

<b>Status</b>	Finished
<b>Started</b>	Wednesday, 22 October 2025, 8:45 PM
<b>Completed</b>	Wednesday, 22 October 2025, 9:31 PM
<b>Duration</b>	46 mins 2 secs
<b>Marks</b>	6.80/8.00
<b>Grade</b>	2.83 out of 3.33 (85%)

**Question 1**

Correct

Mark 1.00 out of 1.00

Which of the following vectors  $|\psi_i\rangle \in \mathbb{C}^2$  are quantum qubit states?

- ☐ a.  $|\psi_1\rangle := \frac{|0\rangle + |1\rangle}{2}$
- ☒ b.  $|\psi_4\rangle = \frac{-2}{3} |0\rangle + \frac{\sqrt{2} + i\sqrt{3}}{3} |1\rangle$  ✓
- ☒ c.  $|\psi_2\rangle := \frac{e^{i\pi/5}}{\sqrt{3}} |0\rangle + \frac{\sqrt{2}}{\sqrt{3}} |1\rangle$  ✓
- ☐ d.  $|\psi_0\rangle := \frac{|0\rangle - |+\rangle}{\sqrt{2}}$
- ☒ e.  $|\psi_3\rangle = \frac{3 - 4i}{5} |0\rangle$  ✓

Votre réponse est correcte.

The correct answers are:

$$|\psi_2\rangle := \frac{e^{i\pi/5}}{\sqrt{3}} |0\rangle + \frac{\sqrt{2}}{\sqrt{3}} |1\rangle$$

$$|\psi_3\rangle = \frac{3 - 4i}{5} |0\rangle$$

$$|\psi_4\rangle = \frac{-2}{3} |0\rangle + \frac{\sqrt{2} + i\sqrt{3}}{3} |1\rangle$$

## Question 2

Correct

Mark 1.00 out of 1.00

Which of the following vectors  $|\psi_i\rangle \in \mathbb{C}^2$  are quantum qubit states? (The vectors are represented as column matrices in the computational basis)

- ☒ a.  $|\psi_2\rangle := \begin{bmatrix} \sqrt{6/7} \\ \sqrt{-1/7} \end{bmatrix}$  ✓
- ☒ b.  $|\psi_1\rangle := \begin{bmatrix} 12 \\ -5 \end{bmatrix} \frac{1}{13}$  ✓
- ☒ c.  $|\psi_3\rangle := \frac{1}{\sqrt{2}} \begin{bmatrix} e^{i\pi} \\ -e^{7i} \end{bmatrix}$  ✓
- ☐ d.  $|\psi_4\rangle := i \begin{bmatrix} e^\pi \\ 0 \end{bmatrix}$
- ☒ e.  $|\psi_0\rangle := \sqrt{2} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$  ✓

Votre réponse est correcte.

The correct answers are:

$$|\psi_0\rangle := \sqrt{2} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$

$$|\psi_1\rangle := \begin{bmatrix} 12 \\ -5 \end{bmatrix} \frac{1}{13}$$

$$|\psi_2\rangle := \begin{bmatrix} \sqrt{6/7} \\ \sqrt{-1/7} \end{bmatrix}$$

$$|\psi_3\rangle := \frac{1}{\sqrt{2}} \begin{bmatrix} e^{i\pi} \\ -e^{7i} \end{bmatrix}$$

## Question 3

Correct

Mark 1.00 out of 1.00

Which of the following operators  $U_i : \mathbb{C}^2 \rightarrow \mathbb{C}^2$  are unitary? (The operators are represented as matrices in the computational basis)

- ☒ a.  $U_0 := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  ✓
- ☐ b.  $U_3 := \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
- ☒ c.  $U_2 := \begin{bmatrix} 0 & 1 \\ e^{i\pi} & 0 \end{bmatrix}$  ✓
- ☐ d.  $U_4 := \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
- ☒ e.  $U_1 := \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$  ✓

Votre réponse est correcte.

The correct answers are:

$$U_0 := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$U_1 := \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix},$$

$$U_2 := \begin{bmatrix} 0 & 1 \\ e^{i\pi} & 0 \end{bmatrix}$$

**Question 4**

Correct

Mark 1.00 out of 1.00

Which of the following operators  $U_i : \mathbb{C}^2 \rightarrow \mathbb{C}^2$  are unitary?

