POSL: A Parallel-Oriented Solver Language

Thesis for the Degree of
Doctor of Computer Science

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Part I

POSL: Parallel Oriented Solver Language

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A Parallel-Oriented Language for Modeling Meta-Heuristic-Based Solvers

In this chapter POSL *is introduced as the main contribution of this thesis, and a new way to solve CSPs. Its characteristics and advantages are summarized, and a general procedure to be followed is described, in order to build parallel solvers using* POSL, *followed by a detailed description of each of the single steps.*

Contents

1.1 Introduction

Meta-heuristic methods, despite showing very good results solving *Constraint Satisfaction Problems*, they are frequently not enough for solve them, when they are applied to problem instances with extremely large search spaces. Most of these methods are sensible to their large number of parameters. For that reason, a first direction of this thesis was tackling the one of the weakest points of meta-heuristic methods: the parameters. In Chapter **??** a performed study applying ParamILS to *Adaptive Search* in order to find a general parameter settings was presented. This experiment did not produce encouraging results, and for that reason it was decided to abandon the idea as the main direction of the thesis. However, I believe that it can be an idea to be considered as a future work.

With the development of parallelism, opening new ways to tackle constrained problems, the accessibility to this technology to a broad public has also increased. It is available through multi-core personal computers, Xeon Phi cards and GPU video cards. For that reason it was decided focusing completely on the parallel approach. In Chapter **??** it was presented a study in which the problem-subdivision approach was applied to the resolution of *K-Medoids Problem*. The main goal of this work was generalizing the proposed ideas to similar problems. It was only a theoretical study because it was realised in parallel with what would latter be the main scientific contribution of this thesis.

After analyzing all weak point of the most important previews works, another issue arises, frequently undervalued: the codding time, that is always long when codding parallel programs. This was the main motivation to start searching techniques for implementing parallel solution strategies with or without communication in a fast and easy way. The main goal was creating a tool providing:

1. An simple way to create *flexible* solvers, i.e., solvers ables to be modified with a few effort.

2. Fast and simple mechanisms to connect solvers, ables to exchange information.

3. A way to create numerous and different parallel strategies designs, where different communicating and not communicating solvers can be combined, exploiting to the maximum computation resources.

1.1.1 Precedents

During the development process, some inspired ideas were taken into account. HYPERION[2] [62] is a java framework for meta– and hyper–heuristics built providing generic templates for a variety of local search and evolutionary computation algorithms, allowing quick prototyping with the possibility of reusing source code. A similar idea was proposed by Fukunaga [7], introducing an evolutionary approach that uses a simple composition operator to automatically discover new local search heuristics for SAT and to visualize them as combinations of blocks. The goal of this thesis is to create a tool offering the same advantages, but providing also a mechanism to define communication protocols between solvers. It must also provide a way to create an abstract solver by combining simple functions that we call *modules*.

In [8] is presented a framework to facilitate the development of search procedures by using *combinators* to design features commonly found in search procedures as standard bricks and joining them. This approach can speed up the development and experimentation of search procedures when developing a specific solver based on local search. Martin et al. [9] propose an approach of using cooperating meta–heuristic based local search processes, using an asynchronous message passing protocol. The cooperation is based on the general strategies of pattern matching and reinforcement learning. The tool developed for this thesis, uses the combination of both ideas, where search process features can be combined and reused, and it is also possible to design communication strategies between solvers.

1.1.2 POSL

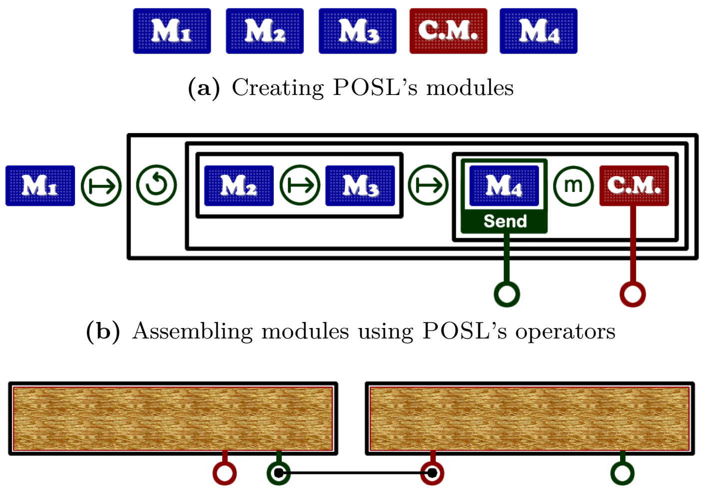
In this chapter is presented POSL, the main contribution of this thesis, as well as the different steps to build communicating parallel solvers with. It is proposed as a new way to implement *solution algorithms* to solve *Constraint Satisfaction Problems*, through local-search meta-heuristics using the multi-walk parallel approach. It is based on improving step by step an initial configuration, driven by a *cost function* provided by the user through the model. The implementation must follow the following stages.

1. The conceived *solution algorithm* to solve the target problem is decomposed it into small modules of computation, which are implemented as separated *functions*. We name them *computation modules* (see Figure 1.1a, blue shapes). At this point it is crucial to find a good decomposition of its *solution algorithm*, because it will have a significant impact in its future re-usage.

2. Deciding which information is interesting to *receive* from other solvers. This information is encapsulated into another kind of component called *communication module*, allowing data transmission between solvers (see Figure 1.1a, red shapes).

3. A third stage is to ensemble the modules through POSL's inner language to create independent solvers.

4. The parallel-oriented language based on operators provided by POSL (see Figure 1.1b, green shapes) allows the information exchange, and executing modules in parallel. In this stage the information that is interesting to be shared with other solvers is sent using operators. After that we can connect them using *communication operators*. We call this final entity a *solver set* (see Figure 1.1c).



(a) Creating POSL's modules

(b) Assembling modules using POSL's operators

(c) Connecting POSL solvers to create *communication strategies*

Figure 1.1: Solver construction process using POSL

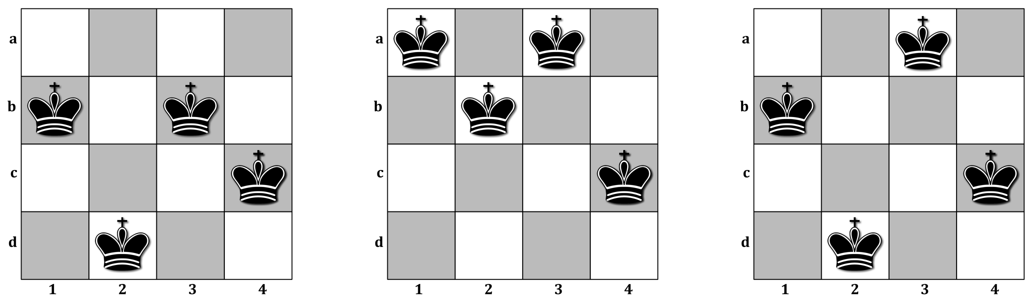
In the following sections all these steps are explained in details, but first, I explain how to model the target benchmark using POSL.

1.2 Modeling the target benchmark

Target problems are modeled in POSL using the C++ programing language, respecting some rules of the object-oriented design. First of all, the benchmark must inherit from the **Benchmark** class provided by POSL. This class does not have any method to be overridden or implemented, but receives in its constructor three objects, instances of classes the user must create. Those classes must inherit from **SolutionCostStrategy**, **RelativeCostStrategy** and **ShowStrategy**, respectively. In these classes the most important functionalities of the benchmark model are defined.

SolutionCostStrategy: In this class the strategy computing the *cost* of a configuration is implemented. This *cost function* must return an integer taking into account the problem constraints. Given a configuration s, the *cost function*, as a mandatory rule, must return 0 if and only if s is a solution of the problem, i.e., s fulfills all the problem constraints. Otherwise, it must return an integer describing "how long" is the given configuration from a solution. An example of *cost function* is the one returning the number of violated constraints. However, the more expressive the cost function is, the better the performance of POSL is, leading to the solution.

> Let us take the example of the *4-Queens Problem*. This problem is about placing 4 queens on a 4×4 chess board so that none of them can hit any other in one move. A configuration for this benchmark is a vector of 4 integer indicating the row where a queens is placed on each column. So, the configuration $s_a = (1, 3, 1, 2)$ corresponds to the example in Figure 1.2a.
>
> Now, let us suppose two different *cost functions*:
>
> 1. $f_1(s) = c$ if and only if c is the maximum number of queens hitting another.
> 2. $f_2(s) = c$ if and only if c is the sum of the number of queens that each queen hits.
>
> Tacking these two functions into account, it is easy to see that $f_1(s_a) = 3$ and $f_2(s_a) = 4$. If we take the example in Figure 1.2b, the corresponding configuration is $s_b = (0, 1, 0, 2)$ with $f_1(s_b) = 3$ and $f_2(s_b) = 6$. In this case, according to the *cost function* f_1 both configurations have the same opportunity of being selected, because they have the same cost. However, applying the *cost function* f_2, the best configuration is s_b in which a solution can be obtained just moving the queen *b3* to *a3*.
>
> In that sense, f_2 is *more expressive* than f_1.



(a) (b) (c)

Figure 1.2: 4-Queens examples

The method to be implemented in this class is:

- `int solutionCost(std::vector<int> & c)` → Computes the cost of a given configuration `c`.

RelativeCostStrategy: In this class the user implements the strategy to compute the *cost* of a given configuration with respect to another, with the help of some stored information.

> Coming back to the previews example, let us suppose that the current configuration is $s_a = (1, 3, 1, 2)$ corresponding to the Figure 1.2a. Taking the *cost function* f_2, the cost of this configurations is $f_2(s_a) = 4$. If we want to compute the cost of $s_c = (1, 3, 0, 2)$ (Figure 1.2c), knowing that the only change with respect to the current configuration is the queen in the column 3, we can use the following *relative cost function*:
>
> $$\begin{aligned} rf(s_c) &= c - 2 \cdot q + a \\ &= 4 - 2 \cdot 2 + 0 \\ &= 0 \end{aligned}$$
>
> where c is the current cost, q is the number of queens that the queen in column 3 hits (an information that can be stored), and a the number of queens that the queen in the column 3 hits in the new position (*a3*).

The methods to implement in this class are:

- `void initializeCostData(std::vector<int> & c)` → Initializes the information related to the cost (auxiliary data structures, the current configuration `c`, the current cost, etc.)
- `void updateConfiguration(std::vector<int> & c)` → Updates the information related to the cost.
- `int relativeSolutionCost(std::vector<int> & c)` → Returns the relative cost of the configuration `c` with respect to the current configuration.
- `int currentCost()` → Property that returns the cost of the current configuration.
- `int costOnVariable(int variable_index)` → Returns a measure of the contribution of a variable to the total cost of a configuration.
- `int sickestVariable()` → Returns the variable contributing the most to the cost.

