Introduction to statistics

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Problems session 6 (Tema 6)

Tema 6: Summary

Given a random sample $X_1, \dots X_n$ of size n, with $E(X)=\mu$ and $V(X)=\sigma^2$ we can compute:

- $Y=\sum_{j=1..n} X_j$
- $\bar{X}=\frac{1}{n}\sum_{j=1..n} X_j$
- $S^2=\frac{1}{n-1}\sum_{i=1}^n (X_i-\bar{X})^2$

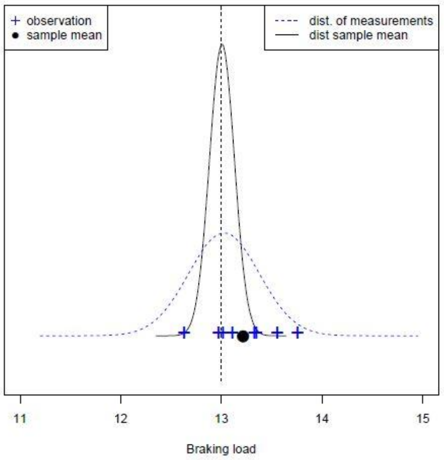
In general, no matter how X distributes:

- $E(Y)=n\cdot\mu$ and $V(Y)=n\cdot\sigma^2$
- $E(\bar{X})=\mu$ and $V(\bar{X})=\frac{\sigma^2}{n}$
- $E(S^2)=\sigma^2$

If $X \hookrightarrow N(\mu,\sigma^2)$ then

- $Y \hookrightarrow N(n\cdot\mu, n\cdot\sigma^2)$
- $\bar{X} \hookrightarrow N(\mu, \sigma^2/n)$ (also used in a good approximation when $n>30$ is large no matter how X distributes: CLT)
- $\frac{S^2(n-1)}{\sigma^2} \hookrightarrow \chi^2(n-1)$

Tema 6: Summary



Tema 6: Summary

Definition

Given a random sample $X_1, \dots X_n$ a **statistic** is any real value function of the random variables that define the random sample: $f(X_1, \dots X_n)$

- $\bar{X} = \frac{1}{N} \sum_{j=1..N} X_j$
- $S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2$
- $\max X_1, X_n$

are statistics.

- $\bar{X}$ is an **unbiased** estimator of μ because $E(\bar{X}) = \mu$
- S^2 is an **unbiased** estimator of σ^2 because $E(S^2) = \sigma^2$

Tema 6: Problem 1

- $E(X) = \mu = 100$
- $\sigma = 30h$
- $n = 50$

Therefore

- $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = 30h/\sqrt{50}$

a. compute: $P(|\bar{X} - \mu| \leq 1)$

$P(|\bar{X} - \mu| \leq 1) = P(-1 \leq \bar{X} - \mu \leq 1)$

Tema 6: Problem 1

Let's devide by $\sigma_{\bar{X}}$
$= P(\frac{-1}{\sigma_{\bar{X}}} \leq \frac{\bar{X}-\mu}{\sigma_{\bar{X}}} \leq \frac{1}{\sigma_{\bar{X}}})$

since $n > 30$ by CLT then $Z = \frac{\bar{X}-\mu}{\sigma_{\bar{X}}} \hookrightarrow N(\mu = 0, \sigma^2 = 1)$

Tema 6: Problem 1

We use $\Phi(z)$

$P(|\bar{X}-\mu| \leq 1) = \Phi(\frac{1}{\sigma_{\bar{X}}}) - \Phi(-\frac{1}{\sigma_{\bar{X}}}) =$

$= \Phi(0.24) - \Phi(-0.24)$

$=$ `pnorm(0.24)-pnorm(-0.24)` $= 0.189$

Tema 6: Problem 1

b.

