POSL: A Parallel-Oriented Solver Language

Thesis for the Degree of
Doctor of Computer Science

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Part I

POSL: Parallel Oriented Solver Language

1

A Parallel-Oriented Language for Modeling Meta-Heuristic-Based Solvers

In this chapter POSL *is introduced as the main contribution, and a new way to solve CSPs. Its characteristics and advantages are summarized, and a general procedure to be followed is described, in order to build parallel solvers using* POSL*, followed by a detailed description of each of the single steps.*

Contents

1.1 Modeling the target benchmark **4**

1.2 First Stage: Creating POSL**'s modules** **6**

1.2.1 Computation Module 7

1.2.2 Communication modules 8

1.3 Second Stage: Assembling POSL**'s modules** **10**

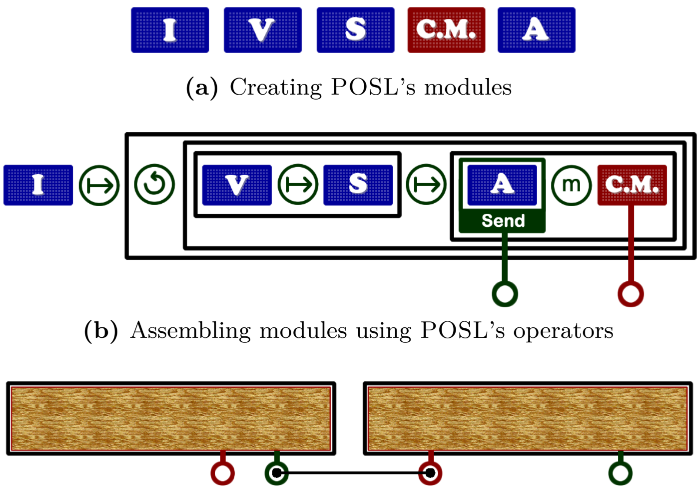
1.4 Third Stage: Creating POSL **solvers** **18**

1.5 Forth Stage: Connecting the solvers **19**

1.5.1 Solver name space expansion 22

1.6 Step-by-step POSL **code example** **23**

In this chapter we present the different steps to build communicating parallel solvers with POSL. First of all, the algorithm we have conceived to solve the target problem is decomposed it into small modules of computation, which are implemented as separated *functions*. We name them *computation modules* (see Figure 1.1a, blue shapes). At this point it is crucial to find a good decomposition of its algorithm, because it will have a significant impact in its future re-usage and variability. The next step is to decide which information is interesting to *receive* from other solvers. This information is encapsulated into another kind of component called *communication module*, allowing data transmission between solvers (see Figure 1.1a, red shapes). A third stage is to ensemble the modules through POSL's inner language (the interested reader is referred to Appendix [...]) to create independent solvers. The parallel-oriented language based on operators provided by POSL (see Figure 1.1b, green shapes) allows not only the information exchange, but also executing components in parallel. In this stage the information that is interesting to be shared with other solvers is sent using operators. After that we can connect them using *communication operators*. We call this final entity a *solvers set* (see Figure 1.1c).



(a) Creating POSL's modules

(b) Assembling modules using POSL's operators

(c) Connecting POSL solvers to create *communication strategies*

Figure 1.1: Solver construction process using POSL

In the following sections all these steps are explained in details, but first, I explain how to model the target benchmark using POSL.

1.1 Modeling the target benchmark

Target problems are modeled in POSL using the C++ programing language, respecting some rules of the object-oriented design. First of all, the benchmark must inherit from the class **Benchmark** provided by POSL. This class does not have any method to be overridden or implemented, but receives in its constructor three objects, instances from classes that the user must create. Those classes must inherit from **SolutionCostStrategy**, **RelativeCostStrategy** and **ShowStrategy**, respectively. In these classes the most important functionalities of the benchmark model are defined.

SolutionCostStrategy: In this class the strategy to compute the *cost* of a configuration is implemented. POSL is based on improving step by step an initial configuration, taking into account a *cost function* provided by the user through the model (by implementing the function *solutionCost(dots)*). The kind of problems that POSL solves is the class of *Constraint Satisfaction Problems*, so this *cost function* must return an integer taking into account the problem constraints. Given a configuration s, the *cost function*, as a mandatory rule, must return 0 if and only if s is a solution of the problem, i.e., s fulfill all the problem constraints. An example of *cost function* is one that returns the number of violated constraints. However, the more expressive the function cost is, the better the performance of POSL leading to the solution.

The method to be implemented in this class is:

- `int solutionCost(std::vector<int> & c)` → Computes the cost of a given configuration (`c`).

RelativeCostStrategy: In this class the user implements the strategy to compute the *cost* of a given configuration with respect to another. If the cost of some configuration has been calculated, sometimes it is possible to store some information in order to compute the cost of another configuration, if the differences between them are known. If it is possible, the algorithms is defined in this class. If it is not possible, this class must have the same functionality of **SolutionCostStrategy**.

The methods to implement in this class are:

- `void initializeCostData(std::vector<int> & c)` → Initializes the information related to the cost (auxiliary data structures, the current configuration (`c`), the current cost, etc.)
- `void updateConfiguration(std::vector<int> & c)` → Updates the information related to the cost.
- `int relativeSolutionCost(std::vector<int> & c)` → Returns the relative cost of the configuration `c` with respect to the current configuration.
- `int currentCost()` → Property that returns the cost of the current configuration.
- `int costOnVariable(int variable_index)` → Returns a measure of the contribution of a variable to the total cost of a configuration.

- `int sickestVariable()` → Returns the variable contributing the most to the cost.

SolutionCostStrategy: This class represents the way a benchmark shows a configuration, in order to provide more information about the structure. For example, a configuration of the instance 3–3–2 of the *Social Golfers Problem* (see bellow for more details about this benchmark) can be written as follows:

```
[1, 2, 3, 4, 5, 6, 7, 8, 9, 3, 4, 5, 6, 7, 8, 9, 1, 2]
```

This text is, nevertheless, very difficult to be read if the instance is larger. Therefore, it is recommended that the user implements this class in order to give more details and to make it easier to interpret the configuration. For example, for the same instance of the problem, a solution could be presented as follows:

```
Golfers: players-3, groups-3, weeks-2
6        8        7
1        3        5
4        9        2
--
7        2        3
4        8        1
5        6        9
--
```

The method to be implemented in this class is:

- `std::string showSolution(std::shared_ptr<Solution> s)` → Returns a string to be written in the standard output.

Once we have modeled the target benchmark, it can be solved using POSL. In the following sections we describe how to use this parallel-oriented language to solve *Constraint Satisfaction Problems*.

1.2 First stage: creating POSL's modules

There exist two types of basic modules in POSL: *computation modules* and *communication modules*. A *computation module* is a function which received an input, then executes an internal algorithm, and returns an output. A *communication module* is also a function receiving and returning information, but in contrast, the *communication module* can receive information from two different sources: through input parameters or from outside, i.e., by communicating with a module from another solver.

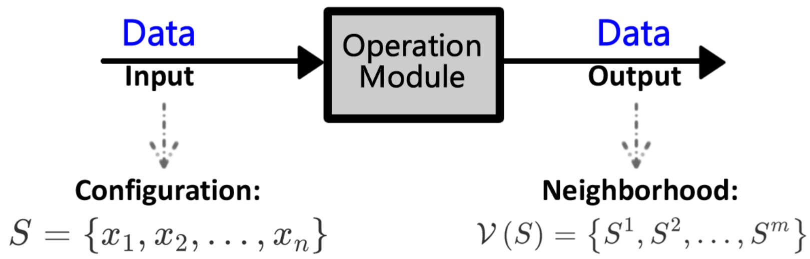


Figure 1.2: An example of a computation module computing a neighborhood

1.2.1 Computation Module

A *computation module* is the most basic and abstract way to define a piece of computation. It is a function which receives an instance of a POSL data type as input, then executes an internal algorithm, and returns an instance of a POSL data type as output. The input and output types will characterize the computation module signature. It can be dynamically replaced by (or combined with) other computation modules, since they can be shared among solvers working in parallel. They are joined through *abstract solvers*.

Definition 1 ***(Computation Module)*** *A computation module Cm is a mapping defined by:*

$$Cm : D \to I \tag{1.1}$$

where D and I can be either a set of configurations, a set of sets of configurations, a set of values of some data type, etc.

Consider a local search meta-heuristic solver. One of its *computation modules* can be the function returning the set of configurations composing the neighborhood of a given configuration:

$$Cm_{neighborhood} : D_1 \times D_2 \times \cdots \times D_n \to 2^{D_1 \times D_2 \times \cdots \times D_n}$$

where D_i represents the definition domains of each variable of the input configuration.

Figure 1.2 shows an example of *computation module*: which receives a configuration S and then computes the set $\mathcal{V}$ of its neighbor configurations $\{S^1, S^2, \dots, S^m\}$.