ShowStrategy: This class represents the way a benchmark shows a configuration, in order to provide more information about the structure.

For example, a configuration of the instance 3–3–2 of the *Social Golfers Problem* (see bellow for more details about this benchmark) can be written as follows:

```
[1, 2, 3, 4, 5, 6, 7, 8, 9, 3, 4, 5, 6, 7, 8, 9, 1, 2]
```

This text is, nevertheless, very difficult to be read if the instance is larger. Therefore, it is recommended that the user implements this class in order to give more details and to make it easier to interpret the configuration. For example, for the same instance of the problem, a solution could be presented as follows:

```
Golfers: players-3, groups-3, weeks-2
6        8        7
1        3        5
4        9        2
--
7        2        3
4        8        1
5        6        9
--
```

The method to be implemented in this class is:

- `std::string showSolution(std::shared_ptr<Solution> s)` → Returns a string to be written in the standard output.

Once we have modeled the target benchmark, it can be solved using POSL. In the following sections we describe how to use this parallel-oriented language to solve *Constraint Satisfaction Problems*.

1.3 First stage: creating POSL's modules

There exist two types of basic modules in POSL: *computation modules* and *communication modules*. A *computation module* is basically a function and a *communication module* is also a function, but in contrast, the it can receive information from two different sources: through input parameters or from outside, i.e., by communicating with a module from another solver.

1.3.1 Computation module

A *computation module* is the most basic and abstract way to define a piece of computation. It is a function which receives an instance of a POSL data type as input, then executes an internal algorithm, and returns an instance of a POSL data type as output. The input and output types will characterize the computation module signature. It can be dynamically replaced by (or combined with) other computation modules, since they can be transmitted to other solvers working in parallel. They are joined through operators defined in Section 1.4.

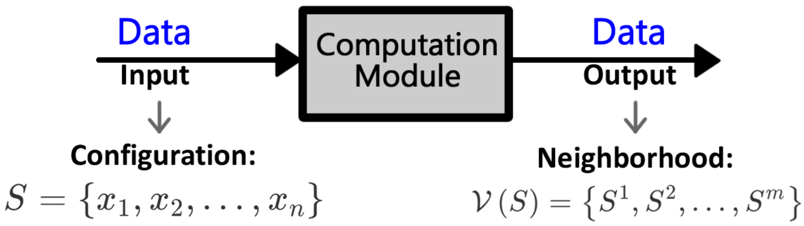


Figure 1.3: An example of a computation module computing a neighborhood

Definition 1 ***(Computation Module)*** *A computation module Cm is a mapping defined by:*

$$Cm : I \to O \tag{1.1}$$

where I and O, for instance, can be independently a set of configurations, a set of sets of configurations, a set of values of some data type, etc.

Consider a local search meta-heuristic solver. One of its *computation modules* can be the function returning the set of configurations composing the neighborhood of a given configuration:

$$Cm_{neighborhood} : I_1 \times I_2 \times \cdots \times I_n \to 2^{I_1 \times I_2 \times \cdots \times I_n}$$

where I_i represents the definition domains of each variable of the input configuration.

Figure 1.3 shows an example of *computation module*: which receives a configuration S and then computes the set $\mathcal{V}$ of its neighbor configurations $\{S^1, S^2, \ldots, S^m\}$.

1.3.1.1 Creating new *computation modules*

To create new *computation modules* we use C++ programing language. POSL provides a hierarchy of data types to work with (See annexes) and some abstract classes to inherit from, depending on the type of *computation module* the user wants to create. These abstract classes represent *abstract computation module* and define a type of action to be executed. In the following we present the most important ones:

- **ACM_FirstConfigurationGeneration** → Represents *computation modules* generating a first configuration. The user must implement the method *execute*(***ComputationData***) which returns a pointer to a **Solution**, that is, an object containing all the information concerning a partial solution (configuration, variable domains, etc.)

- **ACM_NeighborhoodFunction** → Represent *computation modules* creating a neighborhood of a given configuration. The user must implement the method *execute*(***Solution***) which returns a pointer to an object **Neighborhood**, containing a set of configurations which constitute the neighborhood of a given configuration, according to certain criteria. These configurations are efficiently stored in term of space.

- **ACM_SelectionFunction** → Represents *computation modules* selecting a configuration from a neighborhood. The user must implement the method *execute*(***Neighborhood***) which returns a pointer to an object **DecisionPair**, containing two solutions: the current and the selected one.

- **ACM_DecisionFunction** → Represents *computation modules* deciding which of the two solutions will be the current configuration for the next iteration. The user must implement the method *execute*(***DecisionPair***) which returns a pointer to an object **Solution**.

1.3.2 Communication modules

A *communication module* is the component managing the information reception in the communication between solvers (I talk about information transmission in Section 1.4). They can interact with *computation modules* through operators (see Figure 1.4).

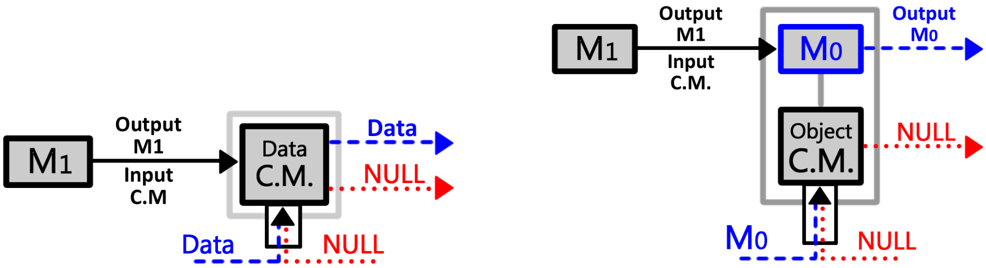
A *communication module* can receive two types of information from an external solver: data or *computation modules*. It is important to notice that by sending/receiving *computation modules*, I mean sending/receiving the required information to identify and being able to instantiate the *computation module*. For instance, an integer identifier.

In order to distinguish from the two types of *communication modules*, I will call Data Communication Module the *communication module* responsible for the data reception (Figure 1.4a), and Object Communication Module the one responsible for the reception and instantiation of *computation modules* (Figure 1.4b).

Definition 2 ***(Data Communication Module)*** *A* Data Communication Module *Ch is a module that produces a mapping defined as follows:*

$$Ch : I \times \{D \cup \{NULL\}\} \rightarrow D \cup \{NULL\} \tag{1.2}$$

No matter what the input I is, it returns the information D coming from an external solver.



(a) Data *communication module* **(b)** Object *communication module*

Figure 1.4: Communication module

Definition 3 ***(Object Communication Module)*** *If we denote by* $\mathbb{M}$ *the space of all the computation modules defined by Definition 1.1, then an* Object Communication Module Ch *is a module that produces and executes a computation module coming from an external solver as follows:*

$$Ch : I \times \{\mathbb{M} \cup \{NULL\}\} \rightarrow O \cup \{NULL\} \tag{1.3}$$

It returns the output O *of the execution of the computation module coming from an external solver, using* I *as the input.*

Users can implement new computation and connection modules but POSL already contains many useful modules for solving a broad range of problems.

Due to the fact that *communication modules* receive information coming from outside without having control on them, it is necessary to define the *NULL* information, in order to denote the absence of information. If a Data Communication Module receives information, it is returned automatically. If a Object Communication Module receives a *computation module*, it is instantiated and executed with the *communication module*'s input and its result is returned. In both cases, if no available information exists (no communications performed), the *communication module* returns the *NULL* object.

1.4 Second stage: assembling POSL's modules

Modules mentioned above are defined respecting the signature of some predefined abstract module. For example, the module showed in Figure 1.3 is a *computation module* based on an abstract module that receives a configuration and returns a neighborhood. In that sense, an example of a concrete *computation module* (or just *computation module*) can be a function receiving a configuration, and returning a neighborhood constituted by N configurations which only differ from the input configuration in one entry.

In this stage an *abstract solver* is coded using POSL. It takes abstract modules as *parameters* and combines them through operators. Through the *abstract solver*, we can also decide which information to send to other solvers by using some operators to send the result of a computation module (see below). In the following we present a formal and more detailed specification of POSL's operators.

The *abstract solver* is the solver's backbone. It joins the *computation modules* and the *communication modules* coherently. It is independent from the *computation modules* and *communication modules* used in the solver. It means that they can be changed or modified during the execution, without altering the general algorithm, but still respecting the main structure. Each time we combine some of them using POSL's operators, we are creating a *compound module*. Here we formally define the concept of *module* and *compound module*.

Definition 4 *A* **module** *is (and it is denoted by the letter $\mathcal{M}$):*

1. *a computation module or*
2. *a communication module or*
3. $[\mathcal{M}_1 \ OP \ \mathcal{M}_2]$*, which is the composition of two modules* $\mathcal{M}_1$ *and* $\mathcal{M}_2$ *to be executed sequentially, returning an output depending on the nature of the operator* OP*; or*
4. $[\![\mathcal{M}_1 \ OP \ \mathcal{M}_2]\!]_p$*, which is the composition of two modules* $\mathcal{M}_1$ *and* $\mathcal{M}_2$ *to be executed, returning an output depending on the nature of the operator* OP*. These two modules will be executed in parallel if and only if* OP *supports parallelism, (i.e. some modules will be executed sequentially although they were grouped this way); or sequentially otherwise.*

We denote the space of the modules by $\mathbb{M}$ *and call compound modules to the composition of modules described in 3. and 4..*

For a better understanding of Definition 4, Figure 1.5 shows graphically the structure of a *compound module*.