- $Y = \sum X_i$ number of components on stock

therefore

- $\sigma_Y^2 = n\sigma^2$

Compute n such that $P(Y \geq 2750) = 0.95$

$P(Y \geq 2750) = P(\frac{Y}{n} \geq \frac{2750}{n}) = P(\bar{X} \geq \frac{2750}{n})$

standardize

$P(Y \geq 2750) = P(\frac{\bar{X}-100}{\sigma/\sqrt{n}} \geq \frac{\frac{2750}{n}-100}{\sigma/\sqrt{n}}) = 1 - P(Z < \frac{\frac{2750}{n}-100}{\sigma/\sqrt{n}})$

let's assume $n > 30$ then $Z \hookrightarrow N(\mu = 0, \sigma^2 = 1)$

Tema 6: Problem 1

$1 - P(Z < \frac{\frac{2750}{n}-100}{\sigma/\sqrt{n}}) = 1 - \Phi(\frac{\frac{2750}{n}-100}{\sigma/\sqrt{n}}) = 0.95$

applying Φ^{-1}

$\Phi^{-1}(0.05) =$ `qnorm(0.05)` $= \frac{\frac{2750}{n}-100}{\sigma/\sqrt{n}}$

Replacing the values and solving for n we have the equation

$100n - 49.35\sqrt{n} - 2750 = 0$ of for $t = \sqrt{n}$

$100t^2 - 49.35t - 2750 = 0$ with roots $t = 5.49, -5.00$

then $n = t^2 = 30.20 \sim 31$

Tema 6: Problem 2

Consider:

- X weight of one unit
- $E(X) = \mu = 250gr$
- $\sigma = 20gr$
- $n = 50$
- $Y = \sum_{i=1}^{50} X_i$ the weight of 50 units

a. What is $f(Y)$?, we don't know what $f(X)$ is, but n is big (> 30), then

$Y \hookrightarrow N(\mu_Y, \sigma_Y^2)$

by CLT

where $\mu_Y = E(Y) = n \cdot \mu$ and $\sigma_Y = V(Y) = n \cdot \sigma^2$

$Y \hookrightarrow N(n\mu, n\sigma^2) = N(12.5kg, 0.02kg^2)$

Tema 6: Problem 2

b. Compute $P(Y > 12.75)$

$P(Y > 12.75) = 1 - P(Y \leq 12.75)$

Let’s standardize

$1 - P(\frac{Y-\mu_Y}{\sigma_Y} \leq \frac{12.75-\mu_Y}{\sigma_Y})$

$= 1 - P(Z \leq \frac{12.75-12.5}{0.1414})$

$= 1 - \Phi(1.768) =$ `1-pnorm(1.768)` $= 0.0383$

Tema 6: Problem 2

c. Find $\mu = \mu_Y/n$ such that $P(Y \leq 11.5) = 0.95$

Standardize

$P(\frac{Y-\mu_Y}{\sigma_Y} \leq \frac{11.5-\mu_Y}{0.1414}) = 0.95$

apply $F^{-1} = \Phi^{-1}$

$\frac{11.5-\mu_Y}{0.1414} = \Phi^{-1}(0.95) =$ `qnorm(0.95)` $= 1.644854$

solve for μ_Y

$\mu_Y = 11.5 - 1.644854 * 0.1414 = 11.26742$

$\mu = \mu_Y/n = 11.26742/50 = 0.2253Kg$

Tema 6: Problem 3

Consider:

- X height of a student $X \hookrightarrow N(\mu, \sigma^2)$
- $\mu = 174.5$
- $\sigma = 6.9$

if

- sample of $n = 25$
- $\bar{X} = \frac{1}{25}\sum_{i=1}^{25} X_i$ the sample mean of the height of 25 students

then

- $\bar{X} \hookrightarrow N(\mu_{\bar{X}}, \sigma^2_{\bar{X}})$
- $\mu_{\bar{X}} = E(\bar{X}) = E(X) = \mu = 174.5$
- $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = 6.9/\sqrt{25} = 1.38$

Tema 6: Problem 3

b. compute $P(\bar{X} < 173)$

standardize

$P(\bar{X} < 173) = P(Z < \frac{173-174.5}{1.38}) = P(Z < -1.09)$

because $Z \hookrightarrow N(0,1)$ then

$P(Z < -1.09) =$ `pnorm(-1.09)` $= 0.1378566$

c. consider a set of $m = 200$ from the variable $\bar{X}$

- Y the number of samples with $\bar{X} < 1.73$: event A with $p = 0.1378$

then $Y \hookrightarrow Bin(p, m)$ and $E(Y) = mp = 200 * 0.1378 = 27.6$

Tema 6: Problem 3

d. find n for which $E(Y) = mp = 9$, then $p = 9/200 = 0.045$

standardize

$p = P(\bar{X} < 173) = P(Z < \frac{173-174.5}{6.9/\sqrt{n}}) = \Phi(\frac{173-174.5}{6.9/\sqrt{n}}) = 0.045$

