POSL: A Parallel-Oriented Solver Language

Thesis for the Degree of
Doctor of Computer Science

Alejandro Reyes Amaro

Doctoral School STIM

Academic advisors:
Eric Monfroy[1], Florian Richoux[2]

[1]Department of Informatics
Faculty of Science
University of Nantes
France

[2]Department of Informatics
Faculty of Science
University of Nantes
France

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Assessment committee:

Prof. (1)

Institution (1)

Prof. (2)

Institution (2)

Prof. (3)

Institution (3)

ISBN ??

Part I

POSL: Parallel Oriented Solver Language

1

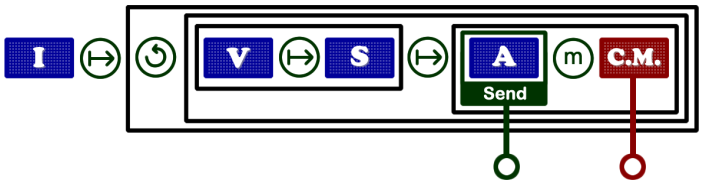
A Parallel-Oriented Language for Modeling Meta-Heuristic-Based Solvers

In this chapter POSL *is introduced as the main contribution, and a new way to solve CSPs. Its characteristics and advantages are summarized, and a general procedure to be followed is described, in order to build parallel solvers using* POSL*, followed by a detailed description of each of the single steps.*

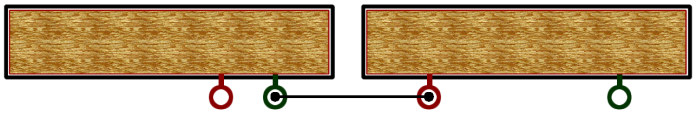
Contents

In this chapter we present the different steps to build communicating parallel solvers with POSL. First of all, the algorithm we have conceived to solve the target problem is decomposed it into small modules of computation, which are implemented as separated *functions*. We name them *computation modules* (see Figure 1.1a, blue shapes). At this point it is crucial to find a good decomposition of its algorithm, because it will have a significant impact in its future re-usage and variability. The next step is to decide which information is interesting to *receive* from other solvers. This information is encapsulated into another kind of component called *communication module*, allowing data transmission between solvers (see Figure 1.1a, red shapes). A third stage is to ensemble the modules through POSL's inner language (the interested reader is referred to Appendix [...]) to create independent solvers. The parallel-oriented language based on operators provided by POSL (see Figure 1.1b, green shapes) allows not only the information exchange, but also executing components in parallel. In this stage the information that is interesting to be shared with other solvers is sent using operators. After that we can connect them using *communication operators*. We call this final entity a *solvers set* (see Figure 1.1c).

(a) Creating POSL's modules



(b) Assembling modules using POSL's operators



(c) Connecting POSL solvers to create *communication strategies*

Figure 1.1: Solver construction process using POSL

In the following sections all these steps are explained in details, but first, I explain how to model the target benchmark using POSL.

1.1 Modeling the target benchmark

Target problems are modeled in POSL using the C++ programing language, respecting some rules of the object-oriented design. First of all, the benchmark must inherit from the class **Benchmark** provided by POSL. This class does not have any method to be

overridden or implemented, but receives in its constructor three objects, instances from classes that the user must create. Those classes must inherit from **SolutionCostStrategy**, **RelativeCostStrategy** and **ShowStrategy**, respectively. In these classes the most important functionalities of the benchmark model are defined.

SolutionCostStrategy: In this class the strategy to compute the *cost* of a configuration is implemented. POSL is based on improving step by step an initial configuration, taking into account a *cost function* provided by the user through the model (by implementing the function *solutionCost(dots)*). The kind of problems that POSL solves is the class of *Constraint Satisfaction Problems*, so this *cost function* must return an integer taking into account the problem constraints. Given a configuration s, the *cost function*, as a mandatory rule, must return 0 if and only if s is a solution of the problem, i.e., s fulfill all the problem constraints. An example of *cost function* is one that returns the number of violated constraints. However, the more expressive the function cost is, the better the performance of POSL leading to the solution.

The method to be implemented in this class is:

- `int solutionCost(std::vector<int> & c)` → Computes the cost of a given configuration (`c`).

RelativeCostStrategy: In this class the user implements the strategy to compute the *cost* of a given configuration with respect to another. If the cost of some configuration has been calculated, sometimes it is possible to store some information in order to compute the cost of another configuration, if the differences between them are known. If it is possible, the algorithms is defined in this class. If it is not possible, this class must have the same functionality of **SolutionCostStrategy**.

The methods to implement in this class are:

- `void initializeCostData(std::vector<int> & c)` → Initializes the information related to the cost (auxiliary data structures, the current configuration (`c`), the current cost, etc.)
- `void updateConfiguration(std::vector<int> & c)` → Updates the information related to the cost.
- `int relativeSolutionCost(std::vector<int> & c)` → Returns the relative cost of the configuration `c` with respect to the current configuration.
- `int currentCost()` → Property that returns the cost of the current configuration.
- `int costOnVariable(int variable_index)` → Returns a measure of the contribution of a variable to the total cost of a configuration.

- `int sickestVariable()` → Returns the variable contributing the most to the cost.

SolutionCostStrategy: This class represents the way a benchmark shows a configuration, in order to provide more information about the structure. For example, a configuration of the instance 3–3–2 of the *Social Golfers Problem* (see bellow for more details about this benchmark) can be written as follows:

```
[1, 2, 3, 4, 5, 6, 7, 8, 9, 3, 4, 5, 6, 7, 8, 9, 1, 2]
```

This text is, nevertheless, very difficult to be read if the instance is larger. Therefore, it is recommended that the user implements this class in order to give more details and to make it easier to interpret the configuration. For example, for the same instance of the problem, a solution could be presented as follows:

```
Golfers: players-3, groups-3, weeks-2
6         8         7
1         3         5
4         9         2
--
7         2         3
4         8         1
5         6         9
--
```

The method to be implemented in this class is:

- `std::string showSolution(std::shared_ptr<Solution> s)` → Returns a string to be written in the standard output.

Once we have modeled the target benchmark, it can be solved using POSL. In the following sections we describe how to use this parallel-oriented language to solve *Constraint Satisfaction Problems*.

1.2 First stage: creating POSL's modules

There exist two types of basic modules in POSL: *computation modules* and *communication modules*. A *computation module* is a function which received an input, then executes an internal algorithm, and returns an output. A *communication module* is also a function receiving and returning information, but in contrast, the *communication module* can receive information from two different sources: through input parameters or from outside, i.e., by communicating with a module from another solver.

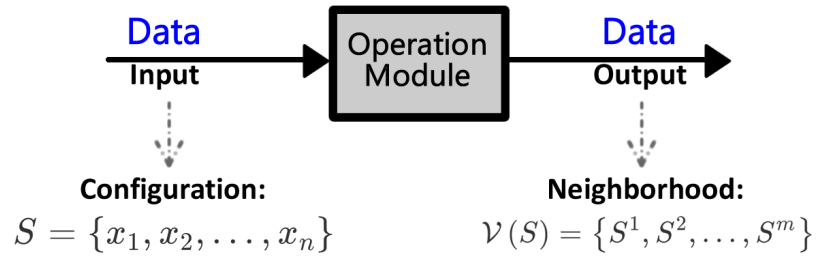


Figure 1.2: An example of a computation module computing a neighborhood

1.2.1 Computation Module

A *computation module* is the most basic and abstract way to define a piece of computation. It is a function which receives an instance of a POSL data type as input, then executes an internal algorithm, and returns an instance of a POSL data type as output. The input and output types will characterize the computation module signature. It can be dynamically replaced by (or combined with) other computation modules, since they can be shared among solvers working in parallel. They are joined through *abstract solvers*.

Definition 1 ***(Computation Module)*** *A computation module Cm is a mapping defined by:*

$$Cm : D \rightarrow I \tag{1.1}$$

where D and I can be either a set of configurations, a set of sets of configurations, a set of values of some data type, etc.

Consider a local search meta-heuristic solver. One of its *computation modules* can be the function returning the set of configurations composing the neighborhood of a given configuration:

$$Cm_{neighborhood} : D_1 \times D_2 \times \cdots \times D_n \rightarrow 2^{D_1 \times D_2 \times \cdots \times D_n}$$

where D_i represents the definition domains of each variable of the input configuration.

Figure 1.2 shows an example of *computation module*: which receives a configuration S and then computes the set $\mathcal{V}$ of its neighbor configurations $\{S^1, S^2, \ldots, S^m\}$.

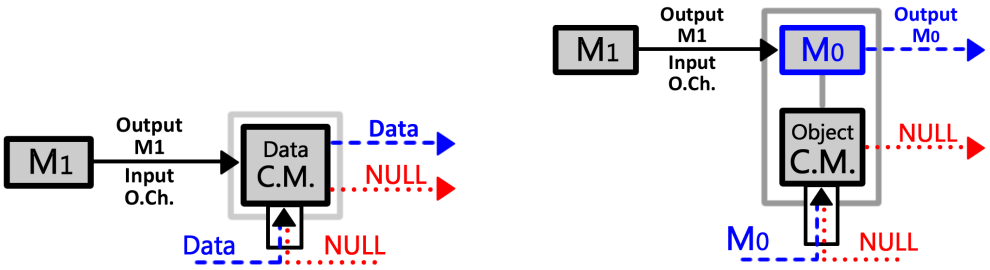
1.2.1.1 Creating new *computation modules*

To create new *computation modules* we use C++ programing language. POSL provides a hierarchy of data types to work with (See anexes) and some abstract classes to inherit from, depending on the type of *computation module* that the user wants to create. These abstract classes represent *abstract computation module* and define a type of action to be executed. In the following we present the most important ones:

- **AOM_FirstConfigurationGeneration** → Represents *computation modules* generating a first configuration. The user must implement the method *spcf_execute(**ComputationData**)* which returns a pointer to a **Solution**, that is, an object containing all the information concerning a partial solution (configuration, variable domains, etc.)
- **AOM_NeighborhoodFunction** → Represent *computation modules* creating a neighborhood of a given configuration. The user must implement the method *spcf_execute(**Solution**)* which returns a pointer to an object **Neighborhood**, containing a set of configurations which constitute the neighborhood of a given configuration, according to certain criteria. These configuration are efficiently stored.
- **AOM_SelectionFunction** → Represents *computation modules* selecting a configuration from a neighborhood. The user must implement the method *spcf_execute(**Neighborhood**)* which returns a pointer to an object **DecisionPair**, containing two solutions: the current and the selected one.
- **AOM_DecisionFunction** → Represents *computation modules* deciding which of the two solutions will be the current configuration for the next iteration. The user must implement the method *spcf_execute(**DecisionPair**)* which returns a pointer to an object **Solution**.

