ARCH models for Value at Risk forecasting in Latin American stock and Forex markets*

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Abstract

Using stock and Forex markets daily returns, a set of ARCH models with three different mean specifications and four error distributions are used to forecast the one-day ahead Value at Risk at 1% and 5% confidence level for a group of Latin American countries. The main results are: (i) in general, APARCH volatility process and leptokurtic distributions are able to produce better one-step-ahead VaR forecasts, (ii) the models that best fit the full series in-sample are not necessarily the ones that obtain the most accurate VaR forecasts out-of-sample, (iii) the data never validates a $\mathcal{N}$ distribution and, among the leptokurtic distributions, a $\mathcal{GED}$ is preferred over a $\mathcal{S}$ distribution, (iv) the models producing the most accurate forecasts vary by market and country.

JEL Classification: C22, C52, C53.

Keywords: ARCH models, Value at Risk, Backtest, Volatility, Latin American markets, Stock, Forex.

*The source data, results tables and procedures to replicate the results in this research are available in the supplementary material.

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1 Introduction

Volatility is a key input in financial risk management methods like Value at Risk (VaR) so its study has been a field of interest within finance and econometrics. One of the most popular class of models for study the volatility are the autoregressive conditional heteroskedasticity (ARCH) that since its development by Engle (1982) have been widely extended to capture the complexity of financial data. The better the model fits and forecast the financial series, the more accurate the forecasted risk metric will be. In this context, the empirical literature has identified a set of stylized facts of the financial returns, some of these are *i*) correlated volatility, *ii*) leptokurtic distribution of the disturbance term, *iii*) leverage effect, *iv*) negative skewness and *v*) long memory in r_t^δ or other transformations; see Franses and Dijk (2000).

In this research a group of five ARCH volatility models with three different specifications for the mean equation, and four error distributions are used to model and forecast stock and Forex market return volatility of a group of Latin American countries. Then, from a risk management perspective, the forecasted volatility and its distribution parameters is used to estimate the one-day ahead Value at Risk (VaR) at 99% and 95% confidence level. The accuracy of the forecasts are evaluated through backtest procedures from which the following results are obtained: (i) in general, APARCH volatility process and leptokurtic distributions are able to produce better one-step-ahead VaR forecasts (ii) the models that best fit the full series in-sample are not necessarily the ones that obtain the most accurate VaR forecasts out-of-sample and (iii) the models producing the most accurate forecasts vary by market and country.

The document proceeds as follows. Section 2 contains the literature review. Section 3 briefly presents the models and backtest procedures that are used in the empirical section. Section 4 analyze the empirical findings.

2 Literature review

The theoretical literature on the study of volatility with ARCH models is extensive and a great interest to financial risk managers[1]. The starting point is often considered to be Engle (1982), who formally introduces the autoregressive conditional heteroskedasticity (ARCH) univariate model to explain the dynamic of inflation in the United Kingdom. Following this line, Bollerslev (1986) developed a generalization of the ARCH model by allowing past conditional variances to be integrated as regressors within the current conditional variance equation.

The GARCH model has been extended to capture additional stylized facts observed in financial returns. One of the most studied is the so-called financial leverage effect, which refers that volatility responds asymmetrically to negative and positive past returns. In this sense, it is more likely to observe high volatility when past returns have been negative; see Black (1976). In this regard, Nelson (1991) developed the exponential GARCH (EGARCH), which captures the leverage effects through a dummy variable and remove the restrictions of the non-negativity of the variance present in the GARCH model. In the same line, Glosten et al. (1993) developed the GJR model witch incorporates an asymmetry parameter in the GARCH equation that exacerbates the volatility when past returns were negative.

Another stylized fact is the long memory of volatility. To capture this effect Baillie et al. (1996) introduce the fractionally integrated GARCH (FIGARCH) which allows to conditional variance decay at a slow hyperbolic rate determined by the parameter of fractional differentiation. Similarly, Ding et al. (1993) proposed the asymmetric power ARCH (APARCH) that allows an estimation of the long memory parameter in the volatility.

On the side of empirical research, Vlaar (2000) found that the GARCH-$\mathcal{N}$ outperform other distributions such as student-$\mathcal{S}$ and the historical simulation models in VaR estimation for the Dutch bond interest rates. On the other hand, Brooks and Persand (2003) compare symmetric and asymmetric arch-type models on VaR accuracy for a selection of Southeast Asian stock indices. They found that models ignoring asymmetrical effects lead to inappropriately small VaR estimates compared to models taking the asymmetries in account. In addition, Angelidis et al. (2004) evaluate through backtest procedures the performance of an extensive family of ARCH models with different error distributions and sample sizes in forecasting one-day ahead VaR of a selection of major stock indices. The results don't reveal a clearly superior model but concluded that heavy-tail

[1] See Engle and Manganelli (2004) and Bollerslev (2008) for a survey on the theoretical development of the ARCH models, and Poon and Granger (2003) and Andersen et al. (2005) for empirical application in forecasting volatility.

distributions lead to more accurate VaR forecasts than the Normal distribution and that there is no consistent relation between the sample sizes and the optimal models. Ane (2006) compares different VaR specifications in an Asymmetric Power ARCH framework for the japanesse stock market and found that include a optimal power parameter provides little improvements for risk management. Orhan and Koksal (2012) compare a comprehensive list of ARCH models in quantifying VaR for of stock market indices of emerging (Brazil and Turkey) and developed (Germany and the USA) markets. They concluded that leptokurtic error distributions yielded the best results for VaR estimations.

Other documents with empirical applications of ARCH models in VaR forecast are Fan et al. (2008), who analyze the crude oil price incorporating a GED distribution; Liu and Hung (2010), who study the Standard & Poor's 100 with asymmetric distributions and ARCH models; and Ardia and Hoogerheide (2014), who analyze the impact of the estimation frequency on one-day ahead forecasts of risk metrics.

3 Methodology

A set of ARCH models is used to forecast the one-day ahead VaR at 99% and 95% confidence level for a group of Latin American countries. This is done through a four-step out-of-sample analysis. First, an ARCH-type model is estimated for a given window of 1000. Second, with the fitted model the one-day-ahead volatility and distribution of returns is obtained. Third, the VaR is computed from forecasted volatility and distribution parameters. Fourth, the accuracy of the VaR forecasts is analyzed by backtest tests. Additionally, as a previous step, an in-sample analysis is performed where the models are fitted with the full available series.

The estimation of the parameters is carried out in a frequentist way through a Quasi-Maximum Likelihood; see Bollerslev and Wooldridge (1992). All the parameters are estimated simultaneously by maximizing the log-likelihood function. The optimal parameters are obtained solving the first-order conditions.

3.1 ARCH models

A complete ARCH model is divided into three components: mean specification, volatility model and distribution of the disturbance term. This research consider three different mean equation specifications (zero, constant, AR(1)), five volatility process (ARCH, GARCH, GJR, FIGARCH and APARCH) and four error distributions (Normal, $\mathcal{S}$-Student, Skeweed $\mathcal{S}$-Student, and Generalized error distribution).

3.1.1 Mean specifications

Given y_t a series of returns, the following mean specifications are considered:

$$y_t = \begin{cases} \epsilon_t & \text{Zero} \\ \mu + \epsilon_t & \text{Constant} \\ \mu + \theta y_{t-1} + \epsilon_t & \text{AR(1)} \end{cases} \tag{1}$$

Where $\epsilon_t = \sigma_t z_t$ are the errors with a zero mean and a conditional variance of σ_t^2. Additionally, z_t is an independent and identically distributed process with $E(e_t) = 0$ and $Var(e_t) = 1$.