apply Φ^{-1}

$\frac{173-174.5}{6.9/\sqrt{n}} =$ `qnorm(0.045)` $= -1.695398$ and

solve for n

$n = \left(-1.695398\frac{6.9}{173-174.5}\right)^2 = 60.82 \sim 60$

Tema 6: Problem 4

Consider:

- $E(X) = \mu = 7$
- $\sigma = 1$
- $n = 9$
- $X \hookrightarrow N(\mu, \sigma^2)$

a. compute $P(6.4 < \bar{X} < 7.2)$

Standardize

$P(\frac{6.4-\mu_{\bar{X}}}{\sigma_{\bar{X}}} < \frac{\bar{X}-\mu_{\bar{X}}}{\sigma_{\bar{X}}} < \frac{7.2-\mu_{\bar{X}}}{\sigma_{\bar{X}}})$

Tema 6: Problem 4

- $\mu_{\bar{X}} = \mu = 7$
- $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{1}{3}$
- $\bar{X} \hookrightarrow N(\mu_{\bar{X}}, \sigma^2_{\bar{X}})$

$P(\frac{6.4-\mu_{\bar{X}}}{\sigma_{\bar{X}}} < Z < \frac{7.2-\mu_{\bar{X}}}{\sigma_{\bar{X}}}) = \Phi(0.6) - \Phi(-1.8) =$ `pnorm(0.6)-pnorm(-1.8)` $= 0.6898166$

Tema 6: Problem 4

b. Compute $P(\bar{X} > \theta) = 0.15$

Standardize

$P(\frac{\bar{X}-\mu_{\bar{X}}}{\sigma_{\bar{X}}} > \frac{\theta-\mu_{\bar{X}}}{\sigma_{\bar{X}}}) = 0.15$

$P(Z > \frac{\theta-\mu_{\bar{X}}}{\sigma_{\bar{X}}}) = 0.15$

$P(Z \leq \frac{\theta-\mu_{\bar{X}}}{\sigma_{\bar{X}}}) = 1 - 0.15$

$\Phi(\frac{\theta-\mu_{\bar{X}}}{\sigma_{\bar{X}}}) = 0.85$

Tema 6: Problem 4

then

$\frac{\theta-\mu_{\bar{X}}}{\sigma_{\bar{X}}} = \Phi^{-1}(0.85) =$ `qnorm(0.85)` $= 1.036433$ then after substitution of the mean and standard deviation of $\bar{X}$

$\frac{\theta-7}{1/3} = 1.036433$ solving for θ then $\theta = 7.35$

Tema 6: Problem 5

Consider:

- $E(X_i) = \mu_{X_i} = 10$
- $\sigma_{X_i} = 1$
- $n = 100$

We don't know the probability density function fo X_i but because $n > 30$ than by the CLT:

$X = \sum_i^{100} X_i \hookrightarrow N(\mu, \sigma^2)$

where

- $E(X) = n \cdot \mu_{X_i} = 100 \times 10 = 1000$
- $\sigma^2 = n \cdot \sigma_{X_i}^2 = 10 \times 1 = 100$

Tema 6: Problem 5

a. compute $P(X > 1025)$

$P(X > 1025) = 1 - P(X \leq 1025) = 1 - P(\frac{X-\mu}{\sigma} \leq \frac{1025-\mu}{\sigma})$

substitution of μ and the **standard deviation** σ

$P(X > 1025) = 1 - \Phi(2.5) =$ `1-pnorm(2.5)` $= 0.00620$

Tema 6: Problem 5

b. find n for which $P(X \leq 655.68) = 0.975$

Standardize

$P(\frac{X-\mu}{\sigma} \leq \frac{655.68-\mu}{\sigma}) = 0.975$

consider:

- $E(X) = n \cdot \mu_{Xi} = n \cdot 10$
- $\sigma^2 = n \cdot \sigma^2_{X_i}$ then $\sigma = \sqrt{n}\sigma_{X_i} = \sqrt{n}$