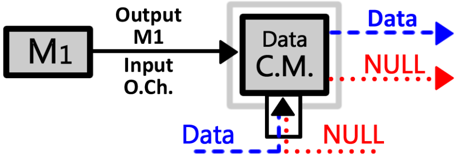
1.2.1.1 Creating new *computation modules*

To create new *computation modules* we use C++ programing language. POSL provides a hierarchy of data types to work with (See anexes) and some abstract classes to inherit from, depending on the type of *computation module* that the user wants to create. These abstract classes represent *abstract computation module* and define a type of action to be executed. In the following we present the most important ones:

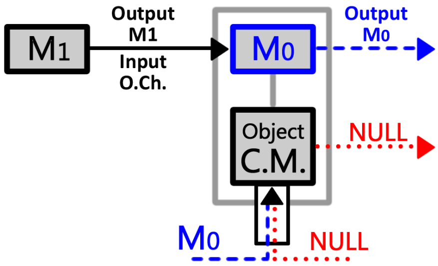
- **AOM_FirstConfigurationGeneration** → Represents *computation modules* generating a first configuration. The user must implement the method *spcf_execute(**ComputationData**)* which returns a pointer to a **Solution**, that is, an object containing all the information concerning a partial solution (configuration, variable domains, etc.)
- **AOM_NeighborhoodFunction** → Represent *computation modules* creating a neighborhood of a given configuration. The user must implement the method *spcf_execute(**Solution**)* which returns a pointer to an object **Neighborhood**, containing a set of configurations which constitute the neighborhood of a given configuration, according to certain criteria. These configuration are efficiently stored.
- **AOM_SelectionFunction** → Represents *computation modules* selecting a configuration from a neighborhood. The user must implement the method *spcf_execute(**Neighborhood**)* which returns a pointer to an object **DecisionPair**, containing two solutions: the current and the selected one.
- **AOM_DecisionFunction** → Represents *computation modules* deciding which of the two solutions will be the current configuration for the next iteration. The user must implement the method *spcf_execute(**DecisionPair**)* which returns a pointer to an object **Solution**.

1.2.2 Communication modules

A *communication module* is also a function receiving and returning information, but in contrast, the *communication module* can also receive information by communicating with a module from another solver. A *communication module* is the component managing the information reception in the communication between solvers (we will talk about information transmission in the next section). They can interact with *computation modules* through operators (see Figure 1.3).



(a) Data *communication module*



(b) Object *communication module*

Figure 1.3: Communication module

A *communication module* can receive two types of information from an external solver: data or *computation modules*. It is important to notice that by sending/receiving *computation modules*, we mean sending/receiving only required information to identify and being able to instantiate the *computation module*.

In order to distinguish from the two types of *communication modules*, we will call Data Communication Module to the *communication module* responsible for the data reception (Figure 1.3a), and Object Communication Module to the one responsible for the reception and instantiation of *computation modules* (Figure 1.3b).

Definition 2 ***(Data Communication Module)*** *A* Data Communication Module *Ch is a component that produces a mapping defined as follows:*

$$Ch : U \rightarrow I \tag{1.2}$$

It returns the information I coming from an external solver, no matter what the input U is.

Definition 3 ***(Object Communication Module)*** *If we denote by $\mathbb{M}$ the space of all the computation modules defined by Definition 1.1, then an* Object Communication Module *Ch is a component that produces a computation module coming from an external solver as follows:*

$$Ch : \mathbb{M} \rightarrow \mathbb{M} \tag{1.3}$$

Users can implement new computation and connection modules but POSL already contains many useful modules for solving a broad range of problems.

Due to the fact that *communication modules* receive information coming from outside without having control on them, it is necessary to define the *NULL* information, in order to denote the absence of information. If a Data Communication Module receives a piece of information, is returned automatically. If a Object Communication Module receives a *computation module*, it is instantiated and executed with the *communication module*'s input and its result is returned. In both cases, if no available information exists (no communications performed), the *communication module* returns the *NULL* object.

1.3 Second stage: assembling POSL's modules

Modules mentioned above are defined respecting the signature of some predefined abstract module. For example, the module showed in Figure 1.2 is a *computation module* based on an abstract module that receives a configuration and returns a neighborhood. In that sense, an example of a concrete *computation module* (or just *computation module*) can be a function receiving a configuration, and returning a neighborhood constituted by N configurations which only differ from the input configuration in one entry.

In this stage an *abstract solver* is coded using POSL. It takes abstract modules as *parameters* and combines them through operators. Through the *abstract solver*, we can also decide which information to send to other solvers by using some operators to send the result of a computation module (see below). In the following we present a formal and more detailed specification of POSL's operators.

The *abstract solver* is the solver's backbone. It joins the *computation modules* and the *communication modules* coherently. It is independent from the *computation modules* and *communication modules* used in the solver. It means that they can be changed or modified during the execution, without altering the general algorithm, but still respecting the main structure. Each time we combine some of them using POSL's operators, we are creating a *compound module*. Here we formally define the concept of *module* and *compound module*.

Definition 4 *A* **module** *is (and it is denoted by the letter $\mathcal{M}$):*

a) a computation module or

b) a communication module or

c) $[\mathcal{M}_1 \ OP \ \mathcal{M}_2]$, which is the composition of two modules $\mathcal{M}_1$ and $\mathcal{M}_2$ to be executed sequentially, returning an output depending on the nature of the operator OP*; or*

d) $[\![\mathcal{M}_1 \ OP \ \mathcal{M}_2]\!]_p$, which is the composition of two modules $\mathcal{M}_1$ and $\mathcal{M}_2$ to be executed, returning an output depending on the nature of the operator OP*. These two modules will be executed in parallel if and only if* OP *supports parallelism, (i.e. some modules will be executed sequentially although they were grouped this way); or sequentially otherwise.*

We denote the space of the modules by $\mathbb{M}$ *and call compound modules to the composition of modules described in c) and d).*

For a better understanding of Definition 4, Figure 1.4 shows graphically the structure of a *compound module.*

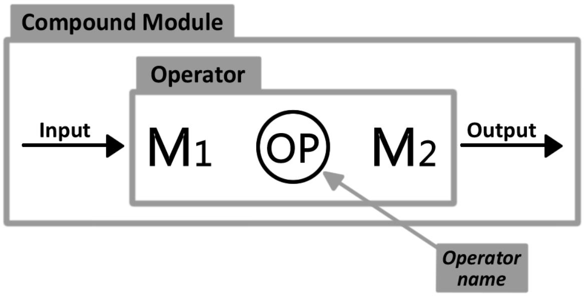


Figure 1.4: A *compound module*

As mentioned before, the *abstract solver* is independent from the *computation modules* and *communication modules* used in the solver. It means that one *abstract solver* can be used to construct many different solvers, by implementing it using different modules (see below the related concept of *abstract solver* instantiation). This is the reason why the *abstract solver* is defined only using abstract modules. Formally, we define an *abstract solver* as follows:

Definition 5 ***(Abstract Solver)*** *An* Abstract Solver *AS is a triple* $(\mathbf{M}, \mathcal{L}^m, \mathcal{L}^c)$, *where:* $\mathbf{M}$ *is a compound module (also called* root *compound module),* $\mathcal{L}^m$ *a list of abstract computation modules appearing in* $\mathcal{M}$, *and* $\mathcal{L}^c$ *a list of communication modules appearing in* $\mathcal{M}$.

The *root compound module* can be defined also as a free-context grammar as follows:

Definition 6 (root *compound module***'s grammar)** $G_{POSL} = (\mathbf{V}, \Sigma, \mathbf{S}, \mathbf{R})$, *where:*

a) $\mathbf{V} = \{CM, OP\}$ *is the set of variables,*

b) $\Sigma = \left\{\alpha, \beta, be, [,], [\![,]\!]_p, (,), \{,\}, (\!|, |\!)^m, |\!)^o, \mapsto, ?, \circlearrowleft, \rho, \vee, \wedge, M, m, \downarrow, \cup, \cap\right\}$ *is the set of terminals,*

c) $\mathbf{S} = \{CM\}$ *is the set of start variables,*

d) and $\mathbf{R} =$

$$CM \longmapsto \alpha \mid \beta \mid (\!|CM|\!)^o \mid (\!|CM|\!)^m \mid [OP] \mid [\![OP]\!]_p$$
$$OP \longmapsto CM \mapsto CM \mid CM\, ?\, CM \mid CM\, \rho\, CM \mid CM \vee CM \mid CM \wedge CM$$
$$OP \longmapsto CM\, M\, CM \mid CM\, m\, CM \mid CM \downarrow CM \mid CM \cup CM \mid CM \cap CM$$
$$OP \longmapsto CM \circlearrowleft (be) CM$$

is a set of rules

In the following I explain some of the concepts in Definition 6:

- The variables CM and OP are two very important entities in the language, as it can be seen in the grammar. We name them *compound module* and *operator*, respectively.
- The terminals α and β represent a *computation module* and a *communication module*, respectively.
- The terminal *be* is a boolean expression.
- The terminals $[\ ]$, $[\![\ ]\!]_p$ are symbols for grouping and defining the way the involved *compound modules*are executed. Depending on the nature of the operator, this can be either sequentially or in parallel:

  a) [OP]: The involved operator is executed sequentially.

  b) $[\![\text{OP}]\!]_p$: The involved operator is executed in parallel if and only if *OP* supports parallelism. Otherwise, an exception is thrown.
- The terminals (and) are symbols for grouping the boolean expression in some operators.
- The terminals $(\!|.|\!)^m$, $(\!|.|\!)^o$, are operators to send information to other solvers (explained bellow).
- The rest of terminals are POSL operators.