1.2.2 Communication modules

A *communication module* is also a function receiving and returning information, but in contrast, the *communication module* can also receive information by communicating with a module from another solver. A *communication module* is the component managing the information reception in the communication between solvers (we will talk about information transmission in the next section). They can interact with *computation modules* through operators (see Figure 1.3).



(a) Data *communication module* **(b)** Object *communication module*

Figure 1.3: Communication module

A *communication module* can receive two types of information from an external solver: data or *computation modules*. It is important to notice that by sending/receiving *computation modules*, we mean sending/receiving only required information to identify and being able to instantiate the *computation module*.

In order to distinguish from the two types of *communication modules*, we will call Data Communication Module to the *communication module* responsible for the data reception (Figure 1.3a), and Object Communication Module to the one responsible for the reception and instantiation of *computation modules* (Figure 1.3b).

Definition 2 ***(Data Communication Module)*** *A* Data Communication Module *Ch is a component that produces a mapping defined as follows:*

$$Ch : U \rightarrow I \tag{1.2}$$

It returns the information I coming from an external solver, no matter what the input U is.

Definition 3 ***(Object Communication Module)*** *If we denote by* $\mathbb{M}$ *the space of all the computation modules defined by Definition 1.1, then an* Object Communication Module *Ch is a component that produces a computation module coming from an external solver as follows:*

$$Ch : \mathbb{M} \rightarrow \mathbb{M} \tag{1.3}$$

Users can implement new computation and connection modules but POSL already contains many useful modules for solving a broad range of problems.

Due to the fact that *communication modules* receive information coming from outside without having control on them, it is necessary to define the *NULL* information, in order to denote the absence of information. If a Data Communication Module receives a piece of information, is returned automatically. If a Object Communication Module receives a *computation module*, it is instantiated and executed with the *communication module*'s input and its result is

returned. In both cases, if no available information exists (no communications performed), the *communication module* returns the *NULL* object.

1.3 Second stage: assembling POSL's modules

Modules mentioned above are defined respecting the signature of some predefined abstract module. For example, the module showed in Figure 1.2 is a *computation module* based on an abstract module that receives a configuration and returns a neighborhood. In that sense, an example of a concrete *computation module* (or just *computation module*) can be a function receiving a configuration, and returning a neighborhood constituted by N configurations which only differ from the input configuration in one entry.

In this stage an *abstract solver* is coded using POSL. It takes abstract modules as *parameters* and combines them through operators. Through the *abstract solver*, we can also decide which information to send to other solvers by using some operators to send the result of a computation module (see below). In the following we present a formal and more detailed specification of POSL's operators.

The *abstract solver* is the solver's backbone. It joins the *computation modules* and the *communication modules* coherently. It is independent from the *computation modules* and *communication modules* used in the solver. It means that they can be changed or modified during the execution, without altering the general algorithm, but still respecting the main structure. Each time we combine some of them using POSL's operators, we are creating a *compound module*. Here we formally define the concept of *module* and *compound module*.

Definition 4 *A* **module** *is (and it is denoted by the letter $\mathcal{M}$):*

a) a computation module or

b) a communication module or

c) $[\mathcal{M}_1\ OP\ \mathcal{M}_2]$, which is the composition of two modules $\mathcal{M}_1$ and $\mathcal{M}_2$ to be executed sequentially, returning an output depending on the nature of the operator OP*; or*

d) $[\![\mathcal{M}_1\ OP\ \mathcal{M}_2]\!]_p$, which is the composition of two modules $\mathcal{M}_1$ and $\mathcal{M}_2$ to be executed, returning an output depending on the nature of the operator OP*. These two modules will be executed in parallel if and only if* OP *supports parallelism, (i.e. some modules will be executed sequentially although they were grouped this way); or sequentially otherwise.*

We denote the space of the modules by $\mathbb{M}$ *and call compound modules to the composition of modules described in c) and d).*

For a better understanding of Definition 4, Figure 1.4 shows graphically the structure of a *compound module*.

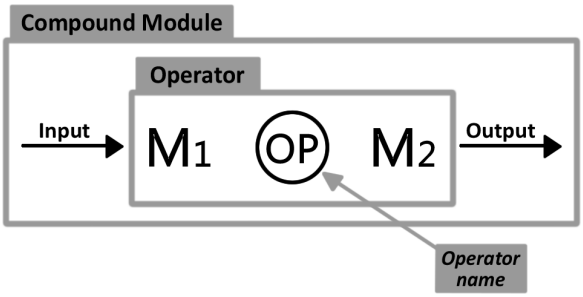


Figure 1.4: A *compound module*

As mentioned before, the *abstract solver* is independent from the *computation modules* and *communication modules* used in the solver. It means that one *abstract solver* can be used to construct many different solvers, by implementing it using different modules (see below the related concept of *abstract solver* instantiation). This is the reason why the *abstract solver* is defined only using abstract modules. Formally, we define an *abstract solver* as follows:

Definition 5 ***(Abstract Solver)*** *An* Abstract Solver *AS is a triple* $(\mathbf{M}, \mathcal{L}^m, \mathcal{L}^c)$*, where:* $\mathbf{M}$ *is a compound module (also called* root *compound module),* $\mathcal{L}^m$ *a list of abstract computation modules appearing in* $\mathcal{M}$*, and* $\mathcal{L}^c$ *a list of communication modules appearing in* $\mathcal{M}$*.*

The *root compound module* can be defined also as a free-context grammar as follows:

Definition 6 (root *compound module***'s grammar)** $G_{POSL} = (\mathbf{V}, \Sigma, \mathbf{S}, \mathbf{R})$*, where:*

a) $\mathbf{V} = \{CM, OP\}$ *is the set of variables,*

b) $\Sigma = \left\{\alpha, \beta, be, [,], [\![,]\!]_p, (,), \{,\}, (\!|,|\!)^m, |\!)^o, \textcircled{\mapsto}, \textcircled{?}, \circlearrowleft, \textcircled{\rho}, \textcircled{\vee}, \textcircled{\wedge}, \textcircled{M}, \textcircled{m}, \textcircled{\downarrow}, \textcircled{\cup}, \textcircled{\cap}\right\}$ *is the set of terminals,*

c) $\mathbf{S} = \{CM\}$ *is the set of start variables,*

d) and $\mathbf{R} =$

$$CM \longmapsto \alpha \mid \beta \mid (\!|CM|\!)^o \mid (\!|CM|\!)^m \mid [OP] \mid [\![OP]\!]_p$$
$$OP \longmapsto CM \textcircled{\mapsto} CM \mid CM \textcircled{?} CM \mid CM \textcircled{\rho} CM \mid CM \textcircled{\vee} CM \mid CM \textcircled{\wedge} CM$$
$$OP \longmapsto CM \textcircled{M} CM \mid CM \textcircled{m} CM \mid CM \textcircled{\downarrow} CM \mid CM \textcircled{\cup} CM \mid CM \textcircled{\cap} CM$$
$$OP \longmapsto CM \circlearrowleft (be) CM$$

is a set of rules

In the following I explain some of the concepts in Definition 6:

- The variables CM and OP are two very important entities in the language, as it can be seen in the grammar. We name them *compound module* and *operator*, respectively.
- The terminals α and β represent a *computation module* and a *communication module*, respectively.
- The terminal *be* is a boolean expression.
- The terminals $[\ ], [\![\ ]\!]_p$ are symbols for grouping and defining the way the involved *compound modules*are executed. Depending on the nature of the operator, this can be either sequentially or in parallel:
  - *a)* [OP]: The involved operator is executed sequentially.
  - *b)* $[\![\text{OP}]\!]_p$: The involved operator is executed in parallel if and only if OP supports parallelism. Otherwise, an exception is thrown.
- The terminals (and) are symbols for grouping the boolean expression in some operators.
- The terminals $(\!|.|\!)^m, (\!|.|\!)^o$, are operators to send information to other solvers (explained bellow).
- The rest of terminals are POSL operators.

In the following we define POSL operators. In order to group modules, like in Definition $4(c)$) and $4(d)$), we will use |.| as generic grouper. In order to help the reader to easely understand how to use the operators, I use an example of a solver that I build step by step, while presenting the definitions.

POSL creates solvers based on local search meta-heuristics algorithms. These algorithms have a common structure: 1. They start by initializing some data structures (e.g., a *tabu list* for *Tabu Search* [34], a *temperature* for *Simulated Annealing* [32], etc.). 2. An initial configuration s is generated. 3. A new configuration s' is selected from the neighborhood $\mathcal{V}(s)$. 4. If s' is a solution for the problem P, then the process stops, and s' is returned. If not, the data structures are updated, and s' is accepted or not for the next iteration, depending on a certain criterion. An example of such data structure is the penalizing features of local optima defined by Boussaïd et al [31] in their algorithm *Guided Local Search*.

Abstract computation modules composing *local search meta–heuristics* are:

$Abstract\ Computation\ module - 1$	I: Generating a configuration s
$Abstract\ Computation\ module - 2$	V: Defining the neighborhood $\mathcal{V}(s)$

Abstract Computation module − 3	S: Selecting $s' \in \mathcal{V}(s)$
Abstract Computation module − 4	A: Evaluating an acceptance criterion for s'

The list of modules to be used in the examples have been presented. Now I present the POSL operators.

Definition 7 (Operator Sequential Execution) *Let*

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$,

be modules, where $\mathcal{I}_1 \subseteq \mathcal{D}_2$. *Then the operation* $\left|\mathcal{M}_1 \textcircled{\mapsto} \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_{seq}$ *as the result of executing* $\mathcal{M}_1$ *followed by executing* $\mathcal{M}_2$*:*

$$\mathcal{M}_{seq} : \mathcal{D}_1 \rightarrow \mathcal{I}_2$$

This is an example of an operator that does not support the execution of its involved *compound modules* in parallel, because the input of the second *compound module* is the output of the first one.