Tema 6: Problem 5

then

$P(Z \leq \frac{655.68-10n}{\sqrt{n}}) = \Phi(\frac{655.68-10n}{\sqrt{n}}) = 0.975$

$\frac{655.68-10n}{\sqrt{n}} = \Phi^{-1}(0.975) =$ `qnorm(0.975)` $= 1.959964$

solving for $n$we have the quadratic equation for $\sqrt{n}$ $10n + 1.96\sqrt{n} - 655.68 = 0$ with solutions $\sqrt{n} = -8.19, 8.0$ then

$n = 64$

Tema 6: Problem 6

Consider

- $p = 0.4$
- $n = 2000$
- $X \hookrightarrow Bin(n = 2000, p = 0.4)$

then

- $E(X) = np = 800$
- $V(X) = np(1-p) = 480$

Since $n = 2000 > 30$, $np = 800 > 5$ and $nq = 1200 > 5$ then we can approximate the binomial probability function to the normal density function

$X \hookrightarrow N(\mu = 800, \sigma^2 = 480)$

Tema 6: Problem 6

a. compute $P(791 \leq X \leq 809)$

standardize

$P(791 \leq X \leq 809) = P(\frac{791-\mu}{\sigma} \leq \frac{X-\mu}{\sigma} \leq \frac{809-\mu}{\sigma})$
$= \Phi(\frac{809-800}{\sqrt{480}}) - \Phi(\frac{791-800}{\sqrt{480}})$
$=$ `pnorm(0.4107919)-pnorm(-0.4107919)` $= 0.3187749$

Tema 6: Problem 7

Consider:

- $E(R) = \mu_R = 40$
- $\sigma_R = 2$
- $n = 36$

a. for $\bar{R} = \frac{1}{36}\sum_{i=1}^{36} R_i$ Compute $\mu_{\bar{R}}$ and $\sigma_{\bar{R}}$

- $E(\bar{R}) = \mu_R = 40$
- $\sigma_{\bar{R}}^2 = \frac{\sigma_R^2}{n} = \frac{4}{36}$ or $\sigma_{\bar{R}} = 2/6 = 1/3$

Tema 6: Problem 7

b. compute $P(\bar{R} < 39.5)$

standardizing

$P(\bar{R} < 39.5) = P(\frac{\bar{R}-40}{1/3} < \frac{39.5-40}{1/3})$

$P(Z < \frac{39.5-40}{1/3}) = P(Z < -1.5) = 1 - \Phi(1.5)$

$=$ `1-pnorm(1.5)` $= 0.06681$

Tema 6: Problem 7

c. compute: $P(R_T > 1458)$

$P(R_T > 1458) = P(\frac{\sum_{i=1}^{36} R_i}{36} > \frac{1458}{36})$

$P(\bar{R} > \frac{1458}{36}) = P(\bar{R} > 40.5)$

Standardizing

$P(\frac{\bar{R}-40}{1/3} > \frac{40.5-40}{1/3}) = P(Z > 1.5)$

$= 1 - P(Z \leq 1.5) =$ `1-pnorm(1.5)` $= 0.0668072$

Tema 6: Problem 8

Consider:

- $\sigma = 16$

a. if $n = 10$ compute $P(-2 \leq \bar{X} - \mu \leq 2) = 0.9$

Since we don't know $f(X)$, we don't know $f(\bar{X})$. We could approximate $f(\bar{X})$ by a normal distribution $N(\mu, \frac{\sigma^2}{n})$ if $n > 30$ but this is not the case.

We **cannot** compute the probability.

