ARCH models for Value at Risk forecasting in Latin American stock and Forex markets

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Estructura

An adequate VaR forecast can help agents and institutions to reduce financial uncertainty, prevent liquidity problems, and optimize provisioning mechanisms.

Identify the optimal ARCH-type model to forecast the one-day ahead Value at Risk (VaR) at 1% and 5% confidence level for a group of Latin American countrys

Literature Review

Stylized facts of financial returns: Franses and van Dijk (2000), Engle (2004) and Teräsvirta (2009)

- Clustering of extreme returns
- Non normality in distribution
- Leverage effect: high volatility after extreme negative returns
- Long memory in r_t^δ or other transformations

Literature Review

- Engle (1982) proposes the ARCH model and Bollerslev (1986) generalizes it to GARCH.
- Arch-type models have been widely developed. Key references are Glosten el at. (1993), Ding et al. (1993) and Baillie et al. (1996).
- Volatility as a key input in Value at Risk: Angelidis et al. (2004) for the major stock indices, Fan et al. (2008) for crude oil price, Liu and Hung (2010) for S&P 100, Brooks and Persand (2003), Ardia and Hoogerheide (2014), among others.

Methodology

A set of ARCH models is used to forecast the one-day ahead VaR at 1% and 5% confidence level for a group of Latin American countrys.

1. An ARCH-type model parameters are estimated for a given rolling-window of 250 observations.
2. With the fitted model the one-day-ahead volatility.
3. VaR is computed from forecasted volatility and distribution parameters.
4. The accuracy of the VaR forecasts is analyzed by backtest tests.

Additionally, as a previous step, an in-sample analysis is performed where the models are specified and its parameters estimated with the full available series.

Methodology

A complete ARCH model is divided into three components:

- a mean model: zero, constant, AR(1)
- a volatility process: ARCH, GARCH, GJR, FIGARCH, APARCH.
- a distribution for the standardized residuals: Normal, $\mathcal{S}$-Student, Skeweed, and generalized error distribution (GED).

The ARCH model of Engle (1982)

$$\sigma_t^2 = \omega + \sum_{p=1}^{P} \alpha_p \epsilon_{t-P}^2 \tag{1}$$

The GARCH model of Bollerslev (1986)

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{2}$$

The GJR (1993) model

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \gamma_i I_{[\varepsilon_{t-i}<0]} \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{3}$$

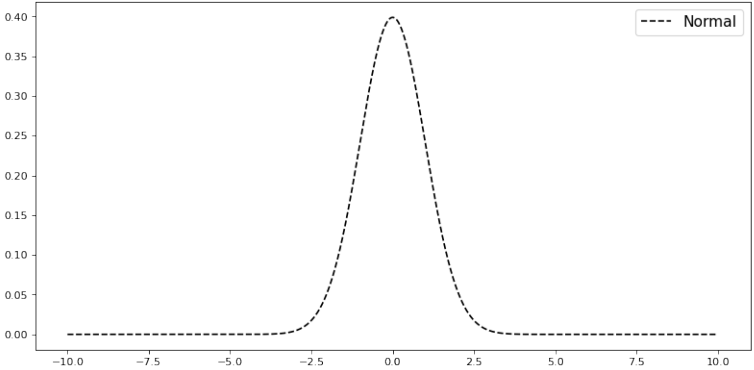
The FIGARCH of Baillie et al. (1986)

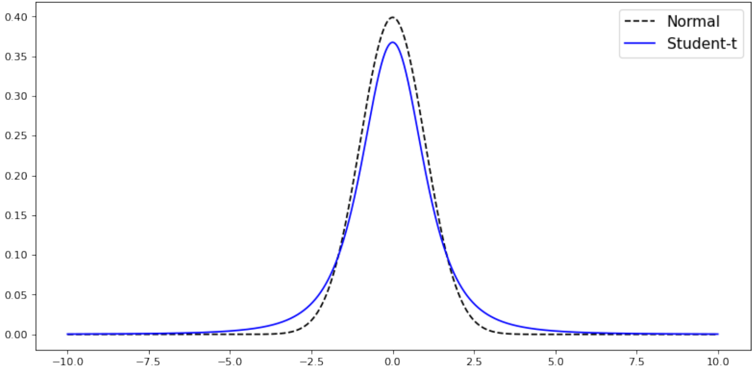
$$\sigma_t^2 = \omega + \left[1 - \beta L - \phi L (1 - L)^d\right] \varepsilon_t^2 + \beta \sigma_{t-1}^2 \tag{4}$$