- ☒ a.  $U_2 := |+\rangle\langle 0| + |-\rangle\langle 1|$  ✓
- ☒ b.  $U_4 := |+\rangle\langle +| + |-\rangle\langle -|$  ✓
- ☒ c.  $U_0 := |0\rangle\langle 0| + |1\rangle\langle 1|$  ✓
- ☐ d.  $U_1 := |0\rangle\langle 0| - 2i |1\rangle\langle 1|$
- ☐ e.  $U_3 := |+\rangle\langle 0| - |+\rangle\langle 1|$

Votre réponse est correcte.

The correct answers are:

$$U_0 := |0\rangle\langle 0| + |1\rangle\langle 1|$$

$$U_2 := |+\rangle\langle 0| + |-\rangle\langle 1|$$

$$U_4 := |+\rangle\langle +| + |-\rangle\langle -|$$

## Question 5

Partially correct

Mark 0.80 out of 1.00

Select the statements which are true.

- ☒ a. Let  $A : \mathbb{C}^d \rightarrow \mathbb{C}^d$  be a linear operator. There exist vectors  $|\psi_i\rangle \in \mathbb{C}^d$  such that  $A = \sum_{i=0}^{d-1} |\psi_i\rangle\langle i|$ . ✔
- ☒ b. Let  $|\psi_i\rangle \in \mathbb{C}^d$  be vectors respecting  $\langle \psi_i | \psi_j \rangle = \delta_{ij}$ . The linear operator  $U = \sum_{i=0}^{d-1} |\psi_i\rangle\langle i|$  is unitary. ✔
- ☐ c. Let  $\{|\psi_i\rangle\}_{i=0}^{d-1}$  be a set of vectors in  $\mathbb{C}^d$ . There exists a unitary operator  $U : \mathbb{C}^d \rightarrow \mathbb{C}^d$  such that  $U|i\rangle = |\psi_i\rangle$ .
- ☒ d. Let  $|\psi_i\rangle \in \mathbb{C}^d$  be vectors respecting  $\langle \psi_i | \psi_j \rangle = \delta_{ij}$ . The linear operator  $U = \sum_{i=0}^{d-1} |i\rangle\langle \psi_i|$  is unitary. ✔
- ☒ e. Let  $U : \mathbb{C}^d \rightarrow \mathbb{C}^d$  be a linear operator. If  $U$  is unitary, there exist vectors  $|\psi_i\rangle \in \mathbb{C}^d$  respecting  $\langle \psi_i | \psi_j \rangle = \delta_{ij}$  such that  $U = \sum_{i=0}^{d-1} |\psi_i\rangle\langle i|$ . ✔
- ☐ f. Let  $|\psi_0\rangle := \frac{3|0\rangle - i4|1\rangle}{5}$  and  $|\psi_1\rangle := \frac{4|0\rangle + 3i|1\rangle}{5}$  be qubit states. There exists a unitary operator  $U : \mathbb{C}^2 \otimes \mathbb{C}^2 \rightarrow \mathbb{C}^2 \otimes \mathbb{C}^2$  such that  $U|\psi_i\rangle|0\rangle = |\psi_i\rangle|\psi_i\rangle$  for every  $i \in \{0, 1\}$ .

Votre réponse est partiellement correcte.

You have correctly selected 4.

The correct answers are:

Let  $A : \mathbb{C}^d \rightarrow \mathbb{C}^d$  be a linear operator. There exist vectors  $|\psi_i\rangle \in \mathbb{C}^d$  such that  $A = \sum_{i=0}^{d-1} |\psi_i\rangle\langle i|$ .

Let  $U : \mathbb{C}^d \rightarrow \mathbb{C}^d$  be a linear operator. If  $U$  is unitary, there exist vectors  $|\psi_i\rangle \in \mathbb{C}^d$  respecting  $\langle \psi_i | \psi_j \rangle = \delta_{ij}$  such that  $U = \sum_{i=0}^{d-1} |\psi_i\rangle\langle i|$ .

Let  $|\psi_i\rangle \in \mathbb{C}^d$  be vectors respecting  $\langle \psi_i | \psi_j \rangle = \delta_{ij}$ .  
The linear operator  $U = \sum_{i=0}^{d-1} |\psi_i\rangle\langle i|$  is unitary.

