POSL: A Parallel-Oriented Solver Language

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Contents

Part I

Presentation

1

Introduction

In this Chapter, the introduction of the work is presented. I describe the target problems, and the approaches implemented so far to solve them. The necessity of a new approach to exploit the new era of parallelism is introduced. In this section are presented the contributions of the thesis, and POSL *is introduced as a new parallel approach including others and novel features. Finally, we describe the structure of the document.*

Contents

Combinatorial optimization is the act of obtaining the best result for a problem from a finite set of possibilities. These possibilities are the different combinations of values that the variables can take (configurations), taking into account their domains (integer values), and sometimes a given finite set of restrictions (constraints). These problems have several applications in many fields. In airlines, the crew scheduling is an example of *Combinatorial Optimization Problem*. It consists of covering flights of the company scheduled in a given time window, with minimum cost, making an efficient and realistic use of the available personal. The problem of task assignment for parallel programs, is one of the most important in parallelism, because it represents the core of a good program performance running in several machines [1]. *Combinatorial Optimization Problems* are also present in electrical engineering, for example, when an efficient circuit layout design is needed [2].

In some cases, all feasible solutions, i.e., configurations fulfilling all constraints, are equally important, hence, the main goal is only to find one of these solutions. This is the case of *Constraint Satisfaction Problems (CSP)*. In other words, a solution is an assignment of variables satisfying the constraint set. There exist many different techniques to solve such problems, mainly classified into two categories. In the first category are tree-search based algorithms, exploring the full search space. Two of the most important methods in this category are: a) The *constraint propagation* method. It proceeds as follows: when a given variable is assigned a value, the algorithm recomputes the possible value sets and assigned values of all other (not yet assigned) variables. The process continues recursively until there are no more changes in the model. These way, an equivalent but smaller and easier to solve problem is obtained. b) The *backtracking* method. It incrementally constructs candidates to the solutions, by assigning values to variables. Each time an assignment cannot possibly be completed to a valid solution, the previews assignation is discarded.

In practice, *Constraint Satisfaction Problems* are intractable. Their search space is huge enough to make tree-search based algorithms useless to solve them. In contrast, in the second category are the *meta-heuristics* methods, algorithms that have been shown to be effective in the resolution of these kind of problems. They are an iterative generation process which guides algorithms by combining intelligently different mechanisms for exploring and exploiting the search space, in order to find efficiently near-optimal solutions [3]. In this category, two groups of methods can be found [4]: a) Single-solution based methods (also known as *local search*). They start with an initial solution and move trying to improve it, inside the search space. b) Population-based methods. in these methods, a set of solutions is modified through operators (recombination, mutation, etc.). Both are nature-inspired methods.

On the other hand, the development of computer architecture is leading us toward massively multi/many–core computers. These architectures unlock new algorithmic possibilities to tackle problems sequential algorithms cannot handle, reducing the search times. However,

this development must go hand by hand with the development of parallel algorithms. At the time of writing a solution algorithm in parallel, some important decisions have to be taken. The way of organizing the search, either by dividing the search space, or by dividing the problem into smaller and easier to solve sub-problems, can provide an important speedup to the algorithm. Nevertheless, the multi-walk parallel scheme, where solvers work with the whole problem but searching in different zones of the search space, have shown very good results also. Other important decision is whether to use communication between process as a mechanism of cooperation. Theoretically, sharing information between solvers helps to the search process, but in practice, an equilibrium between the contribution of the communication and its inherent overheads is needed.

Many results can be showed in parallel computing. Adaptive Search [5] is an efficient method reaching linear speed-ups on hundreds and even thousands of cores (depending of the problem), using an independent multi-walk local search parallel scheme. Munera et al [6] present another implementation of this algorithm using communication between search engines, showing the efficiency of cooperative multi-walks strategies. All these results use a multi-walk parallel approach and show the robustness and efficiency of this parallel scheme. Although, they all concluded there is room for improvements.

With all these elements, the idea of obtaining a tool to rapidly prototyping solution and communication strategies emerges. In that sense, the **main goal** of this work is to provide a framework to build/use easily and rapidly:

a) *Computation modules*: simple functions easily reusable, which can be used to built meta-heuristic-based algorithm by joining them.

b) *Abstract solvers*: algorithms templates, which can be used to build many different solvers, instantiating different *computation modules*.

c) Different *communication strategies*.

1.1 Goals and contributions

The first contribution of this thesis is proposed:

POSL (pronounced "puzzle")
Parallel-Oriented Solver Language

It is a framework based on the creation/utilization of *modules* interconnected through operators, to create local-search meta-heuristic-based solvers. These solvers work in parallel using

the multi-walk parallel scheme, and they are connected to each other through communication operators, allowing information sharing. It is well-known software programming is a very time-consuming activity. This is even more the case while developing a parallel software, where debugging is an extremely difficult task. That is why POSL is based on re-usability to propose to *CSP* solver designers/programmers a parallel framework to quickly build parallel prototypes, speeding-up the design process.

POSL is a language designed to combine *modules* available in the framework, or to create new ones. There exist two types of *modules*: *computation modules*, simple functions receiving an input, then executing an internal algorithm and returning an output, and *communication modules*, responsible for the information sharing. The created/chosen *modules* are joined through operators (the POSL's language) to create independent solvers. After that, created solvers can be connected each other using communication operators. This final entity is called a *solver set*. POSL also provides a framework specification to implement the benchmark (problems to solve), respecting some requirements.

This framework was inspired by a similar idea proposed in [7] without communication, introducing an evolutionary approach that uses a simple composition operator to automatically discover new local search heuristics for SAT and to visualize them as combinations of a set of building blocks. Renaud De Landtsheer et al. present in [8] a framework to facilitate the development of search procedures by using *combinators* to design features commonly found in search procedures as standard bricks and joining them. This approach can speed-up the development and experimentation of search procedures when developing a specific solver based on local search. In [9] is proposed an approach of using cooperating meta-heuristic-based local search processes, using an asynchronous message passing protocol. The cooperation is based on the general strategies of pattern matching and reinforcement learning. POSL uses the combination of both ideas, by combining features of the search process through provided operators, but it also provides an operator-based mechanism to connect solvers, creating *communication strategies*.

The second contribution of the present work is a detailed study of the solution process of some *Constraint Satisfaction Problems* chosen as benchmarks, using POSL. In this study some different strategies are proposed, in order to show how the communication can play an important role in the solution of *CSPs*.

A first study was made using the *Social Golfers Problem*, in which a *standard* communication strategy is used: the communication of the current configuration. This strategy shows to be effective, because it helps to preserve the equilibrium between exploration and exploitation, necessary in the efficient resolution of this problem.

With the *Costas Array Problem*, a similar study was performed, with the slightly difference that the configuration was transmitted in different places of the algorithm.

Another study was performed using the *N-Queens Problem*, in which it was observed that a standard communication strategy is not enough to improve the results without communication. However, with this benchmark a strategy of search-pace partitioning was implemented. This strategy was able to accelerate the beginning of the search, hence the final result in terms of runtime and iterations.

Finally, the *Golomb Ruler Problem* was used to study a different communication strategy, in which the current configuration is communicated, but in contrast to the previews strategies, this configuration is not used to be improved, but to be avoided. The principle is to communicate potential local minima to some solvers, and they will avoid them every time they perform a restart.

In every case, it was possible to show the positive effect of the inter-processes communication. Despite intrinsic overheads, the solver cooperation scheme can help significantly in the search process, if it is studied and chosen correctly. For that reason this study has allowed the validation of the effectiveness of POSL to this purpose.

1.2 Structure of the document

Chapter 2 presents an overview to the state of the art of *Combinatorial Optimization Problems*. Its definition and the link with *Constraint Satisfaction Problems* (*CSP*) are presented, as well as the principal methods to solve them. This chapter is a travel among basic techniques, like *Constraint Propagation*, *meta– and hyper–heuristic methods*; advanced techniques, like *hybridization*, *parallel computing*, and *Solvers cooperation*; and ıparameter setting techniques.

In Chapter 3 prior works leading to POSL are presented. The problem subdivision approach was adopted to divide the domain of a given problem in parallel, in particular, to solve the *K-Medoids Problem*. It contains also a performed study applying the ParamILS tool to find the optimum parameter configuration to *Adaptive Search* solver. ParamILS (version 2.3) is a tool for parameter optimization for parametrized algorithms.

In Chapter 4 is presented formally POSL, the Parallel-Oriented Solver Language that is the heart of this thesis and its main contribution to the community. Its characteristics, main advantages, and a general procedure to be followed in order to use it to solve *Constraint Satisfaction Problems* is presented.

Results for each study using POSL to build *communication strategies* to solve the proposed benchmarks are presented in Chapter 5. In each section, a benchmark problem is defined,

the used *solver set* for each communication strategy are presented, and results are analyzed (details of experiments can be found in Appendices).

Finally, the main results of this thesis are summarized in Chapter 6, where possible new lines of investigation are also discussed.

2

State of the art

This chapter presents an overview to the state of the art of Combinatorial Optimization Problems and different approaches to tackle them. In Section 2.1 the definition of a Combinatorial Optimization Problem and its link with Constraint Satisfaction Problems (CSP) are introduced, where I concentrate our main efforts, and I give some examples. The basic techniques used to solve these problems are introduced, like Constraint Propagation (2.2), meta– and hyper–heuristic methods (Sections 2.3 and 2.4). I also present some advanced techniques like hybridization in Section 2.5, parallel computing in Section 2.6, and Solvers cooperation in Section 2.7. Finally, before ending the chapter with a brief summary, I present parameter setting techniques in Section 2.8.

Contents

2.1 Combinatorial Optimization

An *Optimization Problem* consists in finding the best solution among all possible ones, subject or not, to a set of constraints, depending on whether it is a restricted or an unrestricted problem. The suitable values for the involved variables belong to a set called *domain*. When this domain contains only discrete values, we are facing a *Combinatorial Optimization Problem*, and its goal is to find the best possible solution satisfying a global criterion, named *objective function*. *Resource Allocations* [10], *Task Scheduling* [11], *Master-keying* [12], *Traveling Salesman*, *Knapsack Problem*, among others, are well-known examples of *Combinatorial Optimization Problems* [13].

Sometimes, the main goal is not to find the best solution, but finding one feasible solution. This is the case of *Constraint Satisfaction Problems*. Formally, we present the definition of a *CSP* (sometimes also called *Constraint Network*).

Definition 1 (Constraint Satisfaction Problem) *A Constraint Satisfaction Problem (CSP, denoted by $\mathcal{P}$) is a triple $\langle X, D, C\rangle$, where:*

- $X = \{X_1, \ldots, X_n\}$ *is finite a set of variables,*
- $D = \{D_1, \ldots, D_n\}$ *is the set of associated domains. Each domain D_i specifies the set of possible values to the variable X_i.*
- $C = \{c_1, \ldots, c_m\}$ *is a set of constraints. Each constraint is defined involving a set of variables, and specifies the possible combinations of values for these variables.*

In *CSPs*, a *configuration* $s \in D_1 \times D_2 \times \cdots \times D_n$ is a combination of values for the variables in X. Following we define the concept of solution, which is in other words, a configuration satisfying all the constraints $c_i \in C$.

Definition 2 (Solution of a CSP) *Given a CSP $\mathcal{P} = \langle X, D, C\rangle$ and a configuration $S \in D_1 \times D_2 \times \cdots \times D_n$ we say that it is a solution if and only if:*

$$c_i\,(S) \;\; \textit{is true}\; \forall c_i \in C$$

The set of all solutions of $\mathcal{P}$ is denoted by $Sol(\mathcal{P})$

This field, also called *Constraint Programing* is a famous research topic developed by the field of artificial intelligence in the middle of the 70's, and a programming paradigm since the end of the 80's. A *CSP* can be considered as a special case of *Combinatorial Optimization*

Problems, where the objective function is to reduce to the minimum the number of violated constraints in the model. A solution is then obtained when the number of violated constraints reach the value zero. We focus our work in solving this particular case of problems.

CSPs find a lot of "real-world" applications in the industry. In practice, these problems are intractable for classical constraint programming approaches, like *tree search-based solvers* or *backtracking-based solvers* [14], exploring the whole solution space, wish is huge. For that reason, these kind of problems are mostly tackled by *meta-heuristic methods* or hybrid approaches, like *Monte Carlo Tree Search* methods, which combine precision (tree search) with randomness (meta-heuristic) showing good results in artificial intelligence for games [15, 16].

2.2 Constraint propagation

Constraint propagation techniques are methods used to modify a *Constraint Satisfaction Problem* in order to reduce its variables domains, and turning the problem into one that is equivalent, but usually easier to solve [17]. The main goal is to choose one (or some) constraint(s) and analyzing *local consistency*, which means trying to find values in the variables domain which make constraint unsatisfiable, in order to remove them from the domain. The applied procedure to reduce the variable domains is called *reduction function*, and it is applied until a new, "smaller" and easier to solve is obtained, and it can not be further reduced: a *fixed point*.

Chaotic Iterations is a technique, that comes from numerical analysis and adapted for computer science needs, used for computing limits of iterations of finite sets of functions [18, 19]. In [20, 21] a formalization of constraint propagation is proposed through *chaotic iterations*. In [22], a coordination-based chaotic iteration algorithm for constraint propagation is proposed. It is a scalable, flexible and generic framework for constraint propagation using coordination languages, not requiring special modeling of *CSPs*. We can find an implementation of this algorithm in DICE (Distributed Constraint Environment) [23] using the Manifold coordination language. Coordination services implement existing protocols for constraint propagation, termination detection and splitting of *CSPs*. DICE combines these protocols with support for parallel search and the grouping of closely related components into cooperating solvers.

Manifold is a strongly-typed, block-structured, event-driven language for managing events, dynamically changing interconnections among sets of independent, concurrent and cooperative processes. A Manifold application consists of a number of processes running on a

heterogeneous network. Processes in the same application may be written in different programming languages. MANIFOLD has been successfully used in a broad range of applications [24].

In [25] is proposed an implementation of constraint propagation by composition of reductions. It is a general algorithmic approach to tackle strategies that can be dynamically tuned with respect to the current state of constraint propagation, using composition operators. A composition operator models a sub–sequence of an iteration, in which the ordering of application of reduction functions is described by means of combinators for sequential, parallel or fixed–point computation, integrating smoothly the strategies to the model. This general framework provides a good level of abstraction for designing an object-oriented architecture of constraint propagation. Composition can be handled by the *Composite Design Pattern* [26], supporting inheritance between elementary and compound reduction functions. The propagation mechanism uses the *Observer (Listener) Design Pattern* [27], that makes the connection between domain modifications and re–invocation of reduction functions (event-based relations between objects); and the generic algorithm has been implemented using the *Strategy Design Pattern* [28], that allows to parametrize parts of algorithms.

A propagation engine prototype with a *Domain Specific Language* (DSL) was implemented in [29]. It is a solver–independent language able to configure constraint propagations at the modeling stage. The main contributions are a DSL to ease configure constraint propagation engines, and the exploitation of the basic properties of DSL in order to ensure both completeness and correctness of the produced propagation engine, like: i) *Solver independent description*: The DSL does not rely on specific solver requirements (but assuming that solvers provide full access to variable and propagator properties), ii) *Expressivity*: The DSL covers commonly used data structures and characteristics, iii) *Extensibility*: New attributes can be introduced to make group definition more concise. New collections and iterators can provide new propagation schemes, iv) *Unique propagation*: The top-bottom left-right evaluation of the DSL ensures that each arc is only represented once in the propagation engine.

Some characteristics are required to fully benefit from the DSL. Due to their positive impact on efficiency, modern constraint solvers already implement these techniques: i) Propagators are discriminated thanks to their priority (deciding which propagator to run next): lighter propagators (in the complexity sense) are executed before heavier ones. ii) A controller propagator is attached to each group of propagators. iii) Open access to variable and propagator properties: for instance, variable cardinality, propagator arity or propagator priority.

To be more flexible and more accurate, they assume that all arcs from the current *CSP*, are explicitly accessible. This is achieved by explicitly representing all of them and associating

them with *watched literals* [30] (controlling the behavior of variable–value pairs to trigger propagation) or *advisors* [31] (a method for supporting incremental propagation in propagator–centered setting). *Advisors* in [31] are used to modify propagator state and to decide whether a propagator must be propagated or "scheduled". They also present a concrete implementation of the DSL based on *Choco* [32] (an open source java constraint programming library) and an extension of the *MiniZinc*, a simple but expressive constraint programming modeling language which is suitable for modeling problems for a range of solvers. It is the most used language for codding *CSP*s [33].

However, we can not solve some *CSPs* only applying constraint propagation techniques. It is necessary to combine them with other methods.

2.3 Meta-heuristic methods

Meta-heuristic Methods are non problem-specific techniques that efficiently explore the search space in order to find the solution, and can often find them with less computational effort than iterative methods, so an effective way to face the *CSPs*. Their algorithms are approximate and usually non-deterministic.

A *Meta-heuristic Method* is formally defined as an iterative generation process which guides a subordinate heuristic by combining smartly different concepts for *exploring* (also called *diversification*, is guiding the search process through a much larger portion of the search space with the hope of finding promising solutions that are not yet visited) and *exploiting* (also called *intensification*, is guiding the search process into a limited, but promising, region of the search space with the hope of improving a promising already found solution) the search space (the finite set of candidate solutions or configurations) [3], avoiding getting trapped in lost areas of the search space (local minimums). Sometimes they may make use of domain-specific knowledge in the form of heuristics that are controlled by the upper level strategy. Nowadays more advanced meta-heuristics use search experience to guide the search [34].

They are often nature-inspired and are divided in two groups [4]:

a) Single Solution Based: more exploitation oriented, intensifying the search in some specific areas. (this work focuses its attention on this first group)

b) Population Based: more exploration oriented, identifying areas of the search space where there are (or where there could be) the best solutions.

2.3.1 Single Solution Based Meta-heuristic

Methods of the first group are also called *trajectory methods*, and they are based on choosing a solution taking into account some criterion (usually random), and they move from a solution to his *neighbor*, following a trajectory into the search space. They can be seen as an intelligent extension of *Local Search Methods* [4]. Local Search Methods are the most widely used approaches to solve *Combinatorial Optimization Problems* because they often produces high–quality solutions in reasonable time.

Simulated Annealing (SA) [35] is one of the first algorithms with an explicit strategy to scape from local minima. Is a method inspired by the annealing technique used by the metallurgists to obtain a "well ordered" solid state of minimal energy. Its main feature is to allow moves resulting in solutions of worse quality than the current solution, in order to scape from local minima, under certain probability, which is decreased during the search process [34]. In [36] is presented a SA algorithm (TTSA) for the Traveling Tournament Problem (TPP) that explores both feasible and infeasible schedules that includes advanced techniques such as strategic oscillation to balance the time spent in the feasible and infeasible regions, by varying the penalty for violations; and reheats (increasing the temperature again) to balance the exploration of the feasible and infeasible regions and to escape local minima.

Tabu Search (TS) [37], is among the most used meta-heuristics for *Combinatorial Optimization Problems*. It explicitly maintain a history of the search, as a short term memory keeping track of the most recently visited solutions, to scape from local minima, to avoid cycles, and to deeply explore the search space. A TB meta-heuristic guides the search on the approach presented in [38] to solve instances of the *Social Golfers* problem.

The idea of *Guided Local Search* (GLS) [39] is to dynamically change the objective function to help the search to gradually scape from local minima, by changing the search landscape. The set of solutions and the neighborhood are fixed, while the objective function is dynamically changed with the aim of making the current local optimum less attractive [34]. In [40] an implementation of a GLS is used to solve the satisfiability (SAT) problem, which is a special case of a *CSP* where the variables take booleans values an the constraints are disjunctions (logical OR) of literals (variables or theirs negations).

The *Variable Neighborhood Search* (VNS) is another meta-heuristic that systematically changes the size of neighborhood during the search process. These neighborhoods can be arbitrarily chosen, but often a sequence $|\mathcal{N}_1| < |\mathcal{N}_2| < \cdots < |\mathcal{N}_{k_{max}}|$ of neighborhoods with increasing cardinality is defined. The choice of neighborhoods of increasing cardinality yields a progressive diversification of the search [41, 34]. In [42] is introduced a *generalized Variable*

Neighborhood Search for *Combinatorial Optimization Problems*, and in [43] is presented a model combining integer programming and VNS for *Constrained Nurse Rostering* problems.

One meta-heuristic that can be efficiently implement on parallel processors is *Greedy Randomized Adaptive Search Procedures* (GRASP). GRASP is an iterative randomized sampling technique in which each iteration provides a solution to the target problem at hand through two phases (constructive and search) within each iteration: the first smartly constructs an initial solution via an adaptive randomized greedy function, and the second applies a local search procedure to the constructed solution in to find an improvement [44]. GRASP does not make any smart use of the history of the search process. It only stores the problem instance and the best found solution. That is why GRASP is often outperformed by other meta-heuristics [34]. However, in [45] some extensions like alternative solution construction mechanisms and techniques to speed up the search are presented.

Galinier et al. present in [46] a general approach for solving constraint based problems by local search. In this work, authors present the concept of *penalty functions*, that we pick up in order to write a *CSP* as an *Unrestricted Optimization Problem* (UOP). This formulation was useful in this thesis for modeling the tackled benchmarks. In this formulation, the *objective function* of this new problem must be such that its set of optimal solutions is equal to the solution set of the original (associated) *CSP*.

Definition 3 (Local penalty function) *Let a* ***CSP*** $\mathcal{P}\langle X, D, C\rangle$ *and a configuration* S *be. We define the operator* **local penalty function** *as follow:*

$$\omega_i : D(X) \times 2^{D(X)} \rightarrow \mathbb{R}^+ \cup 0 \text{ where:}$$
$$\omega_i(S, c_i) = \begin{cases} 0 & \text{if} \quad c_i(S) \text{ is true} \\ k \in \mathbb{R}^+ & \text{if not} \end{cases}$$

This penalty function defines the cost of a configuration with respect to a given constraint. In consequence, we define the *global penalty function*, to define the cost of a configuration with respect to all constraint on a *CSP*:

Definition 4 (Global penalty function) *Let a* ***CSP*** $\mathcal{P}\langle X, D, C\rangle$ *and a configuration* S*. We define the operator* **global penalty function** *as follows:*

$$\Omega : D(X) \times 2^{D(X)} \rightarrow \mathbb{R}^+ \cup 0 \text{ where:}$$
$$\Omega(S, C) = \sum_{i=1}^{m} \omega_i(S, c_i)$$

We can now formulate a *Constraint Satisfaction Problem* as an *UOP*:

Definition 5 (CSP's Associated Unrestricted Optimization Problem) *Given a* ***CSP*** $\mathcal{P}\langle X, D, C\rangle$ *we define its* **associated Unrestricted Optimization Problem** $\mathcal{P}_{opt}\langle X, D, f\rangle$ *as follows:*

$$\min_{X} f(X, C)$$

Where: $f(X, C) \equiv \Omega(X, C)$ *is the objective function to be minimized over the variable* X

It is important to note that a given S is optimum if and only if $f(S, C) = 0$, which means that S satisfies all the constrains in the original *CSP* $\mathcal{P}$.

Adaptive Search is also another efficient algorithm based *local search method*, that takes advantage of the structure of the problem in terms of constraints and variables. It uses also the concept of *penalty function* and relies on iterative repair, based on this information, seeking to reduce the *error* (a projected cost of a variable, as a measure of how responsible is the variable in the cost of a configuration) on the worse variable so far. It computes the penalty function of each constraint, then combines for each variable the *errors* of all constraints in which it appears. This allows to chose the variable with the maximal *error* will be chosen as a "culprit" and thus its value will be modified for the next iteration with the best value, that is, the value for which the total error in the next configuration is minimal [5, 47, 48]. In [49] Munera et al. based their solution method in *Adaptive Search* to solve the *Stable Marriage with Incomplete List and Ties* problem [50], a natural variant of the *Stable Marriage Problem* [51].

Michel and Van Hentenryck [52] propose a constraint-based, object-oriented, architecture to reduce the development time of local search algorithms significantly. The architecture consists of two main components: a declarative component which models the application in terms of constraints and functions, and a search component which specifies the meta-heuristic. Its main feature is to allow practitioners to focus on high-level modeling issues and to relieve them from many tedious and error-prone aspects of local search. The architecture is illustrated using COMET, an optimization platform that provides a Java-like programming language to work with constraint and objective functions (a high level constraint programming) [53, 54], that supports the local search architecture with a number of novel concepts, abstractions, and control structures.

2.3.2 Population Based Meta-heuristic

Also there exist heuristic methods based on populations. These methods do not work with a single solution, but with a set of solutions named *population*. In this other group we can find the *Evolutionary Algorithms*. This is the general definition to name the algorithms inspired

by the "Darwin's principle", that says that only the best adapted individuals will survive. They involve *operators* to handle the population to guide it through the search process. The evolutionary algorithm's operators are another branch of study, because they have to be selected properly according to the specific problem, due to they will play an important roll in the algorithm behavior [55].

Probably the most popular in this group are the *Genetic Algorithms* [56], and theirs operators are based on the simulation of the genetic variation process to achieve individuals (solutions in this case) more adapted; and the *Ant Colony* algorithms [57], that simulate the behavior of an ant swarm to find the shortest path from the food source to the nest.

2.4 Hyper-heuristic Methods

Hyper-heuristics are automated methodologies for selecting or generating heuristics to solve hard computational problems [58]. This can be achieved with a learning mechanism that evaluates the quality of the algorithm solutions, in order to become general enough to solve new instances of a given problem. *Hyper-heuristics* are related with the *Algorithm Selection Problem*, so they establish a close relationship between a problem instance, the algorithm to solve it and its performance [59].

Hyper-heuristic frameworks are also known as Algorithm-Portfolio-based frameworks, and their goal is predicting the running time of algorithms using statistical regression. Then the fastest predicted algorithm is used to solved the problem until a suitable solution is found or a time-out is reached [60]. In [61] is presented a *Simple Neighborhood-based Algorithm Portfolio* written in *Python* (Snappy), a very recent framework. Its aim is to provide a tool that can improve its own performances through on-line learning. Instead of using the traditional off-line training step, a neighborhood search predicts the performance of the algorithms.

HYPERION^2 [62] is a Java^{TM} framework for meta– and hyper– heuristics which allows the analysis of the trace taken by an algorithm and its constituent components through the search space, built with the principles of interoperability, generality and efficiency. The main goals of HYPERION^2 are:

a) Promoting interoperability via component interfaces,

b) Allowing rapid prototyping of meta– and hyper– heuristics, with the potential to use the same source code in either case,

c) Providing generic templates for a variety of local search and evolutionary computation algorithms,

d) Making easier the construction of novel meta- and hyper- heuristics by hybridization (via interface interoperability) or extension (subtype polymorphism),

e) *Only pay for what you use* – a design philosophy that attempts to ensure that generality doesn't necessarily imply inefficiency.

hMod is inspired by the previous frameworks, and using a new object-oriented architecture, encodes the core of the hyper-heuristic in several modules, referred as algorithm containers. *hMod* directs the programmer to define the heuristic using two separate XML files; one for the heuristic selection process and another for the acceptance criteria [63].

Evolving evolutionary algorithms are specialized hyper-heuristic method which attempt to readjust an evolutionary algorithm to the problem needs. An Evolutionary Algorithm (EA) discover the rules and knowledge, to find the best algorithm to solve a problem. In [64] is used linear genetic programming and multi-expression genetic programming, to optimize the EA solving unimodal mathematical functions and another EA adjusts the sequence of genetic and reproductive operators. A solution consists of a new evolutionary algorithm capable of outperforming genetic algorithms when solving a specific class of unimodal test functions.

2.5 Hybridization

The *Hybridization* approach is the one who combine different approaches into the same solution strategy, and recently, it leads to very good results in the constraint satisfaction field. For example, constraint propagation may find a solution to a problem, but they can fail even if the problem is satisfiable, because of its local nature. At each step, the value of a small number of variables are changed, with the overall aim of increasing the number of constraints satisfied by this assignment, and applying other techniques to avoid local solutions, for example adding a stochastic component to choose variables to affect. Integrations of global search (complete search) with local search have been developed, leading to hybrid algorithms.

Hooker J.N. presents in [65] some ideas to illustrate the common structure present in exact and heuristic methods, to encourage the exchange of algorithmic techniques between them. The goal of this approach is to design solution methods ables to smoothly transform its strategy from exhaustive to non-exhaustive search as the problem becomes more complex.

In [66] a taxonomy of hybrid optimization algorithms is presented in an attempt to provide a mechanism to allow qualitative comparison of hybrid optimization algorithms, combining meta-heuristics with other optimization algorithms from mathematical programming, machine learning and constraint programming.

Monfroy et al. present in [67, 68] a general hybridization framework, proposed to combine complete constraints resolution techniques with meta-heuristic optimization methods in order to reduce the problem through domain reduction functions, ensuring not loosing solutions. Other interesting ideas are TEMPLAR, a framework to generate algorithms changing predefined components using hyper-heuristics methods [69]; and *ParadisEO*, a framework to design parallel and distributed hybrid meta-heuristics showing very good results [70], including a broad range of reusable features to easily design evolutionary algorithms and local search methods.

Another technique has been developed, the called *autonomous search*, based on the supervised or controlled learning. This systems improve their functioning while they solve problems, either modifying their internal components to take advantage of the opportunities in the search space, or to adequately chose the solver to use (*portfolio point of view*) [71].

In [72] is proposed another portfolio-based technique, *time splitting*, to solve optimization problems. Given a problem P and a schedule $Sch = [(\Sigma_1, t_1), \ldots, (\Sigma_n, t_n)]$ of n solvers, the corresponding time-split solver is defined as a particular solver such that:

a) runs Σ_1 on P for a period of time t_1,

b) then, for $i = 1, \ldots, n-1$, runs Σ_{i+1} on P for a period of time t_{i+1} exploiting or not the best solution found by the previous solver Σ_i t_i units of time.

In [73] is proposed a tool (`xcsp2mzn`) for converting problem instances from the XCSP[i] format [74] to MINIZINC that is a simple but expressive constraint programming modeling language which is suitable for modeling problems for a range of solvers. It is the most used language for codding *CSPs* [33]. The second contribution of this work is the development of `mzn2feat` a tool to extract static and dynamic features from the MINIZINC representation, with the help of the GECODE[ii] [75] interpreter, and allows a better and more accurate selection of the solvers to be used according to the instances to solve. Some results are showed proposing that the performances that can be obtained using these features are competitive with state of the art on *CSP* portfolio techniques.

[i] Is a XML-like language for coding *CSPs*. Is not more used than MINIZINC but although it was mainly used as the standard in the *International Constraint Solver Competition* (ended in 2009), the *ICSC* dataset is for sure the biggest dataset of CSP instances existing today

[ii] Is an efficient open source environment for developing constraint-based system and applications.

2.6 Parallel computing

Parallel computing is a way to solve problems using some calculus resources at the same time. It is a powerful alternative to solve problems which would require too much time by using the traditional ways, i.e., sequential algorithms [76]. That is why this field is in constant development and it is the topic where I put most of our effort.

For a couple of years, all processors in modern machines are multi-core. Massively parallel architectures, previously expensive and so far reserved for super–computers, become now a trend available to a broad public through hardware like the Xeon Phi or GPU cards. The power delivered by massively parallel architectures allow us to treat faster these problems [77]. However this architectural evolution is a non-sense if algorithms do not evolve at the same time: the development and the implementation of algorithms should take this into account and tackling the problems with very different methods, changing the sequential reasoning of researchers in Computer Science [78, 79]. We can find in [80] a survey of the different parallel programming models and available tools, emphasizing on their suitability for high-performance computing.

Falcou propose in [81] a programming model: *Parallel Algorithmic Skeletons* (along with a C++ implementation called Quaff to make parallel application development easier. Writing efficient code for parallel machines is less trivial, as it usually involves dealing with low-level APIs such as OpenMP, message-passing interfaces (MPI), among others. However, years of experience have shown that using those frameworks is difficult and error-prone. Usually many undesired behaviors (like deadlocks) make parallel software development very slow compared to the classic, sequential approach. In that sense, this model is a high-order pattern to hide all low-level, architecture or framework dependent code from the user, and provides a decent level of organization. Quaff is a skeleton-based parallel programming library, which has demonstrated its efficiency and expressiveness solving some application from computer vision, that relies on C++ template meta-programming to reduce the overhead traditionally associated with object-oriented implementations of such libraries: the code generation is done at compilation time.

The contribution in terms of hardware has been crucial, achieving powerful technologies to perform large–scale calculations. But the development of the techniques and algorithms to solve problems in parallel is also visible, focusing the main efforts in three fundamentals concepts:

a) Problem subdivision,

b) Scalability and

c) Inter-process communication.

In a preliminary review of literature on parallel constraint solving [82], addressing the literature in constraints on exploitation of parallel systems for constraint solving, is starting first by looking at the justification for the multi-core architecture. It presents an analysis of some limiting factors on performance such as *Amdahl*'s law, and then reviews recent literature on parallel constraint programming, grouping the paper in four areas: i) parallelizing the search process, ii) parallel and distributed arc-consistency, iii) multi-agent and cooperative search and iv) combined parallel search and parallel consistency.

The issue of sub-dividing a given problem in some smaller sub-problems is sometimes not easy to address. Even when we can do it, the time needed by each process to solve its own part of the problem is rarely balanced. In [83] are proposed some techniques to tackle this problem, taking into account that sometimes, the more can be sub-divided a problem, the more balanced will be the execution times of the process. In [84] is presented an comparison between Transposition-table Driven Scheduling (TDS) and a parallel implementation of a best-first search strategy (Hash Distributed A*), that uses the standard approach of of *Work Stealing* for partitioning the search space. This technique is based on maintaining a local work queue, (provided by a *root process* through hash-based distribution that assign an unique processor to each work) accessible to other process that "steal" work from if they become unoccupied. The same approach is used in [85] to evaluate *Zobrist Hashing*, an efficient hash function designed for table games like chess and Go, to mitigate communication overheads.

In [86] is presented a study of the impact of space-partitioning techniques on the performance of parallel local search algorithms to tackle the *k-medoids* clustering problem. Using a parallel local search, this work aims to improve the scalability of the sequential algorithm, which is measured in terms of the quality of the solution within the same time with respect to the sequential algorithm. Two main techniques are presented for domain partitioning: first, *space-filling curves*, used to reduce any N-dimensional representation into a one-dimension space (this technique is also widely used in the nearest-neighbor-finding problem [87]); and second, *k-Means* algorithm, one of the most popular clustering algorithms [88].

