# REPASO DE FM

## 0 Sumatoris i productoris

$$\sum_{i=m}^{n} f(i) = f(m) + f(m+1) \dots + f(n)$$

$$\prod_{i=m}^{n} f(i) = f(m) \cdot f(m+1) \dots f(n-1) \cdot f(n)$$

### 0.1. Propietats

$$\sum_{i=m}^{n} (f(i) + g(i)) = \sum_{i=m}^{n} f(i) + \sum_{i=m}^{n} g(i)$$

$$\sum_{i=m}^{n} c \cdot f(i) = c \cdot \sum_{i=m}^{n} f(i)$$

### 0.2. Progressions aritmètiques

$\left[a_i = a_0 + id\right] \to$ cada terme $a_{i+1}$ s'obté de l'anterior + una quantitat $d$. $a_{i+1} = a_i + d$.

### 0.3. Progressions geomètriques

$\left[a_i = a_0 r^i\right] \to$ cada terme $a_{i+1}$ s'obté de l'anterior * una quantitat $r$ $a_{i+1} = r a_i$

## 1. Lògica i demostracions

### 1.1. Lògica proposicional

| $\varphi$ | $\psi$ | $\wedge$ | $\vee$ | $\to$ | $\longleftrightarrow$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |

### 1.2. Equivalència de fórmules

| | |
|---|---|
| DIST | $\varphi \wedge (\psi \vee \theta) \equiv (\varphi \wedge \psi) \vee (\varphi \wedge \theta)$ |
| MORG. | $\neg(\varphi \wedge \psi) \equiv \neg\varphi \vee \neg\psi$ |
| ABS. | $\varphi \wedge (\varphi \vee \psi) \equiv \varphi$ |
| IDEM. | $\varphi \wedge \varphi \equiv \varphi$ |
| COM. | $\varphi \wedge \psi \equiv \psi \wedge \varphi$ |
| ASSO | $\varphi \wedge (\psi \wedge \theta) \equiv (\varphi \wedge \psi) \wedge \theta$ |
| NEU. | $\varphi \wedge 1 \equiv \varphi$ ; $\varphi \vee 0 \equiv \varphi$ |
| E.ABS. | $\varphi \vee 1 \equiv 1$ ; $\varphi \wedge 0 \equiv 0$ |
| COMP. | $\varphi \vee \neg\varphi \equiv 1$ ; $\varphi \wedge \neg\varphi \equiv 0$ |
| ¬¬N | $\neg\neg\varphi \equiv \varphi$ |
| T. → | $\varphi \to \psi \equiv \neg\varphi \vee \psi$ ; $\neg(\varphi \to \psi) \equiv \varphi \wedge \neg\psi$ |
| T ↔ | $\varphi \leftrightarrow \psi \equiv (\varphi \to \psi) \wedge (\psi \to \varphi)$ |
| | $\neg(\varphi \leftrightarrow \psi) \equiv (\varphi \wedge \neg\psi) \vee (\psi \wedge \neg\varphi)$ |
| C.R | $\varphi \to \psi \equiv \neg\psi \to \neg\varphi$ |
| RA | $\varphi \equiv \neg\varphi \to 0$ |
| V. CONS. | $\varphi \to (\psi \vee \theta) \equiv (\varphi \wedge \neg\psi) \to \theta$ |
| V. ANT. | $(\psi \vee \theta) \to \varphi \equiv (\psi \to \varphi) \wedge (\theta \to \varphi)$ |

### 1.3. Lògica de primer ordre

$\forall$ : Pertot

$\exists$ : Existeix

#### 1.3.1. Equivalències

$\neg\forall x\, \varphi \equiv \exists x\, \neg\varphi$

$\forall x\, \forall y\, \varphi \equiv \forall y\, \forall x\, \varphi$

$\forall x\, (\varphi \wedge \psi) \equiv \forall x\, \varphi \wedge \forall x\, \psi$

$\exists x\, (\varphi \vee \psi) \equiv \exists x\, \varphi \vee \exists x\, \psi$