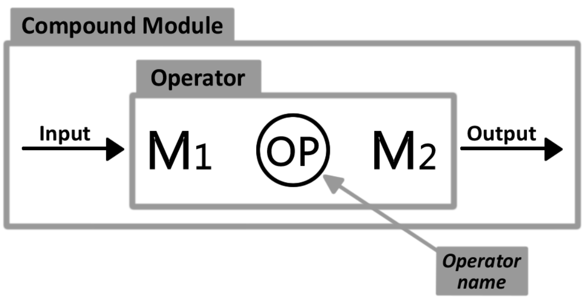


Figure 1.5: A *compound module*

As mentioned before, the *abstract solver* is independent from the *computation modules* and *communication modules* used in the solver. It means that one *abstract solver* can be used to construct many different solvers, by implementing it using different modules (see below the related concept of *abstract solver* instantiation). This is the reason why the *abstract solver* is defined only using abstract modules. Formally, we define an *abstract solver* as follows:

Definition 5 ***(Abstract Solver)*** *An* Abstract Solver *AS is a triple $(\mathbf{M}, \mathcal{L}^m, \mathcal{L}^c)$, where: $\mathbf{M}$ is a compound module (also called* root *compound module), $\mathcal{L}^m$ a list of abstract computation modules appearing in $\mathcal{M}$, and $\mathcal{L}^c$ a list of communication modules appearing in $\mathcal{M}$.*

The *root compound module* can be defined also as a free-context grammar as follows:

Definition 6 (root *compound module***'s grammar)** *$G_{POSL} = (\mathbf{V}, \Sigma, \mathbf{S}, \mathbf{R})$, where:*

1. *$\mathbf{V} = \{CM, OP\}$ is the set of variables,*
2. *$\Sigma = \left\{\alpha, \beta, be, [,], [\![,]\!]_p, (,), \{,\}, (\!|, |\!)^m, |\!)^o, \mapsto, ?, \circlearrowleft, \rho, \vee, \wedge, M, m, \downarrow, \cup, \cap\right\}$ is the set of terminals,*
3. *$\mathbf{S} = \{CM\}$ is the set of start variables,*
4. *and $\mathbf{R} =$*

$$CM \longmapsto \alpha \mid \beta \mid (\!|CM|\!)^o \mid (\!|CM|\!)^m \mid [OP] \mid [\![OP]\!]_p$$
$$OP \longmapsto CM \mapsto CM \mid CM\, ?\, CM \mid CM\, \rho\, CM \mid CM \vee CM \mid CM \wedge CM$$
$$OP \longmapsto CM\, M\, CM \mid CM\, m\, CM \mid CM \downarrow CM \mid CM \cup CM \mid CM \cap CM$$
$$OP \longmapsto CM \circlearrowleft (be) CM$$

is a set of rules

In the following I explain some of the concepts in Definition 6:

- The variables CM and OP are two very important entities in the language, as it can be seen in the grammar. We name them *compound module* and *operator*, respectively.
- The terminals α and β represent a *computation module* and a *communication module*, respectively.
- The terminal *be* is a boolean expression.
- The terminals $[\,]$, $[\![\,]\!]_p$ are symbols for grouping and defining the way the involved *compound modules*are executed. Depending on the nature of the operator, this can be either sequentially or in parallel:

1. [OP]: The involved operator is executed sequentially.

2. $[\![\mathrm{OP}]\!]_p$: The involved operator is executed in parallel if and only if OP supports parallelism. Otherwise, an exception is thrown.

- The terminals (and) are symbols for grouping the boolean expression in some operators.

- The terminals $(\!|.|\!)^m$, $(\!|.|\!)^o$, are operators to send information to other solvers (explained bellow).

- The rest of terminals are POSL operators.

In the following we define POSL operators. In order to group modules, like in Definition 4(3.) and 4(4.), we will use $|.|$ as generic grouper. In order to help the reader to easely understand how to use the operators, I use an example of a solver that I build step by step, while presenting the definitions.

POSL creates solvers based on local search meta-heuristics algorithms. These algorithms have a common structure: 1. They start by initializing some data structures (e.g., a *tabu list* for *Tabu Search* [37], a *temperature* for *Simulated Annealing* [35], etc.). 2. An initial configuration s is generated. 3. A new configuration s' is selected from the neighborhood $\mathcal{V}(s)$. 4. If s' is a solution for the problem P, then the process stops, and s' is returned. If not, the data structures are updated, and s' is accepted or not for the next iteration, depending on a certain criterion. An example of such data structure is the penalizing features of local optima defined by Boussaïd et al [4] in their algorithm *Guided Local Search*.

Abstract computation modules composing *local search meta–heuristics* are:

$\boxed{Abstract\ Computation\ module - 1}$	I: Generating a configuration s
$\boxed{Abstract\ Computation\ module - 2}$	V: Defining the neighborhood $\mathcal{V}(s)$
$\boxed{Abstract\ Computation\ module - 3}$	S: Selecting $s' \in \mathcal{V}(s)$
$\boxed{Abstract\ Computation\ module - 4}$	A: Evaluating an acceptance criterion for s'

The list of modules to be used in the examples have been presented. Now I present the POSL operators.

Definition 7 (Operator Sequential Execution) *Let*

1. $\mathcal{M}_1 : \mathcal{D}_1 \to \mathcal{I}_1$ *and*

2. $\mathcal{M}_2 : \mathcal{D}_2 \to \mathcal{I}_2$, *be modules, where* $\mathcal{I}_1 \subseteq \mathcal{D}_2$. *Then the operation* $\left|\mathcal{M}_1 \mapsto \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_{seq}$ *as the result of executing* $\mathcal{M}_1$ *followed by executing* $\mathcal{M}_2$*:*

$$\mathcal{M}_{seq} : \mathcal{D}_1 \rightarrow \mathcal{I}_2$$

This is an example of an operator that does not support the execution of its involved *compound modules* in parallel, because the input of the second *compound module* is the output of the first one.

> Coming back to the example, I can use defined *abstract computation modules* to create a *compound module* that perform only one iteration of a local search, using the operator **Sequential Execution**. I create a *compound module* to execute sequentially I and V (see Figure 1.6a), then I create an other *compound module* to execute sequentially the *compound module* already created and S (see Figure 1.6b), and finally this *compound module* and the *computation module* A are executed sequentially (see Figure 1.6c). The *compound module* presented in Figure 1.6c can be coded as follows:
>
> $$\left[\left[\left[I \mapsto V\right] \mapsto S\right] \mapsto A\right]$$

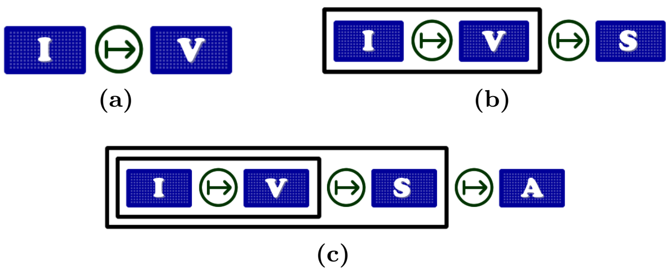


Figure 1.6: Using **sequential execution** operator

The following operator is very useful to execute modules sequentially creating bifurcations, subject to some boolean condition:

Definition 8 (Operator Conditional Execution) *Let*

1. $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*
2. $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$,

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$. *Then the operation* $\left|\mathcal{M}_1 ?_{<cond>} \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_{cond}$ *as result of the sequential execution of* $\mathcal{M}_1$ *if* $< cond >$ *is* **true** *or* $\mathcal{M}_2$*, otherwise:*

$$\mathcal{M}_{cond} : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

This operator can be used in the example if I want to execute two different *selection computation modules* (S_1 and S_2) depending on certain criterion (see Figure 1.7):

$$\left[\left[\left[I \textcircled{\mapsto} V\right] \textcircled{\mapsto} \left[S_1 \textcircled{?} S_2\right]\right] \textcircled{\mapsto} A\right]$$

In examples I remove the clause $< cond >$ for simplification.

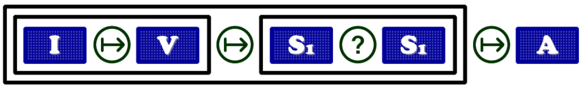


Figure 1.7: Using **conditional execution** operator

We can execute modules sequentially creating also cycles.

Definition 9 (Operator Cyclic Execution) *Let $\mathcal{M} : \mathcal{D} \to \mathcal{I}$ be a module, where $\mathcal{I} \subseteq \mathcal{D}$. Then, the operation $|\circlearrowleft_{<cond>} \mathcal{M}|$ defines the compound module $\mathcal{M}_{cyc}$ as result of the sequential execution of $\mathcal{M}$ repeated while $< cond >$ remains* **true***:*

$$\mathcal{M}_{cyc} : \mathcal{D} \to \mathcal{I}$$

Using this operator I can model a local search algorithm, by executing the *abstract computation module* I and then the other *computation modules* (V, S and A) cyclically, until finding a solution (i.e, a configuration with cost equal to zero) (see Figure 1.8):

$$\left[I \textcircled{\mapsto} \left[\circlearrowleft \left[\left[V \textcircled{\mapsto} S\right] \textcircled{\mapsto} A\right]\right]\right]$$

In the examples, I remove the clause $< cond >$ for simplification.

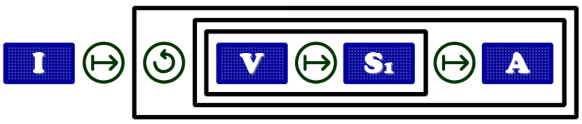


Figure 1.8: Using **cyclic execution** operator

Definition 10 (Operator Random Choice) *Let*

1. $\mathcal{M}_1 : \mathcal{D}_1 \to \mathcal{I}_1$ *and*

2. $\mathcal{M}_2 : \mathcal{D}_2 \to \mathcal{I}_2$,

be modules, where $\mathcal{D}_1 \subset \mathcal{D}_2$ and a real value ρ. Then the operation $\left|M_1 \textcircled{\rho} \mathcal{M}_2\right|$ defines the compound module $\mathcal{M}_{rho}$ that executes and returns the output of $\mathcal{M}_1$ with probability ρ, or executes and returns the output of $\mathcal{M}_2$ with probability $(1 - \rho)$:

$$\mathcal{M}_{rho} : \mathcal{D}_1 \cap \mathcal{D}_2 \to \mathcal{I}_1 \cup \mathcal{I}_2$$

In the example I can create a *compound module* to execute two *abstract computation modules* A_1 and A_2 following certain probability ρ using the operator **random execution** as follows (see Figure 1.9):

$$\left[I \mapsto \left[\circlearrowleft \left[\left[V \mapsto S\right] \mapsto \left[A_1 \, (\rho) \, A_2\right]\right]\right]\right]$$

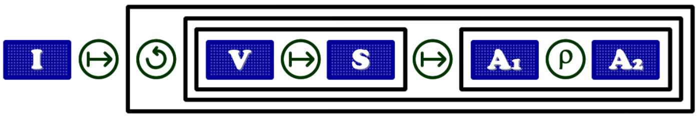


Figure 1.9: Using **random execution** operator

The following operator is very useful if the user needs to use a *communication module* inside an *abstract solver*. As explained before, if a *communication module* does not receive any information from another solver, it returns *NULL*. This may cause the undesired termination of the solver if this case is not considered correctly. Next, I introduce the operator **Operator Not** *NULL* **Execution** and illustrate how to use it in practice with an example.