3.1.2 Volatility process

Regarding the volatility process, two symmetric models are implemented: the ARCH model of Engle (1982) and the GARCH of Bollerslev (1986). Fist, the ARCH(1) model can be expressed as:

$$\sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2 \tag{2}$$

where ω represents the constant in the volatility equation and α is the coeficient of the lagged squared residuals. In order for the ARCH(1) process to be well defined $\forall t$ has to be positive. The conditions of sufficiency to assure the positivity of the variance are given by $\omega > 0$ and $\alpha \geq 0$

Secondly, the GARCH(1,1) model can be given by:

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{3}$$

Where β is the coeficient of the lagged conditional variance. This model reduces the number of estimated parameters by imposing the restrictions $\omega > 0$, $\alpha \geq 0$ and $\beta \geq 0$ to ensure a positive conditional variance, and $\alpha + \beta < 1$ to ensure that the unconditional variance is defined.

On the other hand, two asymmetric in volatility models are considered: the GJR model of Glosten et al. (1993) and the APARCH model of Ding et al. (1993). The GJR(1,1,1) incorporates a dummy variable to capture the positive and negative asymmetry of the innovations. This model may be expressed as:

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \gamma I_{\{\epsilon_{t-1}<0\}}\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{4}$$

Where γ is the parameter of asymmetry and $I_{\{.\}}$ is a dummy variable related to the fulfillment of the condition.

The APARCH(1,1,1) model parameterizes the non-linearity including a δ parameter to estimate. It can be expressed as:

$$\sigma_t^\delta = \omega + \alpha\left(\left|\epsilon_{t-i}^2\right| - \gamma I_{\{\epsilon_{t-1}<0\}}\epsilon_{t-1}\right)^\delta + \beta_i\sigma_{t-i}^\delta \tag{5}$$

where $\delta > 0$ is a Box-Cox transformation of σ_t. Moreover, $-1 < \gamma < 1$, $\omega > 0$, $\delta \geq 0$ and $\beta \geq 0$. The APARCH model nests other seven univariate ARCH model as special cases. These are i) the ARCH model when $\delta = 2$, $\gamma = 0$ and $\beta = 0$; ii) the GARCH model when $\delta = 2$ and $\gamma = 0$; iii) the GJR model when $\delta = 2$; iv) the Absolute-Value-GARCH (AVARCH) of Taylor (1986) and Schwert (1990) when $\delta = 1$ and $\gamma = 0$; v) the Threshold ARCH (TARCH) of Zakoian (1994) when $\delta = 1$; vi) the Nonlinear GARCH (NGARCH) of Bera and Higgins (1993) for $\gamma = 0$ and $\beta = 0$; vii) and the log-ARCH of Geweke (1986) and Pantula (1986) when $\delta \Rightarrow 0$.

Finally, in FIGARCH(1,1) of Baillie et al. (1996) model the hyperbolic rate of decay for the lagged squared or absolute innovations in the conditional variance function using fractional orders of integration in the autoregressive polynomial. It can given by:

$$\sigma_t^2 = \omega + \left[1 - \beta L - \phi L\left(1 - L\right)^d\right]\epsilon_t^2 + \beta\sigma_{t-1}^2 \tag{6}$$

where L is the lag operator, d is the fractional differencing parameter and ϕ is the parameter that measure the sensitivity to the long-memory component. Adicionally, the FIGARCH nest the IGARCH model of Engle and Bollerslev (1986) as special case when $d = 1$.

3.1.3 Distribution of the errors

Regarding the error distributions, three symmetric distributions around the mean are used. First, a normal distribution ($\mathcal{N}$), because it is the most used among practitioners and its practical advantages. The corresponding density function given by:

$$f\left(z_t\right) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}z^2\right) \tag{7}$$

Second, a student-$\mathcal{S}$ ($\mathcal{S}$)distribution is considered because its thick-tailed and higher peak seems to correspond more closely to financial returns. Under this distribution the density function is given by:

$$f\left(z;\nu\right) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{\pi\left(\nu-2\right)}}\left(1 + \frac{z^2}{\nu-2}\right)^{-\left(\frac{\nu+1}{2}\right)}$$

Where ν is the kurtosis parameter or the number of degrees of freedom, and $\Gamma(.)$ is the Gamma function. The variance of the $\mathcal{S}$ distribution is defined if $\nu > 0$.

Third, Generalized error distribution ($\mathcal{GED}$) is included following the Nelson (1991) recomendation because have finite unconditional moments. The density of a $\mathcal{GED}$ can be expressed by:

$$\mathcal{GED}\left(z_t;\nu\right) = \frac{\nu\exp\left(-\frac{1}{2}\left|\frac{z_t}{\lambda}\right|^2\right)}{\lambda 2^{(1+1/\nu)}\Gamma\left(\frac{1}{\nu}\right)} \tag{8}$$

where $\nu > 0$ and $\lambda = \left(\frac{\Gamma(1/\nu)}{4^{1/\nu}\Gamma(3/\nu)}\right)^{\frac{1}{2}}$. The $\mathcal{GED}$ includes a $\mathcal{N}$ distribution when $\nu = 2$, and a double exponential distribution when $\nu = 1$. For $\nu < 2$ indicates the presence of tails heavier than the $\mathcal{N}$.

Finally, the skewed Student's t-distribution proposed by Hansen (1994) is considered to capture asymmetry in conjunction with leptokurtosis. The corresponding density function expressed by

$$f(z_t; \nu, \lambda) = \begin{cases} bc\left(1 + \frac{1}{\nu-2}\left(\frac{b\epsilon_t + \sigma_t a}{\sigma_t(1-\lambda)}\right)^2\right)^{-(\nu+1)/2} & \epsilon_t < -a/b \\ bc\left(1 + \frac{1}{\nu-2}\left(\frac{b\epsilon_t + \sigma_t a}{\sigma_t(1+\lambda)}\right)^2\right)^{-(\nu+1)/2} & \epsilon_t \geq -a/b \end{cases} \tag{9}$$

Where λ parameter controls the skewness. In addition, $2 < \nu < \infty$, and $-1 < \lambda < 1$. The constants a, b and c are given by

$$a = 4\lambda c\left(\frac{\nu-2}{\nu-1}\right) \tag{10}$$

$$b = \left(1 - 3\lambda^2 - a^2\right)^{\frac{1}{2}} \tag{11}$$

$$c = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi(\nu-2)}\Gamma\left(\frac{\nu}{2}\right)} \tag{12}$$

If $\lambda > 0$, the mode of the density is to the left of zero and the variable is skewed to the right, and vice-versa when $\lambda < 0$. When $\lambda = 0$ the distribution is identical to a standarized Student-$\mathcal{S}$.

3.2 Value at Risk

Regarding the risk management perspective, the Value-at-Risk (VaR) is a market risk metric defined as the maximum loss that will be incurred on an asset or portfolio with a given level of significance α over a specified period. It is given by

$$\mathrm{VaR}_{t+1}^{\alpha} = -\hat{\mu}_{t+1} - \widehat{\sigma}_{t+1} q_\alpha \tag{13}$$

Where $\widehat{\sigma}_{t+1}$ is the forecasted 1-day ahead volatility that will be obtained through the ARCH models, and q_α corresponds to the α^{th} quantile of the cumulative distribution function of the errors distribution.

3.3 Backtest procedures

Backtesting are a group of procedures developed to determine the accuracy of a VaR model involves the comparison of the estimated VaR to the realized profit or loss; see Jorion (2001). The accuracy of VaR forecasts are measured in terms of unconditional coverage and independence of the failures. A failure occurs when the return is less than the forecasted VaR value. In this line, in a VaR at 95% confidence level a failure is expected to occur only in 5% of the time. In this research the backtest procedures tu use are four: the Unconditional coverage (UC) test of Kupiec (1995)[2] witch is used to analyze unconditional coverage, the Conditional coverage independence (CCI) test of Christoffersen (1998) to analyze the independence independence of the failures, and the Conditional Coverage (CC) test of Christoffersen (1998)[3] and the Dynamic Quantile (DQ) test of Engle and Manganelli (2004) to jointly analyze the unconditional coverage and independence.