The APARCH of Ding et al. (1993)

$$\sigma_t^\delta = \omega + \sum_{i=1}^{q} \alpha_i \left(|\varepsilon_{t-i}| - \gamma_i I_{[o \geq i]} \varepsilon_{t-i}\right)^\delta + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^\delta \tag{5}$$

Methodology
Distributions




Normal
Student-t
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
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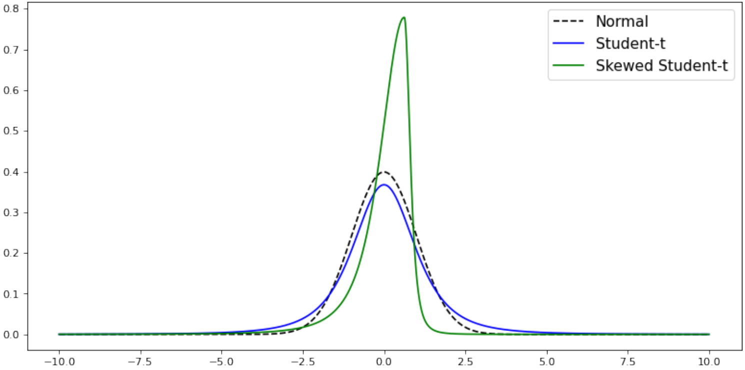
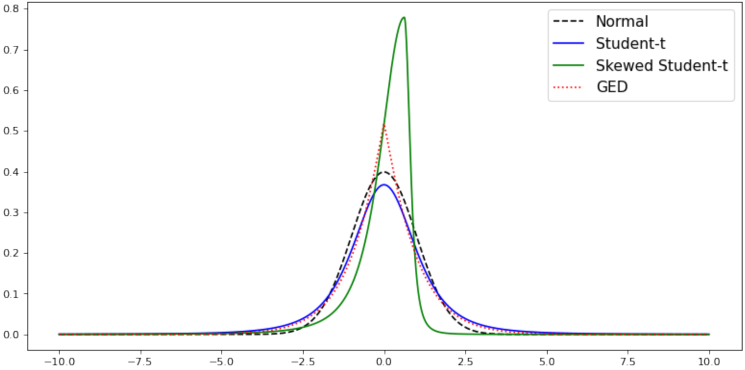
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0

Methodology
Distributions



Methodology

Distributions



$$\widehat{VaR}_{t+1}^{\alpha} = -\hat{\mu}_{t+1} - \widehat{\sigma}_{t+1} q_{\alpha} \quad (6)$$

backtest procedures:

- Unconditional coverage (UC) test of Kupiec (1995)
- Conditional Coverage (CC) test of Christoffersen (1998)
- Dynamic quantile (DQ) test of Engle and Manganelli (2004).

Empirical Evidence: Data

- Countries: Argentina, Brazil, Chile, Colombia, Mexico and Peru
- Markets: stocks and Forex
- Frecuency: daily
- Source: Bloomberg Financial Data
- Returns are calculated as $y_t = 100 * [\log(p_t) - \log(p_{t-1})]$

Table 1: Descriptive Statistics for Stock and Forex Markets Returns

Country	Security ID	Start Date	End Date	Obs.	Mean	Std,	Min	Max	Skew	Kurt
(a) Stock										
Argentina	MERVAL	1991-12-26	2021-08-31	7744	0.06	2.30	-47.69	16.12	-1.43	27.89
Brasil	IBOV	1995-03-16	2021-08-31	6904	0.05	1.96	-17.23	28.82	0.09	14.05
Chile	IPSA	1990-08-09	2021-08-31	8104	0.05	1.14	-15.22	11.80	-0.32	12.83
Colombia	IGBC	2001-07-26	2021-08-31	5244	0.05	1.14	-11.05	14.69	-0.18	15.73
México	MEXBOL	1994-03-31	2021-08-31	7154	0.04	1.40	-14.31	12.15	0.03	7.56
Perú	SPBLPGPT	2002-02-07	2021-08-31	5104	0.05	1.35	-13.29	12.82	-0.59	12.69
(b) Forex										
Argentina	ARS	2014-03-06	2021-08-31	1954	0.13	1.14	-6.03	30.80	12.32	294.71
Brasil	BRL	1999-06-03	2021-08-31	5804	0.02	1.04	-10.34	7.11	0.07	5.90
Chile	CLP	1990-01-18	2021-08-31	8249	0.01	0.60	-4.33	4.68	0.25	7.54
Colombia	COP	1992-09-03	2021-08-31	7564	0.02	0.66	-7.60	6.02	0.17	10.31
México	MXN	1996-05-09	2021-08-31	6604	0.01	0.71	-6.65	7.98	0.85	11.43
Perú	PEN	1995-05-25	2021-08-31	6854	0.01	0.30	-2.85	3.55	0.08	14.54