### 1.4. Demostracions

* DIRECTA $A \Rightarrow B$
* CONTRARECÍPROC $p \to q \equiv \neg q \to \neg p$
* REDUCCIÓ A L'ABSURD

$\begin{cases} p \equiv \neg p \to 0 \\ \neg A \Rightarrow \dots \Rightarrow \text{contradicció} \end{cases}$

$\begin{cases} p \to q \equiv (p \wedge \neg q) \to 0 \\ A, \neg B \Rightarrow \dots \Rightarrow \text{contradicció} \end{cases}$

* PROVA D'UNA DISJUNCIÓ

$(q \vee r) \equiv (\neg q \to r)$

$B \vee C$ ; $\neg B \Rightarrow \dots \Rightarrow C$

- Conseqüent:

$p \to (q \vee r) \equiv (p \wedge \neg q \to r)$

$A, \neg B \Rightarrow \dots \Rightarrow C$

- Antecedent:

$(q \vee r) \to p \equiv (q \to p) \wedge (r \to p)$

$B \Rightarrow \dots \Rightarrow A$

$C \Rightarrow \dots \Rightarrow A$

* PER CASOS $(p_1 \vee \dots \vee p_n) \to (p \leftrightarrow (p_1 \to p) \wedge \dots)$

## 2. Inducció simple

$\forall n \geq n_0 \; P(n)$

* PAS BASE: $P(n_0)$
* PAS INDUCTIU:
  - Hipòtesi: $P(n)$
  - Tesi: $P(n+1)$

## 3. Conjunts i relacions

$A = \{x \mid P(x)\} \rightarrow \forall x \in A \Leftrightarrow P(x)$

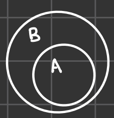
$A = \{x \in B \mid P(x)\} \rightarrow \forall x \in B \Rightarrow x \in A \Leftrightarrow P(x)$

$\emptyset = \{\} = \{x \mid x \neq x\}$

### 3.1. Igualtat

$A = B \Leftrightarrow \forall x \, (x \in A \leftrightarrow x \in B)$

### 3.2. Inclusió ($\subseteq$)



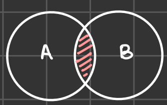

$A \subseteq B \Leftrightarrow \forall x \, (x \in A \rightarrow x \in B)$

$A = B \Leftrightarrow A \subseteq B \, ; \, B \subseteq A$

#### 3.2.1. Propietats

* $\emptyset \subseteq A$
* $A \subseteq A$
* $A \subseteq B \wedge B \subseteq C \Rightarrow A \subseteq C$

### 3.3. Unió ($\cup$)




$A \cup B = \{x \mid x \in A \vee x \in B\}$

$x \in A \cup B \Leftrightarrow x \in A \vee x \in B$

#### 3.3.1. Propietats

* $A \cup A = A$
* $A \cup \emptyset = A$
* $A \cup B = B \cup A$
* $A \cup (B \cup C) = (A \cup B) \cup C$
* $A \subseteq A \cup B, \; B \subseteq A \cup B$
* $A \subseteq B \Leftrightarrow A \cup B = B$
* $A \cup B \subseteq C \Leftrightarrow A \subseteq C, B \subseteq C$

### 3.4 Intersecció ($\cap$)

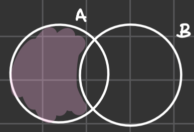
$A \cap B = \{x \mid x \in A \wedge x \in B\}$

$x \in A \cap B \Leftrightarrow x \in A \wedge x \in B$

#### 3.4.1. Propietats

* $A \cap A = A$
* $A \cap \emptyset = \emptyset$
* $A \cap B = B \cap A$
* $A \cap (B \cap C) = (A \cap B) \cap C$
* $A \cap B \subseteq A \, ; \; A \cap B \subseteq B$
* $A \subseteq B \Leftrightarrow A \cap B = A$
* $C \subseteq A \cap B \Leftrightarrow C \subseteq A$ i $C \subseteq B$

