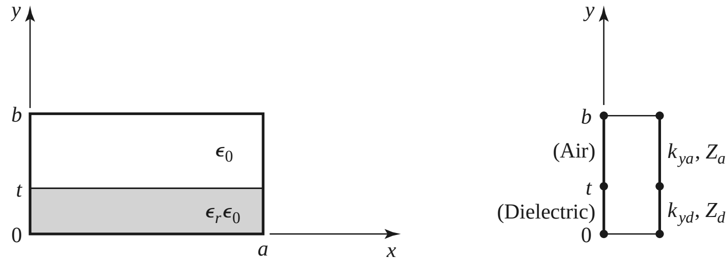

FIGURE 3.28 A rectangular waveguide partially filled with dielectric and the transverse resonance equivalent circuit.

and has a transverse propagation constant k_{yd} and a characteristic impedance for TE modes given by

$$Z_d = \frac{k\eta}{k_{yd}} = \frac{k_0\eta_0}{k_{yd}}, \tag{3.207a}$$

where $k_0 = \omega\sqrt{\mu_0\epsilon_0}$ and $\eta_0 = \sqrt{\mu_0/\epsilon_0}$. For $t < y < b$, the guide is air filled and has a transverse propagation constant k_{ya} and an equivalent characteristic impedance given by

$$Z_a = \frac{k_0\eta_0}{k_{ya}}. \tag{3.207b}$$

Applying condition (3.206) yields

$$k_{ya} \tan k_{yd}t + k_{yd} \tan k_{ya}(b - t) = 0. \tag{3.208}$$

This equation contains two unknowns, k_{ya} and k_{yd}. An additional equation is obtained from the fact that the longitudinal propagation constant, β, must be the same in both regions for phase matching of the tangential fields at the dielectric interface. Thus, with $k_x = 0$,

$$\beta = \sqrt{\epsilon_r k_0^2 - k_{yd}^2} = \sqrt{k_0^2 - k_{ya}^2},$$

or

$$\epsilon_r k_0^2 - k_{yd}^2 = k_0^2 - k_{ya}^2. \tag{3.209}$$

Equations (3.208) and (3.209) can be solved (numerically or graphically) to obtain k_{yd} and k_{ya}. There will be an infinite number of solutions, corresponding to the n dependence (number of variations in y) of the TE_{0n} mode.

3.10 WAVE VELOCITIES AND DISPERSION

We have so far encountered two types of velocities related to the propagation of electromagnetic waves:

- The speed of light in a medium $(1/\sqrt{\mu\epsilon})$
- The phase velocity $(v_p = \omega/\beta)$

The speed of light in a medium is the velocity at which a plane wave would propagate in that medium, while the phase velocity is the speed at which a constant phase point travels. For a TEM plane wave, these two velocities are identical, but for other types of guided wave propagation the phase velocity may be greater or less than the speed of light.

If the phase velocity and attenuation of a line or guide are constants that do not change with frequency, then the phase of a signal that contains more than one frequency component will not be distorted. If the phase velocity is different for different frequencies, then the individual frequency components will not maintain their original phase relationships as they propagate down the transmission line or waveguide, and signal distortion will occur. Such an effect is called *dispersion* since different phase velocities allow the "faster" waves to lead in phase relative to the "slower" waves, and the original phase relationships will gradually be dispersed as the signal propagates down the line. In such a case, there is no single phase velocity that can be attributed to the signal as a whole. However, if the bandwidth of the signal is relatively small or if the dispersion is not too severe, a *group velocity* can be defined in a meaningful way. This velocity can be used to describe the speed at which the signal propagates.

Group Velocity

As discussed earlier, the physical interpretation of group velocity is the velocity at which a narrowband signal propagates. We will derive the relation of group velocity to the propagation constant by considering a signal $f(t)$ in the time domain. The Fourier transform of this signal is defined as

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt, \tag{3.210a}$$

and the inverse transform is

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega. \tag{3.210b}$$

Now consider the transmission line or waveguide on which the signal $f(t)$ is propagating as a linear system, with a transfer function $Z(\omega)$ that relates the output, $F_o(\omega)$, of the line to the input, $F(\omega)$, of the line, as shown in Figure 3.29. Thus,

$$F_o(\omega) = Z(\omega)F(\omega). \tag{3.211}$$

For a lossless matched transmission line or waveguide, the transfer function $Z(\omega)$ can be expressed as

$$Z(\omega) = Ae^{-j\beta z} = |Z(\omega)|e^{-j\psi}, \tag{3.212}$$

where A is a constant and β is the propagation constant of the line or guide.