Tema 6: Problem 8

b. compute n such that $P(-2 \leq \bar{X} - \mu \leq 2) = 0.9$

- We will assume $n > 30$ and then $\bar{X} \hookrightarrow N(\mu, \frac{\sigma^2}{n})$

substitute $\mu = \mu_{\bar{X}}$ and divide by by $\sigma_{\bar{X}}$ to form a standardized variable Z

$P(\frac{-2}{\sigma_{\bar{X}}} \leq \frac{\bar{X}-\mu_{\bar{X}}}{\sigma_{\bar{X}}} \leq \frac{2}{\sigma_{\bar{X}}}) = 0.9$

substitute $\sigma_{\bar{X}} = \sigma/\sqrt{n}$

$P(\frac{-2}{\sigma/\sqrt{n}} \leq Z \leq \frac{2}{\sigma/\sqrt{n}}) = 0.9$

$\Phi(\frac{2}{\sigma/\sqrt{n}}) - \Phi(\frac{-2}{\sigma/\sqrt{n}}) = 0.9$

Tema 6: Problem 8

$\Phi(\frac{2}{\sigma/\sqrt{n}}) - \Phi(\frac{-2}{\sigma/\sqrt{n}}) = 0.9$

remember $\Phi(z) = 1 - \Phi(-z)$

$\Phi(\frac{2}{\sigma/\sqrt{n}}) - 1 + \Phi(\frac{2}{\sigma/\sqrt{n}}) = 0.9$

$2\Phi(\frac{2}{\sigma/\sqrt{n}}) - 1 = 0.9$

$\Phi(\frac{2}{\sigma/\sqrt{n}}) = 0.95$

$\frac{2}{\sigma/\sqrt{n}} = \Phi^{-1}(0.95) =$ `qnorm(0.95)` $= 1.644854$ and solve for n

$n = (1.644854 \times 16/2)^2$ or $n = 174$

Tema 6: Problem 9

Consider:

- $E(X) = \mu = 3000$
- $\sigma = 696$
- $n = 36$

a. Compute $\mu_{\bar{X}}$ and $\sigma_{\bar{X}}$

- $E(\bar{X}) = \mu = 3000$
- $\sigma^2_{\bar{X}} = \frac{\sigma^2}{n} = \frac{696^2}{36}$ or $\sigma_{\bar{X}} = \sqrt{\frac{696^2}{36}} = 696/6 = 116$

since $n > 30$ then $\bar{X} \hookrightarrow N(\mu_{\bar{X}}, \sigma^2_{\bar{X}}) = N(\mu, \frac{\sigma^2}{n})$

Tema 6: Problem 9

b. compute: $P(2670.56 \leq \bar{X} \leq 2809.76)$

standardize

$P(2670.56 \leq \bar{X} \leq 2809.76) = P(\frac{2670.56-\mu_{\bar{X}}}{\sigma_{\bar{X}}} \leq \frac{\bar{X}-\mu_{\bar{X}}}{\sigma_{\bar{X}}} \leq \frac{2809.76-\mu_{\bar{X}}}{\sigma_{\bar{X}}})$

$= P(-2.84 \leq Z \leq -1.64) =$ `pnorm(-1.64)- pnorm(-2.84)` $= 0.04824691$

Tema 6: Problem 9

c. Find n such that $P(3219.2 \leq \bar{X}) = 0.01$

We

1. standardize
2. substitute the values of $\mu_{\bar{X}} = \mu$ and $\sigma_{\bar{X}} = \sigma/\sqrt{n}$
3. use the standard distribution Φ

Tema 6: Problem 9

- standardize

$$P(\frac{3219.24-\mu_{\bar{X}}}{\sigma_{\bar{X}}} \leq \frac{\bar{X}-\mu_{\bar{X}}}{\sigma_{\bar{X}}}) = 0.01$$

- substitute the values of $\mu_{\bar{X}} = \mu$ and $\sigma_{\bar{X}} = \sigma/\sqrt{n}$

$$P(\frac{3219.24-\mu_{\bar{X}}}{\sigma_{\bar{X}}} \leq \frac{\bar{X}-\mu_{\bar{X}}}{\sigma_{\bar{X}}}) = 0.01$$

$$P(\frac{219.24}{696/\sqrt{n}} \leq Z) = 0.01$$

Tema 6: Problem 9

- use the standard distribution Φ since $Z \hookrightarrow N(0,1)$

$$P(\frac{219.24}{696/\sqrt{n}} \leq Z) = 1 - \Phi(\frac{219.24}{696/\sqrt{n}}) = 0.01$$

solve for n

$\frac{219.24}{696/\sqrt{n}} = \Phi^{-1}(0.99) = \texttt{qnorm(0.99)} = 2.32$
then $n = (2.32 * 696/219.24)^2 = 54.24439$ then $n \sim 55$