In the following we define POSL operators. In order to group modules, like in Definition $4(c)$) and $4(d)$), we will use $|.|$ as generic grouper. In order to help the reader to easely understand how to use the operators, I use an example of a solver that I build step by step, while presenting the definitions.

> POSL creates solvers based on local search meta-heuristics algorithms. These algorithms have a common structure: 1. They start by initializing some data structures (e.g., a *tabu list* for *Tabu Search* [34], a *temperature* for *Simulated Annealing* [32], etc.). 2. An initial configuration s is generated. 3. A new configuration s' is selected from the neighborhood $\mathcal{V}(s)$. 4. If s' is a solution for the problem P, then the process stops, and s' is returned. If not, the data structures are updated, and s' is accepted or not for the next iteration, depending on a certain criterion. An example of such data structure is the penalizing features of local optima defined by Boussaïd et al [31] in their algorithm *Guided Local Search*.
>
> *Abstract computation modules* composing *local search meta–heuristics* are:
>
> $\boxed{Abstract\ Computation\ module-1}$ I: Generating a configuration s
>
> $\boxed{Abstract\ Computation\ module-2}$ V: Defining the neighborhood $\mathcal{V}(s)$

Abstract Computation module – 3	S: Selecting $s' \in \mathcal{V}(s)$
Abstract Computation module – 4	A: Evaluating an acceptance criterion for s'

The list of modules to be used in the examples have been presented. Now I present the POSL operators.

Definition 7 (Operator Sequential Execution) *Let*

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$*,*

be modules, where $\mathcal{I}_1 \subseteq \mathcal{D}_2$*. Then the operation* $\left|\mathcal{M}_1 \ominus\!\!\!\!\mapsto \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_{seq}$ *as the result of executing* $\mathcal{M}_1$ *followed by executing* $\mathcal{M}_2$*:*

$$\mathcal{M}_{seq} : \mathcal{D}_1 \rightarrow \mathcal{I}_2$$

This is an example of an operator that does not support the execution of its involved *compound modules* in parallel, because the input of the second *compound module* is the output of the first one.

Coming back to the example, I can use defined *abstract computation modules* to create a *compound module* that perform only one iteration of a local search, using the operator **Sequential Execution.** I create a *compound module* to execute sequentially I and V (see Figure 1.5a), then I create an other *compound module* to execute sequentially the *compound module* already created and S (see Figure 1.5b), and finally this *compound module* and the *computation module* A are executed sequentially (see Figure 1.5c). The *compound module* presented in Figure 1.5c can be coded as follows:

$$\left[\left[\left[I \circledcirc V\right] \circledcirc S\right] \circledcirc A\right]$$

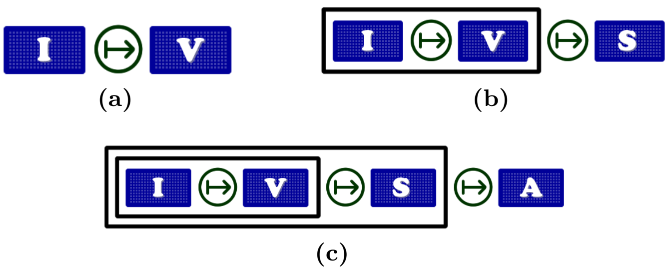


Figure 1.5: Using **sequential execution** operator

The following operator is very useful to execute modules sequentially creating bifurcations, subject to some boolean condition:

Definition 8 (Operator Conditional Execution) *Let*

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$*,*

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$*. Then the operation* $\left|\mathcal{M}_1 \textcircled{?}_{<cond>} \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_{cond}$ *as result of the sequential execution of* $\mathcal{M}_1$ *if* $< cond >$ *is* **true** *or* $\mathcal{M}_2$*, otherwise:*

$$\mathcal{M}_{cond} : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

This operator can be used in the example if I want to execute two different *selection computation modules* (S_1 and S_2) depending on certain criterion (see Figure 1.6):

$$\left[\left[\left[I \textcircled{\mapsto} V\right] \textcircled{\mapsto} \left[S_1 \textcircled{?} S_2\right]\right] \textcircled{\mapsto} A\right]$$

In examples I remove the clause $< cond >$ for simplification.

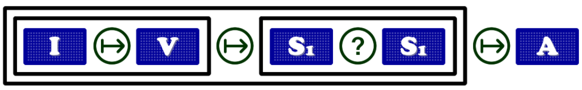


Figure 1.6: Using **conditional execution** operator

We can execute modules sequentially creating also cycles.

Definition 9 (Operator Cyclic Execution) *Let* $\mathcal{M} : \mathcal{D} \rightarrow \mathcal{I}$ *be a module, where* $\mathcal{I} \subseteq \mathcal{D}$*. Then, the operation* $|\circlearrowleft_{<cond>} \mathcal{M}|$ *defines the compound module* $\mathcal{M}_{cyc}$ *as result of the sequential execution of* $\mathcal{M}$ *repeated while* $< cond >$ *remains* **true***:*

$$\mathcal{M}_{cyc} : \mathcal{D} \rightarrow \mathcal{I}$$

Using this operator I can model a local search algorithm, by executing the *abstract computation module* I and then the other *computation modules* (V, S and A) cyclically, until finding a solution (i.e, a configuration with cost equal to zero) (see Figure 1.7):

$$\left[I \textcircled{\mapsto} \left[\circlearrowleft \left[\left[V \textcircled{\mapsto} S\right] \textcircled{\mapsto} A\right]\right]\right]$$

In the examples, I remove the clause $< cond >$ for simplification.

Definition 10 (Operator Random Choice) *Let*

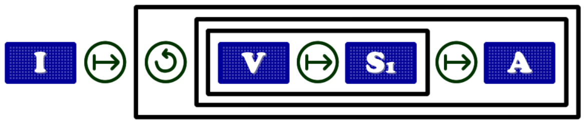


Figure 1.7: Using **cyclic execution** operator

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$*,*

be modules, where $\mathcal{D}_1 \subset \mathcal{D}_2$ *and a real value* ρ*. Then the operation* $\left| M_1 \textcircled{\rho} \mathcal{M}_2 \right|$ *defines the compound module* $\mathcal{M}_{rho}$ *that executes and returns the output of* $\mathcal{M}_1$ *with probability* ρ*, or executes and returns the output of* $\mathcal{M}_2$ *with probability* $(1 - \rho)$*:*

$$\mathcal{M}_{rho} : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

> In the example I can create a *compound module* to execute two *abstract computation modules* A_1 and A_2 following certain probability ρ using the operator **random execution** as follows (see Figure 1.8):
>
> $$\left[I \textcircled{\mapsto} \left[\circlearrowleft \left[\left[V \textcircled{\mapsto} S \right] \textcircled{\mapsto} \left[A_1 \textcircled{\rho} A_2 \right] \right] \right] \right]$$

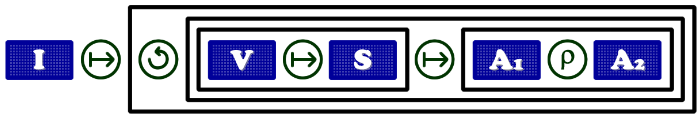


Figure 1.8: Using **random execution** operator

The following operator is very useful if the user needs to use a *communication module* inside an *abstract solver*. As explained before, if a *communication module* does not receive any information from another solver, it returns *NULL*. This may cause the undesired termination of the solver if this case is not considered correctly. Next, I introduce the operator **Operator Not** *NULL* **Execution** and illustrate how to use it in practice with an example.

Definition 11 (Operator Not *NULL* **Execution)** *Let*

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$*,*

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$*. Then, the operation* $\left| \mathcal{M}_1 \textcircled{\vee} \mathcal{M}_2 \right|$ *defines the compound module* $\mathcal{M}_{non}$ *that executes* $\mathcal{M}_1$ *and returns its output if it is not NULL, or executes* $\mathcal{M}_2$ *and returns its output otherwise:*

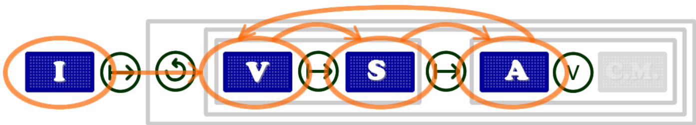
$$\mathcal{M}_{non} : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

Let us make consider a slightly more complex example: When applying the acceptance criterion, suppose that we want to receive a configuration from other solver to combine the *computation module A* with a *communication module*:

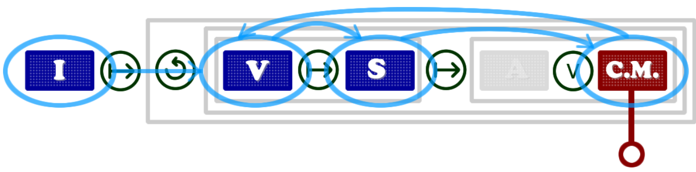
$\boxed{Communication\ module - 1:}$ $C.M.$: Receiving a configuration.

