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REYES AMARO

Mémoire présenté en vue de l'obtention du
grade de Docteur de l'Université de Nantes
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École doctorale : Sciences et technologies de l'information, et mathématiques

Discipline : Informatique et applications, section CNU 27
Unité de recherche : Laboratoire d'informatique de Nantes-Atlantique (LINA)

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POSL: A Parallel-Oriented Solver Language

JURY

Président :	**M. Frédéric LARDEUX**, Maître de conférences, Université d'Angers
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Short abstract:
The multi-core technology and massive parallel architectures are nowadays more accessible for a broad public through hardware like the Xeon Phi or GPU cards. This architecture strategy has been commonly adopted by processor manufacturers to stick with Moore's law. However, this new architecture implies new ways of designing and implementing algorithms to exploit their full potential. This is in particular true for constraint-based solvers dealing with combinatorial optimization problems.

Furthermore, the developing time needed to code parallel solvers is often underestimated. In fact, conceiving efficient algorithms to solve certain problems takes a considerable amount of time. In this thesis we present POSL, a Parallel-Oriented Solver Language for building solvers based on meta-heuristic, in order to solve Constraint Satisfaction Problems (CSP) in parallel. The main goal of this thesis is to obtain a system with which solvers can be easily built, reducing therefore their development effort, by proposing a mechanism of code reusing between solvers. It provides a mechanism to implement solver-independent communication strategies. We also present a detailed analysis of the results obtained when solving some CSPs. The goal is not to outperform the state of the art in terms of efficiency, but showing that it is possible to rapidly prototyping with POSL in order to experiment different communication strategies.

Keywords: Constraint satisfaction, meta-heuristics, parallel, inter-process communication, language.

Contents

Part I

Study and evaluation of POSL

1

Experiments design and results

In this Chapter, I expose all details about the evaluation process of POSL*, i.e. all experiments performed. For each benchmark, I explain strategies used in the evaluation process and what are the experiment environments before exposing a complete analysis of the obtained results.*

Contents

In this chapter, I illustrate and analyze the versatility of POSL studying different ways to solve constraint problems based on local search meta-heuristics. I have chosen the Social Golfers Problem, the N-Queens Problem, the Costas Array Problem and the Golomb Ruler Problem as benchmarks since they are challenging yet differently structured problems. Social Golfers Problem has the structure of tournament problems, where the scheduling of matches between players along a given period of time. The constraints, related to how many times a player can participate in a match the same week, make the problem more complex as the number of weeks increases. N-Queens and Costas Array are similarly modeled in these experiments, since they are represented as permutation problems. However, they have a very interesting characteristic which differentiate them from each other: from certain order on, the number of solutions with respect to the order increases for the case of N-Queens Problem, and for decrease drastically for Costas Array Problem. Golomb Ruler Problem was chosen for two main reasons. Its solution representation is very different from the other studied problems, and because during the search process, POSL describes a different behavior: it performs many restarts.

First results using POSL to solve constraint problems were published in [126] where it was used to solve the Social Golfers Problem and to study some communication strategies. It was the first version of POSL, and it was able to solve relatively easy instances only. However, results suggested that the communication can play an important role if we are able to find the proper communication strategy, and they encourage us to go even further on this direction. In this first version were implemented simple communciation mechanisms only (communication one to one and one to N, but only in one direction.)

In the sencond and current version of POSL[i], other communication operators were incorporated, and those already existent were improved, in order to be able to build more sophisticated communication strategies. With this version, I decided to start performing more detailed studies.

1.1 Methodology

Some terms are necessary to be defined for simplification sake. They are the *sequential environment* and the *parallel environment*, which are the description of the computation resources used for experimentation. Experiments were performed on an Intel® Xeon™ E5-2680 v2, 10×4 cores, 2.80GHz. This server is called CURIOSIPHI and is located at the *Laboratoire d'Informatique de Nantes Atlantique* of the Université de Nantes.

[i]POSL source code is available on GitHub: https://github.com/alejandro-reyesamaro/POSL

Definition 1 *We say that we launch an experiment using a* ***sequential environment*** *if we execute a solver set into a single process of* CURIOSIPHI.

Definition 2 *We say that we launch an experiment using a* ***parallel environment*** *if we execute a solver set in parallel (multi-walk) using the maximum of available processes in* CURIOSIPHI.

With the aim of being as exhaustive as possible in the experimentation process, a methodology based on four stages is proposed:

1. **Algorithm selection** In this stage some experiments are launched, using the sequential environment, to ensure choosing the right computation modules, and the right design of the abstract solver. The following statistical analysis is performed: A set of 30 runs for each setup are performed, and used to a) build box-plot diagrams and bars graphs with some additional information about winner solvers, presented in Appendixes **??**, **??**, **??** and **??**; b) compute means and standard deviation for run-times and iterations, showed in tables, in columns labeled **T** (run-time in seconds), **It.** (number of iterations), **T(sd)** and **It.(sd)** (their respective standard deviations). In some tables, the column labeled **% success** indicates the percentage of solver sets finding a solution before reaching a time–out of 5 minutes (imperative when dealing with meta-heuristics).

2. **Algorithm evaluation in the parallel environment** The selected algorithm is launched using the parallel environment. It is performed a similar statistical analysis to the one described in the previews stage, and results are compared.

3. **Communication strategies selection** After a detailed study of the search process and the behavior of the designed solver sets, some changes in the solver set are proposed in order to design a communication strategy:

   - replacing some computation modules for others based on the originals, but with some modifications according to the new demands of the proposed communication strategy;
   - adding some communication modules depending on the information that we intend to share;
   - a new abstract solver is coded, whose modifications are the strictly necessary to incorporate communication modules;
   - the structure of the communication is designed in order to chose the right communication operators.

4. **Communication strategy evaluation** The designed communication strategy is launched suing the parallel environment, and a statistical analysis is performed. Communication strategies are compared each others based on obtained results in order to select the right one. These results are also compared to those obtained during the stage 2., to be able to draw conclusions about the success of the cooperative approach.

It is important to point out that POSL is not designed to obtain the best results in terms of performance, and much less to outperform the state-of-the-art solutions, but to give the possibility of rapidly prototyping and studying different cooperative or non cooperative search strategies.

1.2 A dynamic configuration exchange strategy (Social Golfers)

In this section, I present the performed study using Social Golfers Problem (SGP) as a benchmark. The communication strategy analyzed here consists in applying a mechanism of cost descending acceleration, exchanging the current configuration between two solvers with different characteristics. Final obtained results show that this communication strategy works well for this problem.

1.2.1 Problem definition

The Social Golfers Problem (SGP) consists in scheduling $g \times p$ golfers into g groups of p players every week for w weeks, such that two players play in the same group at most once. An instance of this problem can be represented by the triple $g - p - w$. This problem, and other closely related problems, arise in many practical applications such as encoding, encryption, and covering problems [127].

Its structure is very attractive, because it is very similar to other problems. For example, the *Kirkman's Schoolgirl Problem* has almost the same formulation, where a number n of girls (analogous to the total number of players) walk in rows of 3 girls (analogous to the number of players per group) with the requirement that no pair of girls walk in the same row twice. An other example is the *Sports Tournament Scheduling* which has a similar structure of the solution, where the number of players per group is 2, and the goal is to schedule a tournament of n players over $n - 1$ weeks.

As it was explained in Chapter **??**, all benchmarks in this chapter were modeled as unconstrained optimization problems, where the objective functions is a linear combination of penalty functions associated to each constraint. The proposed model of SGP has $g \times p \times w$ variables: $\{v_1, v_2, \ldots, v_{g \times p \times w}\}$. Their domains are the same: $D_{v_i} = \{1, \ldots, p \times g\}$.

The cost function (objective function to be minimized) for this benchmark assumes the following structure of a configuration: $s = (W_1, W_2, \ldots, W_w)$, where W_i are integer vectors of size $p \times g$.

This function assumes that each vector W_i has the structure $W_i = \left(G_1^i, G_2^i, \ldots, G_g^i\right)$, where G_j^i are vectors of size p. Based on this structure, the cost c_s of a configuration is:

$$c_s = \sum \max\left(0, \left|G_i^k \cap G_j^l\right| - 1\right), \forall\ i, j \in [1 \ldots g] \text{ and } \forall\ k, l \in [1 \ldots w], k \neq l \tag{1.1}$$

The cost c_s is penalized if some vector W_i does not have its values all different.

For example, let the instace 3–3–2 of SGP, and a configuration

$$s = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 4, 7, 2, 5, 6, 3, 8, 9\}$$

be. The structure of this configuration is the following:

$$s = \left\{ \overbrace{G_1^1, G_2^1, G_3^1}^{W_1}, \overbrace{G_1^2, G_2^2, G_3^2}^{W_2} \right\}$$

$$s = \left\{ \overbrace{1,2,3}^{G_1^1}, \overbrace{4,5,6}^{G_2^1}, \overbrace{7,8,9}^{G_3^1}, \overbrace{1,4,7}^{G_1^2}, \overbrace{2,5,6}^{G_2^2}, \overbrace{3,8,9}^{G_3^2} \right\}$$

If we use the cost function defined in (1.1), it is clear to see that the cost $c_s = 2$.

The cost function for this problem was implemented making an efficient use of the stored information about the cost of the previews configuration. Using integers to work with bit-flags (i.e. using integer's bits to represent boolean values), a table to store the partners of each player in each week can be filled in $O\left(p^2 \cdot g \cdot w\right)$. So, if a configuration has $n = (p \cdot g \cdot w)$ elements, this table can be filled in $O\left(p \cdot n\right)$. This table is filled from scratch only one time in the search process. Then, every cost of a new configuration is calculated based on this information and the number c of performed changes between the new configuration and the stored one. This relative cost is calculated in $O\left(c \cdot g\right)$.

1.2.2 Experiments design and results

Here, I present the abstract solver designed for this problem as well as concrete computation modules composing the different solvers I have tested:

1. Generation abstract module I:

   I_{BP}: Returns a random configuration s, respecting the structure of the problem, i.e. the configuration is a set of w permutations of the vector $[1..P]$, where $P = g \times p$.

2. Neighborhood abstract modules V:

   V_{std}: Given a configuration, returns the neighborhood $\mathcal{V}(s)$ swapping players among groups in the same week, i.e. performs all possible swaps between players in G_i^k and G_j^k for all $i, j \in [1...g]$ with $i < j$, and for all $k \in [1..w]$.