Definition 11 (Operator Not *NULL* **Execution)** *Let*

1. $\mathcal{M}_1 : \mathcal{D}_1 \to \mathcal{I}_1$ *and*
2. $\mathcal{M}_2 : \mathcal{D}_2 \to \mathcal{I}_2$,

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$. *Then, the operation* $\left|\mathcal{M}_1 \, (\vee) \, \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_{non}$ *that executes* $\mathcal{M}_1$ *and returns its output if it is not NULL, or executes* $\mathcal{M}_2$ *and returns its output otherwise:*

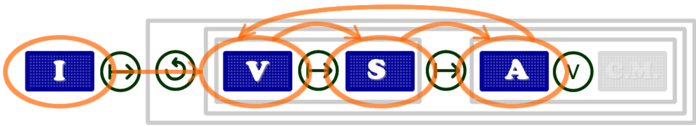
$$\mathcal{M}_{non} : \mathcal{D}_1 \cap \mathcal{D}_2 \to \mathcal{I}_1 \cup \mathcal{I}_2$$

Let us make consider a slightly more complex example: When applying the acceptance criterion, suppose that we want to receive a configuration from other solver to combine the *computation module* A with a *communication module*:

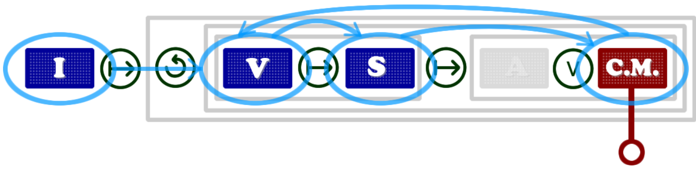
$\boxed{Communication\ module - 1 :}$ $C.M.$: Receiving a configuration.

Figure 1.10 shows how to combine a *communication module* with the *computation module* A through the operator $(\vee)$. Here, the *computation module* A will be executed as long as the *communication module* remains *NULL*, i.e., there is no information coming from outside. This behavior is represented in Figure 1.10a by the orange lines. If some data has been received through the *communication module*, the later is executed instead of the module A, represented in Figure 1.10b by blue lines. The code can be written as follows:

$$\left[I \;\text{\textcircled{$\mapsto$}}\; \left[\circlearrowleft \left[\left[V \;\text{\textcircled{$\mapsto$}}\; S\right] \;\text{\textcircled{$\mapsto$}}\; \left[A \;\text{\textcircled{$\vee$}}\; C.M.\right]\right]\right]\right]$$



(a) The solver executes the computation module **A** if no information is received through the connection module



(b) The solver uses the information coming from an external solver

Figure 1.10: Two different behaviors within the same solver

This is *short-circuit* operator. It means that if the first argument (module) does not return *NULL*, the second will not be executed. POSL provides another operator with the same functionality but not *short-circuit*:

Definition 12 (Operator *BOTH* Execution) *Let*

1. $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*
2. $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$,

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$. *Then the operation* $\left|\mathcal{M}_1 \text{\textcircled{$\wedge$}} \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_{both}$ *that executes both* $\mathcal{M}_1$ *and* $\mathcal{M}_2$, *then returns the output of* $\mathcal{M}_1$ *if it is not NULL, or the output of* $\mathcal{M}_2$ *otherwise:*

$$\mathcal{M}_{both} : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

In the following definitions, the concepts of *cooperative parallelism* and *competitive parallelism* are implicitly included. We say that cooperative parallelism exists when two or more processes are running separately, they are independent, and the general result will be some combination of the results of all the involved processes (e.g. Definitions 13 and 14). On the other hand, competitive parallelism arise when the general result is the result of the process ending first (e.g. Definition 15).

Definition 13 (Operator Minimum) *Let*

1. $\mathcal{M}_1 : \mathcal{D}_1 \to \mathcal{I}_1$ *and*
2. $\mathcal{M}_2 : \mathcal{D}_2 \to \mathcal{I}_2$*,*

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$*. Let also* o_1 *and* o_2 *be the outputs of* $\mathcal{M}_1$ *and* $\mathcal{M}_2$*, respectively. Assume that there exists some order criteria between them. Then the operation* $\left|\mathcal{M}_1 \textcircled{m} \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_{min}$ *that executes* $\mathcal{M}_1$ *and returns* $\min\{o_1, o_2\}$*:*

$$\mathcal{M}_{min} : \mathcal{D}_1 \cap \mathcal{D}_2 \to \mathcal{I}_1 \cup \mathcal{I}_2$$

Similarly we define the operator **Maximum**:

Definition 14 (Operator Maximum) *Let*

1. $\mathcal{M}_1 : \mathcal{D}_1 \to \mathcal{I}_1$ *and*
2. $\mathcal{M}_2 : \mathcal{D}_2 \to \mathcal{I}_2$*,*

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$*. Let also* o_1 *and* o_2 *be the outputs of* $\mathcal{M}_1$ *and* $\mathcal{M}_2$*, respectively. Assume that there exists some order criteria between them. Then the operation* $\left|\mathcal{M}_1 \textcircled{M} \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_{max}$ *that executes* $\mathcal{M}_1$ *and returns* $\max\{o_1, o_2\}$*:*

$$\mathcal{M}_{max} : \mathcal{D}_1 \cap \mathcal{D}_2 \to \mathcal{I}_1 \cup \mathcal{I}_2$$

Comming back to the previews example, the **minimum operator** can be applied to obtain a more interesting behavior in the solver: When applying the acceptance criteria, suppose that we want to receive a configuration from other solver, to compare it with ours and select the one with the lowest cost. We can do that by applying the operator (m) to combine the *computation module A* with a *communication module C.M.* (see Figure1.11):

$$\left[I \textcircled{\mapsto} \left[\circlearrowleft \left[\left[V \textcircled{\mapsto} S\right] \textcircled{\mapsto} \left[\!\left[A \textcircled{m} C.M.\right]\!\right]_p\right]\right]\right]$$

Notice that in this example, I can use the grouper $[\![.]\!]_p$ since the **minimum operator** supports parallelism.

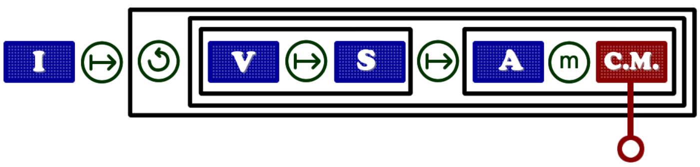


Figure 1.11: Using **minimum** operator

Definition 15 (Operator Race) *Let*

1. $\mathcal{M}_1 : \mathcal{D}_1 \to \mathcal{I}_1$ *and*

2. $\mathcal{M}_2 : \mathcal{D}_2 \to \mathcal{I}_2$*,*

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$ *and* $\mathcal{I}_1 \subset \mathcal{I}_2$*. Then the operation* $\left|\mathcal{M}_1 \textcircled{\downarrow} \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_{race}$ *that executes both modules* $\mathcal{M}_1$ *and* $\mathcal{M}_2$*, and returns the output of the module ending first:*

$$\mathcal{M}_{race} : \mathcal{D}_1 \cap \mathcal{D}_2 \to \mathcal{I}_1 \cup \mathcal{I}_2$$

> Sometimes nighborhood functions are slow depending on the configuration. In that case two neighborhood *computation modules* can be executed and take into account the output of the module ending first (see Figure1.12):
>
> $$\left[I \textcircled{\mapsto} \left[\circlearrowleft \left[\left[\left[\!\left[V_1 \textcircled{\downarrow} V_2\right]\!\right]_p \textcircled{\mapsto} S\right] \textcircled{\mapsto} \left[\!\left[A \textcircled{m} C.M.\right]\!\right]_p\right]\right]\right]$$

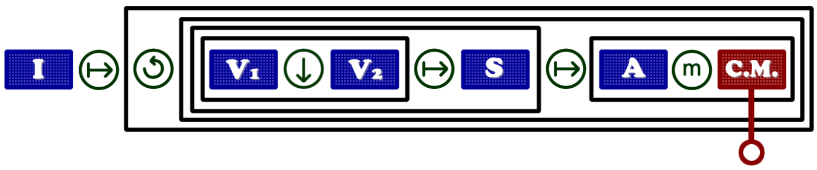


Figure 1.12: Using **race** operator

Some others operators can be useful when dealing with *sets*.

Definition 16 (Operator Union) *Let*

1. $\mathcal{M}_1 : \mathcal{D}_1 \to \mathcal{I}_1$ *and*

2. $\mathcal{M}_2 : \mathcal{D}_2 \to \mathcal{I}_2$*,*

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$*. Let also* V_1 *and* V_2 *be the outputs of* $\mathcal{M}_1$ *and* $\mathcal{M}_2$*, respectively. Then the operation* $\left|\mathcal{M}_1 \textcircled{\cup} \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_\cup$ *that executes both modules* $\mathcal{M}_1$ *and* $\mathcal{M}_2$*, and returns* $V_1 \cup V_2$*:*

$$\mathcal{M}_\cup : \mathcal{D}_1 \cap \mathcal{D}_2 \to \mathcal{I}_1 \cup \mathcal{I}_2$$

Similarly we define the operators **Intersection** and **Subtraction**:

Definition 17 (Operator Intersection) *Let*

1. $\mathcal{M}_1 : \mathcal{D}_1 \to \mathcal{I}_1$ *and*

2. $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$,

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$*. Let also* V_1 *and* V_2 *be the outputs of* $\mathcal{M}_1$ *and* $\mathcal{M}_2$*, respectively. Then the operation* $\left|\mathcal{M}_1 \bigcirc\!\!\!\!\cap\, \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_\cap$ *that executes both modules* $\mathcal{M}_1$ *and* $\mathcal{M}_2$*, and returns* $V_1 \cap V_2$*:*

$$\mathcal{M}_\cap : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

Definition 18 (Operator Subtraction) *Let*

1. $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*
2. $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$,

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$*. Let also* V_1 *and* V_2 *be the outputs of* $\mathcal{M}_1$ *and* $\mathcal{M}_2$*, respectively. Then the operation* $\left|\mathcal{M}_1 \ominus \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_-$ *that executes both modules* $\mathcal{M}_1$ *and* $\mathcal{M}_2$*, and returns* $V_1 - V_2$*:*

$$\mathcal{M}_- : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

Now, I define the operators which allows to send information to other solvers. Two types of information can be sent: i) the output of the *computation module* and send its output, or ii) the *computation module* itself. . This utility is very useful in terms of sharing behaviors between solvers.