In [89] is proposed a mechanism to create sub-*CSPs* (whose union contains all the solutions of the original *CSP*) by splitting the domain of the variables though communication between processes. The contribution of this work is explained in details in Section 2.7.

Related to the search process, we can find two main approaches. First, the *single walk* approach, in which all the processes try to follow the same path towards the solution, solving their corresponding part of the problem, with or without cooperation (communication). The other is known as *multi walk*, and it proposes the execution of various independent processes to find the solution. Each process applies its own strategies (portfolio approach) or simply

explores different places inside the search space. Although this approach may seem too trivial and not so smart, it is fair to say that it is in fashion due to the good obtained results using it [5].

Scalability is the ability of a system to handle the increasing growth of workload. A system which has improved over time its performance after adding work resources, and it is capable of doing it proportionally is called *scalable*. The increase has not been only in terms of calculus resources, but also in the amount of sub-problems coming from the sub-division of the original problem. The more we can divide a problem into smaller sub-problems, the faster we can solve it [90]. *Adaptive Search* is a good example of local search method that can scale up to a larger number of cores, e.g., a few hundreds or even thousands [5]. For this algorithm, an implementation of a cooperative multi-walks strategy has been published in [6]. In this framework, the processes are grouped in teams to achieve search intensification, which cooperate with others teams through a head node (process) to achieve search diversification. Using an adaptation of this method, Munera et al. propose a parallel solution strategy able to solve hard instances of *Stable Marriage with Incomplete List and Ties Problem* quickly. In [91] is presented a combination of this method with an *Extremal Optimization* procedure: a nature-inspired general-purpose meta-heuristic [92].

A lot of studies have been published around this topic. A parallel solver for numerical *CSPs* is presented in [93] showing good results scaling on a number of cores. In [94], an estimation of the speed-up (a performance measure of a parallel algorithm) through statistical analysis of its sequential algorithm is presented. This is a very interesting result because it a way to have a rough idea of the resources needed to solve a given problem in parallel.

Another issue to treat is the interprocess communication. Many times a close collaboration between process is required, in order to achieve the solution. But the first inconvenient is the slowness of the communication process. Some work have achieved to identify what information is viable to share. One example is [95] where an idea to include low-level reasoning components in the SAT problems resolution is proposed. This approach allow us to perform the clause-shearing, controlling the exchange between any pair of process.

In [6] is presented a new paradigm that includes cooperation between processes, in order to improve the independent multi-walk approach. In that case, cooperative search methods add a communication mechanism to the independent walk strategy, to share or exchange information between solver instances during the search process. This proposed framework is oriented towards distributed architectures based on clusters of nodes, with the notion of *teams* running on nodes and controlling several search engines (*explorers*) running on cores, and the idea that all teams are distributed and thus have limited inter–node communication. The communication between teams ensures diversification, while the communication between explorers is needed for intensification. This framework is oriented towards distributed

architectures based on clusters of nodes, where teams are mapped to nodes and explorers run on cores. This framework was developed using the *X10 programming language*, which is a novel language for parallel processing developed by IBM Research, giving more flexibility than traditional approaches, e.g., MPI communication package.

In [96] have been presented an implementation of the meta-solver framework which coordinates the cooperative work of arbitrary pluggable constraint solvers. This approach intents to integrate arbitrary, new or pre–existing constraint solvers, to form a system capable of solving complex mixed–domain constraint problems. The existing increased cooperation overhead is reduced through problem-specific cooperative solving strategies.

In [97] is proposed the first *Deterministic Parallel DPLL* (A complete, backtracking-based search algorithm for deciding the satisfiability of propositional logic formulas in conjunctive normal form) engine. The experimental results show that their approach preserves the performance of the parallel portfolio approach while ensuring full reproducibility of the results. Parallel exploration of the search space, defines a controlled environment based on a total ordering of solvers interactions through synchronization barriers. To maximize efficiency, information exchange (conflict-clauses) and check for termination are performed on a regular basis. The frequency of these exchanges greatly influences the performance of the solver. The paper explores the trade off between frequent synchronizing which allows the fast integration of foreign conflict–clauses at the cost of more synchronizing steps, and infrequent synchronizing at the cost of delayed foreign conflict–clauses integration.

Considering the problem of parallelizing restarted back–track search, in [98] was developed a simple technique for parallelizing restarted search deterministically and it demonstrates experimentally that it can achieve near–linear speed–ups in practice, when the number of processors is constant and the number of restarts grows to infinity. They propose the following: Each parallel search process has its own local copy of a scheduling class which assigns restarts and their respective fail–limits to processors. This scheduling class computes the next *Luby* restart fail–limit and adds it to the processor that has the lowest number of accumulated fails so far, following an *earliest–start–time–first strategy*. Like this, the schedule is filled and each process can infer which is the next fail–limit that it needs to run based on the processor it is running on – without communication. Overhead is negligible in practice since the scheduling itself runs extremely fast compared to CP search, and communication is limited to informing the other processes when a solution has been found.

In [99], were explored the two well–known principles of diversification and intensification in portfolio–based parallel SAT solving. To study their trade–off, they define two roles for the computational units. Some of them classified as *Masters* perform an original search strategy, ensuring diversification. The remaining units, classified as *Slaves* are there to intensify their master's strategy. There are some important questions to be answered: i) what

information should be given to a slave in order to intensify a given search effort?, ii) how often, a subordinated unit has to receive such information? and iii) the question of finding the number of subordinated units and their connections with the search efforts? The results lead to an original intensification strategy which outperforms the best parallel SAT solver *ManySAT*, and solves some open SAT instances.

Multi-objective optimization problems involve more than one objective function to be optimized simultaneously. Usually these problems do not have an unique optimal solution because the exist a trade-off between one objective function and the others. For that reason, in a multi-objective optimization problem, the concept of Pareto optimal points is used. A Pareto optimal point is a solution that improving one objective function value, implies the deterioration of at least one of the other objective function. A collection of Pareto optimal points defines a Pareto front. In [100], is proposed a new search method, called *Multi-Objective Embarrassingly Parallel Search* (MO–EPS) to solve multi-objective optimization problems, based on: i) Embarrassingly Parallel Search (EPS): where the initial problem is split into a number of independent sub-problems, by partitioning the domain of decision variables [83, 101]; and ii) Multi-Objective optimization adding cuts (MO–AC): an algorithm that transforms the multi-objective optimization problem into a feasibility one, searches a feasible solution and then the search is continued adding constraints to the problem until either the problem becomes infeasible or the search space gets entirely explored [102].

A component-based constraint solver in parallel is proposed in [103]. In this work, a parallel solver coordinates autonomous instances of a sequential constraint solver, which is used as a software component. The component solvers achieve load balancing of tree search through a time-out mechanism. It is implemented a specific mode of solver cooperation that aims at reducing the turn-around time of constraint solving through parallelization of tree search. The main idea is to try to solve a *CSP* before a time-out. If it can not find a solution, the algorithm defines a set of disjoint sub-problems to be distributed among a set of solvers running in parallel. The goal of the time-out mechanism is to provide an implicit load balancing: when a solver is busy, and there are no subproblems available, another solver produces new sub-problems when its time-out elapses.

2.7 Solvers cooperation

The interaction between solvers exchanging some information is called *solver cooperation* and it is very popular in this field due to their good results. Its main goal is to improve some kind of limitations or inefficiency imposed by the use of unique solver. In practice, each solver runs in a computation unit, i.e. thread or processor. The cooperation is performed

through inter–process communication, by using different methods: *signals*, asynchronous notifications between processes in order to notify an event occurrence; *semaphore*, an abstract data type for controlling access, by multiple processes, to a common resource; *shared memory*, a memory simultaneously accessible by multiple processes; *message passing*, allowing multiple programs to communicate using messages; among others.

Kishimoto et al. present in [104] a parallelization of the an algorithm A* (Hash Distributed A*) for *optimal sequential planning* [105], exploiting distributed memory computers clusters, to extract significant speedups from the hardware. In classical planning solving, both the memory and the CPU requirements are main causes of performance bottlenecks, so parallel algorithms have the potential to provide required resources to solve changeling instances. In [84], authors study scalable parallel best-first search algorithms, using MPI, a paradigm of *Message Passing Interface* that allows parallelization, not only in distributed memory based architectures, but also in shared memory based architectures and mixed environments (clusters of multi-core machines) [106].

In [107] is presented a paradigm that enables the user to properly separate computation strategies from the search phases in solver cooperations. The cooperation must be supervised by the user, through *cooperation strategy language*, which defines the solver interactions in order to find the desired result.

In [95], an idea to include low-level reasoning components in the SAT problems resolution is proposed, dynamically adjusting the size of shared clauses to reduce the possible blow up in communication. [107] presents a paradigm that enables the user to properly separate strategies combining solver applications in order to find the desired result, from the way the search space is explored.

Meta–S is an implementation of a theoretical framework proposed in [96], which allows to tackle problems, through the cooperation of arbitrary domain–specific constraint solvers. *Meta–S* [96] is a practical implementation and extension of a theoretical framework, which allows the user to attack problems requiring the cooperation of arbitrary domain–specific constraint solvers. Through its modular structure and its extensible strategy specification language it also serves as a test–bed for generic and problem–specific (meta–)solving strategies, which are employed to minimize the incurred cooperation overhead. Treating the employed solvers as black boxes, the meta–solver takes constraints from a global pool and propagates them to the individual solvers, which are in return requested to provide newly gained information (i.e., constraints) back to the meta–solver, through variable projections. The major advantage of this approach lies in the ability to integrate arbitrary, new or pre–existing constraint solvers, to form a system that is capable of solving complex mixed–domain constraint problems, at the price of increased cooperation overhead. This overhead can however be reduced through more intelligent and/or problem–specific cooperative solving

strategies. HYPERION [62] is an already mentioned framework for meta– and hyper–heuristics built with the principle of interoperability, generality by providing generic templates for a variety of local search and evolutionary computation algorithms and efficiency, allowing rapid prototyping with the possibility of reusing source code.

Arbab and Monfory propose in [89] a technique to guide the search by splitting the domain of variables. A *Master* process builds the network of variables and domain reduction functions, and sends this informations to the worker agents. They workers concentrate their efforts on only one sub-CSP and the *Master* collects solutions. The main advantage is that by changing only the search agent, different kinds of search can be performed. The coordination process is managing using the MANIFOLD coordination language [24].

2.8 Parameter setting techniques

Most of these methods to tackle combinatorial problems, involve a number of parameters that govern their behavior, and they need to be well adjusted, and most of the times they depend on the nature of the specific problem, so they require a previous analysis to study their behavior [108]. That is way another branch of the investigation arises: *parameter tuning*. It is also known as a meta optimization problem, because the main goal is to find the best solution (parameter configuration) for a program, which will try to find the best solution for some problem as well. In order to measure the quality of some found parameter setting for a program (solver), one of these criteria are taken into consideration: the speed of the run or the quality of the found solution for the problem that it solves.

The are tow classes to classify these methods:

a) *Off-line tunning*: Also known just as parameter tuning, were parameters are computed before the run.

b) *On-line tunning*: Also known as parameter control, were parameters are adjusted during the run, and

2.8.1 Off-line tunning

The technique of parameter tuning or off-line tunning, is used to computed the best parameter configuration for an algorithm before the run (solving a given instance of a problem), to obtain the best performance. Most of algorithms are very sensible to their parameters. This

is the case of Evolutionary Algorithms (EA), were some parameters define the behavior of the algorithm. In [109] is presented a study of methods to tune these algorithms.

In [110] is presented *EVOCA*, a tool which allows meta-heuristics designers to obtain good results searching a good parameter configuration with no too much effort, by using the tool during the iterative design process. Holger H. Hoos highlights in [111] the efficacy of the technique named *racing procedure*, that is based on choosing a set of model problems and adjusting the parameters through a certain number of solver runs, discarding configurations that show a behavior substantially worse than the best already obtained so far.

ParamsILS (version 2.3) is a tool for parameter optimization for parametrized algorithms, which uses powerful stochastic local search methods and it has been applied with success in many combinatorial problems in order to find the best parameter configuration [112]. It is an open source program written in *Ruby*, and the public source include some examples and a detailed and complete User Guide with a compact explanation about how to use it with a specific solver [113].

Revac is a method based on information theory to measure parameter relevance, that calibrates the parameters of EAs in a robust way. Instead of estimating the performance of an EA for different parameter values, the method estimates the expected performance when parameter values are chosen from a given probability density distribution C. The method iteratively refines the probability distribution C over possible parameter sets, and starting with a uniform distribution C_0 over the initial parameter space $\mathcal{X}$, the method gives a higher and higher probability to regions of $\mathcal{X}$ that increase the expected performance of the target EA [114]. In [115] is presented a case study demonstrating that using the Revac the "world champion" EA (the winner of the CEC-2005 competition) can be improved with few effort.

Another technique was successfully used to tune automatically parameters for EAs, through a model based on a *case-based reasoning* system. It attempts to imitate the human behavior in solving problems: look in the memory how we have solved a similar problem [116] .

2.8.2 On-line tunning

Although parameter tunning shows to be an effective way to adjust parameters to sensibles algorithms, in some problems the optimal parameter settings may be different for various phases of the search process. This is the main motivation to use on-line tuning techniques to find the best parameter setting, also called *Parameter Control Techniques*. Parameter control techniques are further divided into i) *deterministic parameter control*, where the value of a strategy parameter is altered by some deterministic rule, ignoring any feedback; ii) *adaptive parameter control*, which continually update their parameters using feedback

from the population or the search, and this feedback is used to determine the direction or magnitude of the parameter changes; and iii) *self-adaptive parameter control*, which assign different parameters to each individual, Here the parameters to be adapted are coded into the chromosomes that undergo mutation and recombination, but these parameters are coded into the chromosomes that undergo mutation and recombination [117].

Differential Evolution (DE) algorithm has been demonstrated to be an efficient, effective and robust optimization method. However, its performance is very sensitive to the parameters setting, and this dependency changes from problem to problem. The selection of proper parameters for a particular optimization problem is a quite complicate subject, especially in the multi-objective optimization field. This is the reason why many researchers are motivated to develop techniques to set the parameters automatically.

Liu et al. propose in [118] an adaptive approach which uses fuzzy logic controllers to guide the search parameters, with the novelty of changing the mutation control parameter and the crossover during the optimization process. A self-adaptive DE (SaDE) algorithm is proposed in [119], where both trial vector generation strategies and their associated control parameter values are gradually adjusted by learning from the way they have generated their previous promising solutions, eliminating this way the time-consuming exhaustive search for the most suitable parameter setting. This algorithm has been generalized to multi-objective realm, with objective-wise learning strategies (OW-MOSaDE) [120].

Drozdik et al. present in [121] a study of various approaches to find out if one can find an inherently better one in terms of performance and whether the parameter control mechanisms can find favorable parameters in problems which can be successfully optimized only with a limited set of parameters. They focused in the most important parameters: 1) the *scaling factor*, which controls the structure of new invidious; and 2) the *crossover probability*.

Meta-GAs [122] is a genetic self-adapting algorithm, adjusting genetic operators of genetic algorithms. In this paper the authors propose an approach of moving towards a Genetic Algorithm that does not require a fixed and predefined parameter setting, because it evolves during the run.

2.9 Summary and discussion

In this chapter I have presented an overview of the different techniques to solve *Constraint Satisfaction Problems*. Special attention was given to the *local-search meta-heuristics*, as well as *parallel computing*, which are directly related to this investigation.

In contrast with tree-based methods (complete methods), *Meta-heuristic methods* have shown good results solving large and complex *CSPs*, where the search space is huge. They are algorithms applying different techniques to guide the search as direct as possible through the solution. The main contribution of this thesis is presented in Chapter 4, where is proposed a framework to build local-search meta-heuristics combining small functions (*computation modules*) through an operator-based language. *Hybridization* is also an important point in this investigation due to their good results in solving *CSPs*. With the proposed framework, many different solvers can be created using solvers templates (*abstract solvers*), that can be instantiated with different *computation modules*.

The era of multi/many-core computers, and the development of parallel algorithms have opened new ways to solve constraint problems. In this field, the solver cooperation has become a very popular technique. In general, the main goal of parallelism is to improve some limitations imposed by the use of unique solver. The present investigation attempts to show the importance and the success of this technique, by proposing a deep study of some parallel *communication strategies* in Chapter 5.

3

Prior works leading to POSL

In this chapter are presented prior works leading to POSL. *In Section 3.1 we present a brief work where we applied the problem subdivision approach to solve the k-medoids problem in parallel, as a first attempt aiming for the right direction in order to find the proper approach. Finally we present in Section 3.2 a study applying the* ParamILS *tool in order to find the optimum parameter configuration to Adaptive Search solver.*

Contents

3.1 Domain Split

Usually, to solve some problem using parallelism, our first thought is either partitioning the problem into a set of sub-problems, or dividing its search space. In both cases, the idea is to solve a set of problems, all of them smaller and easer than the original one, and combining all the solution to obtain the solution of the original problem. In [86] a study of the impact of space-partitioning techniques on the performance of parallel local search algorithms to tackle the *k-medoids* clustering problem is presented. The authors use a parallel local search, in order to improve the scalability of the sequential algorithm, which is measured, in terms of the quality of the solution, with respect to the sequential algorithm. In this work two main techniques for domain partitioning are presented: *space-filling curves*, used to reduce any n-dimensional representation into a one-dimension space; and *k-Means* algorithm. We found that the used methods for domain partitioning do not take into account the number of clients associated to each new sub-domain. This results in an unbalanced distributions of workload phenomenon. For that reason the goal of this study was designing some ideas to tackle the same problem. [i]

The *k-medoids* problem aims to select a subset M of k points (the medoids) from a set of points S, such that the average distance from any point to its closest medoid is minimized. It finds a lot of applications in the industry, like in resource allocation, data mining, among others. It is quite similar to *k-means* problem, except that the set of medoids M is forced to be a subset of S.

We propose Algorithm 1, which represents the backbone of our idea. This algorithm takes a set of $\mathbb{R}^2$ *points* (representing the locations of the clients) and returns a partition of size K. Such a set of points is called a *domain* and the partition a *sub-domain*. At each intermediate step i, we have a set (list) of sub-domains. The algorithm takes the most populated one, splits it into two (or four, depending on the strategy) new sub-domains and includes them in

[i] This work falls within the framework of the Ulysses project between France and Ireland.

the list. The stop condition will depends on the fallowed approach (see below).

Algorithm 1: Domain_Split

input : U: Set of client locations

output : Q = $\{Q_i\}_{i=1...K}$: K subsets of U

```
1 A ← U
2 Q.Insert(A)
3 repeat
4 |   A ← Q.GetNext() /* It also removes the returned element */
5 |   [a1, a2] ← Split(A)
6 |   Q.Insert(a1, a2)
7 until <some condition>
```

First of all, we make clear some details of the algorithm exposed above:

- `Insert(...)` Inserts a set (or two) into the data structure.
- `GetNext()` Returns the next sub-set to be divided, tacking into account the *split strategy* (see below).
- `Split(...)` Returns two sub-domains as a subdivision of the given domain (parameter).

In the next sub-sections we answer two main question arising at this point:

a) How to split the each sub-domain? (<*Split*> function on line 5) It refers to, given a set of points (locations), how to decide which of them will be included into one sub-domain and which of them into the other.

b) How much to split each sub-domain? (<*some condition*> on line 3) It refers to decide when to stop splitting the domain.

3.1.1 Domain Splitting. General point of view

In order to split the domain, we can think in some approaches, taking into account the number of available cores and the number of metro-nodes we want to place in the system. In the article of A. Arbelaez and L. Quesada a domain split taking into account the number of available cores for parallel calculus is proposed. In our approach we intend to extend this idea keeping in mind also the number of metro-nodes to allocate. Following, we propose three variants to face the problem:

a) **one metro-node per core**: In this variant we can assign one metro-node to each core, and in this case, splitting the domain in K sub-domains (K is the number of

cores). It means that the algorithm will compute the best position for a metro-node in a current sub-domain.

- In this case we only have to replace the line 3 by something like: **for** $i \leftarrow 1$ **to** N **do** ..., where N is the number of metro-nodes.
- The ideal scenario here is when $N = K$ which is not probable at all. So we only should study the case when they are different
- In that case, we need to distribute efficiently the metro-nodes into the domain subdivisions, but here, one possible scenario arise: it can be happen that, depending on the followed domain-split strategy, we were trying to allocate a metro-node into an area with a few clients. This produce a very local point of view of the problem. That is the reason why we propose the following *second variant*.

b) **Incomplete partition**: To split a sub-domain if it can generate sub-sub-domains containing at least C clients. It means, for example, if there exist in list a sub-domain with 8 clients, and the number C is fixed in $C = 5$, this sub-domain can not be divided, because it will generate for sure a sub-sub-domain with less that 5 clients. In this case more than one metro-node would be allocated in some of those specific sub-domains, because we will have more metro-nodes than sub-domains in our model.

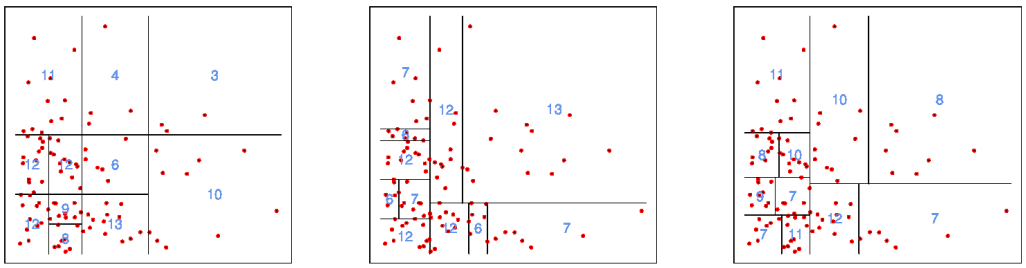
- Of course, using this variant we can find situations described in the first variant. In that case we should proceed consequently.

c) **Combination**: This variant is just a combination of the two previous variants. Then, we will be working with two parameters:

- $C \rightarrow$ Lower bound of clients for new sub-domains. It means that a sub-domain can be divided iff the new produced sub-domains will contains more than C clients.
- $M \rightarrow$ Lower bound of metro-nodes to be allocated in a sub-domain. In this case we will split the domain while it can be ensure that at least M metro-nodes will be assigned to each sub-domain.

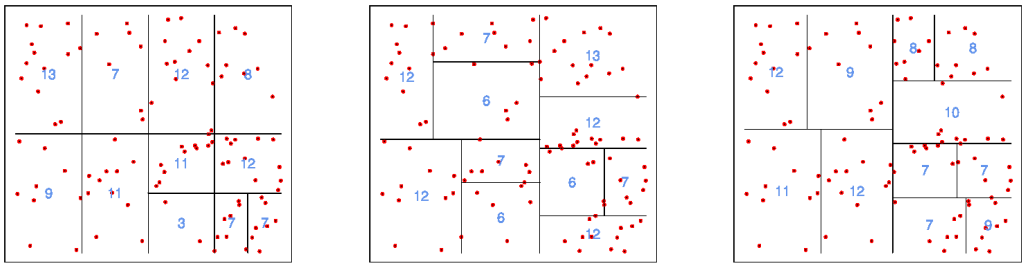
3.1.2 Split strategies

In this sub-section we first discuss three ideas to split the domain. They take a sub-domain and produce other two, dividing the space vertically or horizontally, depending on the shape of the current sub–domain. In other section we expose another strategy to follow, in which the subdivision of a sub-domain produces four sub-domains.



(a) Geometrical Split **(b)** Cardinal Split **(c)** Mean Split

Figure 3.1: Split domain of point normally distributed $\mathcal{N}(0, 0.35)$



(a) Geometrical Split **(b)** Cardinal Split **(c)** Mean Split

Figure 3.2: Split domain of point uniformly distributed

In all of them the number of clients in each sub-domain is taken into account, but in different ways. A common feature between them is that, at the moment of splitting a given sub-domain, it will be done in a perpendicular way (either to the x–axis or to the y–axis, depending on the characteristic of the sub-domain to be divided). The reference point to divide the sub-domain is called the *cut point* of the sub-domain.

a) **Geometrical Split**:

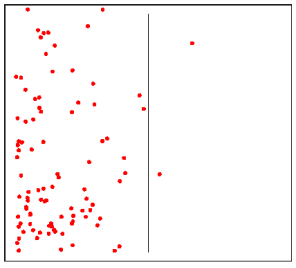


Figure 3.3: If we assume $C = 3$ then this set can't be divided

- Split criterion: *Geometrical* → Dividing the region in tow parts with the same area.
- Cut point: The middle point of the x–axis (or y–axis, depending on the shape)
- Result: Two equal regions, but with different amount of clients (see Figure 3.1a). In the next step, the next sub-domain to divide will be, from those already divided, the most populated one (the sub-domain with more clients).
- Problem: Maybe we need to divide a sub-domain because it has a lot of client, but in the other hand it generate a sub-domain with a few clients, as you can see in the Figure 3.3.
- Benefit: Very fast split. If we attack scenarios with point uniformly distributed, the behavior is pretty much desirable, as we can see in the Figure 3.2a.

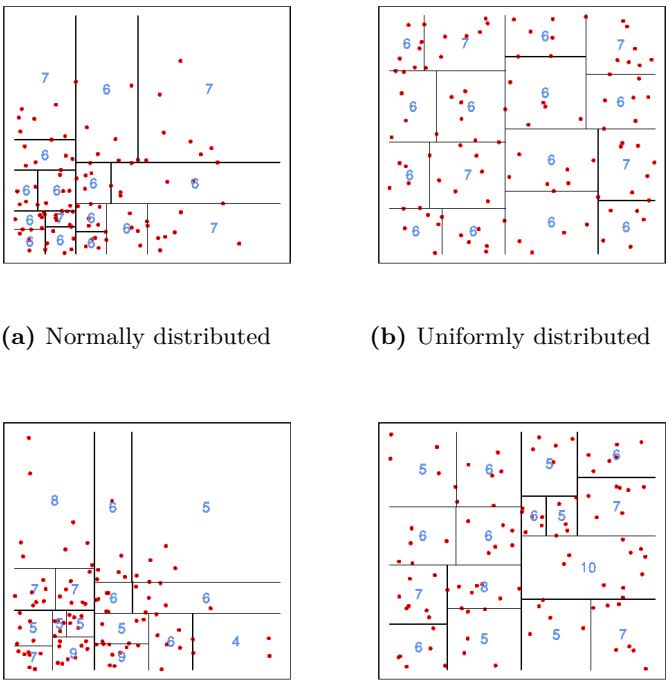
b) **Cardinal Split**

- Split criterion: The number of clients, i.e., a sub-domain is divided in such a way that in the resulting sub-sub-domains there will be the same number of clients.
- Cut point: The current sub-domain is ordered and the x–axis (or y–axis, depending on the shape) of the element (location of the client) right in the center is selected to be the *cut point*
- Result: Two regions with the same (± 1) amount of clients (see Figure 3.1a). In the next step, the next sub-domain to divide will be, from those already divided, the most populated one (the sub-domain with more clients).
- Problem: The subdivision process is a bit more costly: we need to group the clients on both sides of a *perfect pivot*[ii].
- Benefit: It guarantees the same cardinality in both new sub-sub-domains.

c) **Mean Split**

- Split criterion: This is a mid-point strategy between the two previous. The goal is to find a *cut point* to group the elements of the current sub-domain, but in a easy way (fast), for that reason we do not compute the exact middle point to produce two sub-sub-domains with the same cardinality, as in the previous approach, instead of that we work with his expected value: the arithmetic mean.
- Cut point: We compute the mean of the x–axis (or y–axis) of the elements (locations) of the current sub-domain, and it will be the *cut point*

[ii] Element of a set with the same number of elements lower and greater than him



(a) Normally distributed **(b)** Uniformly distributed

(c) Normally distributed **(d)** Uniformly distributed

Figure 3.4: Domain divided in $2^4 = 16$ sub-domains: (a) - (b) Cardinal; (c) - (d) Mean.

- Result: Two regions with not exact information about their sizes or their cardinality.
- Problem: *Idem.*
- Benefit: The *cut point* can be obtained by a $O(n)$ number of operations. As we can see in the preliminary results[iii] (Figures 3.4c and 3.4d), the behavior of this technique is near to the *cardinal split* technique, at least for normally and uniformly distributed sets of point.

3.1.3 Conclusion

In this section we present a theoretical and not validated work where we applied the *search space subdivision* approach to solve *k-medoids problem* in parallel. We have proposed some different strategies and we have showed graphically some characteristics of them.

[iii] The experiments are coded in R[123].

3.2 Tunning methods for local search algorithms

In this section we present our results applying ParamILS (version 2.3)[iv]. ParamILS (first introduced by Hutter, Hoos and St utzle in 2007), is a stochastic local search approach for automated algorithm configuration. The source is available on internet and includes some examples that you can run and see how the tool works. In addition, it brings a complete User Guide with a compact explanation about how to use it with a specific solver [113, 112]. In this study we used it to tune *Adaptive Search* solver[v].

The first step was building a *wrapper* in C++ language, in order to tune more than one problem with the same code. The goal of doing this is using the tool to tune the solver, i.e., finding the best parameter configuration for a specific problem, but also the best parameter configuration to solve any kind of benchmark (a general parameter configuration).

Following we present in Table 3.1 the parameter list that we worked with:

Table 3.1: Adaptive Search parameters

Parameter	Type	Description
-P	PERCENT	probability to select a local min (instead of staying on a plateau)
-f	NUMBER	freeze variables of local min for NUMBER swaps
-F	NUMBER	freeze variables swapped for NUMBER swaps
-l	LIMIT	reset some variables when LIMIT variable are frozen
-p	PERCENT	reset PERCENT of variables

In this section, we explain in details the implementation and the experimentation process.

3.2.1 Using ParamILS

To use the tool ParamILS, we have installed Ruby! 1.8.7 in our computer. We used a laptop Dell XPS 15(Intel Core i7-4702HQ 2.2 GHz, 16384 MB, Dual-channel DDR3L 1600 MHz) with Ubuntu 14.4. To run the tool, we needed to use the following command line:

```
>> ruby param_ils_2_3_run.rb -numRun 0 -scenariofile /.../<scenario_file> -validN 100
```

Where <scenario_file> is the name of the file where we have to put all the information that ParamILS needs to tune the solver (the *tuning scenario file*). We explain its content in the next section.

[iv] Open source program (project) in *Ruby*, available at http://cs.ubc.ca/labs/beta/Projects/ParamILS

[v] An implementation from Daniel Díaz available at https://sourceforge.net/projects/adaptivesearch/

3.2.2 Tuning scenario files

The *tuning scenario file* is a text file with all needed information to tune the solver using ParamILS. It includes where to find the solver binary file, the parameters domains, etc. In our case, the *tuning scenario file* looks like the following:

```
algo = ./QtWrapper_wrapper
execdir = /.../src
deterministic = 0
run_obj = runtime
overall_obj = mean
cutoff_time = 50.0
cutoff_length = max
tunerTimeout = 3600
paramfile = instances/all_intervals-params.txt
outdir = instances/all_intervals-paramils-out
instance_file = instances/.../all_intervals-lower-instances.txt
test_instance_file = instances/.../all_intervals-upper-instances.txt
```

We explain in details each line in this file:

- **algo** → An algorithm executable or a call to a wrapper script around an algorithm that aims the input/output format of *ParamILS* (the wrapper).
- **execdir** → Directory to execute **algo** from: "cd <execdir>; <algo>"
- **run_obj** → A scalar quantifying how "good" a single algorithm execution is, such as its required runtime.
- **overall_obj** → While **run_obj** defines the objective function for a single algorithm run, **overall_obj** defines how those single objectives are combined to reach a single scalar value to compare two parameter configurations. Implemented examples include **mean**, **median**, **q90** (the 90% quantile), **adj__mean** (a version of the mean accounting for unsuccessful runs: total runtime divided by number of successful runs), **mean1000** (another version of the mean accounting for unsuccessful runs: (total runtime of successful runs + 1000 x runtime of unsuccessful runs) divided by number of runs – this effectively maximizes the number of successful runs, breaking ties by the runtime of successful runs; it is the criterion used in most of Frank experiments), and **geomean** (geometric mean, primarily used in combination with **run_obj** = **speedup**. The

empirical statistic of the cost distribution (across multiple instances and seeds) to be minimized, such as the mean (of the single run objectives). [vi]

- **`cutoff_time`** → The time after which a single algorithm execution will be terminated unsuccessfully. This is an important parameter: if chosen too high, lots of time will be wasted with unsuccessful runs. If chosen too low the optimization is biased to perform well on easy instances only.
- **`tunerTimeout`** → The timeout of the tuner. Validation of the final best found parameter configuration starts after the timeout.
- **`paramfile`** → Specifies the file with the parameters of the algorithms.
- **`outdir`** → Specifies the directory *ParamILS* should write its results to.
- **`instance_file`** → Specifies the file with a list of training instances.
- **`test_instance_file`** → Specifies the file with a list of test instances.

Another important file that we have to compose properly is the *algorithm parameter file*, just following the instruction from [113] *–[...] each line lists one parameter, in curly parentheses the possible values considered, and in square parentheses the default value [...].* Our *algorithm parameter file* looks like follows:

```
P {20, 25, 30, 35, 40, 45, 50, 55, 60} [50]
f {0, 1, 2, 3} [1]
F {0, 1, 2, 3} [0]
l {0, 1, 2, 3} [1]
p {1, 2, 3, 5, 10, 20} [5]
```

In the current *Adaptive Search* implementation, the solver binary file and the problem instance are the same thing. It means that we only have to use the following command to solve the *All–intervals* problem of size K, for example:

```
>> ./all-intervals K
```

So, to use ParamILS we modified a little the code: now our solver takes the size parameter from a text file. That way, the instance file is a text file only containing a number.

The solver we want to tune using ParamILS (*Adaptive Search* in this case), must aims specific input/output rules. For that reason instead of modifying the current code of *Adaptive Search* implementation, we preferred to build a wrapper.