Coming back to the example, I can use defined *abstract computation modules* to create a *compound module* that perform only one iteration of a local search, using the operator **Sequential Execution**. I create a *compound module* to execute sequentially I and V (see Figure 1.5a), then I create an other *compound module* to execute sequentially the *compound module* already created and S (see Figure 1.5b), and finally this *compound module* and the *computation module* A are executed sequentially (see Figure 1.5c). The *compound module* presented in Figure 1.5c can be coded as follows:

$$\Big[\Big[\Big[I \textcircled{\mapsto} V\Big] \textcircled{\mapsto} S\Big] \textcircled{\mapsto} A\Big]$$

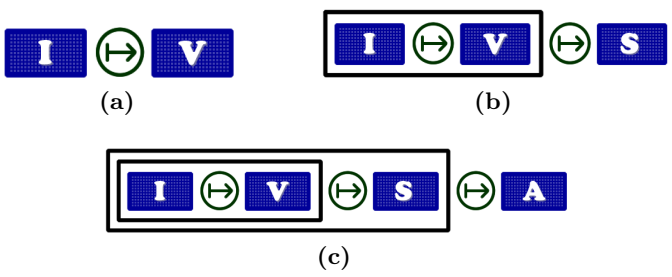


Figure 1.5: Using **sequential execution** operator

The following operator is very useful to execute modules sequentially creating bifurcations, subject to some boolean condition:

Definition 8 (Operator Conditional Execution) *Let*

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$,

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$. *Then the operation* $\left|\mathcal{M}_1 \textcircled{?}_{<cond>} \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_{cond}$ *as result of the sequential execution of* $\mathcal{M}_1$ *if* $< cond >$ *is* **true** *or* $\mathcal{M}_2$, *otherwise:*

$$\mathcal{M}_{cond} : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

This operator can be used in the example if I want to execute two different *selection computation modules* (S_1 and S_2) depending on certain criterion (see Figure 1.6):

$$\left[\left[\left[I \textcircled{\mapsto} V\right] \textcircled{\mapsto} \left[S_1 \textcircled{?} S_2\right]\right] \textcircled{\mapsto} A\right]$$

In examples I remove the clause $< cond >$ for simplification.

Figure 1.6: Using **conditional execution** operator

We can execute modules sequentially creating also cycles.

Definition 9 (Operator Cyclic Execution) *Let* $\mathcal{M} : \mathcal{D} \rightarrow \mathcal{I}$ *be a module, where* $\mathcal{I} \subseteq \mathcal{D}$. *Then, the operation* $|\circlearrowleft_{<cond>} \mathcal{M}|$ *defines the compound module* $\mathcal{M}_{cyc}$ *as result of the sequential execution of* $\mathcal{M}$ *repeated while* $< cond >$ *remains* **true**:

$$\mathcal{M}_{cyc} : \mathcal{D} \rightarrow \mathcal{I}$$

Using this operator I can model a local search algorithm, by executing the *abstract computation module* I and then the other *computation modules* (V, S and A) cyclically, until finding a solution (i.e, a configuration with cost equal to zero) (see Figure 1.7):

$$\left[I \textcircled{\mapsto} \left[\circlearrowleft \left[\left[V \textcircled{\mapsto} S\right] \textcircled{\mapsto} A\right]\right]\right]$$

In the examples, I remove the clause $< cond >$ for simplification.

Definition 10 (Operator Random Choice) *Let*

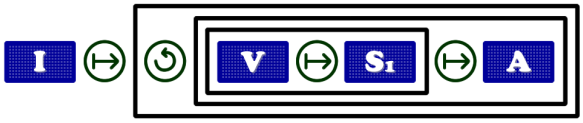


Figure 1.7: Using **cyclic execution** operator

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$,

be modules, where $\mathcal{D}_1 \subset \mathcal{D}_2$ *and a real value* ρ*. Then the operation* $\left|M_1 (\rho) \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_{rho}$ *that executes and returns the output of* $\mathcal{M}_1$ *with probability* ρ*, or executes and returns the output of* $\mathcal{M}_2$ *with probability* $(1 - \rho)$*:*

$$\mathcal{M}_{rho} : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

In the example I can create a *compound module* to execute two *abstract computation modules* A_1 and A_2 following certain probability ρ using the operator **random execution** as follows (see Figure 1.8):

$$\left[I \mapsto \left[\circlearrowleft \left[\left[V \mapsto S\right] \mapsto \left[A_1 \ (\rho) \ A_2\right]\right]\right]\right]$$

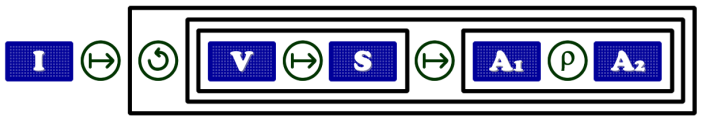


Figure 1.8: Using **random execution** operator

The following operator is very useful if the user needs to use a *communication module* inside an *abstract solver*. As explained before, if a *communication module* does not receive any information from another solver, it returns *NULL*. This may cause the undesired termination of the solver if this case is not considered correctly. Next, I introduce the operator **Operator Not** *NULL* **Execution** and illustrate how to use it in practice with an example.

Definition 11 (Operator Not *NULL* **Execution)** *Let*

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$,

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$*. Then, the operation* $\left|\mathcal{M}_1 (\vee) \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_{non}$ *that executes* $\mathcal{M}_1$ *and returns its output if it is not NULL, or executes* $\mathcal{M}_2$ *and returns its output otherwise:*

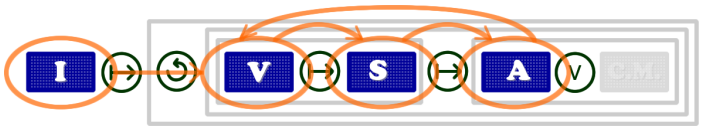
$$\mathcal{M}_{non} : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

Let us make consider a slightly more complex example: When applying the acceptance criterion, suppose that we want to receive a configuration from other solver to combine the *computation module* A with a *communication module*:

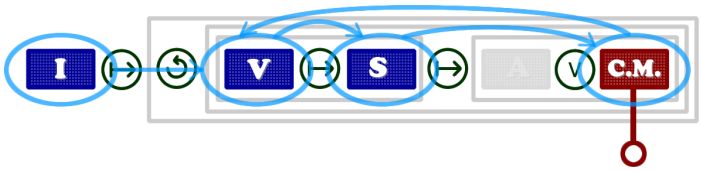
$\boxed{Communication\ module - 1:}$ $C.M.$: Receiving a configuration.

Figure 1.9 shows how to combine a *communication module* with the *computation module* A through the operator $\bigcirc\!\!\!\!\vee$. Here, the *computation module* A will be executed as long as the *communication module* remains *NULL*, i.e., there is no information coming from outside. This behavior is represented in Figure 1.9a by the orange lines. If some data has been received through the *communication module*, the later is executed instead of the module A, represented in Figure 1.9b by blue lines. The code can be written as follows:

$$\left[I \bigcirc\!\!\!\!\mapsto \left[\circlearrowleft \left[\left[V \bigcirc\!\!\!\!\mapsto S\right] \bigcirc\!\!\!\!\mapsto \left[A \bigcirc\!\!\!\!\vee C.M.\right]\right]\right]\right]$$



(a) The solver executes the computation module **A** if no information is received through the connection module



(b) The solver uses the information coming from an external solver

Figure 1.9: Two different behaviors within the same solver

This is *short-circuit* operator. It means that if the first argument (module) does not return *NULL*, the second will not be executed. POSL provides another operator with the same functionality but not *short-circuit*:

Definition 12 (Operator *BOTH* Execution) *Let*

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$,

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$. *Then the operation* $\left|\mathcal{M}_1 \bigcirc\!\!\!\!\wedge \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_{both}$ *that executes both* $\mathcal{M}_1$ *and* $\mathcal{M}_2$, *then returns the output of* $\mathcal{M}_1$ *if it is not NULL, or the output of* $\mathcal{M}_2$ *otherwise:*

$$\mathcal{M}_{both} : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

In the following definitions, the concepts of *cooperative parallelism* and *competitive parallelism* are implicitly included. We say that cooperative parallelism exists when two or more processes are running separately, they are independent, and the general result will be some combination of the results of all the involved processes (e.g. Definitions 13 and 14). On the other hand, competitive parallelism arise when the general result is the result of the process ending first (e.g. Definition 15).

Definition 13 (Operator Minimum) *Let*

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$,

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$. *Let also* o_1 *and* o_2 *be the outputs of* $\mathcal{M}_1$ *and* $\mathcal{M}_2$, *respectively. Assume that there exists some order criteria between them. Then the operation* $\left|\mathcal{M}_1 \textcircled{m} \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_{min}$ *that executes* $\mathcal{M}_1$ *and returns* $\min\{o_1, o_2\}$*:*

$$\mathcal{M}_{min} : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

Similarly we define the operator **Maximum**:

Definition 14 (Operator Maximum) *Let*

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$,

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$. *Let also* o_1 *and* o_2 *be the outputs of* $\mathcal{M}_1$ *and* $\mathcal{M}_2$, *respectively. Assume that there exists some order criteria between them. Then the operation* $\left|\mathcal{M}_1 \textcircled{M} \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_{max}$ *that executes* $\mathcal{M}_1$ *and returns* $\max\{o_1, o_2\}$*:*

$$\mathcal{M}_{max} : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

Comming back to the previews example, the **minimum operator** can be applied to obtain a more interesting behavior in the solver: When applying the acceptance criteria, suppose that we want to receive a configuration from other solver, to compare it with ours and select the one with the lowest cost. We can do that by applying the operator (m) to combine the *computation module* A with a *communication module* $C.M.$ (see Figure1.10):

$$\left[I \textcircled{\mapsto} \left[\circlearrowleft \left[\left[V \textcircled{\mapsto} S\right] \textcircled{\mapsto} \left[\!\left[A \textcircled{m} C.M.\right]\!\right]_p\right]\right]\right]$$

Notice that in this example, I can use the grouper $[\![.]\!]_p$ since the **minimum operator** supports parallelism.

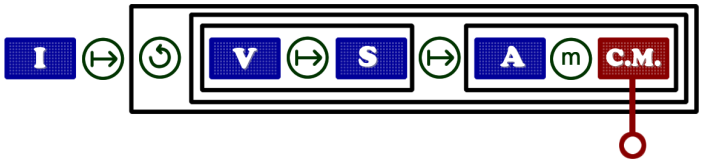


Figure 1.10: Using **minimum** operator

Definition 15 (Operator Race) *Let*

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$,

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$ *and* $\mathcal{I}_1 \subset \mathcal{I}_2$. *Then the operation* $\left|\mathcal{M}_1 \textcircled{\downarrow} \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_{race}$ *that executes both modules* $\mathcal{M}_1$ *and* $\mathcal{M}_2$, *and returns the output of the module ending first:*

$$\mathcal{M}_{race} : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

Sometimes nighborhood functions are slow depending on the configuration. In that case two neighborhood *computation modules* can be executed and take into account the output of the module ending first (see Figure1.11):

$$\left[I \textcircled{\mapsto} \left[\circlearrowleft \left[\left[\left[\!\left[V_1 \textcircled{\downarrow} V_2 \right]\!\right]_p \textcircled{\mapsto} S \right] \textcircled{\mapsto} \left[\!\left[A \textcircled{m} C.M. \right]\!\right]_p \right] \right] \right]$$

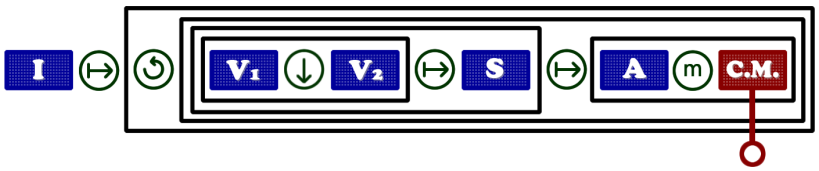


Figure 1.11: Using **race** operator

Some others operators can be useful when dealing with *sets*.