3.3.1 Unconditional coverage test

The UC test of Kupiec (1995) is a likelihood ratio test that assess if the frequency of VaR failures is consistent with the VaR confidence level. The test can be expressed as:

$$\mathrm{LR}_{UC}^{\alpha} = -2\left[(N-x)\log\left(\frac{N(1-p)}{N-x}\right) + x\log\left(\frac{N\cdot p}{x}\right)\right] \tag{14}$$

[2]Also known as the Proportion of Failures (POF) test.
[3]Also known as the Conditional interval forecast evaluation.

It is χ^2-distributed with one degree of freedom where x is the number of VaR failures, N the number of observations and $p = 1 - \alpha$ the VaR confidence level. The null hypothesis is that the observed proportion of tail losses is consistent with the significance level of the model.

3.3.2 Conditional Coverage Independence (CCI) test

On the other hand, independence refers that the failures are serially not correlated and is assessed with the CCI test of Christoffersen (1998). The test can be written as:

$$\mathrm{LR}^{\alpha}_{CCI} = -2\log\left(\frac{(1-\pi_2)^{(n_{00}+n_{10})}\,\pi_2^{(n_{01}+n_{11})}}{(1-\pi_{01})^{n_{00}}\,\pi_{01}^{n_{01}}\,(1-\pi_{11})^{n_{10}}\,\pi_{11}^{n_{11}}}\right) \tag{15}$$

where

$$\pi_{01} = \frac{n_{01}}{n_{00}+n_{01}},\ \pi_{11} = \frac{n_{11}}{n_{10}+n_{11}},\ \pi_2 = \frac{n_{01}+n_{11}}{n_{00}+n_{01}+n_{10}+n_{11}}$$

and n_{ij} is the number of i values followed by a j value in the I_t series with $i, j = 0, 1$. The CCI test is distributed as a χ^2 with 1 degree of freedom under the null hypothesis of serial independence against the alternative of first-order Markov dependence.

3.3.3 Conditional Coverage (CC) test

The CC tests the sum of the individual test statistics as $\mathrm{LR}^{\alpha}_{CCI} = \mathrm{LR}^{\alpha}_{UC} + \mathrm{LR}^{\alpha}_{CCI}$, where $\mathrm{LR}^{\alpha}_{CCI}$ is asymptotically distributed as a χ^2 with 2 degree of freedom under the joint null hypothesis of unconditional coverage and the independence of the VaR failures. For example, given a p-value of 0.06, the null hypothesis is not rejected at the 95% confidence level and the model will be correct in terms of unconditional covarage and independence of the failures.

3.3.4 Dynamic Quantile (DQ) test

The Dynamic Quantile is a test based on a linear regression model of the hits variable on a set of explanatory variables including a constant, the lagged values of the hit variable, and an error term[4]:

$$\mathrm{Hit}^{\alpha}_t = \delta_0 + \sum_{l=1}^{L} \delta_l \mathrm{Hit}^{\alpha}_{t-1} + \delta_{L+1}\mathrm{VaR}^{\alpha}_{t-1} + \epsilon_t$$

In this line, testing for the null hypothesis of conditional coverage is equivalent to testing the joint nullity of the vector of $2K+1$ coefficients $\widehat{\delta} = \left(\hat{\delta}_0, ..., \hat{\delta}_{L+1}\right)'$. Therefore, the test statistic satisfies the following relation:

$$\mathrm{DQ}^{\alpha} = \frac{\hat{\delta}'\boldsymbol{Z}'\boldsymbol{Z}\hat{\delta}}{\alpha\,(1-\alpha)} \tag{16}$$

Where $\boldsymbol{Z}$ is the matrix of explanatory variables of the linear regression model. The test is χ^2 distributed with L + 2 degrees of freedom. Like Engle and Manganelli (2004), $L = 4$ lags are chosen.

4 Empirical evidence

4.1 Data

The daily closing price of the stock indices and exchange rates of Argentina, Brazil, Chile, Colombia, Mexico and Peru are obtained from Bloomberg Financial Data. Returns are calculated as the percentage change in logarithms of the price (p_t) according to $y_t = 100 * [\log(p_t) - \log(p_{t-1})]$. The series are taken from different dates according to availability and following Ataurima and Rodriguez (2020). For stock markets the start dates are December 26, 1991 (Argentina); March

[4]Other explanatory variables can be included such as past returns, transformations of the past returns or the implicit volatility.

16, 1995 (Brazil); August 9, 1990 (Chile); July 26, 2001 (Colombia); March 31, 1994 (Mexico) and February 7, 2002 (Peru). In case of Forex markets the start dates are March 6, 2014 (Argentina); June 03, 1999 (Brazil); January 18, 1990 (Chile); September 3, 1992 (Colombia); May 9, 1996 (Mexico) and May 25, 1995 (Perú). The samples for all countries and markets end on August 31, 2021.

Table 1 shows the descriptive statistics of the series. Panel a show the information about stock indices returns where the extreme values are between a minimum of -47.69 (Argentina) and a maximum of 28.82 (Brazil). Additionally, the most volatile stock market index in terms of standard deviation is Argentina and the least volatile is Mexico. Only the stock index of Argentina and Mexico show positive asymmetries. Panel b show the information about Forex returns where the extreme values are between a minimum of -10.34 (Brasil) and a maximum of 30.80 (Argentina). The most volatile stock market index in terms of standard deviation is Argentina and the least volatile is Mexico.

Regarding the comparison between both markets, a characteristic in common is, firstly, the excess of kurtosis, a well-known stylized fact of the presence of an asymmetric distribution with heavy tails. Second, for all countries except Argentina the extreme values of the stock market are grater than in the Forex markets. In addition, a characteristic that differentiates both markets is that unlike stock markets, all skew coefficients for Forex markets are positive. In terms of standard deviation, the Forex markets are less volatile than the stock markets.

Figure 4.2.2, 4.2.2, 4.2.2 and 4.2.2 show the evolution of Latam stock and Forex markets returns and volatilities respectively. We can see typical stylized facts such as volatility clusters. For example, periods of higher volatility during the 2008–2009 Global Financial crisis and the COVID-19 pandemic.

4.2 Results

A set of five ARCH models with three different mean specifications and four error distributions is implemented. This implies estimating 60 models for each market across all countries. Since we have six stock markets and six Forex markets, the total number of models to estimate is 720.

The results in this paper were obtained using Python 3.6.9 with the ARCH package version 5.1.0 by Sheppard et al. (2021). Additionally, since no backtest procedures implemented in Python were found, these were programmed with the equations from the original papers. The empirical results were compared with those of the Risk Management Toolbox[5] in MATLAB, and the GAS package[6] byArdia et al. (2019) and the rugarch package[7] by Ghalanos (2020) in R 3.6.1 to ensure that the implementation was correct. Computations were performed on virtual machine Windows 2019 x64 (64-bit) with Intel(R) Xeon(R) CPU 3.10 GHz hosted on the Google Cloud platform.