### 3.5. Resta




$A - B = \{x \mid x \in A \wedge x \notin B\}$

$x \in A - B \Leftrightarrow x \in A$ i $x \in B$

#### 3.5.1. Propietats

* $A - A = \emptyset$
* $A - \emptyset = A$
* $\emptyset - A = \emptyset$
* $A - B \subseteq A$
* $(A - B) \cap B = \emptyset$
* $A \subseteq B \Leftrightarrow A - B = \emptyset$
* $C \subseteq A - B \Leftrightarrow C \subseteq A, \; C \cap B = \emptyset$

### 3.6. Complementari ($\neg A$)

$A^c = \Omega - A = \{x \in \Omega \mid x \notin A\}$

$x \in A^c \Leftrightarrow x \in \Omega \wedge x \notin A$

$\forall x \in \Omega \Rightarrow (x \in A^c \Leftrightarrow x \notin A)$

### 3.7. Parts d'un conjunt

$P(A) = \{x \mid x \subseteq A\}$

$x \in P(A) \Leftrightarrow x \subseteq A$

### 3.8. Parella ordenada

$(a,b) = (c,d) \Leftrightarrow a = c, \; b = d$

### 3.9 Producte cartesia

$A \times B = \{(a,b) \mid a \in A, \; b \in B\}$

$(a,b) \in A \times B \Leftrightarrow a \in A \wedge b \in B$

$|A \times B| = |A| \cdot |B|$

### 3.10. Demostracions

$\left.\begin{array}{l} A = B \\ A \subseteq B \end{array}\right\} \rightarrow A \subseteq B \, ; \; B \subseteq A$

$x \in A \Leftrightarrow \ldots \Leftrightarrow x \in B$

3.11. R (equivalència)

Reflex: $\forall x \in A \quad x R x$

Simètrica: $\forall x,y \in A \ (xRy \rightarrow yRx)$

Transitiva: $\forall x,y,z \in A \ (xRy \wedge xRz \rightarrow xRz)$

3.12. Clases

$\bar{a} = \{x \in A \mid xRa\}$

$x \in \bar{a} \Leftrightarrow xRa$

$A/R = \{x \mid x = \bar{a}$ per un cert $a \in A\} = \{\bar{a} = a \in A\}$

5. Divisibilitat

TEOREMA D'EUCLIDES

$mcd\,(a,b) = mcd\,(a - \mu b, b)$

DIVISIÓ EUCLIDIANA

dividend → $a \mid b$ ← divisor

residu ↗ $r \mid q$ ← quocient

ID. DE BEZOUT

$mcd\,(a,b) = x \cdot a + y \cdot b$

se obtiene con algoritmo de euclides

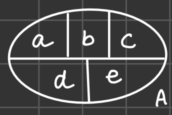
LEMA DE GAUSS

$\left.\begin{array}{l} \text{si } a \mid b \cdot c \\ a, b \text{ primers entre si} \end{array}\right\} \Rightarrow a \mid c$

LEMA DE EUCLIDES

$\left.\begin{array}{l} \text{si } a \text{ primer} \\ a \mid bc \end{array}\right\} \Rightarrow a \mid b \vee a \mid c$

3.13 Particions




$A = \{a, b, c, d, e\}$

4 Funcions

INJECTIVA

$f(x) = f(x') \Rightarrow x = x'$

EXHAUSTIVA

$\forall y \exists x$ tq $f(x) = y$

BIJECTIVA

inj + exhaustiva

6. Congruencias

$a \equiv b \,(\mathrm{mod}\ m) \Leftrightarrow \begin{cases} 1. \ a \underline{|m} \ \text{ i } \ b \underline{|m} \\ 2. \ a - b = \dot{m} \end{cases}$

TEOREMA PETIT FERMAT

$\bar{1} = \overline{a^{p-1}}$ en $\mathbb{Z}_p$

INVERSA

$\bar{a} \cdot \bar{a}^{-1} = \bar{1}$