   V_{BAS}: Given a configuration, returns the neighborhood $\mathcal{V}(s)$ swapping the most *culprit player* with other players from the same week but in different groups. It means that if a variable v_i is selected as a culprit player and we said that for each week, it belongs to the group G_i^k, then V_{BAS} performs all possible swaps between the player v_i and all players in G_j^k for all $j \in [1...g]$ with $i \neq j$, and for all $k \in [1..w]$. If a variable shares the same group with another variable more than once, it is called a culprit player. A variable is more culprit than other, if it shares the same group with more variables than the other.

   $V_{BP}(w)$: Given a configuration, selects a set of w weeks randomly, and returns the neighborhood $\mathcal{V}(s)$ by swapping the most culprit player with other players in the same week, for all selected weeks.

3. Selection abstract modules S:

   S_{first}: Given a neighborhood, selects the first configuration $s' \in V(s)$ improving the current cost and returns it together with the current one into the pair (s', s). These two configurations are returned together in order to separate two concepts: selecting a configuration to be the current one in the next iteration, and accepting it. If there are no configurations improving the cost, the first found configuration with the same cost is returned.

   S_{best}: Given a neighborhood, selects the best configuration $s' \in V(s)$ improving the current cost and returns it together with the current one into the pair (s', s). If there are no configurations improving the cost, the first found configuration with the same cost is returned.

Algorithm 1: Simple solvers for SGP

```
abstract solver as_simple                                   // Itr → number of iterations
computation : I, V, S, A
begin
    [↺ (Itr < K1) I ↦ [↺ (Itr % K2) [V ↦ S ↦ A] ] ]
end
solver Solver_Std implements as_simple
    computation : I_BP, V_std, S_best, A_AI
solver Solver_AS implements as_simple
    computation : I_BP, V_BAS, S_best, A_AI
```

Solver	T	T(sd)	It.	It.(sd)
Solver_{AS}	**1.06**	0.79	352	268
Solver_{rho}	41.53	26.00	147	72
Solver_{Std}	87.90	41.96	146	58

Table 1.1: Social Golfers: Instance 10–10–3 in parallel

S_{rand}: Given a neighborhood, selects randomly a configuration $s' \in V(s)$ and returns it together with the current one, into the pair (s', s).

4. Acceptance abstract module A:

   A_{AI}: Given a pair (s', s), returns always the configuration s'

These concrete modules can be directicly reused to solve tournament-like problems like *Sports Tournament Scheduling* and the *Kirkman's Schoolgirl*, because they are problems that can be modeled the same way as SGP.

In a first stage of experiments, I use the operator-based language provided by POSL to build and test many different non-communicating strategies. The goal is to select the best concrete modules to run tests performing communication. A very first experiment was performed to select the best neighborhood function to solve the problem, comparing a basic solver using V_{std}; a new solver using V_{BAS}; and a combination of V_{std} and V_{BAS} by applying the operator ρ, already introduced in the previous chapter. Algorithms 1 and 2 present solvers for each case, respectively. In these algorithms, K_1 represents the maximum number of *restarts*, and K_2 the maximum number of iterations before each *restart*.

Algorithm 2: Solvers combining neighborhood functions using operator *RHO*

```
abstract solver as_rho                                      // Itr → number of iterations
computation : I, V1, V2, S, A
begin
    [↺ (Itr < K1) I ↦ [↺ (Itr % K2) [[V1 ρ V2] ↦ S ↦ A] ] ]
end
solver Solver_rho implements as_rho
    computation : I_BP, V_std, V_BAS, S_best, A_AI
```

Results in Table 1.1 are easy to interpret. V_{Std} uses no additional information to build the neighborhood. It performs every possible swap between two players in different groups, every week, so it allows a more complete and organized search because the set of neighbors is "pseudo-deterministic", i.e. the construction criteria is always the same but the order in which configurations are stored is random. On the other hand, the neighborhood module V_{BAS} selects the most culprit variable (i.e. a player), that is, the variable the most responsible for constraints violation, and permutes this variable value with the value of each other variable, in all groups and all weeks. The most culprit player is calculated as the player which has played with more other players more than once. This neighborhood is $g \times p$ times smaller than the previous one, with g the number of groups and p the number of players per group. Their elements are more promising configurations, for that reason is more efficient, but its size is smaller, hence solvers perform more iterations. Furthermore, it is based on the *Adaptive Search* algorithm, which has shown very good results [5]. V_{BAS} neighborhood function takes random decisions more frequently (e.g. if there are more than one *most culprit player*, one is randomly selected), and the order of the configurations is random as well. I also tested a solver combining these modules using the (ρ) operator. This operator executes its first or second parameter depending on a given probability ρ. This combination spent more time searching the best configuration among the neighborhood, although with a lower number of iterations than V_{BAS}. Since the V_{BAS} neighborhood function was clearly faster, I have chosen it for next experiments, even if it shows higher values of standard deviation: 0.75 for SOLVER_{AS} versus 0.62 for SOLVER_{Std}, considering the ratio $\frac{T(sd)}{T}$.

***

Once the neighborhood computation module has been selected, I have focused the experiment on choosing the best *selection* computation module. Solvers mentioned above were too slow to solve instances of the problem with more than three weeks: they were very often trapped into local minima. For that reason, another solver implementing the abstract solver described in Algorithm 3 have been created, using V_{BAS} and combining S_{best} and S_{rand}: it tries a number of times to improve the cost, and if it is not possible, it picks a random neighbor for the next iteration. We also compared the S_{first} and S_{best} selection modules. The computation module S_{best} selects the best configuration inside the neighborhood, so it spend more time searching a better configuration. The second computation module S_{first} selects the first configuration inside the neighborhood improving the current cost. Using this module, solvers favor exploration over intensification and of course spend clearly less time searching into the neighborhood. In this algorithm, K_3 represent the maximum number of iterations with the

Instance	Best improvement				First improvement			
	T	T(sd)	It.	It.(sd)	T	T(sd)	It.	It.(sd)
5–3–7	0.45	0.70	406	726	0.23	0.14	142	67
8–4–7	0.37	0.11	68	13	0.28	0.07	93	13
9–4–8	0.87	0.13	95	17	0.60	0.16	139	18

Table 1.2: Social Golfers: comparing selection functions in parallel

same current cost.

Algorithm 3: Solver for SGP to scape from local minima

```
abstract solver as_eager                          // ITR → number of iterations
computation : I, V, S1, S2, A                     // SCI → number of iterations with the same cost
begin
    [(↺) (ITR < K1) I (↦) [(↺) (ITR % K2) [V (↦) [S1 (?)SCI<K3 S2] (↦) A] ] ]
end
solver SOLVER_best implements as_eager
    computation : I_BP, V_std, V_BAS, S_best, S_rand, A_AI
solver SOLVER_first implements as_eager
    computation : I_BP, V_std, V_BAS, S_first, S_rand, A_AI
```

Instance	T	T(sd)	It.	It.(sd)
5–3–7	1.25	1.05	2,907	2,414
8–4–7	0.60	0.33	338	171
9–4–8	1.04	0.72	346	193

Table 1.3: Social Golfers: a single sequential solver using first improvement

Tables 1.2 and 1.3 present results of this experiment, showing that a local exploration-oriented strategy is better for the SGP. If we compare results of Tables 1.2 column *First improvement* and 1.3 with respect to the standard deviation, we see some gains in robustness with parallelism, using the same ratio $\frac{T(sd)}{T}$. The reason is simple: launching a solver set of 40 independent solvers in parallel, is equivalent to launch 40 times a sequential solver and keep the best result. The spread in the running times and iterations for the instance 5–3–7 is 24% lower (0.84 sequentially versus 0.60 in parallel), for 8–4–7 is 30% lower (0.55 sequentially versus 0.25 in parallel) and for 9–4–8 (the hardest one) is 43% lower (0.69 sequentially versus 0.26 in parallel).

***

The conclusion of the last experiment was that the fastest solver to solve SGP using POSL is the one using a neighborhood computation module based on *Adaptive Search* algorithm (V_{BAS}) and a selection computation module selecting the first configuration improving the cost. Using this solver as a base, the next step was to design a simple communication strategy

Algorithm 4: Communicating abstract solver for SGP (sender)

```
abstract solver as_eager_sender                          // ITR → number of iterations
computation : I, V, S1, S2, A                            // SCI → number of iterations with the same cost
begin
    [↻ (ITR < K1)  I ↦ [↻ (ITR % K2)  [V ↦ [S1 ?SCI<K3 S2] ↦ ⟦A⟧^o]  ]  ]
end
solver SOLVER_sender implements as_eager_sender
    computation : I_BP, V_BAS, S_first, S_rand, A_AI
```

Algorithm 5: Communicating abstract solver for SGP (receiver)

```
abstract solver as_eager_receiver                        // ITR → number of iterations
computation : I, V, S1, S2, A                            // SCI → number of iterations with the same cost
communication : C.M.
begin
    [↻ (ITR < K1)
        I ↦ [↻ (ITR % K2)  V ↦ [S1 ?SCI<K3 S2] ↦ [A (m) C.M.]  ]
    ]
end
solver SOLVER_receiver implements as_eager_receiver
    computation : I_BP, V_BAS, S_first, S_rand, A_AI
    communication : CM_last
```

where the shared information is the current configuration. Algorithms 4 and 5 show that the communication is performed while applying the acceptance criterion of the new configuration for the next iteration. Here, receiver solvers receive a configuration from a sender solver, match it with their current configuration, and keep the configuration with the lowest global cost. This operation is coded using the *minimum* operator (m) in Algorithm 5. This way, the receiver solver continues the search from a more promising place into the search space. Different communication strategies were designed, either executing a full connected solvers set, or a tuned combination of connected and unconnected solvers. Between connected solvers, two different connections operations were applied: connecting each sender solver with one receiver solver (one to one), or connecting each sender solver with all receiver solvers (one to N). The code for the different communication strategies are presented in Algorithms 6 to 11. In these algorithms, integer values are related to the syntactic sugar explained in Chapter **??** to declare set of solvers easily and fast.

Algorithm 6: Communication strategy one to one 100%

```
[SOLVER_sender • A] |→| [SOLVER_receiver • C.M.] 20;
```

In Algorithm 5, the abstract communication module $C.M.$ was instantiated with the concrete communication module CM_{last}, which takes into account the last received configuration at the time of its execution.