[vi] We use **mean** but maybe we can experiment with other values

3.2.3 Building the wrapper

The algorithm executable must follow the input/output criteria presented below:

Launch command:

```
>> <algo_exectuable> <instance_name> <instance-specific_information> ...
<cutoff_time> <cutoff_length> <seed> <params>
```

- <algo_exectuable> Solver
- <instance_name> In our case, a text file containing only the problem size
- <instance-specific_information> We don't use it
- <cutoff_time> Cut off time for each run of the solver (see above)
- <cutoff_length> We don't use it
- <seed> Random seed
- <params> Parameters and its values

Exmaple:

```
>> ./QtWrapper_320.txt "" 50.0 214483647 524453158 -p 5 -l 1 -f 1 -P 50 -F 0
```

Output:

```
>> <solved>, <runtime>, <runlength>, <best_sol>, <seed>
```

- <**solved**> `SAT` if the algorithm terminates successfully. `TIMESOUT` if the algorithm times out.
- <**runtime**> Runtime
- <**runlength**> -1 (as Frank Hutter recommended)
- <**best_sol**> -1 (as Frank Hutter recommended)
- <**cutoff_length**> We don't use it
- <**seed**> Used random seed

Exmaple:

```
>> SAT, 2.03435, -1, -1, 524453158
```

To build the wrapper we have followed a simple algorithm: launch two concurrent process. In the parent process we translate the input of the wrapper to the input of *Adaptive Search* solver. The solver is executed, and the runtime is measured. After that we post the output properly. In the child process a *sleep* command is executed for <`runtime`> seconds and after that, if the parent process has not finished yet, it is killed, posting a time-out message. See Algorithm 2 for more details.

Algorithm 2: Costas Wrapper

input : Pth_{π}: problem instance path, k: cut off time, s: random seed, θ: parameters configuration
output: $PiLS_{out}$: Output in a ParamILS way

```
1  fork() /* Divide the execution in two processes                    */
2  if <in child process> then
3      t_0 ← clock_TIC()
4      strCall ← build_str(" ./AS_Wrapper %1 -s %2 %3", Pth_π, s, θ)
5      systemCall(strCall)
6      t_e ← clock_TOC()
7      killProcess(<parent process>)
8      t ← t_e- t_0
9      return paramilsOutput(SAT, t, s)
10 else
11     sleep(k)
12     killProcess(<child process>)
13     return paramilsOutput(TIMESOUT, k, s)
14 end
```

3.2.4 Using the wrapper

In this section we explain how to use our wrapper to be able to tune easily instances of *All-Interval Series* and *Costas Array* problems. The *All-Interval Series Problem*[vii] is the problem of finding a vector $s = (s_1, \ldots, s_n)$, given $n \in \mathbb{N}$, such that s is a permutation of the vector $(0, 1, \ldots, n-1)$ and the interval vector $v = (|S_2 - s_1|, |S_3 - s_2|, \ldots, |S_n - S_{n-1}|)$ (called an all-interval series of size n) is a permutation of the vector $(1, 2, \ldots, n-1)$. The *Costas Array Problem* consists in finding a Costas array, which is an $n \times n$ grid containing n marks such that there is exactly one mark per row and per column and the $n(n-1)/2$ vectors joining each couple of marks are all different (see below for more details about this problems).

[vii] CSPlib:007 (`http://www.csplib.org/Problems/prob007/`)

3.2.4.1 Factory call

The first step is to implement the class ICallFactory. Here, the string-binary-name for the command call is statically obtained. We present, as example, the class All_IntervalCallFactory:

```
// all_interval_call_factory.h
class All_IntervalCallFactory:  public ICallFactory
{
    public:
       std::string BuildCall();
       std::string BuildDefaultCall();
};
```

```
// all_interval_call_factory.cpp
#define ALGO_EXECUTABLE "./all-interval"
#define DEFAULT_CALL "./all-interval _100.txt"

std::string All_IntervalCallFactory::BuildCall()
{
    return ALGO_EXECUTABLE;
}
std::string All_IntervalCallFactory::BuildDefaultCall()
{
    return DEFAULT_CALL;
}
```

All we have to do is to define our new macro **ALGO_EXECUTABLE** (**DEFAULT_CALL** is not being used)

3.2.4.2 Main method

Let's suppose now that we want to run an algorithm called *mySolver* that receives a file as parameter, called *my_instance_size.txt* (this is mandatory). We have to create (as we've explained before) the class My_SolverCallFactory and defining the macro as follows:

```
#define ALGO_EXECUTABLE "./mySolver"
```

Now, the main method would be exactly like this:

```
int main( int argc,  char* argv[])
{
   shared_ptr<ICallFactory> problem =
      make_shared<My_SolverCallFactory>();
   shared_ptr<TuningData> td =
      (make_shared<TuningData>(argc, argv, problem));

   shared_ptr<ADWrapper> w (make_shared<ADWrapper>());
   string output = w->tune(td);

   cout << output << endl;
    return 0;
}
```

3.2.5 Results

In this section we present the results of applying ParamILS to the resolution of *All-Interval Series* and *Costas Array* problems through *Adaptive Search*. In both cases, we need to chose a set of *training instances*, to train the tuner, and a set of *test instances*, used to obtain the parameter setting.

3.2.5.1 Tuning *Adaptive Search* for *All-Intervals Series Problem*

Study cases:

a) The *training instances set* is composed by instances of *All–Intervals* problems of order N with

$$N \in \{100, 110, 120, 130, 140, 150, 160, 170, 180\}$$

b) The *test instances set* is composed by instances of *All–Intervals* problems of order N with

$$N \in \{190, 200, 210, 220, 230, 240, 250, 260, 265\}$$

c) The time-out for each run is 50.0 seconds

d) The test quality is based on 100 runs

In a **First Experiment** we use the following *parameters domains*:

- **P** `{41, 46, 51, 56, 60, 66, 71, 76, 80}`
- **F, f, l** `{0, 1, 2, 3}`
- **p** `{5, 10, 15, 20, 25, 30, 35}`

Table 3.2 shows results using a time-out of 5.5 hours (20,000 seconds), and Table 3.3 shows results using a time-out of 1 hour. In the second case we where able to perform more runs, due to the available time, but in both cases the training qualities are not so different. However, we can se the difference int the test qualities, and conclude that results using 5 hours of time-out are more reliables.

Initial configuration					**Final best configuration**					**Training quality**	**Number of runs**	**Test quality**
F	P	f	l	p	F	P	f	l	p			
0	66	1	1	25	0	80	2	1	35	0.79666	1780	8.274
2	56	2	2	20	1	80	1	1	10	0.795	1637	5.508
0	41	0	0	5	1	80	3	0	15	0.789	1547	5.8478
3	80	3	3	35	1	80	2	0	10	0.880686	1258	6.15398

Table 3.2: *All-Intervals Series*: `tunerTimeout` = 20,000 seconds

Initial configuration					**Final best configuration**					**Training quality**	**Number of runs**	**Test quality**
F	P	f	l	p	F	P	f	l	p			
0	66	1	1	25	0	80	0	1	25	0.815	384	5.8191
0	66	1	1	25	1	80	1	1	35	0.737	452	6.267
0	66	1	1	25	1	56	0	1	35	1.03	371	9.056
0	66	1	1	25	0	76	0	1	20	0.814	385	4.915
0	66	1	1	25	0	80	3	1	20	0.76	469	5.417
2	56	2	2	20	0	41	0	1	10	0.919	239	18.364
2	56	2	2	20	0	56	1	1	20	0.819	407	5.409
2	56	2	2	20	1	80	1	1	35	0.772	457	5.43
2	56	2	2	20	1	80	0	1	10	0.858	504	5.566
2	56	2	2	20	0	80	1	1	10	0.7845	562	18.944
0	41	0	0	5	0	41	1	0	10	0.9749	367	5.97813
0	41	0	0	5	0	41	1	0	10	0.885	450	5.706
0	41	0	0	5	0	41	1	0	10	0.906	335	18.707
0	41	0	0	5	0	41	1	0	10	0.995	335	19.558
0	41	0	0	5	0	41	0	0	5	0.855	404	5.686
3	80	3	3	35	0	66	3	1	25	0.9118	230	26.585
3	80	3	3	35	0	80	1	1	10	0.732	310	7.875
3	80	3	3	35	0	80	0	1	20	0.816	303	7.2896
3	80	3	3	35	1	80	3	1	35	0.821	327	6.812
3	80	3	3	35	0	80	0	1	30	0.9203	443	5.401

Table 3.3: *All-Intervals Series*: `tunerTimeout` = 3,600 seconds

In a **Second Experiment** we decide to enlarge a bit more the parameters domains and use a time-out of 5 hours. The **Parameters domains** are the following:

- **P** `{10, 20, 30, 40, 50, 60, 70, 80, 90}`
- **F, f, l** `{0, 1, 2, 3, 4, 5, 6, 7, 8}`
- **p** `{10, 20, 30, 40, 50, 60, 70}`

Initial configuration					**Final best configuration**					**Training quality**	**Number of runs**	**Test quality**
F	P	f	l	p	F	P	f	l	p			
0	10	0	0	10	0	40	7	0	50	0.883188	936	6.3191
0	10	0	0	10	0	80	2	1	40	0.774659	1584	5.45674
0	10	0	0	10	0	40	2	0	10	0.96885	1104	6.82643
4	60	4	4	40	0	60	8	1	40	0.90358	1566	5.48127
4	50	4	4	40	0	80	5	1	20	0.78536	1662	11.5649
3	50	4	2	30	0	90	6	1	70	0.79440	1395	5.08108
0	90	0	0	10	1	90	6	1	10	0.859569	1379	5.4286
0	90	0	0	10	1	90	6	1	30	0.80738	1117	5.47126
8	90	8	8	60	0	80	5	1	10	0.834934	1384	5.5377
5	30	2	3	60	0	90	1	0	20	0.862013	1707	5.21837
3	20	2	4	60	0	80	6	1	10	0.805604	1630	5.4467
6	70	1	3	50	0	80	5	1	10	0.792600	1344	5.46558
6	40	1	3	30	1	80	7	0	20	0.822703	1977	5.41185

Table 3.4: *All-Intervals Series*: `tunerTimeout` = 18,000 seconds

The results presented in Table 3.4 show better results in terms of test quality with respect to Table 3.2. For that reason, in the **FINAL Experiment**, only the results obtained in those tables were took into account (also because they were obtained by using longer times-out). As it can be observed in those tables, *Adaptive Search* seems to show a good behavior if the parameters **F**, **P** and **l** are in the following sets: **F** $\in$ `{ 0, 1}`, **P** $\in$ `{ 80, 90}` and **l** $\in$ `{ 0, 1}`.

In that sense, a specific configuration was extracted from the results above, and 60 runs of *Adaptive Search* were performed solving *All–Intervals* ($N = 600$) benchmark:

- 30 using the default parameter configuration (`-F 0 -P 66 -f 1 -l 1 -p 25`)
- 30 with an optimal parameter configuration extracted from the Tables 3.2, 3.4 (`-F 0 -P 80 -f 6 -l 1 -p 10`)

Table 3.5 shows results by using the default parameter settings, and Table 3.6 shows the results by using the parameter configuration found by ParamILS, and it is clear that the default configuration shows better results than *ParamILS*'s one, in terms both of runtime mean and standard deviation Using the default parameter settings, *Adaptive Search* can obtains best results int terms of *mean* and *slowest run*. However, using the ParamILS found

parameter settings, it reached a *fastest* run two times faster than the one using the default parameter settings.

37.210	411.300	112.510	171.000	73.770
327.880	214.910	124.910	482.740	530.440
212.660	99.370	287.400	533.540	**18.410**
197.290	1016.950	110.230	566.480	**1362.010**
94.860	819.700	434.460	620.600	95.920
80.580	333.370	121.590	489.700	248.370
mean: 341.005333				
spread: 310.444635				

Table 3.5: *All-Intervals Series*: Default configuration runtimes (secs)

154.460	264.530	169.840	26.990	108.790
550.210	104.900	31.100	**9.870**	1242.900
678.760	475.570	201.200	622.410	297.960
526.930	375.620	293.380	598.850	350.270
540.290	252.940	673.630	423.030	589.210
32.080	254.640	**2034.020**	571.100	207.090
mean: 422.085667				
spread: 404.618226				

Table 3.6: *All-Intervals Series*: ParamILS configuration runtimes (secs)

3.2.5.2 Tuning *Adaptive Search* for *Costas Array Problem*

Study cases:

a) The *training instances set* is composed by instances of *Costas Array* problems of order N with $9 \leq N \leq 15$

b) The *test instances set* is composed by instances of *Costas Array* problems of order N with $14 \leq N \leq 19$

c) The cutoff for each run was 60.0 seconds

d) The test quality is based on 100 runs

The **First Experiments** with this benchmark was using the following parameter domains:

- **P** `{10, 20, 30, 40, 50, 60, 70, 80, 90}`
- **F, f, l** `{0, 1, 2, 3, 4, 5, 6, 7, 8}`

- **p** `{5, 10, 20, 30, 40, 50, 60, 70}`

Table 3.7 shows results selecting directly a time-out of 5 hours (18,000 seconds). In this case the training quality of the solutions is better, but do not observe any improvement in the test quality. We can see also how *Adaptive Search* seems to be not sensitive to parameters **F** and **p**, i.e. they don't change during the tuning process. On the other hand, the tuner seems to find some optimum values for the other parameters: **P** ∈ `{ 80, 90}`, **f** ∈ `{ 4, 5}` and **l** = `2`.

In that case also, an specific configuration was extracted from the results showed in Table 3.7, and 60 runs of *Adaptive Search* were performed solving *Costas Array* ($N = 20$) benchmark:

- 30 using the default parameter configuration (`-F 0 -P 50 -f 1 -l 0 -p 5`)

- 30 with an optimal parameter configuration extracted from the Table 3.7 (`-F 3 -P 90 -f 5 -l 2 -p 30`)

Initial configuration					**Final best configuration**					**Training quality**	**Number of runs**	**Test quality**
F	P	f	l	p	F	P	f	l	p			
0	10	0	0	5	2	90	2	2	5	0.0493699	957	5.8461
0	10	0	0	5	0	90	5	2	5	0.0509388	1783	6.52742
0	10	0	0	5	0	90	5	2	5	0.049901	1759	5.21828
3	40	4	4	30	3	90	5	2	30	0.053974	856	6.3539
4	50	3	5	20	4	90	5	2	20	0.0500355	2000	5.4047
4	60	5	3	50	4	60	5	3	50	0.0520575	2000	6.09106
8	90	8	8	70	8	80	4	2	70	0.052685	550	3.85682
8	90	8	8	70	8	80	4	2	70	0.054104	536	4.17855
8	90	8	8	70	8	80	4	2	70	0.0497819	1284	3.90945
3	10	1	6	60	3	90	5	2	60	0.054934	2000	6.81675
5	70	6	1	10	5	90	4	2	10	0.0499895	2000	4.07365
1	30	5	7	5	1	90	4	2	5	0.0525747	1237	2.70091
7	80	2	0	70	7	90	5	2	70	0.0502264	212	5.2637

Table 3.7: *Costas Array*: `tunerTimeout` = 18,000 seconds

Table 3.8 shows the results by using the default parameter configuration, and Table 3.9 shows the results by using the parameter configuration found by *ParamILS*. One more time, "in the mean", the default configuration outperforms ParamILS's.

3.2.6 Conclusion

The conclusion of this study is that the tunning process by hand in this case was more effective than using ParamILS. Results show that default parameters used in the current

452.980	91.420	31.510	**827.860**	96.670
635.030	295.830	272.360	151.040	170.660
183.550	161.340	91.240	426.470	62.020
138.090	236.030	**2.850**	187.240	21.510
165.370	90.440	195.580	15.390	229.720
170.840	174.210	30.520	6.570	115.880
mean: 191.007				
spread: 185.362				

Table 3.8: Default configuration runtimes (secs)

546.260	263.230	17.200	29.220	495.940
237.340	187.760	**7.810**	43.120	94.370
59.930	128.690	247.810	265.010	231.260
209.640	465.340	21.840	8.740	**1264.610**
57.700	122.890	450.610	229.580	174.540
414.080	402.250	91.150	677.190	58.640
mean: 250.125				
spread 263.539				

Table 3.9: ParamILS configuration runtimes (secs)

Adaptive Search implementation are more effective and consistent than those found by ParamILS for both benchmarks (*All-Interval Series* and *Costas Array* problems).

Part II

POSL: Parallel Oriented Solver Language

4

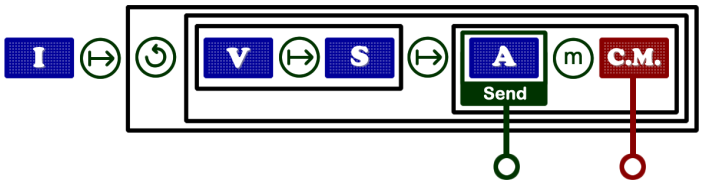
A Parallel-Oriented Language for Modeling Meta-Heuristic-Based Solvers

In this chapter POSL *is introduced as the main contribution, and a new way to solve CSPs. Its characteristics and advantages are summarized, and a general procedure to be followed is described, in order to build parallel solvers using* POSL*, followed by a detailed description of each of the single steps.*

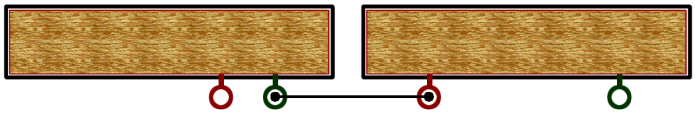
Contents

In this chapter we present the different steps to build communicating parallel solvers with POSL. First of all, the algorithm we have conceived to solve the target problem is decomposed it into small modules of computation, which are implemented as separated *functions*. We name them *computation modules* (see Figure 4.1a, blue shapes). At this point it is crucial to find a good decomposition of its algorithm, because it will have a significant impact in its future re-usage and variability. The next step is to decide which information is interesting to *receive* from other solvers. This information is encapsulated into another kind of component called *communication module*, allowing data transmission between solvers (see Figure 4.1a, red shapes). A third stage is to ensemble the modules through POSL's inner language (the interested reader is referred to Appendix [...]) to create independent solvers. The parallel-oriented language based on operators provided by POSL (see Figure 4.1b, green shapes) allows not only the information exchange, but also executing components in parallel. In this stage the information that is interesting to be shared with other solvers is sent using operators. After that we can connect them using *communication operators*. We call this final entity a *solvers set* (see Figure 4.1c).

(a) Creating POSL's modules



(b) Assembling modules using POSL's operators



(c) Connecting POSL solvers to create *communication strategies*

Figure 4.1: Solver construction process using POSL

In the following sections all these steps are explained in details, but first, I explain how to model the target benchmark using POSL.

4.1 Modeling the target benchmark

Target problems are modeled in POSL using the C++ programing language, respecting some rules of the object-oriented design. First of all, the benchmark must inherit from the class **Benchmark** provided by POSL. This class does not have any method to be

overridden or implemented, but receives in its constructor three objects, instances from classes that the user must create. Those classes must inherit from **SolutionCostStrategy**, **RelativeCostStrategy** and **ShowStrategy**, respectively. In these classes the most important functionalities of the benchmark model are defined.

SolutionCostStrategy: In this class the strategy to compute the *cost* of a configuration is implemented. POSL is based on improving step by step an initial configuration, taking into account a *cost function* provided by the user through the model (by implementing the function *solutionCost(dots)*). The kind of problems that POSL solves is the class of *Constraint Satisfaction Problems*, so this *cost function* must return an integer taking into account the problem constraints. Given a configuration s, the *cost function*, as a mandatory rule, must return 0 if and only if s is a solution of the problem, i.e., s fulfill all the problem constraints. An example of *cost function* is one that returns the number of violated constraints. However, the more expressive the function cost is, the better the performance of POSL leading to the solution.

The method to be implemented in this class is:

- `int solutionCost(std::vector<int> & c)` → Computes the cost of a given configuration (`c`).

RelativeCostStrategy: In this class the user implements the strategy to compute the *cost* of a given configuration with respect to another. If the cost of some configuration has been calculated, sometimes it is possible to store some information in order to compute the cost of another configuration, if the differences between them are known. If it is possible, the algorithms is defined in this class. If it is not possible, this class must have the same functionality of **SolutionCostStrategy**.

The methods to implement in this class are:

- `void initializeCostData(std::vector<int> & c)` → Initializes the information related to the cost (auxiliary data structures, the current configuration (`c`), the current cost, etc.)
- `void updateConfiguration(std::vector<int> & c)` → Updates the information related to the cost.
- `int relativeSolutionCost(std::vector<int> & c)` → Returns the relative cost of the configuration `c` with respect to the current configuration.
- `int currentCost()` → Property that returns the cost of the current configuration.
- `int costOnVariable(int variable_index)` → Returns a measure of the contribution of a variable to the total cost of a configuration.

- `int sickestVariable()` → Returns the variable contributing the most to the cost.

SolutionCostStrategy: This class represents the way a benchmark shows a configuration, in order to provide more information about the structure. For example, a configuration of the instance 3–3–2 of the *Social Golfers Problem* (see bellow for more details about this benchmark) can be written as follows:

```
[1, 2, 3, 4, 5, 6, 7, 8, 9, 3, 4, 5, 6, 7, 8, 9, 1, 2]
```

This text is, nevertheless, very difficult to be read if the instance is larger. Therefore, it is recommended that the user implements this class in order to give more details and to make it easier to interpret the configuration. For example, for the same instance of the problem, a solution could be presented as follows:

```
Golfers: players-3, groups-3, weeks-2
6         8         7
1         3         5
4         9         2
--
7         2         3
4         8         1
5         6         9
--
```

The method to be implemented in this class is:

- `std::string showSolution(std::shared_ptr<Solution> s)` → Returns a string to be written in the standard output.

Once we have modeled the target benchmark, it can be solved using POSL. In the following sections we describe how to use this parallel-oriented language to solve *Constraint Satisfaction Problems*.

4.2 First stage: creating POSL's modules

There exist two types of basic modules in POSL: *computation modules* and *communication modules*. A *computation module* is a function which received an input, then executes an internal algorithm, and returns an output. A *communication module* is also a function receiving and returning information, but in contrast, the *communication module* can receive information from two different sources: through input parameters or from outside, i.e., by communicating with a module from another solver.

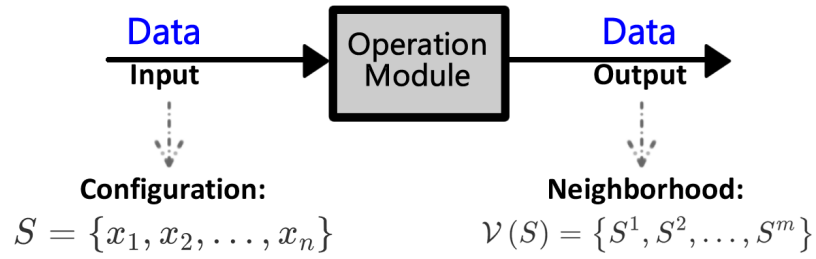


Figure 4.2: An example of a computation module computing a neighborhood

4.2.1 Computation Module

A *computation module* is the most basic and abstract way to define a piece of computation. It is a function which receives an instance of a POSL data type as input, then executes an internal algorithm, and returns an instance of a POSL data type as output. The input and output types will characterize the computation module signature. It can be dynamically replaced by (or combined with) other computation modules, since they can be shared among solvers working in parallel. They are joined through *abstract solvers*.

Definition 6 ***(Computation Module)*** *A computation module Cm is a mapping defined by:*

$$Cm : D \rightarrow I \tag{4.1}$$

where D and I can be either a set of configurations, a set of sets of configurations, a set of values of some data type, etc.

Consider a local search meta-heuristic solver. One of its *computation modules* can be the function returning the set of configurations composing the neighborhood of a given configuration:

$$Cm_{neighborhood} : D_1 \times D_2 \times \cdots \times D_n \rightarrow 2^{D_1 \times D_2 \times \cdots \times D_n}$$

where D_i represents the definition domains of each variable of the input configuration.

Figure 4.2 shows an example of *computation module*: which receives a configuration S and then computes the set $\mathcal{V}$ of its neighbor configurations $\{S^1, S^2, \ldots, S^m\}$.

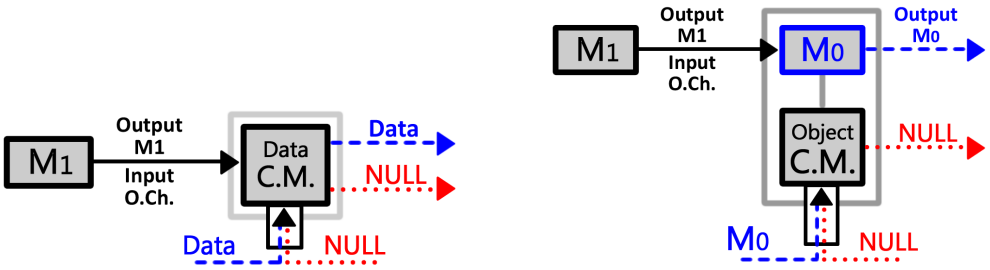
4.2.1.1 Creating new *computation modules*

To create new *computation modules* we use C++ programing language. POSL provides a hierarchy of data types to work with (See anexes) and some abstract classes to inherit from, depending on the type of *computation module* that the user wants to create. These abstract classes represent *abstract computation module* and define a type of action to be executed. In the following we present the most important ones:

- **AOM_FirstConfigurationGeneration** → Represents *computation modules* generating a first configuration. The user must implement the method *spcf_execute(**ComputationData**)* which returns a pointer to a **Solution**, that is, an object containing all the information concerning a partial solution (configuration, variable domains, etc.)
- **AOM_NeighborhoodFunction** → Represent *computation modules* creating a neighborhood of a given configuration. The user must implement the method *spcf_execute(**Solution**)* which returns a pointer to an object **Neighborhood**, containing a set of configurations which constitute the neighborhood of a given configuration, according to certain criteria. These configuration are efficiently stored.
- **AOM_SelectionFunction** → Represents *computation modules* selecting a configuration from a neighborhood. The user must implement the method *spcf_execute(**Neighborhood**)* which returns a pointer to an object **DecisionPair**, containing two solutions: the current and the selected one.
- **AOM_DecisionFunction** → Represents *computation modules* deciding which of the two solutions will be the current configuration for the next iteration. The user must implement the method *spcf_execute(**DecisionPair**)* which returns a pointer to an object **Solution**.

4.2.2 Communication modules

A *communication module* is also a function receiving and returning information, but in contrast, the *communication module* can also receive information by communicating with a module from another solver. A *communication module* is the component managing the information reception in the communication between solvers (we will talk about information transmission in the next section). They can interact with *computation modules* through operators (see Figure 4.3).



(a) Data *communication module* **(b)** Object *communication module*

Figure 4.3: Communication module

A *communication module* can receive two types of information from an external solver: data or *computation modules*. It is important to notice that by sending/receiving *computation modules*, we mean sending/receiving only required information to identify and being able to instantiate the *computation module*.

In order to distinguish from the two types of *communication modules*, we will call Data Communication Module to the *communication module* responsible for the data reception (Figure 4.3a), and Object Communication Module to the one responsible for the reception and instantiation of *computation modules* (Figure 4.3b).

Definition 7 ***(Data Communication Module)*** *A* Data Communication Module *Ch is a component that produces a mapping defined as follows:*

$$Ch : U \rightarrow I \tag{4.2}$$

It returns the information I coming from an external solver, no matter what the input U is.

Definition 8 ***(Object Communication Module)*** *If we denote by* $\mathbb{M}$ *the space of all the computation modules defined by Definition 4.1, then an* Object Communication Module *Ch is a component that produces a computation module coming from an external solver as follows:*

$$Ch : \mathbb{M} \rightarrow \mathbb{M} \tag{4.3}$$

Users can implement new computation and connection modules but POSL already contains many useful modules for solving a broad range of problems.

Due to the fact that *communication modules* receive information coming from outside without having control on them, it is necessary to define the *NULL* information, in order to denote the absence of information. If a Data Communication Module receives a piece of information, is returned automatically. If a Object Communication Module receives a *computation module*, it is instantiated and executed with the *communication module*'s input and its result is

returned. In both cases, if no available information exists (no communications performed), the *communication module* returns the *NULL* object.

4.3 Second stage: assembling POSL's modules

Modules mentioned above are defined respecting the signature of some predefined abstract module. For example, the module showed in Figure 4.2 is a *computation module* based on an abstract module that receives a configuration and returns a neighborhood. In that sense, an example of a concrete *computation module* (or just *computation module*) can be a function receiving a configuration, and returning a neighborhood constituted by N configurations which only differ from the input configuration in one entry.

In this stage an *abstract solver* is coded using POSL. It takes abstract modules as *parameters* and combines them through operators. Through the *abstract solver*, we can also decide which information to send to other solvers by using some operators to send the result of a computation module (see below). In the following we present a formal and more detailed specification of POSL's operators.

The *abstract solver* is the solver's backbone. It joins the *computation modules* and the *communication modules* coherently. It is independent from the *computation modules* and *communication modules* used in the solver. It means that they can be changed or modified during the execution, without altering the general algorithm, but still respecting the main structure. Each time we combine some of them using POSL's operators, we are creating a *compound module*. Here we formally define the concept of *module* and *compound module*.

Definition 9 *A* **module** *is (and it is denoted by the letter* $\mathcal{M}$*):*

a) a computation module or

b) a communication module or

c) $[\mathcal{M}_1\ OP\ \mathcal{M}_2]$*, which is the composition of two modules* $\mathcal{M}_1$ *and* $\mathcal{M}_2$ *to be executed sequentially, returning an output depending on the nature of the operator* OP*; or*

d) $[\![\mathcal{M}_1\ OP\ \mathcal{M}_2]\!]_p$*, which is the composition of two modules* $\mathcal{M}_1$ *and* $\mathcal{M}_2$ *to be executed, returning an output depending on the nature of the operator* OP*. These two modules will be executed in parallel if and only if* OP *supports parallelism, (i.e. some modules will be executed sequentially although they were grouped this way); or sequentially otherwise.*

We denote the space of the modules by $\mathbb{M}$ *and call compound modules to the composition of modules described in c) and d).*

For a better understanding of Definition 9, Figure 4.4 shows graphically the structure of a *compound module*.

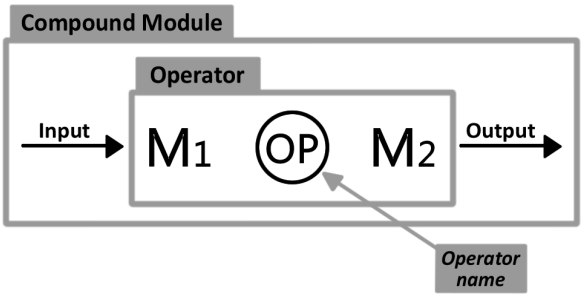


Figure 4.4: A *compound module*

As mentioned before, the *abstract solver* is independent from the *computation modules* and *communication modules* used in the solver. It means that one *abstract solver* can be used to construct many different solvers, by implementing it using different modules (see below the related concept of *abstract solver* instantiation). This is the reason why the *abstract solver* is defined only using abstract modules. Formally, we define an *abstract solver* as follows:

Definition 10 ***(Abstract Solver)*** *An* Abstract Solver *AS is a triple* $(\mathbf{M}, \mathcal{L}^m, \mathcal{L}^c)$*, where:* $\mathbf{M}$ *is a compound module (also called* root *compound module),* $\mathcal{L}^m$ *a list of abstract computation modules appearing in* $\mathcal{M}$*, and* $\mathcal{L}^c$ *a list of communication modules appearing in* $\mathcal{M}$*.*

The *root compound module* can be defined also as a free-context grammar as follows:

Definition 11 (root *compound module***'s grammar)** $G_{POSL} = (\mathbf{V}, \Sigma, \mathbf{S}, \mathbf{R})$*, where:*

a) $\mathbf{V} = \{CM, OP\}$ *is the set of variables,*

b) $\Sigma = \left\{\alpha, \beta, be, [,], [\![,]\!]_p, (,), \{,\}, (\!|,|\!)^m, |\!)^o, \textcircled{\mapsto}, \textcircled{?}, \circlearrowleft, \textcircled{\rho}, \textcircled{\vee}, \textcircled{\wedge}, \textcircled{M}, \textcircled{m}, \textcircled{\downarrow}, \textcircled{\cup}, \textcircled{\cap}\right\}$ *is the set of terminals,*

c) $\mathbf{S} = \{CM\}$ *is the set of start variables,*

d) and $\mathbf{R} =$

$$CM \longmapsto \alpha \mid \beta \mid (\!|CM|\!)^o \mid (\!|CM|\!)^m \mid [OP] \mid [\![OP]\!]_p$$
$$OP \longmapsto CM\textcircled{\mapsto}CM \mid CM\textcircled{?}CM \mid CM\textcircled{\rho}CM \mid CM\textcircled{\vee}CM \mid CM\textcircled{\wedge}CM$$
$$OP \longmapsto CM\textcircled{M}CM \mid CM\textcircled{m}CM \mid CM\textcircled{\downarrow}CM \mid CM\textcircled{\cup}CM \mid CM\textcircled{\cap}CM$$
$$OP \longmapsto CM \circlearrowleft (be)CM$$

is a set of rules

In the following I explain some of the concepts in Definition 11:

- The variables CM and OP are two very important entities in the language, as it can be seen in the grammar. We name them *compound module* and *operator*, respectively.
- The terminals α and β represent a *computation module* and a *communication module*, respectively.
- The terminal *be* is a boolean expression.
- The terminals $[\ ], [\![\ ]\!]_p$ are symbols for grouping and defining the way the involved *compound modules*are executed. Depending on the nature of the operator, this can be either sequentially or in parallel:
  - *a)* [OP]: The involved operator is executed sequentially.
  - *b)* $[\![\text{OP}]\!]_p$: The involved operator is executed in parallel if and only if OP supports parallelism. Otherwise, an exception is thrown.
- The terminals (and) are symbols for grouping the boolean expression in some operators.
- The terminals $(\!|.|\!)^m, (\!|.|\!)^o$, are operators to send information to other solvers (explained bellow).
- The rest of terminals are POSL operators.

In the following we define POSL operators. In order to group modules, like in Definition $9(c)$) and $9(d)$), we will use |.| as generic grouper. In order to help the reader to easely understand how to use the operators, I use an example of a solver that I build step by step, while presenting the definitions.

POSL creates solvers based on local search meta-heuristics algorithms. These algorithms have a common structure: 1. They start by initializing some data structures (e.g., a *tabu list* for *Tabu Search* [37], a *temperature* for *Simulated Annealing* [35], etc.). 2. An initial configuration s is generated. 3. A new configuration s' is selected from the neighborhood $\mathcal{V}(s)$. 4. If s' is a solution for the problem P, then the process stops, and s' is returned. If not, the data structures are updated, and s' is accepted or not for the next iteration, depending on a certain criterion. An example of such data structure is the penalizing features of local optima defined by Boussaïd et al [4] in their algorithm *Guided Local Search*.