4.2.1 In-sample estimation

First, an in–sample analysis is considered. In this, the models are fitted to the full available series. Table 2a and 2b show the results for the estimated parameters for the stock and Forex markets respectively[8]. The best models are selected using the log likelihood criteria and, in case of controversy, the Bayesian information criterion (BIC) and the significance of the parameters. Common result between stocks and Forex markets are as follows. Fist, integrating an AR(1) component in the mean equation clearly improves the log-like and the BIC. In this regard, the θ coefficient of the autoregressive term is significant and different from zero in most of the cases. Second, the symmetric (ARCH, GARCH) or purely asymmetric (GJR) models in the variance are outperformed by more flexible models as the APARCH and the FIGARCH. Third, the data never validates a $\mathcal{N}$ distribution. Also, among the leptokurtic distributions, a $\mathcal{GED}$ is preferred over a $\mathcal{S}$ distribution. Market-specific results are as follow.

Table 2a show the results for stock markets. In this the AR(1) mean specification is selected for all countries. Also, the reported results includes an constant mean for Argentina and Brazil, and a zero mean for Argentina as alternative mean specifications. In terms of the volatility process, the APARCH model is validated by data for all markets except for Peru, where a FIGARCH is

[5]https://mathworks.com/products/risk-management.html
[6]https://CRAN.R-project.org/package=GAS
[7]https://CRAN.R-project.org/package=rugarch
[8]Due the large number of models, Table 2a, 2b, 3a and 3b only report five of the 60 best performing models for each serie. The complete tables are available in the supplementary material.

selected. The alternative volatility models includes a GJR for all countries and a GARCH for Colombia and Peru. In terms of distribution, a $\mathcal{GED}$ is chosen for all markets except for Chile, where a sk$\mathcal{S}$ is selected. The $\mathcal{S}$ is an alternative distribution for Chile, Colombia, Mexico and Peru.

Regarding the parameter analysis, θ is positive and significant at 1% for all countries except for Brazil, which supports the inclusion of an AR(1) component in the mean equation. The parameter of asymmetry in the volatility γ is positive and significant at 1% in most of the cases. This validates the stylized fact for the stock markets that negative past returns increases the volatility. Further, the ν parameter related to the kurtosis is in most cases significant and close to the unit, reflecting the presence of heavy tails.

Table 2b show the results for Forex markets. The AR(1) mean specification is selected for Argentina, Brazil, Colombia and Mexico, and the Zero mean is selected for Chile and Peru. The constant mean is present as an alternative in Argentina, Brazil, Mexico and Peru. Regarding the volatility process, the APARCH is selected model Argentina, Brazil, Colombia and Mexico; and a FIGARCH is selected for Colombia and Peru. The GJR is an alternative for all countries except for Chile and the GARCH for Chile and Colombia. Likewise, a $\mathcal{GED}$ distribution is selected for Brazil, Colombia and Peru; and a sk$\mathcal{S}$ for Argentina, Chile and Mexico. A $\mathcal{S}$ distribution is appear as an alternative for Argentina, Chile and Peru. The volatility asymmetry parameter

Regarding the parameter analysis, the parameter of asymmetry in the volatility γ is negative and significant at 1% in most of the cases. This can be interpreted as that volatility increases before positive returns in the Forex market (depreciation of the local currency). This is an effect opposite to that of the stock market. In most of the cases when a sk$\mathcal{S}$ distribution is chosen, the bias parameter λ is significant and positive, which indicates that the episodes of depreciation of the local currency against the US dollar predominate. The fractional parameter ϕ estimated for Colombia (0.2619) and Peru (0.1765) is significant at 1% wich sugest the presence of long memory in those series.

4.2.2 Out-of-sample forecasting and backtesting

Secondly, in the case of out-of-sample analysis, a rolling–window of 1000 observations is used to forecast one–day ahead VaR through a suite of ARCH models[9]. The model parameters are updated daily. Other estimation frequencies were not considered since in the context of ARCH-type models, the performance of VaR forecasts is not affected significantly when changing the updating frequency; see Ardia and Hoogerheide (2014), Ardia et al. (2018) and Omari et al. (2020).

The accuracy of the forecasts are measured in terms of conditional coverage through the UC test, independence through the CCI tests and both conditional coverage and independence simultaneously through the CC test and DQ test.

Table 4 (stock) and 5 (Forex) shows the results for p-values of the backtest tests used for VaR at 95% (left panel) and 99% (right panel) confidence level. Overall, the backtest results indicate that the models that had the best fit in the in-sample analysis are not necessarily the most effective in the out-of-sample application. For example, at 5% confidence, the specification of the selected mean does not coincide in any case, for the volatility process it only coincides in one (Peru) of six cases; and regarding the distribution in two (Brazil and Peru) of six cases. Further, it is observed that models that capture asymmetries and long memory (GJR, FIGARCH, APARCH) outperform the standard GARCH and ARCH models; and a fat-tailed distribution of the errors leads to a substantial reduction in the rejection frequencies. Finally, it is observed that the models producing the most accurate forecasts vary by market and country.

[9]The results for a 250 observations window size were also obtained. However, given the poor performance in terms of backtest, these results have been ruled out. This paper is following the standart in most of the related research witch is between 1000 and 1500 observations window size; see Angelidis et al. (2004) Ardia et al. (2016) and Ardia et al. (2019).

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Table 1: Descriptive Statistics for Stock and Forex Markets Returns

Country	Security ID	Start Date	End Date	Obs.	Mean	Std,	Min	Max	Skew	Kurt
(a) Stock Market Returns										
Argentina	MERVAL	1991-12-26	2021-08-31	7744	0.06	2.30	-47.69	16.12	-1.43	27.89
Brasil	IBOV	1995-03-16	2021-08-31	6904	0.05	1.96	-17.23	28.82	0.09	14.05
Chile	IPSA	1990-08-09	2021-08-31	8104	0.05	1.14	-15.22	11.80	-0.32	12.83
Colombia	IGBC	2001-07-26	2021-08-31	5244	0.05	1.14	-11.05	14.69	-0.18	15.73
México	MEXBOL	1994-03-31	2021-08-31	7154	0.04	1.40	-14.31	12.15	0.03	7.56
Perú	SPBLPGPT	2002-02-07	2021-08-31	5104	0.05	1.35	-13.29	12.82	-0.59	12.69
(b) Forex Market Returns										
Argentina	ARS	2014-03-06	2021-08-31	1954	0.13	1.14	-6.03	30.80	12.32	294.71
Brasil	BRL	1999-06-03	2021-08-31	5804	0.02	1.04	-10.34	7.11	0.07	5.90
Chile	CLP	1990-01-18	2021-08-31	8249	0.01	0.60	-4.33	4.68	0.25	7.54
Colombia	COP	1992-09-03	2021-08-31	7564	0.02	0.66	-7.60	6.02	0.17	10.31
México	MXN	1996-05-09	2021-08-31	6604	0.01	0.71	-6.65	7.98	0.85	11.43
Perú	PEN	1995-05-25	2021-08-31	6854	0.01	0.30	-2.85	3.55	0.08	14.54

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Table 2a: Estimated Parameters for daily Latin American Stock Markets Return

