)	8
(3,6)	12	(7,3)	13
(3,7)	11	(2,8)	25
(4,2)	4	(3,8)	10
(4,3)	0	(4,8)	15
(4,5)	7	(5,8)	7
(4,6)	5	(6,8)	20
(4,7)	9	(7,8)	10

1. *Formulate an ILP model to define the itinerary to use to route the 50 tons of garbage material from 1 to 8, by respecting the given constraints and minimizing the number of traveled streets.*
2. *Formulate a new ILP model by also considering the following additional constraint:*
   a. If link (1,3) is selected, then at least 3 tons of garbage material have to be transported on link (5,7).
3. *Implement the two models via the modeling language AMPL and solve them by means of the optimization solver CPLEX, by then comparing the optimal solutions found.*

Kind of Problem:

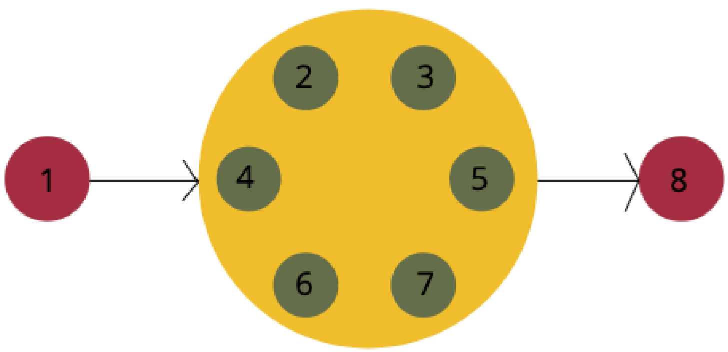
The problem we want to analyze belongs to the category of Minimum Cost Flow Problem (MCFP) and the goal is to transport an amount of flow (garbage) from the node 1 to the node 8 by minimizing the total number of arches which will be used.
If we define the cost of using a certain arc as 1, minimizing the cost with which the garbage is transported from point 1 to point 8 means minimizing the number of used arches.

In this MCFP problem the cost is not represented by a quantitative variable (for instance, transportation costs or travelling time among nodes), but by a boolean variable. Consequently, the aim of this optimization problem is not to minimize the sum of quantitative variables, but to minimize the sum of boolean variables.

Map of the problem:

The map is divided in 2 sets:

The nodes in the yellow set are not in a specific order



- 1 and 8 are the origin and destination nodes, they are connected to the nodes in the yellow set in different directions, but they are non connected to each other
- The yellow part indicates the set of nodes where each element of the set is connected to each other in both way:
from *i* to *j* and from *j* to *i*

Input data:
Let *G=(N,A)* be the network in which the garbage is transported. *N* defines the set of nodes,while *A* is the set of arcs.
N = (1,2,3,4,5,6,7,8)
where 1 and 8 are the origin and destination
2,...,7 are the intermediate nodes
Subset of NxN:
A =
(1,2) (5,2) (1,3) (5,4) (1,4) (5,3) (1,5) (5,6) (1,6) (5,7) (1,7) (6,2) (2,3) (6,4) (2,4) (6,5) (2,5) (6,3) (2,6) (6,7) (2,7) (7,2) (3,2) (7,4) (3,4) (7,5) (3,5) (7,6) (3,6) (7,3) (3,7) (2,8) (4,2) (3,8) (4,3) (4,8) (4,5) (5,8) (4,6) (6,8) (4,7) (7,8)

$c_{i,j}$ **:** indicates the maximum capacity of each link $\forall i, j \epsilon N$3r24tyerw3qewfsdv
(we'll see that two nodes of our problem must have the full capacity satisfied)

Decision variables:

$x_{i,j}$: integer number of tons transported from the nodes *i* to the nodes *j*

$y_{i,j}$: boolean variable =*1* if the arch from *i* to *j* is used

=*0* if the arch form *i* to *j* is not used

Objective Function:

$min \mid \{y_{i,j} \mid y_{i,j} = 1 \;\; ; \;\; (i,j) \epsilon A\} \mid$

We want to minimize the cardinality of this set

Constraints:

1. Non-negativity constraint**:** $x_{i,j} \geqslant 0 \quad \forall (i,j) \epsilon A$

   (The amount of garbage routed on each arch can't have a negative value).

2. Capacity constraint: $x_{i,j} \leqslant c_{i,j} \;\; \forall (i,j) \epsilon A$

   (Each amount that enters in a node must not exceed the capacity of that node).

3. Balance of the origin $x_{1,i}: \sum_{i\epsilon\,(N-\{1,8\})} - x_{1,i} = \;\; - 50$

   (All the 50 tons of garbage have to leave this node and nothing can enter).

4. Balance destination $x_{i,8}: \sum_{i\epsilon\,(N-\{1,8\})} x_{i,8} = \;\; + 50$

   (All the 50 tons of garbage have to reach the destination node).

5. Balance of the middle nodes:

$$\sum_{(i,j)\epsilon\,(BS(j)-\{(i,8)\,\mid\, i\epsilon N\})\;\; i\neq j} x_{i,j} \; - \; \sum_{(i,j)\epsilon\,(FS(i)-\{(1,j)\mid\, j\epsilon N\})\;\; i\neq j} x_{j,i} \;\; = \;\; 0$$

Where BS(j) is the Backward star of j (the set of arcs from any node $j{\neq}i$ to the node i) and FS(i) is the Forward star (the set of arcs from the node i to any node $j{\neq}i$)

BS(i) FS(i)

We need also to specify that the arches are of the entering and leaving type, because we can't only swap the index *i* and *j*. This because $c_{i,j} \neq c_{j,i}$.