Table 2a: Estimated Parameters for daily Latin American stock Markets Return

Model	μ	ω	α	β	γ	δ	ν	λ	BIC	log-lik
Argentina (MERVAL)										
μ-APARCH-GED	0.0462^a	0.1573^a	0.1120^a	0.8564^a	0.2148^a	1.9847^a	1.0919^a		31888.194	-15912.755
μ-GJR-GED	0.0462^a	0.1592^a	0.0692^a	0.8558^a	0.0955^a		1.0921^a		31879,24598	-15912,758
Brazil (IBOV)										
μ-APARCH-GED	0.0503^a	0.0701^a	0.0759^a	0.8980^a	0.4245^a	1.7129^a	1.3592^a		26273.472	-13105.796
μ-GJR-GED	0.0510^a	0.0814^a	0.0275^a	0.8933^a	0.1067^a		1.3594^a		26266,91876	-13106,939
Chile (IPSA)										
μ-APARCH-GED	0.0304^a	0.0295^a	0.1411^a	0.8475^a	0.1485^a	1.7939^a	1.3146^a		21853.744	-10895.372
μ-FIGARCH-GED	0.0372^a	0.0400^a	0.2548^a	0.5357^a		0.4905^a	1.3183^a		21845,49687	-10895,7481
Colombia (IGBC)										
APARCH-GED		0.0045^a	0.0943^a	0.9203^a	0.2276^a	0.3190^a	1.0853^a		5889.822	-2919.216
μ-APARCH-GED	-0.0000	0.0000	0.1932	0.8068	0.0230	1.8244	1.0100		6023.314	-2981.680
Mexico (MEXBOL)										
μ-APARCH-GED	0.0316^a	0.0160^a	0.0786^a	0.9214^a	0.4290^a	1.4718^a	1.3037^a		22136.973	-11037.422
μ-GJR-GED	0.0331^a	0.0167^a	0.0312^a	0.9131^a	0.0988^a		1.3015^a		22138,60164	-11042,674
Peru (SPBLPGPT)										
μ-FIGARCH-GED	0.0397^a	0.0846^a	0.0867	0.3316^a		0.4235^a	1.1342^a		14482.021	-7215.397
μ-APARCH-GED	0.0375^b	0.0394^a	0.1473^a	0.8337^a	0.0743^b	1.9825^a	1.1281^a		14524,71567	-7232,475

a, b, c denote significance level at 1%, 5% and 10% respectively

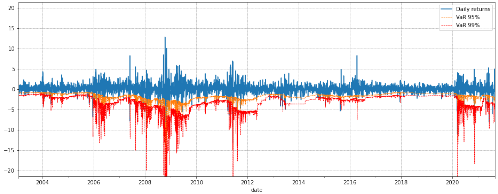
Table 2a: Estimated Parameters for daily Latin American Forex Markets Return

Model	μ	ω	α	β	γ	δ	ν	λ	BIC	log-lik
					Argentina (ARS)					
μ-FIGARCH-sk$\mathcal{S}$	0.0539^a	0.0003	0.0541	0.6814^b		0.8918^a	3.0927^a	0.0909^b	500.480	-223.718
μ-FIGARCH-$\mathcal{S}$	0.0501	0.0003	0.0768	0.6077^a		0.8463^a	3.0500^a		505,101	-229,8176
					Brazil (BRL)					
μ-APARCH-GED	0.0022	0.0139^a	0.0968^a	0.9032^a	-0.3064^a	1.4421^a	1.3863^a		14806.536	-7372.936
GJR-GED		0.0101^a	0.1438^a	0.8940^a	-0.0872^a		1.3768^a		14800,927	-7378,798
					Chile (CLP)					
APARCH-$\mathcal{S}$		0.0270^a	0.1300^a	0.8686^a	-0.4153^a	0.2499^a	2.8537^a		8009.367	-3977.630
APARCH-sk$\mathcal{S}$		0.0177^a	0.0884^a	0.9099^a	0.2023^a	0.3396^a	3.0470^a	0.0971^a	8837.625	-4387.2502
					Colombia (COP)					
μ-FIGARCH-GED	0.0000	0.0024^c	0.2664^a	0.5728^a		0.4672^a	1.0100^a		10252.889	-5099.651
FIGARCH-GED		0.0024^c	0.2662^a	0.5730^a		0.4675^a	1.0100^a		10244,165	-5099,754
					Mexico (MXN)					
μ-APARCH-sk$\mathcal{S}$	0.0117^a	0.0120^a	0.0966^a	0.9034^a	-0.4849^a	1.1410^a	7.3379^a	0.1336^a	11378.144	-5653.890
μ-GJR-GED	0.0331^a	0.0167^a	0.0312^a	0.9131^a	0.0988^a		1.3015^a		22138,601	-11042,674
					Peru (PEN)					
μ-FIGARCH-GED	-0.0000	0.0002	0.1766^a	0.6098^a		0.6468^a	1.0100^a		-2873.629	1463.312
μ-APARCH-GED	-0.0000	0.0004	0.1618^a	0.8382^a	-0.1393	2.0107	1.0100^a		-2844,943	1453,385