Model	c	θ	ω	α	β	γ	δ	ϕ	ν	λ	BIC	log-lik
Argentina (MERVAL)												
AR-APARCH-$\mathcal{GED}$	0.053^a	0.035^a	0.164^a	0.113^a	0.851^a	0.216^a	2.023^a		1.104^a		31879.643	-15904.003
AR-GJR-$\mathcal{GED}$	0.053^a	0.035^a	0.162^a	0.069^a	0.852^a	0.099^a			1.104^a		31870.701	-15904.009
c-APARCH-$\mathcal{GED}$	0.046^a		0.157^a	0.112^a	0.856^a	0.214^a	1.984^a		1.092^a		31888.194	-15912.755
c-GJR-$\mathcal{GED}$	0.046^a		0.159^a	0.069^a	0.855^a	0.095^a			1.092^a		31879.245	-15912.758
GJR-$\mathcal{GED}$			0.164^a	0.067^a	0.857^a	0.102^a			1.065^a		31877.421	-15916.324
Brazil (IBOV)												
AR-APARCH-$\mathcal{GED}$	0.051^a	-0.005	0.069^a	0.075^a	0.898^a	0.422^a	1.711^a		1.357^a		26274.450	-13101.866
AR-GJR-$\mathcal{GED}$	0.052^a	-0.005	0.081^a	0.027^a	0.893^a	0.106^a			1.357^a		26267.942	-13103.032
c-APARCH-$\mathcal{GED}$	0.050^a		0.070^a	0.075^a	0.898^a	0.424^a	1.712^a		1.359^a		26273.472	-13105.796
AR-APARCH-sk$\mathcal{S}$	0.055^a	-0.002	0.061^a	0.073^a	0.904^a	0.418^a	1.648^a		7.945^a	-0.051^a	26293.180	-13106.811
c-GJR-$\mathcal{GED}$	0.051^a		0.081^a	0.027^a	0.893^a	0.106^a			1.359^a		26266.918	-13106.939
Chile (IPSA)												
AR-APARCH-sk$\mathcal{S}$	0.033^a	0.198^a	0.029^a	0.135^a	0.846^a	0.173^a	1.848^a		7.178^a	0.014	21614.095	-10766.547
AR-GJR-sk$\mathcal{S}$	0.033^a	0.198^a	0.029^a	0.092^a	0.842^a	0.089^a			7.178^a	0.013	21605.886	-10766.943
AR-APARCH-$\mathcal{GED}$	0.025^a	0.182^a	0.028^a	0.138^a	0.846^a	0.163^a	1.830^a		1.370^a		21605.965	-10766.982
AR-APARCH-$\mathcal{S}$	0.031^a	0.198^a	0.029^a	0.136^a	0.846^a	0.172^a	1.853^a		7.202^a		21605.967	-10766.983
AR-GJR-$\mathcal{S}$	0.031^a	0.198^a	0.029^a	0.092^a	0.842^a	0.089^a			7.202^a		21597.702	-10767.351
Colombia (IGBC)												
AR-APARCH-$\mathcal{GED}$	0.031^c	0.102^a	0.069^a	0.181^a	0.790^a	0.181^a	1.554^a		1.115^a		13031,085	-6481,657
AR-GJR-$\mathcal{GED}$	0.031^a	0.101^a	0.067^a	0.125^a	0.777^a	0.108^a			1.114^a		13027,349	-6484,024
AR-GARCH-$\mathcal{GED}$	0.038^a	0.094^a	0.059^a	0.175^a	0.788^a				1.103^a		13038,955	-6494,063
AR-APARCH-sk$\mathcal{S}$	0.041^a	0.139^a	0.067^a	0.198^a	0.786^a	0.175^a	1.477^a		4.802^a	-0.032^c	13077,244	-6500,500
AR-APARCH-$\mathcal{S}$	0.051^a	0.139^a	0.067^a	0.198^a	0.786^a	0.177^a	1.466^a		4.756^a		13071,841	-6502,035
Mexico (MEXBOL)												
AR-APARCH-$\mathcal{GED}$	0.029^a	0.076^a	0.016^a	0.079^a	0.920^a	0.456^a	1.450^a		1.320^a		22096.751	-11012.874
AR-GJR-$\mathcal{GED}$	0.031^a	0.076^a	0.017^a	0.030^a	0.911^a	0.104^a			1.317^a		22098.343	-11018.108
AR-APARCH-sk$\mathcal{S}$	0.029^b	0.084^a	0.016^a	0.079^a	0.921^a	0.458^a	1.457^a		6.715^a	-0.028^b	22135.738	-11027.930
AR-APARCH-$\mathcal{S}$	0.036^a	0.085^a	0.015^a	0.078^a	0.921^a	0.459^a	1.450^a		6.652^a		22130.155	-11029.576
AR-GJR-sk$\mathcal{S}$	0.032^a	0.084^a	0.016^a	0.030^a	0.912^a	0.103^a			6.635^a	-0.027^b	22138.780	-11033.889
Peru (SPBLPGPT)												
AR-FIGARCH-$\mathcal{GED}$	0.049^a	0.114^a	0.079^a		0.340^a		0.438^a	0.066	1.174^a		14420.632	-7180.434
AR-APARCH-$\mathcal{GED}$	0.045^a	0.117^a	0.037^a	0.146^a	0.840^a	0.087^a	1.819^a		1.174^a		14460.025	-7195.862
AR-GJR-$\mathcal{GED}$	0.045^a	0.116^a	0.038^a	0.122^a	0.833^a	0.046^b			1.173^a		14452.178	-7196.207
AR-GARCH-$\mathcal{GED}$	0.049^a	0.114^a	0.037^a	0.145^a	0.834^a				1.168^a		14450.534	-7199.654
AR-FIGARCH-$\mathcal{S}$	0.059^a	0.134^a	0.093^a		0.358^a		0.446^a	0.077	4.754^a		14461.002	-7200.619

