Then from (3.133) the attenuation due to conductor loss is

$$\alpha_c = \frac{R_s}{ak\eta\beta}\left(k_c^2 + \frac{k^2}{p_{11}'^2 - 1}\right) = 0.0672 \text{ Np/m} = 0.583 \text{ dB/m}.$$

The total attenuation is $\alpha = \alpha_d + \alpha_c = 2.07$ dB/m, and the loss in the 30 cm length of guide is

$$\text{attenuation (dB)} = \alpha(\text{dB/m}) \times L \text{ (m)} = (2.07)(0.3) = 0.62 \text{ dB}.$$ ■

3.5 COAXIAL LINE

TEM Modes

Although we have already discussed TEM mode propagation on a coaxial line in Chapter 2, we will briefly reconsider it here in the context of the general framework that is being used in this chapter.

The coaxial transmission line geometry is shown in Figure 3.15, where the inner conductor is at a potential of V_o volts and the outer conductor is at zero volts. From Section 3.1 we know that the fields can be derived from a scalar potential function, $\Phi(\rho, \phi)$, which is a solution to Laplace's equation (3.14). In cylindrical coordinates Laplace's equation takes the form

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\Phi(\rho,\phi)}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\Phi(\rho,\phi)}{\partial\phi^2} = 0. \tag{3.143}$$

This equation must be solved for $\Phi(\rho, \phi)$ subject to the boundary conditions

$$\Phi(a, \phi) = V_o, \tag{3.144a}$$

$$\Phi(b, \phi) = 0. \tag{3.144b}$$

By the method of separation of variables, let $\Phi(\rho, \phi)$ be expressed in product form as

$$\Phi(\rho, \phi) = R(\rho)P(\phi). \tag{3.145}$$

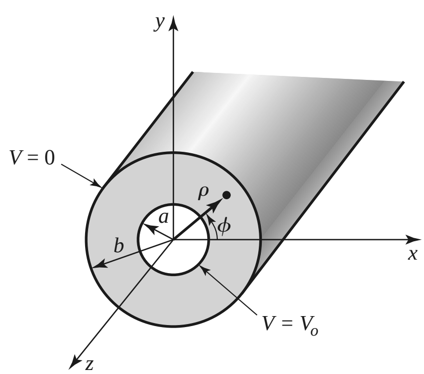


FIGURE 3.15 Coaxial line geometry.

Substituting (3.145) into (3.143) and dividing by RP gives

$$\frac{\rho}{R}\frac{\partial}{\partial\rho}\left(\rho\frac{dR}{d\rho}\right)+\frac{1}{P}\frac{d^2P}{d\phi^2}=0. \tag{3.146}$$

By the usual separation-of-variables argument, the two terms in (3.146) must be equal to constants, so that

$$\frac{\rho}{R}\frac{\partial}{\partial\rho}\left(\rho\frac{dR}{d\rho}\right)=-k_\rho^2, \tag{3.147}$$

$$\frac{1}{P}\frac{d^2P}{d\phi^2}=-k_\phi^2, \tag{3.148}$$

$$k_\rho^2+k_\phi^2=0. \tag{3.149}$$

The general solution to (3.148) is

$$P(\phi)=A\cos n\phi+B\sin n\phi, \tag{3.150}$$

where $k_\phi=n$ must be an integer since increasing ϕ by a multiple of 2π should not change the result. Now, because the boundary conditions of (3.144) do not vary with ϕ, the potential $\Phi(\rho,\ \phi)$ should not vary with ϕ. Thus, n must be zero. By (3.149), this implies that k_ρ must also be zero, so that the equation for $R(\rho)$ in (3.147) reduces to

$$\frac{\partial}{\partial\rho}\left(\rho\frac{dR}{d\rho}\right)=0.$$

The solution for $R(\rho)$ is then

$$R(\rho)=C\ln\rho+D,$$

and so

$$\Phi(\rho,\phi)=C\ln\rho+D. \tag{3.151}$$

Applying the boundary conditions of (3.144) gives two equations for the constants C and D:

$$\Phi(a,\phi)=V_o=C\ln a+D, \tag{3.152a}$$

$$\Phi(b,\phi)=0=C\ln b+D. \tag{3.152b}$$

After solving for C and D, we can write the final solution for $\Phi(\rho,\phi)$ as

$$\Phi(\rho,\phi)=\frac{V_o\ln b/\rho}{\ln b/a}. \tag{3.153}$$

The $\bar{E}$ and $\bar{H}$ fields can now be found using (3.13) and (3.18), and the voltage, current, and characteristic impedance can be determined as in Chapter 2. Attenuation due to dielectric or conductor loss has already been treated in Chapter 2.