Definition 19 (Sending Data Operator) *Let* $\mathcal{M} : \mathcal{D} \rightarrow \mathcal{I}$ *be a module. Then the operation* $|(\!|\mathcal{M}|\!)^o|$ *defines the compound module* $\mathcal{M}_{sendD}$ *that executes the module* $\mathcal{M}$ *and sends its output outside:*

$$\mathcal{M}_{sendD} : \mathcal{D} \rightarrow \mathcal{I}$$

Similarly we define the operator **Send Module**:

Definition 20 (Sending Module Operator) *Let* $\mathcal{M} : \mathcal{D} \rightarrow \mathcal{I}$ *be a module. Then the operation* $|(\!|\mathcal{M}|\!)^m|$ *defines the compound module* $\mathcal{M}_{sendM}$ *that executes the module* $\mathcal{M}$*, then returns its output and sends the module itself outside:*

$$\mathcal{M}_{sendM} : \mathcal{D} \rightarrow \mathcal{I}$$

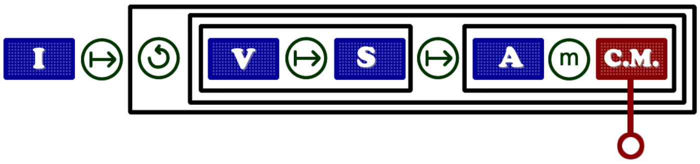
In the following example, I use one of the *compound modules* already presented in the previews examples, using a *communication module* to receive a configuration (see Figure 1.13a):

$$\left[I \,\text{\textcircled{$\mapsto$}}\, \left[\circlearrowleft \left[\left[V \,\text{\textcircled{$\mapsto$}}\, S \right] \,\text{\textcircled{$\mapsto$}}\, \llbracket A \,\text{\textcircled{m}}\, C.M. \rrbracket_p \right] \right] \right]$$

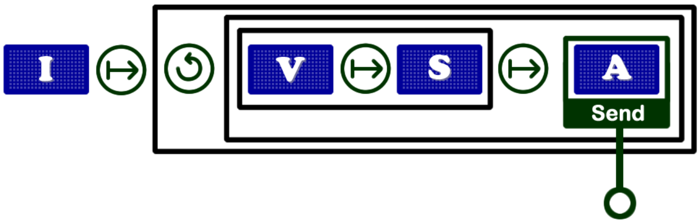
I also build another, as its complement: sending the accepted configuration to outside, using the **sending data operator** (see Figure1.13b):

$$\left[I \,\text{\textcircled{$\mapsto$}}\, \left[\circlearrowleft \left[\left[V \,\text{\textcircled{$\mapsto$}}\, S \right] \,\text{\textcircled{$\mapsto$}}\, (\!| A |\!)^o \right] \right] \right]$$

In the Section 1.6 I explain how to connect solvers to each other.



(a)



(b)

Figure 1.13: Sender and receiver behaviors

Once all desired abstract modules are linked together with operators, we obtain the *root compound module*, an important part of an *abstract solver*. To implement a concrete solver from an *abstract solver*, one must instantiate each abstract module with a concrete one respecting the required signature. From the same *abstract solver*, one can implement many different concrete solvers simply by instantiating abstract modules with different concrete modules.

An *abstract solver* is defined as follows: after declaring the **abstract solver**'s name, the first line defines the list of abstract *computation modules*, the second one the list of abstract *communication modules*, then the algorithm of the solver is defined as the solver's body (the root *compound module*), between **begin** and **end**.

An *abstract solver* can be declared through the simple regular expression:

$$\textbf{abstract solver}\ \textit{name}\ \textbf{computation}:\ L^m\ (\textbf{communication}:\ L^c)?\ \textbf{begin}\ \mathcal{M}\ \textbf{end}$$

where:

- *name* is the identifier of the *abstract solver*,
- L^m is the list of abstract *computation modules*,
- L^c is the list of abstract *communication modules*, and
- $\mathcal{M}$ is the root *compound module*.

For instance, Algorithm 1 illustrates the abstract solver corresponding to Figure 1.1b.

Algorithm 1: POSL pseudo-code for the *abstract solver* presented in Figure 1.1b

abstract solver *as_01*
computation : I, V, S, A
connection: $C.M.$
 begin
  $I \; \bigcirc\!\!\!\!\mapsto$
  $[\circlearrowleft (\text{ITR} \,\%\, K_1)$
   $\left[V \bigcirc\!\!\!\!\mapsto S \bigcirc\!\!\!\!\mapsto \left[C.M. \; (m) \; (\!|A|\!)^o\right]\right]$
  $]$
 end

1.5 Third stage: creating POSL solvers

With *computation* and *communication modules* composing an *abstract solver*, one can create solvers by instantiating *modules*. This is simply done by specifying that a given **solver** must **implements** a given *abstract solver*, followed by the list of *computation* then *communication modules*. These modules must match signatures required by the *abstract solver*.

In the following example, I describe some concrete *computation modules* that can be used to implement the *abstract solver* declared in Algorithm 1:

$\boxed{Computation\ module - 1}$ I_{rand} generates a random configuration s

$\boxed{Computation\ module - 2}$ V_{1ch} defines the neighborhood $\mathcal{V}(s)$ changing only one element

$\boxed{Computation\ module - 3}$ S_{best} selects the best configuration $s' \in \mathcal{V}(s)$ improving the current cost.

$\boxed{Computation\ module - 4}$ A_{alw} evaluates an acceptance criterion for s'. We have chosen the classical module, selecting the configuration with the lowest global cost, *i.e.*, the one which is likely the closest to a solution.

I use also the following concrete *communication module*:

$\boxed{Communication\ module - 1}$ CM_{last} returns the last configuration arrived, if at the time of its execution, there is more than one configuration waiting to be received.

These modules are used and explained in details in the Chapter ?? of this document. Algorithm 2 implements Algorithm 1 by instantiating its modules.

Algorithm 2: An instantiation of the *abstract solver* presented in Algorithm 1

solver solver_01 **implements** as_01
computation : $I_{rand}, V_{1ch}, S_{best}, A_{alw}$
connection: CM_{last}

1.6 Forth stage: connecting the solvers

We call *solver set* to the pool of (concrete) solvers that we plan to use in parallel to solve a problem. Once we have our solvers set, the last stage is to connect the solvers to each other. Up to this point, solvers are disconnected, but they are ready to establish the communication. POSL provides a platform to the user such that cooperative strategies can be easily defined.

In the following we present two important concepts necessary to formalize the *communication operators*.

Definition 21 (Communication Jack) *Let $\mathcal{S}$ be a solver. Then the operation $\mathcal{S} \cdot \mathcal{M}$ opens an outgoing connection from the solver $\mathcal{S}$, sending to the outside either a) the output of $\mathcal{M}$, if it is affected by a sending data operator as presented in Definition 19, or b) $\mathcal{M}$ itself, if it is affected by a sending module operator as presented in Definition 20.*

Definition 22 (Communication Outlet) *Let $\mathcal{S}$ be a solver. Then, the operation $\mathcal{S} \cdot \mathcal{CM}$ opens an ingoing connection to the solver $\mathcal{S}$, receiving from the outside either a) the output of some computation module, if $\mathcal{CM}$ is a data communication module, or b) a computation module, if $\mathcal{CM}$ is an object communication module.*

The communication is established by following the following rules guideline:

1. Each time a solver sends any kind of information by using a *sending* operator, it creates a *communication jack*.
2. Each time a solver defines a *communication module*, it creates a *communication outlet*.

3. Solvers can be connected to each other by linking *communication jacks* to *communication outlets*.

Following, we define the *connection operators* that POSL provides.

Definition 23 (Connection Operator One-to-One) *Let*

1. $\mathcal{J} = [\mathcal{S}_0 \cdot \mathcal{M}_0, \mathcal{S}_1 \cdot \mathcal{M}_1, \ldots, \mathcal{S}_{N-1} \cdot \mathcal{M}_{N-1}]$ *be the list of communication jacks, and*
2. $\mathcal{O} = [\mathcal{Z}_0 \cdot \mathcal{CM}_0, \mathcal{Z}_1 \cdot \mathcal{CM}_1, \ldots, \mathcal{Z}_{N-1} \cdot \mathcal{CM}_{N-1}]$ *be the list of communication outlets*

Then the operation

$$\mathcal{J} \; \textcircled{\rightarrow} \; \mathcal{O}$$

connects each communication jack $\mathcal{S}_i \cdot \mathcal{M}_i \in \mathcal{J}$ *with the corresponding communication outlet* $\mathcal{Z}_i \cdot \mathcal{CM}_i \in \mathcal{O}$, $\forall 0 \leq i \leq N-1$ *(see Figure 1.14a).*

Definition 24 (Connection Operator One-to-N) *Let*

1. $\mathcal{J} = [\mathcal{S}_0 \cdot \mathcal{M}_0, \mathcal{S}_1 \cdot \mathcal{M}_1, \ldots, \mathcal{S}_{N-1} \cdot \mathcal{M}_{N-1}]$ *be the list of communication jacks, and*
2. $\mathcal{O} = [\mathcal{Z}_0 \cdot \mathcal{CM}_0, \mathcal{Z}_1 \cdot \mathcal{CM}_1, \ldots, \mathcal{Z}_{M-1} \cdot \mathcal{CM}_{M-1}]$ *be the list of communication outlets*

Then the operation

$$\mathcal{J} \; \textcircled{\rightsquigarrow} \; \mathcal{O}$$

connects each communication jack $\mathcal{S}_i \cdot \mathcal{M}_i \in \mathcal{J}$ *with every communication outlet* $\mathcal{Z}_j \cdot \mathcal{CM}_j \in \mathcal{O}$, $\forall 0 \leq i \leq N-1$ *and* $0 \leq j \leq M-1$ *(see Figure 1.14b).*

Definition 25 (Connection Operator Ring) *Let*

1. $\mathcal{J} = [\mathcal{S}_0 \cdot \mathcal{M}_0, \mathcal{S}_1 \cdot \mathcal{M}_1, \ldots, \mathcal{S}_{N-1} \cdot \mathcal{M}_{N-1}]$ *be the list of communication jacks, and*
2. $\mathcal{O} = [\mathcal{S}_0 \cdot \mathcal{CM}_0, \mathcal{S}_1 \cdot \mathcal{CM}_1, \ldots, \mathcal{S}_{N-1} \cdot \mathcal{CM}_{N-1}]$ *be the list of communication outlets*

Then the operation

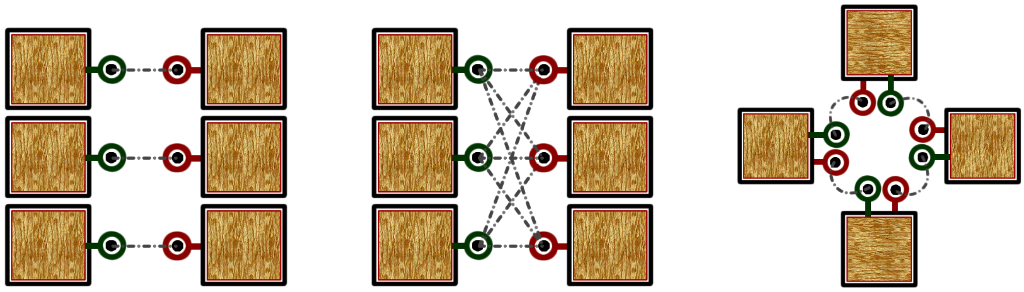
$$\mathcal{J} \; \textcircled{\leftrightarrow} \; \mathcal{O}$$

connects each communication jack $\mathcal{S}_i \cdot \mathcal{M}_i \in \mathcal{J}$ *with the corresponding communication outlet* $\mathcal{Z}_{(i+1)\%N} \cdot \mathcal{CM}_{(i+1)\%N} \in \mathcal{O}$, $\forall 0 \leq i \leq N-1$ *(see Figure 1.14c).*

POSL also allows to declare non-communicating solvers to be executed in parallel, declaring only the list of solver names:

$$[\mathcal{S}_0, \mathcal{S}_1, \ldots, \mathcal{S}_{N-1}]$$

When we apply a connection operator $\textcircled{\text{op}}$ between a *communication jacks* list $\mathcal{J}$ and a *communication outlets* list $\mathcal{O}$, internally we are assigning an *abstract computation unit*



(a) Communication 1 to 1 **(b)** Communication 1 to N **(c)** Cyclic communication

Figure 1.14: Graphic representation of communication operators

(typically a thread) to each solver that we declare in each list. This assignment receives the name of *Solver Scheduling*. Before running the *solver set*, this *abstract unit of computation* is just an integer $\tau \in [0..N]$ identifying uniquely each of the solvers. When the *solver set* is launched, the solver with the identifier τ runs into the computation unit τ. This identifier assignation remains independent of the real availability of resources of computation. It just takes into account the user declaration. This means that, if the user declares 30 solvers (15 senders and 15 receivers) and the *solver set* is launched using 20 cores, only the first 20 solvers will be executed, and in consequence, there will be 10 solvers sending information to nowhere. Users should take this into account when declaring the *solver set*.