Algorithm 7: Communication strategy one to N 100%

$[\text{SOLVER}_{sender} \bullet A(20)] \boxed{\rightsquigarrow} [\text{SOLVER}_{receiver} \bullet C.M.(20)]$;

Algorithm 8: Communication strategy one to one 50%

$[\text{SOLVER}_{sender} \bullet A] \boxed{\rightarrow} [\text{SOLVER}_{receiver} \bullet C.M.]$ 10;
$[\text{SOLVER}_{first}]$ 20;

Each time a POSL meta-solver is launched, many independent search solvers are executed. We call "good" configuration a configuration with a cost strictly lesser than the current one. Once a good configuration is found in a sender solver, it is transmitted to a receiver. At this moment, if the information is accepted by the receiver, there are some solvers searching in the same subset of the search space (i.e. they continue the search from the same configuration), and the search process becomes more exploitation–oriented. This can be problematic if this process makes solvers converging too often towards local minima. In that case, we waste more than one solver trapped into a local minima: we waste all solvers that have been attracted to this part of the search space because of communications. This phenomenon is avoided through a simple (but effective) play: if a solver is not able to find a better configuration inside the neighborhood (executing S_{first}), it selects a random one at the next iteration (executing S_{rand}).

After the selection of the proper modules to study different communication strategies, I proceeded to tune parameters K_1, K_2 and K_3. Only a few runs were necessaries to conclude that the mechanism of using the computation module S_{rand} to escape from local minima was enough. For that reason, since the solver never perform restarts, the parameter K_1 was irrelevant. So the reader can assume $K_1 = 1$ for every experiment. With the certainty that solvers do not perform restarts during the search process, I selected the same value for $K_2 = 5000$ in order to be able to use the same abstract solver for all instances. This value was selected after an experimental estimation of the upper bound of the number of iterations for the more complex instance (9–4–8). Finally, in the tuning process of K_3, I notice only slightly differences between using the values 5, 10, and 15. So I decided to use $K_3 = 5$. In this version of POSL, each time that we have to use different values for K_i we have to change the abstract solver. One of the goals for the next version is to include these values as parameters of the solver, in order to be even more modular.

This communication strategy produces some gain in terms of runtime (Table 1.2 with respect to Tables 1.4, 1.5 and 1.6). Having many solvers searching in different places of the search

Algorithm 9: Communication strategy one to N 50%

$[\text{SOLVER}_{sender} \bullet A(10)] \boxed{\rightsquigarrow} [\text{SOLVER}_{receiver} \bullet C.M.(10)]$;
$[\text{SOLVER}_{first}]$ 20;

Algorithm 10: Communication strategy one to one 25%

$[\text{SOLVER}_{sender} \bullet A] \boxed{\rightarrow} [\text{SOLVER}_{receiver} \bullet C.M.]\, 5;$
$[\text{SOLVER}_{first}]\, 30;$

Algorithm 11: Communication strategy one to N 25%

$[\text{SOLVER}_{sender} \bullet A(5)] \boxed{\rightsquigarrow} [\text{SOLVER}_{receiver} \bullet C.M.(5)]\,;$
$[\text{SOLVER}_{first}]\, 30;$

space, the probability that one of them reaches a promising place is higher. Then, when a solver finds a good configuration, it can be communicated, and receiving the help of one or more solvers in order to find the solution. Using this strategy, the spread ratio in the running times and iterations calculated from values in Table 1.2 was reduced for the instance 9–4–8 (0.22 using communication one to one and 50% of communication solvers, versus 0.26), but not for instances 5–3–7 and 8–4–7 (0.70 using communication one to N and 50% of communicating solvers versus 0.60, and 0.28 using communication one to one and 50% of communicating solvers versus 0.25, respectively).

Another communication strategy was analyzed in the resolution of this problem, with no success, based on the sub-division of the work by weeks, i.e. solvers trying to improve a configuration only working with one week. I called *circular strategy* and it consists in having K solvers trying to improve a configuration during a given number of iteration, only working on one week. When no improvement is obtained, the current configuration is communicated to the next solver (circularly), which tries to do the same working on the next week (see Figure 1.1). This strategy did not show better results than previews strategies: more than two time worse than sequential results, for every instance. The reason is because, although the communication in POSL is asynchronous, most of the times solvers were trapped "waiting" for a configuration coming from its neighbor solver.

***

One last experiment using this benchmark was implementing a communication strategy which applies a mechanism of cost descending acceleration, exchanging the current configuration

Instance	Communication 1 to 1				Communication 1 to N			
	T	T(sd)	It.	It.(sd)	T	T(sd)	It.	It.(sd)
5–3–7	0.20	0.20	165	110	0.20	0.17	144	108
8–4–7	0.27	0.09	88	28	0.24	0.05	95	12
9–4–8	0.52	0.14	117	25	0.55	0.14	126	20

Table 1.4: Social Golfers: 100% of communicating solvers

Instance	Communication 1 to 1				Communication 1 to N			
	T	T(sd)	It.	It.(sd)	T	T(sd)	It.	It.(sd)
5–3–7	0.18	0.13	125	88	0.17	0.12	139	81
8–4–7	0.21	0.06	89	18	0.22	0.06	90	20
9–4–8	0.49	0.11	119	24	0.51	0.15	124	21

Table 1.5: Social Golfers: 50% of communicating solvers

Instance	Communication 1 to 1				Communication 1 to N			
	T	T(sd)	It.	It.(sd)	T	T(sd)	It.	It.(sd)
5–3–7	0.22	0.20	181	130	0.23	0.16	143	80
8–4–7	0.24	0.08	95	22	0.29	0.09	93	12
9–4–8	0.55	0.14	134	21	0.55	0.11	130	20

Table 1.6: Social Golfers: 25% of communicating solvers

between two solvers with different characteristics. Results show that this communication strategy works pretty well for this problem.

For this strategy, new solvers were built reusing same modules used for the communication strategies exposed before, and another different neighborhood computation module: $V_{BP}(w)$, which given a configuration, returns the neighborhood $\mathcal{V}(s)$ by swapping the culprit player chosen for all w randomly selected weeks with other players in the same week. This new solver was called *companion solver*, and it descends quicker the cost of its current solution at the beginning because its neighborhood generates less values, but the convergence is slower and yet not certain. It works toguether with a similar solver used for the communication strategy exposed before. It is called *standard solver*, and converges in a stable way to the solution. So, the companion solver uses the same neighborhood function that the standard solver, but is parametrized in such a way that it builds neighbors only swapping players among two weeks.

The idea of the communication strategy is to communicate a configuration from the companion solver to the standard solver, in order to be able to continue the search from a more promising place into the search space. After some iterations (depending on the instance), the standard solver sends its configuration to the companion solver. The companion solver takes this received configuration and starts its search from there and finds quickly a much better configuration to send back to the standard solver again. To force the companion solver to take the received configurations over its own, we use the *not null* operator together with the communication module $C.M.$ (Algorithm 13). This process is repeated until a solution is found.

Figure 1.2 shows a single standard solver's run versus a single companion solver's run. In this chart we can see that, at the beginning of the run, found configurations by the companion solver have costs significantly lower than those found by the standard solver. At the 60-th

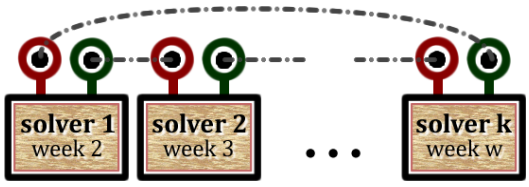

Figure 1.1: Unsuccessful communication strategies to solve SGP

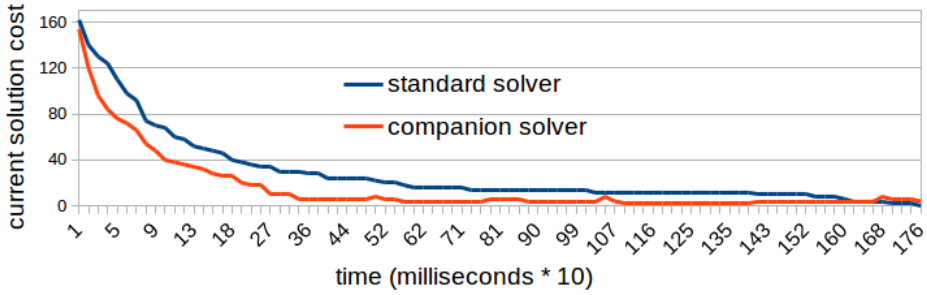

Figure 1.2: Companion solver vs. standard solver (solving Social Golfers Problem)

millisecond the standard solver current configuration has cost 123, and the companion solver's one, 76. So for example, the communication at this time, can accelerate the process significantly.

Algorithm 12: Standard solver for SGP

abstract solver *as_standard*
computation : I, V, S_1, S_2, A
communication : $C.M.$
begin
$I \mapsto [\circlearrowleft (\text{Itr} < K_1)\ \ V \mapsto [S_1\ ?_{\text{Sci}\%K_2}\ S_1] \mapsto [C.M.\ m\ (\!|A|\!)^d]\ ]$
end
solver $\text{Solver}_{standard}$ **implements** as_standard
computation : $I_{BP}, V_{BAS}, S_{first}, S_{rand}, A_{AI}$
communication : CM_{last}

Algorithm 13: Companion solver for SGP

abstract solver *as_companion*
computation : I, V, S_1, S_2, A
communication : $C.M.$
begin
$I \mapsto [\circlearrowleft (\text{Itr} < K_1)\ \ V \mapsto [S_1\ ?_{\text{Sci}\%K_2}\ S_2] \mapsto [C.M.\ \vee\ (\!|A|\!)^d]\ ]$
end
solver $\text{Solver}_{companion}$ **implements** as_companion
computation : $I_{BP}, V_{BP}(2), S_{first}, S_{rand}, A_{AI}$
communication : CM_{last}

We also design different communication strategies, combining connected and unconnected solvers in different percentages, and applying two different communication operators: one to one and one to N. The code for the communication strategy of 100% of communicating solvers is presented in Algorithm 14 and for 50% of communicating solvers in

Algorithm 15.

The main idea of this strategy follows the following steps:

Step 1

The computation module I_{BP} generates an initial configuration s respecting the structure of the problem's model, i.e. the configuration s is a set of w permutations of the vector $[1..P]$, where $P = g \times p$. The configuration s is the input of the computation module in the Step 2.

Step 2

The computation module $V_{BP}(w)$ takes a configuration s as input and selects a set of w weeks randomly. Then it returns the neighborhood $\mathcal{V}(s)$ by swapping the most culprit player with other players in the same week, for all selected weeks.