Higher Order Modes

The coaxial line, like the parallel plate waveguide, can also support TE and TM waveguide modes in addition to the TEM mode. In practice, these modes are usually cut off (evanescent), and so have only a reactive effect near discontinuities or sources, where they may be excited. It is important in practice, however, to be aware of the cutoff frequency of the lowest order waveguide-type modes to avoid the propagation of these modes. Undesirable effects can occur if two or more modes with different propagation constants are propagating at the same time. Avoiding propagation of higher order modes sets an upper limit on the size of a coaxial cable or, equivalently, an upper limit on the frequency of operation for a given cable. This also affects the power handling capacity of a coaxial line (see the Point of Interest on power capacity of transmission lines).

We will derive the solution for the TE modes of the coaxial line; the TE_{11} mode is the dominant waveguide mode of the coaxial line and so is of primary importance.

For TE modes, $E_z = 0$, and H_z satisfies the wave equation of (3.112):

$$\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2} + k_c^2\right) h_z(\rho, \phi) = 0, \tag{3.154}$$

where $H_z(\rho, \phi, z) = h_z(\rho, \phi)e^{-j\beta z}$, and $k_c^2 = k^2 - \beta^2$. The general solution to this equation, as derived in Section 3.4, is given by the product of (3.118) and (3.120):

$$h_z(\rho, \phi) = (A \sin n\phi + B \cos n\phi)(C J_n(k_c\rho) + DY_n(k_c\rho)). \tag{3.155}$$

In this case, $a \leq \rho \leq b$, so we have no reason to discard the Y_n term. The boundary conditions are

$$E_\phi(\rho, \phi, z) = 0 \text{ for } \rho = a, b. \tag{3.156}$$

Using (3.110b) to find E_ϕ from H_z gives

$$E_\phi = \frac{j\omega\mu}{k_c}(A \sin n\phi + B \cos n\phi)[C J_n'(k_c\rho) + DY_n'(k_c\rho)]e^{-j\beta z}. \tag{3.157}$$

Applying (3.156) to (3.157) gives two equations:

$$C J_n'(k_c a) + DY_n'(k_c a) = 0, \tag{3.158a}$$

$$C J_n'(k_c b) + DY_n'(k_c b) = 0. \tag{3.158b}$$

Because this is a homogeneous set of equations, the only nontrivial ($C \neq 0,\ D \neq 0$) solution occurs when the determinant is zero. Thus we must have

$$J_n'(k_c a)Y_n'(k_c b) = J_n'(k_c b)Y_n'(k_c a). \tag{3.159}$$

This is a characteristic (or eigenvalue) equation for k_c. The values of k_c that satisfy (3.159) then define the TE_{nm} modes of the coaxial line.

Equation (3.159) is a transcendental equation, which must be solved numerically for k_c. Figure 3.16 shows the result of such a solution for $n = 1$ for various b/a ratios. An approximate solution that is often used in practice is

$$k_c = \frac{2}{a + b}.$$

Once k_c is known, the propagation constant or cutoff frequency can be determined. Solutions for the TM modes can be found in a similar manner; the required determinantal equation is the same as (3.159), except for the derivatives. Field lines for the TEM and TE_{11} modes of the coaxial line are shown in Figure 3.17.

EXAMPLE 3.3 HIGHER ORDER MODE OF A COAXIAL LINE

Consider a RG-401U semirigid coaxial cable, with inner and outer conductor diameters of 0.0645 in. and 0.215 in., and a Teflon dielectric with $\epsilon_r = 2.2$. What is the highest usable frequency before the TE_{11} waveguide mode starts to propagate?