a, b, c denote significance level at 1%, 5% and 10% respectively

Figure: One-step ahead VaR forecast for Peru (SPBLPGPT)

μ-ARCH-$\mathcal{S}$



μ-FIGARCH-GED

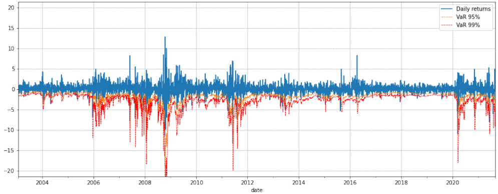


Table 3: Accuracy of VaR predictions for Stock and Forex Markets Returns

VaR 5% risk level					VaR 1% risk level				
Model	% Failures	UC	CC	DQ	Model	% Failures	UC	CC	DQ
(a) Stock Market Returns									
Argentina (MERVAL)									
AR(1)-GARCH-$\mathcal{S}$	0.0653	0.0373*	0.0078*	0.0000*	AR(1)-FIGARCH-GED	0,0179	0.0272*	0.0518	0.0000*
AR(1)-GJR-$\mathcal{N}$	0.0674	0.0185*	0.0061*	0.0011*	GARCH-$\mathcal{N}$	0,0189	0.0133*	0.0331*	0.0000*
Brazil (IBOV)									
μ-GJR-GED	0.055285	0.0749	0.0066*	0.0155*	APARCH-$\mathcal{S}$	0.0167	0.0000*	0.0000*	0.0000*
μ-APARCH-GED	0.055286	0.0748	0.0067*	0.0155*	APARCH-GED	0.0167	0.0000*	0.0000*	0.0000*
Chile (IPSA)									
GJR-$\mathcal{S}$	0,0568	0.0058*	0.0000*	0.0000*	FIGARCH-sk$\mathcal{S}$	0,0166	0.0000*	0.0000*	0.0000*
APARCH-$\mathcal{S}$	0,0569	0.0058*	0.0000*	0.0000*	GARCH-GED	0,0166	0.0000*	0.0000*	0.0000*
Mexico (MEXBOL)									
FIGARCH-$\mathcal{S}$	0,044	0.3030	0.0047*	0.0121*	GJR-$\mathcal{N}$	0,0146	0.0690	0.0003*	0.0000*
FIGARCH-GED	0,045	0.3030	0.0003*	0.0002*	APARCH-$\mathcal{N}$	0,0147	0.0690	0.0002*	0.0000*
Peru (SPBLPGPT)									
FIGARCH-GED	0,0569	0.0312*	0.0000*	0.0000*	FIGARCH-sk$\mathcal{S}$	0,0165*	0.0000*	0.0000*	0.0000*
μ-GARCH-$\mathcal{S}$	0,0571	0.0266*	0.0000*	0.0000*	FIGARCH-GED	0,0173*	0.0000*	0.0000*	0.0000*
(b) Forex Market Returns									

Conclusions (for stock markets)

1. In general, FIGARCH volatility process and leptokurtic distributions are able to produce better one-step-ahead VaR forecasts
2. The models that best fit the full series in-sample are not necessarily the ones that obtain the most accurate VaR forecasts out-of-sample
3. The models producing the most accurate forecasts vary by market and country.

Agenda

- Obtain the results also for the Forex market
- Compare the results for the stock market and the Forex market.

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