Definition 16 (Operator Union) *Let*

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$,

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$. *Let also* V_1 *and* V_2 *be the outputs of* $\mathcal{M}_1$ *and* $\mathcal{M}_2$, *respectively. Then the operation* $\left|\mathcal{M}_1 \textcircled{\cup} \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_\cup$ *that executes both modules* $\mathcal{M}_1$ *and* $\mathcal{M}_2$, *and returns* $V_1 \cup V_2$:

$$\mathcal{M}_\cup : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

Similarly we define the operators **Intersection** and **Subtraction**:

Definition 17 (Operator Intersection) *Let*

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$,

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$. *Let also* V_1 *and* V_2 *be the outputs of* $\mathcal{M}_1$ *and* $\mathcal{M}_2$, *respectively. Then the operation* $\left|\mathcal{M}_1 \textcircled{\cap} \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_\cap$ *that executes both modules* $\mathcal{M}_1$ *and* $\mathcal{M}_2$, *and returns* $V_1 \cap V_2$:

$$\mathcal{M}_\cap : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

Definition 18 (Operator Subtraction) *Let*

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$,

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$. *Let also* V_1 *and* V_2 *be the outputs of* $\mathcal{M}_1$ *and* $\mathcal{M}_2$, *respectively. Then the operation* $\left|\mathcal{M}_1 \textcircled{-} \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_-$ *that executes both modules* $\mathcal{M}_1$ *and* $\mathcal{M}_2$, *and returns* $V_1 - V_2$:

$$\mathcal{M}_- : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

Now, I define the operators which allows to send information to other solvers. Two types of information can be sent: i) the output of the *computation module* and send its output, or ii) the *computation module* itself. . This utility is very useful in terms of sharing behaviors between solvers.

Definition 19 (Sending Data Operator) *Let* $\mathcal{M} : \mathcal{D} \rightarrow \mathcal{I}$ *be a module. Then the operation* $|(\!|\mathcal{M}|\!)^o|$ *defines the compound module* $\mathcal{M}_{sendD}$ *that executes the module* $\mathcal{M}$ *and sends its output outside:*

$$\mathcal{M}_{sendD} : \mathcal{D} \rightarrow \mathcal{I}$$

Similarly we define the operator **Send Module**:

Definition 20 (Sending Module Operator) *Let $\mathcal{M} : \mathcal{D} \to \mathcal{I}$ be a module. Then the operation $|(\!|\mathcal{M}|\!)^m|$ defines the compound module $\mathcal{M}_{sendM}$ that executes the module $\mathcal{M}$, then returns its output and sends the module itself outside:*

$$\mathcal{M}_{sendM} : \mathcal{D} \to \mathcal{I}$$

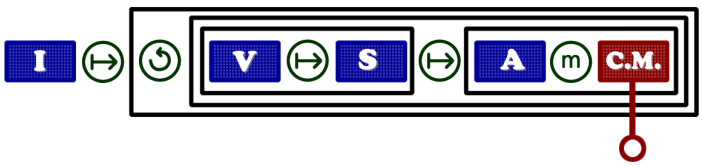
In the following example, I use one of the *compound modules* already presented in the previews examples, using a *communication module* to receive a configuration (see Figure 1.12a):

$$\left[I \textcircled{\mapsto} \left[\circlearrowleft \left[\left[V \textcircled{\mapsto} S \right] \textcircled{\mapsto} \left[\!\left[A \textcircled{m} C.M. \right]\!\right]_p \right] \right] \right]$$

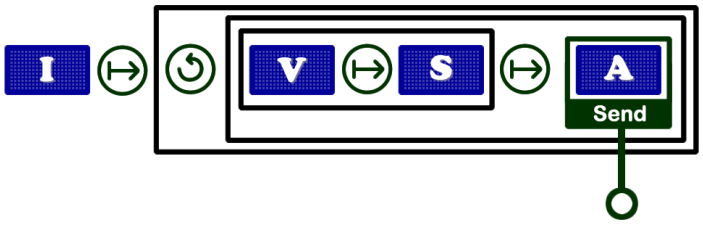
I also build another, as its complement: sending the accepted configuration to outside, using the **sending data operator** (see Figure1.12b):

$$\left[I \textcircled{\mapsto} \left[\circlearrowleft \left[\left[V \textcircled{\mapsto} S \right] \textcircled{\mapsto} (\!|A|\!)^o \right] \right] \right]$$

In the Section 1.5 I explain how to connect solvers to each other.



(a)



(b)

Figure 1.12: Sender and receiver behaviors

Once all desired abstract modules are linked together with operators, we obtain the *root compound module*, an important part of an *abstract solver*. To implement a concrete solver from an *abstract solver*, one must instantiate each abstract module with a concrete one respecting the required signature. From the same *abstract solver*, one can implement many different concrete solvers simply by instantiating abstract modules with different concrete modules.

An *abstract solver* is defined as follows: after declaring the **abstract solver**'s name, the first line defines the list of abstract *computation modules*, the second one the list of abstract *communication modules*, then the algorithm of the solver is defined as the solver's body (the root *compound module*), between **begin** and **end**.

An *abstract solver* can be declared through the simple regular expression:

abstract solver *name* **computation**: L^m (**communication**: L^c)? **begin** $\mathcal{M}$ **end**

where:

- *name* is the identifier of the *abstract solver*,
- L^m is the list of abstract *computation modules*,
- L^c is the list of abstract *communication modules*, and
- $\mathcal{M}$ is the root *compound module*.

For instance, Algorithm 1 illustrates the abstract solver corresponding to Figure 1.1b.

Algorithm 1: POSL pseudo-code for the *abstract solver* presented in Figure 1.1b

```
abstract solver as_01
computation : I, V, S, A
connection: C.M.
    begin
        I ⟼
        [↻ (Itr % K_1)
            [V ⟼ S ⟼ [C.M. (m) (|A|)^o]]
        ]
    end
```

1.4 Third stage: creating POSL solvers

With *computation* and *communication modules* composing an *abstract solver*, one can create solvers by instantiating *modules*. This is simply done by specifying that a given **solver** must **implements** a given *abstract solver*, followed by the list of *computation* then *communication modules*. These modules must match signatures required by the *abstract solver*.

> In the following example, I describe some concrete *computation modules* that can be used to implement the *abstract solver* declared in Algorithm 1:

$\boxed{Computation\ module-1}$ I_{rand} generates a random configuration s

$\boxed{Computation\ module-2}$ V_{1ch} defines the neighborhood $\mathcal{V}(s)$ changing only one element

$\boxed{Computation\ module-3}$ S_{best} selects the best configuration $s' \in \mathcal{V}(s)$ improving the current cost.

$\boxed{Computation\ module-4}$ A_{alw} evaluates an acceptance criterion for s'. We have chosen the classical module, selecting the configuration with the lowest global cost, *i.e.*, the one which is likely the closest to a solution.

I use also the following concrete *communication module*:

$\boxed{Communication\ module-1}$ CM_{last} returns the last configuration arrived, if at the time of its execution, there is more than one configuration waiting to be received.

These modules are used and explained in details in the Chapter 2 of this document. Algorithm 2 implements Algorithm 1 by instantiating its modules.

Algorithm 2: An instantiation of the *abstract solver* presented in Algorithm 1

solver solver_01 **implements** as_01
computation : $I_{rand}, V_{1ch}, S_{best}, A_{alw}$
connection: CM_{last}

1.5 Forth stage: connecting the solvers

We call *solver set* to the pool of (concrete) solvers that we plan to use in parallel to solve a problem. Once we have our solvers set, the last stage is to connect the solvers to each other. Up to this point, solvers are disconnected, but they are ready to establish the communication. POSL provides a platform to the user such that cooperative strategies can be easily defined.

In the following we present two important concepts necessary to formalize the *communication operators*.

Definition 21 (Communication Jack) *Let $\mathcal{S}$ be a solver. Then the operation $\mathcal{S} \cdot \mathcal{M}$ opens an outgoing connection from the solver $\mathcal{S}$, sending to the outside either a) the output of $\mathcal{M}$, if it is affected by a sending data operator as presented in Definition 19, or b) $\mathcal{M}$ itself, if it is affected by a sending module operator as presented in Definition 20.*

Definition 22 (Communication Outlet) *Let $\mathcal{S}$ be a solver. Then, the operation $\mathcal{S} \cdot \mathcal{CM}$ opens an ingoing connection to the solver $\mathcal{S}$, receiving from the outside either a) the output of some computation module, if $\mathcal{CM}$ is a data communication module, or b) a computation module, if $\mathcal{CM}$ is an object communication module.*

The communication is established by following the following rules guideline:

a) Each time a solver sends any kind of information by using a *sending* operator, it creates a *communication jack*.

b) Each time a solver defines a *communication module*, it creates a *communication outlet*.

c) Solvers can be connected to each other by linking *communication jacks* to *communication outlets*.

Following, we define the *connection operators* that POSL provides.

Definition 23 (Connection Operator One-to-One) *Let*

a) $\mathcal{J} = [\mathcal{S}_0 \cdot \mathcal{M}_0, \mathcal{S}_1 \cdot \mathcal{M}_1, \ldots, \mathcal{S}_{N-1} \cdot \mathcal{M}_{N-1}]$ *be the list of communication jacks, and*

b) $\mathcal{O} = [\mathcal{Z}_0 \cdot \mathcal{CM}_0, \mathcal{Z}_1 \cdot \mathcal{CM}_1, \ldots, \mathcal{Z}_{N-1} \cdot \mathcal{CM}_{N-1}]$ *be the list of communication outlets*

Then the operation

$$\mathcal{J} \textcircled{\rightarrow} \mathcal{O}$$

connects each communication jack $\mathcal{S}_i \cdot \mathcal{M}_i \in \mathcal{J}$ with the corresponding communication outlet $\mathcal{Z}_i \cdot \mathcal{CM}_i \in \mathcal{O}$, $\forall 0 \leq i \leq N-1$ (see Figure 1.13a).