a, b, c denote significance level at 1%, 5% and 10% respectively

Table 2b: Estimated Parameters for daily Latin American Forex Markets Return

Model	c	θ	ω	α	β	γ	δ	ϕ	ν	λ	BIC	log-lik
						Argentina (ARS)						
AR-APARCH-sk$\mathcal{S}$	0.051^{a}	0.1668^{a}	0.009^{c}	0.121^{a}	0.879^{a}	-0.148^{b}	0.050		2.134^{a}	0.115^{a}	414.609	-173.207
AR-FIGARCH-sk$\mathcal{S}$	0.043^{a}	0.1790^{a}	0.001^{c}		0.573^{a}		0.843^{a}	0.078	3.010^{a}	0.081^{a}	451.117	-195.250
c-APARCH-sk$\mathcal{S}$	0.063^{a}		0.008^{c}	0.119^{a}	0.880^{a}	-0.062	0.050		2.208^{a}	0.133^{a}	461.976	-200.677
AR-FIGARCH-$\mathcal{S}$	0.039^{a}	0.1813^{a}	0.001^{c}		0.519^{b}		0.809^{a}	0.095	2.966^{a}		454.976	-200.968
AR-GJR-sk$\mathcal{S}$	0.047^{a}	0.1558^{a}	0.000^{b}	0.183^{a}	0.820^{a}	-0.005			3.063^{a}	0.121^{a}	484.886	-212.134
						Brazil (BRL)						
AR-APARCH-$\mathcal{GED}$	0.002	-0.036^{a}	0.013^{a}	0.096^{a}	0.904^{a}	-0.295^{a}	1.440^{a}		1.386^{a}		14806.689	-7368.680
AR-APARCH-sk$\mathcal{S}$	0.011	-0.039^{a}	0.011^{a}	0.091^{a}	0.908^{a}	-0.283^{a}	1.516^{a}		8.228^{a}	0.049^{a}	14819.322	-7370.663
c-APARCH-$\mathcal{GED}$	0.002		0.013^{a}	0.096^{a}	0.903^{a}	-0.306^{a}	1.442^{a}		1.386^{a}		14806.536	-7372.936
APARCH-$\mathcal{GED}$			0.013^{a}	0.096^{a}	0.903^{a}	-0.304^{a}	1.442^{a}		1.385^{a}		14797.971	-7372.986
AR-GJR-$\mathcal{GED}$	0.001	-0.036^{a}	0.010^{a}	0.142^{a}	0.894^{a}	-0.084^{a}			1.377^{a}		14809.844	-7374.590
						Chile (CLP)						
APARCH-sk$\mathcal{S}$			0.019^{a}	0.068^{a}	0.931^{a}	-0.296^{a}	0.122^{a}		2.050^{a}	0.005	5723.089	-2829.982
APARCH-$\mathcal{S}$			0.031^{a}	0.091^{a}	0.900^{a}	-0.272^{a}	0.140^{a}		2.328^{a}		7637.957	-3791.925
! FIGARCH-$\mathcal{GED}$			0.001		0.555^{c}		0.416^{a}	0.291	1.010^{a}		10322.129	-5138.520
GARCH-$\mathcal{GED}$			0.000	0.060^{a}	0.939^{a}				1.010^{a}		10342.756	-5153.342
APARCH-$\mathcal{GED}$			0.000	0.068^{a}	0.932^{a}	-0.146	1.786^{a}		1.010^{a}		10391.979	-5168.936
						Colombia (COP)						
AR-FIGARCH-$\mathcal{GED}$	0.0040	0.0274^{a}	0.002^{c}		0.5736^{a}		0.470^{a}	0.261^{a}	1.010^{a}		10258.037	-5097.759941
FIGARCH-$\mathcal{GED}$			0.002^{c}		0.5730^{a}		0.467^{a}	0.266^{a}	1.010^{a}		10244.165	-5099.754812
AR-GJR-$\mathcal{GED}$	0.0045^{a}	0.0275^{a}	0.002^{a}	0.128^{a}	0.8841^{a}	-0.025^{b}			1.010^{a}		10326.135	-5131.808960
AR-GARCH-$\mathcal{GED}$	0.0043^{a}	0.0282^{a}	0.001^{a}	0.114^{a}	0.8854^{a}				1.010^{a}		10321.883	-5134.148673
APARCH-$\mathcal{GED}$			0.002	0.115^{a}	0.8846^{a}	-0.064^{c}	1.962^{a}		1.010^{a}		10322.193	-5134.303423
						Mexico (MXN)						
AR-APARCH-sk$\mathcal{S}$	0.009	-0.060^{a}	0.011^{a}	0.094^{a}	0.905^{a}	-0.450^{a}	1.140^{a}		7.176^{a}	0.139^{a}	11364.875	-5642.859
c-APARCH-sk$\mathcal{S}$	0.011^{a}		0.012^{a}	0.096^{a}	0.903^{a}	-0.484^{a}	1.141^{a}		7.337^{a}	0.133^{a}	11378.144	-5653.890
AR-FIGARCH-sk$\mathcal{S}$	-0.001	-0.072^{a}	0.015^{a}		0.398^{a}		0.370^{a}	0.144^{b}	7.212^{a}	0.136^{a}	11380.398	-5655.017
APARCH-sk$\mathcal{S}$			0.011^{a}	0.097^{a}	0.902^{a}	-0.462^{a}	1.150^{a}		7.299^{a}	0.124^{a}	11372.984	-5655.708
AR-GJR-sk$\mathcal{S}$	0.007	-0.062^{a}	0.006^{a}	0.150^{a}	0.886^{a}	-0.096^{a}			7.048^{a}	0.138^{a}	11394.038	-5661.838
						Peru (PEN)						
FIGARC-$\mathcal{GED}$			0.000		0.610^{a}		0.647^{a}	0.176^{a}	1.010^{a}		-2882.379	1463.271
GJR-$\mathcal{GED}$			0.001^{b}	0.199^{a}	0.839^{a}	-0.079^{a}			1.010^{a}		-2858.820	1451.491
AR-FIGARCH-$\mathcal{S}$	-0.0028^{b}	0.0182	0.000		0.601^{a}		0.647^{a}	0.177^{a}	3.755^{a}		-2634.874	1348.350
c-FIGARCH-$\mathcal{S}$	-0.0029^{b}		0.000		0.601^{a}		0.644^{a}	0.178^{a}	3.753^{a}		-2640.998	1346.997

a, b, c denote significance level at 1%, 5% and 10% respectively

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Table 3a: Accuracy of VaR predictions for Stock Markets Returns

Model	VaR level	% Fails	UC	CCI	CC	DQ
Argentina (MERVAL)						
FIGARCH-sk$\mathcal{S}$	0.01	0.011	0.362	0.000	0.000	0.000
FIGARCH-$\mathcal{GED}$	0.01	0.011	0.303	0.000	0.000	0.000
FIGARCH-$\mathcal{S}$	0.01	0.011	0.252	0.000	0.000	0.000
c-FIGARCH-sk$\mathcal{S}$	0.01	0.011	0.206	0.000	0.000	0.000
c-FIGARCH-$\mathcal{S}$	0.01	0.011	0.168	0.000	0.000	0.000
FIGARCH-sk$\mathcal{S}$	0.05	0.049	0.730	0.000	0.000	0.000
FIGARCH-$\mathcal{S}$	0.05	0.049	0.772	0.000	0.000	0.000
FIGARCH	0.05	0.049	0.816	0.000	0.000	0.000
c-FIGARCH-sk$\mathcal{S}$	0.05	0.049	0.860	0.000	0.000	0.000
c-FIGARCH-$\mathcal{S}$	0.05	0.049	0.993	0.000	0.000	0.000
Brazil (IBOV)						
GARCH-$\mathcal{S}$	0.01	0.010	0.607	0.243	0.444	0.404
GARCH-$\mathcal{GED}$	0.01	0.010	0.607	0.243	0.444	0.049
c-GARCH-$\mathcal{N}$	0.01	0.010	0.607	0.243	0.444	0.016
GARCH-$\mathcal{N}$	0.01	0.010	0.521	0.236	0.403	0.015
GARCH-sk$\mathcal{S}$	0.01	0.010	0.521	0.236	0.403	0.174
GJR-GARCH-$\mathcal{GED}$	0.05	0.047	0.465	0.226	0.368	0.298
c-APARCH-$\mathcal{N}$	0.05	0.048	0.502	0.359	0.525	0.748
c-GJR-GARCH	0.05	0.048	0.542	0.249	0.428	0.337
GJR-GARCH	0.05	0.048	0.582	0.167	0.331	0.364
GJR-GARCH-$\mathcal{S}$	0.05	0.048	0.624	0.274	0.487	0.428
Colombia (IGBC)						
c-GARCH-$\mathcal{S}$	0.01	0.013	0.032	0.048	0.014	0.006
AR(1)-GARCH-$\mathcal{S}$	0.01	0.013	0.032	0.048	0.014	0.006
FIGARCH-$\mathcal{S}$	0.01	0.013	0.045	0.006	0.003	0.000
FIGARCH-sk$\mathcal{S}$	0.01	0.013	0.045	0.006	0.003	0.000
c-FIGARCH-$\mathcal{N}$	0.01	0.012	0.063	0.005	0.003	0.000
c-FIGARCH	0.05	0.050	0.841	0.000	0.000	0.000
AR(1)-FIGARCH-$\mathcal{S}$	0.05	0.050	0.841	0.000	0.000	0.000
c-FIGARCH-$\mathcal{GED}$	0.05	0.051	0.631	0.000	0.000	0.000
AR(1)-FIGARCH-$\mathcal{GED}$	0.05	0.051	0.631	0.000	0.000	0.000
c-FIGARCH	0.05	0.048	0.715	0.000	0.000	0.000
Mexico (MEXBOL)						
APARCH-$\mathcal{N}$	0.05	0.052	0.437	0.014	0.036	0.027
c-APARCH-$\mathcal{N}$	0.05	0.052	0.314	0.019	0.039	0.014
AR(1)-APARCH-$\mathcal{N}$	0.05	0.052	0.314	0.019	0.039	0.014
APARCH-$\mathcal{N}$	0.05	0.052	0.342	0.002	0.006	0.013
APARCH-$\mathcal{S}$	0.05	0.052	0.372	0.002	0.006	0.011
FIGARCH-sk$\mathcal{S}$	0.01	0.010	0.489	0.214	0.364	0.016
c-FIGARCH-$\mathcal{GED}$	0.01	0.010	0.489	0.214	0.364	0.011
AR(1)-FIGARCH-$\mathcal{GED}$	0.01	0.010	0.489	0.214	0.364	0.011
FIGARCH-$\mathcal{GED}$	0.01	0.011	0.414	0.226	0.344	0.015
c-FIGARCH-$\mathcal{N}$	0.01	0.011	0.414	0.226	0.344	0.014
Peru (SPBLPGPT)						
GARCH-$\mathcal{S}$	0.01	0.014	0.012	0.000	0.000	0.000
GARCH-sk$\mathcal{S}$	0.01	0.014	0.012	0.000	0.000	0.000
GJR-GARCH-sk$\mathcal{S}$	0.01	0.014	0.012	0.000	0.000	0.000
GJR-GARCH-$\mathcal{GED}$	0.01	0.014	0.012	0.000	0.000	0.000
c-GARCH-$\mathcal{S}$	0.01	0.014	0.012	0.000	0.000	0.000
c-GJR-GARCH-$\mathcal{S}$	0.05	0.055	0.141	0.000	0.000	0.000
AR(1)-GJR-GARCH-$\mathcal{S}$	0.05	0.055	0.141	0.000	0.000	0.000
GJR-GARCH-sk$\mathcal{S}$	0.05	0.055	0.107	0.000	0.000	0.000
c-GJR-GARCH-sk$\mathcal{S}$	0.05	0.055	0.107	0.000	0.000	0.000
AR(1)-GJR-GARCH-sk$\mathcal{S}$	0.05	0.055	0.107	0.000	0.000	0.000