Figure 1.9 shows how to combine a *communication module* with the *computation module A* through the operator $\bigcirc\!\!\!\!\vee$. Here, the *computation module A* will be executed as long as the *communication module* remains *NULL*, i.e., there is no information coming from outside. This behavior is represented in Figure 1.9a by the orange lines. If some data has been received through the *communication module*, the later is executed instead of the module A, represented in Figure 1.9b by blue lines. The code can be written as follows:

$$\left[I \bigcirc\!\!\!\!\mapsto \left[\circlearrowleft \left[\left[V \bigcirc\!\!\!\!\mapsto S \right] \bigcirc\!\!\!\!\mapsto \left[A \bigcirc\!\!\!\!\vee C.M. \right] \right] \right] \right]$$



(a) The solver executes the computation module **A** if no information is received through the connection module



(b) The solver uses the information coming from an external solver

Figure 1.9: Two different behaviors within the same solver

This is *short-circuit* operator. It means that if the first argument (module) does not return *NULL*, the second will not be executed. POSL provides another operator with the same functionality but not *short-circuit*:

Definition 12 (Operator *BOTH* Execution) *Let*

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$,

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$. *Then the operation* $\left| \mathcal{M}_1 \bigcirc\!\!\!\!\wedge \mathcal{M}_2 \right|$ *defines the compound module* $\mathcal{M}_{both}$ *that executes both* $\mathcal{M}_1$ *and* $\mathcal{M}_2$, *then returns the output of* $\mathcal{M}_1$ *if it is not NULL, or the output of* $\mathcal{M}_2$ *otherwise:*

$$\mathcal{M}_{both} : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

In the following definitions, the concepts of *cooperative parallelism* and *competitive parallelism* are implicitly included. We say that cooperative parallelism exists when two or more processes are running separately, they are independent, and the general result will be some combination of the results of all the involved processes (e.g. Definitions 13 and 14). On the other hand, competitive parallelism arise when the general result is the result of the process ending first (e.g. Definition 15).

Definition 13 (Operator Minimum) *Let*

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$,

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$. *Let also* o_1 *and* o_2 *be the outputs of* $\mathcal{M}_1$ *and* $\mathcal{M}_2$, *respectively. Assume that there exists some order criteria between them. Then the operation* $\left|\mathcal{M}_1 \, (m) \, \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_{min}$ *that executes* $\mathcal{M}_1$ *and returns* $\min\{o_1, o_2\}$:

$$\mathcal{M}_{min} : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

Similarly we define the operator **Maximum**:

Definition 14 (Operator Maximum) *Let*

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$,

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$. *Let also* o_1 *and* o_2 *be the outputs of* $\mathcal{M}_1$ *and* $\mathcal{M}_2$, *respectively. Assume that there exists some order criteria between them. Then the operation* $\left|\mathcal{M}_1 \, (M) \, \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_{max}$ *that executes* $\mathcal{M}_1$ *and returns* $\max\{o_1, o_2\}$:

$$\mathcal{M}_{max} : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

Comming back to the previews example, the **minimum operator** can be applied to obtain a more interesting behavior in the solver: When applying the acceptance criteria, suppose that we want to receive a configuration from other solver, to compare it with ours and select the one with the lowest cost. We can do that by applying the operator (m) to combine the *computation module* A with a *communication module* $C.M.$ (see Figure1.10):

$$\left[I \, (\mapsto) \left[\circlearrowleft \left[\left[V \, (\mapsto) \, S \right] (\mapsto) \left[\!\left[A \, (m) \, C.M. \right]\!\right]_p \right]\right]\right]$$

> Notice that in this example, I can use the grouper $[\![.]\!]_p$ since the **minimum operator** supports parallelism.

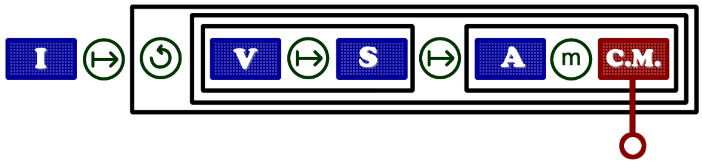


Figure 1.10: Using **minimum** operator

Definition 15 (Operator Race) *Let*

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$,

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$ *and* $\mathcal{I}_1 \subset \mathcal{I}_2$*. Then the operation* $\left|\mathcal{M}_1 \textcircled{\downarrow} \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_{race}$ *that executes both modules* $\mathcal{M}_1$ *and* $\mathcal{M}_2$*, and returns the output of the module ending first:*

$$\mathcal{M}_{race} : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

> Sometimes nighborhood functions are slow depending on the configuration. In that case two neighborhood *computation modules* can be executed and take into account the output of the module ending first (see Figure1.11):
>
> $$\left[I \mapsto \left[\circlearrowleft \left[\left[\left[\!\left[V_1 \downarrow V_2\right]\!\right]_p \mapsto S\right] \mapsto \left[\!\left[A \; m \; C.M.\right]\!\right]_p\right]\right]\right]$$

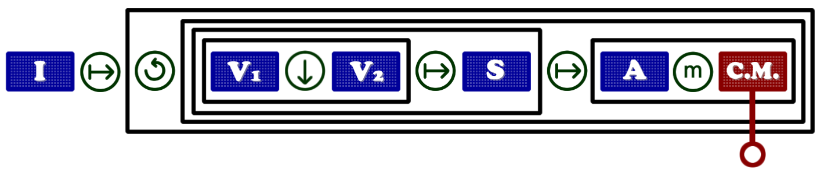


Figure 1.11: Using **race** operator

Some others operators can be useful when dealing with *sets*.

Definition 16 (Operator Union) *Let*

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$, *be modules, where* $\mathcal{D}_1 \subseteq \mathcal{D}_2$. *Let also* V_1 *and* V_2 *be the outputs of* $\mathcal{M}_1$ *and* $\mathcal{M}_2$*, respectively. Then the operation* $\left|\mathcal{M}_1 \textcircled{\cup} \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_\cup$ *that executes both modules* $\mathcal{M}_1$ *and* $\mathcal{M}_2$*, and returns* $V_1 \cup V_2$*:*

$$\mathcal{M}_\cup : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

Similarly we define the operators **Intersection** and **Subtraction**:

Definition 17 (Operator Intersection) *Let*

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$*,*

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$. *Let also* V_1 *and* V_2 *be the outputs of* $\mathcal{M}_1$ *and* $\mathcal{M}_2$*, respectively. Then the operation* $\left|\mathcal{M}_1 \textcircled{\cap} \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_\cap$ *that executes both modules* $\mathcal{M}_1$ *and* $\mathcal{M}_2$*, and returns* $V_1 \cap V_2$*:*

$$\mathcal{M}_\cap : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

Definition 18 (Operator Subtraction) *Let*

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$*,*

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$. *Let also* V_1 *and* V_2 *be the outputs of* $\mathcal{M}_1$ *and* $\mathcal{M}_2$*, respectively. Then the operation* $\left|\mathcal{M}_1 \textcircled{-} \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_-$ *that executes both modules* $\mathcal{M}_1$ *and* $\mathcal{M}_2$*, and returns* $V_1 - V_2$*:*

$$\mathcal{M}_- : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

Now, I define the operators which allows to send information to other solvers. Two types of information can be sent: i) the output of the *computation module* and send its output, or ii) the *computation module* itself. . This utility is very useful in terms of sharing behaviors between solvers.