Definition 24 (Connection Operator One-to-N) *Let*

a) $\mathcal{J} = [\mathcal{S}_0 \cdot \mathcal{M}_0, \mathcal{S}_1 \cdot \mathcal{M}_1, \ldots, \mathcal{S}_{N-1} \cdot \mathcal{M}_{N-1}]$ *be the list of communication jacks, and*

b) $\mathcal{O} = [\mathcal{Z}_0 \cdot \mathcal{CM}_0, \mathcal{Z}_1 \cdot \mathcal{CM}_1, \ldots, \mathcal{Z}_{M-1} \cdot \mathcal{CM}_{M-1}]$ *be the list of communication outlets*

Then the operation

$$\mathcal{J} \textcircled{\rightsquigarrow} \mathcal{O}$$

connects each communication jack $\mathcal{S}_i \cdot \mathcal{M}_i \in \mathcal{J}$ with every communication outlet $\mathcal{Z}_j \cdot \mathcal{CM}_j \in \mathcal{O}$, $\forall 0 \leq i \leq N-1$ and $0 \leq j \leq M-1$ (see Figure 1.13b).

Definition 25 (Connection Operator Ring) *Let*

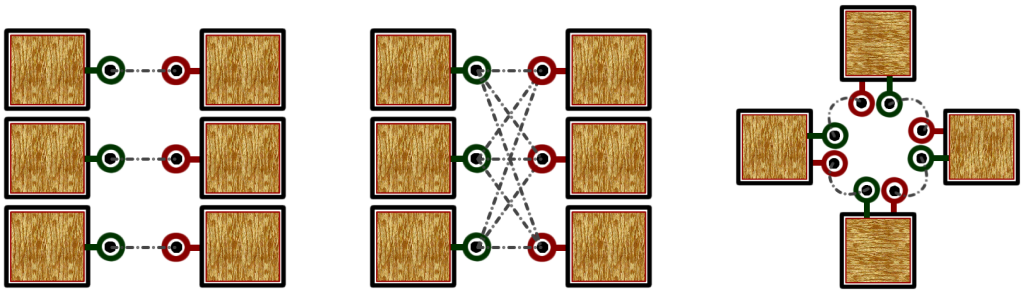
a) $\mathcal{J} = [\mathcal{S}_0 \cdot \mathcal{M}_0, \mathcal{S}_1 \cdot \mathcal{M}_1, \ldots, \mathcal{S}_{N-1} \cdot \mathcal{M}_{N-1}]$ *be the list of communication jacks, and*

b) $\mathcal{O} = [\mathcal{S}_0 \cdot \mathcal{CM}_0, \mathcal{S}_1 \cdot \mathcal{CM}_1, \ldots, \mathcal{S}_{N-1} \cdot \mathcal{CM}_{N-1}]$ *be the list of communication outlets*

Then the operation

$$\mathcal{J} \circledast \mathcal{O}$$

connects each communication jack $\mathcal{S}_i \cdot \mathcal{M}_i \in \mathcal{J}$ *with the corresponding communication outlet* $\mathcal{Z}_{(i+1)\%N} \cdot \mathcal{CM}_{(i+1)\%N} \in \mathcal{O}$, $\forall 0 \leq i \leq N-1$ *(see Figure 1.13c).*



(a) Communication 1 to 1 **(b)** Communication 1 to N **(c)** Cyclic communication

Figure 1.13: Graphic representation of communication operators

POSL also allows to declare non-communicating solvers to be executed in parallel, declaring only the list of solver names:

$$[\mathcal{S}_0, \mathcal{S}_1, \ldots, \mathcal{S}_{N-1}]$$

When we apply a connection operator $\circledast$ between a *communication jacks* list $\mathcal{J}$ and a *communication outlets* list $\mathcal{O}$, internally we are assigning an *abstract computation unit* (typically a thread) to each solver that we declare in each list. This assignment receives the name of *Solver Scheduling*. Before running the *solver set*, this *abstract unit of computation* is just an integer $\tau \in [0..N]$ identifying uniquely each of the solvers. When the *solver set* is launched, the solver with the identifier τ runs into the computation unit τ. This identifier assignation remains independent of the real availability of resources of computation. It just takes into account the user declaration. This means that, if the user declares 30 solvers (15 senders and 15 receivers) and the *solver set* is launched using 20 cores, only the first 20 solvers will be executed, and in consequence, there will be 10 solvers sending information to nowhere. Users should take this into account when declaring the *solver set*.

The connection process depends on the applied connection operator. In each case the goal is to assign, to the sending operator ($(\!|.|\!)^o$ or $(\!|.|\!)^m$) inside the *abstract solver*, the identifier of the solver (or solvers, depending on the connection operator) where the information will be

sent. Algorithm 3 presents the connection process.

Algorithm 3: Scheduling and connection main algorithm

```
input  : 𝒥 list of communication jacks,
         𝒪 list of communication outlets
1 while no available jacks or outlets do
2 |   S_jack ← GetNext(𝒥)
3 |   R_oulet ← GetNext(𝒪)
4 |   S ← GetSolverFromConnector(S_jack)
5 |   R ← GetSolverFromConnector(R_oulet)
6 |   Schedule(S)
7 |   R_id ← Schedule(R)
8 |   Connect(root(S),S_jack, R_id)
9 end
```

In Algorithm 3:

- `GetNext(...)` returns the next available solver-jack (or solver-outlet) in the list, depending on the connection operator, e.g., for the connection operator One-to-N, each *communication jack* in $\mathcal{J}$ must be connected with each *communication outlet* in $\mathcal{O}$.
- `GetSolverFromConnector(...)` returns the solver name given a connector declaration.
- `Schedule(...)` schedules a solver and returns its identifier.
- `Root(...)` returns the *root compound module* of a solver.
- `Connect(...)` searches the *computation module* S_{jack} recursively inside the *root compound module* of S and places the identifier R_{id} into its list of destination solvers.

Let us suppose that we have declared two solvers S and Z, both implementing the *abstract solver* in Algorithm 1, so they can be either sender or receiver. The following code connects them using the operator **1 to N**:

$$[S \cdot A] \overset{\rightsquigarrow}{\bigcirc} [Z \cdot C.M.]$$

If the operator **1 to N** is used with only with one solver in each list, the operation is equivalent to applying the operator **1 to 1**. However, to obtain a communication strategy like the one showed in Figure 1.13b, six solvers (three senders and three receivers) have to be declared to be able to apply the following operation:

$$[S_1 \cdot A, S_2 \cdot A, S_3 \cdot A] \overset{\rightsquigarrow}{\bigcirc} [Z_1 \cdot C.M., Z_2 \cdot C.M., Z_3 \cdot C.M.]$$

POSL provides a mechanism to make this easier, through *namespace expansions*.

1.5.1 Solver namespace expansion

One of the goals of POSL is to provide a way to declare sets of solvers to be executed in parallel fast and easily. For that reason, POSL provides two forms of namespace expansion, in order to create sets of solvers using already declared ones:

Solver name expansion - Uses an integer K to denote how many times the solver name S will appear in the declaration. $[\dots S_i \cdot \mathcal{M}(K), \dots]$ expands as $\left[\dots S_i \cdot \mathcal{M}, S_i^2 \cdot \mathcal{M}, \dots S_i^K \cdot \mathcal{M} \dots\right]$
and all new solvers $S_i^j, j \in [2..K]$ are created using the same solver declaration of solver S_i.

Connection declaration expansion - Uses an integer K to denote how many times the connection will be repeated in the declaration. Let a) $[\mathcal{S}_1 \cdot \mathcal{M}_1, \dots, \mathcal{S}_N \cdot \mathcal{M}_N]$ and b) $[\mathcal{R}_1 \cdot \mathcal{CM}_1, \dots, \mathcal{R}_M \cdot \mathcal{CM}_M]$ be the list of *communication jacks* and *communication outlets*, respectively, and c) $\textcircled{\text{op}}$ a connection operator. Then

$$[\mathcal{S}_1 \cdot \mathcal{M}_1, \dots, \mathcal{S}_N \cdot \mathcal{M}_N] \ \textcircled{op} \ [\mathcal{R}_1 \cdot \mathcal{CM}_1, \dots, \mathcal{R}_M \cdot \mathcal{CM}_M]\, K$$

expands as

$$\begin{gathered}
[\mathcal{S}_1 \cdot \mathcal{M}_1, \dots, \mathcal{S}_N \cdot \mathcal{M}_N] \ \textcircled{op} \ [\mathcal{R}_1 \cdot \mathcal{CM}_1, \dots, \mathcal{R}_N \cdot \mathcal{CM}_N] \\
\left[\mathcal{S}_1^2 \cdot \mathcal{M}_1, \dots, \mathcal{S}_N^2 \cdot \mathcal{M}_N\right] \ \textcircled{op} \ \left[\mathcal{R}_1^2 \cdot \mathcal{CM}_1, \dots, \mathcal{R}_N^2 \cdot \mathcal{CM}_N\right] \\
\dots \\
\left[\mathcal{S}_1^K \cdot \mathcal{M}_1, \dots, \mathcal{S}_N^K \cdot \mathcal{M}_N\right] \ \textcircled{op} \ \left[\mathcal{R}_1^K \cdot \mathcal{CM}_1, \dots, \mathcal{R}_N^K \cdot \mathcal{CM}_N\right]
\end{gathered}$$

and all new solvers $S_i^k, i \in [1..N]$ and $R_j^k, j \in [1..M]$, $k \in [2..K]$, are created using the same solver declaration of solvers S_i and R_j, respectively.

> Now, suppose that I have created solvers S and Z mentioned in the previews example. As a communication strategy, I want to connect them through the operator **1 to N**, using S as sender and Z as receiver. Then, using **namespace expansions**, I need to declare how many solvers I want to connect. Algorithm 4 shows the desired communication strategy. Notice in this example that the connection operation is affected also by the number 2 at the end of the line, as **connection declaration expansion**. In that sense, and supposing that 12 units of computation are available, a *solver set* working on parallel following the topology described in Figure 1.14 can be obtained.

Algorithm 4: A communication strategy

1 [S · *A* (3)] ⇝ [Z · *C.M.* (3)] 2 ;

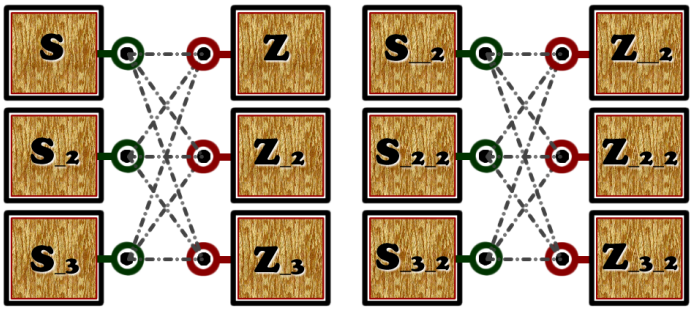


Figure 1.14: An example of connection strategy for 12 units of computation

1.6 Summarize

In this Chapter POSL have been formally presented, as a Parallel–Oriented Solver Language to build meta-heuristic-based solver to solve *Constraint Satisfaction Problems*. This language provides a set of *computation modules* useful to solve a wide range of problems. It is also possible to create new ones if needed, using a low-level framework in C++ programming language. POSL also provides a set of *communication modules*, essential features to share information between solvers.