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Table 3b: Accuracy of VaR predictions for Forex Markets Returns

Model	VaR level	% Fails	UC	CCI	CC	DQ
Argentina (ARS)						
c-FIGARCH-$\mathcal{S}$	0.05	0.038	0.100	0.000	0.000	0.000
c-FIGARCH-$\mathcal{GED}$	0.05	0.039	0.137	0.003	0.004	0.003
c-APARCH-sk$\mathcal{S}$	0.05	0.051	0.841	0.012	0.043	0.004
c-APARCH-sk$\mathcal{S}$	0.05	0.051	0.841	0.012	0.043	0.004
AR(1)-APARCH-$\mathcal{GED}$	0.05	0.061	0.103	0.010	0.009	0.011
c-FIGARCH-sk$\mathcal{S}$	0.01	0.009	0.861	0.678	0.903	0.999
c-FIGARCH-$\mathcal{GED}$	0.01	0.009	0.861	0.678	0.903	0.999
AR(1)-FIGARCH-$\mathcal{S}$	0.01	0.009	0.861	0.678	0.903	0.998
c-FIGARCH-$\mathcal{S}$	0.01	0.008	0.608	0.712	0.819	0.997
AR(1)-FIGARCH-$\mathcal{GED}$	0.01	0.011	0.640	0.612	0.788	0.995
Brazil (BLR)						
AR(1)-GARCH-$\mathcal{GED}$	0.05	0.049	0.992	0.551	0.837	0.328
AR(1)-GARCH-$\mathcal{N}$	0.05	0.050	0.955	0.572	0.851	0.377
AR(1)-GARCH-$\mathcal{S}$	0.05	0.051	0.748	0.660	0.862	0.281
GARCH-$\mathcal{GED}$	0.05	0.051	0.605	0.528	0.717	0.440
GARCH-$\mathcal{N}$	0.05	0.051	0.605	0.528	0.717	0.291
c-APARCH-$\mathcal{S}$	0.01	0.013	0.038	0.266	0.063	0.255
c-APARCH-sk$\mathcal{S}$	0.01	0.013	0.038	0.059	0.019	0.035
AR(1)-APARCH-$\mathcal{GED}$	0.01	0.012	0.052	0.054	0.024	0.003
APARCH-sk$\mathcal{S}$	0.01	0.012	0.052	0.054	0.024	0.003
GARCH-$\mathcal{N}$	0.01	0.012	0.071	0.050	0.028	0.002
Chile (CLP)						
GARCH-$\mathcal{N}$	0.01	0.011	0.224	0.088	0.112	0.029
GARCH-$\mathcal{GED}$	0.01	0.011	0.184	0.355	0.270	0.034
c-GARCH-$\mathcal{N}$	0.01	0.011	0.184	0.094	0.102	0.021
AR(1)-GARCH-$\mathcal{N}$	0.01	0.011	0.184	0.094	0.102	0.021
c-FIGARCH-sk$\mathcal{S}$	0.01	0.012	0.096	0.113	0.071	0.128
GARCH-sk$\mathcal{S}$	0.05	0.057	0.004	0.000	0.000	0.000
FIGARCH-$\mathcal{S}$	0.05	0.057	0.004	0.000	0.000	0.000
c-GARCH-$\mathcal{S}$	0.05	0.057	0.004	0.001	0.000	0.000
GARCH-$\mathcal{S}$	0.05	0.057	0.004	0.001	0.000	0.000
GARCH-$\mathcal{S}$	0.05	0.057	0.003	0.000	0.000	0.000
Colombia (CLP)						
GARCH-$\mathcal{N}$	0.01	0.011	0.136	0.016	0.018	0.000
GARCH-$\mathcal{GED}$	0.01	0.011	0.136	0.016	0.018	0.000
APARCH-$\mathcal{N}$	0.01	0.011	0.136	0.016	0.018	0.000
c-GARCH-$\mathcal{N}$	0.01	0.011	0.136	0.016	0.018	0.000
AR(1)-GARCH-$\mathcal{N}$	0.01	0.011	0.136	0.016	0.018	0.000
c-FIGARCH-sk$\mathcal{S}$	0.05	0.054	0.106	0.079	0.058	0.011
AR(1)-FIGARCH-sk$\mathcal{S}$	0.05	0.054	0.106	0.079	0.058	0.011
c-FIGARCH-$\mathcal{GED}$	0.05	0.054	0.085	0.055	0.036	0.002
AR(1)-FIGARCH-$\mathcal{GED}$	0.05	0.054	0.085	0.055	0.036	0.002
FIGARCH-sk$\mathcal{S}$	0.05	0.055	0.066	0.100	0.048	0.007
Mexico (MXN)						
AR(1)-GARCH-$\mathcal{S}$	0.05	0.049	0.751	0.885	0.941	0.959
AR(1)-GARCH-$\mathcal{S}$	0.05	0.049	0.751	0.885	0.941	0.959
AR(1)-GARCH-$\mathcal{GED}$	0.05	0.049	0.895	0.953	0.989	0.956
AR(1)-GARCH-$\mathcal{GED}$	0.05	0.049	0.895	0.953	0.989	0.956
AR(1)-FIGARCH-$\mathcal{S}$	0.05	0.049	0.943	0.798	0.965	0.886
AR(1)-GARCH-$\mathcal{S}$	0.01	0.009	0.996	0.287	0.568	0.913
AR(1)-GARCH-$\mathcal{S}$	0.01	0.009	0.996	0.287	0.568	0.913
c-GARCH-sk$\mathcal{S}$	0.01	0.010	0.896	0.279	0.551	0.904
AR(1)-GARCH-sk$\mathcal{S}$	0.01	0.010	0.896	0.279	0.551	0.905
c-GARCH-sk$\mathcal{S}$	0.01	0.010	0.896	0.279	0.551	0.904