Abstract computation modules composing *local search meta–heuristics* are:

$Abstract\ Computation\ module - 1$	I: Generating a configuration s
$Abstract\ Computation\ module - 2$	V: Defining the neighborhood $\mathcal{V}(s)$

Abstract Computation module − 3	S: Selecting $s' \in \mathcal{V}(s)$
Abstract Computation module − 4	A: Evaluating an acceptance criterion for s'

The list of modules to be used in the examples have been presented. Now I present the POSL operators.

Definition 12 (Operator Sequential Execution) *Let*

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$,

be modules, where $\mathcal{I}_1 \subseteq \mathcal{D}_2$. *Then the operation* $\left|\mathcal{M}_1 \textcircled{\mapsto} \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_{seq}$ *as the result of executing* $\mathcal{M}_1$ *followed by executing* $\mathcal{M}_2$*:*

$$\mathcal{M}_{seq} : \mathcal{D}_1 \rightarrow \mathcal{I}_2$$

This is an example of an operator that does not support the execution of its involved *compound modules* in parallel, because the input of the second *compound module* is the output of the first one.

Coming back to the example, I can use defined *abstract computation modules* to create a *compound module* that perform only one iteration of a local search, using the operator **Sequential Execution**. I create a *compound module* to execute sequentially I and V (see Figure 4.5a), then I create an other *compound module* to execute sequentially the *compound module* already created and S (see Figure 4.5b), and finally this *compound module* and the *computation module* A are executed sequentially (see Figure 4.5c). The *compound module* presented in Figure 4.5c can be coded as follows:

$$\left[\left[\left[I \textcircled{\mapsto} V\right] \textcircled{\mapsto} S\right] \textcircled{\mapsto} A\right]$$

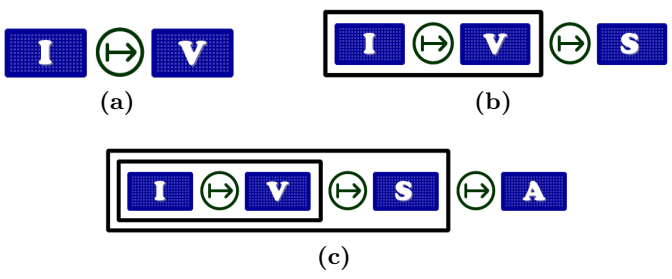


Figure 4.5: Using **sequential execution** operator

The following operator is very useful to execute modules sequentially creating bifurcations, subject to some boolean condition:

Definition 13 (Operator Conditional Execution) *Let*

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$,

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$. *Then the operation* $\left|\mathcal{M}_1 \textcircled{?}_{<cond>} \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_{cond}$ *as result of the sequential execution of* $\mathcal{M}_1$ *if* $< cond >$ *is* **true** *or* $\mathcal{M}_2$, *otherwise:*

$$\mathcal{M}_{cond} : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

This operator can be used in the example if I want to execute two different *selection computation modules* (S_1 and S_2) depending on certain criterion (see Figure 4.6):

$$\left[\left[\left[I \textcircled{\mapsto} V\right] \textcircled{\mapsto} \left[S_1 \textcircled{?} S_2\right]\right] \textcircled{\mapsto} A\right]$$

In examples I remove the clause $< cond >$ for simplification.

Figure 4.6: Using **conditional execution** operator

We can execute modules sequentially creating also cycles.

Definition 14 (Operator Cyclic Execution) *Let* $\mathcal{M} : \mathcal{D} \rightarrow \mathcal{I}$ *be a module, where* $\mathcal{I} \subseteq \mathcal{D}$. *Then, the operation* $|\circlearrowleft_{<cond>} \mathcal{M}|$ *defines the compound module* $\mathcal{M}_{cyc}$ *as result of the sequential execution of* $\mathcal{M}$ *repeated while* $< cond >$ *remains* **true**:

$$\mathcal{M}_{cyc} : \mathcal{D} \rightarrow \mathcal{I}$$

Using this operator I can model a local search algorithm, by executing the *abstract computation module* I and then the other *computation modules* (V, S and A) cyclically, until finding a solution (i.e, a configuration with cost equal to zero) (see Figure 4.7):

$$\left[I \textcircled{\mapsto} \left[\circlearrowleft \left[\left[V \textcircled{\mapsto} S\right] \textcircled{\mapsto} A\right]\right]\right]$$

In the examples, I remove the clause $< cond >$ for simplification.

Definition 15 (Operator Random Choice) *Let*

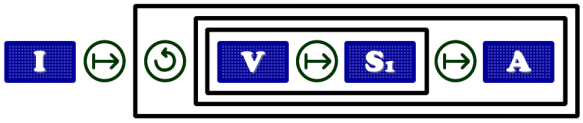


Figure 4.7: Using **cyclic execution** operator

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$,

be modules, where $\mathcal{D}_1 \subset \mathcal{D}_2$ *and a real value* ρ*. Then the operation* $\left|M_1 (\rho) \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_{rho}$ *that executes and returns the output of* $\mathcal{M}_1$ *with probability* ρ*, or executes and returns the output of* $\mathcal{M}_2$ *with probability* $(1 - \rho)$*:*

$$\mathcal{M}_{rho} : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

In the example I can create a *compound module* to execute two *abstract computation modules* A_1 and A_2 following certain probability ρ using the operator **random execution** as follows (see Figure 4.8):

$$\left[I \, (\mapsto) \left[\circlearrowleft \left[\left[V \, (\mapsto) \, S\right] \, (\mapsto) \, \left[A_1 \, (\rho) \, A_2\right]\right]\right]\right]$$

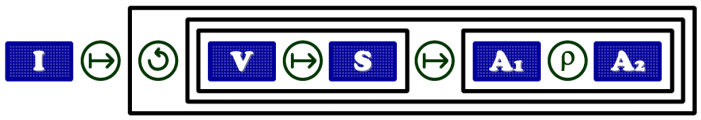


Figure 4.8: Using **random execution** operator

The following operator is very useful if the user needs to use a *communication module* inside an *abstract solver*. As explained before, if a *communication module* does not receive any information from another solver, it returns *NULL*. This may cause the undesired termination of the solver if this case is not considered correctly. Next, I introduce the operator **Operator Not** *NULL* **Execution** and illustrate how to use it in practice with an example.

Definition 16 (Operator Not *NULL* **Execution)** *Let*

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$,

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$*. Then, the operation* $\left|\mathcal{M}_1 (\vee) \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_{non}$ *that executes* $\mathcal{M}_1$ *and returns its output if it is not NULL, or executes* $\mathcal{M}_2$ *and returns its output otherwise:*

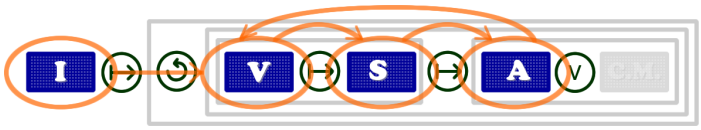
$$\mathcal{M}_{non} : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

Let us make consider a slightly more complex example: When applying the acceptance criterion, suppose that we want to receive a configuration from other solver to combine the *computation module* A with a *communication module*:

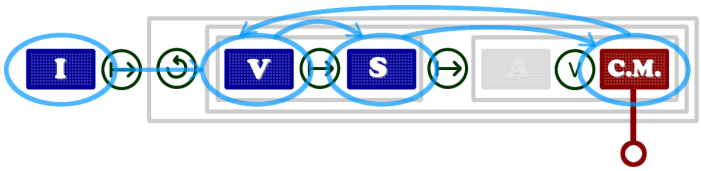
$\boxed{Communication\ module - 1:}$ $C.M.$: Receiving a configuration.

Figure 4.9 shows how to combine a *communication module* with the *computation module* A through the operator $\bigcirc\!\!\!\!\vee$. Here, the *computation module* A will be executed as long as the *communication module* remains *NULL*, i.e., there is no information coming from outside. This behavior is represented in Figure 4.9a by the orange lines. If some data has been received through the *communication module*, the later is executed instead of the module A, represented in Figure 4.9b by blue lines. The code can be written as follows:

$$\left[I \bigcirc\!\!\!\!\mapsto \left[\circlearrowleft \left[\left[V \bigcirc\!\!\!\!\mapsto S\right] \bigcirc\!\!\!\!\mapsto \left[A \bigcirc\!\!\!\!\vee C.M.\right]\right]\right]\right]$$



(a) The solver executes the computation module **A** if no information is received through the connection module



(b) The solver uses the information coming from an external solver

Figure 4.9: Two different behaviors within the same solver

This is *short-circuit* operator. It means that if the first argument (module) does not return *NULL*, the second will not be executed. POSL provides another operator with the same functionality but not *short-circuit*:

Definition 17 (Operator *BOTH* Execution) *Let*

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$,

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$. *Then the operation* $\left|\mathcal{M}_1 \bigcirc\!\!\!\!\wedge \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_{both}$ *that executes both* $\mathcal{M}_1$ *and* $\mathcal{M}_2$, *then returns the output of* $\mathcal{M}_1$ *if it is not NULL, or the output of* $\mathcal{M}_2$ *otherwise:*

$$\mathcal{M}_{both} : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

In the following definitions, the concepts of *cooperative parallelism* and *competitive parallelism* are implicitly included. We say that cooperative parallelism exists when two or more processes are running separately, they are independent, and the general result will be some combination of the results of all the involved processes (e.g. Definitions 18 and 19). On the other hand, competitive parallelism arise when the general result is the result of the process ending first (e.g. Definition 20).

Definition 18 (Operator Minimum) *Let*

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$,

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$. *Let also* o_1 *and* o_2 *be the outputs of* $\mathcal{M}_1$ *and* $\mathcal{M}_2$, *respectively. Assume that there exists some order criteria between them. Then the operation* $\left|\mathcal{M}_1 \textcircled{m} \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_{min}$ *that executes* $\mathcal{M}_1$ *and returns* $\min\{o_1, o_2\}$*:*

$$\mathcal{M}_{min} : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

Similarly we define the operator **Maximum**:

Definition 19 (Operator Maximum) *Let*

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$,

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$. *Let also* o_1 *and* o_2 *be the outputs of* $\mathcal{M}_1$ *and* $\mathcal{M}_2$, *respectively. Assume that there exists some order criteria between them. Then the operation* $\left|\mathcal{M}_1 \textcircled{M} \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_{max}$ *that executes* $\mathcal{M}_1$ *and returns* $\max\{o_1, o_2\}$*:*

$$\mathcal{M}_{max} : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

Comming back to the previews example, the **minimum operator** can be applied to obtain a more interesting behavior in the solver: When applying the acceptance criteria, suppose that we want to receive a configuration from other solver, to compare it with ours and select the one with the lowest cost. We can do that by applying the operator (m) to combine the *computation module* A with a *communication module* $C.M.$ (see Figure4.10):

$$\left[I \textcircled{\mapsto} \left[\circlearrowleft \left[\left[V \textcircled{\mapsto} S\right] \textcircled{\mapsto} \left[\!\left[A \textcircled{m} C.M.\right]\!\right]_p\right]\right]\right]$$

Notice that in this example, I can use the grouper $[\![.]\!]_p$ since the **minimum operator** supports parallelism.

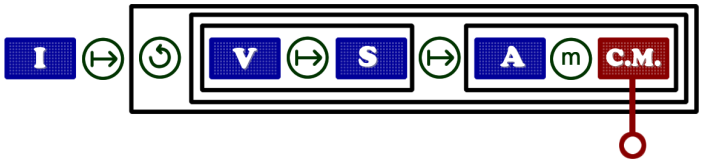


Figure 4.10: Using **minimum** operator

Definition 20 (Operator Race) *Let*

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$,

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$ *and* $\mathcal{I}_1 \subset \mathcal{I}_2$. *Then the operation* $\left|\mathcal{M}_1 \textcircled{\downarrow} \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_{race}$ *that executes both modules* $\mathcal{M}_1$ *and* $\mathcal{M}_2$, *and returns the output of the module ending first:*

$$\mathcal{M}_{race} : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

Sometimes nighborhood functions are slow depending on the configuration. In that case two neighborhood *computation modules* can be executed and take into account the output of the module ending first (see Figure4.11):

$$\left[I \textcircled{\mapsto} \left[\circlearrowleft \left[\left[\left[\!\left[V_1 \textcircled{\downarrow} V_2 \right]\!\right]_p \textcircled{\mapsto} S \right] \textcircled{\mapsto} \left[\!\left[A \textcircled{m} C.M. \right]\!\right]_p \right] \right] \right]$$

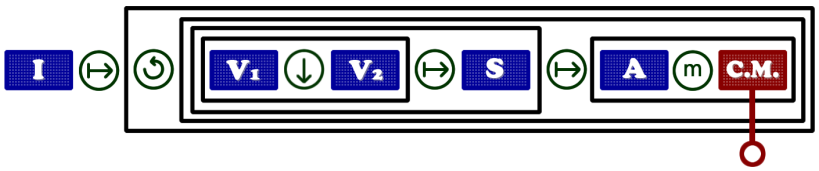


Figure 4.11: Using **race** operator

Some others operators can be useful when dealing with *sets*.

Definition 21 (Operator Union) *Let*

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$,

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$*. Let also* V_1 *and* V_2 *be the outputs of* $\mathcal{M}_1$ *and* $\mathcal{M}_2$*, respectively. Then the operation* $\left|\mathcal{M}_1 \textcircled{\cup} \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_\cup$ *that executes both modules* $\mathcal{M}_1$ *and* $\mathcal{M}_2$*, and returns* $V_1 \cup V_2$*:*

$$\mathcal{M}_\cup : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

Similarly we define the operators **Intersection** and **Subtraction**:

Definition 22 (Operator Intersection) *Let*

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$*,*

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$*. Let also* V_1 *and* V_2 *be the outputs of* $\mathcal{M}_1$ *and* $\mathcal{M}_2$*, respectively. Then the operation* $\left|\mathcal{M}_1 \textcircled{\cap} \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_\cap$ *that executes both modules* $\mathcal{M}_1$ *and* $\mathcal{M}_2$*, and returns* $V_1 \cap V_2$*:*

$$\mathcal{M}_\cap : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

Definition 23 (Operator Subtraction) *Let*

a) $\mathcal{M}_1 : \mathcal{D}_1 \rightarrow \mathcal{I}_1$ *and*

b) $\mathcal{M}_2 : \mathcal{D}_2 \rightarrow \mathcal{I}_2$*,*

be modules, where $\mathcal{D}_1 \subseteq \mathcal{D}_2$*. Let also* V_1 *and* V_2 *be the outputs of* $\mathcal{M}_1$ *and* $\mathcal{M}_2$*, respectively. Then the operation* $\left|\mathcal{M}_1 \textcircled{-} \mathcal{M}_2\right|$ *defines the compound module* $\mathcal{M}_-$ *that executes both modules* $\mathcal{M}_1$ *and* $\mathcal{M}_2$*, and returns* $V_1 - V_2$*:*

$$\mathcal{M}_- : \mathcal{D}_1 \cap \mathcal{D}_2 \rightarrow \mathcal{I}_1 \cup \mathcal{I}_2$$

Now, I define the operators which allows to send information to other solvers. Two types of information can be sent: i) the output of the *computation module* and send its output, or ii) the *computation module* itself. . This utility is very useful in terms of sharing behaviors between solvers.

Definition 24 (Sending Data Operator) *Let* $\mathcal{M} : \mathcal{D} \rightarrow \mathcal{I}$ *be a module. Then the operation* $|(\!|\mathcal{M}|\!)^o|$ *defines the compound module* $\mathcal{M}_{sendD}$ *that executes the module* $\mathcal{M}$ *and sends its output outside:*

$$\mathcal{M}_{sendD} : \mathcal{D} \rightarrow \mathcal{I}$$

Similarly we define the operator **Send Module**:

Definition 25 (Sending Module Operator) *Let $\mathcal{M} : \mathcal{D} \to \mathcal{I}$ be a module. Then the operation $|(\!|\mathcal{M}|\!)^m|$ defines the compound module $\mathcal{M}_{sendM}$ that executes the module $\mathcal{M}$, then returns its output and sends the module itself outside:*

$$\mathcal{M}_{sendM} : \mathcal{D} \to \mathcal{I}$$

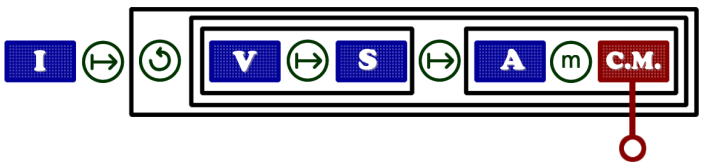
In the following example, I use one of the *compound modules* already presented in the previews examples, using a *communication module* to receive a configuration (see Figure 4.12a):

$$\left[I \textcircled{\mapsto} \left[\circlearrowleft \left[\left[V \textcircled{\mapsto} S \right] \textcircled{\mapsto} \left[\!\left[A \textcircled{m} C.M. \right]\!\right]_p \right] \right] \right]$$

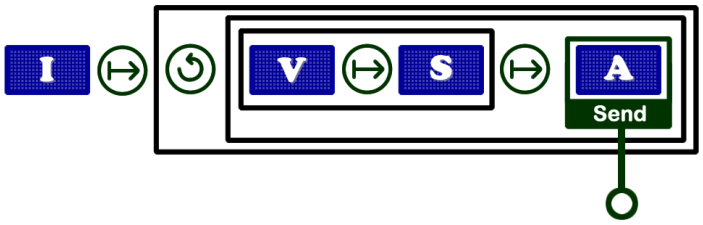
I also build another, as its complement: sending the accepted configuration to outside, using the **sending data operator** (see Figure4.12b):

$$\left[I \textcircled{\mapsto} \left[\circlearrowleft \left[\left[V \textcircled{\mapsto} S \right] \textcircled{\mapsto} (\!|A|\!)^o \right] \right] \right]$$

In the Section 4.5 I explain how to connect solvers to each other.



(a)



(b)

Figure 4.12: Sender and receiver behaviors

Once all desired abstract modules are linked together with operators, we obtain the *root compound module*, an important part of an *abstract solver*. To implement a concrete solver from an *abstract solver*, one must instantiate each abstract module with a concrete one respecting the required signature. From the same *abstract solver*, one can implement many different concrete solvers simply by instantiating abstract modules with different concrete modules.

An *abstract solver* is defined as follows: after declaring the **abstract solver**'s name, the first line defines the list of abstract *computation modules*, the second one the list of abstract *communication modules*, then the algorithm of the solver is defined as the solver's body (the root *compound module*), between **begin** and **end**.

An *abstract solver* can be declared through the simple regular expression:

abstract solver *name* **computation**: L^m (**communication**: L^c)? **begin** $\mathcal{M}$ **end**

where:

- *name* is the identifier of the *abstract solver*,
- L^m is the list of abstract *computation modules*,
- L^c is the list of abstract *communication modules*, and
- $\mathcal{M}$ is the root *compound module*.

For instance, Algorithm 3 illustrates the abstract solver corresponding to Figure 4.1b.

Algorithm 3: POSL pseudo-code for the *abstract solver* presented in Figure 4.1b

```
1 abstract solver as_01
2 computation : I, V, S, A
3 connection: C.M.
4     begin
5         I (↦)
6         [↺ (ITR % K_1)
7             [V (↦) S (↦) [C.M. (m) (|A|)^o]]
8         ]
9     end
```

4.4 Third stage: creating POSL solvers

With *computation* and *communication modules* composing an *abstract solver*, one can create solvers by instantiating *modules*. This is simply done by specifying that a given **solver** must **implements** a given *abstract solver*, followed by the list of *computation* then *communication modules*. These modules must match signatures required by the *abstract solver*.

> In the following example, I describe some concrete *computation modules* that can be used to implement the *abstract solver* declared in Algorithm 3:

$\boxed{Computation\ module-1}$ I_{rand} generates a random configuration s

$\boxed{Computation\ module-2}$ V_{1ch} defines the neighborhood $\mathcal{V}(s)$ changing only one element

$\boxed{Computation\ module-3}$ S_{best} selects the best configuration $s' \in \mathcal{V}(s)$ improving the current cost.

$\boxed{Computation\ module-4}$ A_{alw} evaluates an acceptance criterion for s'. We have chosen the classical module, selecting the configuration with the lowest global cost, *i.e.*, the one which is likely the closest to a solution.

I use also the following concrete *communication module*:

$\boxed{Communication\ module-1}$ CM_{last} returns the last configuration arrived, if at the time of its execution, there is more than one configuration waiting to be received.

These modules are used and explained in details in the Chapter 5 of this document. Algorithm 4 implements Algorithm 3 by instantiating its modules.

Algorithm 4: An instantiation of the *abstract solver* presented in Algorithm 3

```
1 solver solver_01 implements as_01
2 computation : I_rand, V_1ch, S_best, A_alw
3 connection: CM_last
```

4.5 Forth stage: connecting the solvers

We call *solver set* to the pool of (concrete) solvers that we plan to use in parallel to solve a problem. Once we have our solvers set, the last stage is to connect the solvers to each other. Up to this point, solvers are disconnected, but they are ready to establish the communication. POSL provides a platform to the user such that cooperative strategies can be easily defined.

In the following we present two important concepts necessary to formalize the *communication operators*.

Definition 26 (Communication Jack) *Let $\mathcal{S}$ be a solver. Then the operation $\mathcal{S} \cdot \mathcal{M}$ opens an outgoing connection from the solver $\mathcal{S}$, sending to the outside either a) the output of $\mathcal{M}$, if it is affected by a sending data operator as presented in Definition 24, or b) $\mathcal{M}$ itself, if it is affected by a sending module operator as presented in Definition 25.*

Definition 27 (Communication Outlet) *Let $\mathcal{S}$ be a solver. Then, the operation $\mathcal{S} \cdot \mathcal{CM}$ opens an ingoing connection to the solver $\mathcal{S}$, receiving from the outside either a) the output of some computation module, if $\mathcal{CM}$ is a data communication module, or b) a computation module, if $\mathcal{CM}$ is an object communication module.*

The communication is established by following the following rules guideline:

a) Each time a solver sends any kind of information by using a *sending* operator, it creates a *communication jack*.

b) Each time a solver defines a *communication module*, it creates a *communication outlet*.

c) Solvers can be connected to each other by linking *communication jacks* to *communication outlets*.

Following, we define the *connection operators* that POSL provides.

Definition 28 (Connection Operator One-to-One) *Let*

a) $\mathcal{J} = [\mathcal{S}_0 \cdot \mathcal{M}_0, \mathcal{S}_1 \cdot \mathcal{M}_1, \ldots, \mathcal{S}_{N-1} \cdot \mathcal{M}_{N-1}]$ *be the list of communication jacks, and*

b) $\mathcal{O} = [\mathcal{Z}_0 \cdot \mathcal{CM}_0, \mathcal{Z}_1 \cdot \mathcal{CM}_1, \ldots, \mathcal{Z}_{N-1} \cdot \mathcal{CM}_{N-1}]$ *be the list of communication outlets*

Then the operation

$$\mathcal{J} \textcircled{\rightarrow} \mathcal{O}$$

connects each communication jack $\mathcal{S}_i \cdot \mathcal{M}_i \in \mathcal{J}$ with the corresponding communication outlet $\mathcal{Z}_i \cdot \mathcal{CM}_i \in \mathcal{O}$, $\forall 0 \leq i \leq N-1$ (see Figure 4.13a).

Definition 29 (Connection Operator One-to-N) *Let*

a) $\mathcal{J} = [\mathcal{S}_0 \cdot \mathcal{M}_0, \mathcal{S}_1 \cdot \mathcal{M}_1, \ldots, \mathcal{S}_{N-1} \cdot \mathcal{M}_{N-1}]$ *be the list of communication jacks, and*

b) $\mathcal{O} = [\mathcal{Z}_0 \cdot \mathcal{CM}_0, \mathcal{Z}_1 \cdot \mathcal{CM}_1, \ldots, \mathcal{Z}_{M-1} \cdot \mathcal{CM}_{M-1}]$ *be the list of communication outlets*

Then the operation

$$\mathcal{J} \textcircled{\rightsquigarrow} \mathcal{O}$$

connects each communication jack $\mathcal{S}_i \cdot \mathcal{M}_i \in \mathcal{J}$ with every communication outlet $\mathcal{Z}_j \cdot \mathcal{CM}_j \in \mathcal{O}$, $\forall 0 \leq i \leq N-1$ and $0 \leq j \leq M-1$ (see Figure 4.13b).

Definition 30 (Connection Operator Ring) *Let*

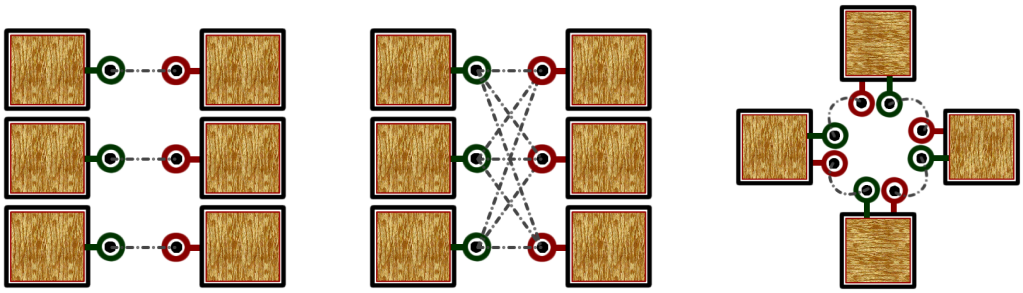
a) $\mathcal{J} = [\mathcal{S}_0 \cdot \mathcal{M}_0, \mathcal{S}_1 \cdot \mathcal{M}_1, \ldots, \mathcal{S}_{N-1} \cdot \mathcal{M}_{N-1}]$ *be the list of communication jacks, and*

b) $\mathcal{O} = [\mathcal{S}_0 \cdot \mathcal{CM}_0, \mathcal{S}_1 \cdot \mathcal{CM}_1, \ldots, \mathcal{S}_{N-1} \cdot \mathcal{CM}_{N-1}]$ *be the list of communication outlets*

Then the operation

$$\mathcal{J} \textcircled{\leftrightarrow} \mathcal{O}$$

connects each communication jack $\mathcal{S}_i \cdot \mathcal{M}_i \in \mathcal{J}$ *with the corresponding communication outlet* $\mathcal{Z}_{(i+1)\%N} \cdot \mathcal{CM}_{(i+1)\%N} \in \mathcal{O}$, $\forall 0 \leq i \leq N-1$ *(see Figure 4.13c).*



(a) Communication 1 to 1 **(b)** Communication 1 to N **(c)** Cyclic communication

Figure 4.13: Graphic representation of communication operators

POSL also allows to declare non-communicating solvers to be executed in parallel, declaring only the list of solver names:

$$[\mathcal{S}_0, \mathcal{S}_1, \ldots, \mathcal{S}_{N-1}]$$

When we apply a connection operator $\textcircled{\text{op}}$ between a *communication jacks* list $\mathcal{J}$ and a *communication outlets* list $\mathcal{O}$, internally we are assigning an *abstract computation unit* (typically a thread) to each solver that we declare in each list. This assignment receives the name of *Solver Scheduling*. Before running the *solver set*, this *abstract unit of computation* is just an integer $\tau \in [0..N]$ identifying uniquely each of the solvers. When the *solver set* is launched, the solver with the identifier τ runs into the computation unit τ. This identifier assignation remains independent of the real availability of resources of computation. It just takes into account the user declaration. This means that, if the user declares 30 solvers (15 senders and 15 receivers) and the *solver set* is launched using 20 cores, only the first 20 solvers will be executed, and in consequence, there will be 10 solvers sending information to nowhere. Users should take this into account when declaring the *solver set*.

The connection process depends on the applied connection operator. In each case the goal is to assign, to the sending operator ($(\!|.|\!)^o$ or $(\!|.|\!)^m$) inside the *abstract solver*, the identifier of the solver (or solvers, depending on the connection operator) where the information will be

sent. Algorithm 5 presents the connection process.

Algorithm 5: Scheduling and connection main algorithm

```
input  : 𝒥 list of communication jacks,
         𝒪 list of communication outlets
1 while no available jacks or outlets do
2 |  S_jack ← GetNext(𝒥)
3 |  R_oulet ← GetNext(𝒪)
4 |  S ← GetSolverFromConnector(S_jack)
5 |  R ← GetSolverFromConnector(R_oulet)
6 |  Schedule(S)
7 |  R_id ← Schedule(R)
8 |  Connect(root(S),S_jack, R_id)
9 end
```

In Algorithm 5:

- `GetNext(...)` returns the next available solver-jack (or solver-outlet) in the list, depending on the connection operator, e.g., for the connection operator One-to-N, each *communication jack* in $\mathcal{J}$ must be connected with each *communication outlet* in $\mathcal{O}$.
- `GetSolverFromConnector(...)` returns the solver name given a connector declaration.
- `Schedule(...)` schedules a solver and returns its identifier.
- `Root(...)` returns the *root compound module* of a solver.
- `Connect(...)` searches the *computation module* S_{jack} recursively inside the *root compound module* of S and places the identifier R_{id} into its list of destination solvers.

Let us suppose that we have declared two solvers S and Z, both implementing the *abstract solver* in Algorithm 3, so they can be either sender or receiver. The following code connects them using the operator **1 to N**:

$$[S \cdot A] \circledcirc [Z \cdot C.M.]$$

If the operator **1 to N** is used with only with one solver in each list, the operation is equivalent to applying the operator **1 to 1**. However, to obtain a communication strategy like the one showed in Figure 4.13b, six solvers (three senders and three receivers) have to be declared to be able to apply the following operation:

$$[S_1 \cdot A, S_2 \cdot A, S_3 \cdot A] \circledcirc [Z_1 \cdot C.M., Z_2 \cdot C.M., Z_3 \cdot C.M.]$$

POSL provides a mechanism to make this easier, through *namespace expansions*.

4.5.1 Solver namespace expansion

One of the goals of POSL is to provide a way to declare sets of solvers to be executed in parallel fast and easily. For that reason, POSL provides two forms of namespace expansion, in order to create sets of solvers using already declared ones:

Solver name expansion - Uses an integer K to denote how many times the solver name S will appear in the declaration. $[\dots S_i \cdot \mathcal{M}(K), \dots]$ expands as $\left[\dots S_i \cdot \mathcal{M}, S_i^2 \cdot \mathcal{M}, \dots S_i^K \cdot \mathcal{M} \dots\right]$
and all new solvers $S_i^j, j \in [2..K]$ are created using the same solver declaration of solver S_i.

Connection declaration expansion - Uses an integer K to denote how many times the connection will be repeated in the declaration. Let a) $[\mathcal{S}_1 \cdot \mathcal{M}_1, \dots, \mathcal{S}_N \cdot \mathcal{M}_N]$ and b) $[\mathcal{R}_1 \cdot \mathcal{CM}_1, \dots, \mathcal{R}_M \cdot \mathcal{CM}_M]$ be the list of *communication jacks* and *communication outlets*, respectively, and c) $\textcircled{\text{op}}$ a connection operator. Then

$$[\mathcal{S}_1 \cdot \mathcal{M}_1, \dots, \mathcal{S}_N \cdot \mathcal{M}_N] \ \textcircled{op} \ [\mathcal{R}_1 \cdot \mathcal{CM}_1, \dots, \mathcal{R}_M \cdot \mathcal{CM}_M]\, K$$

expands as

$$\begin{gathered}
[\mathcal{S}_1 \cdot \mathcal{M}_1, \dots, \mathcal{S}_N \cdot \mathcal{M}_N] \ \textcircled{op} \ [\mathcal{R}_1 \cdot \mathcal{CM}_1, \dots, \mathcal{R}_N \cdot \mathcal{CM}_N] \\
\left[\mathcal{S}_1^2 \cdot \mathcal{M}_1, \dots, \mathcal{S}_N^2 \cdot \mathcal{M}_N\right] \ \textcircled{op} \ \left[\mathcal{R}_1^2 \cdot \mathcal{CM}_1, \dots, \mathcal{R}_N^2 \cdot \mathcal{CM}_N\right] \\
\dots \\
\left[\mathcal{S}_1^K \cdot \mathcal{M}_1, \dots, \mathcal{S}_N^K \cdot \mathcal{M}_N\right] \ \textcircled{op} \ \left[\mathcal{R}_1^K \cdot \mathcal{CM}_1, \dots, \mathcal{R}_N^K \cdot \mathcal{CM}_N\right]
\end{gathered}$$

and all new solvers $S_i^k, i \in [1..N]$ and $R_j^k, j \in [1..M]$, $k \in [2..K]$, are created using the same solver declaration of solvers S_i and R_j, respectively.

> Now, suppose that I have created solvers S and Z mentioned in the previews example. As a communication strategy, I want to connect them through the operator **1 to N**, using S as sender and Z as receiver. Then, using **namespace expansions**, I need to declare how many solvers I want to connect. Algorithm 6 shows the desired communication strategy. Notice in this example that the connection operation is affected also by the number 2 at the end of the line, as **connection declaration expansion**. In that sense, and supposing that 12 units of computation are available, a *solver set* working on parallel following the topology described in Figure 4.14 can be obtained.

Algorithm 6: A communication strategy

1 [S · *A* (3)] ⇝ [Z · *C.M.* (3)] 2 ;

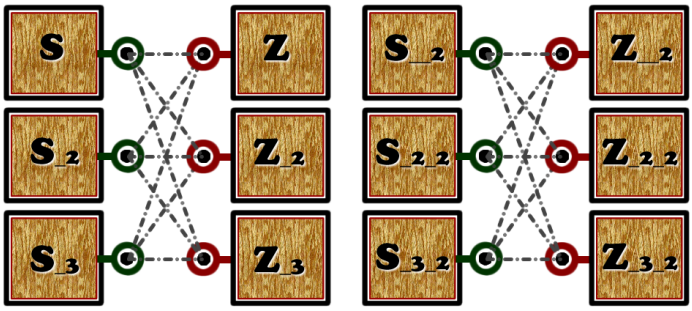


Figure 4.14: An example of connection strategy for 12 units of computation

4.6 Summarize

In this Chapter POSL have been formally presented, as a Parallel–Oriented Solver Language to build meta-heuristic-based solver to solve *Constraint Satisfaction Problems*. This language provides a set of *computation modules* useful to solve a wide range of problems. It is also possible to create new ones if needed, using a low-level framework in C++ programming language. POSL also provides a set of *communication modules*, essential features to share information between solvers.

One of the advantages of POSL is to create *abstract solvers* using a operator-based language, that remains independent of the used *computation* and *communication modules*. That is why it is possible to create many different solvers using the same solution strategy (the *abstract solver*) only instantiating it with different modules (*computation* and *communication modules*). It is also possible to create different communication strategies using the *connection operators* that POSL provides.