The connection process depends on the applied connection operator. In each case the goal is to assign, to the sending operator ($(\!|.|\!)^o$ or $(\!|.|\!)^m$) inside the *abstract solver*, the identifier of the solver (or solvers, depending on the connection operator) where the information will be sent. Algorithm 3 presents the connection process.

Algorithm 3: Scheduling and connection main algorithm

input : $\mathcal{J}$ list of *communication jacks*,
$\mathcal{O}$ list of *communication outlets*

while *no available jacks or outlets* **do**
  $S_{jack} \leftarrow$ `GetNext(`$\mathcal{J}$`)`
  $R_{oulet} \leftarrow$ `GetNext(`$\mathcal{O}$`)`
  $S \leftarrow$ `GetSolverFromConnector(`S_{jack}`)`
  $R \leftarrow$ `GetSolverFromConnector(`R_{oulet}`)`
  `Schedule(`S`)`
  $R_{id} \leftarrow$ `Schedule(`R`)`
  `Connect(root(`S`),`S_{jack}, R_{id}`)`
end

In Algorithm 3:

- `GetNext(...)` returns the next available solver-jack (or solver-outlet) in the list, depending on the connection operator, e.g., for the connection operator One-to-N, each *communication jack* in $\mathcal{J}$ must be connected with each *communication outlet* in $\mathcal{O}$.

- `GetSolverFromConnector(...)` returns the solver name given a connector declaration.
- `Schedule(...)` schedules a solver and returns its identifier.
- `Root(...)` returns the *root compound module* of a solver.
- `Connect(...)` searches the *computation module* S_{jack} recursively inside the *root compound module* of S and places the identifier R_{id} into its list of destination solvers.

Let us suppose that we have declared two solvers S and Z, both implementing the *abstract solver* in Algorithm 1, so they can be either sender or receiver. The following code connects them using the operator **1 to N**:

$$[S \cdot A] \ \rightsquigarrow \ [Z \cdot C.M.]$$

If the operator **1 to N** is used with only with one solver in each list, the operation is equivalent to applying the operator **1 to 1**. However, to obtain a communication strategy like the one showed in Figure 1.14b, six solvers (three senders and three receivers) have to be declared to be able to apply the following operation:

$$[S_1 \cdot A, S_2 \cdot A, S_3 \cdot A] \ \rightsquigarrow \ [Z_1 \cdot C.M., Z_2 \cdot C.M., Z_3 \cdot C.M.]$$

POSL provides a mechanism to make this easier, through *namespace expansions*.

1.6.1 Solver namespace expansion

One of the goals of POSL is to provide a way to declare sets of solvers to be executed in parallel fast and easily. For that reason, POSL provides two forms of namespace expansion, in order to create sets of solvers using already declared ones:

Solver name expansion - Uses an integer K to denote how many times the solver name S will appear in the declaration. $[\dots S_i \cdot \mathcal{M}(K), \dots]$ expands as $\left[\dots S_i \cdot \mathcal{M}, S_i^2 \cdot \mathcal{M}, \dots S_i^K \cdot \mathcal{M} \dots\right]$
and all new solvers $S_i^j, j \in [2..K]$ are created using the same solver declaration of solver S_i.

Connection declaration expansion - Uses an integer K to denote how many times the connection will be repeated in the declaration. Let a) $[\mathcal{S}_1 \cdot \mathcal{M}_1, \dots, \mathcal{S}_N \cdot \mathcal{M}_N]$ and b) $[\mathcal{R}_1 \cdot \mathcal{CM}_1, \dots, \mathcal{R}_M \cdot \mathcal{CM}_M]$ be the list of *communication jacks* and *communication outlets*, respectively, and c) op a connection operator. Then

$$[\mathcal{S}_1 \cdot \mathcal{M}_1, \dots, \mathcal{S}_N \cdot \mathcal{M}_N] \ op \ [\mathcal{R}_1 \cdot \mathcal{CM}_1, \dots, \mathcal{R}_M \cdot \mathcal{CM}_M] \, K$$

expands as

$$
\begin{array}{c}
[\mathcal{S}_1 \cdot \mathcal{M}_1, \ldots, \mathcal{S}_N \cdot \mathcal{M}_N] \; \textcircled{\scriptsize op} \; [\mathcal{R}_1 \cdot \mathcal{CM}_1, \ldots, \mathcal{R}_N \cdot \mathcal{CM}_N] \\
\left[\mathcal{S}_1^2 \cdot \mathcal{M}_1, \ldots, \mathcal{S}_N^2 \cdot \mathcal{M}_N\right] \; \textcircled{\scriptsize op} \; \left[\mathcal{R}_1^2 \cdot \mathcal{CM}_1, \ldots, \mathcal{R}_N^2 \cdot \mathcal{CM}_N\right] \\
\ldots \\
\left[\mathcal{S}_1^K \cdot \mathcal{M}_1, \ldots, \mathcal{S}_N^K \cdot \mathcal{M}_N\right] \; \textcircled{\scriptsize op} \; \left[\mathcal{R}_1^K \cdot \mathcal{CM}_1, \ldots, \mathcal{R}_N^K \cdot \mathcal{CM}_N\right]
\end{array}
$$

and all new solvers $S_i^k, i \in [1..N]$ and $R_j^k, j \in [1..M]$, $k \in [2..K]$, are created using the same solver declaration of solvers S_i and R_j, respectively.

> Now, suppose that I have created solvers S and Z mentioned in the previews example. As a communication strategy, I want to connect them through the operator $\mathbf{1 \text{ to } N}$, using S as sender and Z as receiver. Then, using **namespace expansions**, I need to declare how many solvers I want to connect. Algorithm 4 shows the desired communication strategy. Notice in this example that the connection operation is affected also by the number 2 at the end of the line, as **connection declaration expansion**. In that sense, and supposing that 12 units of computation are available, a *solver set* working on parallel following the topology described in Figure 1.15 can be obtained.

Algorithm 4: A communication strategy

```
[ S·A (3) ] (⇝) [ Z·C.M. (3) ] 2 ;
```

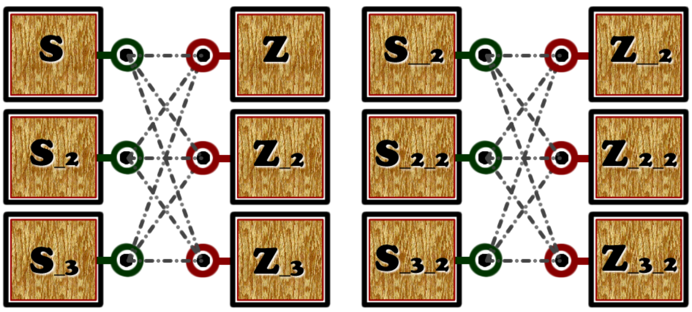


Figure 1.15: An example of connection strategy for 12 units of computation

1.7 Summarize

In this chapter POSL have been formally presented, as a Parallel–Oriented Solver Language to build meta-heuristic-based solver to solve *Constraint Satisfaction Problems*. This language provides a set of *computation modules* useful to solve a wide range of constrained problems. It is also possible to create new ones, through the low-level framework in C++ programming language. POSL also provides a set of *communication modules*, essential features to share information between solvers.

One of the most important advantages of POSL is the possibility of creating *abstract solvers* using an operator-based language, that remains independent from used *computation* and *communication modules*. That is the reason why it is possible to create many different solvers using the same solution strategy (the *abstract solver*) by instantiating it with different modules (*computation* and *communication modules*). It is also possible to create different *communication strategies* by using *connection operators* that POSL provides.

In the next chapter, a detailed study of various communicating and non-communicating strategies is presented, using some *Constraint Satisfaction Problems* as benchmarks. In this study, is showed the efficacy of POSL to analyze quickly and easily these strategies.

2

Conclusion and future works

In this chapter, the conclusions of the work is presented, emphasizing on our contribution and obtained results. Future branches to follow are also discussed.

Contents

2.1 Conclusions

The era of parallel computing has opened new and more efficient ways to solve constraint problems. This development is leading us to the multi/many–core technology and massive parallel architectures, which are nowadays more accessible for a broad public through hardware like the Xeon Phi or GPU cards. For that reason, this new architecture implies new ways for designing and implementing algorithms to exploit its full potential.

In this thesis I have presented as a main contribution a Parallel-Oriented Solver Language (POSL) focused in the solution of *Constraint Satisfaction Problems*, which are very complicated. These problems have huge search spaces, making them intractable through tree-search techniques. POSL propose a language to build meta-heuristic-based solvers, tacking into account the success of these methods solving *CSPs*. This meta-heuristics are built using the POSL's language following rigorous but well detailed steps, based on the re-usability and coupling small pieces of computation and communication (*computation modules* and *communication modules*), designed to the resolution of a broad range of *CSPs*.

Meta-heuristic methods have some times a lot of parameters to be adjusted. Prior to the POSL's design, Chapter **??** (Section **??**) contains a study in which the tool ParamILS was used to tune *Adaptive Search* to solve *Costas Array* and *All-Interval Series* problems. The main goals of that work were studying the performance of the tool, and finding a new and more efficient parameters setting that allow a faster resolution of the mentioned benchmark problems. However, the conclusion, after a comparison between obtained results using default parameters found through manually experiments, and obtained results using ParamILS, were that, for this implementation of *Adaptive Search* the tool is not able to find parameter settings improving obtained results using default parameters. This corroborates the practical intuition that, when the parameters set is not so large, the experience of the scientist is crucial and more accurate that using this kind of tools.