Figure 1: Latin American Stock Markets Returns

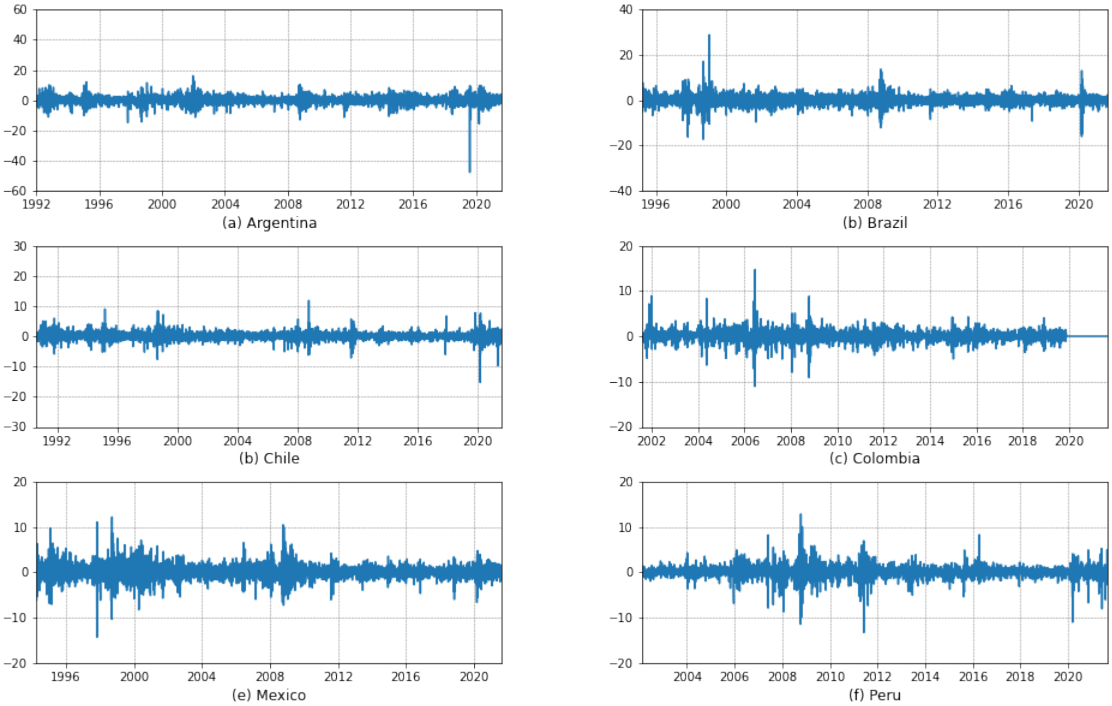

Figure 2: Latin American Forex Markets Returns

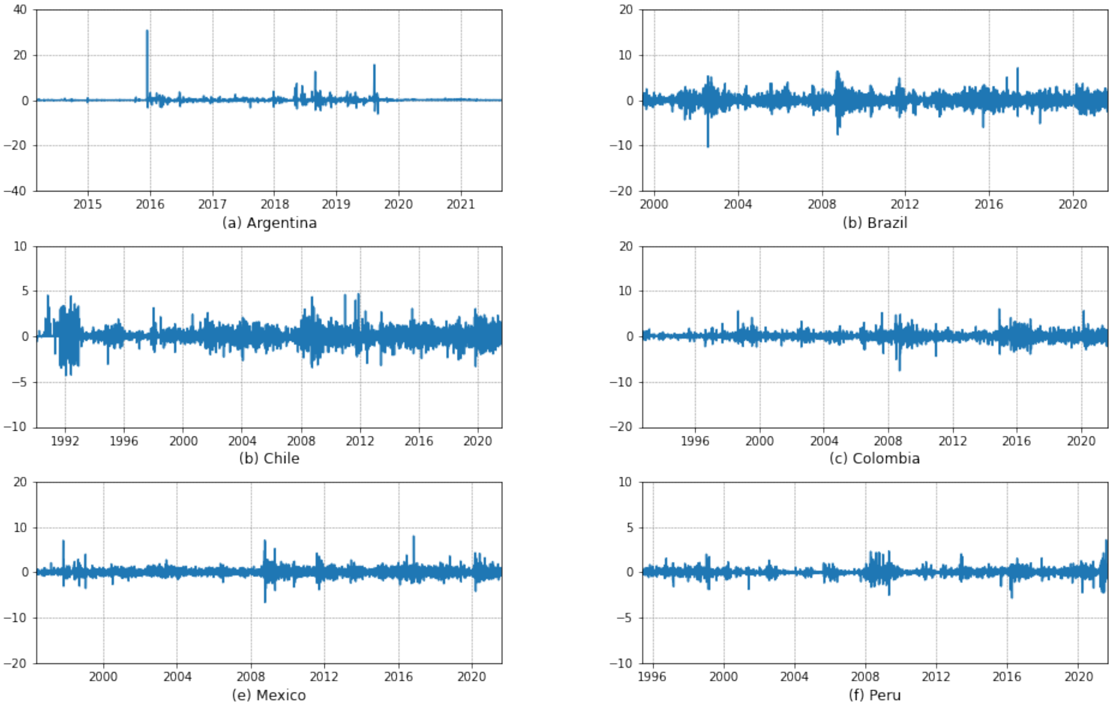

Figure 3: Squared of Latin American Stock Markets Returns

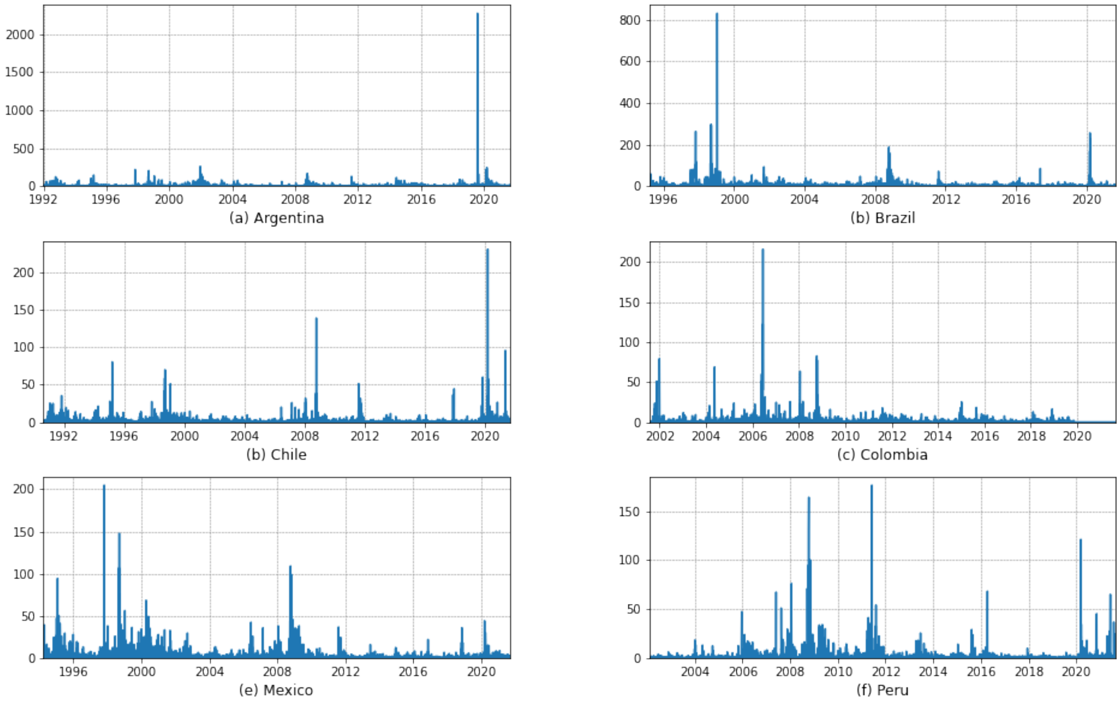

Figure 4: Squared of Latin American Forex Markets Returns