Definition 19 (Sending Data Operator) *Let* $\mathcal{M} : \mathcal{D} \rightarrow \mathcal{I}$ *be a module. Then the operation* $|(\!|\mathcal{M}|\!)^o|$ *defines the compound module* $\mathcal{M}_{sendD}$ *that executes the module* $\mathcal{M}$ *and sends its output outside:*

$$\mathcal{M}_{sendD} : \mathcal{D} \rightarrow \mathcal{I}$$

Similarly we define the operator **Send Module**:

Definition 20 (Sending Module Operator) *Let $\mathcal{M} : \mathcal{D} \rightarrow \mathcal{I}$ be a module. Then the operation $|(\!|\mathcal{M}|\!)^m|$ defines the compound module $\mathcal{M}_{sendM}$ that executes the module $\mathcal{M}$, then returns its output and sends the module itself outside:*

$$\mathcal{M}_{sendM} : \mathcal{D} \rightarrow \mathcal{I}$$

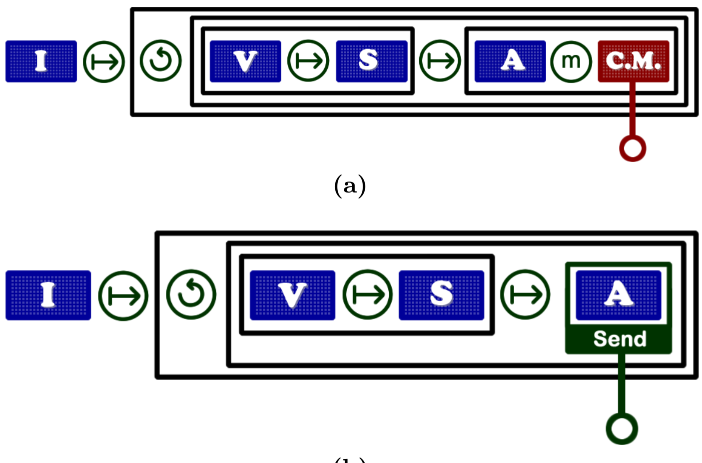
In the following example, I use one of the *compound modules* already presented in the previews examples, using a *communication module* to receive a configuration (see Figure 1.12a):

$$\left[I \textcircled{$\mapsto$} \left[\circlearrowleft \left[\left[V \textcircled{$\mapsto$} S \right] \textcircled{$\mapsto$} [\![A \textcircled{m} C.M.]\!]_p \right] \right] \right]$$

I also build another, as its complement: sending the accepted configuration to outside, using the **sending data operator** (see Figure1.12b):

$$\left[I \textcircled{$\mapsto$} \left[\circlearrowleft \left[\left[V \textcircled{$\mapsto$} S \right] \textcircled{$\mapsto$} (\!|A|\!)^o \right] \right] \right]$$

In the Section 1.5 I explain how to connect solvers to each other.



(a)

(b)

Figure 1.12: Sender and receiver behaviors

Once all desired abstract modules are linked together with operators, we obtain the *root compound module*, an important part of an *abstract solver*. To implement a concrete solver from an *abstract solver*, one must instantiate each abstract module with a concrete one respecting the required signature. From the same *abstract solver*, one can implement many different concrete solvers simply by instantiating abstract modules with different concrete modules.

An *abstract solver* is defined as follows: after declaring the **abstract solver**'s name, the first line defines the list of abstract *computation modules*, the second one the list of abstract *communication modules*, then the algorithm of the solver is defined as the solver's body (the root *compound module*), between **begin** and **end**.

An *abstract solver* can be declared through the simple regular expression:

$$\textbf{abstract solver}\ \textit{name}\ \textbf{computation}\textbf{:}\ L^m\ (\textbf{communication}\textbf{:}\ L^c)?\ \textbf{begin}\ \mathcal{M}\ \textbf{end}$$

where:

- *name* is the identifier of the *abstract solver*,
- L^m is the list of abstract *computation modules*,
- L^c is the list of abstract *communication modules*, and
- $\mathcal{M}$ is the root *compound module*.

For instance, Algorithm 1 illustrates the abstract solver corresponding to Figure 1.1b.

Algorithm 1: POSL pseudo-code for the *abstract solver* presented in Figure 1.1b

abstract solver *as_01*
computation *:* I, V, S, A
connection*:* $C.M.$
  begin
    $I \ (\mapsto)$
    $[\circlearrowleft (\text{I}{\scriptstyle\text{TR}}\ \%\ K_1)$
      $\left[V \ (\mapsto)\ S\ (\mapsto)\ \left[C.M.\ (m)\ (\!|A|\!)^o\right]\right]$
    $]$
  end

1.4 Third stage: creating POSL solvers

With *computation* and *communication modules* composing an *abstract solver*, one can create solvers by instantiating *modules*. This is simply done by specifying that a given **solver** must **implements** a given *abstract solver*, followed by the list of *computation* then *communication modules*. These modules must match signatures required by the *abstract solver*.

> In the following example, I describe some concrete *computation modules* that can be used to implement the *abstract solver* declared in Algorithm 1:

$\boxed{Computation\ module - 1}$ I_{rand} generates a random configuration s

$\boxed{Computation\ module - 2}$ V_{1ch} defines the neighborhood $\mathcal{V}(s)$ changing only one element

$\boxed{Computation\ module - 3}$ S_{best} selects the best configuration $s' \in \mathcal{V}(s)$ improving the current cost.

$\boxed{Computation\ module - 4}$ A_{alw} evaluates an acceptance criterion for s'. We have chosen the classical module, selecting the configuration with the lowest global cost, *i.e.*, the one which is likely the closest to a solution.

I use also the following concrete *communication module*:

$\boxed{Communication\ module - 1}$ CM_{last} returns the last configuration arrived, if at the time of its execution, there is more than one configuration waiting to be received.

These modules are used and explained in details in the Chapter 2 of this document. Algorithm 2 implements Algorithm 1 by instantiating its modules.

Algorithm 2: An instantiation of the *abstract solver* presented in Algorithm 1

solver solver_01 **implements** as_01
computation : $I_{rand}, V_{1ch}, S_{best}, A_{alw}$
connection: CM_{last}

1.5 Forth stage: connecting the solvers

We call *solver set* to the pool of (concrete) solvers that we plan to use in parallel to solve a problem. Once we have our solvers set, the last stage is to connect the solvers to each other. Up to this point, solvers are disconnected, but they are ready to establish the communication. POSL provides a platform to the user such that cooperative strategies can be easily defined.

In the following we present two important concepts necessary to formalize the *communication operators*.

Definition 21 (Communication Jack) *Let $\mathcal{S}$ be a solver. Then the operation $\mathcal{S} \cdot \mathcal{M}$ opens an outgoing connection from the solver $\mathcal{S}$, sending to the outside either a) the output of $\mathcal{M}$, if it is affected by a sending data operator as presented in Definition 19, or b) $\mathcal{M}$ itself, if it is affected by a sending module operator as presented in Definition 20.*

Definition 22 (Communication Outlet) *Let $\mathcal{S}$ be a solver. Then, the operation $\mathcal{S} \cdot \mathcal{CM}$ opens an ingoing connection to the solver $\mathcal{S}$, receiving from the outside either a) the output of some computation module, if $\mathcal{CM}$ is a data communication module, or b) a computation module, if $\mathcal{CM}$ is an object communication module.*

The communication is established by following the following rules guideline:

a) Each time a solver sends any kind of information by using a *sending* operator, it creates a *communication jack*.

b) Each time a solver defines a *communication module*, it creates a *communication outlet*.

c) Solvers can be connected to each other by linking *communication jacks* to *communication outlets*.

Following, we define the *connection operators* that POSL provides.

Definition 23 (Connection Operator One-to-One) *Let*

a) $\mathcal{J} = [\mathcal{S}_0 \cdot \mathcal{M}_0, \mathcal{S}_1 \cdot \mathcal{M}_1, \dots, \mathcal{S}_{N-1} \cdot \mathcal{M}_{N-1}]$ *be the list of communication jacks, and*

b) $\mathcal{O} = [\mathcal{Z}_0 \cdot \mathcal{CM}_0, \mathcal{Z}_1 \cdot \mathcal{CM}_1, \dots, \mathcal{Z}_{N-1} \cdot \mathcal{CM}_{N-1}]$ *be the list of communication outlets*

Then the operation

$$\mathcal{J} \ \textcircled{\rightarrow} \ \mathcal{O}$$

connects each communication jack $\mathcal{S}_i \cdot \mathcal{M}_i \in \mathcal{J}$ with the corresponding communication outlet $\mathcal{Z}_i \cdot \mathcal{CM}_i \in \mathcal{O}$, $\forall 0 \leq i \leq N-1$ (see Figure 1.13a).

Definition 24 (Connection Operator One-to-N) *Let*

a) $\mathcal{J} = [\mathcal{S}_0 \cdot \mathcal{M}_0, \mathcal{S}_1 \cdot \mathcal{M}_1, \dots, \mathcal{S}_{N-1} \cdot \mathcal{M}_{N-1}]$ *be the list of communication jacks, and*

b) $\mathcal{O} = [\mathcal{Z}_0 \cdot \mathcal{CM}_0, \mathcal{Z}_1 \cdot \mathcal{CM}_1, \dots, \mathcal{Z}_{M-1} \cdot \mathcal{CM}_{M-1}]$ *be the list of communication outlets*

Then the operation

$$\mathcal{J} \ \textcircled{\rightsquigarrow} \ \mathcal{O}$$

connects each communication jack $\mathcal{S}_i \cdot \mathcal{M}_i \in \mathcal{J}$ with every communication outlet $\mathcal{Z}_j \cdot \mathcal{CM}_j \in \mathcal{O}$, $\forall 0 \leq i \leq N-1$ and $0 \leq j \leq M-1$ (see Figure 1.13b).