The most important characteristic of POSL is allowing the construction of many solvers to work in parallel using the *multi-walk* approach, which has shown very good results solving constrained problems. Into another work prior to POSL's design, I have presented a study of some techniques to improve the performance of algorithms proposed in [86] were a study of the impact of space-partitioning techniques on the performance of parallel local search algorithms is proposed to tackle the *K-Medoids Clustering Problem*. The basic idea of their specific problem is to how allocate communication metronodes in order to maximize the client covering. Their solution is based on domain partitioning techniques like *space-filling curves*, and *k-Means* algorithm, but they do not take into account the number of clients associated to each new sub-domain. For that reason, in Chapter **??** (Section **??**) are proposed a set of ideas/hypothesis to improve the performance, based on geometrical balancing of the search space. This work was not validated, because it was performed in parallel with the first ideas of POSL, which finally was the main direction of this thesis.

In Chapter 1 was dedicated to POSL, the main contribution of this thesis, a Parallel-Oriented Solver Language to build interconnected meta-heuristic-based solvers working in parallel. The language was formally presented by defining each provided operator, as well as the benchmark codification method, and the process of creation/usage of the *computation* and *communication modules*.

The most important advantage of POSL is allowing the codding, easily and fast, of many different solvers through a mechanism of module re-usability, and communication strategies through communication operators, which are also formally defined. Hence, as other contribution of this thesis, is presented in Chapter **??** a detailed study of various communication strategies to analyze the behavior and relevance of the information sharing solving constraint problems.

Solving *Social Golfers Problem*, it was successfully applied an exploitation-oriented communication strategy, in which the current configuration is communicated to focus various solvers in a more promising area. The same idea was applied to solve the *N-Queens Problem*, showing no better results than obtained without communication. However, a deep study of the POSL's behavior during the search process allows to design a communication strategy able to improve the results obtained using non-communicating strategies. It was based on crating *partial* solvers (solvers only searching into a portion of the search space) to accelerate other's solvers search, by communicating the current configuration at the beginning of the search process. The *Costas Array Problem* is a very complicated constrained problem, and very sensitive to the methods to solve it. Thanks to some studies of different communication strategies, based on the communication of the current configuration at different times (places) in the algorithm, it was possible to find a communication strategy to improve the performance, in comparison with those obtained without communication. Finally, the *Golomb Ruler Problem* was chosen to study a different and innovative communication strategy in which the communicated information is a potential local minimum to be avoided. This new communication strategy showed to be effective to solve these kind of problems.

Thanks to the operator-based language provided by POSL it was possible to test many different strategies (communicating and non-communicating). The process of building solvers implementing different solution strategies is complex and tedious, but POSL gives the possibility to make communicating and non-communicating solver prototypes and to study them with few efforts. It was possible to show that a good selection and management of inter-solvers communication can play an important role during the search process, working with constrained problems, most of them very complicated.

2.2 Future works

POSL is a tool entirely developed within the context of this thesis. It was completely new and yet under optimization. Although it has shown its first results, I believe that there is still a long way to go.

One of the first steps to do when solving *Constraint Satisfaction Problems*using POSL is precisely the problem modeling. POSL at the language level does not provide a mechanism for benchmark modeling. It currently handles this issue through the low-level framework in C++ programing language, but the creation or integration of problem definition languages is one of the next goals.

During the resolution of *Social Golfers Problem* the success of a *communication strategy* combining intensification and exploration was showed. In that direction, many other strategies can be analyzed. For instance, the study of the cost history during the search process, in order to find a lower bound for the cost, that indicates the time to communicate the current configuration to all solvers. This strategy allows to focus the search in the same area, launching a generalized intensification.

The *communication strategy* applied to the resolution of *N-Queens Problem*, where *partial solvers* transmit the configuration to *full solvers*, accelerates the search process at the beginning. However, once the a configuration is accepted by a *full solver*, *partial solvers* are no longer able to find better configurations to be sent. The first idea coming into our minds is to create *cyclic communication strategies* in which *full solvers* return their improved configurations to *partial solvers.* This way the search process can be accelerated at various moments.

One of the most costly process during the resolution of *Golomb Ruler Problem* was finding the right *parameters* for the *tabu list.* A set of values for each parameter was proposed, an after a tuning process, those showing the best behavior were chosen. Nevertheless, is clear that in this part there is a lot of room for improvement. The key of the good performance of this strategy is the right choosing of the proximity criterion between configurations. For that reason this issue deserves a deep study in the future.

POSL already has an important library of ready–to–use computation and connection modules, based on a deep study of classical meta-heuristics algorithms for solving combinatorial problems. In the near future we plan to make it grow, in order to increase possibilities of POSL. In such a way, building new algorithms by using POSL will be easier. At the same time we plan to enrich the language by proposing new operators. It is necessary, for example, to improve the solver definition language, allowing to build sets of many new solvers faster and easier. Furthermore, we are aiming to expand the communication definition language, in order to create versatile and more complex and dynamic communication strategies, to allow a communication strategy to change during runtime.

The operators described above only give the possibility to define static communication strategies. However, we aim to improve POSL with more expressive operators in terms of communication between solvers to allow dynamic modifications of communication strategies, that is, having such strategies adapting themselves during runtime. This way, many different communication strategies would be defined in the same *solver set*. Then, after some time of calculation, an evaluation would be performed in order to make all solvers able to adopt the best strategy until the end of the search process.

In most of the performed experiments, the shared information was the best found configuration. So far, there are no results showing what "a good information to communicate" is. Actually, [48] shows that in fact, the current configuration is not always a relevant information to share among solvers. That is why this subject deserves a deep study. We plan in the near future to investigate other informations to be communicated, such as search directions, search space features, among others.