Step 3

In this step a compound module which combines the computation modules S_{first} and S_{rand} through the operator $(?)_{\text{SCI}\%K_2}$ is executed. The operator takes the input of the Step 2 and executes either S_{first}, which selects the first configuration $s' \in V(s)$ improving the current cost, if the current cost have been the same during the last K_2 iterations, at the time of executing the operator; or S_{rand}, which selects randomly a configuration $s' \in V(s)$, otherwise. This compound module returns the configuration s' together with the current one, into the pair (s', s).

Step 4

Standard solver: In this step a compound module which combines the compound module of the send data operator returning the output of A_{AI} and the communication module CM_{last} through the operator (m) is executed. The minimum operator takes the input of the Step 3 and executes A_{AI} and returns its output s' (and sends s' to external solvers); and executes CM_{last}, which returns the received configuration s''. The compound module returns the configuration with the lowest cost between s' and s'', then the process returns to the Step 2.

Companion solver: In this step a compound module which combines the compound module of the send data operator returning the output of A_{AI} and the communication module CM_{last} through the operator $(\vee)$ is executed. The not null operator takes the input of the Step 3 and executes A_{AI} and returns the selected configuration s'(and sends s' to external solvers); and CM_{last}, which returns the received configuration s''. The compound module returns either the configuration s' if the received configuration s'' is NULL, or s'' otherwise. Then, the process returns to the Step 2.

This communication strategy produces some gain in terms of runtime as we can see in Tables 1.7 and 1.8 with respect to Table 1.2. It produces also more robust results in terms of runtime. The spread of results in iterations show higher variances, because there are

Algorithm 14: Companion communication strategy 100% communication

$[\text{SOLVER}_{companion} \bullet A] \boxed{\rightarrow} [\text{SOLVER}_{standard} \bullet C.M.]\ 20;$
$[\text{SOLVER}_{standard} \bullet A] \boxed{\rightarrow} [\text{SOLVER}_{companion} \bullet C.M.]\ 20;$

Algorithm 15: Companion communication strategy 50% communication

$[\text{SOLVER}_{companion} \bullet A] \boxed{\rightarrow} [\text{SOLVER}_{standard} \bullet C.M.]\ 10;$
$[\text{SOLVER}_{standard} \bullet A] \boxed{\rightarrow} [\text{SOLVER}_{companion} \bullet C.M.]\ 10;$
$[\text{SOLVER}_{first}]\ 20;$

included also results of companion solvers, which performs many times more iterations that the standard solvers. The percentage of the receiver solvers that were able to find the solution before the others did, was significant: the 73% of the receiver solvers, as a mean among the three instances, were the winners (see Appendix **??**, Figures **??**, **??** and **??**), showing that the communication played an important role during the search, despite inter–process communication's overheads (reception, information interpretation, making decisions, etc).

1.3 A cyclic communication strategy (N-Queens)

In this section N-Queens Problem (NQP) is selected as a benchmark to study a communication strategy consisting in exchanging cyclically the configuration between solvers using different neighborhood functions, in order to accelerate the process of generating very promising configurations. Final obtained results show that this communication strategy works well for some instances of this problem.

1.3.1 Problem definition

The N-Queens Problem (NQP) asks how to place N queens on a chess board so that none of them can hit any other queen in one move. This problem was introduced in 1848 by the chess player Max Bezzelas as the *8-queen problem*, and years latter it was generalized as

Instance	Comm. one to N				(Comm. one to N)/2				(Comm. one to N)/4			
	T	T(sd)	It.	It.(sd)	T	T(sd)	It.	It.(sd)	T	T(sd)	It.	It.(sd)
5–3–7	0.14	0.08	102	53	0.14	0.07	97	73	0.12	0.08	175	162
8–4–7	0.30	0.13	101	24	0.22	0.06	92	29	0.22	0.06	88	45
9–4–8	0.55	0.15	125	20	0.53	0.14	107	20	0.40	0.14	101	70

Table 1.7: Companion communication strategy with communication one to N

Instance	Comm. one to one (100%)				Comm. one to one (50%)				Comm. one to one (25%)			
	T	T(sd)	It.	It.(sd)	T	T(sd)	It.	It.(sd)	T	T(sd)	It.	It.(sd)
5–3–7	0.10	0.05	98	75	**0.08**	0.04	139	122	0.11	0.05	190	142
8–4–7	**0.14**	0.05	100	64	0.22	0.06	119	74	0.21	0.5	101	64
9–4–8	0.37	0.14	86	65	**0.36**	0.12	144	92	0.45	0.11	150	96

Table 1.8: Companion communication strategy with communication one to one

N-queen problem by Franz Nauck. Since then many mathematicians, including Gauss, have worked on this problem. It can be directly applied to diverse fields, such as parallel memory storage schemes, traffic control, deadlock prevention, neural networks, constraint satisfaction problems, among others [128]. Some studies suggest that the number of solution grows exponentially with the number of queens (N), but local search methods have been shown very good results for this problem [129]. For that reason we tested some communication strategies using POSL, to solve a problem relatively easy to solve using non communication strategies.

Benchmarks in this chapter were also modeled as unconstrained optimization problems. The proposed model for NQP has N variables: $\{v_1, v_2, \ldots, v_N\}$. Their domains are the same: $D_{v_i} = \{1, \ldots, N\}$.

The cost function for this benchmark was implemented in C++ based on the current implementation of Adaptive Search[ii]. It assumes that a configuration s for this problem is an integer permutation of the set $\{1...N\}$, i.e. $s_i = j$ (the i^{th} variable has the value j) means that there is a queen placed in column i and row j. In that sense, the cost function does not verify whether values in s are all different.

Assuming this structure in the configuration, the cost function only has to check whether there are diagonal *collisions* between queens. To do that two vectors Err^{d1} and Err^{d2} are created of size $2 \cdot N - 1$ (the number of diagonals in a $N \times N$ matrix) to store the number of queens placed on each diagonal. So, $Err^{d1}_{i+N-1-j}$ contains the number of queens on such diagonal, taking into account that j is the position (row) of the queen placed on the column i, for all $i \in [1...N]$. Analogously, Err^{d2}_{i+j} contains the number of queens on such diagonal (on the other direction). Based on this structure, the cost c_s of a configuration is:

$$c_s = \sum_{d=1}^{2N-1} \mathcal{F}(Err^{d1}_d) + \mathcal{F}(Err^{d2}_d) \tag{1.2}$$

where:

$$\mathcal{F}(x) = \begin{cases} 0 & x \leq 1 \\ \text{x} & \text{otherwise} \end{cases}$$

[ii] It is based on the code from Daniel Díaz available at https://sourceforge.net/projects/adaptivesearch/

1.3.2 Experiments design and results

To handle this problem, some modules used for the Social Golfers Problem have been reused: the selection computation modules S_{first} and S_{best}, and the acceptance computation module A_{AI}. It uses a simple abstract solver presented in Algorithm 16. For this problem, POSL does not perform restarts, so the value K_1 was fixed in 15000 in order to be able to use the same abstract solver for all instances.

Algorithm 16: Abstract solver for NQP

```
abstract solver as_simple                          // ITR → number of iterations
computation : I, V, S, A
begin
    I (↦) [(↻) (ITR < K1) V (↦) S (↦) A ]
end
solver SOLVER_as implements as_simple
    computation : I_perm, V_AS, S_first, A_AI
solver SOLVER_selective implements as_simple
    computation : I_perm, V_PAS(m), S_first, A_AI
```

Solvers used for the experiments without communications are presented in Algorithm 16, where the abstract solver is instantiated by the solver SOLVER_{as} with the neighborhood computation module V_{AS}. Given a configuration, this module returns a neighborhood $V(s)$ swapping the variable which contributes the most to the cost, with all others. This solver was able to find solutions but taking too much time (a minute for 6000-queens, for example). For that reason a neighborhood computation module $S_{PAS}(m)$ has been implemented similar to S_{AS} but instead of generating neighbors swapping the most culprit variable with all others, it is swapped only with a percentage m of them. Solver $\text{SOLVER}_{selective}$ instantiates the abstract solver with this computation module, showing much better results than SOLVER_{as}.

Table 1.9 presents results of sequential and parallel runs, using $\text{SOLVER}_{selective}$ with a tuned value of $m = 2.5$. Results show that the improvement of the parallel scheme using POSL is not significant. While the number of solutions of this problem is only known for the very small value of $N = 27$, studies suggest that the number of solutions grows significantly with N, i.e. the number of new solutions is about 10 times bigger. It means that the complexity of the problem grows with its order, but not excessively. However, the number of new possible configurations is N times bigger. It implies that as the problem grows in order, solutions inside the search space are farer away from each other. So, in a search space much bigger and with scattered solutions, the probability of starting the search close to a solution decreases, and, as we can deduce from results, it does not increase considerably when

Instance	Sequential				Parallel			
	T	T(sd)	It.	It.(sd)	T	T(sd)	It.	It.(sd)
250	0.29	0.072	8,898	2,158	0.19	0.003	4,139	913
500	0.35	0.087	4,203	998	0.24	0.036	2,675	366
1000	0.35	0.126	2,766	445	0.30	0.037	2,102	222
3000	1.50	0.138	2,191	77	1.33	0.055	2,168	71
6000	4.71	0.183	3,339	51	4.57	0.123	3,323	43

Table 1.9: Results for NQP (sequential and parallel without communication)

applying the parallel approach. This explains also the decrease in the standard deviation when N increases: all solvers start the search from configurations "similarly far" from the solutions, so their courses tent to be similar.