Definition 25 (Connection Operator Ring) *Let*

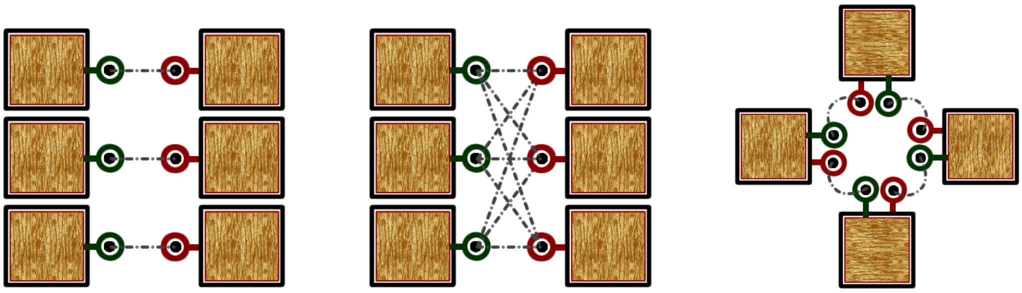
a) $\mathcal{J} = [\mathcal{S}_0 \cdot \mathcal{M}_0, \mathcal{S}_1 \cdot \mathcal{M}_1, \dots, \mathcal{S}_{N-1} \cdot \mathcal{M}_{N-1}]$ *be the list of communication jacks, and*

b) $\mathcal{O} = [\mathcal{S}_0 \cdot \mathcal{CM}_0, \mathcal{S}_1 \cdot \mathcal{CM}_1, \dots, \mathcal{S}_{N-1} \cdot \mathcal{CM}_{N-1}]$ *be the list of communication outlets*

Then the operation

$$\mathcal{J} \; \textcircled{\leftrightarrow} \; \mathcal{O}$$

connects each communication jack $\mathcal{S}_i \cdot \mathcal{M}_i \in \mathcal{J}$ *with the corresponding communication outlet* $\mathcal{Z}_{(i+1)\%N} \cdot \mathcal{CM}_{(i+1)\%N} \in \mathcal{O}$, $\forall 0 \leq i \leq N-1$ *(see Figure 1.13c).*



(a) Communication 1 to 1 **(b)** Communication 1 to N **(c)** Cyclic communication

Figure 1.13: Graphic representation of communication operators

POSL also allows to declare non-communicating solvers to be executed in parallel, declaring only the list of solver names:

$$[\mathcal{S}_0, \mathcal{S}_1, \ldots, \mathcal{S}_{N-1}]$$

When we apply a connection operator $\textcircled{\text{op}}$ between a *communication jacks* list $\mathcal{J}$ and a *communication outlets* list $\mathcal{O}$, internally we are assigning an *abstract computation unit* (typically a thread) to each solver that we declare in each list. This assignment receives the name of *Solver Scheduling*. Before running the *solver set*, this *abstract unit of computation* is just an integer $\tau \in [0..N]$ identifying uniquely each of the solvers. When the *solver set* is launched, the solver with the identifier τ runs into the computation unit τ. This identifier assignation remains independent of the real availability of resources of computation. It just takes into account the user declaration. This means that, if the user declares 30 solvers (15 senders and 15 receivers) and the *solver set* is launched using 20 cores, only the first 20 solvers will be executed, and in consequence, there will be 10 solvers sending information to nowhere. Users should take this into account when declaring the *solver set*.

The connection process depends on the applied connection operator. In each case the goal is to assign, to the sending operator ($(\!|.|\!)^o$ or $(\!|.|\!)^m$) inside the *abstract solver*, the identifier of the solver (or solvers, depending on the connection operator) where the information will be sent. Algorithm 3 presents the connection process.

Algorithm 3: Scheduling and connection main algorithm

input : $\mathcal{J}$ list of *communication jacks*,
$\mathcal{O}$ list of *communication outlets*

while *no available jacks or outlets* **do**
    $S_{jack} \leftarrow$ `GetNext(`$\mathcal{J}$`)`
    $R_{oulet} \leftarrow$ `GetNext(`$\mathcal{O}$`)`
    $S \leftarrow$ `GetSolverFromConnector(`S_{jack}`)`
    $R \leftarrow$ `GetSolverFromConnector(`R_{oulet}`)`
    `Schedule(`S`)`
    $R_{id} \leftarrow$ `Schedule(`R`)`
    `Connect(root(`S`),`S_{jack}, R_{id}`)`
end

In Algorithm 3:

- `GetNext(...)` returns the next available solver-jack (or solver-outlet) in the list, depending on the connection operator, e.g., for the connection operator One-to-N, each *communication jack* in $\mathcal{J}$ must be connected with each *communication outlet* in $\mathcal{O}$.
- `GetSolverFromConnector(...)` returns the solver name given a connector declaration.
- `Schedule(...)` schedules a solver and returns its identifier.
- `Root(...)` returns the *root compound module* of a solver.
- `Connect(...)` searches the *computation module* S_{jack} recursively inside the *root compound module* of S and places the identifier R_{id} into its list of destination solvers.

Let us suppose that we have declared two solvers S and Z, both implementing the *abstract solver* in Algorithm 1, so they can be either sender or receiver. The following code connects them using the operator **1 to N**:

$$[S \cdot A] \rightsquigarrow [Z \cdot C.M.]$$

If the operator **1 to N** is used with only with one solver in each list, the operation is equivalent to applying the operator **1 to 1**. However, to obtain a communication strategy like the one showed in Figure 1.13b, six solvers (three senders and three receivers) have to be declared to be able to apply the following operation:

$$[S_1 \cdot A, S_2 \cdot A, S_3 \cdot A] \rightsquigarrow [Z_1 \cdot C.M., Z_2 \cdot C.M., Z_3 \cdot C.M.]$$

POSL provides a mechanism to make this easier, through *namespace expansions*.

1.5.1 Solver namespace expansion

One of the goals of POSL is to provide a way to declare sets of solvers to be executed in parallel fast and easily. For that reason, POSL provides two forms of namespace expansion, in order to create sets of solvers using already declared ones:

Solver name expansion - Uses an integer K to denote how many times the solver name S will appear in the declaration. $[\dots S_i \cdot \mathcal{M}(K), \dots]$ expands as $\left[\dots S_i \cdot \mathcal{M}, S_i^2 \cdot \mathcal{M}, \dots S_i^K \cdot \mathcal{M} \dots\right]$
and all new solvers $S_i^j, j \in [2..K]$ are created using the same solver declaration of solver S_i.

Connection declaration expansion - Uses an integer K to denote how many times the connection will be repeated in the declaration. Let a) $[\mathcal{S}_1 \cdot \mathcal{M}_1, \dots, \mathcal{S}_N \cdot \mathcal{M}_N]$ and b) $[\mathcal{R}_1 \cdot \mathcal{CM}_1, \dots, \mathcal{R}_M \cdot \mathcal{CM}_M]$ be the list of *communication jacks* and *communication outlets*, respectively, and c) op a connection operator. Then

$$[\mathcal{S}_1 \cdot \mathcal{M}_1, \dots, \mathcal{S}_N \cdot \mathcal{M}_N] \; op \; [\mathcal{R}_1 \cdot \mathcal{CM}_1, \dots, \mathcal{R}_M \cdot \mathcal{CM}_M] \, K$$

expands as

$$\begin{gathered}
[\mathcal{S}_1 \cdot \mathcal{M}_1, \dots, \mathcal{S}_N \cdot \mathcal{M}_N] \; op \; [\mathcal{R}_1 \cdot \mathcal{CM}_1, \dots, \mathcal{R}_N \cdot \mathcal{CM}_N] \\
\left[\mathcal{S}_1^2 \cdot \mathcal{M}_1, \dots, \mathcal{S}_N^2 \cdot \mathcal{M}_N\right] \; op \; \left[\mathcal{R}_1^2 \cdot \mathcal{CM}_1, \dots, \mathcal{R}_N^2 \cdot \mathcal{CM}_N\right] \\
\dots \\
\left[\mathcal{S}_1^K \cdot \mathcal{M}_1, \dots, \mathcal{S}_N^K \cdot \mathcal{M}_N\right] \; op \; \left[\mathcal{R}_1^K \cdot \mathcal{CM}_1, \dots, \mathcal{R}_N^K \cdot \mathcal{CM}_N\right]
\end{gathered}$$

and all new solvers $S_i^k, i \in [1..N]$ and $R_j^k, j \in [1..M]$, $k \in [2..K]$, are created using the same solver declaration of solvers S_i and R_j, respectively.

> Now, suppose that I have created solvers S and Z mentioned in the previews example. As a communication strategy, I want to connect them through the operator **1 to N**, using S as sender and Z as receiver. Then, using **namespace expansions**, I need to declare how many solvers I want to connect. Algorithm 4 shows the desired communication strategy. Notice in this example that the connection operation is affected also by the number 2 at the end of the line, as **connection declaration expansion**. In that sense, and supposing that 12 units of computation are available, a *solver set* working on parallel following the topology described in Figure 1.14 can be obtained.