In the next Chapter, a detailed study of various communicating and non-communicating strategies, using some *Constraint Satisfaction Problems* as benchmarks. In this study, the efficacy of POSL to study easily and fast these strategies, is showed.

Part III

Study and evaluation of POSL

5

Experiments design and results

In this Chapter I expose all details about the evaluation process of POSL*, i.e., all experiments I perform. For each benchmark, I explain used strategies in the evaluation process and the used environments were the runs were performed (Curiosiphi server). I describe all the experiments and I expose a complete analysis of the obtained result.*

Contents

In this chapter I illustrate and analyze the versatility of POSL studying different ways to solve constraint problems based on local search meta-heuristics. I have chosen the *Social Golfers Problem*, the *N-Queens Problem*, the *Costas Array Problem* and the *Golomb Ruler Problem* as benchmarks since they are challenging yet differently structured problems. In this Chapter I present formally each benchmark, I explain the structure of POSL's solvers that I have generated for experiments and present a detailed analysis of obtained results.

The experiments[i] were performed on an Intel® Xeon™ E5-2680 v2, 10×4 cores, 2.80GHz. This server is called *Coriosiphi* and is located at *Laboratoire d'Informatique de Nantes Atlantique*, at the Universiti of Nantes. Showed results are the means of 30 runs for each setup, presented in columns labeled **T**, corresponding to the run-time in seconds, and **It.** corresponding to the number of iterations; and their respective standard deviations (**T(sd)** and **It.(sd)**). In some tables, the column labeled **% success** indicates the percentage of solvers finding a solution before reaching a time–out (5 minutes).

The experiments in this Chapter are multi-walk runs. Parallel experiments use 40 cores for all problem instances. It is important to point out that POSL is not designed to obtain the best results in terms of performance, but to give the possibility of rapidly prototyping and studying different cooperative or non cooperative search strategies.

All benchmarks were coded using the POSL low-level framework in C++.

First results using POSL to solve constraint problems were published in [125] were we used POSL to solve the *Social Golfers Problem* and study some communication strategies. It was the first version of POSL, therefore it was able to solve only relatively easy instances. However, the efficacy of the communication was showed using this tool.

With the next and more optimized version of POSL, I decide to start to perform more detailed studies using the benchmark mentioned before and some others.

5.1 Solving the *Social Golfers Problem*

In this section I present the performed study using *Social Golfers Problem* (*SGP*) as a benchmark.

[i] POSL source code is available on GitHub:https://github.com/alejandro-reyesamaro/POSL

5.1.1 Problem definition

The *Social Golfers Problem* (*SGP*) consists in scheduling $g \times p$ golfers into g groups of p players every week for w weeks, such that two players play in the same group at most once. An instance of this problem can be represented by the triple $g - p - w$. This problem, and other closely related problems, arise in many practical applications such as encoding, encryption, and covering problems [126]. Its structure is very attractive, because it is very similar to other problems, like *Kirkman's Schoolgirl Problem* and the *Steiner Triple System*, so efficient modules to solve a broad range of problems can be built.

The cost function for this benchmark was implemented making an efficient use of the stored information about the cost of the previews configuration. Using integers to work with bit-flags, a table to store the information about the partners of each player in each week can be filled in $O(p^2 \cdot g \cdot w)$. So, if a configuration has $n = (p \cdot g \cdot w)$ elements, this table can be filled in $O(p \cdot n)$. This table is filled from scratch only one time in the search process (I explain in the next section why). Then, every cost of a new configuration, is calculated based on this information and the performed changes between the new configuration and the stored one. This relative cost is calculated in $O(c \cdot g)$, where c is the number of performed changed in the new configuration with respect to the stored one.

5.1.2 Experiment design

Here, I give the *abstract solver* designed for this problem as well as concrete *computation modules* composing the different solvers I have tested:

a) Generation module:

I: Generates a random configuration s, respecting the structure of the problem, *i.e.*, the configuration is a set of w permutations of the vector $[1..n]$.

b) Neighborhood modules:

V_{Std}: Defines the neighborhood $\mathcal{V}(s)$ swapping players among groups.

V_{AS}: Defines the neighborhood $\mathcal{V}(s)$ swapping the most culprit player with other players from the same week. It is based on the *Adaptive Search* algorithm.

c) Selection modules:

S_{First}: Selects the first configuration $s' \in \mathcal{V}(s)$ improving the current cost.

S_{Best}: Selects the best configuration $s' \in \mathcal{V}(s)$ improving the current cost.

S_{Rand}: Selects a random configuration $s' \in \mathcal{V}(s)$.

d) Acceptance module:

A: Evaluates an acceptance criteria for s'. We have chosen the classical module selecting the configuration with the lowest global cost, *i.e.*, the one which is likely the closest to a solution.

A very first experiment was performed to select the best neighborhood function to solve the problem, comparing a basic solver using V_{Std}; a new solver using V_{AS}; and a combination of V_{Std} and V_{AS} by applying the operators ρ, already introduced in the previous chapter. Algorithms 7, 8 and 9 present the *abstract solver* for each case, respectively.

Algorithm 7: Standard *abstract solver* for *SGP*

```
1 abstract solver as_union                          // Itr → number of iterations
2 computation : I, V, S, A
3 begin
4     [↻ (Itr < K1)
5         I ↦ [↻ (Itr % K2) [V ↦ S ↦ A] ]
6     ]
7 end
```

Algorithm 8: *Abstract solver* combining neighborhood functions using operator *RHO*

```
1 abstract solver as_union                          // Itr → number of iterations
2 computation : I, V1, V2, S, A
3 begin
4     [↻ (Itr < K1)
5         I ↦ [↻ (Itr % K2) [[V1 ρ V2] ↦ S ↦ A] ]
6     ]
7 end
```

Algorithm 9: *Abstract solver* combining neighborhood functions using operator *Union*

```
1 abstract solver as_union                          // Itr → number of iterations
2 computation : I, V1, V2, S, A
3 begin
4     [↻ (Itr < K1)
5         I ↦ [↻ (Itr % K2) [[V1 ∪ V2] ↦ S ↦ A] ]
6     ]
7 end
```

Solvers mentioned above were too slow to solve instances of the problem with more than 3 weeks, so another solver implementing the *abstract solver* described in Algorithm 10 have been created, using V_{AS} and combining S_{First} and S_{Rand}: it tries a number of times to improve the cost, and if it is not possible, it picks a random neighbor for the next iteration. We also compared the S_{First} and S_{Best} selection modules.

Algorithm 10: *Abstract solver* for *SGP* to scape from local minima

```
1 abstract solver as_eager                          // Itr → number of iterations
2 computation : I, V, S1, S2, A
3 begin
4     [↻ (Itr < K1)
5         I ↦ [↻ (Itr % K2) [V ↦ [S1 ?Sci<K3 S2] ↦ A] ]
6     ]
7 end
```

After that, the best solver to be communicating solvers to compare their performance with the non communicating strategies was chosen. The shared information is the current configuration. Algorithms 11 and 12 show that the communication is performed while applying the acceptance criterion of the new configuration for the next iteration. Here, solvers receive a configuration from an outer solver, and match it with their current configuration. Then solvers select the configuration with the lowest global cost. We design different communication strategies. Either we execute a full connected solvers set, or a tuned combination of connected and unconnected solvers. Between connected solvers, we applied two different connections operations: connecting each sender solver with one receiver solver (*1 to 1*), or connecting each sender solver with all receiver solvers (*1 to N*).

Algorithm 11: Communicating *abstract solver* for *SGP* (sender)

```
1 abstract solver as_eager_sender                   // Itr → number of iterations
2 computation : I, V, S1, S2, A      // Sci → number of iterations with the same cost
3 begin
4     [↻ (Itr < K1)
5         I ↦ [↻ (Itr % K2) [V ↦ [S1 ?Sci<K3 S2] ↦ (|A|)^o] ]
6     ]
7 end
```

In all Algorithms ins this section, three parameter can be found: 1. K_1: the maximum number of *restarts*, 2. K_2: the maximum number of iterations in each *restart*, and K_3: the maximum number of iterations with the same current cost. 3.

After the selection of the proper modules to study different communication strategies, I proceeded to tune these parameter. Only a few runs were necessaries to conclude that the mechanism of using the *computation module* S_{rand} to scape from local minima was enough.

Algorithm 12: Communicating *abstract solver* for *SGP* (receiver)

```
1  abstract solver as_eager_receiver                    // Itr → number of iterations
2  computation : I, V, S1, S2, A      // Sci → number of iterations with the same cost
3  communication : C.M.
4  begin
5      [↻ (Itr < K1)
6          I ↦
7          [↻ (Itr % K2)
8              V ↦ [S1 ?_{Sci<K3} S2] ↦ [A m C.M.]
9          ]
10     ]
11 end
```

For that reason, since the solver never perform restarts, the parameter K_1 was irrelevant. So the reader can assume $K_1 = 1$ for every experiment.

With the certainty that solvers do not performs restarts during the search process, I select the same value for $K_2 = 5000$ in order to be able to use the same *abstract solver* for all instances.

Finally, in the tuning process of K_3, I notice only slightly differences between using the values 5, 10, and 15. So I decided to use $K_3 = 5$.

5.1.3 Analysis of results

Table 5.1 showed results of launching *solver set*s to solve each instance of the problem sequentially. Not surprisingly, the means of sequential runtimes and iterations (Table 5.1) are bigger than those means of parallel runs, with or without communication (all other tables).

Instance	T	T(sd)	It.	It.(sd)	% success
5–3–7	8.31	7.64	17,347	15,673	100.00
8–4–7	16.92	15.15	7,829	7,019	100.00
9–4–8	79.60	64.07	20,779	16,537	94.28
11–7–5	3.37	2.16	664	380	100.00

Table 5.1: *Social Golfers*: a single sequential solver

In a first stage of the experiments I use the operator-based language provided by POSL to build and test many different non communicating strategies. The goal is to select the best concrete modules to run tests performing communication. In particular, I have tested two kind of computation modules: the one computing the neighborhood of a given configuration and the one choosing the current configuration for the next solver iteration.

Abstract solvers	T	T(sd)	It.	It.(sd)
Adaptive Search (AS)	**1.06**	0.79	352	268
Std (ρ) AS	41.53	26.00	147	72
Std $(\cup)$ AS	59.65	55.01	198	110
Standard (Std)	87.90	41.96	146	58

Table 5.2: *Social Golfers*: Instance 10–10–3 in parallel

I focused on choosing the right neighborhood function. In the case of the *Social Golfers Problem*, this experiment was launched using a basic abstract solver showed in Algorithm 7. Solvers implemented from this abstract solver was too slow to solve instances beyond three weeks: they were very often trapped into local minima. This is the reason why we perform this first experiment with the instance 10–10–3 whereas next experiments scale above 3 weeks. This was not a problem though, since the goal of this first experiment was only to find the right concrete neighborhood module.

Results in Table 5.2 are not surprising. The neighborhood module V_{AS} is based on the *Adaptive Search* algorithm, which has shown very good results [5]. It selects the most culprit variable (i.e., a player), that is, the variable to most responsible for constraints violation. Then, it permutes this variable value with the value of each other variable, in all groups and all weeks. Each permutation gives a neighbor of the current configuration. V_{Std} uses no additional information, so it performs every possible swap between two players in different groups, every week. It means that this neighborhood is $g \times p$ times bigger than the previous one, with g the number of groups and p the number of players per group. It allows for more organized search because the set of neighbors is pseudo-deterministic, i.e., the construction criteria is always the same but the order of the configuration is random. On the other hand, *Adaptive Search* neighborhood function takes random decisions more frequently, and the order of the configurations is random as well. We also tested abstract solvers with different combinations of these modules, using the (ρ) and the $(\cup)$ operators. The (ρ) operator executes its first or second parameter depending on a given probability ρ, and the $(\cup)$ operator returns the union of its parameters output. All these combinations spent more time searching the best configuration among the neighborhood, although with a lower number of iterations than V_{AS}. The V_{AS} neighborhood function being clearly faster, we have chosen it for our experiments, even if it shown a more spread standard deviation: 0.75 for AS versus 0.62 for Std, considering the ratio $\frac{T(sd)}{T}$.

With the selected neighborhood function, I focused on choosing the best *selection* function. I compared two different concrete modules used within the abstract solver in Algorithm 10, which combines selection modules (S_{First} or S_{Best}) with S_{Rand}, to avoid being trapped into local minima: it tries to improve the cost in a limited number of iterations. If it is not possible, it selects a random neighbor for the next iteration. The first module was S_{Best} that

Instance	O.M. Best Improvement				O.M. First Improvement			
	T	T(sd)	It.	It.(sd)	T	T(sd)	It.	It.(sd)
5–3–7	4.99	4.43	4,421	3,938	**1.32**	0.68	1322	676
8–4–7	5.10	1.77	954	334	**1.82**	0.84	445	191
9–4–8	12.37	5.40	1,342	591	**6.43**	4.60	873	591
11–7–5	5.19	1.67	351	114	**2.22**	0.69	273	58

Table 5.3: *Social Golfers*: comparing selection functions

Instance	Communication 1 to 1				Communication 1 to N			
	T	T(sd)	It.	It.(sd)	T	T(sd)	It.	It.(sd)
5–3–7	1.19	0.64	1,156	608	1.11	0.49	1,067	484
8–4–7	**1.30**	0.72	**317**	161	1.46	0.57	347	128
9–4–8	4.38	2.72	**597**	347	5.51	3.06	736	389
11–7–5	1.76	0.41	214	44	**1.62**	0.34	**202**	30

Table 5.4: *Social Golfers*: test with 100% of communication

selects the best configuration inside the neighborhood. It not only spent more time searching a better configuration, but also is more sensitive to become trapped into local minima. The second module was S_{First} which selects the first configuration inside the neighborhood improving the current cost. Using this module, solvers favor exploration over intensification and of course spend clearly less time computing the neighborhood. Table 5.3 presents results of this experiment, showing that an exploration-oriented strategy is better for the *SGP*. If we compare results of Tables 5.1 and 5.3 with respect to the standard deviation, we can some gains in robustness with parallelism. The spread in the running times and iterations for the instance 9–4–8 (the hardest one) is 10% lower (0.80 sequentially versus 0.71 in parallel), and for the others, it is around 40% lower (0.91, 0.89 and 0.64 sequentially versus 0.51, 0.45 and 0.31 in parallel, for 5–3–7, 8–4–7 and 11–7–5 respectively, with the same ratio $\frac{T(sd)}{T}$).

Then we ran experiments to study POSL's behavior solving target problems in communicating scenarios. Some compositions of solvers set were taken into account: i. the structure of the communication (with/without communication or a mix), and ii. the used communication operator.

Each time a POSL meta-solver is launched, many independent search solvers are executed. We call "good" configuration a configuration with the lowest cost within the current con-

Instance	Communication 1 to 1				Communication 1 to N			
	T	T(sd)	It.	It.(sd)	T	T(sd)	It.	It.(sd)
5–3–7	1.04	0.45	1,019	456	1.04	0.53	1,031	530
8–4–7	1.40	0.57	337	122	1.43	0.76	353	167
9–4–8	4.64	2.17	637	279	5.75	3.06	776	389
11–7–5	1.81	0.40	220	33	1.82	0.39	222	39

Table 5.5: *Social Golfers*: test with 50 % of communication

Instance	Communication 1 to 1				Communication 1 to N			
	T	T(sd)	It.	It.(sd)	T	T(sd)	It.	It.(sd)
5–3–7	**0.90**	0.51	**881**	492	1.19	0.67	1,170	655
8–4–7	1.39	0.43	341	94	1.46	0.43	352	96
9–4–8	**4.33**	1.92	599	248	4.53	2.01	625	251
11–7–5	1.99	0.54	242	51	1.63	0.35	224	28

Table 5.6: *Social Golfers*: test with 25% of communication

figuration neighborhood and with a cost strictly lesser than the current one. Once a good configuration is found in a sender solver, it is transmitted to the receiver one. At this moment, if the information is accepted, there are some solvers searching in the same subset of the search space, and the search process becomes more exploitation–oriented. This can be problematic if this process makes solvers converging too often towards local minima. In that case, we waste more than one solver trapped into a local minima: we waste all solvers that have been attracted to this part of the search space because of communications. I avoid this phenomenon through a simple (but effective) play: if a solver is not able to find a better configuration inside the neighborhood (executing S_{First}), it selects a random one at the next iteration (executing S_{Rand}). This strategy, using communication between solvers, produces some gain in terms of runtime (Table 5.3 with respect to Tables 5.4, 5.5 and 5.6. The percentage of the receiver solvers that were able to find the solution before the others did, was significant (see Appendix B). That shows that the communication played an important role during the search, despite inter–process communication's overheads (reception, information interpretation, making decisions, etc). Having many solvers searching in different places of the search space, the probability that one of them reaches a promising place is higher. Then, when a solver finds a good configuration, it can be communicated, and receiving the help of one or more solvers in order to find the solution. For this problem we have reduced the spread in the running times and iterations of the results for the two last instances (9–4–8 and 11–7–5) applying the communication strategy (0.71 without communication versus 0.44 with communication, for 9–4–8, and 0.31 without communication versus 0.20 with communication for 11–7–5).

Other two strategies were analyzed in the resolution of this problem, with no success, both based on the sub-division of the work by weeks, i.e., solvers trying to improve a configuration only working with one or some weeks. To this end two strategies were designed:

A **Circular strategy:** K solvers try to improve a configuration during a during a number of iteration, only working on one week. When no improvement is obtained, the current configuration is communicated to the next solver (circularly), which tries to do the same working on the next week (see Figure 5.1a).

This strategy does not show better results than previews strategies. The reason is

because, although the communication in POSL is asynchronous, most of the times solvers were trapped waiting for a configuration coming from its neighbor solver.

B **Dichotomy strategy:** Solvers are divided by levels. Solvers in level 1, only work on one week, solvers on level 2, only work on 2 consecutive weeks, and so on, until the solver that works on all (except the first one) weeks. Solvers in level 1 improve a configuration during some number of iteration, then this configuration is sent to the corresponding solver. A solver in level 2 do the same, but working on weeks k to $k+1$. It means that it receives configurations from the solver working on week k and from the solver working on week $k+1$, and sends its configuration to the corresponding solver working on weeks k to $k+3$; and so on. The solver in the last level works on all (except the first one) weeks and receive configuration from the solver working on weeks 2 to $w/2$ and from the solver working on weeks $w/2+1$ to w (see Figure5.1b). We tested this strategy with all possible levels.

The goal of this strategy was testing if focused searches rapidly communicated can help at the beginning of the search. However, The failure of this strategy is in the fact that most of the time the sent information arrives to late to the receiver solver.

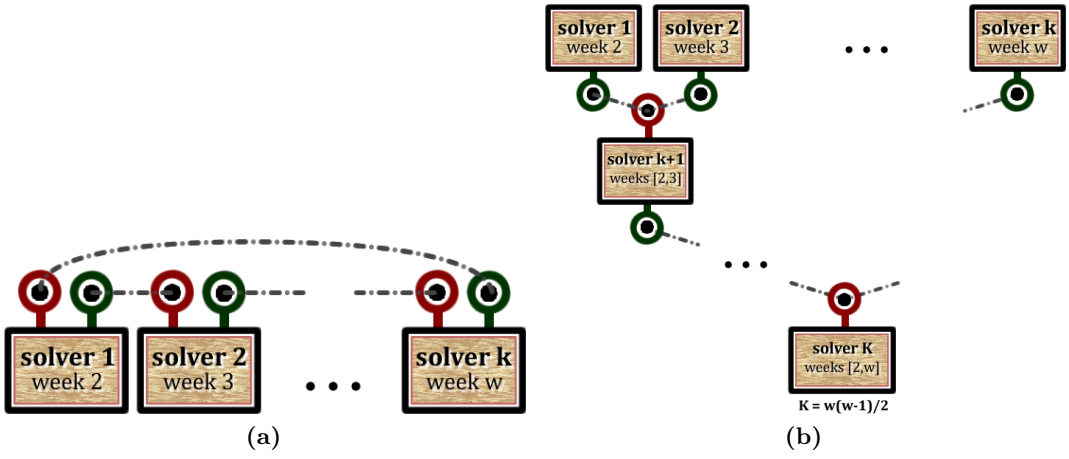


Figure 5.1: Unsuccessful communication strategies to solve *SGP*

5.2 Solving the *N-Queens Problem*

In this section I present the performed study using *N-Queens Problem* (*NQP*) as a benchmark.

5.2.1 Problem definition

The *N-Queens Problem* (*NQP*) asks how to place N queens on a chess board so that none of them can hit any other in one move. This problem was introduced in 1848 by the chess player Max Bezzelas as the *8-queen problem*, and years latter it was generalized as *N-queen problem* by Franz Nauck. Since then many mathematicians, including Gauss, have worked on this problem. It finds a lot of applications, e.g., parallel memory storage schemes, traffic control, deadlock prevention, neural networks, constraint satisfaction problems, among others [127]. Some studies suggest that the number of solution grows exponentially with the number of queens (N), but local search methods have been shown very good results for this problem [128]. For that reason we tested some communication strategies using POSL, to solve a problem relatively easy to solve using non communication strategies.

The cost function for this benchmark was implemented in C++ based on the current implementation of *Adaptive Search*[ii].

5.2.2 Experiment design Nr. 1

To handle this problem, I reused some modules used for the *Social Golfers Problem*: the *Selection* and *Acceptance* modules. The new module is:

a) Neighborhood module:

V_{AS}: Defines the neighborhood $V(s)$ swapping the variable which contributes the most to the cost with other.

Fors this problem I used a simple *abstract solver* showing good results with no communication, based on the idea introduced in the section 5.1, using the *computation module* S_{rand} to scape from local minima. The *abstract solver* is presented in Algorithm 13.

Algorithm 13: *Abstract solver* for *NQP*

```
1 abstract solver as_eager                          // Itr → number of iterations
2 computation : I, V, S1, S2, A      // Sci → number of iterations with the same cost
3 begin
4     I ↦ [ ↻ (Itr < K1) V ↦ [ S1 ?_{Sci<K2} S2 ] ↦ A ]
5 end
```

Using solvers implementing this *abstract solver* we create communicating solvers to compare their performance with the non communicating strategies. The shared information is the

[ii] It is based on the code from Daniel Díaz available at https://sourceforge.net/projects/adaptivesearch/

current configuration. Algorithms 14 and 15 show that the communication is performed while selecting a new configuration for the next iteration. We design different communication strategies. Either I execute a full connected solvers set, or a tuned combination of connected and unconnected solvers. Between connected solvers, I have applied two different connections operations: connecting each sender solver with one receiver solver (*1 to 1*), or connecting each sender solver with all receiver solvers (*1 to N*).

Algorithm 14: *Abstract solver* for *NQP* (sender)

```
1 abstract solver as_eager_sender                    // ITR → number of iterations
2 computation : I, V, S1, S2, A                      // SCI → number of iterations with the same cost
3 begin
4     I ↦ [↻ (ITR < K1) V ↦ [(|S1|)^o ?_{SCI<K2} S2] ↦ A ]
5 end
```

Algorithm 15: *Abstract solver* for *NQP* (receiver)

```
1 abstract solver as_eager_receiver                  // ITR → number of iterations
2 computation : I, V, S1, S2, A                      // SCI → number of iterations with the same cost
3 communication : C.M.
4 begin
5     I ↦
6     [↻ (ITR < K1)
7         V ↦ [[S1 ?_{ITR%K2} [S1 m C.M.]] ?_{SCI<K3} S2] ↦ A
8     ]
9 end
```

5.2.3 Results analysis of experiment Nr. 1

I use directly the neighborhood module V_{AS} based on the *Adaptive Search* algorithm, and the selection module S_{First} which selects the first configuration inside the neighborhood improving the current cost, to create solvers, and studying communicating and non communicating strategies.

Instance	Sequential (1 core)				No Comm. (40 cores)			
	T	T(sd)	It.	It.(sd)	T	T(sd)	It.	It.(sd)
2000	6.20	0.12	947	21	6.15	0.20	952	20
3000	14.19	0.21	1,415	22	14.06	0.33	1,413	25
4000	25.63	0.36	1,900	28	25.46	0.51	1,898	34
5000	41.37	0.44	2,367	26	40.57	0.91	2,377	32
6000	60.42	0.52	2,837	31	60.10	0.70	2,849	43

Table 5.7: Results for *NQP* (no communication)

Instance	25% Comm.				50% Comm.				All Comm.			
	T	T(sd)	It.	It.(sd)	T	T(sd)	It.	It.(sd)	T	T(sd)	It.	It.(sd)
2000	6.05	0.25	934	36	6.01	0.19	920	41	5.92	0.17	885	49
3000	13.89	0.28	1,387	48	13.91	0.30	1,368	51	13.67	0.39	1,346	40
4000	25.26	0.63	1,868	43	25.14	0.50	1,855	50	25.11	0.39	1,834	58
5000	40.38	0.93	2,338	71	40.33	0.66	2,312	69	39.62	1.07	2,287	44
6000	59.28	1.34	2,794	78	58.97	1.19	2,775	67	58.97	1.38	2,729	78

Table 5.8: Results for *NQP* (40 cores, communication 1 to 1)

Instance	25% Comm.				50% Comm.				All Comm.			
	T	T(sd)	It.	It.(sd)	T	T(sd)	It.	It.(sd)	T	T(sd)	It.	It.(sd)
2000	6.07	0.15	925	41	5.98	0.19	915	41	6.01	0.19	887	57
3000	13.97	0.34	1,402	49	13.96	0.31	1,386	52	13.79	0.32	1,365	65
4000	25.30	0.57	1,867	52	25.29	0.42	1,851	66	25.17	0.47	1,838	65
5000	40.45	0.80	2,338	80	40.37	0.56	2,312	56	39.88	0.71	2,291	51
6000	59.77	1.50	2,824	49	59.53	0.98	2,773	69	59.16	1.37	2,773	57

Table 5.9: Results for *NQP* (40 cores, communication 1 to N)

POSL, as it can be seen in Tables 5.8 and 5.9, works very well without communication, for instances relatively big. This confirms once again the success of the *computation module* V_{AS} based on *Adaptive Search* algorithm to solve these kind of problems. That is the reason why the parallel approach does not outperforms significantly the sequential one, as we can see in Table 5.7. However, the communication improve the non communicating results in terms of runtime and iterations, but this improvement is not significant. In contrast to *SGP*, POSL does not get trapped so often into local minima during the resolution of *NQP*. For that reason, the shared information, once received and accepted by the receivers solvers, does not improves largely the current cost.

We can see the improvement with respect to the percentage of communicating solvers in Figure 5.2. The bigger the instance is, the more significant the observed improvement is. This phenomenon suggests that a deeper study and an efficient implementation can make the communication playing a more significant role in the solution process. For that reason, I decided to design another experiment to try to improve the results using communicating strategies using POSL.

5.2.4 Experiment design Nr. 2

The strategy of work sub-division proposed in the previews section with *Social Golfers Problem* seemed interesting to me and I did not want to give up on it. So I tried to apply it to *N-Queens Problem*.

In some experimental runs, I launched some *partial* solvers (i.e., solvers only performing permutations between variables into certain range), together with a *full* solver (i.e., a solver

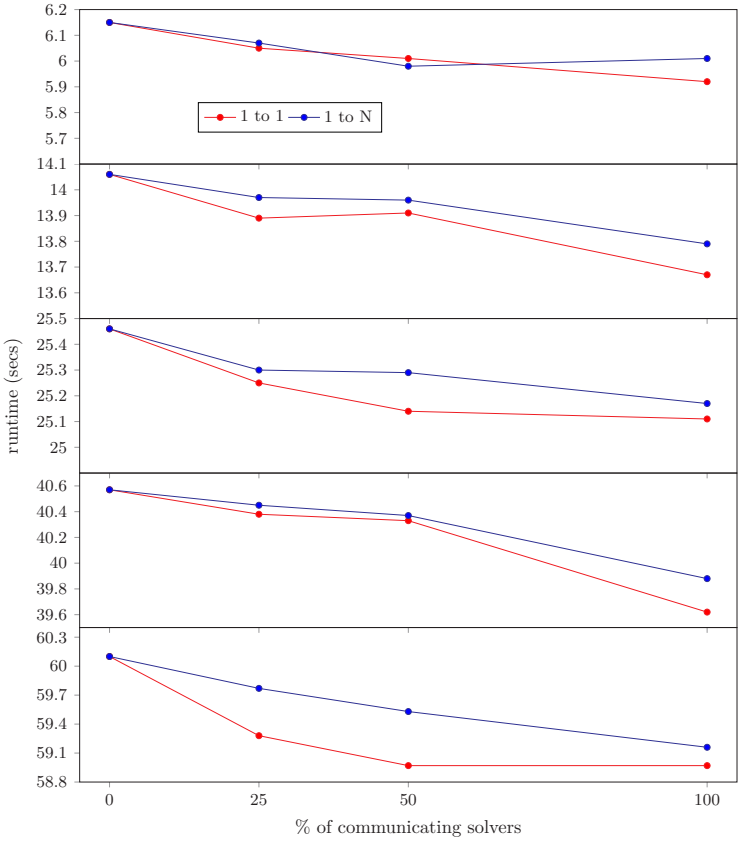


Figure 5.2: Runtime means of instances 2000-, 3000-, 4000-, 5000- and 6000-queens

working with the entire configuration). I used the instance *4000-queens* as test and I built the following solvers:

a) Solver S_1 only permuting the first 1000 variables

b) Solver S_2 only permuting the first 2000 variables

c) Solver S_3 only permuting the first 3000 variables

d) Solver S_4 a *full* solver.

Obviously, the first three solvers were not able to find a solution to the problem, but at the beginning of runs, it was possible to observe that these solvers were able to obtain configurations with costs considerably lower with respect to the *full* solver S_4. For that reason I put in practice the idea of connecting *partial* solvers together with *full* solvers. This way, the search process can be accelerated at the beginning.

Before designing the solution strategy (*abstract solver*) many experiments were launched to select: 1. The number of sub-divisions of the configuration, i.e., how many *partial* solvers works in different sections of the configuration. They are connected to the *full* solver. 2. The size of the section where the *partial* solvers work in.

After many runs of these experiments, it was decided to work with two *partial* sender solvers (implementing the *abstract solver* in Algorithm 16). In this algorithm a and b are parameters of the module $V_{[a,b]}$ used in the *partial* solvers. They represent the variables defining the range of the configuration where the *partial* solver works. They were chosen such that $b - a = \frac{n}{4}$ These solvers send their configurations to the *full* solver that implements the *abstract solver* in Algorithm 17.

Algorithm 16: *Abstract solver* for *NQP* (partial solver sender)

```
1 abstract solver as_partial_sender                    // Itr → number of iterations
2 computation : I, V_[a,b], S, A      // Sci → number of iterations with the same cost
3 begin
4     [↺ (Itr < K_1)
5         I ↦ [↺ (Itr % K_2 || Sci < K_3) [V_[a,b] ↦ S ↦ [(|A|)^o ?_{Itr%K_4} A]] ]
6     ]
7 end
```

Algorithm 17: *Abstract solver* for *NQP* (full solver receiver)

```
1 abstract solver as_full_receiver                     // Itr → number of iterations
2 computation : I, V, S, A            // Sci → number of iterations with the same cost
3 communication : C.M.
4 begin
5     I ↦
6     [↺ (Itr < K_1)
7         [V ↦ S ↦ [A ?_{Itr%K_2} [A m C.M.]]]
8     ]
9 end
```

5.2.5 Results analysis of experiment Nr. 2

Results in Table 5.10 show that this strategy is effective to solve the *N-Queens Problem* improving the runtimes already obtained in the previews experiment. In the resolution of this problem, the improvement rate of the current configuration cost is very slow (yet stable). The *partial* solvers work only on a section of the configuration, and for that reason, they are able to obtain configuration with costs considerably lower than the obtained by the *full* solver more quickly. This characteristic is taken into account: *partial* solvers send their obtained configurations to the *full* solvers. By doing this, the improvement rate of the current configuration can be accelerated at the beginning of the search.

Instance	T	T(sd)	It.	It.(sd)
2000	**5.11**	0.83	**841**	37
3000	**11.55**	1.96	**1,275**	67
4000	**21.27**	3.76	**1,656**	108
5000	**34.77**	4.99	**2,082**	108
6000	**51.72**	5.73	**2,501**	176

Table 5.10: Results for *NQP* (40 cores, communication partial-full solvers)

5.3 Solving the *Costas Array Problem*

In this section I present the performed study using *Costas Array Problem* (*CAP*) as a benchmark.

5.3.1 Problem definition

The *Costas Array Problem* (*CAP*) consists in finding a *costas array*, which is an $n \times n$ grid containing n marks such that there is exactly one mark per row and per column and the $n(n-1)/2$ vectors joining each couple of marks are all different. This is a very complex problem that finds useful application in some fields like sonar and radar engineering. It also presents an interesting characteristic: although the search space grows factorially, from order 17 the number of solutions drastically decreases [129].

The cost function for this benchmark was implemented in C++ based on the current implementation of *Adaptive Search*[iii].

5.3.2 Experiment design

To handle this problem, I have reused all modules used for solving the *N-Queens Problem*. First attempts to solve this problems were using the same strategies (*abstract solvers*) used to solve the *Social Golfers Problem* and *N-Queens Problem*, without success: POSL was not able to solve instances larger than $n = 8$ in a reasonable amount of time (seconds). After many unsuccessful attempts to find the rights parameter of *maximum number of restarts*, *maximum number of iterations*, and *maximum number of iterations with the same cost*, I decided to implement the mechanism used by Daniel Díaz in the current implementation of *Adaptive Search* to escape from local minima: I have added a *Reset* module (R).

[iii] It is based on the code from Daniel Díaz available at https://sourceforge.net/projects/adaptivesearch/

The basic solver I use to solve this problem is presented in Algorithm 18, and I take it as a base to build all the different communication strategies. Basically, it is a classical local search iteration, where instead of performing restarts, it performs resets. After a deep analysis of this implementation and results of some runs, I decided to use $K_1 = 24000$ (maximum number of iterations) big enough to solve the chosen instance $n = 17$; and $K_2 = 3$ (the number of iteration before performing the next *reset*).

Algorithm 18: Reset-based *abstract solver* for *CAP*

```
1 abstract solver as_hard                          // ITR → number of iterations
2 computation : I, R, V, S, A
3 begin
4     I (↦)
5     [↻ (ITR < K1)
6         R (↦) [↻ (ITR % K2) [V (↦) S (↦) A] ]
7     ]
8 end
```

The *abstract solver* for the sender solver is presented in Algorithm 19. Like for the *Social Golfers Problem*, we design different communication strategies combining different percentages of communicating solvers and our two communication operators (*1 to 1* and *1 to N*). However for this problem, we study the behavior of the communication performed at two different moments: while applying the acceptance criteria (Algorithm 20), and while performing a

reset (Algorithms 20, 21 and 22).