***

In order to test the cooperative approach with this problem, a first and simple experiment was performed. Using the previous defined abstract solver, communicating solvers were built to create a simple communication strategy in which the shared information is the current configuration, and is communicated in one direction (from sender solvers to receivers). Algorithms 17 and 18 show that the communication is performed while applying the acceptance criterion. We design different communication strategies:

- a set of sender solvers sending information to receiver solvers, using operator one to one (see see Algorithm 19) and operator one to N (see Algorithm 20 with $K = 1$)
- some sets of sender solvers sending information to receiver solvers, using operator one to N (see see Algorithm 20), with $K \in \{2, 4\}$

Algorithm 17: Sender solver for NQP (simple communication strategy)

abstract solver *as_sender* // ITR → number of iterations
computation : I, V, S, A
begin
$I \ (\mapsto) \ [(\circlearrowleft) \ (\text{ITR} < K_1) \ V \ (\mapsto) \ S \ (\mapsto) \ (\!|A|\!)^d \ ]$
end
solver SOLVER$_{sender}$ **implements** as_sender
computation : $I_{perm}, V_{PAS}(m), S_{first}, A_{AI}$

Algorithm 19: Simple communication strategy one to one for NQP

$[\text{SOLVER}_{sender} \bullet A] \ \boxed{\rightarrow} \ [\text{SOLVER}_{receiver} \bullet C.M.] \ 20;$

Algorithm 20: Simple communication strategy one to N for NQP

$\left[\text{SOLVER}_{sender} \bullet A(\frac{20}{K})\right] \ \boxed{\rightsquigarrow} \ \left[\text{SOLVER}_{receiver} \bullet C.M.(\frac{20}{K})\right] \ K;$

Algorithm 18: Receiver solver for NQP (simple communication strategy)

abstract solver *as_receiver* // ITR → number of iterations
computation *: I, V, S, A*
communication *: C.M.*
begin
$I \ (\mapsto) \ [(\circlearrowleft) \ (\text{ITR} < K_1) \ V \ (\mapsto) \ S \ (\mapsto) \ [A \ (?)_{\text{ITR}\%K_2} \ [A \ (m) \ C.M.]] \ ]$
end
solver SOLVER$_{receiver}$ **implements** as_receiver
computation : $I_{perm}, V_{PAS}(m), S_{first}, A_{AI}$
communication: CM_{last}

Instance	Communication 1-1			
	T	T(sd)	It.	It.(sd)
250	0.18	0.040	3,433	697
500	0.25	0.047	2,216	427
1000	0.26	0.056	1,735	424
3000	1.21	0.088	1,873	227
6000	4.38	0.111	3,178	121

Table 1.10: Simple communication strategy one to one for NQP

Tables 1.10 and 1.11 show that the communication improvement with respect to non communicating results in terms of runtime and iterations was not significant. In contrast to SGP, POSL does not get trapped so often into local minima during the resolution of NQP. For that reason, the shared information, once received and accepted by the receivers solvers, does not improve largely the current cost.

***

In the following experiment, another communication strategy was implemented. Solvers into the solver set use the same neighborhood computation module $V_{PAS}(m)$ but with different values of m, and different selection functions. In this communication strategy solvers are both senders and receivers (see Algorithm 21), so a cyclic exchange of the current configuration is performed between these two different solvers. A set of solvers, using the neighborhood computation module $V_{PAS}(m)$ with a small value of m and using the selection computation module S_{best}, have the role of *companion* solvers, i.e. the small value of m provides to these solvers a slow convergence, but given a configuration with high cost, they are able to obtain another with smaller cost very quick. Other set of solvers is composed by standard solvers, i.e. solver with a structure very similar to the one used for non communicating experiments: the same value of m and the selection computation module is S_{best}.

During the design process of the communication strategies (Algorithms 22, and 23), many experiments were launched to select 1. the percentage of variables that the companion solver

Instance	Communication 1-n				Communication (1-n)×2				Communication (1-n)×4			
	T	T(sd)	It.	It.(sd)	T	T(sd)	It.	It.(sd)	T	T(sd)	It.	It.(sd)
250	0.16	0.032	2,621	894	0.15	0.036	2,459	892	0.15	0.036	2,494	547
500	0.20	0.040	1,592	428	0.19	0.053	1,521	539	0.18	0.057	1,719	593
1000	0.26	0.055	1,329	286	0.25	0.046	1,435	369	0.23	0. 056	1,400	426
3000	1.26	0.078	1,657	212	1.22	0.101	1,598	249	1.20	0.078	1,704	252
6000	4.40	0.118	2,771	197	4.35	0.127	2,840	148	4.33	0.120	2,975	188

Table 1.11: Simple communication strategy one to N in one, two and four groups, for NQP

swaps with the culprit one, when executing the neighborhood computation module (m, it was decided to be 1), 2. the number of companion solvers to connect with the standard one, for the communication strategy using operator one to N (it was decided to be 2 as we can see in Algorithm 23), and 3. the frequency of the communication, i.e. the value K_2 in Algorithm 21 (it was decided to be 5).

Algorithm 21: Solvers for cyclic communication strategy to solve NQP

abstract solver *as_cyc* // ITR → number of iterations
computation : I, V, S, A
communication : $C.M.$
begin
$I \ (\mapsto) \ [(\circlearrowleft) \ (\text{ITR} < K_1) \ V \ (\mapsto) \ S \ (\mapsto) \ \left[A \ (?)_{\text{ITR}\%K_2} \ \left[(\!|A|\!)^d \ (m) \ C.M.\right]\right] \ ]$
end
solver SOLVER$_{standard}$ **implements** as_cyc
computation : $I_{perm}, V_{PAS}(2.5), S_{first}, A_{AI}$
communication: CM_{last}
solver SOLVER$_{companion}$ **implements** as_cyc
computation : $I_{perm}, V_{PAS}(1), S_{best}, A_{AI}$
communication: CM_{last}

Algorithm 22: Cyclic communication strategy one to one for NQP

$[\text{SOLVER}_{companion} \bullet A] \boxed{\rightarrow} [\text{SOLVER}_{standard} \bullet C.M.] \ 20;$
$[\text{SOLVER}_{standard} \bullet A] \boxed{\rightarrow} [\text{SOLVER}_{companion} \bullet C.M.] \ 20;$

Algorithm 23: Cyclic communication strategy one to N for NQP

$[\text{SOLVER}_{companion} \bullet A(2)] \boxed{\rightsquigarrow} [\text{SOLVER}_{standard} \bullet C.M.] \ 13;$
$[\text{SOLVER}_{standard} \bullet A] \boxed{\rightsquigarrow} [\text{SOLVER}_{companion} \bullet C.M.(2)] \ 13;$

The main idea of the proposed communication strategy follows the following steps:

Step 1

The computation module I_{perm} generates an initial configuration s respecting the structure of the problem's model, i.e. the configuration s is a permutation of the vector $[1..N]$. The configuration s is the input of the computation module in the Step 2.

Step 2

The computation module $V_{PAS}(m)$ takes a configuration s as input and returns the neighborhood $\mathcal{V}(s)$ generated by swapping the most culprit variable with a percentage m of all other variables.

Step 3

The computation module S_{first} selects the first configuration $s' \in V(s)$ improving the current cost, and returns it together with the current configuration, into the pair (s', s).

Step 4

This step executes a compound module which combines the computation module A_{AI} and a compound module in charge of the communication, through the operator $(?)_{\text{SCI}\%K_2}$. The operator takes the input of the Step 3 and executes either A_{AI}, which always returns the selected configuration s', if the current cost have been the same during the last K_2 iterations, at the time of executing the operator; or the compound module in charge of the communication, otherwise. The compound module in charge of the communication is the combination of a send data operator, which sends the output of the computation module A_{AI}, and the communication module CM_{last}, which returns the received configuration. Both modules are joined through an operator (m), which returns the configuration with the lowest cost. Then, the process returns to the Step 2.

Instance	Communication 1-1				Communication 1-n			
	T	T(sd)	It.	It.(sd)	T	T(sd)	It.	It.(sd)
250	**0.09**	0.021	1,169	254	0.10	0.021	1,224	254
500	**0.14**	0.027	864	121	0.15	0.030	977	220
1000	0.22	0.041	889	247	**0.21**	0.056	807	196
3000	1.25	0.090	1,602	90	**1.02**	0.145	1,613	206
6000	4.83	0.121	2,938	746	**4.24**	0.746	2,537	779

Table 1.12: Cyclic communication strategy for NQP

	Instances				
	250	500	1000	3000	6000
I.R	2.00	1.65	1.39	1.17	1.01
P.E.C.	77	67	67	60	57

Table 1.13: Improvement rate and percentage of winner receiver solvers for NQP

With this experiment, it was possible to find a communication strategy which improves runtimes significantly, but only for small instances of the problem. Results in Table 1.12 confirm experimentally the hypothesis already introduced, which propose the higher the size of the problem is, (and hence, the number ef solutions inside the search space with respect to N) the lower the gain using communication during the search process is.

Table 1.13 shows how the *improvement ratio* (column **I.R.**) decreases with the instance order

N. This ratio was computed using the following equation:

$$\textbf{I.R.} = \frac{\text{runtime without communication}}{\frac{(\text{runtime using communication 1-1}+\text{runtime using communication 1-n})}{2}}$$

For all instances, the POSL's behavior solving this problem is the same: the current configuration's cost descrives a steady dcline, until the search get trapped into an *apparent* local minimum, hard to escape from. At this point, the communication takes place, providing an intensification factor, decisive to escaping from the apparent local minimum: the configuration of the trapped (standard) solver is sent to all companion solvers and they provide an improved configuration fast enough back to the standard solver. The difference between behaviors of different instances is the following: the smaller the instance is, the faster the standard solver gets trapped into this *apparent* local minimum. It implies that the communication takes place earlier and then it is more effective.

The percentage of the receiver solvers that were able to find the solution before the others did, was significant. It endorses hypotesis of the improvement thanks to the communication, as we can see in Table 1.13 (see also Appendix **??**, Figures **??** and **??**). Furthermore, the row **P.E.C** (*Percentage of Effective Communication*) shows how this percentage decreases together with N.

1.4 A simple communication strategy (Costas Array)

In this section I present a performed study using Costas Array Problem (CAP) as a benchmark, by testing a simple communication strategy, in which the information to communicate between solvers is the current configuration received at different times of the algorithm. Results show that for this problem, this strategy improve the parallel non cooperative approach.

1.4.1 Problem definition

The Costas Array Problem (CAP) consists in finding a *Costas array*, which is an $n \times n$ grid containing n marks such that there is exactly one mark per row and per column and the $n(n-1)/2$ vectors joining each couple of marks are all different. This is a very complex problem that finds useful application in some fields like sonar and radar engineering.

CAP has been studied for several years, and yet many questions remain open, like for example the existence of solutions for all n, the existence of other construction algorithms, among others [130]. It also presents an interesting characteristic: although the search space grows factorially, from order 17 the number of solutions drastically decreases [131]. For example, using the Golomb [132] and Welch [133] constructions, Drakakis et al. present in [134] all Costas arrays for $n = 29$, and they show that among the 29! permutations, there are only 164 Costas arrays. Nowadays, after many decades of research, the problem of knowing whether it exists any Costas array for $n = 32$ remains open [47].

For modeling this benchmark as an unconstrained optimization problem, the proposed model has N variables: $\{v_1, v_2, \ldots, v_n\}$, and their domains are the same: $D_{v_i} = \{1, \ldots, n\}$.