Algorithm 4: A communication strategy

[S $\cdot$ A (3)] $\rightsquigarrow$ [Z $\cdot$ $C.M.$ (3)] 2 ;

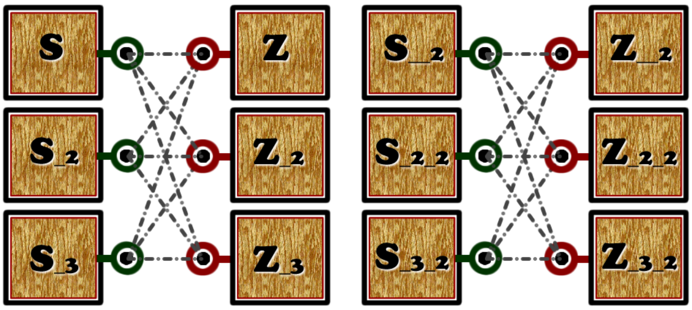


Figure 1.14: An example of connection strategy for 12 units of computation

1.6 Summarize

In this Chapter POSL have been formally presented, as a Parallel–Oriented Solver Language to build meta-heuristic-based solver to solve *Constraint Satisfaction Problems*. This language provides a set of *computation modules* useful to solve a wide range of problems. It is also possible to create new ones if needed, using a low-level framework in C++ programming language. POSL also provides a set of *communication modules*, essential features to share information between solvers.

One of the advantages of POSL is to create *abstract solvers* using a operator-based language, that remains independent of the used *computation* and *communication modules*. That is why it is possible to create many different solvers using the same solution strategy (the *abstract solver*) only instantiating it with different modules (*computation* and *communication modules*). It is also possible to create different communication strategies using the *connection operators* that POSL provides.

In the next Chapter, a detailed study of various communicating and non-communicating strategies, using some *Constraint Satisfaction Problems* as benchmarks. In this study, the efficacy of POSL to study easily and fast these strategies, is showed.

Part II

Study and evaluation of POSL

2

Experiments design

In this chapter I expose all details about the process of evaluation of POSL, *i.e., all experiments I perform. For each benchmark, I explain also used strategies in the evaluation process.*

In this chapter I illustrate and analyze the versatility of POSL studying different ways to solve constraint problems based on local search meta-heuristics. I have chosen the *Social Golfers Problem*, the *N-Queens Problem*, the *Costas Array Problem* and the *Golomb Ruler Problem* as benchmarks since they are two challenging yet differently structured problems. In this chapter I present formally each benchmark and explain the structure of POSL's solvers that I generated for experiments.

2.1 Solving the *Social Golfers Problem*

The *Social Golfers Problem* (*SGP*) consists in scheduling $n = g \times p$ golfers into g groups of p players every week for w weeks, such that two players play in the same group at most once. An instance of this problem can be represented by the triple $g - p - w$. This problem, and other closely related problems, arise in many practical applications such as encoding, encryption, and covering problems [121]. Its structure is very attractive, because it is very similar to other problems, like *Kirkman's Schoolgirl Problem* and the *Steiner Triple System*, so efficient modules to solve a broad range of problems ca be built.

Here, I give the abstract solver designed for this problem as well as concrete computation modules composing the different solvers I have tested:

a) Generation module:

I: Generates a random configuration s, respecting the structure of the problem, *i.e.*, the configuration is a set of w permutations of the vector $[1..n]$.

b) Neighborhood modules:

V_{Std}: Defines the neighborhood $\mathcal{V}(s)$ swapping players among groups.

V_{AS}: Defines the neighborhood $\mathcal{V}(s)$ swapping the most culprit player with other players from the same week. It is based on the *Adaptive Search* algorithm.

c) Selection modules:

S_{First}: Selects the first configuration $s' \in \mathcal{V}(s)$ improving the current cost.

S_{Best}: Selects the best configuration $s' \in \mathcal{V}(s)$ improving the current cost.

S_{Rand}: Selects a random configuration $s' \in \mathcal{V}(s)$.

d) Acceptance module:

A: Evaluates an acceptance criteria for s'. We have chosen the classical module selecting the configuration with the lowest global cost, *i.e.*, the one which is likely the closest to a solution.

A very first experiment was performed to select the best neighborhood function to solve the problem, comparing a basic solver using V_{Std}; a new solver using V_{AS}; and a combination of V_{Std} and V_{AS} by applying the operators ρ, already introduced in the previous chapter. Algorithms 5, 6 and 7 present the *abstract solver* for each case, respectively.

Algorithm 5: Standard *abstract solver* for *SGP*

```
abstract solver as_union                          // ITR → number of iterations
computation : I, V, S, A
begin
    [↻ (ITR < K1)
        I (↦) [↻ (ITR % K2) [V (↦) S (↦) A] ]
    ]
end
```

Algorithm 6: *Abstract solver* combining neighborhood functions using operator *RHO*

```
abstract solver as_union                          // ITR → number of iterations
computation : I, V1, V2, S, A
begin
    [↻ (ITR < K1)
        I (↦) [↻ (ITR % K2) [[V1 (ρ) V2] (↦) S (↦) A] ]
    ]
end
```

Algorithm 7: *Abstract solver* combining neighborhood functions using operator *Union*

```
abstract solver as_union                          // ITR → number of iterations
computation : I, V1, V2, S, A
begin
    [↻ (ITR < K1)
        I (↦) [↻ (ITR % K2) [[V1 (∪) V2] (↦) S (↦) A] ]
    ]
end
```

Solvers mentioned above were too slow to solve instances of the problem with more than 3 weeks, so another solver implementing the *abstract solver* described in Algorithm 8 have been created, using V_{AS} and combining S_{First} and S_{Rand}: it tries a number of times to improve the cost, and if it is not possible, it picks a random neighbor for the next iteration. We also compared the S_{First} and S_{Best} selection modules.

Algorithm 8: *Abstract solver* for *SGP* to scape from local minima

```
abstract solver as_eager                           // ITR → number of iterations
computation : I, V, S1, S2, A
begin
    [↻ (ITR < K1)
        I (↦) [↻ (ITR % K2) [V (↦) [S1 (?)SCI<K3 S2] (↦) A] ]
    ]
end
```

After that, the best solver to be communicating solvers to compare their performance with the non communicating strategies was chosen. The shared information is the current configuration. Algorithms 9 and 10 show that the communication is performed while applying the acceptance criterion of the new configuration for the next iteration. Here, solvers receive a configuration from an outer solver, and match it with their current configuration. Then solvers select the configuration with the lowest global cost. We design different communication strategies. Either we execute a full connected solvers set, or a tuned combination of connected and unconnected solvers. Between connected solvers, we applied two different connections operations: connecting each sender solver with one receiver solver (*1 to 1*), or connecting each sender solver with all receiver solvers (*1 to N*).

Algorithm 9: Communicating *abstract solver* for *SGP* (sender)

```
abstract solver as_eager_sender                    // ITR → number of iterations
computation : I, V, S1, S2, A         // SCI → number of iterations with the same cost
begin
    [↻ (ITR < K1)
        I (↦) [↻ (ITR % K2) [V (↦) [S1 (?)SCI<K3 S2] (↦) ⦇A⦈o] ]
    ]
end
```

Algorithm 10: Communicating *abstract solver* for *SGP* (receiver)

```
abstract solver as_eager_receiver                  // ITR → number of iterations
computation : I, V, S1, S2, A         // SCI → number of iterations with the same cost
communication : C.M.
begin
    [↻ (ITR < K1)
        I (↦)
        [↻ (ITR % K2)
            V (↦) [S1 (?)SCI<K3 S2] (↦) [A (m) C.M.]
        ]
    ]
end
```

2.2 Solving the *N-Queens Problem*

The *N-Queens Problem* (*NQP*) asks how to place N queens on a chess board so that none of them can hit any other in one move. This problem was introduced in 1848 by the chess player Max Bezzelas the *8-queen problem*, and years latter it was generalized as *N-queen problem* by Franz Nauck. Since then many mathematicians, including Gauss, have worked on this problem. It finds a lot of applications, e.g., parallel memory storage schemes, traffic control, deadlock prevention, neural networks, constraint satisfaction problems, among others [123]. Some studies suggest that the number of solution grows exponentially with the number of queens (N), but local search methods have been shown very good results for this problem [124]. For that reason we tested some communication strategies using POSL, to solve a problem relatively easy to solve using non communication strategies.

To handle this problem, we reused some modules used for the *Social Golfers Problem*: the *Selection* and *Acceptance* modules. The new module is:

a) Neighborhood module:

V_{AS}: Defines the neighborhood $V(s)$ swapping the variable which contributes the most to the cost with other.

Fors this problem we used a simple *abstract solver* showing good results with no communication, based on the idea introduced in the section 2.1, using the *computation module* S_{rand} to scape from local minima. The *abstract solver* is presented in Algorithm 11.

Algorithm 11: *Abstract solver* for *NQP*

abstract solver *as_eager* // ITR → number of iterations
computation : I, V, S_1, S_2, A // SCI → number of iterations with the same cost
begin
$I \bigcirc\!\!\!\!\mapsto\; [\circlearrowleft (\text{ITR} < K_1)\; V \bigcirc\!\!\!\!\mapsto \left[S_1 \;\textcircled{?}_{\text{SCI}<K_2}\; S_2\right] \bigcirc\!\!\!\!\mapsto A\,]$
end

Using solvers implementing this *abstract solver* we create communicating solvers to compare their performance with the non communicating strategies. The shared information is the current configuration. Algorithms 12 and 13 show that the communication is performed while selecting a new configuration for the next iteration. We design different communication strategies. Either we execute a full connected solvers set, or a tuned combination of connected and unconnected solvers. Between connected solvers, we applied two different connections operations: connecting each sender solver with one receiver solver (*1 to 1*), or connecting each sender solver with all receiver solvers (*1 to N*).