Algorithm 19: Reset-based *abstract solver* for *CAP* (sender)

```
1 abstract solver as_hard_sender                    // Itr → number of iterations
2 computation : I, R, V, S, A
3 begin
4     I (↦)
5     [↺ (Itr < K1)
6         R (↦) [↺ (Itr % K2) [V (↦) S (↦) (|A|)^o] ]
7     ]
8 end
```

Algorithm 20: Reset-based *abstract solver* for *CAP* (receiver, variant A)

```
1 abstract solver as_hard_receiver_a                // Itr → number of iterations
2 computation : I, R, V, S, A
3 communication : C.M.
4 begin
5     I (↦)
6     [↺ (Itr < K1)
7         R (↦) [↺ (Itr % K2) [V (↦) S (↦) [A (m) C.M.]] ]
8     ]
9 end
```

Algorithm 21: Reset-based *abstract solver* for *CAP* (receiver, variant B)

```
1 abstract solver as_hard_receiver_b                // Itr → number of iterations
2 computation : I, R, V, S, A       // Sci → number of iterations with the same cost
3 communication : C.M.
4 begin
5     I (↦)
6     [↺ (Itr < K1)
7         [R (?)_{Sci<K3} [R (m) C.M.]] (↦) [↺ (Itr % K2) [V (↦) S (↦) A] ]
8     ]
9 end
```

Algorithm 22: Reset-based *abstract solver* for *CAP* (receiver, variant C)

```
1 abstract solver as_hard_receiver_c                // Itr → number of iterations
2 computation : I, R, V, S, A
3 communication : C.M.
4 begin
5     I (↦)
6     [↺ (Itr < K1)
7         [R (m) C.M.] (↦) [↺ (Itr % K2) [V (↦) S (↦) A] ]
8     ]
9 end
```

5.3.3 Analysis of results

I present in Table 5.11 results of launching *solver sets* to solve each instance of *Costas Array Problem* sequentially. Runtimes and iteration means showed in this table are bigger than those presented in Table 5.12, confirming once again the success of the parallel approach.

STRATEGY	T	T(ds)	It.	It.(sd)	% success
Sequential (1 core)	2.12	0.87	44,453	18,113	42.00
Parallel (40 cores)	0.73	0.46	9,556	6,439	100.00

Table 5.11: *Costas Array* 17: no communication

I chose directly the neighborhood module (V_{AS}), the selection module (S_{First}) and the acceptance module A, to create solvers. I ran experiments to study parallel communicating strategies taken into account the structure of the communication, and the communication operator used, but in this problem, I perform the communication at two different times: at the time of applying the acceptance criteria, and at the time of performing the *reset*.

Table 5.12 shows that the *abstract solver A* (receiving the configuration at the time of applying the acceptance criteria) is more effective. The reason is that the others, interfere with the proper performance of the *reset*, that is a very important step in the algorithm. This step can be performed on three different ways:

a) Trying to shift left/right all sub-vectors starting or ending by the variable which contributes the most to the cost, and selecting the configuration with the lowest cost.

b) Trying to add a constant (circularly) to each element in the configuration.

c) Trying to shift left from the beginning to some culprit variable (i.e., a variable contributing to the cost).

Then, one of these 3 generated configuration has the sabe probability of being selected, to be the result of the *reset* step. In that sense, some different *resets* can be performed for the same configuration. Here is when the communication play its important role: receiver and

	100% COMM				50% COMM			
STRATEGY	T	T(sd)	It.	It.(sd)	T	T(sd)	It.	It.(sd)
Str A: 1 to 1	**0.41**	0.30	4,973	3,763	0.55	0.43	8,179	7,479
Str A: 1 to N	0.43	0.31	5,697	4,557	0.57	0.46	8,420	7,564
Str B: 1 to 1	0.48	0.41	6,546	5,562	0.51	0.49	8,004	7,998
Str B: 1 to N	0.45	0.46	5,701	6,295	0.48	0.51	7,245	8,379
Str C: 1 to 1	0.48	0.43	6,954	6,706	0.58	0.43	8,329	6,593
Str C: 1 to N	0.49	0.38	6,457	5,875	0.58	0.50	8,077	8,319

Table 5.12: *Costas Array* 17: with communication

sender solvers apply different *reset* in the same configuration, and results showed the efficacy of this communication strategy.

Table 5.12 shows also high values of standard deviation. This is not surprising, due to the highly random nature of the neighborhood function and the selecting criterion, as well as the execution of many resets during the search process.

5.4 Solving the *Golomb Ruler Problem*

In this section I present the performed study using *Golomb Ruler Problem* (*GRP*) as a benchmark.

5.4.1 Problem definition

The *Golomb Ruler Problem* (*GRP*) problem consists in finding an ordered vector of n distinct non-negative integers, called *marks*, $m_1 < \cdots < m_n$, such that all differences $m_i - m_j$ $(i > j)$ are all different. An instance of this problem is the pair (o, l) where o is the order of the problem, (i.e., the number of *marks*) and l is the length of the ruler (i.e., the last *mark*). We assume that the first *mark* is always 0. This problem has been applied to radio astronomy, x-ray crystallography, circuit layout and geographical mapping [130]. When I apply POSL to solve an instance of this problem sequentially, I can notice that it performs many *restarts* before finding a solution. For that reason I have chosen this problem to study a new communication strategy.

The cost function is implemented based on the storage of a counter for each measure present in the rule (configuration). I also store all distances where a variable is participating. This information is usefull to compute the more culprit variable (the variable that interfiers less in the representd measures), in case of the user wants to apply algorithms like *Adaptive Search*. This cost is calculated in $O\left(o^2 + l\right)$.

5.4.2 Experiment design

I use *Golomb Ruler Problem* instances to study a different communication strategy. This time I communicate the current configuration, to avoid its neighborhood, i.e., a *tabu* configuration. I have reused some modules used in the resolution of *Social Golfers* and *Costas Array*

problems to design the solvers: the *Selection* and *Acceptance* modules. The new modules are:

a) Generation module:

I: Generates a random configuration s, respecting the structure of the problem, i.e., the configuration is an ordered vector of integers. This module takes into account a set of *tabu* configurations arrived from the same solver, and also via solver-communication through a *communication module* $C.M.$ that receives a set of configurations. This module constructs the new configuration far enough from the *tabu* configurations.

b) Neighborhood module:

V: Defines the neighborhood $\mathcal{V}(s)$ by changing one value while keeping the order, i.e., replacing the value s_i by all possible values $s'_i \in D_i$ that satisfy $s_{i-1} < s'_i < s_{i+1}$.

I also add a module to insert a configuration into a *tabu* list inside the solver. In Algorithm 23 the *abstract solver* used to send information (sender *abstract solver*) is presented. When the module T is executed, the solver is unable to find a better configuration around the current one, so it is assumed to be a local minimum, and it is sent to the receiver solver. Algorithm 24 presents an *abstract solver* used to receive information (receiver *abstract solver*). Based on the connection operator used in the communication strategy, this solver might receives one or many configurations. These configurations are the input of the generation module (I), and this module inserts all received configurations into a *tabu* list, and then generates a new first configuration, far from all configurations in the *tabu* list.

Algorithm 23: *Abstract solver* for *GRP* (sender)

```
1 abstract solver as_golomb_sender                    // Itr → number of iterations
2 computation : I, V, S, A, T
3 begin
4     [↺ (Itr < K1)
5         I ↦ [↺ (Itr % K2) [V ↦ S ↦ A] ] ↦ (|T|)^o
6     ]
7 end
```

Algorithm 24: *Abstract solver* for *GRP* (receiver)

```
1 abstract solver as_golomb_receiver                  // Itr → number of iterations
2 computation : I, V, S, A, T
3 connection : C.M.
4 begin
5     [↺ (Itr < K1)
6         [C.M. ↦ I] ↦ [↺ (Itr % K2) [V ↦ S ↦ A] ] ↦ (|T|)^o
7     ]
8 end
```

In this communcation strategy there are some parameters to be tuned. The first ones are: 1. K_1, the number of restarts, and 2. K_2, the number of iterations by restart. Both are instance dependent, so, after many experimental runs, I choose them as follows:

- *Golomb Ruler* 8–34: $K_1 = 300$ and $K_2 = 200$
- *Golomb Ruler* 10–55: $K_1 = 1000$ and $K_2 = 1500$
- *Golomb Ruler* 11-72: $K_1 = 1000$ and $K_2 = 3000$

The other parameters are related to the behavior of the *tabu list*:

The idea of this strategy (*abstract solver*) follows the following steps:

Step 1

The *computation module* generates an initial configuration tacking into account a set of configurations into a *tabu list*. The configuration arriving to this *tabu list* come from the same solver (Step 3) or from outside (other solvers) depending on the strategy (non-communicating or communicating).

This module applies some other modules provided by POSL to solve the *Sub-Sum Problem* in order to generates *valid* configurations for *Golomb Ruler Problem*. A valid configuration s for *Golomb Ruler Problem* is a configuration that fulfills the following constraints:

- $s = (a_1, \ldots, a_o)$ where $a_i < a_j, \forall i < j$, and
- all $d_i = a_{i+1} - a_i$ are all different, for all $d_i, i \in [1...o-1]$

The *Sub-sum Problem* is defined as follows: Given a set E of integers, with $|E| = N$, finding a sub set e of n elements that sums exactly z. In that sense, a solution $S_{sub-sum} = \{s_1, \ldots, s_{o-1}\}$ of the *Sub-sum problem* with $E = \left\{1, \ldots, l - \frac{(o-2)(o-1)}{2}\right\}$, $n = o - 1$ and $z = l$, can be traduced to a *valid* configuration C_{grp} for *Golomb Ruler Problem* as follows:

$$C_{grp} = \{c_1, c_1 + s_1, \ldots, c_{o-1} + s_{o-1}\}$$

where $c_1 = 0$.

In the selection module applied inside the module I, the selection step of the search process selects a configuration from the neighborhood *far* from the $tabu$ configurations, with respect to certain vectorial norm and an epsilon. In other words, a configuration C is selected if and only if:

a) the cost of the configuration C is lower than the current cost, and

b) $||C - C_t||_p > \varepsilon$, for all $tabu$ configuration C_t

where p and ε are parameters.

I experimented with 3 different values for p. Each value defines a different type of norm of a vector $x = \{x_1, \ldots, x_n\}$:

- $p = 1$: $||x||_1 = \sum_{i=0}^{n} |x_i|$
- $p = 2$: $||x||_2 = \sqrt{\sum_{i=0}^{n} |x_i|^2}$
- $p = \infty$: $||x||_\infty = \max(x)$

After many experimental runs with these values I choose $p = \infty$ and $\varepsilon = 4$ for the communication strategy study. I also made experiments trying to find the right size for the $tabu$ list and the conclusion was that the right sizes were 15 for non-communicating strategies and 40 for communicating strategies, taking into account that in the latter, I work with 20 receivers solvers.

Step 2

After generating the first configuration, the next step is to apply a local search to improve it. In this step I use the neighborhood *computation module* V, that creates neighborhood $\mathcal{V}(s)$ by changing one value while keeping the order in the configuration, and the other modules (selection and acceptance). The local search is executed a number K_2 of times, or until a solution is obtained.

Step 3

If no improvement is reached, the current configuration is classified as a *potential local minimum* and inserted into the $tabu$ list. Then, the process returns to the Step 1.

5.4.3 Analysis of results

The benefit of the parallel approach is also proved for the *Golomb Ruler Problem* (see Table 5.13 with respect to 5.14, 5.15, 5.16 and 5.17). But the main goal of choosing this benchmark was to study a different communication strategy, since for solving this problem, POSL needs to perform some restarts. In this communication strategy, solvers do not communicate the current configuration to have more solvers searching in its neighborhood, but a configuration that we assume is a local minimum to be avoided. We consider that the current configuration is a local minimum since the solver (after a given number of iteration) is not able to find a better configuration in its neighborhood, so it will communicate this configuration just before performing the restart.

The first experiment compares the runs of non communitaing solvers not using a *tabu* list with non communicating solvers using a *tabu* list. The results in Tables 5.14 and 5.15 demonstrate that using a *tabu* list can help the search process. Without communication, the improvement is not substantial (8% for 8–34, 7% for 10–55 and 5% for 11–72). The reason is because only one configuration is inserted in the *tabu* list after each restart. When we use *1 to 1* communication, after the restart k, the receiving solver has twice the number of configurations in the *tabu* list (one *tabu* configuration from itself and the received one after each restart). Table 5.16 shows that this strategy is not sufficient for some instances, but when we use *1 to N* communication, the number of *tabu* configurations after the restart k, in the receiving solver is considerably higher, e.g., after the restart k a receiving solver has $k(N+1)$ configurations in his *tabu* list (one *tabu* configuration from itself and N received from the other solvers, each restart). Hence, these solvers can generate configurations far enough from many potentially local minima. This phenomenon is more visible when the problem order increases. Table 5.17 shows that the improvement for the higher case (11-72) is about 32% with respect to non communicating solvers without using a *tabu* list (Table 5.14), and about 29% with respect to non communicating solvers using a *tabu* list (Table 5.15).

Instance	T	T(sd)	It.	It.(sd)	R	R(sd)	% success
8–34	0.79	0.66	13,306	11,154	66	55.74	100.00
8–34 (t)	0.66	0.63	10,745	10,259	53	51.35	100.00
10–55	66.44	49.56	451,419	336,858	301	224.56	80.00
10–55 (t)	67.89	50.02	446,913	328,912	297	219.30	88.00
11–72	160.34	96.11	431,623	272,910	143	90.91	26.67
11–72 (t)	117.49	85.62	382,617	275,747	127	91.85	30.00

Table 5.13: *Golomb Ruler*: a single sequential solver

Instance	T	T(sd)	It.	It.(sd)	R	R(sd)
8–34	0.47	34.82	436	330.10	2	1.63
10–55	5.31	38.63	22,577	16,488	15	11.00
11–72	89.76	55.85	164,763	102,931	54	34.32

Table 5.14: *Golomb Ruler*: parallel, without tabu list.

Instance	T	T(sd)	It.	It.(sd)	R	R(sd)
8–34	0.43	0.37	349	334	1	1.64
10–55	4.92	4.68	20,504	19,742	13	13.07
11–72	85.02	67.22	155,251	121,928	51	40.64

Table 5.15: *Golomb Ruler*: parallel, with tabu list.

Instance	T	T(sd)	It.	It.(sd)	R	R(sd)
8–34	0.44	0.31	309	233	1	1.23
10–55	3.90	3.22	15,437	12,788	10	8.52
11–72	85.43	52.60	156,211	97,329	52	32.43

Table 5.16: *Golomb Ruler*: parallel, communication 1 to 1.

Instance	T	T(sd)	It.	It.(sd)	R	R(sd)
8–34	0.43	0.29	283	225	1	1.03
10–55	3.16	2.82	12,605	11,405	8	7.61
11–72	60.35	43.95	110,311	81,295	36	27.06

Table 5.17: *Golomb Ruler*: parallel, communication 1 to n.

5.5 Summarizing

In this Chapter I have chosen various *Constraint Satisfaction Problems* as benchmarks to 1. evaluate the POSL behavior solving these kind of problems, and 2. to study different solution strategies, specially communication strategies.To this end, I have chosen benchmarks with different characteristics, to help me having a wide view of the POSL behavior.

In the solution process of *Social Golfers Problem*, it was showed the success of an exploitation-oriented communication strategy, in which the current configuration is communicated to ask other solvers for help, concentrating the effort in a more promising area. I was able also to study some other unsuccessful strategies, to show that strategies based on the sub-division of the effort by weeks, is not a good idea.

It was showed that simple communication strategies as they applied to solve *Social Golfers Problem* does not improve enough the results without communication for the *N-Queens Problem*. However, a deep study of the POSL's behavior during the search process allows to design a communication strategy able to improve the results obtained using non-communicating strategies.

The *Costas Array Problem* is a very complicated constrained problem, and very sensitive to the methods to solve it. For that reason I used some ideas from already existent algorithms. However, thanks to some studies of different communication strategies, based on the configuration of the current communication at different times (places) in the algorithm, it was possible to find a communication strategy to improve the performance.

During the solution process of the *Golomb Ruler Problem*, POSL needs to perform many restarts. For that reason, this problem was chosen to study a different (and innovative up tu my knowledge) communication strategy, in which the communicated information is a

potential local minimum to be avoided. This new communication strategy showed to be effective to solve these kind of problems.

In all cases, thanks to the operator-based language provided by POSL it was possible to test many different strategies (communicating and non-communicating) fast and easily. Whereas creating solvers implementing different solution strategies can be complex and tedious, POSL gives the possibility to make communicating and non-communicating solver prototypes and to evaluate them with few efforts. In this Chapter it was possible to show that a good selection and management of inter-solvers communication can largely help to the search process, working with complex constrained problems.

Part IV

Conclusions and future works

6

Conclusion and future works

In this chapter, the conclusions of the work is presented, emphasizing on our contribution and obtained results. Future branches to follow are also discussed.

Contents

6.1 Conclusions

The era of parallel computing has opened new and more efficient ways to solve constraint problems. This development is leading us to the multi/many–core technology and massive parallel architectures, which are nowadays more accessible for a broad public through hardware like the Xeon Phi or GPU cards. For that reason, this new architecture implies new ways for designing and implementing algorithms to exploit its full potential.

In this thesis I have presented as a main contribution a Parallel-Oriented Solver Language (POSL) focused in the solution of *Constraint Satisfaction Problems*, which are very complicated. These problems have huge search spaces, making them intractable through tree-search techniques. POSL propose a language to build meta-heuristic-based solvers, tacking into account the success of these methods solving *CSPs*. This meta-heuristics are built using the POSL's language following rigorous but well detailed steps, based on the re-usability and coupling small pieces of computation and communication (*computation modules* and *communication modules*), designed to the resolution of a broad range of *CSPs*.

Meta-heuristic methods have some times a lot of parameters to be adjusted. Prior to the POSL's design, Chapter 3 (Section 3.2) contains a study in which the tool ParamILS was used to tune *Adaptive Search* to solve *Costas Array* and *All-Interval Series* problems. The main goals of that work were studying the performance of the tool, and finding a new and more efficient parameters setting that allow a faster resolution of the mentioned benchmark problems. However, the conclusion, after a comparison between obtained results using default parameters found through manually experiments, and obtained results using ParamILS, were that, for this implementation of *Adaptive Search* the tool is not able to find parameter settings improving obtained results using default parameters. This corroborates the practical intuition that, when the parameters set is not so large, the experience of the scientist is crucial and more accurate that using this kind of tools.

The most important characteristic of POSL is allowing the construction of many solvers to work in parallel using the *multi-walk* approach, which has shown very good results solving constrained problems. Into another work prior to POSL's design, I have presented a study of some techniques to improve the performance of algorithms proposed in [86] were a study of the impact of space-partitioning techniques on the performance of parallel local search algorithms is proposed to tackle the *K-Medoids Clustering Problem*. The basic idea of their specific problem is to how allocate communication metronodes in order to maximize the client covering. Their solution is based on domain partitioning techniques like *space-filling curves*, and *k-Means* algorithm, but they do not take into account the number of clients associated to each new sub-domain. For that reason, in Chapter 3 (Section 3.1) are proposed

a set of ideas/hypothesis to improve the performance, based on geometrical balancing of the search space. This work was not validated, because it was performed in parallel with the first ideas of POSL, which finally was the main direction of this thesis.

In Chapter 4 was dedicated to POSL, the main contribution of this thesis, a Parallel-Oriented Solver Language to build interconnected meta-heuristic-based solvers working in parallel. The language was formally presented by defining each provided operator, as well as the benchmark codification method, and the process of creation/usage of the *computation* and *communication modules*.

The most important advantage of POSL is allowing the codding, easily and fast, of many different solvers through a mechanism of module re-usability, and communication strategies through communication operators, which are also formally defined. Hence, as other contribution of this thesis, is presented in Chapter 5 a detailed study of various communication strategies to analyze the behavior and relevance of the information sharing solving constraint problems.

Solving *Social Golfers Problem*, it was successfully applied an exploitation-oriented communication strategy, in which the current configuration is communicated to focus various solvers in a more promising area. The same idea was applied to solve the *N-Queens Problem*, showing no better results than obtained without communication. However, a deep study of the POSL's behavior during the search process allows to design a communication strategy able to improve the results obtained using non-communicating strategies. It was based on crating *partial* solvers (solvers only searching into a portion of the search space) to accelerate other's solvers search, by communicating the current configuration at the beginning of the search process. The *Costas Array Problem* is a very complicated constrained problem, and very sensitive to the methods to solve it. Thanks to some studies of different communication strategies, based on the communication of the current configuration at different times (places) in the algorithm, it was possible to find a communication strategy to improve the performance, in comparison with those obtained without communication. Finally, the *Golomb Ruler Problem* was chosen to study a different and innovative communication strategy in which the communicated information is a potential local minimum to be avoided. This new communication strategy showed to be effective to solve these kind of problems.

Thanks to the operator-based language provided by POSL it was possible to test many different strategies (communicating and non-communicating). The process of building solvers implementing different solution strategies is complex and tedious, but POSL gives the possibility to make communicating and non-communicating solver prototypes and to study them with few efforts. It was possible to show that a good selection and management of inter-solvers communication can play an important role during the search process, working with constrained problems, most of them very complicated.

6.2 Future works

POSL already has an important library of ready–to–use computation and connection modules, based on a deep study of classical meta-heuristics algorithms for solving combinatorial problems. In the near future we plan to make it grow, in order to increase possibilities of POSL. In such a way, building new algorithms by using POSL will be easier. At the same time we plan to enrich the language by proposing new operators. It is necessary, for example, to improve the solver definition language, allowing to build sets of many new solvers faster and easier. Furthermore, we are aiming to expand the communication definition language, in order to create versatile and more complex and dynamic communication strategies, to allow a communication strategy to change during runtime.

The operators described above only give the possibility to define static communication strategies. However, we aim to improve POSL with more expressive operators in terms of communication between solvers to allow dynamic modifications of communication strategies, that is, having such strategies adapting themselves during runtime. This way, many different communication strategies would be defined in the same *solver set*. Then, after some time of calculation, an evaluation would be performed in order to make all solvers able to adopt the best strategy until the end of the search process.

In the performed experiments, the shared information was in every case the best found configuration. So far, there are no results showing what "a good information to communicate" is. Actually, [48] shows that in fact, the current configuration is not always a relevant information to share among solvers. That is why this subject deserves a deep study. We plan in the near future to investigate other informations to be communicated, such as really costly configurations, in order to avoid similar ones; search directions, to be avoided or taken into account; among others.

7

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Part V

Appendix

A

Results of experiments with *Social Golfers Problem*

In this Chapter we presents results values of experiments with Social Golfers Problem.

In the following tables: ...

Std: Simple Swap		AS: Adaptive Search swap		A000 / AS (rho) SS		A000 / AS U SS	
Time	Iter.	Time	Iter.	Time	Iter.	Time	Iter.
62,160	108	980	274	65,470	244	64,750	192
60,050	102	1,100	338	90,400	219	65,350	215
60,900	104	400	126	22,760	99	55,930	225
164,690	245	2,510	814	33,230	103	102,410	300
57,950	115	2,380	866	20,860	90	112,620	298
137,180	221	280	96	24,470	104	63,870	191
139,970	233	320	103	23,710	119	3,670	84
60,600	114	250	98	25,650	112	11,580	105
73,670	120	2,760	986	27,540	94	63,830	200
60,480	104	1,860	654	65,820	217	9,670	90
64,240	124	690	285	28,980	98	58,400	219
155,360	227	500	190	31,630	114	162,710	436
56,860	105	560	187	69,000	232	182,930	397
58,050	109	980	347	75,730	239	4,720	96
148,500	236	320	118	80,710	240	130,950	333
65,760	106	650	200	74,160	201	124,350	318
56,050	111	240	106	95,450	357	9,260	98
149,560	211	1,080	311	17,390	87	4,500	89
65,510	101	1,420	483	16,010	75	3,500	88
69,260	108	300	98	18,940	86	9,790	106
50,830	102	240	86	33,360	106	132,650	325
57,180	107	320	100	68,100	221	123,940	361
54,430	100	270	98	23,190	114	8,410	109
153,100	243	2,130	660	32,040	109	9,220	87
59,940	110	1,070	332	16,390	93	58,940	220
72,780	114	1,310	337	21,700	109	11,360	82
154,310	229	2,180	754	20,190	99	10,700	91
55,590	112	960	316	23,570	93	121,510	305
63,300	112	1,900	598	21,640	110	8,780	84
148,800	241	1,880	607	77,870	229	59,280	204
87,902	**146**	**1,061**	**352**	**41,532**	**147**	**59,653**	**198**
41,961	**58**	**791**	**268**	**26,001**	**72**	**55,019**	**110**

Table SGP 1: Comparing neighborhood functions. Social Golfers 10-10-3 (40 cores)

Caption: **mean**
Standard Deviation

Best Improvement		First Improvement		First Improvement (sequential)	
Time	Iter.	Time	Iter.	Time	Iter.
3,020	2,714	1,370	1,308	12,950	27,257
3,860	3,433	1,380	1,308	8,210	17,417
880	751	2,850	2,849	11,310	23,687
15,040	13,458	1,010	958	15,110	31,674
6,470	5,454	2,160	2,193	8,470	17,890
2,160	1,878	450	444	2,090	4,464
4,350	3,756	1,060	1,087	3,390	7,244
3,080	2,908	2,680	2,677	16,140	34,172
4,770	4,272	1,770	1,729	2,780	5,925
3,170	2,842	480	504	17,940	37,708
15,150	13,470	1,710	1,769	9,440	19,294
1,910	1,690	780	784	360	774
7,350	6,544	850	871	4,510	9,558
3,190	2,895	440	482	13,740	28,960
1,450	1,256	2,290	2,329	36,550	74,451
1,680	1,541	1,640	1,549	3,780	8,027
2,430	2,127	540	547	5,160	10,899
4,640	4,021	420	439	1,920	3,377
3,570	3,266	1,920	1,874	21,140	42,813
19,450	17,174	2,020	1,979	9,470	19,832
1,340	1,222	910	931	5,900	12,503
5,100	4,559	1,510	1,517	1,290	2,754
3,680	2,987	1,770	1,715	4,810	10,186
3,680	2,987	1,970	2,013	2,430	4,997
1,960	1,740	1,150	1,090	940	1,996
1,970	1,712	630	664	10,130	21,330
4,740	4,166	960	956	2,060	4,370
8,780	7,878	600	622	7,940	16,710
2,770	2,449	1,390	1,330	4,060	8,656
8,340	7,475	1,120	1,130	5,380	11,473
4,999	**4,421**	**1,328**	**1,322**	**8,313**	**17,347**
4,430	**3,938**	**682**	**676**	**7,640**	**15,673**

Table SGP 2: Comparing selection functions. Social Golfers 5-3-7 (40 cores, and sequential)

Caption: **mean**
Standard Deviation

100%/ com. 1-1		100% / com. 1-N		50% / com. 1-1		50% / com. 1-n	
Time	Iter.	Time	Iter.	Time	Iter.	Time	Iter.
1,080	1,083	520	506	870	848	690	735
1,600	1,562	860	845	1,060	1,087	1,020	965
740	723	580	575	580	580	840	844
860	856	820	763	870	841	710	715
520	609	2,130	2,139	480	484	570	557
2,340	2,352	1,270	1,151	1,000	953	1,340	1,248
1,720	1,650	1,030	979	2,460	2,458	310	282
870	834	820	791	610	579	980	989
1,100	1,068	520	454	1,610	1,559	210	209
1,280	1,317	730	682	1,710	1,744	1,150	1,112
100	116	1,220	1,234	1,280	1,300	900	921
90	116	1,790	1,649	1,580	1,527	380	387
1,730	1,670	540	517	1,100	1,090	1,620	1,549
740	680	1,160	1,056	1,260	1,195	1,600	1,613
1,380	1,291	860	761	780	746	1,000	1,025
1,390	1,271	1,230	1,199	340	342	1,120	1,078
2,670	2,488	1,790	1,727	1,210	1,156	2,360	2,364
1,160	1,163	1,990	1,976	730	741	1,020	961
1,410	1,381	1,880	1,793	1,140	1,042	960	961
1,730	1,657	810	780	720	737	1,340	1,233
1,860	1,727	1,820	1,584	1,240	1,189	740	738
2,000	1,946	1,290	1,231	620	541	1,160	1,171
590	601	1,180	1,080	1,300	1,205	280	277
1,980	1,828	490	458	750	702	1,810	1,742
770	776	1,370	1,278	830	826	2,000	2,037
590	604	890	858	1,400	1,302	650	618
1,590	1,493	1,080	1,095	250	227	2,020	2,050
320	289	690	678	1,470	1,419	680	676
1,050	961	480	464	1,000	1,036	1,040	1,002
580	568	1,740	1,700	1,140	1,127	880	870
1,195	1,156	1,119	1,067	1,046	1,019	1,046	1,031
646	608	499	484	459	456	533	530

Table SGP 3: Comparing communication strategies. Social Golfers 5-3-7 (40 cores)

Caption: **mean**

Standard Deviation

sender solver

receiver solver

receiver solver not tacking into account the received information

non-communicating solver

25% / com. 1-1		25% / com. 1-n	
Time	Iter.	Time	Iter.
1,820	1,745	510	518
980	905	1,330	1,266
610	547	990	994
2,030	1,884	890	844
410	419	1,350	1,264
330	321	380	363
1,710	1,675	1,430	1,305
390	382	820	807
1,560	1,451	1,090	1,079
790	774	2,180	2,072
520	450	730	727
470	481	1,100	1,163
1,350	1,370	810	849
550	563	830	819
820	776	2,040	2,005
580	587	910	871
280	312	990	926
450	444	1,440	1,388
1,310	1,282	690	643
570	542	2,740	2,622
1,210	1,251	960	904
1,370	1,330	450	441
1,200	1,187	680	708
400	393	150	169
260	272	2,820	2,805
870	850	1,550	1,527
1,610	1,616	610	682
470	483	1,380	1,372
1,260	1,195	2,380	2,385
930	932	1,600	1,594
904	881	1,194	1,170
514	492	675	655

Table SGP 4: Comparing communication strategies. Social Golfers 5-3-7 (40 cores)

Caption: **mean**

Standard Deviation

sender solver

receiver solver

receiver solver not tacking into account the received information

non-communicating solver

B001 / Best Impr.		B001 / First Impr.		First Improvement (sequential)	
Time	Iter.	Time	Iter.	Time	Iter.
6,300	1,192	1,720	424	12,760	5,994
5,220	991	1,620	385	1,560	739
3,130	573	2,990	718	22,930	10,742
5,450	1,023	1,390	361	6,120	2,888
2,690	502	4,700	1,104	3,330	1,560
5,810	1,100	1,450	373	5,470	2,631
3,250	623	1,170	300	7,700	3,443
3,880	724	1,780	417	10,140	4,541
6,270	1,163	1,220	308	11,960	5,574
2,880	541	2,320	551	1,280	640
1,520	279	1,660	406	11,450	5,302
4,180	783	2,100	518	48,190	22,314
5,460	980	880	220	18,640	8,735
8,270	1,564	2,190	537	11,670	5,413
4,240	797	1,470	361	16,190	6,423
4,650	836	1,530	387	770	419
8,230	1,548	1,580	380	2,780	1,333
4,790	885	940	255	37,270	17,453
4,920	907	4,110	962	30,880	14,246
7,850	1,455	2,280	537	14,470	6,841
6,340	1,197	1,600	390	24,590	11,291
3,260	605	1,450	343	22,820	10,632
3,470	653	2,090	498	20,230	9,409
7,380	1,384	950	261	58,420	27,138
5,170	973	1,630	405	3,470	1,637
3,200	594	2,020	491	16,220	7,521
5,500	1,046	1,660	401	25,120	11,640
7,920	1,466	1,190	313	500	264
5,570	1,035	1,330	324	49,260	22,696
6,340	1,189	1,720	419	11,470	5,409
5,105	**954**	**1,825**	**445**	**16,922**	**7,829**
1,776	**334**	**840**	**191**	**15,151**	**7,019**

Table SGP 5: Comparing selection functions. Social Golfers 8-4-7 (40 cores, and sequential)

Caption: **mean**

Standard Deviation

100%/ com. 1-1		100% / com. 1-N		50% / com. 1-1		50% / com. 1-n	
Time	Iter.	Time	Iter.	Time	Iter.	Time	Iter.
1,250	287	1,500	358	890	229	2,060	472
1,390	327	1,450	362	1,900	416	2,210	520
1,320	321	1,990	455	1,930	449	770	201
2,220	522	1,010	253	740	206	1,070	276
850	213	2,130	466	760	206	1,800	418
2,200	499	1,010	247	640	151	860	214
1,040	271	820	178	960	239	840	224
2,860	667	1,040	258	870	229	4,120	936
2,710	630	1,370	309	1,680	415	1,550	365
320	97	2,250	532	1,040	251	1,230	308
410	102	1,930	431	180	61	300	112
1,140	283	950	209	2,040	485	1,380	357
870	215	1,580	356	1,860	417	730	203
960	254	1,210	277	370	127	990	249
710	184	2,620	620	1,040	250	1,230	318
2,150	468	1,470	339	2,520	556	1,230	318
800	207	1,160	304	1,050	279	2,230	531
2,720	642	1,720	441	1,510	375	1,770	424
490	152	2,470	587	1,600	378	830	223
460	134	1,770	393	1,290	343	2,450	581
1,340	335	970	234	1,810	425	820	206
1,220	292	490	148	1,380	382	2,360	516
2,040	496	740	197	2,160	507	930	248
680	175	1,050	256	2,060	449	700	187
1,590	382	2,210	502	1,790	417	1,080	281
1,790	435	2,440	562	1,960	421	1,160	300
1,030	257	930	220	1,890	468	1,350	336
660	160	1,370	336	1,140	289	1,330	321
1,040	283	920	221	1,720	400	1,360	334
820	208	1,420	349	1,210	297	2,420	596
1,303	317	1,466	347	1,400	337	1,439	353
724	161	572	128	579	122	768	167

Table SGP 6: Comparing communication strategies. Social Golfers 8-4-7 (40 cores)