3

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Part II

Appendix

4

POSL Class diagrams

In this chapter class diagrams of POSL *are presented.*

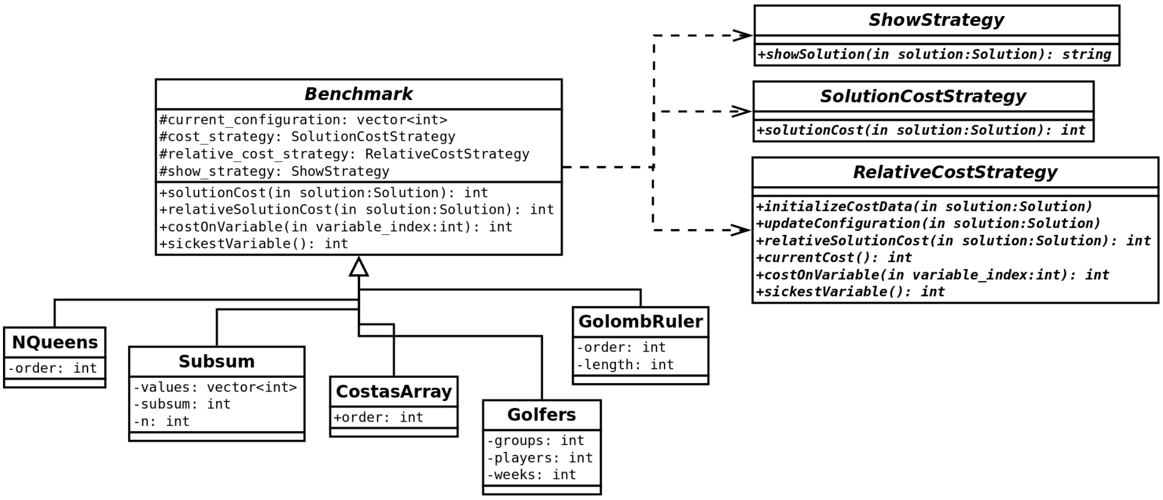

Figure 4.1: Benchmark class diagram

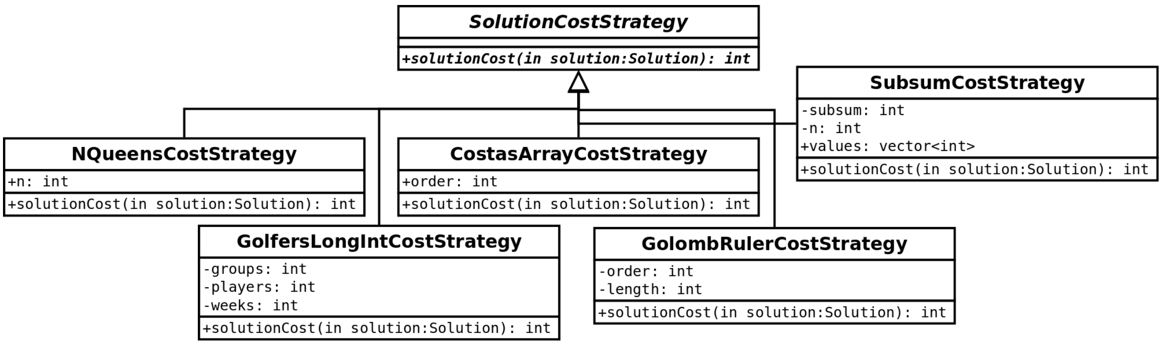

Figure 4.2: Cost strategy class diagram

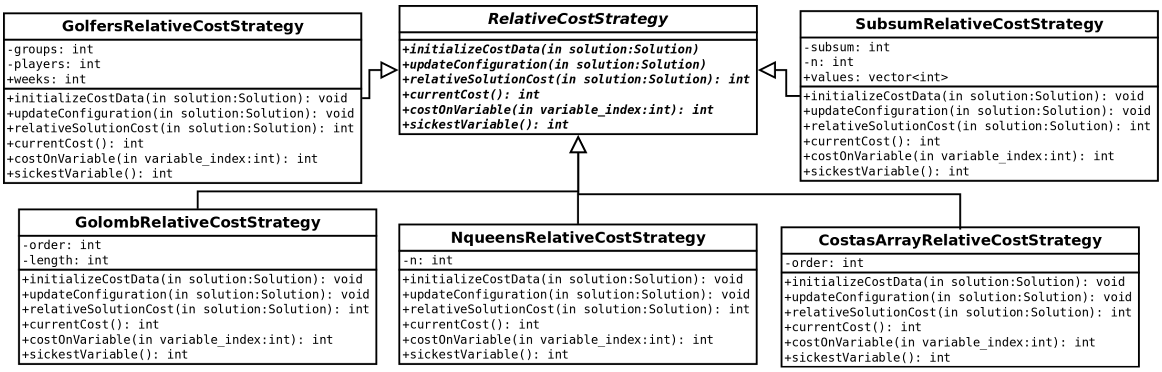

Figure 4.3: Relative cost strategy class diagram

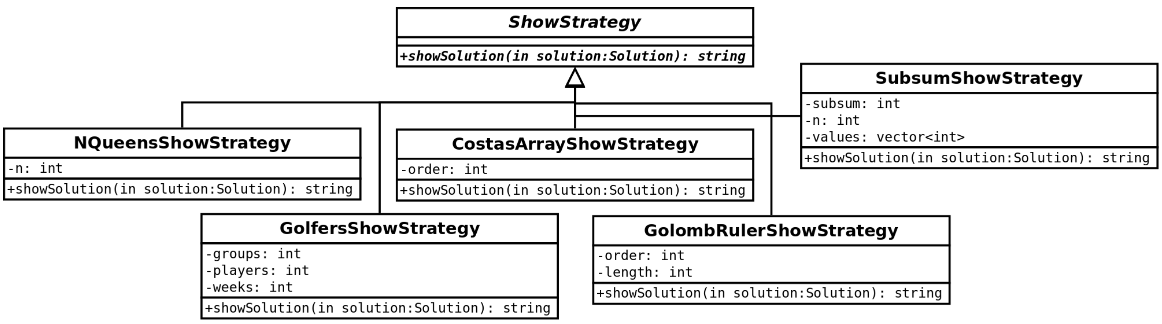

Figure 4.4: Show solution strategy class diagram

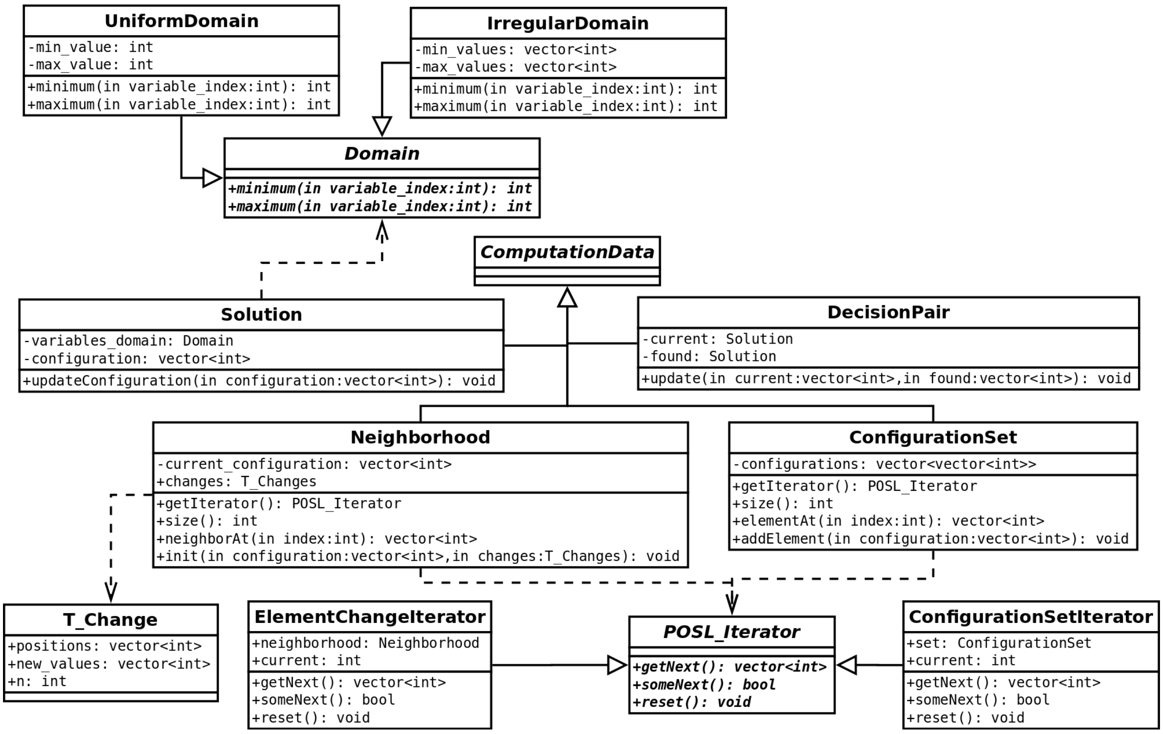

Figure 4.5: POSL data class diagram

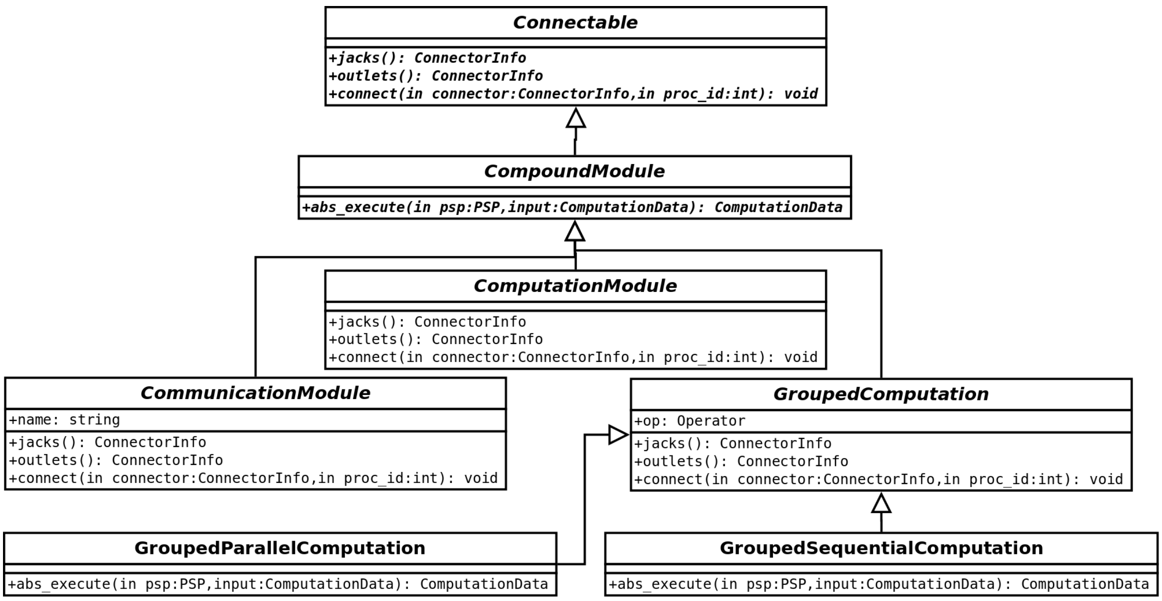

Figure 4.6: Compound modules class diagram

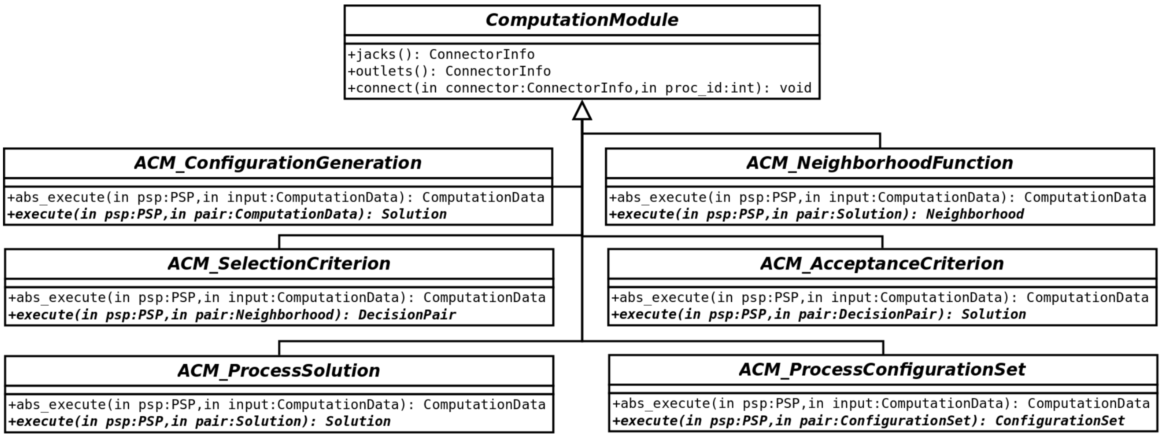

Figure 4.7: Abstract computation modules class diagram

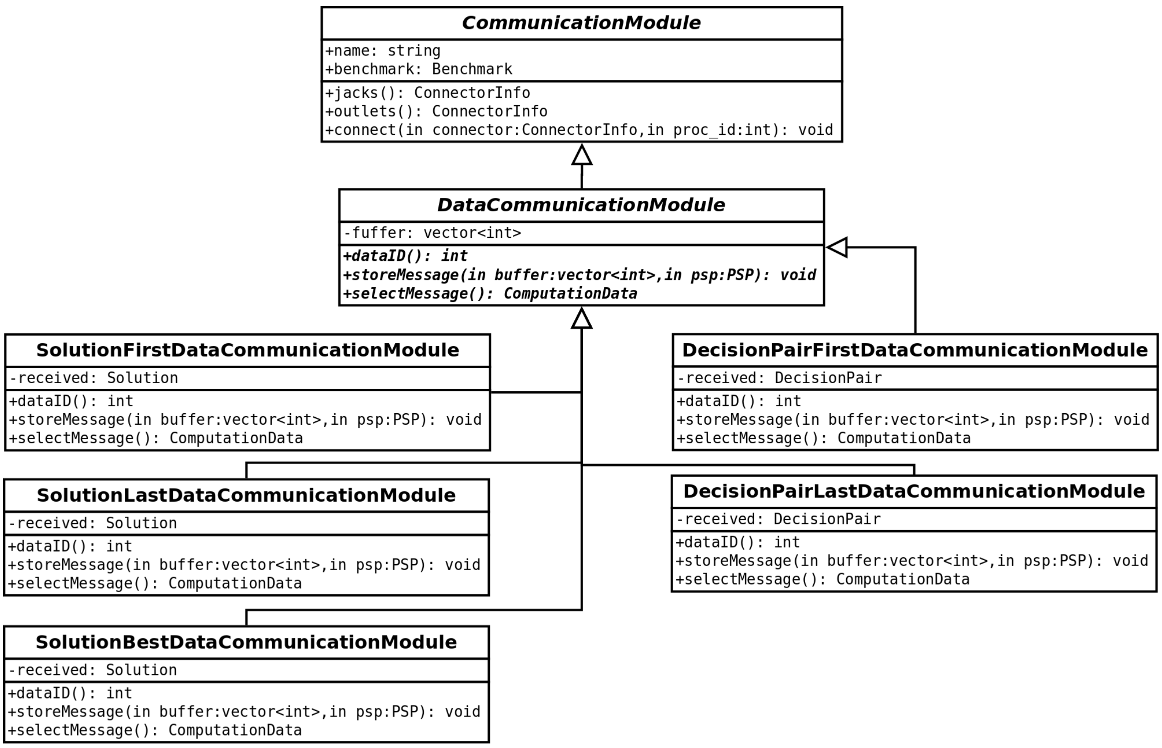

Figure 4.8: Communication modules class diagram