One of the advantages of POSL is to create *abstract solvers* using a operator-based language, that remains independent of the used *computation* and *communication modules*. That is why it is possible to create many different solvers using the same solution strategy (the *abstract solver*) only instantiating it with different modules (*computation* and *communication modules*). It is also possible to create different communication strategies using the *connection operators* that POSL provides.

In the next Chapter, a detailed study of various communicating and non-communicating strategies, using some *Constraint Satisfaction Problems* as benchmarks. In this study, the efficacy of POSL to study easily and fast these strategies, is showed.

Part II

Study and evaluation of POSL

2

Experiments design and results

In this chapter I expose all details about the process of evaluation of POSL*, i.e., all experiments I perform. For each benchmark, I explain also used strategies in the evaluation process. I explain the used environments were we run the experiments (description of my desktop machine, Curiosiphi server). I describe all the experiments and I expose a complete analysis of the obtained result.*

In this chapter I illustrate and analyze the versatility of POSL studying different ways to solve constraint problems based on local search meta-heuristics. I have chosen the *Social Golfers Problem*, the *N-Queens Problem*, the *Costas Array Problem* and the *Golomb Ruler Problem* as benchmarks since they are two challenging yet differently structured problems. In this chapter I present formally each benchmark, I explain the structure of POSL's solvers that I generated for experiments and present a detailed analysis of obtained results.

The experiments[i] were performed on an Intel® Xeon™ E5-2680 v2, 10×4 cores, 2.80GHz. Showed results are the means of 30 runs for each setup, presented in columns labeled **T**, corresponding to the run-time in seconds, and **It.** corresponding to the number of iterations; and their respective standard deviations (**T(sd)** and **It.(sd)**). In some tables, the column labeled **% success** indicates the percentage of solvers finding a solution before reaching a time–out (5 minutes).

The experiments in this section are multi-walk runs using the same solver main structure (except differents w.r.t. communication operations). Parallel experiments use 40 cores for all problem instances. It is important to point out that POSL is not designed to obtain the best results in terms of performance, but to give the possibility of rapidly prototyping and studying different cooperative or non cooperative search strategies.

All benchmarks were coded using the POSL low-level framework in C++.

First results using POSL to solve constraint problems were published in [126] were we used POSL to solve the *Social Golfers Problem* and study some communication strategies. It was the first version of POSL, therefore it was able to solve only relatively easy instances. However, the efficacy of the communication was showed using this tool.

With the next and more optimized version of POSL, I decide to start to perform more detailed studies using the benchmark mentioned before and some others.

2.1 Solving the *Social Golfers Problem*

In this section I present the performed study using *Social Golfers Problem* (*SGP*) as a benchmark.

[i] POSL source code is available on GitHub:https://github.com/alejandro-reyesamaro/POSL

2.1.1 Problem definition

The *Social Golfers Problem* (*SGP*) consists in scheduling $g \times p$ golfers into g groups of p players every week for w weeks, such that two players play in the same group at most once. An instance of this problem can be represented by the triple $g - p - w$. This problem, and other closely related problems, arise in many practical applications such as encoding, encryption, and covering problems [121]. Its structure is very attractive, because it is very similar to other problems, like *Kirkman's Schoolgirl Problem* and the *Steiner Triple System*, so efficient modules to solve a broad range of problems can be built.

The cost function for this benchmark was implemented make an efficient use of the stored information about the cost of the previews configuration. Using integers to work with bit-flags, a table to store the information about the partners of each player in each week can be filled in $O(p^2 \cdot g \cdot w)$. So if a configuration has $n = (p \cdot g \cdot w)$ elements, this table can be filled in $O(p \cdot n)$. This table is filled from scratch only one time in the search process (I explain in the next section why). Then, every cost of a new configuration, is calculated based on this information and the performed changes between the new configuration an the stored one. This relative cost is calculated in $O(c \cdot g)$, where c is the number of performed changed in the new configuration with respect to the stored one.

2.1.2 Experiment design

Here, I give the abstract solver designed for this problem as well as concrete computation modules composing the different solvers I have tested:

a) Generation module:

I: Generates a random configuration s, respecting the structure of the problem, *i.e.*, the configuration is a set of w permutations of the vector $[1..n]$.

b) Neighborhood modules:

V_{Std}: Defines the neighborhood $\mathcal{V}(s)$ swapping players among groups.

V_{AS}: Defines the neighborhood $\mathcal{V}(s)$ swapping the most culprit player with other players from the same week. It is based on the *Adaptive Search* algorithm.

c) Selection modules:

S_{First}: Selects the first configuration $s' \in \mathcal{V}(s)$ improving the current cost.

S_{Best}: Selects the best configuration $s' \in \mathcal{V}(s)$ improving the current cost.

S_{Rand}: Selects a random configuration $s' \in \mathcal{V}(s)$.

d) Acceptance module:

A: Evaluates an acceptance criteria for s'. We have chosen the classical module selecting the configuration with the lowest global cost, *i.e.*, the one which is likely the closest to a solution.

A very first experiment was performed to select the best neighborhood function to solve the problem, comparing a basic solver using V_{Std}; a new solver using V_{AS}; and a combination of V_{Std} and V_{AS} by applying the operators (ρ), already introduced in the previous chapter. Algorithms 5, 6 and 7 present the *abstract solver* for each case, respectively.

Algorithm 5: Standard *abstract solver* for *SGP*

```
1 abstract solver as_union                          // Itr → number of iterations
2 computation : I, V, S, A
3 begin
4     [↺ (Itr < K1)
5         I (↦) [↺ (Itr % K2) [V (↦) S (↦) A] ]
6     ]
7 end
```

Algorithm 6: *Abstract solver* combining neighborhood functions using operator *RHO*

```
1 abstract solver as_union                          // Itr → number of iterations
2 computation : I, V1, V2, S, A
3 begin
4     [↺ (Itr < K1)
5         I (↦) [↺ (Itr % K2) [[V1 (ρ) V2] (↦) S (↦) A] ]
6     ]
7 end
```

Algorithm 7: *Abstract solver* combining neighborhood functions using operator *Union*

```
1 abstract solver as_union                          // Itr → number of iterations
2 computation : I, V1, V2, S, A
3 begin
4     [↺ (Itr < K1)
5         I (↦) [↺ (Itr % K2) [[V1 (∪) V2] (↦) S (↦) A] ]
6     ]
7 end
```

Solvers mentioned above were too slow to solve instances of the problem with more than 3 weeks, so another solver implementing the *abstract solver* described in Algorithm 8 have been

created, using V_{AS} and combining S_{First} and S_{Rand}: it tries a number of times to improve the cost, and if it is not possible, it picks a random neighbor for the next iteration. We also compared the S_{First} and S_{Best} selection modules.

Algorithm 8: *Abstract solver* for *SGP* to scape from local minima

```
1 abstract solver as_eager                                  // Itr → number of iterations
2 computation : I, V, S1, S2, A
3 begin
4     [↻ (Itr < K1)
5         I ↦ [↻ (Itr % K2) [V ↦ [S1 ?_{Sci<K3} S2] ↦ A] ]
6     ]
7 end
```

After that, the best solver to be communicating solvers to compare their performance with the non communicating strategies was chosen. The shared information is the current configuration. Algorithms 9 and 10 show that the communication is performed while applying the acceptance criterion of the new configuration for the next iteration. Here, solvers receive a configuration from an outer solver, and match it with their current configuration. Then solvers select the configuration with the lowest global cost. We design different communication strategies. Either we execute a full connected solvers set, or a tuned combination of connected and unconnected solvers. Between connected solvers, we applied two different connections operations: connecting each sender solver with one receiver solver (*1 to 1*), or connecting each sender solver with all receiver solvers (*1 to N*).

Algorithm 9: Communicating *abstract solver* for *SGP* (sender)

```
1 abstract solver as_eager_sender                           // Itr → number of iterations
2 computation : I, V, S1, S2, A                  // Sci → number of iterations with the same cost
3 begin
4     [↻ (Itr < K1)
5         I ↦ [↻ (Itr % K2) [V ↦ [S1 ?_{Sci<K3} S2] ↦ (|A|)^o] ]
6     ]
7 end
```

In all Algorithms ins this section, three parameter can be found: 1. K_1: the maximum number of *restarts*, 2. K_2: the maximum number of iterations in each *restart*, and K_3: the maximum number of iterations with the same current cost. 3.

After the selection of the proper modules to study the different communication strategies, I proceeded to tune these parameter. Only a few runs were necessaries to conclude that the mechanism of using the *computation module* S_{rand} to scape from local minima was enough. For that reason, since the solver never perform restarts, the parameter K_1 was irrelevant. So the reader can assume $K_1 = 1$ for every experiment.

Algorithm 10: Communicating *abstract solver* for *SGP* (receiver)

```
1  abstract solver as_eager_receiver                    // Itr → number of iterations
2  computation : I, V, S1, S2, A          // Sci → number of iterations with the same cost
3  communication : C.M.
4  begin
5      [↻ (Itr < K1)
6          I (↦)
7          [↻ (Itr % K2)
8              V (↦) [S1 (?)Sci<K3 S2] (↦) [A (m) C.M.]
9          ]
10     ]
11 end
```

With the certainty of the solvers do not performs restarts during the search process, I select the same value for $K_2 = 5000$ in order to be able to use the same *abstract solver* for all instances.

Finally, in the tuning process of K_3, I notice only slightly differences between using the values 5, 10, and 15. So I decided to use $K_3 = 5$.

2.1.3 Analysis of results

Table 2.1 showed results of launching *solver set*s to solve each instance of the problem sequentially. Not surprisingly, the means of sequential runtimes and iterations (Table 2.1) are bigger than those means of parallel runs, with or without communication (all other tables).

Instance	T	T(sd)	It.	It.(sd)	% success
5–3–7	8.31	7.64	17,347	15,673	100.00
8–4–7	16.92	15.15	7,829	7,019	100.00
9–4–8	79.60	64.07	20,779	16,537	94.28
11–7–5	3.37	2.16	664	380	100.00

Table 2.1: *Social Golfers*: a single sequential solver

In a first stage of the experiments I use the operator-based language provided by POSL to build and test many different non communicating strategies. The goal is to select the best concrete modules to run tests performing communication. In particular, I have tested two kind of computation modules: the one computing the neighborhood of a given configuration and the one choosing the current configuration for the next solver iteration.