Caption: **mean**

Standard Deviation

sender solver

receiver solver

receiver solver not tacking into account the received information

non-communicating solver

25% / com. 1-1		25% / com. 1-n	
Time	Iter.	Time	Iter.
800	209	1,490	337
1,570	396	790	194
890	221	960	247
1,130	308	1,360	324
1,470	359	1,820	419
1,590	380	1,410	349
2,160	496	1,510	356
2,080	508	2,640	621
880	222	1,660	368
1,770	397	1,410	346
1,070	275	1,050	278
1,230	323	1,120	292
1,220	323	1,830	427
1,610	384	1,810	419
1,090	270	1,620	388
1,910	430	1,220	323
780	215	1,680	436
870	223	1,410	331
1,090	282	1,270	316
1,580	396	1,340	324
1,490	368	1,810	427
2,160	500	1,330	310
1,500	329	1,500	356
1,590	384	1,660	395
1,580	384	760	186
720	188	1,070	247
1,830	435	1,730	417
1,880	433	1,560	385
1,600	374	710	191
820	203	2,450	565
1,399	341	1,466	352
435	94	434	96

Table SGP 7: Comparing communication strategies. Social Golfers 8-4-7 (40 cores)

Caption: **mean**

Standard Deviation

sender solver

receiver solver

receiver solver not tacking into account the received information

non-communicating solver

B001 / Best Impr.		B001 / First Impr.		First Improvement (sequential)	
Time	Iter.	Time	Iter.	Time	Iter.
6,700	722	5,150	697	87,900	23,451
10,500	1,136	9,710	1,312	32,970	8,761
12,320	1,342	1,630	261	42,030	11,055
8,250	891	7,580	923	199,170	53,107
16,480	1,805	2,930	428	200,570	53,327
11,080	1,193	5,100	693	99,330	26,472
10,400	1,124	7,660	1,004	153,110	40,620
12,670	1,385	2,810	420	28,660	7,641
7,630	817	6,440	903	65,230	17,365
20,590	2,250	3,990	558	82,080	21,887
13,150	1,401	4,270	597	8,060	2,168
11,360	1,227	5,410	729	10,520	2,818
3,090	327	7,310	996	109,290	28,731
7,210	773	3,700	534	122,420	32,141
3,390	351	15,430	2,037	11,800	3,068
19,630	2,136	1,280	223	57,280	14,529
19,290	2,105	21,620	2,817	30,700	8,036
8,090	874	2,470	381	76,630	20,459
18,030	1,964	15,710	2,076	55,600	14,763
15,600	1,712	2,130	314	78,660	20,925
19,950	2,181	7,210	987	20,690	5,507
14,200	1,538	7,360	985	131,740	34,774
23,010	2,502	10,660	1,403	10,300	2,712
19,090	2,043	3,250	453	145,370	38,315
16,470	1,781	4,260	619	4,570	1,266
9,870	1,075	10,550	1,403	122,070	31,495
6,620	713	3,680	528	41,970	10,294
11,440	1,242	4,330	598	256,760	62,613
6,700	725	6,000	826	55,230	13,420
8,410	917	3,540	488	47,320	11,650
12,374	**1,342**	**6,439**	**873**	**79,601**	**20,779**
5,400	**591**	**4,607**	**591**	**64,072**	**16,537**

Table SGP 8: Comparing selection functions. Social Golfers 9-4-8 (40 cores, and sequential)

Caption: **mean**

Standard Deviation

100%/ com. 1-1		100% / com. 1-N		50% / com. 1-1		50% / com. 1-n	
Time	Iter.	Time	Iter.	Time	Iter.	Time	Iter.
2,050	292	6,020	809	4,590	640	8,980	1,143
4,200	594	4,010	573	6,570	888	14,360	1,909
1,880	281	4,770	641	5,530	774	7,770	999
3,530	516	5,560	755	5,180	714	3,800	529
1,000	175	5,250	686	3,360	477	1,950	282
5,960	793	6,180	832	2,750	401	5,870	759
3,730	532	15,050	2,001	4,050	565	5,570	755
2,610	362	5,660	752	8,980	1,195	2,200	286
3,990	529	8,910	1,150	8,920	1,161	7,180	970
4,760	633	8,610	1,098	2,180	299	5,420	745
3,460	462	1,620	256	6,570	925	1,580	255
9,290	1,187	4,480	608	8,040	1,083	8,860	1,185
2,820	387	5,300	709	2,970	415	6,110	846
10,050	1,357	1,030	166	3,380	469	5,590	787
11,170	1,465	2,690	393	4,390	550	3,300	442
4,200	565	5,970	803	3,130	421	4,090	555
2,340	324	3,100	418	2,840	405	3,930	572
4,140	561	10,100	1,311	3,550	500	5,550	757
4,920	669	3,700	507	2,450	369	4,540	631
2,810	412	800	147	6,140	847	9,990	1,301
7,010	957	7,250	952	1,660	240	11,020	1,427
10,160	1,313	3,790	507	3,840	538	8,060	1,067
3,380	465	6,020	765	3,350	478	4,590	647
1,960	315	4,230	583	5,430	733	3,100	438
4,750	667	6,370	873	3,090	456	2,050	316
2,760	365	2,360	330	2,760	408	3,200	488
2,270	349	4,890	684	5,830	764	4,210	593
6,480	856	3,050	403	7,520	1,004	6,390	862
2,220	331	11,060	1,395	8,510	1,103	3,040	424
1,510	202	7,560	975	1,920	290	10,290	1,324
4,380	597	5,513	736	4,649	637	5,753	776
2,720	347	3,066	389	2,173	279	3,067	389

Table SGP 6: Comparing communication strategies. Social Golfers 8-4-7 (40 cores)

Caption: **mean**

Standard Deviation

sender solver

receiver solver

receiver solver not tacking into account the received information

non-communicating solver

25% / com. 1-1		25% / com. 1-n	
Time	Iter.	Time	Iter.
3,280	472	7,040	923
6,380	876	7,440	994
6,570	882	1,920	315
3,150	405	2,180	348
1,990	293	3,940	510
4,010	545	5,510	754
5,130	658	5,130	689
5,460	750	3,210	465
6,360	856	4,050	565
4,030	564	3,110	434
8,970	1,235	5,160	714
1,660	270	2,580	384
1,710	252	2,360	359
3,530	503	2,450	372
4,130	588	4,370	602
3,950	592	3,660	530
1,630	272	7,130	991
5,660	775	2,790	403
7,110	914	9,110	1,178
5,500	709	2,790	393
3,870	541	2,550	390
2,700	366	4,560	591
3,290	450	6,390	873
6,430	851	2,340	345
7,860	1,083	5,560	766
3,330	486	4,750	651
2,840	412	6,480	843
3,220	459	7,160	950
3,700	512	7,700	1,021
2,520	389	2,630	394
4,332	599	4,535	625
1,920	248	2,019	251

Table SGP 7: Comparing communication strategies. Social Golfers 8-4-7 (40 cores)

Caption:

mean

Standard Deviation

sender solver

receiver solver

receiver solver not tacking into account the received information

non-communicating solver

B001 / Best Impr.		B001 / First Impr.		First Improvement (sequential)	
Time	Iter.	Time	Iter.	Time	Iter.
8,650	565	1,340	179	1,850	320
1,910	129	2,600	302	5,040	964
6,440	447	2,490	280	2,780	556
2,340	160	2,510	297	1,940	406
2,370	160	3,460	350	1,440	324
4,170	280	1,770	235	4,640	929
6,770	469	3,720	402	6,470	1,201
7,720	531	3,730	402	3,190	627
4,270	283	2,520	325	2,650	586
6,490	440	1,860	228	2,440	501
2,920	194	1,300	202	9,190	1,694
6,510	446	2,380	286	3,980	757
6,510	446	1,390	216	1,210	313
4,540	301	1,860	245	3,790	739
6,320	433	3,630	387	5,810	1,065
6,260	429	2,410	278	2,070	438
5,170	339	1,950	265	2,890	599
6,370	438	1,780	233	1,650	392
6,580	427	1,660	255	1,600	356
5,290	361	2,260	275	5,670	1,070
6,480	446	2,690	323	2,750	558
4,520	315	2,300	276	1,980	402
4,670	317	2,280	276	4,060	815
3,570	241	1,750	217	8,560	1,563
3,010	193	1,800	245	2,970	614
6,670	457	2,280	281	4,120	810
4,840	327	2,270	281	520	172
4,080	272	1,500	196	520	172
5,330	348	1,600	265	720	196
4,990	338	1,530	197	3,920	768
5,192	**351**	**2,221**	**273**	**3,347**	**664**
1,671	**114**	**692**	**58**	**2,167**	**380**

Table SGP 11: Comparing selection functions. Social Golfers 11-7-5 (40 cores, and sequential)

Caption: **mean**

Standard Deviation

100%/ – – >		100% / # – >		50% / – – >		50% / # – >	
Time	Iter.	Time	Iter.	Time	Iter.	Time	Iter.
1,330	180	1,790	216	1,010	154	1,390	151
2,240	223	1,530	207	1,960	220	1,810	238
1,590	215	1,200	177	1,540	220	2,030	249
2,090	216	1,580	217	2,640	276	2,260	253
1,790	215	1,340	150	1,970	230	2,030	276
1,590	216	1,530	166	1,640	209	2,080	248
1,710	187	1,080	146	2,160	195	1,540	213
1,010	141	2,570	286	1,640	183	1,510	181
1,790	189	1,580	205	1,640	219	2,280	258
2,370	221	1,530	196	2,290	257	1,390	176
1,990	220	1,300	198	1,800	228	1,300	217
1,990	220	1,540	203	1,800	218	1,640	179
1,320	180	1,640	170	2,670	298	1,340	199
1,890	249	1,330	194	1,640	206	1,400	200
1,130	143	2,140	251	1,620	234	2,170	210
1,250	150	1,720	189	2,160	232	1,940	259
1,390	210	1,190	160	2,130	279	2,040	187
1,210	159	1,910	203	1,900	237	1,900	206
1,240	149	2,130	221	1,630	216	1,600	201
1,830	204	1,900	243	1,840	197	1,280	153
1,610	227	1,790	223	1,540	206	2,550	275
1,710	226	1,430	173	2,290	248	2,500	301
2,420	272	1,360	188	1,200	198	1,830	230
1,820	240	1,360	185	1,410	187	1,360	186
1,830	240	1,990	219	1,910	254	1,570	224
2,220	270	1,400	208	1,680	211	2,660	288
2,280	266	1,520	195	1,570	193	1,920	255
1,680	192	2,270	253	1,050	145	1,890	196
1,810	233	1,770	212	1,890	223	1,430	202
2,730	352	1,340	191	2,250	234	2,020	252
1,762	214	1,625	202	1,816	220	1,822	222
420	45	348	31	401	33	395	39

Table SGP 12: Comparing communication strategies. Social Golfers 11-7-5 (40 cores)

Caption: **mean**

Standard Deviation

sender solver

receiver solver

receiver solver not tacking into account the received information

non-communicating solver

25% / – – >		25% / # – >	
Time	Iter.	Time	Iter.
2,330	268	2,180	250
2,590	300	2,150	258
2,600	300	1,760	237
2,600	300	1,760	260
1,420	211	2,510	277
2,680	280	2,470	271
1,830	221	2,420	286
2,190	261	2,400	286
2,220	268	1,840	220
770	139	2,120	221
1,280	170	1,710	218
2,020	210	2,040	244
1,520	207	2,050	244
2,030	263	2,150	258
2,550	269	1,760	241
1,140	180	1,390	197
2,530	280	1,540	196
2,520	319	1,730	220
1,530	201	2,200	246
2,580	286	1,930	261
1,560	161	1,090	179
1,510	215	1,800	195
1,980	270	1,200	179
2,130	271	1,510	209
1,640	211	1,810	229
2,150	265	1,680	233
1,260	150	1,830	250
1,470	204	1,500	218
2,730	320	2,070	225
2,440	274	2,080	237
1,993	242	1,630	224

Table SGP 12: Comparing communication strategies. Social Golfers 11-7-5 (40 cores)

Caption:

mean

Standard Deviation

sender solver

receiver solver

receiver solver not tacking into account the received information

non-communicating solver

B

Results of experiments with *N-Queens Problem*

In this Chapter we presents results values of experiments with N-Queens Problem.

In the following tables: ...

Sequential		Parallel	
Time	Iter.	Time	Iter.
6,440	920	6,160	933
6,080	942	6,510	957
6,280	975	6,070	957
6,180	964	6,410	975
6,310	949	6,240	949
6,080	913	6,210	943
6,030	929	6,240	948
6,230	912	6,340	931
6,360	940	6,540	906
6,120	926	6,540	983
6,160	942	5,810	940
6,220	959	5,950	943
6,300	970	6,100	974
6,280	990	6,110	927
6,190	959	6,060	938
6,480	943	6,080	976
6,050	949	6,140	957
6,380	925	5,840	1,002
6,100	932	6,210	974
6,240	973	6,200	985
6,150	973	6,080	948
6,060	973	6,100	943
6,320	930	5,900	935
6,050	945	6,330	946
6,040	940	6,150	962
6,120	959	6,180	929
6,280	937	6,130	963
6,340	972	5,650	942
6,080	913	6,140	957
6,300	942	6,160	948
6,208	947	6,153	952
127	21	203	20

Table NQP 1: Non-communicating results. 2000-Queens (40 cores)

Caption: **mean** **Standard Deviation**

100% - com. 1 to 1		100% - com. 1 to N	
Time	Iter.	Time	Iter.
5,710	894	6,220	959
5,790	825	6,160	925
6,150	914	6,180	950
5,520	847	6,150	926
5,790	945	5,760	902
6,290	897	5,850	865
5,880	857	6,330	823
5,820	898	6,010	906
6,030	955	6,280	894
5,970	904	6,040	865
6,060	935	5,690	937
5,960	864	5,810	832
6,050	946	5,890	833
5,870	849	6,240	886
6,090	867	6,140	950
6,150	910	6,120	894
5,900	835	6,000	750
5,710	771	5,740	719
6,010	800	5,750	858
6,010	896	6,020	923
5,990	968	6,020	885
5,960	844	6,140	941
5,790	843	6,280	946
5,990	933	5,920	822
5,570	860	5,590	918
5,880	854	6,040	865
5,960	963	5,920	884
6,150	908	5,880	912
5,740	857	6,150	945
6,000	896	6,240	889
5,926	885	6,019	887
174	49	197	57

Table NQP 2: Communicating results (100%). 2000-Queens (40 cores)

Caption:

mean

Standard Deviation

sender solver

receiver solver

receiver solver not tacking into account the received information

50% - com. 1 to 1		50% - com. 1 to N	
Time	Iter.	Time	Iter.
5,960	945	5,410	910
6,000	935	6,220	941
6,160	918	5,910	949
5,790	923	5,860	967
6,240	967	5,970	948
5,660	796	5,980	858
6,150	933	5,960	915
6,020	919	6,080	957
5,720	918	6,080	920
6,070	907	6,090	951
5,980	937	5,810	965
5,820	970	5,640	929
5,890	919	5,860	886
5,750	958	5,990	991
6,000	864	5,900	875
6,190	887	6,040	904
6,120	952	5,840	906
6,280	958	6,160	866
5,990	897	6,020	898
5,940	946	5,880	862
6,040	935	5,820	864
5,940	938	5,940	950
5,750	805	6,360	930
5,960	955	6,050	962
6,310	906	6,120	904
6,270	899	6,150	894
6,300	948	6,210	941
5,900	885	6,390	896
5,860	919	5,870	911
6,380	955	5,820	808
6,015	920	5,981	915
194	41	199	41

Table NQP 3: Communicating results (50%). 2000-Queens (40 cores)

Caption:

mean

Standard Deviation

sender solver

receiver solver

receiver solver not tacking into account the received information

non-communicating solver

25% - com. 1 to 1		25% - com. 1 to N	
Time	Iter.	Time	Iter.
6,000	931	6,080	891
6,190	937	5,670	938
6,000	992	6,090	931
6,150	940	5,960	930
5,960	894	6,370	949
6,170	960	6,240	913
6,350	890	6,240	927
6,230	949	6,010	945
6,240	995	6,110	903
5,820	964	6,020	890
6,080	946	6,130	936
6,130	929	5,970	931
6,210	938	5,860	869
5,600	932	6,260	878
5,550	907	6,290	972
6,040	920	6,060	877
6,040	914	6,200	931
6,170	956	6,100	948
5,890	977	6,080	936
6,550	856	6,010	929
6,050	950	6,080	962
5,820	915	6,160	769
5,370	905	5,690	927
6,210	1,010	6,160	936
6,120	859	6,090	920
6,000	937	6,170	949
5,970	900	6,140	975
6,440	927	5,890	972
6,070	956	6,030	933
6,300	930	6,080	972
6,057	934	6,075	925
250	36	156	41

Table NQP 4: Communicating results (25%). 2000-Queens (40 cores)

Caption:

mean

Standard Deviation

sender solver

receiver solver

receiver solver not tacking into account the received information

non-communicating solver

Time	Iter.
6,200	793
5,820	768
4,290	896
4,920	754
4,590	860
4,550	830
4,640	826
4,260	897
5,930	840
4,360	872
6,050	871
5,330	846
6,320	833
4,340	832
5,220	883
6,290	799
5,990	821
6,670	853
5,190	844
4,640	857
4,160	850
5,440	835
4,600	857
4,200	828
6,100	825
4,530	846
3,920	840
4,530	877
4,300	915
6,000	787
5,113	841
822	37

Table NQP 5: Communicating results (Partial-full solvers). 2000-Queens (40 cores)

Caption:

mean
Standard Deviation

receiver solver

receiver solver not tacking into account the received information

Sequential		Parallel	
Time	Iter.	Time	Iter.
14,350	1,439	14,600	1,393
14,380	1,424	14,540	1,414
14,210	1,406	13,250	1,387
13,760	1,399	14,370	1,433
14,010	1,419	14,300	1,400
14,150	1,416	13,710	1,423
14,490	1,406	14,190	1,420
14,160	1,412	14,140	1,420
14,030	1,420	13,810	1,467
14,420	1,436	13,830	1,420
13,950	1,383	13,790	1,375
13,990	1,384	14,050	1,383
14,520	1,450	14,160	1,413
14,080	1,411	13,880	1,465
14,280	1,431	13,320	1,411
14,270	1,394	14,010	1,464
14,080	1,474	14,630	1,403
14,180	1,384	13,810	1,394
14,100	1,409	13,990	1,391
14,340	1,387	14,030	1,411
14,200	1,391	14,520	1,394
14,180	1,403	14,480	1,429
14,260	1,419	14,050	1,370
14,350	1,446	13,910	1,400
14,180	1,404	13,830	1,405
14,060	1,418	14,380	1,423
13,890	1,382	14,080	1,409
13,870	1,425	14,110	1,398
14,460	1,432	14,220	1,443
14,690	1,437	13,940	1,434
14,196	1,415	14,064	1,413
211	22	335	25

Table NQP 6: Non-communicating results. 3000-Queens (40 cores)

Caption: **mean**

Standard Deviation

100% - com. 1 to 1		100% - com. 1 to N	
Time	Iter.	Time	Iter.
13,410	1,274	14,320	1,406
13,430	1,402	13,680	1,251
13,940	1,396	13,470	1,423
13,500	1,322	13,830	1,411
13,660	1,328	13,680	1,255
13,670	1,346	13,700	1,375
13,580	1,339	13,930	1,249
13,710	1,371	14,350	1,415
13,220	1,384	14,040	1,381
13,550	1,318	13,580	1,298
13,890	1,365	13,420	1,299
13,690	1,277	13,910	1,375
14,170	1,400	13,850	1,337
13,900	1,359	13,500	1,332
13,410	1,345	13,830	1,342
13,890	1,341	13,530	1,403
13,650	1,246	13,820	1,326
14,070	1,370	14,170	1,410
14,080	1,357	14,320	1,392
14,170	1,335	14,050	1,437
13,850	1,360	13,290	1,342
13,580	1,371	13,740	1,413
13,710	1,409	13,520	1,298
13,800	1,303	13,590	1,369
13,480	1,363	14,390	1,433
12,210	1,293	13,750	1,377
13,950	1,327	13,370	1,440
13,350	1,368	14,220	1,391
14,330	1,378	13,690	1,370
13,360	1,324	13,200	1,411
13,674	1,346	13,791	1,365
391	40	327	56

Table NQP 7: Communicating results (100%). 3000-Queens (40 cores)

Caption:

mean

Standard Deviation

sender solver

receiver solver

receiver solver not tacking into account the received information

50% - com. 1 to 1		50% - com. 1 to N	
Time	Iter.	Time	Iter.
14,310	1,317	13,920	1,260
13,680	1,389	14,300	1,384
13,420	1,408	14,430	1,357
13,910	1,417	14,060	1,441
13,740	1,320	14,140	1,368
13,880	1,345	13,880	1,348
14,010	1,405	13,960	1,405
13,790	1,345	14,470	1,463
13,860	1,289	13,760	1,428
14,310	1,265	13,960	1,381
14,070	1,417	13,740	1,439
14,300	1,360	13,880	1,417
13,960	1,444	14,180	1,399
13,430	1,299	14,150	1,353
14,080	1,423	13,340	1,306
13,820	1,456	13,740	1,421
14,430	1,402	13,860	1,365
14,360	1,403	13,840	1,403
13,350	1,356	13,660	1,272
14,190	1,357	13,160	1,404
13,890	1,442	13,810	1,410
13,980	1,372	14,220	1,352
14,300	1,389	14,060	1,435
13,720	1,418	14,100	1,414
13,430	1,319	14,120	1,437
13,840	1,305	13,790	1,410
14,050	1,400	14,340	1,407
13,710	1,348	13,480	1,270
14,140	1,337	14,430	1,403
13,610	1,301	14,120	1,420
13,919	1,368	13,963	1,386
300	51	311	52

Table NQP 8: Communicating results (50%). 3000-Queens (40 cores)

Caption:

mean

Standard Deviation

sender solver

receiver solver

receiver solver not tacking into account the received information

non-communicating solver

25% - com. 1 to 1		25% - com. 1 to N	
Time	Iter.	Time	Iter.
13,600	1,397	14,020	1,353
13,740	1,442	14,330	1,413
13,930	1,288	13,980	1,213
13,830	1,341	13,690	1,411
13,790	1,465	14,360	1,405
13,860	1,394	13,480	1,393
13,350	1,378	13,910	1,394
13,840	1,375	14,210	1,497
14,240	1,323	13,780	1,416
13,960	1,437	14,600	1,448
13,810	1,415	13,640	1,438
14,500	1,415	14,340	1,379
13,840	1,415	14,390	1,371
13,630	1,337	14,120	1,446
13,700	1,323	14,160	1,362
13,870	1,372	13,650	1,406
14,200	1,415	13,340	1,381
14,380	1,352	13,470	1,426
13,610	1,454	14,410	1,416
13,860	1,388	14,000	1,438
13,770	1,458	13,430	1,456
14,340	1,348	14,380	1,391
13,820	1,387	13,940	1,379
14,180	1,419	14,110	1,449
13,650	1,402	13,940	1,357
14,160	1,356	13,960	1,389
14,430	1,430	13,480	1,432
13,640	1,441	14,320	1,428
13,600	1,349	13,860	1,392
13,850	1,302	14,020	1,384
13,899	1,387	13,977	1,402
283	48	341	49

Table NQP 9: Communicating results (25%). 3000-Queens (40 cores)

Caption:

mean

Standard Deviation

sender solver

receiver solver

receiver solver not tacking into account the received information

non-communicating solver

Time	Iter.
10,340	1,369
10,600	1,297
9,350	1,324
9,370	1,374
11,110	1,254
14,830	1,213
10,170	1,358
10,370	1,284
11,000	1,214
9,680	1,315
14,080	1,260
9,980	1,300
14,050	1,220
9,080	1,361
12,940	1,335
14,130	1,277
8,840	1,190
14,180	1,189
14,300	1,171
10,110	1,290
11,990	1,237
11,850	1,269
10,750	1,258
14,860	1,198
11,540	1,404
14,300	1,210
9,480	1,335
10,210	1,202
10,440	1,362
12,760	1,193
11,556	1,275
1,965	67

Table NQP 10: Communicating results (Partial-full solvers). 3000-Queens (40 cores)

Caption:

mean
Standard Deviation

receiver solver

receiver solver not tacking into account the received information

Sequential		Parallel	
Time	Iter.	Time	Iter.
25,860	1,859	25,300	1,871
25,610	1,947	25,870	1,932
26,140	1,874	25,010	1,861
25,980	1,893	25,040	1,928
26,200	1,937	25,450	1,902
25,430	1,904	26,390	1,862
25,800	1,930	25,800	1,912
25,900	1,887	25,580	1,892
25,690	1,948	25,950	1,951
25,600	1,885	25,050	1,952
25,450	1,885	25,060	1,832
25,310	1,914	25,430	1,935
25,670	1,901	25,290	1,840
25,110	1,908	25,560	1,858
25,680	1,934	25,690	1,881
25,540	1,883	25,150	1,891
26,140	1,890	25,850	1,867
25,100	1,903	26,090	1,907
25,390	1,866	26,060	1,895
25,640	1,949	25,320	1,905
25,130	1,877	25,970	1,936
25,310	1,908	25,640	1,914
25,820	1,901	24,150	1,855
25,070	1,853	25,990	1,914
26,170	1,929	26,040	1,895
25,210	1,931	24,920	1,928
25,210	1,882	25,000	1,915
26,090	1,890	24,490	1,871
25,600	1,855	25,760	1,956
26,310	1,877	24,930	1,873
25,639	1,900	25,461	1,898
369	28	515	34

Table NQP 11: Non-communicating results. 4000-Queens (40 cores)

Caption: **mean**

Standard Deviation

100% - com. 1 to 1		100% - com. 1 to N	
Time	Iter.	Time	Iter.
25,530	1,812	24,790	1,878
25,080	1,717	24,940	1,768
25,470	1,794	25,200	1,771
25,040	1,902	25,100	1,829
24,940	1,815	24,730	1,881
24,740	1,858	25,420	1,676
24,960	1,854	25,510	1,756
24,330	1,811	25,580	1,876
24,970	1,836	26,130	1,849
25,360	1,863	25,010	1,890
24,630	1,888	25,410	1,891
25,260	1,818	24,710	1,820
25,460	1,684	24,850	1,868
25,710	1,827	25,620	1,928
25,090	1,825	25,170	1,758
25,340	1,877	24,900	1,885
24,900	1,912	25,450	1,761
25,310	1,866	25,360	1,840
25,940	1,789	26,280	1,890
24,470	1,861	24,940	1,910
25,710	1,849	25,700	1,870
24,960	1,810	24,480	1,910
25,050	1,737	24,160	1,925
24,550	1,786	24,830	1,925
24,840	1,878	24,640	1,730
25,550	1,945	24,760	1,786
25,000	1,890	25,790	1,835
25,540	1,773	25,200	1,784
24,850	1,879	25,450	1,829
24,730	1,849	25,080	1,824
25,110	1,834	25,173	1,838
396	58	475	65

Table NQP 12: Communicating results (100%). 4000-Queens (40 cores)

Caption:

mean

Standard Deviation

sender solver

receiver solver

receiver solver not tacking into account the received information

50% - com. 1 to 1		50% - com. 1 to N	
Time	Iter.	Time	Iter.
25,130	1,883	25,280	1,927
24,940	1,729	25,790	1,886
25,600	1,916	24,580	1,847
25,940	1,843	25,590	1,916
25,630	1,898	24,750	1,776
24,910	1,823	25,260	1,868
24,360	1,765	26,030	1,864
25,590	1,897	24,970	1,805
24,700	1,849	25,880	1,775
25,540	1,866	24,910	1,864
25,140	1,846	25,150	1,870
25,860	1,897	25,340	1,878
25,030	1,806	25,850	1,952
25,620	1,928	24,670	1,908
24,130	1,792	25,010	1,848
24,360	1,850	25,500	1,876
24,490	1,864	25,070	1,871
25,010	1,824	25,100	1,860
25,730	1,925	25,210	1,802
25,330	1,920	25,970	1,780
25,640	1,807	25,370	1,852
24,920	1,865	25,140	1,914
25,670	1,798	25,750	1,796
24,310	1,859	24,730	1,731
25,420	1,887	24,660	1,756
24,990	1,820	25,480	1,934
24,980	1,815	24,920	1,927
25,010	1,891	25,380	1,693
25,680	1,871	25,770	1,936
24,790	1,909	25,720	1,812
25,148	1,855	25,294	1,851
505	50	424	66

Table NQP 13: Communicating results (50%). 4000-Queens (40 cores)

Caption:

mean

Standard Deviation

sender solver

receiver solver

receiver solver not tacking into account the received information

non-communicating solver

25% - com. 1 to 1		25% - com. 1 to N	
Time	Iter.	Time	Iter.
23,940	1,912	25,340	1,872
25,740	1,900	24,900	1,863
25,770	1,909	25,700	1,931
25,620	1,933	25,100	1,834
25,670	1,835	25,670	1,858
25,790	1,851	24,120	1,740
24,130	1,852	24,550	1,930
25,900	1,884	25,260	1,881
25,670	1,835	25,920	1,935
25,190	1,833	25,720	1,893
24,940	1,922	26,050	1,904
25,620	1,845	26,060	1,849
25,410	1,901	25,610	1,848
24,850	1,889	24,470	1,882
24,270	1,888	25,710	1,909
24,550	1,794	25,570	1,962
25,700	1,853	25,270	1,896
25,570	1,909	24,440	1,844
24,680	1,877	24,210	1,885
25,340	1,911	25,550	1,872
26,530	1,808	25,350	1,730
25,690	1,899	25,520	1,891
25,340	1,758	25,800	1,876
26,020	1,904	25,700	1,845
25,090	1,838	25,520	1,903
24,330	1,799	25,250	1,853
25,640	1,882	24,070	1,844
25,260	1,840	25,440	1,829
25,530	1,889	25,540	1,773
24,270	1,894	25,690	1,883
25,268	1,868	25,303	1,867
635	43	570	52

Table NQP 14: Communicating results (25%). 4000-Queens (40 cores)

Caption:

mean

Standard Deviation

sender solver

receiver solver

receiver solver not tacking into account the received information

non-communicating solver

Time	Iter.
18,490	1,635
23,080	1,761
25,490	1,555
17,000	1,708
25,290	1,349
25,660	1,496
19,870	1,710
24,870	1,630
16,660	1,714
21,990	1,708
16,680	1,605
16,360	1,803
25,250	1,690
15,840	1,764
18,330	1,681
15,980	1,734
25,920	1,550
24,890	1,671
25,520	1,618
24,500	1,475
17,230	1,712
25,300	1,656
24,520	1,723
22,670	1,730
20,920	1,574
17,810	1,789
16,100	1,758
23,600	1,773
23,700	1,468
18,660	1,644
21,273	1,656
3,767	108

Table NQP 15: Communicating results (Partial-full solvers). 4000-Queens (40 cores)

Caption:

mean

Standard Deviation

receiver solver

receiver solver not tacking into account the received information

Sequential		Parallel	
Time	Iter.	Time	Iter.
41,270	2,350	40,360	2,386
41,370	2,384	40,370	2,365
42,150	2,362	41,280	2,346
41,190	2,426	40,400	2,348
41,260	2,356	40,850	2,401
40,980	2,351	41,140	2,381
41,300	2,362	40,550	2,355
41,400	2,304	38,160	2,387
41,890	2,330	40,820	2,436
40,220	2,381	41,930	2,335
41,540	2,403	41,030	2,372
41,170	2,381	41,660	2,414
41,250	2,345	41,430	2,404
41,130	2,363	40,800	2,398
41,710	2,379	41,250	2,367
41,480	2,321	40,440	2,357
40,990	2,375	40,350	2,420
41,770	2,350	40,680	2,389
41,210	2,406	40,930	2,317
40,940	2,347	39,670	2,391
41,910	2,359	39,190	2,339
41,520	2,399	39,950	2,411
40,590	2,391	38,470	2,398
41,260	2,366	41,180	2,404
41,720	2,353	41,260	2,347
41,790	2,350	39,950	2,310
40,700	2,404	39,710	2,406
41,780	2,377	42,130	2,418
41,720	2,375	41,300	2,370
42,150	2,367	40,130	2,340
41,379	2,367	40,579	2,377
446	26	914	32

Table NQP 16: Non-communicating results. 5000-Queens (40 cores)

Caption: **mean**

Standard Deviation

100% - com. 1 to 1		100% - com. 1 to N	
Time	Iter.	Time	Iter.
37,950	2,296	39,240	2,228
39,710	2,299	39,400	2,295
38,270	2,181	38,410	2,301
37,320	2,237	40,120	2,313
41,220	2,289	40,500	2,296
39,310	2,228	40,960	2,353
40,760	2,364	39,660	2,360
39,150	2,292	39,200	2,326
39,620	2,275	39,280	2,265
40,860	2,273	41,440	2,355
38,360	2,332	40,360	2,305
38,760	2,344	39,410	2,315
40,470	2,281	39,940	2,211
39,500	2,313	40,700	2,255
39,840	2,313	40,670	2,246
40,630	2,324	40,220	2,208
38,270	2,371	39,830	2,277
39,570	2,223	39,150	2,276
38,740	2,297	40,660	2,191
37,690	2,232	39,930	2,304
39,280	2,272	40,100	2,306
40,160	2,270	38,920	2,238
39,570	2,320	39,250	2,260
41,330	2,258	38,960	2,266
40,550	2,255	40,920	2,361
40,990	2,322	40,190	2,341
40,080	2,239	39,220	2,241
40,240	2,299	39,570	2,304
40,120	2,274	40,400	2,395
40,270	2,337	39,830	2,328
39,620	2,287	39,881	2,291
1,074	44	717	51

Table NQP 17: Communicating results (100%). 5000-Queens (40 cores)

Caption:

mean

Standard Deviation

sender solver

receiver solver

receiver solver not tacking into account the received information

50% - com. 1 to 1		50% - com. 1 to N	
Time	Iter.	Time	Iter.
41,200	2,379	40,390	2,300
40,910	2,207	40,320	2,380
40,020	2,287	40,800	2,307
40,330	2,365	40,230	2,297
40,510	2,402	40,710	2,304
40,670	2,396	40,090	2,332
38,420	2,145	40,930	2,388
40,170	2,367	40,880	2,283
40,430	2,304	40,510	2,176
40,490	2,184	40,810	2,275
39,570	2,316	40,820	2,335
40,590	2,306	40,010	2,276
40,410	2,300	39,810	2,211
41,390	2,348	39,890	2,260
40,320	2,326	40,410	2,302
40,890	2,359	41,030	2,385
40,580	2,250	39,660	2,333
40,420	2,254	39,940	2,319
40,880	2,291	40,990	2,381
39,680	2,364	41,330	2,321
40,680	2,290	40,120	2,294
39,940	2,192	41,150	2,286
39,020	2,401	38,910	2,333
40,750	2,374	40,060	2,390
39,710	2,347	41,030	2,297
40,130	2,319	39,620	2,246
39,920	2,304	40,280	2,358
39,810	2,331	40,820	2,221
40,730	2,237	39,690	2,391
41,570	2,409	39,940	2,388
40,338	2,312	40,373	2,312
661	69	565	56

