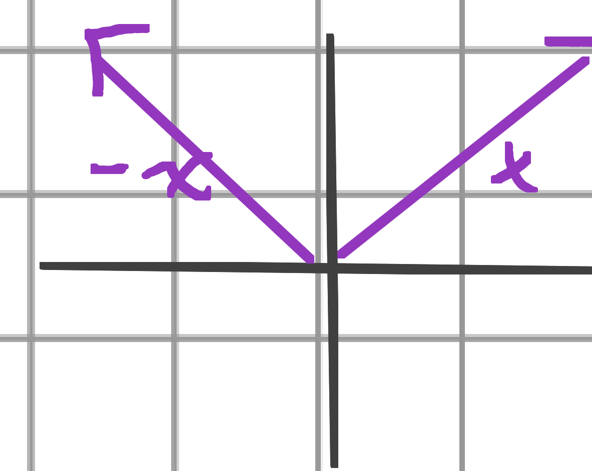
MATH 410 08/28/24

Chapter 02

① Distance

(def) $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

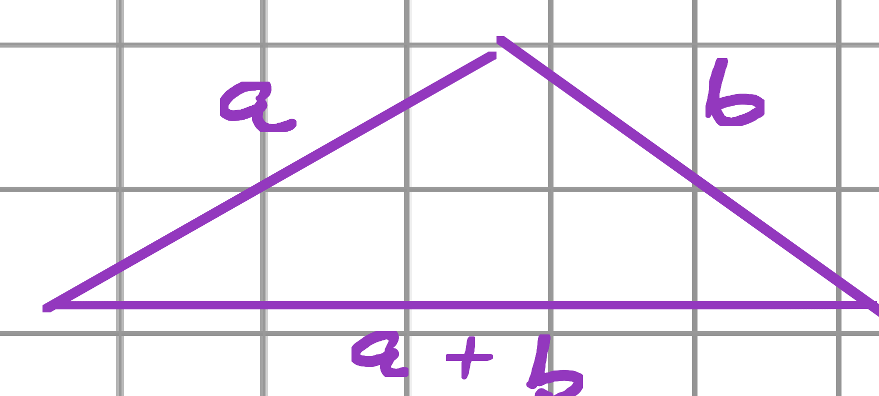


★ used for distance

② Triangle △ Inequality

· $\forall a, b \in \mathbb{R}$

↳ $|a+b| = |a| + |b|$



Proof

Assume $a, b \in \mathbb{R}$ & without loss of generality, $a \geq b$.

Case 01: $a \geq b \geq 0$ (Positivity Axiom)

then $|a+b| = a+b = |a| + |b|$

(definition of $|x|$)

case 02: $a \geq 0 \geq b$ and $a+b \geq 0$. Note $b \leq 0$,

then $b \leq |b|$ (neg pos)

so, $|a+b| = a + b \leq |a| + |b|$

def of $|x|$

Case 03: $a \geq 0 \geq b$ and $a+b < 0$, so

$|a+b| = -(a+b) = -a-b$

$\leq |a| - b$

$= |a| + |b|$

$\Rightarrow |a+b| \leq |a| + |b|$ as needed

Note:

$|a|, a \geq 0$

$-a \leq 0$

$-a \leq |a|$

neg pos

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Case 04: $0 \geq a \geq b$ $(a+b<0)$

so, $|a+b| = -(a+b) = -a-b$

$= -a + -b$

by def of $|x|$

$= |a| + |b|$

so $|a+b| \leq |a| + |b| \ \forall a, b \in \mathbb{R}$. //

③ <u>Boundedness</u>

(def) A subset of S is said to be **bounded above** if there is a real number r such that $s \leq r$ for all $s \in S$.

$S = [0, 1]$