I focused on choosing the right neighborhood function. In the case of the *Social Golfers Problem*, this experiment was launched using a basic abstract solver showed in Algorithm 5.

Abstract solvers	T	T(sd)	It.	It.(sd)
Adaptive Search (AS)	**1.06**	0.79	352	268
Std (ρ) AS	41.53	26.00	147	72
Std $(\cup)$ AS	59.65	55.01	198	110
Standard (Std)	87.90	41.96	146	58

Table 2.2: *Social Golfers*: Instance 10–10–3 in parallel

Instance	O.M. Best Improvement				O.M. First Improvement			
	T	T(sd)	It.	It.(sd)	T	T(sd)	It.	It.(sd)
5–3–7	4.99	4.43	4,421	3,938	**1.32**	0.68	1322	676
8–4–7	5.10	1.77	954	334	**1.82**	0.84	445	191
9–4–8	12.37	5.40	1,342	591	**6.43**	4.60	873	591
11–7–5	5.19	1.67	351	114	**2.22**	0.69	273	58

Table 2.3: *Social Golfers*: comparing selection functions

Solvers implemented from this abstract solver was too slow to solve instances beyond three weeks: they were very often trapped into local minima. This is the reason why we perform this first experiment with the instance 10–10–3 whereas next experiments scale above 3 weeks. This was not a problem though, since the goal of this first experiment was only to find the right concrete neighborhood module.

Results in Table 2.2 are not surprising. The neighborhood neighborhood module V_{AS} is based on the *Adaptive Search* algorithm, which has shown very good results [1]. It selects the most culprit variable (i.e., a player), that is, the variable to most responsible for constraints violation. Then, it permutes this variable value with the value of each other variable, in all groups and all weeks. Each permutation gives a neighbor of the current configuration. V_{Std} uses no additional information, so it performs every possible swap between two players in different groups, every week. It means that this neighborhood is $g \times p$ times bigger than the previous one, with g the number of groups and p the number of players per group. It allows for more organized search because the set of neighbors is pseudo-deterministic, i.e., the construction criteria is always the same but the order of the configuration is random. On the other hand, *Adaptive Search* neighborhood function takes random decisions more frequently, and the order of the configurations is random as well. We also tested abstract solvers with different combinations of these modules, using the (ρ) and the $(\cup)$ operators. The (ρ) operator executes its first or second parameter depending on a given probability ρ, and the $(\cup)$ operator returns the union of its parameters output. All these combinations spent more time searching the best configuration among the neighborhood, although with a lower number of iterations than V_{AS}. The V_{AS} neighborhood function being clearly faster, we have chosen it for our experiments, even if it shown a more spread standard deviation: 0.75 for AS versus 0.62 for Std, considering the ratio $\frac{T(sd)}{T}$.

With the selected neighborhood function, I focused on choosing the best *selection* function.

Instance	Communication 1 to 1				Communication 1 to N			
	T	T(sd)	It.	It.(sd)	T	T(sd)	It.	It.(sd)
5–3–7	1.19	0.64	1,156	608	1.11	0.49	1,067	484
8–4–7	**1.30**	0.72	**317**	161	1.46	0.57	347	128
9–4–8	4.38	2.72	**597**	347	5.51	3.06	736	389
11–7–5	1.76	0.41	214	44	**1.62**	0.34	**202**	30

Table 2.4: *Social Golfers*: test with 100% of communication

Instance	Communication 1 to 1				Communication 1 to N			
	T	T(sd)	It.	It.(sd)	T	T(sd)	It.	It.(sd)
5–3–7	1.04	0.45	1,019	456	1.04	0.53	1,031	530
8–4–7	1.40	0.57	337	122	1.43	0.76	353	167
9–4–8	4.64	2.17	637	279	5.75	3.06	776	389
11–7–5	1.81	0.40	220	33	1.82	0.39	222	39

Table 2.5: *Social Golfers*: test with 50 % of communication

I compared two different concrete modules used within the abstract solver in Algorithm 8, which combines selection modules (S_{First} or S_{Best}) with S_{Rand}, to avoid being trapped into local minima: it tries to improve the cost in a limited number of iterations. If it is not possible, it selects a random neighbor for the next iteration. The first module was S_{Best} that selects the best configuration inside the neighborhood. It not only spent more time searching a better configuration, but also is more sensitive to become trapped into local minima. The second module was S_{First} which selects the first configuration inside the neighborhood improving the current cost. Using this module, solvers favor exploration over intensification and of course spend clearly less time computing the neighborhood. Table 2.3 presents results of this experiment, showing that an exploration-oriented strategy is better for the *SGP*. If we compare results of Tables 2.1 and 2.3 with respect to the standard deviation, we can some gains in robustness with parallelism. The spread in the running times and iterations for the instance 9–4–8 (the hardest one) is 10% lower (0.80 sequentially versus 0.71 in parallel), and for the others, it is around 40% lower (0.91, 0.89 and 0.64 sequentially versus 0.51, 0.45 and 0.31 in parallel, for 5–3–7, 8–4–7 and 11–7–5 respectively, with the same ratio $\frac{T(sd)}{T}$).

Then we ran experiments to study POSL's behavior solving target problems in communicating scenarios. Some compositions of solvers set were taken into account: i. the structure of the

Instance	Communication 1 to 1				Communication 1 to N			
	T	T(sd)	It.	It.(sd)	T	T(sd)	It.	It.(sd)
5–3–7	**0.90**	0.51	**881**	492	1.19	0.67	1,170	655
8–4–7	1.39	0.43	341	94	1.46	0.43	352	96
9–4–8	**4.33**	1.92	599	248	4.53	2.01	625	251
11–7–5	1.99	0.54	242	51	1.63	0.35	224	28

Table 2.6: *Social Golfers*: test with 25% of communication

communication (with/without communication or a mix), and ii. the used communication operator.

Each time a POSL meta-solver is launched, many independent search solvers are executed. We call "good" configuration a configuration with the lowest cost within the current configuration neighborhood and with a cost strictly lesser than the current one. Once a good configuration is found in a sender solver, it is transmitted to the receiver one. At this moment, if the information is accepted, there are some solvers searching in the same subset of the search space, and the search process becomes more exploitation–oriented. This can be problematic if this process makes solvers converging too often towards local minima. In that case, we waste more than one solver trapped into a local minima: we waste all solvers that have been attracted to this part of the search space because of communications. I avoid this phenomenon through a simple (but effective) play: if a solver is not able to find a better configuration inside the neighborhood (executing S_{First}), it selects a random one at the next iteration (executing S_{Rand}). This strategy, using communication between solvers, produces some gain in terms of runtime (Table 2.3 with respect to Tables 2.4, 2.5 and 2.6. The percentage of the receiver solvers that were able to find the solution before the others did, was significant See Anexes. That shows that the communication played an important role during the search, despite inter–process communication's overheads (reception, information interpretation, making decisions, etc). Having many solvers searching in different places of the search space, the probability that one of them reaches a promising place is higher. Then, when a solver finds a good configuration, it can be communicated, and receiving the help of one or more solvers in order to find the solution. For this problem we have reduced the spread in the running times and iterations of the results for the two last instances (9–4–8 and 11–7–5) applying the communication strategy (0.71 without communication versus 0.44 with communication, for 9–4–8, and 0.31 without communication versus 0.20 with communication for 11–7–5).

2.2 Solving the *Costas Array Problem*

In this section I present the performed study using *Costas Array Problem* (*CAP*) as a benchmark.

2.2.1 Problem definition

The *Costas Array Problem* (*CAP*) consists in finding a *costas array*, which is an $n \times n$ grid containing n marks such that there is exactly one mark per row and per column and the $n(n-1)/2$ vectors joining each couple of marks are all different. This is a very complex problem that finds useful application in some fields like sonar and radar engineering. It also presents an interesting characteristic: although the search space grows factorially, from order 17 the number of solutions drastically decreases [122].

The cost function for this benchmark was implemented in C++ based on the current implementation of *Adaptive Search*[ii].

2.2.2 Experiment design

To handle this problem, I have reused some modules used for the *Social Golfers Problem* and *N-Queens Problem*: the *Neighborhood computation module* used for *N-Queens*, and the *Selection* and *Acceptance computation modules* used for both. The new modules are:

a) Generation module:

I: Generates a random configuration s, as a permutation of the vector $[1..n]$.

b) Neighborhood module:

V_{AS}: Defines the neighborhood $V(s)$ swapping the variable which contributes the most to the cost with other.

First attempts to solve this problems were using the same strategy (*abstract solver*) used to solve the *Social Golfers Problem*, without success: POSL was not able to solve instances larger than $n = 8$ in a reasonable amount of time (seconds). After many unsuccessful attempts to find the rights parameter of *maximum number of restarts*, *maximum number of iterations*, and *maximum number of iterations with the same cost*, I decided to implement the mechanism used by Daniel Díaz in the current implementation of *Adaptive Search* to escape from local minima: I have added a *Reset* module (R).

The basic solver I use to solve this problem is presented in Algorithm 11, and I take it as a base to build all the different communication strategies. Basically, it is a classical local search iteration, where instead of performing restarts, it performs resets. After a deep analysis of this implementation and results of some runs, I decided to use $K_1 = 24000$ (maximum

[ii] It is based on the code from Daniel Díaz available at https://sourceforge.net/projects/adaptivesearch/

number of iterations) big enough to solve the chosen instance $n = 17$; and $K_2 = 3$ (the number of iteration before performing the next *reset*).

Algorithm 11: Reset-based *abstract solver* for *CAP*

```
1 abstract solver as_hard                          // ITR → number of iterations
2 computation : I, R, V, S, A
3 begin
4     I (↦)
5     [↺ (ITR < K1)
6         R (↦) [↺ (ITR % K2) [V (↦) S (↦) A] ]
7     ]
8 end
```

The *abstract solver* for the sender solver is presented in Algorithm 12. Like for the *Social Golfers Problem*, we design different communication strategies combining different percentages of communicating solvers and our two communication operators (*1 to 1* and *1 to N*). However for this problem, we study the behavior of the communication performed at two different moments: while applying the acceptance criteria (Algorithm 13), and while performing a

reset (Algorithms 13, 14 and 15).