Table NQP 18: Communicating results (50%). 5000-Queens (40 cores)

Caption:

mean

Standard Deviation

sender solver

receiver solver

receiver solver not tacking into account the received information

non-communicating solver

25% - com. 1 to 1		25% - com. 1 to N	
Time	Iter.	Time	Iter.
41,120	2,333	41,450	2,321
41,560	2,270	38,980	2,346
40,060	2,195	39,330	2,167
41,700	2,419	40,920	2,319
41,590	2,220	40,550	2,425
40,300	2,422	41,140	2,400
40,260	2,268	40,530	2,364
40,990	2,320	38,890	2,336
39,770	2,405	41,030	2,454
40,040	2,213	41,320	2,251
39,030	2,364	40,580	2,123
40,450	2,235	40,540	2,405
38,210	2,259	40,950	2,281
38,900	2,408	41,460	2,364
41,230	2,338	41,290	2,245
41,000	2,368	39,340	2,368
39,680	2,377	40,430	2,313
41,570	2,347	39,970	2,188
39,580	2,417	41,390	2,367
40,420	2,244	40,520	2,289
40,710	2,306	39,100	2,356
39,180	2,304	40,380	2,382
39,280	2,357	39,870	2,420
40,290	2,430	40,220	2,423
39,870	2,356	40,640	2,359
41,010	2,336	40,250	2,380
41,590	2,402	41,700	2,454
40,090	2,415	40,340	2,367
41,650	2,405	39,260	2,342
40,420	2,400	41,210	2,344
40,385	2,338	40,453	2,338
930	71	807	80

Table NQP 19: Communicating results (25%). 5000-Queens (40 cores)

Caption:

mean

Standard Deviation

sender solver

receiver solver

receiver solver not tacking into account the received information

non-communicating solver

Time	Iter.
40,090	2,135
38,090	2,058
25,270	2,155
39,390	1,895
39,880	2,149
26,480	2,192
38,030	2,090
35,710	2,002
39,180	1,910
33,100	2,052
37,230	2,192
38,000	2,133
26,350	2,232
30,290	2,099
39,440	1,800
36,560	2,216
39,660	2,059
38,050	1,918
30,500	2,098
38,450	2,004
39,620	2,203
33,930	2,191
37,370	2,103
39,980	1,910
30,640	2,093
30,090	2,059
27,900	2,189
31,410	2,084
25,620	2,134
37,070	2,105
34,779	2,082
4,993	108

Table NQP 20: Communicating results (Partial-full solvers). 5000-Queens (40 cores)

Caption:

mean

Standard Deviation

receiver solver

receiver solver not tacking into account the received information

Sequential		Parallel	
Time	Iter.	Time	Iter.
60,450	2,865	59,280	2,816
60,590	2,860	59,410	2,941
60,880	2,845	59,380	2,883
60,660	2,827	60,960	2,803
59,560	2,820	60,950	2,893
60,340	2,910	60,110	2,840
59,910	2,864	61,710	2,893
61,480	2,850	59,530	2,792
60,670	2,839	60,270	2,841
60,360	2,811	59,700	2,848
61,030	2,818	59,940	2,844
60,670	2,842	60,390	2,904
60,440	2,904	59,590	2,843
59,830	2,889	60,640	2,774
60,190	2,852	59,660	2,817
60,900	2,826	59,770	2,843
59,950	2,834	60,680	2,807
59,830	2,810	59,250	2,817
61,340	2,861	59,280	2,931
60,070	2,810	60,850	2,865
59,870	2,819	60,040	2,843
60,390	2,857	59,880	2,863
60,650	2,794	60,760	2,847
60,480	2,824	59,220	2,875
60,590	2,807	60,480	2,833
60,590	2,806	60,220	2,854
61,220	2,865	59,680	2,794
59,290	2,810	60,380	2,782
59,800	2,787	61,790	2,912
60,690	2,802	59,360	2,871
60,424	2,837	60,105	2,849
522	31	709	43

Table NQP 21: Non-communicating results. 6000-Queens (40 cores)

Caption: **mean** **Standard Deviation**

100% - com. 1 to 1		100% - com. 1 to N	
Time	Iter.	Time	Iter.
59,830	2,725	57,960	2,868
58,510	2,707	59,790	2,810
60,590	2,660	58,260	2,748
59,080	2,813	58,910	2,729
58,390	2,772	58,270	2,690
59,650	2,769	56,310	2,656
57,350	2,550	59,980	2,811
56,450	2,602	57,280	2,739
57,050	2,792	62,020	2,703
58,520	2,824	57,810	2,833
61,190	2,650	59,850	2,759
61,550	2,820	60,320	2,780
59,050	2,773	59,110	2,801
60,310	2,664	60,290	2,855
60,130	2,764	57,650	2,794
58,640	2,758	59,350	2,811
59,690	2,777	58,120	2,821
56,860	2,604	56,530	2,752
59,270	2,668	58,790	2,680
57,200	2,583	59,840	2,772
60,680	2,757	60,760	2,740
58,340	2,670	58,520	2,853
59,600	2,712	60,060	2,744
57,850	2,814	60,760	2,837
58,080	2,761	61,500	2,853
59,020	2,762	59,140	2,773
59,970	2,703	58,840	2,688
59,870	2,812	59,390	2,796
56,510	2,824	58,540	2,741
60,120	2,793	60,860	2,749
58,978	2,729	59,160	2,773
1,380	78	1,378	57

Table NQP 22: Communicating results (100%). 6000-Queens (40 cores)

Caption:

mean

Standard Deviation

sender solver

receiver solver

receiver solver not tacking into account the received information

50% - com. 1 to 1		50% - com. 1 to N	
Time	Iter.	Time	Iter.
57,800	2,822	57,350	2,862
59,720	2,728	58,890	2,820
59,100	2,851	61,070	2,864
59,870	2,850	59,120	2,928
58,160	2,738	59,720	2,808
60,200	2,881	60,720	2,742
59,050	2,817	59,020	2,765
59,280	2,659	60,250	2,802
58,240	2,722	60,500	2,811
60,980	2,754	60,010	2,804
57,200	2,664	58,010	2,802
57,550	2,726	60,950	2,673
61,860	2,890	60,720	2,658
59,500	2,757	58,270	2,841
59,380	2,732	59,490	2,650
58,550	2,758	59,890	2,696
57,240	2,863	60,060	2,778
57,580	2,829	59,940	2,680
59,120	2,787	59,740	2,672
60,660	2,630	59,980	2,759
59,700	2,812	60,350	2,784
58,060	2,780	58,100	2,832
57,330	2,851	57,760	2,692
57,580	2,676	58,500	2,784
58,090	2,769	60,320	2,805
58,540	2,740	59,570	2,713
59,480	2,763	60,070	2,776
60,020	2,805	58,550	2,803
59,870	2,755	59,570	2,838
59,640	2,826	59,560	2,750
58,978	2,775	59,535	2,773
1,190	67	984	69

Table NQP 23: Communicating results (50%). 6000-Queens (40 cores)

Caption:

mean

Standard Deviation

sender solver

receiver solver

receiver solver not tacking into account the received information

non-communicating solver

25% - com. 1 to 1		25% - com. 1 to N	
Time	Iter.	Time	Iter.
58,670	2,657	60,790	2,752
59,340	2,825	58,690	2,877
57,600	2,814	60,940	2,839
59,410	2,680	60,750	2,863
60,050	2,755	59,480	2,790
59,290	2,826	62,360	2,882
60,380	2,772	60,040	2,871
56,360	2,640	59,820	2,805
60,220	2,684	58,820	2,739
60,790	2,789	60,350	2,792
61,930	2,828	58,060	2,890
57,780	2,819	58,460	2,766
59,530	2,874	59,800	2,847
60,380	2,902	60,890	2,719
58,050	2,814	56,810	2,873
59,410	2,790	59,200	2,878
62,810	2,884	59,890	2,800
59,600	2,863	58,280	2,815
59,540	2,694	59,920	2,830
59,300	2,860	59,720	2,835
58,920	2,895	59,370	2,778
58,940	2,871	62,070	2,888
59,830	2,861	56,560	2,761
58,140	2,657	57,810	2,771
56,860	2,756	60,610	2,847
59,160	2,781	58,190	2,878
60,060	2,838	62,320	2,864
59,520	2,713	61,630	2,824
57,800	2,788	61,340	2,801
58,960	2,892	60,140	2,851
59,288	2,794	59,770	2,824
1,340	78	1,500	49

Table NQP 24: Communicating results (25%). 6000-Queens (40 cores)

Caption:

mean

Standard Deviation

sender solver

receiver solver

receiver solver not tacking into account the received information

non-communicating solver

Time	Iter.
59,000	2,406
54,190	2,511
46,540	2,417
52,710	2,588
48,020	2,546
54,890	2,648
47,130	2,724
55,980	2,529
56,040	2,574
50,920	2,734
41,560	2,515
56,900	2,186
52,780	2,451
51,170	2,479
50,760	2,367
51,710	2,276
56,300	2,691
48,700	2,737
59,710	2,555
45,980	2,807
41,860	2,497
54,060	2,440
43,060	2,509
59,310	2,200
57,750	2,405
57,650	2,127
47,270	2,525
57,200	2,285
38,870	2,810
53,800	2,503
51,727	2,501
5,737	176

Table NQP 25: Communicating results (Partial-full solvers). 6000-Queens (40 cores)

Caption:

mean
Standard Deviation

receiver solver

receiver solver not tacking into account the received information

C

Results of experiments with *Costas Array Problem*

In this Chapter we presents results values of experiments with Costas Array Problem.

In the following tables: ...

Sequential		No Communication	
Time	Iter.	Time	Iter.
3,410	70,278	1,810	19,861
3,140	67,274	250	4,672
2,070	43,561	120	1,234
340	7,381	440	5,205
3,010	61,855	1,570	16,639
3,150	64,966	340	4,840
2,030	43,528	450	4,831
1,280	26,578	120	2,306
2,280	47,955	1,020	10,638
1,340	27,925	200	2,539
2,240	46,846	1,160	14,293
2,610	53,341	910	16,289
740	15,529	550	6,652
2,050	43,846	700	12,914
1,600	33,460	280	2,884
2,640	55,891	1,160	11,834
1,980	41,479	850	14,665
1,440	29,980	390	4,630
3,080	63,730	890	9,382
3,090	65,338	560	9,262
1,100	22,774	1,580	16,999
		1,620	29,008
		310	5,462
		280	2,974
		180	3,367
		380	3,631
		270	3,064
		1,000	10,571
		1,280	14,581
		230	4,099
		100	1,752
		560	6,574
		1,150	12,022
		720	10,345
		1,210	13,006
		280	3,023
		110	1,195
		990	10,448
		910	9,787
		1,020	18,244
		920	10,564
		730	10,939
		780	12,232
		1,430	24,616
		900	15,970
2,125	44,453	727	9,557
871	18,113	469	6,439

Table CAP 1: No communication (1 and 40 cores) Costas Array 17

St_a / 100% com . 1-1		St_a / 100% com. 1-n		St_a / 50% com. 1-1		St_a / 50% com. 1-n	
Time	Iter.	Time	Iter.	Time	Iter.	Time	Iter.
860	9,481	1,000	13,702	330	3,964	760	12,946
810	7,954	170	1,648	550	10,750	140	1,963
830	7,870	350	4,172	840	15,835	120	1,384
240	2,947	350	3,617	280	3,019	780	9,553
180	2,252	210	2,593	960	11,142	720	12,013
300	3,106	200	1,976	1,030	15,841	20	172
280	3,978	160	1,617	340	5,251	570	7,855
190	2,392	370	4,336	330	4,153	110	1,259
200	1,868	1,040	16,596	260	3,313	290	3,496
740	6,670	150	1,891	290	4,834	1,270	22,888
380	6,395	230	2,566	10	119	1,020	20,027
200	2,320	800	9,874	170	3,730	1,430	16,756
700	9,247	920	9,599	370	8,019	460	6,115
170	1,700	130	1,486	1,640	23,068	290	5,632
1,030	10,403	780	8,361	1,060	20,573	40	571
760	13,045	800	10,440	360	5,329	1,700	23,350
760	7,660	340	4,647	410	5,146	430	8,873
500	6,751	1,040	13,030	580	7,987	420	5,695
160	1,360	380	4,243	810	9,737	990	11,596
140	1,528	360	3,763	90	1,112	250	3,478
120	1,198	500	6,985	220	2,644	210	2,500
380	4,804	110	1,312	530	10,198	300	3,691
220	1,981	100	1,066	1,310	13,846	490	7,138
180	2,833	150	2,560	70	1,348	760	7,840
190	1,928	560	5,119	570	6,703	440	7,016
980	8,974	590	9,214	550	6,703	1,540	30,901
1,130	10,750	650	7,919	20	427	1,640	19,264
260	2,572	160	1,922	300	3,779	1,380	27,048
380	3,324	260	2,638	180	2,364	180	3,737
120	1,465	80	1,035	750	8,929	720	8,287
40	689	540	11,224	630	9,292	1,430	24,226
790	13,384	760	13,072	50	681	660	6,959
80	844	370	5,209	600	7,396	160	2,191
610	5,650	110	2,298	1,330	15,478	470	9,328
470	4,748	120	1,182	780	14,206	40	402
100	1,036	420	4,527	790	12,373	350	4,260
50	562	100	1,069	100	1,171	490	10,333
480	7,780	80	1,054	340	4,189	960	11,074
510	4,883	690	7,756	260	3,493	60	616
630	11,797	80	871	620	8,236	30	328
610	6,125	1,100	18,772	510	5,246	380	5,173
260	2,734	80	872	660	6,893	220	2,638
270	3,322	690	7,912	20	241	480	6,091
240	2,500	910	14,065	1,330	21,556	960	12,664
210	2,041	420	7,740	1,920	39,955	320	5,176
416	4,819	431	5,723	559	8,228	588	8,767
296	3,589	317	4,744	443	7,556	477	7,774

Table CAP 2: Strategy A (40 cores) Costas Array 17

Caption: **mean**

Standard Deviation

sender solver

receiver solver

receiver solver not tacking into account the received information

non-communicating solver

St_b / 100% com. 1-1		St_b / 100% com. 1-n		St_b / 50% com. 1-1		St_b / 50% com. 1-n	
Time	Iter.	Time	Iter.	Time	Iter.	Time	Iter.
10	322	2,420	32,411	990	13,492	220	4,852
320	5,560	1,690	17,034	130	1,654	2,190	27,028
790	9,226	650	6,397	960	19,321	810	13,510
40	541	1,200	22,472	570	12,136	80	814
1,280	20,420	640	6,389	230	4,858	680	12,859
1,240	12,358	610	5,670	740	8,590	630	6,412
850	10,123	920	9,427	410	5,498	120	1,390
280	4,840	250	3,058	320	4,429	440	4,309
880	10,507	960	16,715	300	4,039	490	5,614
430	5,125	940	9,898	320	4,123	450	9,082
60	625	100	1,243	1,180	15,841	110	1,252
340	6,875	870	17,608	420	7,598	2,330	42,391
60	583	180	1,885	50	616	140	1,780
980	11,539	790	7,738	1,620	18,894	120	1,557
90	949	470	5,982	1,890	26,893	500	5,965
230	2,281	130	1,366	130	1,924	1,310	27,804
830	12,990	120	1,780	1,280	13,420	500	7,017
550	6,721	170	1,932	160	1,861	430	5,332
330	6,537	40	432	10	211	290	3,736
320	3,317	320	3,547	110	1,369	80	859
760	11,194	200	3,073	120	1,573	160	3,142
420	4,405	80	1,360	100	872	40	507
160	1,955	20	130	60	691	100	1,219
90	1,105	430	7,915	1,050	14,983	580	6,859
200	3,649	30	373	1,670	34,654	100	1,486
300	3,982	210	2,869	490	6,250	170	2,440
160	1,748	50	544	1,560	26,260	200	2,719
790	8,998	470	5,890	10	187	40	382
280	3,715	370	3,932	130	1,246	430	6,370
120	1,318	170	2,164	300	4,066	630	7,480
1,370	14,143	90	1,546	290	3,562	360	4,621
390	4,216	1,160	14,086	1,030	13,999	1,500	17,857
630	7,579	120	1,447	500	10,426	1,240	16,702
170	1,990	120	1,186	300	3,451	660	9,184
90	1,012	300	5,012	110	1,441	160	1,642
1,590	16,567	300	3,133	970	20,263	200	2,668
320	3,976	20	280	40	988	170	2,218
1,310	17,230	470	6,574	320	6,691	260	2,918
120	1,567	260	2,423	480	10,150	620	13,171
190	3,301	90	1,048	640	6,592	1,030	19,681
210	2,143	360	3,436	20	451	90	1,036
470	9,954	600	7,060	90	1,609	1,100	13,264
360	5,307	290	3,217	480	8,611	440	4,471
160	1,691	380	5,341	270	3,601	280	3,415
1,020	17,737	280	2,581	940	15,943	210	2,746
480	6,265	452	5,769	529	8,118	504	7,372
417	5,321	477	6,578	506	8,192	524	8,544

Table CAP 3: Strategy B (40 cores) Costas Array 17

Caption: **mean**

Standard Deviation

sender solver

receiver solver

receiver solver not tacking into account the received information

non-communicating solver

St_c / 100% com. 1-1		St_c / 100% com. 1-n		St_c / 50% com. 1-1		St_c / 50% com. 1-n	
Time	Iter.	Time	Iter.	Time	Iter.	Time	Iter.
40	340	960	18,532	1,990	24,955	70	793
200	1,936	980	10,132	40	551	360	5,321
180	1,966	290	3,196	720	15,226	220	2,374
1,170	11,605	540	5,428	730	7,378	210	2,416
520	10,654	1,300	26,122	1,200	14,257	540	7,306
390	5,275	180	2,005	720	9,265	40	583
380	4,012	140	2,065	870	10,972	410	4,288
660	6,797	170	1,849	270	5,617	870	12,076
340	4,321	180	2,233	190	2,620	580	11,218
240	2,527	400	4,198	1,010	19,234	720	8,242
320	3,799	680	7,546	1,280	15,586	1,600	30,455
310	3,484	780	7,377	330	4,636	1,600	15,751
400	4,138	890	11,849	870	11,847	180	2,164
1,100	21,040	870	9,637	300	4,129	2,080	25,051
1,110	19,507	110	1,406	80	1,066	1,350	18,460
240	3,493	120	1,315	410	5,167	1,380	13,921
1,250	25,261	250	2,574	1,570	29,595	440	4,351
320	6,529	660	9,044	860	9,987	380	4,663
360	4,054	200	2,683	80	1,765	370	5,008
340	3,547	630	12,604	390	3,992	230	4,882
1,070	10,996	630	6,409	310	3,754	320	3,643
320	3,574	570	5,563	960	10,719	130	2,618
340	7,062	800	7,642	970	11,053	400	4,690
360	4,459	500	10,312	910	17,042	230	2,917
1,100	19,301	500	8,659	1,150	15,991	400	4,807
160	1,672	590	6,070	320	3,959	110	1,393
1,410	16,363	130	1,423	120	2,460	260	3,580
150	1,615	1,820	30,643	520	8,194	1,280	25,900
300	3,160	830	16,030	520	5,842	400	5,084
1,310	15,066	720	8,838	590	12,658	300	4,077
1,360	25,852	740	8,437	220	3,193	290	2,944
590	12,271	100	1,027	100	1,184	200	2,211
230	2,788	600	7,387	10	241	210	2,983
10	313	580	6,493	700	8,263	220	2,239
390	8,149	170	1,957	390	5,312	310	3,288
390	6,727	150	1,771	720	11,386	770	7,825
370	7,654	340	4,627	160	2,080	1,790	22,204
20	250	570	6,751	640	12,271	180	1,978
120	1,518	500	5,932	870	10,204	1,710	36,013
100	1,201	20	265	530	5,959	470	4,819
170	2,237	280	3,910	390	8,155	660	12,346
230	2,767	250	2,785	160	1,985	910	10,369
490	6,028	190	2,368	810	10,024	920	11,116
480	8,626	180	3,091	180	3,850	440	4,474
920	9,349	450	5,035	280	3,375	420	5,179
495	7,184	501	6,783	588	8,378	599	8,178
400	6,632	359	6,218	432	6,437	525	8,289

Table CAP 4: Strategy C (40 cores) Costas Array 17

Caption: **mean**

Standard Deviation

sender solver

receiver solver

receiver solver not tacking into account the received information

non-communicating solver

D

Results of experiments with *Golomb Ruler Problem*

In this Chapter we presents results values of experiments with Golomb Ruler Problem.

In the following tables: ...

Using Tabu List (no communication)		
time	iterations	restarts
110	743	3
100	768	3
50	518	2
20	88	0
20	202	1
30	233	1
20	197	0
10	112	0
20	86	0
40	229	1
10	88	0
20	88	0
20	89	0
100	970	4
30	173	0
40	401	2
20	169	0
30	173	0
30	196	0
170	1543	7
90	576	2
60	602	3
60	570	2
60	547	2
10	86	0
10	138	0
20	142	0
10	87	0
50	487	2
20	167	0
43	**349**	**1**
38	**334**	**2**

Without Tabu List (no communication)		
time	iterations	restarts
70	685	3
50	432	2
50	400	1
90	801	4
50	371	1
60	603	3
110	1118	5
30	286	1
40	378	1
30	315	1
30	259	1
50	431	2
20	204	1
130	1290	6
100	961	4
60	604	3
20	263	1
20	194	0
10	112	0
30	290	1
20	166	0
40	343	1
10	114	0
140	1200	5
30	259	1
20	203	1
40	260	1
10	88	0
20	202	1
40	259	1
47	**436**	**2**
35	**330**	**2**

Table GRP 1: No communication (40 cores) Golomb Ruler 8-34

Caption: **mean** **Standard Deviation**

Eps = 4
Norm = 8
tabu size = 15

Using Tabu List (communication 1-1)		
time	iterations	restarts
40	431	2
30	199	0
30	140	0
30	316	1
80	569	2
60	340	1
40	202	1
30	140	0
30	113	0
160	1202	6
90	570	2
60	430	2
30	262	1
50	395	1
30	143	0
40	337	1
30	263	1
40	312	1
20	193	0
20	120	0
30	286	1
30	112	0
20	174	0
10	89	0
80	573	2
20	60	0
30	289	1
70	460	2
10	4	0
90	543	2
44	309	1
31	233	1

Using Tabu List (communication 1-n)		
time	iterations	restarts
20	87	0
10	30	0
20	113	0
80	543	2
70	403	2
10	57	0
70	460	2
40	203	1
10	121	0
10	62	0
80	594	2
40	203	1
30	203	1
10	87	0
70	342	1
90	630	3
40	368	1
10	3	0
80	632	3
60	488	2
20	203	1
10	30	0
70	492	2
40	170	0
80	570	2
40	261	1
100	768	3
10	60	0
50	289	1
10	3	0
43	283	1
30	225	1

Table GRP 2: Communication (40 cores) Golomb Ruler 8-34

Eps = 4
Norm = 8
tabu size = 40

Caption:

mean

Standard Deviation

sender solver

receiver solver

Sequential (without tabu list)		
time	iterations	restarts
80	1232	6
950	15915	79
400	6803	34
600	10316	51
880	14802	74
2050	33658	168
2580	43742	218
200	3313	16
690	10887	54
1640	26832	134
1040	17171	85
210	3486	17
240	4082	20
100	1717	8
130	2483	12
1040	18059	90
1720	28799	143
1070	18138	90
1290	21230	106
40	740	3
460	7604	38
100	1741	8
940	15030	75
110	1887	9
1760	28857	144
420	7172	35
1440	25339	126
780	13258	66
220	3857	19
670	11030	55
795	**13,306**	**66**
668	**11,154**	**56**

Sequential (with tabu list)		
time	iterations	restarts
1090	17140	85
10	233	1
240	3860	19
2190	36030	180
2350	37203	186
700	11366	56
150	2634	13
740	11461	57
870	13795	68
80	1260	6
50	994	4
450	7202	36
970	15796	78
510	8142	40
100	1547	7
1230	20487	102
500	8287	41
560	9311	46
860	14089	70
70	1345	6
510	8343	41
450	7342	36
80	1401	7
260	4340	21
1960	31257	156
260	4086	20
1340	21514	107
250	3942	19
30	546	2
1070	17399	86
664	**10,745**	**53**
639	**10,260**	**51**

Table GRP 3: Sequential (1 core) Golomb Ruler 8-34

Caption: **mean**

Standard Deviation

Eps = 4
Norm = 8
tabu size = 15

Using Tabu List (no communication)		
time	iterations	restarts
4,740	20,069	13
13,990	56,986	37
7,980	33,958	22
6,810	28,162	18
9,300	37,336	24
21,410	89,951	59
3,340	13,506	9
3,190	13,458	8
1,230	4,449	2
2,320	9,792	6
5,520	23,529	15
4,330	17,268	11
3,170	12,892	8
1,050	3,734	2
8,090	34,830	23
5,310	21,741	14
2,950	12,597	8
620	2,892	1
11,370	48,732	32
2,790	11,731	7
2,400	9,733	6
2,110	8,268	5
11,010	47,398	31
1,560	6,140	4
3,270	13,937	9
640	2,355	1
990	3,698	2
480	1,768	1
3,120	12,896	8
2,600	11,322	7
4,923	**20,504**	**13**
4,687	**19,743**	**13**

Without Tabu List (no communication)		
time	iterations	restarts
5,360	21,072	14
1,700	7,295	4
6,590	27,135	18
900	3,466	2
5,710	24,439	16
4,000	16,161	10
4,760	21,600	14
1,480	5,834	3
15,100	63,697	42
12,840	54,234	36
4,270	19,252	12
7,360	32,230	21
7,330	32,695	21
2,370	10,163	6
6,890	29,354	19
1,080	4,570	3
10,340	44,978	29
13,190	56,969	37
8,370	36,197	24
3,260	14,755	9
970	4,450	2
920	4,247	2
4,250	17,070	11
3,630	15,300	10
8,690	36,498	24
2,000	7,965	5
3,760	15,501	10
7,390	29,296	19
3,370	14,731	9
1,430	6,169	4
5,310	**22,577**	**15**
3,863	**16,488**	**11**

Table GRP 4: No communication (40 cores) Golomb Ruler 10-55

Caption: **mean**

Standard Deviation

Eps = 4
Norm = 8
tabu size = 15

Using Tabu List (communication 1-1)		
time	iterations	restarts
1,370	5,957	3
440	1,613	1
6,440	26,337	17
590	2,402	1
310	1,258	0
1,790	7,891	5
2,880	11,075	7
1,700	6,537	4
260	797	0
550	2,409	1
5,750	21,993	14
6,800	26,270	17
9,100	35,963	23
1,070	4,018	2
3,010	11,320	7
4,590	17,455	11
10,120	40,753	27
3,620	14,283	9
3,080	12,199	8
430	1,674	1
9,580	39,655	26
3,060	11,673	7
7,210	28,998	19
1,310	5,008	3
6,620	27,298	18
8,850	35,597	23
2,480	10,118	6
9,170	32,662	21
1,460	5,952	3
3,460	13,935	9
3,903	15,437	10
3,225	12,789	9

Using Tabu List (communication 1-n)		
time	iterations	restarts
8,180	35,159	23
7,920	33,205	22
1,350	5,190	3
3,030	11,119	7
3,460	15,232	10
630	2,326	1
9,300	36,107	24
1,680	6,760	4
170	757	0
5,520	21,400	14
210	758	0
1,100	4,314	2
8,150	32,458	21
1,510	5,813	3
460	1,897	1
2,230	8,359	5
3,650	14,376	9
1,240	4,771	3
580	2,432	1
460	1,743	1
5,560	22,256	14
1,470	5,451	3
960	3,858	2
1,280	5,100	3
4,800	20,616	13
3,760	14,826	9
1,620	6,434	4
1,370	5,199	3
7,910	29,852	19
5,290	20,388	13
3,162	12,605	8
2,824	11,405	8

Table GRP 5: Communication (40 cores) Golomb Ruler 10-55

Eps = 4
Norm = 8
tabu size = 40

Caption:

mean

Standard Deviation

sender solver

receiver solver

Sequential (without tabu list)		
time	iterations	restarts
11,540	85,707	57
106,120	759,399	506
58,220	411,362	274
49,070	342,733	228
20,200	146,027	97
47,620	327,598	218
55,600	372,135	248
90,620	610,539	407
7,650	52,896	35
145,150	966,493	644
33,210	224,551	149
96,990	662,726	441
47,300	318,721	212
15,960	108,936	72
18,560	130,505	87
7,480	50,949	33
68,200	471,494	314
92,600	620,422	413
22,950	156,637	104
45,450	306,110	204
34,570	188,025	125
112,110	768,339	512
142,360	965,061	643
142,300	960,895	640
78,720	528,396	352
56,810	389,701	259
12,120	82,335	54
58,860	387,600	258
208,790	1,417,178	944
106,150	729,103	486
66,443	**451,419**	**301**
49,569	**336,859**	**225**

Sequential (with tabu list)		
time	iterations	restarts
17,720	117,993	78
146,880	993,766	662
63,660	428,626	285
10,090	67,926	45
75,560	514,704	343
53,700	372,893	248
147,140	1,011,234	674
57,570	400,503	267
44,530	295,743	197
40,970	277,320	184
410	2,791	1
40,840	271,494	180
57,140	382,793	255
38,570	256,086	170
134,890	894,727	596
91,410	608,716	405
57,340	389,626	259
105,230	728,128	485
114,190	790,798	527
110,290	758,168	505
23,310	157,930	105
14,320	88,766	59
151,240	1,007,429	671
88,750	600,499	400
39,030	257,327	171
51,060	221,101	147
37,810	198,660	132
24,270	156,268	104
5,610	32,591	21
193,370	1,122,786	748
67,897	**446,913**	**297**
50,024	**328,912**	**219**

Table GRP 6: Sequential (1 core) Golomb Ruler 10-55

Caption: **mean**

Standard Deviation

Eps = 4
Norm = 8
tabu size = 15

Using Tabu List (no communication)		
time	iterations	restarts
32170	58451	19
86650	156935	52
161460	293271	97
220280	398176	132
74050	136241	45
66420	119174	39
179500	327108	109
117350	212795	70
103960	192214	64
9800	17999	5
25290	45418	15
108060	201626	67
119020	219628	73
18550	34456	11
210190	384418	128
53520	100043	33
127630	230691	76
52620	96214	32
214640	383587	127
28080	50999	16
79130	146484	48
55850	103242	34
91340	167080	55
7270	12421	4
184280	338482	112
1620	2999	0
12520	22971	7
3640	6928	2
34220	64652	21
71590	132829	44
85,023	**155,251**	**51**
67,223	**121,929**	**41**

Without Tabu List (no communication)		
time	iterations	restarts
273700	508450	169
30790	56778	18
103280	192006	64
53230	95905	31
41540	78212	26
58890	104483	34
90190	167372	55
133910	248677	82
82350	152068	50
150320	273317	91
1430	2785	0
95300	173065	57
173490	311376	103
51820	95176	31
21590	40349	13
40000	73144	24
123830	232758	77
82010	149688	49
60880	113789	37
98000	181972	60
56980	104485	34
129980	236088	78
98100	180725	60
78750	146276	48
166690	302689	100
82660	151445	50
92160	164589	54
142440	263995	87
54130	98081	32
24350	43139	14
89,760	**164,763**	**54**
55,859	**102,931**	**34**

Table GRP 7: No communication (40 cores) Golomb Ruler 11-72

Caption: **mean**

Standard Deviation

Eps = 4
Norm = 8
tabu size = 15

Using Tabu List (communication 1-1)		
time	iterations	restarts
58150	104687	34
211410	387838	129
5580	10135	3
81170	151964	50
44770	80806	26
92430	163137	54
62460	111108	37
48680	87420	29
50410	89278	29
192780	356580	118
84510	158490	52
174050	321314	107
62130	110587	36
116400	209895	69
69700	127352	42
78790	143793	47
53510	100957	33
42740	76143	25
73150	131271	43
38160	72321	24
12310	22243	7
96690	179268	59
190280	350380	116
78940	143379	47
98090	180525	60
159820	295856	98
96030	175445	58
78610	141931	47
24700	44064	14
86470	158172	52
85,431	156,211	52
52,610	97,330	32

Using Tabu List (communication 1-n)		
time	iterations	restarts
170270	312109	104
62810	111833	37
14140	26681	8
53730	97756	32
16230	28758	9
24030	42519	14
39870	74069	24
59800	107992	35
16660	30007	10
79380	146171	48
95520	175044	58
118760	217352	72
21140	37044	12
19580	34345	11
124340	224683	74
85260	152680	50
10690	19041	6
75000	136039	45
26580	46451	15
154340	287165	95
7170	12523	4
44110	81521	27
90380	168320	56
32580	59889	19
47430	83487	27
114890	215689	71
47620	85957	28
13730	24413	8
77170	144113	48
67510	125686	41
60,357	110,311	36
43,951	81,296	27

Table GRP 8: Communication (40 cores) Golomb Ruler 11-72

Eps = 4
Norm = 8
tabu size = 40

Caption:

mean

Standard Deviation

sender solver

receiver solver

Sequential (without tabu list)		
time	iterations	restarts
279920	536214	178
123870	246321	82
158860	486633	162
59290	187837	62
298830	914400	304
216040	673087	224
98300	301330	100
47670	107164	35
160,348	**431,623**	**143**
96,116	**272,911**	**91**

Sequential (with tabu list)		
time	iterations	restarts
26420	80481	26
126880	414007	138
140600	462828	154
68180	214270	71
272440	859895	286
10300	34398	11
124410	417131	139
72180	243698	81
216000	716845	238
117,490	**382,617**	**127**
85,624	**275,748**	**92**

Table GRP 9: Sequential (1 core) Golomb Ruler 11-72

Caption: **mean**

Standard Deviation

Eps = 4
Norm = 8
tabu size = 15