Tema 6: Problem 10

Consider:

- $E(X) = \mu = 78$
- $\sigma^2 = 169$
- $n = 36$
- $X \hookrightarrow N(\mu, \sigma^2)$

a. Compute $P(\bar{X} \leq 75.7)$

- $E(\bar{X}) = \mu = 78$
- $\sigma^2_{\bar{X}} = \frac{\sigma^2}{n} = \frac{169}{36}$ or $\sigma_{\bar{X}} = \sqrt{\frac{169}{36}} = 13/6 = 2.166667$

since $X \hookrightarrow N(\mu, \sigma^2)$ then $\bar{X} \hookrightarrow N(\mu_{\bar{X}}, \sigma^2_{\bar{X}}) = N(\mu, \frac{\sigma^2}{n})$

Tema 6: Problem 10

a. Compute $P(\bar{X} \leq 75.7)$

Standardizing and using Φ

$P(\frac{\bar{X}-\mu_{\bar{X}}}{\sigma_{\bar{X}}} \leq \frac{\bar{X}-75.7}{\sigma_{\bar{X}}})$

$P(Z \leq -1.06) = \Phi(-1.06) =$ `pnorm(-1.06)` $= 0.1445723$

Tema 6: Problem 10

b. If $R = \sum_{i=1}^{n} X_i$, find n such that $P(R > 3200) < 0.015$

divide by n

$P(R > 3200) =< 0.015$

Remember:

- $\mu_R = n \cdot \mu$ and $\sigma_R^2 = n \cdot \sigma^2$, thas is also $\sigma_R = \sqrt{n} \cdot \sigma$
- standardize, substitute the values of μ_R and σ_R
- and use Φ

$P(\frac{R-\mu_R}{\sigma_R} > \frac{3200-\mu_R}{\sigma_R}) < 0.015$

$P(Z > \frac{3200-78\cdot n}{13\cdot\sqrt{n}}) < 0.015$

$1 - \Phi(\frac{3200-78\cdot n}{13\cdot\sqrt{n}}) < 0.015$

Tema 6: Problem 10

$\Phi(\frac{3200-78\cdot n}{13\cdot\sqrt{n}}) > 0.985$

$\frac{3200-78\cdot n}{13\cdot\sqrt{n}} > \Phi^{-1}(0.985) =$ `qnorm(0.985)` $= 2.17$

then

$78n + 28.21\sqrt{n} - 3200 > 0$

if $t = \sqrt{n}$ then the quadratic equation for t gives the minimun t at $-6.58, 6.22$ selectinng the positive root then $t > 6.22$ or $n > 38.68$ or $n_{min} = 39$

Tema 6: Problem 11

Cables are built with mean traction of $80kg$ and variance of $36kg^2$

- If a sample of 9 cables are selected what is the probability that the mean traction is lower than 79?
- what should be the minimum sample size n for obtaining a probability of 5% that the average traction of the sample is lower than 79 kg?

Tema 6: Problem 11

Consider:

- $E(X) = \mu = 80kg$
- $V(X) = \sigma^2 = 36kg^2$

Compute: n such that $P(\bar{X} \leq 79kg) = 0.05$

For

- $P(\bar{X} \leq 79kg)$ We need $\bar{X} \rightarrow f(\bar{x})$

We know $f(\bar{x})$ when

- it is explicitly mentioned
- when $X \hookrightarrow N(\mu, \sigma^2)$ then $f(\bar{X}) = N(\mu, \frac{\sigma^2}{n})$
- when $n > 30$ then $f(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}) \sim N(0,1)$

Tema 6: Problem 11

We know

- $\mu_{\bar{X}} = 80kg$
- $\sigma^2_{\bar{X}} = 36kg^2$

what is n such that $P(\bar{X} \leq 79kg) = 0.05$

- We assume that $n > 30$ and the CTL approximation would be applicable

Tema 6: Problem 11

Let's standardize

$Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} = \frac{\bar{X}-80}{6/\sqrt{n}} \rightarrow N(0,1)$

$P(\bar{X} \leq 79kg) = P(\frac{\bar{X}-80}{6/\sqrt{n}} \leq \frac{79-80}{6/\sqrt{n}})$