Algorithm 12: Reset-based *abstract solver* for *CAP* (sender)

```
1 abstract solver as_hard_sender                          // Itr → number of iterations
2 computation : I, R, V, S, A
3 begin
4     I (↦)
5     [↺ (Itr < K_1)
6         R (↦) [↺ (Itr % K_2) [V (↦) S (↦) (|A|)^o] ]
7     ]
8 end
```

Algorithm 13: Reset-based *abstract solver* for *CAP* (receiver, variant A)

```
1 abstract solver as_hard_receiver_a                      // Itr → number of iterations
2 computation : I, R, V, S, A
3 communication : C.M.
4 begin
5     I (↦)
6     [↺ (Itr < K_1)
7         R (↦) [↺ (Itr % K_2) [V (↦) S (↦) [A (m) C.M.]] ]
8     ]
9 end
```

Algorithm 14: Reset-based *abstract solver* for *CAP* (receiver, variant B)

```
1 abstract solver as_hard_receiver_b                      // Itr → number of iterations
2 computation : I, R, V, S, A          // Sci → number of iterations with the same cost
3 communication : C.M.
4 begin
5     I (↦)
6     [↺ (Itr < K_1)
7         [R (?)_{Sci<K_3} [R (m) C.M.]] (↦) [↺ (Itr % K_2) [V (↦) S (↦) A] ]
8     ]
9 end
```

Algorithm 15: Reset-based *abstract solver* for *CAP* (receiver, variant C)

```
1 abstract solver as_hard_receiver_c                      // Itr → number of iterations
2 computation : I, R, V, S, A
3 communication : C.M.
4 begin
5     I (↦)
6     [↺ (Itr < K_1)
7         [R (m) C.M.] (↦) [↺ (Itr % K_2) [V (↦) S (↦) A] ]
8     ]
9 end
```

2.2.3 Analysis of results

We present in Table 2.7 results of launching *solver sets* to solve each instance of *Costas Array Problem* sequentially. Runtimes and iteration means showed in this Table are bigger than those presented in Table 2.8, confirming once again the success of the parallel approach.

STRATEGY	T	T(ds)	It.	It.(sd)	% success
Sequential (1 core)	2.12	0.87	44,453	18,113	42.00
Parallel (40 cores)	0.73	0.46	9,556	6,439	100.00

Table 2.7: *Costas Array* 17: no communication

I chose directly the neighborhood module (V_{AS}), the selection module (S_{First}) and the acceptance module A, to create the solvers. I ran experiments to study parallel communicating strategies taken into account the structure of the communication, and the communication operator used, but in this problem, I perform the communication at two different times: at the time of applying the acceptance criteria, and at the time of performing the *reset*.

Table 2.8 shows that the *abstract solver A* (receiving the configuration at the time of applying the acceptance criteria) is more effective. The reason is that the others, interfere with the proper performance of the *reset*, that is a very important step in the algorithm. This step can be performed on three different ways:

a) Trying to shift left/right all sub-vectors starting or ending by the variable which contributes the most to the cost, and selecting the configuration with the lowest cost.

b) Trying to add a constant (circularly) to each element in the configuration.

c) Trying to shift left from the beginning to some culprit variable (i.e., a variable contributing to the cost).

Then, one of these 3 generated configuration has the sabe probability of being selected, to be the result of the *reset* step. In that sense, some different *resets* can be performed for the same configuration. Here is when the communication play its important role: receiver and

	100% COMM				50% COMM			
STRATEGY	T	T(sd)	It.	It.(sd)	T	T(sd)	It.	It.(sd)
Str A: 1 to 1	**0.41**	0.30	4,973	3,763	0.55	0.43	8,179	7,479
Str A: 1 to N	0.43	0.31	5,697	4,557	0.57	0.46	8,420	7,564
Str B: 1 to 1	0.48	0.41	6,546	5,562	0.51	0.49	8,004	7,998
Str B: 1 to N	0.45	0.46	5,701	6,295	0.48	0.51	7,245	8,379
Str C: 1 to 1	0.48	0.43	6,954	6,706	0.58	0.43	8,329	6,593
Str C: 1 to N	0.49	0.38	6,457	5,875	0.58	0.50	8,077	8,319

Table 2.8: *Costas Array* 17: with communication

sender solvers apply different *reset* in the same configuration, and results showed the efficacy of this communication strategy.

Table 2.8 shows also high values of standard deviation. This is not surprising, due to the highly random nature of the neighborhood function and the selecting criterion, as well as the execution of many resets during the search process.

2.3 Solving the *Golomb Ruler Problem*

In this section I present the performed study using *Golomb Ruler Problem* (*GRP*) as a benchmark.

2.3.1 Problem definition

The *Golomb Ruler Problem* (*GRP*) problem consists in finding an ordered vector of n distinct non-negative integers, called *marks*, $m_1 < \cdots < m_n$, such that all differences $m_i - m_j$ $(i > j)$ are all different. An instance of this problem is the pair (o, l) where o is the order of the problem, (i.e., the number of *marks*) and l is the length of the ruler (i.e., the last *mark*). We assume that the first *mark* is always 0. This problem has been applied to radio astronomy, x-ray crystallography, circuit layout and geographical mapping [125]. When I apply POSL to solve an instance of this problem sequentially, I can notice that it performs many *restarts* before finding a solution. For that reason I have chosen this problem to study a new communication strategy.

2.3.2 Experiment design

I use *Golomb Ruler Problem* instances to study a different communication strategy. This time I communicate the current configuration, to avoid its neighborhood, i.e., a *tabu* configuration. I have reused some modules used in the resolution of *Social Golfers* and *Costas Array* problems to design the solvers: the *Selection* and *Acceptance* modules. The new modules are:

a) Generation module:

I: Generates a random configuration s, respecting the structure of the problem, i.e., the configuration is an ordered vector of integers. This module takes into account

a set of *tabu* configurations arrived via solver-communication (and also from the same solver) to construct the new configuration far enough from them.

b) Neighborhood module:

V: Defines the neighborhood $\mathcal{V}(s)$ by changing one value while keeping the order, i.e., replacing the value s_i by all possible values $s'_i \in D_i$ that satisfy $s_{i-1} < s'_i < s_{i+1}$.

I also add a module to insert a configuration into a *tabu* list inside the solver. In Algorithm 16 the *abstract solver* used to send information (sender *abstract solver*) is presented. When the module T is executed, the solver is unable to find a better configuration around the current one, so it is assumed to be a local minimum, and it is sent to the receiver solver. Algorithm 17 presents an *abstract solver* used to receive information (receiver *abstract solver*). Based on the connection operator used in the communication strategy, this solver might receives one or many configurations. These configurations are the input of the generation module (I), and this module inserts all received configurations into a *tabu* list, and then generates a new first configuration, far from all configurations in the *tabu* list.

Algorithm 16: *Abstract solver* for *GRP* (sender)

```
1 abstract solver as_golomb_sender                    // Itr → number of iterations
2 computation : I, V, S, A, T
3 begin
4     [↻ (Itr < K1)
5         I ↦ [↻ (Itr % K2) [V ↦ S ↦ A] ] ↦ (|T|)^o
6     ]
7 end
```

Algorithm 17: *Abstract solver* for *GRP* (receiver)

```
1 abstract solver as_golomb_receiver                  // Itr → number of iterations
2 computation : I, V, S, A, T
3 connection : C.M.
4 begin
5     [↻ (Itr < K1)
6         [C.M. ↦ I] ↦ [↻ (Itr % K2) [V ↦ S ↦ A] ] ↦ (|T|)^o
7     ]
8 end
```

Instance	T	T(sd)	It.	It.(sd)	R	R(sd)	% success
8–34	0.79	0.66	13,306	11,154	66	55.74	100.00
8–34 (t)	0.66	0.63	10,745	10,259	53	51.35	100.00
10–55	66.44	49.56	451,419	336,858	301	224.56	80.00
10–55 (t)	67.89	50.02	446,913	328,912	297	219.30	88.00
11–72	160.34	96.11	431,623	272,910	143	90.91	26.67
11–72 (t)	117.49	85.62	382,617	275,747	127	91.85	30.00

Table 2.9: *Golomb Ruler*: a single sequential solver

2.3.3 Analysis of results

The benefit of the parallel approach is also proved for the *Golomb Ruler Problem* (see Table 2.9 with respect to 2.10, 2.11, 2.12 and 2.13).

For *Golomb Ruler Problem*, the communication strategy that we adopt was different. Solvers do not communicate the current configuration to have more solvers searching in its neighborhood, but a configuration that we assume is a local minimum to be avoided. We consider that the current configuration is a local minimum since the solver (after a given number of iteration) is not able to find a better configuration in its neighborhood.

The first experiment compares the runs of non communitaing solvers not using a *tabu* list with non communicating solvers using a *tabu* list. The results in Tables 2.10 and 2.11 demonstrate that using a *tabu* list can help the search process. Without communication, the improvement is not substantial (8% for 8–34, 7% for 10–55 and 5% for 11–72) because only one configuration is inserted in the *tabu* list after each restart. When we use *one to one* communication, after the restart k, the receiving solver has twice the number of configurations in the *tabu* list (one *tabu* configuration from itself and the received one after each restart). Table 2.12 shows that this strategy is not sufficient for some instances, but when we use *1 to N* communication, the number of *tabu* configurations after the restart k, in the receiving solver is considerably higher, e.g., after the restart k a receiving solver has $k(N+1)$ configurations in his *tabu* list. Hence, these solvers can generate configurations far enough from many potentially local minima. This phenomenon is more visible when the problem order increases. Table 2.13 shows that the improvement for the higher case (11-72) is about 32% with respect to non communicating solvers without using a *tabu* list (Table 2.10), and about 29% with respect to non communicating solvers using a *tabu* list (Table 2.11).

Instance	T	T(sd)	It.	It.(sd)	R	R(sd)
8–34	0.47	34.82	436	330.10	2	1.63
10–55	5.31	38.63	22,577	16,488	15	11.00
11–72	89.76	55.85	164,763	102,931	54	34.32

Table 2.10: *Golomb Ruler*: parallel, without tabu list.

Instance	T	T(sd)	It.	It.(sd)	R	R(sd)
8–34	0.43	0.37	349	334	1	1.64
10–55	4.92	4.68	20,504	19,742	13	13.07
11–72	85.02	67.22	155,251	121,928	51	40.64

Table 2.11: *Golomb Ruler*: parallel, with tabu list.

Instance	T	T(sd)	It.	It.(sd)	R	R(sd)
8–34	0.44	0.31	309	233	1	1.23
10–55	3.90	3.22	15,437	12,788	10	8.52
11–72	85.43	52.60	156,211	97,329	52	32.43

Table 2.12: *Golomb Ruler*: parallel, communication 1 to 1.

Instance	T	T(sd)	It.	It.(sd)	R	R(sd)
8–34	0.43	0.29	283	225	1	1.03
10–55	3.16	2.82	12,605	11,405	8	7.61
11–72	60.35	43.95	110,311	81,295	36	27.06

Table 2.13: *Golomb Ruler*: parallel, communication 1 to n.

3

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