The cost function for CAP was implemented in C++ based on the current implementation of Adaptive Search[iii]. It assumes that a configuration s is an integer permutation of the set $\{1...n\}$, i.e. $s_i = j$ (the i^{th} variable has the value j) means that there is a mark placed in column i and row j. In that sense, the cost function does not verify whether values in s are all different.

The cost is computed by building the triangular matrix of all *diferences* between marks. A difference between two marks m_i and m_j is defined as the vertical shift of m_j with respect to m_i. For example, if the mark m_i (mark placed on column i) is placed on the row 4, and the mark m_j (mark placed on column j) is placed on the row 3, the difference between them is $4 - 3 = 1$. The following example describe a differences triangle for a 5×5 matrix:

Figure 1.3 shows a 5×5 matrix with marks. The corresponding configuration for the Costas Array Problem is the following:

$$s = \{3, 2, 4, 5, 1\}$$

Its differences triangle is defined as follows:

$$\begin{array}{ccccc} 3 & 2 & 4 & 5 & 1 \\ 1 & -2 & -1 & 4 & \\ -1 & -3 & 3 & & \\ -2 & 1 & & & \\ 2 & & & & \end{array} \begin{array}{c} \rightarrow \\ \rightarrow \\ \rightarrow \\ \rightarrow \\ \rightarrow \end{array} \begin{array}{l} d_0 = s \\ d_1 \\ d_2 \\ d_3 \\ d_4 \end{array}$$

In this triangular matrix, row i is denoted by d_i, which represent the difference between elements of s located between each other at a distance i. For example, d_3 are the differences of

[iii] It is based on the code from Daniel Díaz available at https://sourceforge.net/projects/adaptivesearch/

values from s located each other at distance 3, so

$$d_3 = \{s_0 - s_3, s_1 - s_4\}$$
$$d_3 = \{3 - 5, 2 - 1\}$$
$$d_3 = \{-2, 1\}$$

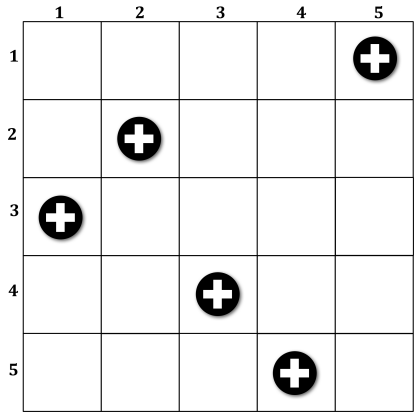

Figure 1.3: Example of marks for CAP

The cost of the configuration s is then the number of equal elements on each row d_i of its corresponding differences triangle. It can be easily proved that this differences triangle is only necessary to be verified until the row d_B, with $B = \lfloor (n-1)/2 \rfloor$.

1.4.2 Experiments design and results

To handle this problem, I have reused all modules used for solving the N-Queens Problem. First attempts to solve this problems were using the same strategies (abstract solvers) used to solve the Social Golfers Problem and N-Queens Problem, without success: POSL was not able to solve instances larger than $n = 8$ in a reasonable amount of time (seconds). After many unsuccessful attempts to find the right parameters of maximum number of restarts, maximum number of iterations, and maximum number of iterations with the same cost, I decided to implement the mechanism used by Daniel Díaz in the current implementation of *Adaptive Search* to escape from local minima: I have added a *Reset* computation module R_{AS} based on the abstract computation module R. The other computation modules were the same used for solving the N-Queens Problem.

Given a configuration s, the basic principle of the reset is build another following four steps:

1. A configuration is obtained by performing left/right shifts to all sub-vectors of s starting or ending by the variable which contributes the most to the cost, and selecting the configuration with the lowest cost.

2. A configurations is obtained by adding a constant (circularly) to each element in the configuration s.

3. A configuration is obtained by shifting left from the beginning of s to some culprit variable (i.e. a variable contributing to the cost).

4. Then, one of these 3 generated configuration has the same probability of being selected, to be the result of the reset algorithm. In that sense, some different resets can be performed for the same configuration.

The basic solver used to solve this problem is presented in Algorithm 24, and it was taken as a base to build all the different communication strategies. Basically, it is a classical local search iteration, where instead of performing restarts, it performs resets. After a deep analysis of this implementation and results of some runs, I decided to use $K_1 = 2,000,000$ (maximum number of iterations) big enough to solve the chosen instance $n = 19$; and $K_2 = 3$ (the number of iteration before performing the next *reset*).

Algorithm 24: Reset-based abstract solver for CAP

```
abstract solver as_hard                          // Itr → number of iterations
computation : I, R, V, S, A
begin
    I ↦ [↺ (Itr < K1)  R ↦ [↺ (Itr % K2) [V ↦ S ↦ A] ] ]
end
solver Solver_single implements as_hard
    computation : I_perm, R_AS, V_AS, S_first, A_AI
```

Table 1.14 shows results of launching solver sets to solve each instance of Costas Array Problem 19 sequentially and in parallel without communication. Runtimes and iteration means showed in this confirm once again the success of the parallel approach.

STRATEGY	T	T(ds)	It.	It.(sd)	% success
Sequential (1 core)	132.73	80.32	2,332,088	1,424,757	40.00
Parallel (40 cores)	25.51	15.75	231,262	143,789	100.00

Table 1.14: Costas Array 19: no communication

***

In order to improve results, a simple communication strategy was applied: communicating the current configuration to other solvers. To do so, we insert a *sending output* operator to the abstract solver in Algorithm 24. This results in the sender solver presented in Algorithm 25.

Algorithm 25: Sender solver for CAP

abstract solver *as_hard_sender*
computation : I, R, V, S, A
begin
$I \ (\mapsto) \ [(\circlearrowleft) \ (\text{ITR} < K_1) \ \ R \ (\mapsto) \ [(\circlearrowleft) \ (\text{ITR} \ \mathbf{\%} \ K_2) \ \left[V \ (\mapsto) \ S \ (\mapsto) \ (\!|A|\!)^d\right] \ ] \ ]$
end
solver SOLVER$_{sender}$ **implements** as_hard_sender
computation : $I_{perm}, R_{AS}, V_{AS}, S_{first}, A_{AI}$

Studying some runs of POSL solving CAP, it was observed that during the search process, the cost of the current configuration of all solvers describes an oscillatory descent due to the repeated resets ($K_2 = 3$ in Algorithm 24). It means that the cost of the current configuration decreases during K_2 iterations and then the current configuration is created by performing a reset which generates generally a more costly configuration. However, in every performed experiment (more than 20 runs with 20 solvers in parallel without communication), the current configuration's cost of the winner solver (first solver finding a solution) described an oscillatory descent also, but not so pronounced (i.e. the difference between the cost of the last current configuration before performing the reset and the cost of the configuration after the reset is not high). For that reason, it was decided to apply a simple communication strategy that shares the current configuration while applying the acceptance criterion. To do so, a communication module using a *minimum* operator (m) together with the abstract computation module A was inserted, as shown in Algorithm 26.

One of the main purpose of this study is to explore different communication strategies. We have then implemented and tested different variations of the strategy exposed above by combining two communication operators (one to one and one to N) and different percentages of communicating solvers. For this problem, it was study also the behavior of the communication performed at two different moments: while applying the acceptance criteria (Algorithm 26), and while performing a *reset* (Algorithm 27).

Algorithm 26: Receiver solver for CAP (variant A)

abstract solver *as_hard_receiver_a* `// ITR → number of iterations`
computation : I, R, V, S, A
communication : $C.M.$
begin
$I \ (\mapsto) \ [(\circlearrowleft) \ (\text{ITR} < K_1) \ \ R \ (\mapsto) \ [(\circlearrowleft) \ (\text{ITR} \ \mathbf{\%} \ K_2) \ \left[V \ (\mapsto) \ S \ (\mapsto) \ \left[A \ (m) \ C.M.\right]\right] \ ] \ ]$
end
solver SOLVER$_{receiverA}$ **implements** as_hard_receiver_a
computation : $I_{perm}, R_{AS}, V_{AS}, S_{first}, A_{AI}$
communication: CM_{last}

The instantiation for receiver solvers instantiates the abstract communication module $C.M.$ with the concrete communication module CM_{last}, which takes into account the last received

Algorithm 27: Receiver solver for CAP (variant B)

```
abstract solver as_hard_receiver_b                    // ITR → number of iterations
computation : I, R, V, S, A
communication : C.M.
begin
    I (↦) [(↻) (ITR < K1) [R (m) C.M.] (↦) [(↻) (ITR % K2) [V (↦) S (↦) A] ] ]
end
solver SOLVER_receiverB implements as_hard_receiver_b
    computation : I_perm, R_AS, V_AS, S_first, A_AI
    connection: CM_last
```

STRATEGY	100% COMM				50% COMM			
	T	T(sd)	It.	It.(sd)	T	T(sd)	It.	It.(sd)
Str A: 1 to 1	11.60	9.17	84,159	68,958	16.78	13.43	148,222	121,688
Str A: 1 to N	**10.83**	8.72	79,551	63,785	13.03	13.46	106,826	120,894
Str B: 1 to 1	14.84	13.54	119,635	112,085	14.51	13.88	125,982	123,261
Str B: 1 to N	22.99	23.82	199,930	207,851	16.62	15.16	138,840	116,858

Table 1.15: Costas Array 19: with communication

configuration at the time of its execution.

Table 1.15 shows that solver sets executing the strategy A (receiving the configuration at the time of applying the acceptance criteria) are more effective. The main reason is because in the communication strategy B, the communication interferes with the proper performance of the *reset*, which is a very important step in the algorithm. Furthermore, the reset provides a very important exploratory factor: when receivers solvers receive the sent configurations and it is accepted by them, they can performed different resets with the same configuration, as it was explained before.

By analyzing the whole information obtained during the experiments, we can observe that the percentage of communicating solvers finding the solution thanks to the received information was high (74%, see Appendix **??**, Figure **??**). That shows that the communicated information was very helpful during the search process. With the simplicity of the operator-based language provided by POSL, we were able to find a simple communication strategy to obtain better results than applying sequential and parallel independent multi-walk approaches.

Algorithm 28 shows the code for the communication strategy of 100% of communicating solvers using the one to N operator [⇝]. As expected, this was the best communication strategy. It finds a proper equilibrium between intensification, by communicating a promising place (configuration) inside the search space to a maximum of solvers; and exploration, by performing stochastic decisions once the configuration is accepted, e.g. the must culprit variable is randomly selected if there are more than one, and the way the reset select the returned configuration (explained before).