$= P(Z \leq -0.16667\sqrt{n})$

$= \Phi(-0.16667\sqrt{n}) = 0.05$

$-0.16667\sqrt{n} = \texttt{qnorm(0.05)} = -1.644854$

and then $\sqrt{n} = 1.644854/0.16667$ or $n > 97.39$ or $n > 98$, which$>> 30$

Tema 6: Problem 11

consider:

- $\bar{X} \rightarrow N(\mu_{\bar{X}}, \sigma^2_{\bar{X}}) = N(\mu, \sigma^2/n)$
- $n = 9$
- $\sigma^2 = 36$

b. find μ such that $P(\bar{X} \leq 79) = 0.05$ it would the minimum value it can take.

let's standardize

$P(\bar{X} \leq 79) = P(\frac{\bar{X}-\mu_{\bar{X}}}{\sigma_{\bar{X}}} \leq \frac{\bar{79}-\mu_{\bar{X}}}{\sigma_{\bar{X}}})$
$= P(Z \leq \frac{79-\mu}{6/\sqrt{9}})$
$= \Phi(\frac{79-\mu}{6/\sqrt{9}}) = 0.05$

$\frac{79-\mu}{6/\sqrt{9}} =$ `qnorm(0.05)` $= -1.644854$

Tema 6: Problem 11

$\frac{79-\mu}{6/\sqrt{9}} = -1.645$

the minimum μ is 82.29 so or $\mu > 82.29$

Tema 6: Problem 12

Consider:

- $E(X) = \mu = 75$
- $\sigma = 15$
- $n = 25$
- $X \hookrightarrow N(\mu, \sigma^2)$

a. Compute $P(|\bar{X} - \mu| \leq 5)$

- $E(\bar{X}) = \mu = 75$
- $\sigma^2_{\bar{X}} = \frac{\sigma^2}{n} = \frac{15^2}{25}$ or $\sigma_{\bar{X}} = \sqrt{\frac{15^2}{25}} = 15/5 = 3$

since $X \hookrightarrow N(\mu, \sigma^2)$ then $\bar{X} \hookrightarrow N(\mu_{\bar{X}}, \sigma^2_{\bar{X}}) = N(\mu, \frac{\sigma^2}{n})$

Tema 6: Problem 12

devide by $\sigma_{\bar{X}}$ to form standardized variable

$P(\frac{-5}{3} \leq \frac{\bar{X}-75}{3} \leq \frac{5}{3}) = P(\frac{-5}{3} \leq Z \leq \frac{5}{3})$

Use Φ

- $\Phi(\frac{5}{3}) - \Phi(\frac{5}{3}) =$ `pnorm(5/3)-pnorm(-5/3)` $= 0.9044193$

compute $P(|\bar{X} - \mu| \leq 5)$ if $n = 100$ then $\sigma_{\bar{X}} = \sqrt{\frac{15^2}{100}} = 15/10 = 3/2$

Tema 6: Problem 11

then

$P(\frac{-5}{3/2} \leq \frac{\bar{X}-75}{3/2} \leq \frac{5}{3}) = P(\frac{-5}{3/2} \leq Z \leq \frac{5}{3/2})$

- $\Phi(\frac{10}{3}) - \Phi(\frac{10}{3}) =$ `pnorm(10/3)-pnorm(-10/3)` $= 0.9991419$

as n increases the probability increases.

Tema 6: Problem 12

b. consider

- $n = 9$

compute: C such that $P(\bar{X} > C) = 0.015$

or $P(\bar{X} \leq C) = 1 - 0.01 = 0.985$

Remember:

- standardize
- substitute the values of $\mu_{\bar{X}} = \mu$ and $\sigma_{\bar{X}} = \sigma/\sqrt{n} = 15/3 = 5$
- use Φ

$P(\frac{\bar{X}-75}{5} \leq \frac{C-75}{5}) = 0.985$

$P(Z \leq \frac{C-75}{5}) = 0.985$

Tema 6: Problem 12

$\Phi(\frac{C-75}{5}) = 0.985$

$\frac{C-75}{5} = \Phi^{-1}(0.985) = \texttt{qnorm(0.985)} = 2.17009$

solving for C then

$C = 2.17009 * 5 + 75 = 85.85045$