The time domain representation of the output signal, $f_o(t)$, can then be written as

$$f_o(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)|Z(\omega)|e^{j(\omega t-\psi)}d\omega. \tag{3.213}$$

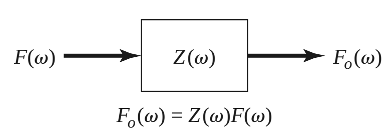

FIGURE 3.29 A transmission line or waveguide represented as a linear system with transfer function $Z(\omega)$.

If $|Z(\omega)| = A$ is a constant and the phase ψ of $Z(\omega)$ is a linear function of ω, say $\psi = a\omega$, the output can be expressed as

$$f_o(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} AF(\omega)e^{j\omega(t-a)}d\omega = Af(t-a), \tag{3.214}$$

which is seen to be a replica of $f(t)$, except for an amplitude factor A and time shift a. Thus, a transfer function of the form $Z(\omega) = Ae^{-j\omega a}$ does not distort the input signal. A lossless TEM wave has a propagation constant $\beta = \omega/c$, which is of this form, so a TEM line is *dispersionless* and does not lead to signal distortion. If the TEM line is lossy, however, the attenuation may be a function of frequency, which could lead to signal distortion.

Now consider a narrowband input signal of the form

$$s(t) = f(t)\cos\omega_0 t = \text{Re}\left\{f(t)e^{j\omega_o t}\right\}, \tag{3.215}$$

which represents an amplitude-modulated carrier wave of frequency ω_o. Assume that the highest frequency component of $f(t)$ is ω_m, where $\omega_m \ll \omega_o$. The Fourier transform, $S(\omega)$, of $s(t)$, is

$$S(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega_o t}e^{j\omega t}dt = F(\omega-\omega_o), \tag{3.216}$$

where we have used the complex form of the input signal as expressed in (3.215). We will need to take the real part of the output inverse transform to obtain the time domain output signal. The spectra of $F(\omega)$ and $S(\omega)$ are depicted in Figure 3.30.

The output signal spectrum is

$$S_o(\omega) = AF(\omega-\omega_o)e^{-j\beta z}, \tag{3.217}$$

and in the time domain,

$$\begin{aligned} s_o(t) &= \frac{1}{2\pi}\text{Re}\int_{-\infty}^{\infty} S_o(\omega)e^{j\omega t}d\omega \\ &= \frac{1}{2\pi}\text{Re}\int_{\omega_o-\omega_m}^{\omega_o+\omega_m} AF(\omega-\omega_o)e^{j(\omega t-\beta z)}d\omega. \end{aligned} \tag{3.218}$$

In general, the propagation constant β may be a complicated function of ω. However, if $F(\omega)$ is narrowband ($\omega_m \ll \omega_o$), then β can often be linearized by using a Taylor series expansion about ω_o:

$$\beta(\omega) = \beta(\omega_o) + \left.\frac{d\beta}{d\omega}\right|_{\omega=\omega_o}(\omega-\omega_o) + \frac{1}{2}\left.\frac{d^2\beta}{d\omega^2}\right|_{\omega=\omega_o}(\omega-\omega_o)^2 + \cdots. \tag{3.219}$$

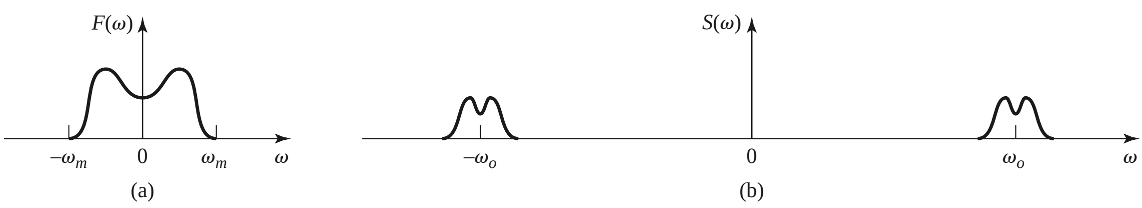

FIGURE 3.30 Fourier spectra of the signals (a) $f(t)$ and (b) $s(t)$.