Let  $|\psi_i\rangle \in \mathbb{C}^d$  be vectors respecting  $\langle \psi_i | \psi_j \rangle = \delta_{ij}$ .  
The linear operator  $U = \sum_{i=0}^{d-1} |i\rangle\langle \psi_i|$  is unitary.

Let  $|\psi_0\rangle := \frac{3|0\rangle - i4|1\rangle}{5}$  and  $|\psi_1\rangle := \frac{4|0\rangle + 3i|1\rangle}{5}$  be qubit states. There exists a unitary operator  $U : \mathbb{C}^2 \otimes \mathbb{C}^2 \rightarrow \mathbb{C}^2 \otimes \mathbb{C}^2$  such that  $U |\psi_i\rangle |0\rangle = |\psi_i\rangle |\psi_i\rangle$  for every  $i \in \{0, 1\}$ .

## Question 6

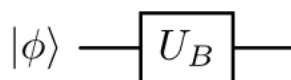
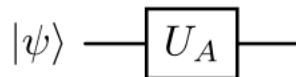
Incorrect

Mark 0.00 out of 1.00

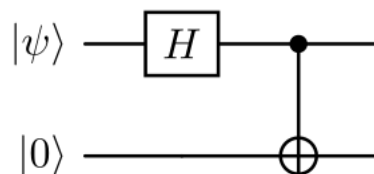
Select the statements which are true.

☒ a.

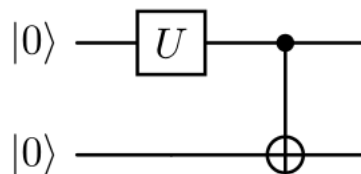
For any qubit unitary operators  $U_A$  and  $U_B$ , qubit states  $|\psi\rangle$  and  $|\phi\rangle$ , the output of the quantum state below is separable.


☒ b.

For any quantum state  $|\psi\rangle \in \mathbb{C}^2$  the quantum circuit below outputs an entangled state.


☐ c.

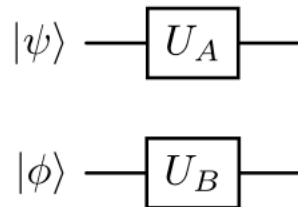
Let  $U : \mathbb{C}^2 \rightarrow \mathbb{C}^2$  be a unitary operator such that  $U|0\rangle \neq e^{i\theta_0}|0\rangle$  for every  $\theta_0 \in \mathbb{R}$  and  $U|0\rangle \neq e^{i\theta_1}|1\rangle$  for every  $\theta_1 \in \mathbb{R}$ . The output of the quantum state below is an entangled state.



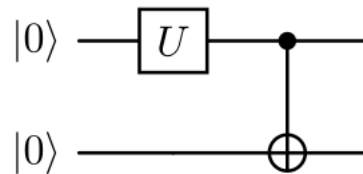
Votre réponse est incorrecte.

The correct answers are:

For any qubit unitary operators  $U_A$  and  $U_B$ , qubit states  $|\psi\rangle$  and  $|\phi\rangle$ , the output of the quantum state below is separable.



Let  $U : \mathbb{C}^2 \rightarrow \mathbb{C}^2$  be a unitary operator such that  $U|0\rangle \neq e^{i\theta_0}|0\rangle$  for every  $\theta_0 \in \mathbb{R}$  and  $U|0\rangle \neq e^{i\theta_1}|1\rangle$  for every  $\theta_1 \in \mathbb{R}$ . The output of the quantum state below is an entangled state.



#### Question 7

Correct

Mark 1.00 out of 1.00

For every two-qubit state  $|\psi_{AB}\rangle$  and every qubit unitary operator  $U$  and any  $\theta \in \mathbb{R}$ , the probability of obtaining the outcome 00 is the same in the two circuits below.



☒ True ✓

☐ False

The correct answer is 'True'.



## Question 8

Correct

Mark 1.00 out of 1.00

For every two-qubit state  $|\psi_{AB}\rangle$ , for every two-qubit unitary  $V$  and every single-qubit unitary operator  $U$ , the probability of obtaining the outcome 00 is the same in the two circuits below.



Recall:

$$= |0\rangle\langle 0| \otimes \mathbb{1} + |1\rangle\langle 1| \otimes e^{i\theta}U.$$

☐ True

☒ False ✓

The correct answer is 'False'.