Algorithm 28: Communication strategy one to N 100% for CAP

$[\text{SOLVER}_{sender} \bullet A(20)] \boxed{\rightsquigarrow} [\text{SOLVER}_{receiverA} \bullet C.M.(20)]$;

The random nature of this solution strategy has showed to be effective. However, it is the explanation to the high values of standard deviation showed in Table 1.15. Abstract solvers used for the resolution of CAP combine computation modules which take many stochastic decisions. The neighborhood computation module compute the neighborhood based on the most culprit variable of the input configuration, which is randomly selected if there exist more than one. This module also generates neighbors by selecting randomly the variables to swap. In addiction, the reset computation module generates configurations in three different ways and equally probable, for a same input configuration.

1.5 A local minima evasion strategy (Golomb Ruler)

Using Golomb Ruler Problem (GRP) as a benchmark, a different communication strategy was tested in this section: the communication of the current configuration in order to avoid its neighborhood, i.e. a *tabu* configuration.

1.5.1 Problem definition

The Golomb Ruler Problem (GRP) problem consists in finding an ordered vector of n distinct non-negative integers, called *marks*, $m_1 < \cdots < m_n$, such that all differences $m_i - m_j$ $(i > j)$ called *measures* are all different. An instance of this problem is the pair (o, l) where o is the order of the problem, (i.e. the number of *marks*) and l is the length of the ruler (i.e. the last *mark*). We assume that the first *mark* is always 0. This problem has been applied to radio astronomy, x-ray crystallography, circuit layout and geographical mapping [135]. When POSL is applied to solve an instance of this problem sequentially, it can be noticed that it performs many *restarts* before finding a solution, as Table 1.16 shows. For that reason this problem was chosen to study a new communication strategy.

The cost function is implemented based on the storage of a counter for each measure present in the rule (configuration). All measure where a variable is participating are also stored. This information is useful to compute the more culprit variable (the variable that interferes less in the represented measures), in case of the user wants to apply algorithms like Adaptive Search. To compute the cost of a configuration, a differences triangle is built similar to the one introduced in previews section stuying Costas Array Problem. This

differeces triangle represents all possible measures present in the rule. Se the following example:

Figure 1.4 shows all possible measures that the rule represented through the configuration $s = \{0, 1, 4, 9, 11\}$ of order o can measure. So, its corresponding differences triangle is the following:

$$\begin{array}{rcl} 0\ \ 1\ \ 4\ \ 9\ \ 11 & \rightarrow & s \\ 1\ \ 3\ \ 5\ \ 2 & \rightarrow & d_1 \\ 4\ \ 8\ \ 7 & \rightarrow & d_2 \\ 9\ \ 10 & \rightarrow & d_3 \\ 11 & \rightarrow & d_4 \end{array}$$

where each measure in d_k are the vector of measures $s_{i+k} - s_i$, for all $k \in [0, o - i - 1]$.

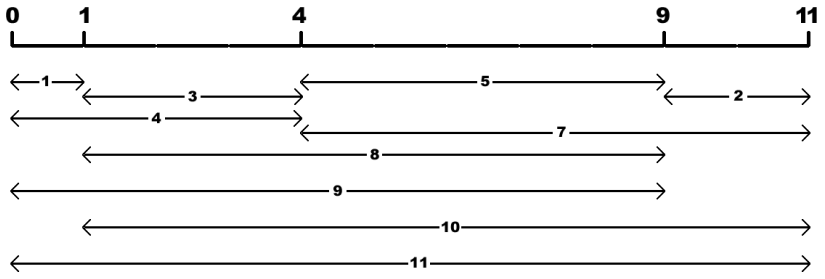

Figure 1.4: Golomb ruler of order 5 and length 11

The cost is calculated as the number of equal measures present in the differences triangle. It is easy to prove that checking this triangle until the row d_B, with $B = \lfloor (o-1)/2 \rfloor$ is enough. So, the cost can be calculated in $O(o^2)$.

1.5.2 Experiments design and results

Golomb Ruler Problem instances were used to study a different communication strategy. This time the current configuration is communicated to avoid its neighborhood, i.e. a *tabu* configuration. Some modules used in the resolution of Social Golfers and Costas Array problems have been reused to design the solvers: the *Selection* and *Acceptance* modules. The new modules are:

1. I_{sort}: returns a random configuration s as an increasing ordered vector of integers. The configuration is generated *far* from the set of *tabu* configurations arrived via solver-communication, in communicating strategies (based on the *generation* abstract module I).

2. V_{sort}: given a configuration, returns the neighborhood $V(s)$ by changing one value while keeping the order, i.e. replacing the value s_i by all possible values $s'_i \in D_i$ satisfying $s_{i-1} < s'_i < s_{i+1}$ (based on the *neighborhood* abstract module V).

An abstract module R for reset was also added: it receives and returns a configuration. The concrete reset module used for this problem (R_{tabu}) inserts the input configuration into a *tabu* list inside the solver and returns the input configuration as-is. This way it cam be assambled inside the cyclical operator, as Algorithm 30 shows. This module is executed just before performing a restart, so if the solver was unable to find a better configuration around the current one, this current configuration is assumed to be a local minimum, and it is inserted into a tabu list. Then, in the next iteration, the solver generates a first configuration far enough from all tabu configurations inside the tabu list. Algorithm 29 and 30 present both solvers, respectively without and with a tabu list. They were used to obtain results presented in Tables 1.16 and 1.17 to show that the approach explained above provides some gain in terms of runtime but also in terms of percentage of success. As we can see, the runtime improvement is not substantial. The reason is because the solver using a tabu list, can insert one configuration only in the tabu list after each restart.

Algorithm 29: Solver without using tabu list, for GRP

```
abstract solver as_golomb_notabu                    // ITR → number of iterations
computation : I, V, S, A
begin
    [↺ (ITR < K1)  I ↦  [↺ (ITR % K2)  [V ↦ S ↦ A]  ]  ]
end
solver SOLVER_notabu implements as_notabu
    computation : I_sort, V_sort, S_first, A_AI
```

Algorithm 30: Solver using tabu list, for GRP

```
abstract solver as_golomb_tabu                      // ITR → number of iterations
computation : I, V, S, A, R
begin
    [↺ (ITR < K1)  I ↦  [↺ (ITR % K2)  [V ↦ S ↦ A]  ]  ↦ R  ]
end
solver SOLVER_tabu implements as_tabu
    computation : I_sort, V_sort, S_first, A_AI, R_tabu
```

Instance	T	T(sd)	It.	It.(sd)	R	R(sd)	% success
8–34	0.79	0.66	13,306	11,154	66	55.74	100.00
10–55	66.44	49.56	451,419	336,858	301	224.56	80.00
11–72	160.34	96.11	431,623	272,910	143	90.91	26.67

Table 1.16: A single sequential solver without using tabu list for GRP

Instance	T	T(sd)	It.	It.(sd)	R	R(sd)	% success
8–34	0.66	0.63	10,745	10,259	53	51.35	100.00
10–55	67.89	50.02	446,913	328,912	297	219.30	88.00
11–72	117.49	85.62	382,617	275,747	127	91.85	30.00

Table 1.17: A single sequential solver using tabu list for GRP

Instance	T	T(sd)	It.	It.(sd)	R	R(sd)
8–34	0.43	0.37	349	334	1	1.64
10–55	4.92	4.68	20,504	19,742	13	13.07
11–72	85.02	67.22	155,251	121,928	51	40.64

Table 1.18: Parallel solvers using tabu list for GRP

***

Results in Table 1.18 show once again the success of the parallel approach. Although this time the improvement with respect to the sequential approach was significant, and tacking into account that POSL needs to perform some restarts for solving this problem, a different communication strategy is proposed. Up to now, in all previews communication strategies the current configuration was communicated in order to have more solvers searching in its neighborhood, as an intensification mechanism. In the following communication strategy the current configuration is communicated when it can be considered as a local minimum. Then, receivers solvers avoid it, by generating starting configurations far enough from it. The current configuration is considered as a local minimum since the solver (after a given number of iteration) is not able to find a better configuration in its neighborhood, so it communicates this configuration just before performing the restart.

Algorithm 31 presents the sender solver and Algorithm 32 presents the receiver solver. Based on the connection operator used in the communication strategy, this solver might receives one or many configurations. These configurations are the input of the generation module (I), and this module inserts all received configurations into a *tabu* list, and then generates a new first configuration, far from all configurations in the *tabu* list.

Algorithm 31: Sender solver for GRP

```
abstract solver as_golomb_sender                    // ITR → number of iterations
computation : I, V, S, A, R
begin
    [(↺) (ITR < K1)  I (↦)  [(↺) (ITR % K2) [V (↦) S (↦) A] ]  (↦) (|R|)^o  ]
end
solver SOLVER_sender implements as_golomb_sender
    computation : I_sort, V_sort, S_first, A_AI, R_tabu
```

In this communication strategy there are some parameters to be tuned. The first ones are: 1. K_1, the number of restarts, and 2. K_2, the number of iterations by restart. Both are instance

Algorithm 32: Receiver solver for GRP

```
abstract solver as_golomb_receiver                    // ITR → number of iterations
computation : I, V, S, A, R
connection : C.M.
begin
    [↻ (ITR < K1) [C.M. ↦ I] ↦ [↻ (ITR % K2) [V ↦ S ↦ A] ] ↦ R ]
end
solver SOLVER_receiver implements as_golomb_receiver
    computation : I_sort, V_sort, S_first, A_AI, R_tabu
    communication: CM_last
```

dependent, so, after many experimental runs, I choose them as follows:

- Golomb Ruler 8–34: $K_1 = 300$ and $K_2 = 200$
- Golomb Ruler 10–55: $K_1 = 1000$ and $K_2 = 1500$
- Golomb Ruler 11-72: $K_1 = 1000$ and $K_2 = 3000$

The idea of this strategy (abstract solver) follows the following steps:

Step 1

The computation module I_{sort} generates an initial configuration tacking into account a set of configurations into a tabu list. The configuration arriving to this tabu list come from the itself (Step 3) and/or from outside (other solvers) depending on the strategy (non-communicating or communicating), and on the type of the solver (sender or receiver).