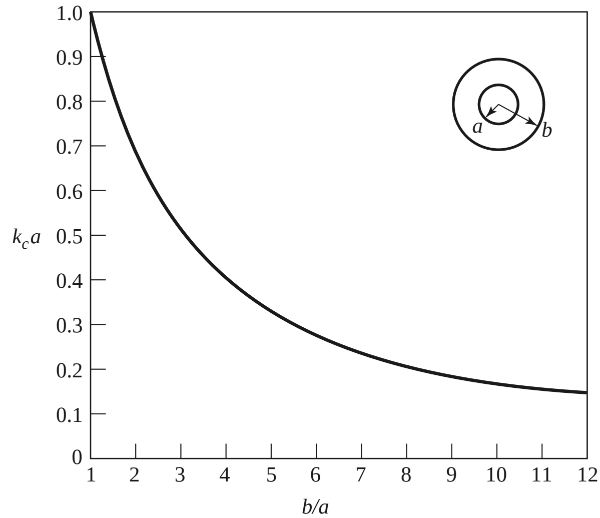


FIGURE 3.16 Normalized cutoff frequency of the dominant TE_{11} waveguide mode for a coaxial line.

Solution
We have

$$\frac{b}{a} = \frac{2b}{2a} = \frac{0.215}{0.0645} = 3.33.$$

From Figure 3.16 this value of b/a gives $k_c a = 0.45$ [the approximate result is $k_c a = 2/(1 + b/a) = 0.462$]. Thus, $k_c = 549.4 \text{ m}^{-1}$, and the cutoff frequency of the TE_{11} mode is

$$f_c = \frac{ck_c}{2\pi\sqrt{\epsilon_r}} = 17.7 \text{ GHz}.$$

In practice, a 5% safety margin is usually recommended, so

$$f_{\max} = (0.95)\,(17.7 \text{ GHz}) = 16.8 \text{ GHz}. \qquad \blacksquare$$

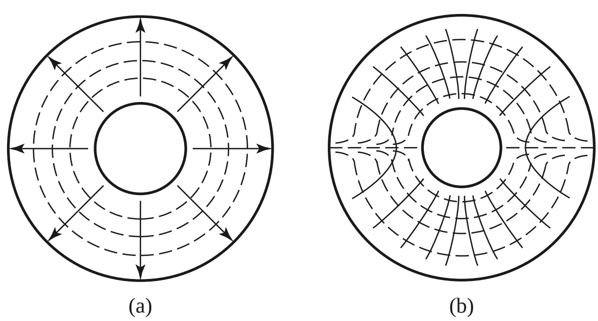
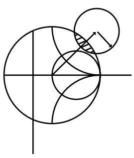


FIGURE 3.17 Field lines for the (a) TEM and (b) TE_{11} modes of a coaxial line.



POINT OF INTEREST: Coaxial Connectors

Most coaxial cables and connectors in common use have a 50 Ω characteristic impedance, with an exception being the 75 Ω cable used in television systems. The reasoning behind these choices is that an air-filled coaxial line has minimum attenuation for a characteristic impedance of about 77 Ω (Problem 2.27), while maximum power capacity occurs for a characteristic impedance of about 30 Ω (Problem 3.28). A 50 Ω characteristic impedance thus represents a compromise between minimum attenuation and maximum power capacity. Other requirements for coaxial connectors include low SWR, higher-order-mode–free operation at a high frequency, high repeatability after a connect–disconnect cycle, and mechanical strength. Connectors are used in pairs, with a male end and a female end (or plug and jack). The accompanying photo shows several types of commonly used coaxial connectors and adapters. From top left: Type-N, TNC, SMA, APC-7, and 2.4 mm.

Type-N: This connector was developed in 1942 and is named after its inventor, P. Neil, of Bell Labs. The outer diameter of the female end is about 0.625 in. The recommended upper frequency limit ranges from 11 to 18 GHz, depending on cable size. This rugged but large connector is often found on older equipment.

TNC: This is a threaded version of the very common BNC connector. Its use is limited to frequencies below 1 GHz.

SMA: The need for smaller and lighter connectors led to the development of this connector in the 1960s. The outer diameter of the female end is about 0.25 in. It can be used up to frequencies in the range of 18–25 GHz and is probably the most commonly used microwave connector today.

APC-7: This is a precision connector (Amphenol Precision Connector) that can repeatedly achieve SWR less than 1.04 at frequencies up to 18 GHz. The connectors are “sexless,” with butt contact between both inner conductors and outer conductors. This connector is most commonly used for measurement and instrumentation applications.