For all the nodes which go from node 2 to node 7, the total entering flow must.
(The constraint doesn't change if we consider also the arcs from i and the arcs to j)

6. Saturation of the arcs (3,4) and (5,6):

$$x_{34} = c_{34} = 20$$
$$x_{56} = c_{56} = 15$$

This is a constraint given by the formulation of the problem

7. Upper bound**:**

$$x_{i,j} \preccurlyeq bigM * y_{i,j}$$

Where bigM is a huge number.
We want to specify 2 cases generated by the value that $y_{i,j}$ assumes :

$$y_{i,j} = 0 \quad \Rightarrow \quad x_{i,j} = 0$$
$$y_{i,j} = 1 \quad \Rightarrow \quad x_{i,j} \preccurlyeq bigM$$

(The constraint doesn' t change if we consider the capacity i,j instead of the bigM)

Solution to 1:

Formulate an ILP model to define the itinerary to use to route the 50 tons of garbage material from 1 to 8, by respecting the given constraints and minimizing the number of traveled streets.

By running problem 1 in AMPL,the solution is:

UseOfLinks [*,*]

:	2	3	4	5	6	7	8	:=
1	1	1	0	1	0	0	.	
2	.	0	0	0	0	0	1	
3	0	.	1	0	0	0	0	
4	0	0	.	0	1	0	1	
5	0	0	0	.	1	0	0	
6	0	0	0	0	.	0	1	
7	0	0	0	0	0	.	0	

Useoflinks is the binary variable we used in the last page as

$y_{i,j}$ =*1* if the arch from *i* to *j* is used

=*0* if the arch form *i* to *j* is not used

The minimum number of arches which is necessary to use in order to route the garbage from the node 1 to the node 8 is 9: $(\sum_{(i,j)\in A)} y_{i,j} = 9)$

x [*,*]

:	eight	five	four	seven	six	three	two	:=
five	0	.	0	0	15	0	0	
four	15	0	.	0	5	0	0	
one	.	15	0	0	0	20	15	
seven	0	0	0	.	0	0	0	
six	20	0	0	0	.	0	0	
three	0	0	20	0	0	.	0	
two	15	0	0	0	0	0	.	

This table indicates the quantity of tons that the optimal solution brings in each link

Solution to 2:

:	2	3	4	5	6	7	8
1	1	1	0	1	0	0	.
2	.	0	0	0	0	0	1
3	0	.	1	0	0	0	0
4	0	0	.	0	1	0	1
5	0	0	0	.	1	1	0
6	0	0	0	0	.	0	1
7	0	0	0	1	0	.	0

Useoflinks

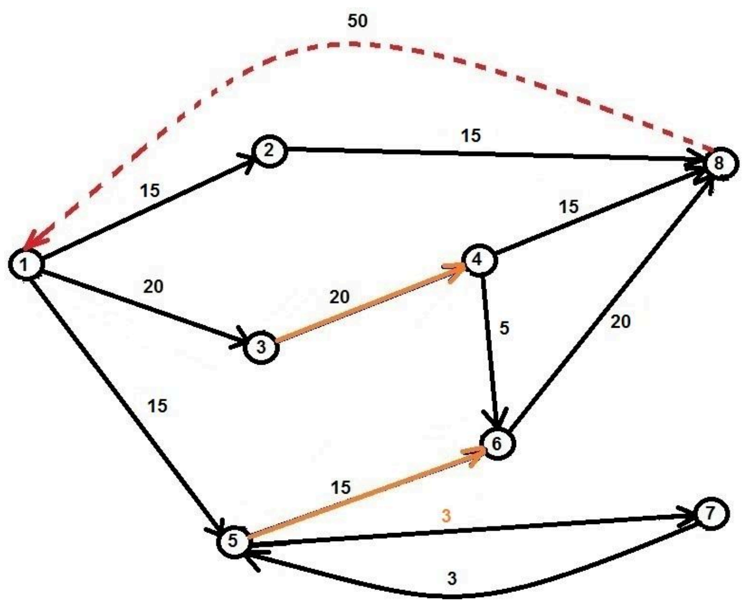
tons [*,*]

:	2	3	4	5	6	7	8
1	15	20	0	15	0	0	.
2	.	0	0	0	0	0	15
3	0	.	20	0	0	0	0
4	0	0	.	0	5	0	15
5	0	0	0	.	15	3	0
6	0	0	0	0	.	0	20
7	0	0	0	3	0	.	0

Tons

$$\left(\sum_{(i,j)\in A} y_{i,j} = 11\right)$$

After we introduced the constraint in problem number 2, we can see that the solution is almost identical. The only difference is that the vehicle travels from the point 5 to 7 and then from 7 to 5.



Initially this solution made us uncertain and perplexed. It seemed to be unrealistic and useless. So we tried to put another constraint on the program:

if (i>=j) then (UseOfLinks[i,j]=0);

This constraint means that the low triangle of the matrix can't be filled with numbers other than 0, preventing AMPL from going back along the same arc.
We found out the number of links used were 12 instead of the 11.
Why didn't AMPL simply use the path from 7 to 8, so the number of used links would remain11?

The answer to this question is not that simple: if we remember the request of the first problem, where AMPL needed to use the path from 5 to 6 entirely, we can realize that the capacity from 1 to 5 is 17 and not 18. So the capacity is not enough to satisfy the request to split 15 tons to link (5,6) and 3 tons to link (5,7).