This module executes some other modules provided by POSL to solve the *Sub-Sum Problem* in order to generates *valid* configurations for Golomb Ruler Problem. A valid configuration s for Golomb Ruler Problem is a configuration that fulfills the following constraints:

- $s = (a_1, \ldots, a_o)$ where $a_i < a_j, \forall i < j$, and
- all $d_i = a_{i+1} - a_i$ are all different, for all $d_i, i \in [1...o-1]$

The *Sub-sum Problem* is defined as follows: Given a set E of integers, with $|E| = N$, finding a subset e of n elements that sums exactly z. In that sense, a solution $S_{sub-sum} = \{s_1, \ldots, s_{o-1}\}$ of the *Sub-sum problem* with $E = \left\{1, \ldots, l - \frac{(o-2)(o-1)}{2}\right\}$, $n = o - 1$ and $z = l$, can be traduced to a *valid* configuration C_{grp} for Golomb Ruler Problem as follows:

$$C_{grp} = \{c_1, c_1 + s_1, \ldots, c_{o-1} + s_{o-1}\}$$

where $c_1 = 0$.

In the selection module applied inside the module I, the selection step of the search process selects a configuration from the neighborhood *far* from the *tabu* configurations, with respect to

certain vectorial norm and an epsilon. In other words, a configuration C is selected if and only if:

1. the cost of the configuration C is lower than the current cost, and
2. $||C - C_t||_p > \varepsilon$, for all $tabu$ configuration C_t

where p and ε are parameters.

I experimented with 3 different values for p. Each value defines a different type of norm of a vector $x = \{x_1, \ldots, x_n\}$:

- $p = 1$: $||x||_1 = \sum_{i=0}^{n} |x_i|$
- $p = 2$: $||x||_2 = \sqrt{\sum_{i=0}^{n} |x_i|^2}$
- $p = \infty$: $||x||_\infty = \max(x)$

After many experimental runs with these values I choose $p = \infty$ and $\varepsilon = 4$ for the study of the communication strategy. I also made experiments trying to find the right size for the $tabu$ list and the conclusion was that the right sizes were 15 for non-communicating strategies and 40 for communicating strategies, taking into account that in the latter, I work with 20 receivers solvers.

Step 2

After generating the first configuration, the next step is to apply a local search to improve it. In this step I use the neighborhood computation module V_{sort}, that creates neighborhood $\mathcal{V}(s)$ by changing one value while keeping the order in the configuration, and the other modules (selection and acceptance). The local search is executed a number K_2 of times, or until a solution is obtained.

Step 3

If no improvement is reached, the current configuration is classified as a *potential local minimum* and inserted into the $tabu$ list, then send it (on the case of sender solvers). Then, the process returns to the Step 1.

The POSL code of the communication strategy using the one to N operator is presented in Algorithm 33.

Algorithm 33: Communication strategy one to N for GRP

$[\text{SOLVER}_{sender} \bullet R(20)] \boxed{\rightsquigarrow} [\text{SOLVER}_{receiver} \bullet C.M.(20)]$;

When we use communication one to one, after k restarts the receiver solver has $2k$ configurations inside its tabu list: its own tabu configurations and the received ones. Table 1.19 shows that this strategy is not sufficient for some instances, but when we use communication one to N, the number of tabu configurations after k restarts in the receiver solver is considerably higher: $k(N+1)$: its own tabu configurations and the others received from N sender solvers the receiver solver is connected with. Hence, these solvers can generate configurations far enough from many potentially local minima. This phenomenon is more visible when the problem order o increases. Table 1.20 shows that the improvement for the higher case (11-72) is about 29% w.r.t non communicating solvers (Table 1.18).

Instance	T	T(sd)	It.	It.(sd)	R	R(sd)
8–34	0.44	0.31	309	233	1	1.23
10–55	3.90	3.22	15,437	12,788	10	8.52
11–72	85.43	52.60	156,211	97,329	52	32.43

Table 1.19: Golomb Ruler: parallel, communication one to one

Instance	T	T(sd)	It.	It.(sd)	R	R(sd)
8–34	**0.43**	0.29	283	225	1	1.03
10–55	**3.16**	2.82	12,605	11,405	8	7.61
11–72	**60.35**	43.95	110,311	81,295	36	27.06

Table 1.20: Golomb Ruler: parallel, communication one to N

1.6 Summarizing

In this chapter various Constraint Satisfaction Problems as benchmarks have been chosen to 1. evaluate the POSL behavior solving these kind of problems, and 2. to study different solution strategies, specially communication strategies. To this end, benchmarks with different characteristics have been selected, to help me having a wide view of the POSL behavior.

In the solution process of Social Golfers Problem, it was studied an exploitation-oriented communication strategy, in which the current configuration is communicated to ask other solvers for help to concentrate the effort in a more promising area. Results show that this communication strategy can provide some gain in terms of runtime. It was also presented results showing the success of a cost descending acceleration communication strategy, exchanging the current configuration between two solvers with different characteristics. Some other unsuccessful communication strategies were studied, showing that the sub-division of the effort by weeks, do not work well. Table 1.21 summarizes the obtained results solving SGP.

Instance	Sequential		Parallel		Cooperation	
	T	It.	T	It.	T	It.
5–3–7	1.25	2,907	0.23	142	**0.08**	139
8–4–7	0.60	338	0.28	93	**0.14**	100
9–4–8	1.04	346	0.60	139	**0.36**	144

Table 1.21: Summarizing results for SGP

It was showed that simple communication strategies as they were applied to solve Social Golfers Problem does not improve enough the results without communication for the N-Queens Problem. However, a deep study of the POSL's behavior during the search process allowed to design a communication strategy able to improve obtained results using non-communicating strategies for small instances. Table 1.22 summarizes the obtained results solving NQP.

Instance	Sequential		Parallel		Cooperation	
	T	It.	T	It.	T	It.
250	0.29	8,898	0.19	4,139	**0.09**	1,169
500	0.35	4,203	0.24	2,675	**0.14**	864
1000	0.35	2,766	0.30	2,102	**0.21**	807
3000	1.50	2,191	1.33	2,168	**1.02**	1,613
6000	4.71	3,339	4.57	3,323	**4.24**	2,537

Table 1.22: Summarizing results for NQP

The Costas Array Problem is a very complicated constrained problem, and very sensitive to the methods to solve it. For that reason I used some ideas from already existing algorithms. However, thanks to some studies of different communication strategies, based on the communication of the current configuration at different times (places) in the algorithm, it was possible to find a communication strategy to improve the performance. Table 1.23 summarizes the obtained results solving CAP.

STRATEGY	T	It.	% success
Sequential	132.73	2,332,088	40.00
Parallel	25.51	231,262	100.00
Cooperative Strategy	**10.83**	79,551	100.00

Table 1.23: Summarizing results for CAP 19

During the solution process of the Golomb Ruler Problem, POSL needs to perform many restarts. For that reason, this problem was chosen to study a different (and innovative up to my knowledge) communication strategy, in which the communicated information is a potential local minimum to be avoided. This new communication strategy showed to be effective to solve these kind of problems. Table 1.24 summarizes the obtained results solving GRP.

Instance	Sequential				Parallel			Cooperation		
	T	It.	R	% success	T	It.	R	T	It.	R
8–34	0.66	10,745	53	100.00	0.43	349	1	**0.43**	283	1
10–55	67.89	446,913	297	88.00	4.92	20,504	13	**3.16**	12,605	8
11–72	117.49	382,617	127	30.00	85.02	155,251	51	**60.35**	110,311	36

Table 1.24: Summarizing results for GRP

In all cases, thanks to the operator-based language provided by POSL it was possible to test many different strategies (communicating and non-communicating) fast and easily. Whereas creating solvers implementing different solution strategies can be complex and tedious, POSL gives the possibility to make communicating and non-communicating solver prototypes and to evaluate them with few efforts. In this chapter it was possible to show that a good selection and management of inter-solvers communication can help to the search process, working with complex constrained problems.

2

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UNIVERSITE BRETAGNE LOIRE

UNIVERSITÉ DE NANTES

Thèse de Doctorat

Alejandro
REYES AMARO

POSL: Un Languaje Orienté Parallèle pour construire des Solveurs de contraintes

POSL: A Parallel-Oriented Solver Language

Résumé
La technologie multi-coeur et les architectures massivement parallèles sont de plus en plus accessibles à tous, à travers des technologies comme le Xeon Phi ou les cartes GPU. Cette stratégie d'architecture a été communément adoptée par les constructeurs pour faire face à la loi de Moore. Or, ces nouvelles architectures impliquent d'autres manières de concevoir et d'implémenter les algorithmes, pour exploiter complètement leur potentiel, en particulier dans le cas des solveurs de contraintes traitant de problèmes d'optimisation combinatoire. De plus, le temps de développement nécessaire pour coder des solveurs en parallèle est souvent sous-estimée, et concevoir des algorithmes efficaces pour résoudre certains problèmes consomme trop de temps. Dans cette thèse nous présentons le langage orienté parallèle POSL, permettant de construire des solveurs de contraintes basés sur des méta-heuristiques qui résolvent des Problèmes de Satisfaction de Contraintes. Le but de ce travail est d'obtenir un système pour facilement construire des solveurs et réduire l'effort de leur développement en proposant un mécanisme de réutilisation de code entre les différents solveurs. Il fournit aussi un mécanisme pour coder des stratégies de communication indépendantes des solveurs. Dans cette thèse, nous présentons aussi une analyse détaillée des résultats obtenus en résolvant plusieurs instances des CSPs. L'idée n'est pas d'améliorer l'état de l'art en terme d'efficacité sur ces instances de CSPs, mais de démontrer qu'il est possible de rapidement écrire des prototypes avec POSL afin d'expérimenter facilement différentes stratégies de communication.

Mots clés
CSP, méta-heuristiques, parallèle, communication entre processus, langage.

Abstract
The multi-core technology and massive parallel architectures are nowadays more accessible for a broad public through hardware like the Xeon Phi or GPU cards. This architecture strategy has been commonly adopted by processor manufacturers to stick with Moore's law. However, this new architecture implies new ways of designing and implementing algorithms to exploit their full potential. This is in particular true for constraint-based solvers dealing with combinatorial optimization problems. Furthermore, the developing time needed to code parallel solvers is often underestimated. In fact, conceiving efficient algorithms to solve certain problems takes a considerable amount of time. In this thesis we present POSL, a Parallel-Oriented Solver Language for building solvers based on meta-heuristic, in order to solve Constraint Satisfaction Problems (CSP) in parallel. The main goal of this thesis is to obtain a system with which solvers can be easily built, reducing therefore their development effort, by proposing a mechanism of code reusing between solvers. It provides a mechanism to implement solver-independent communication strategies. We also present a detailed analysis of the results obtained when solving some CSPs. The goal is not to outperform the state of the art in terms of efficiency, but showing that it is possible to rapidly prototyping with POSL in order to experiment different communication strategies.

Key Words
CSP, meta-heuristics, parallel, inter-process communication, language.