Algorithm 12: *Abstract solver* for *NQP* (sender)

---

abstract solver *as_eager_sender* // $\textsc{Itr} \rightarrow$ number of iterations
computation : I, V, S_1, S_2, A // $\textsc{Sci} \rightarrow$ number of iterations with the same cost
begin
  $I \mapsto [\circlearrowleft (\textsc{Itr} < K_1)\ V \mapsto \left[(\!|S_1|\!)^o\ ?_{\textsc{Sci}<K_2}\ S_2\right] \mapsto A\ ]$
end

Algorithm 13: *Abstract solver* for *NQP* (receiver)

---

abstract solver *as_eager_receiver* // $\textsc{Itr} \rightarrow$ number of iterations
computation : I, V, S_1, S_2, A // $\textsc{Sci} \rightarrow$ number of iterations with the same cost
communication : $C.M.$
begin
  $I \mapsto$
  $[\circlearrowleft (\textsc{Itr} < K_1)$
    $V \mapsto \left[\left[S_1\ ?_{\textsc{Itr}\%K_2}\ \left[S_1\ m\ C.M.\right]\right]\ ?_{\textsc{Sci}<K_3}\ S_2\right] \mapsto A$
  $]$
end

2.3 Solving the *Costas Array Problem*

The *Costas Array Problem* (*CAP*) consists in finding a *costas array*, which is an $n \times n$ grid containing n marks such that there is exactly one mark per row and per column and the $n(n-1)/2$ vectors joining each couple of marks are all different. This is a very complex problem that finds useful application in some fields like sonar and radar engineering. It also presents an interesting characteristic: although the search space grows factorially, from order 17 the number of solutions drastically decreases [122].

To handle this problem, I have reused some modules used for the *Social Golfers Problem* and *N-Queens Problem*: the *Neighborhood computation module* used for *N-Queens*, and the *Selection* and *Acceptance computation modules* used for both. The new modules are:

a) Generation module:

  I: Generates a random configuration s, as a permutation of the vector $[1..n]$.

b) Neighborhood module:

V_{AS}: Defines the neighborhood $V(s)$ swapping the variable which contributes the most to the cost with other.

I have also added a *Reset* module (R), a mechanism to escape from local minima[i]. The basic solver I use to solve this problem is presented in Algorithm 14, and I take it as a base to build all the different communication strategies. Basically, it is a classical local search iteration, where instead of performing restarts, it performs resets.

Algorithm 14: Reset-based *abstract solver* for *CAP*

```
abstract solver as_hard                    // ITR → number of iterations
computation : I, R, V, S, A
begin
    I (↦)
    [↻ (ITR < K₁)
        R (↦) [↻ (ITR % K₂) [V (↦) S (↦) A] ]
    ]
end
```

The *abstract solver* for the sender solver is presented in Algorithm 15. Like for the *Social Golfers Problem*, we design different communication strategies combining different percentages of communicating solvers and our two communication operators (*1 to 1* and *1 to N*). However for this problem, we study the behavior of the communication performed at two different moments: while applying the acceptance criteria (Algorithm 16), and while performing a

[i]It is based on the code from Daniel Díaz available at https://sourceforge.net/projects/adaptivesearch/

reset (Algorithms 16, 17 and 18).

Algorithm 15: Reset-based *abstract solver* for *CAP* (sender)

abstract solver *as_hard_sender* // $\textsc{Itr} \rightarrow$ number of iterations
computation : I, R, V, S, A
begin
  $I \mapsto$
  $[\circlearrowleft (\textsc{Itr} < K_1)$
    $R \mapsto [\circlearrowleft (\textsc{Itr}\ \%\ K_2) \left[V \mapsto S \mapsto (\!|A|\!)^o\right]]$
  $]$
end

Algorithm 16: Reset-based *abstract solver* for *CAP* (receiver, variant A)

abstract solver *as_hard_receiver_a* // $\textsc{Itr} \rightarrow$ number of iterations
computation : I, R, V, S, A
communication : $C.M.$
begin
  $I \mapsto$
  $[\circlearrowleft (\textsc{Itr} < K_1)$
    $R \mapsto [\circlearrowleft (\textsc{Itr}\ \%\ K_2) \left[V \mapsto S \mapsto \left[A\ m\ C.M.\right]\right]]$
  $]$
end

Algorithm 17: Reset-based *abstract solver* for *CAP* (receiver, variant B)

abstract solver *as_hard_receiver_b* // $\textsc{Itr} \rightarrow$ number of iterations
computation : I, R, V, S, A // $\textsc{Sci} \rightarrow$ number of iterations with the same cost
communication : $C.M.$
begin
  $I \mapsto$
  $[\circlearrowleft (\textsc{Itr} < K_1)$
    $\left[R\ ?_{\textsc{Sci}<K_3} \left[R\ m\ C.M.\right]\right] \mapsto [\circlearrowleft (\textsc{Itr}\ \%\ K_2) \left[V \mapsto S \mapsto A\right]]$
  $]$
end

Algorithm 18: Reset-based *abstract solver* for *CAP* (receiver, variant C)

abstract solver *as_hard_receiver_c* // $\textsc{Itr} \rightarrow$ number of iterations
computation : I, R, V, S, A
communication : $C.M.$
begin
  $I \mapsto$
  $[\circlearrowleft (\textsc{Itr} < K_1)$
    $\left[R\ m\ C.M.\right] \mapsto [\circlearrowleft (\textsc{Itr}\ \%\ K_2) \left[V \mapsto S \mapsto A\right]]$
  $]$
end

2.4 Solving the *Golomb Ruler Problem*

The *Golomb Ruler Problem* (*GRP*) problem consists in finding an ordered vector of n distinct non-negative integers, called *marks*, $m_1 < \cdots < m_n$, such that all differences $m_i - m_j$ $(i > j)$ are all different. An instance of this problem is the pair (o, l) where o is the order of the problem, (i.e., the number of *marks*) and l is the length of the ruler (i.e., the last *mark*). We assume that the first *mark* is always 0. This problem has been applied to radio astronomy, x-ray crystallography, circuit layout and geographical mapping [125]. When we apply POSL to solve an instance of this problem sequentially, we can notice that it performs many *restarts* before finding a solution. For that reason we chose this problem to study a new communication strategy.

We use *Golomb Ruler Problem* instances to study a different communication strategy. This time we communicate the current configuration, to avoid its neighborhood, i.e., a *tabu* configuration. We reused some modules used in the resolution of *Social Golfers* and *Costas Array* problems to design the solvers: the *Selection* and *Acceptance* modules. The new modules are:

a) Generation module:

I: Generates a random configuration s, respecting the structure of the problem, i.e., the configuration is an ordered vector of integers. This module takes into account a set of *tabu* configurations arrived via solver-communication (and also from the same solver) to construct the new configuration far enough from them.

b) Neighborhood module:

V: Defines the neighborhood $\mathcal{V}(s)$ by changing one value while keeping the order, i.e., replacing the value s_i by all possible values $s'_i \in D_i$ that satisfy $s_{i-1} < s'_i < s_{i+1}$.

We also add a module to insert a configuration into a *tabu* list inside the solver. In Algorithm 19 we present the *abstract solver* used to send information (sender *abstract solver*). When the module T is executed, the solver is unable to find a better configuration around the current one, so it is assumed to be a local minimum, and it is sent to the receiver solver. Algorithm 20 presents an *abstract solver* used to receive information (receiver *abstract solver*). Based on the connection operator used in the communication strategy, this solver might receives one or many configurations. These configurations are the input of the generation module (I), and this module inserts all received configurations into a *tabu* list, and then generates a new first configuration, far from all configurations in the *tabu* list.

Algorithm 19: *Abstract solver* for *GRP* (sender)

abstract solver *as_golomb_sender* // ITR $\rightarrow$ number of iterations
computation : I, V, S, A, T
begin
$[\circlearrowleft (\text{ITR} < K_1)$
$I \circledmapsto [\circlearrowleft (\text{ITR} \,\%\, K_2) \left[V \circledmapsto S \circledmapsto A\right]\,] \circledmapsto (\!|T|\!)^o$
$]$
end

Algorithm 20: *Abstract solver* for *GRP* (receiver)

abstract solver *as_golomb_receiver* // ITR $\rightarrow$ number of iterations
computation : I, V, S, A, T
connection : $C.M.$
begin
$[\circlearrowleft (\text{ITR} < K_1)$
$\left[C.M. \circledmapsto I\right] \circledmapsto [\circlearrowleft (\text{ITR} \,\%\, K_2) \left[V \circledmapsto S \circledmapsto A\right]\,] \circledmapsto (\!|T|\!)^o$
$]$
end

3

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