# POSL: A Parallel-Oriented Solver Language

Thesis for the Degree of
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# POSL: A Parallel-Oriented Solver Language

**Short abstract:**
For a couple of years, all processors in modern machines are multi-core. Massively parallel architectures, so far reserved for super-computers, become now available to a broad public through hardware like the Xeon Phi or GPU cards. This architecture strategy has been commonly adopted by processor manufacturers, allowing them to stick with Moore's law. However, this new architecture implies new ways to design and implement algorithms to exploit its full potential. This is in particular true for constraint-based solvers dealing with combinatorial optimization problems. Here we propose a Parallel-Oriented Solver Language (POSL, pronounced "puzzle"), a new framework to build interconnected meta-heuristic based solvers working in parallel. The novelty of this approach lies in looking at solver as a set of components with specific goals, written in a parallel-oriented language based on operators. A major feature in POSL is the possibility to share not only information, but also behaviors, allowing solver modifications during runtime. Our framework has been designed to easily build constraint-based solvers and reduce the developing effort in the context of parallel architecture. POSL's main advantage is to allow solver designers to quickly test different heuristics and parallel communication strategies to solve combinatorial optimization problems, usually time-consuming and very complex technically, requiring a lot of engineering.

**Keywords:** Constraint programming, meta-heuristics, language

# Contents

**Resumé:**
Resumé en français ...

# Preface

This thesis was submitted to the Faculty of Science, University of Nantes, as a requirement to obtain the PhD degree. The work presented was carried out in the years 2013-2016 in the laboratory LINA (*Laboratoire d'Informatique de Loire Atlantique*) as part of the Inria-TASC research team, under the supervision of Éric MONFROY and Florian Richoux, and with a French Ministry of Research's Grant. I've always enjoyed my time at Nantes where I spent about two years months. I additionally spent some days uslises

During this time I have participated in some schools schools and I have received some curses like.

## Context

The evolution of computer architecture is leading us toward massively multi-core computers for tomorrow, composed of thousands of computing units. However nowadays, we do not know how to design algorithms able to manage efficiently such a computing power. In particular, this is true for combinatorial optimization algorithms, like algorithms solving constraint-based problems.

There exists several techniques for solving constraint-based problems: constraint programming, linear programming, boolean satisfaction methods, and local search methods to give an non-exhaustive list. The latter is often among the most efficient techniques to solve large size problems. Nowadays and up to our knowledge, there exists only one algorithm showing very good performances scaling to thousands of cores. However its parallel scheme does not include inter-processes cooperative communications. Moreover, the rising of more and more complex algorithms leads to an number of parameters which become intractable to manage by hand, and parallel algorithms emphasize this trend.

## Thesis objectives

In this context, this Ph.D. topic has two major objectives :

*a)* Obj 1

*b)* Obj 2

## Problem presentation

Optimization Problems are classical problems in Applied Mathematics and Computer Science. Their main goal is to find the best solution to a given mathematical model, which can have restrictions (constraints) or not. Variables composing the problem take their value from continuous or discrete domains. In the latter case, we are talking about of Combinatorial Optimization. Combinatorial Optimization has important applications in several fields, including machine learning, artificial intelligence, and software engineering. For example, some common problems involving combinatorial optimization are the traveling salesman problem (TSP) and the minimum spanning tree problem (MST). In some cases (problems), the main goal is only to find one solution, not the best, which is the case of the Constraint Satisfaction Problems (CSP), i.e., a solution will be a configuration of variables that complies with the constraints set. In other words: finding one feasible solution is enough.

The CSPs are one of the most trendiest in the Combinatorial Optimization field, and they find a lot of application in the industry. In theoretical terms, it means that new and more complex (also bigger) problems appear. For that reason, a lot of techniques and methods are applied to the resolution of these problems. Although many of these techniques, like meta-heuristics, have shown themselves to be effective, sometimes the real problems we want to solve are too large, i.e., the search space is huge, and in most cases too long execution time is needed to find a solution. However, the development of the super-computing has opened a new way to find solutions for these problems in a more feasible manner, reducing the search times. Therefore, in this thesis we will focus in finding new technologies and methods for the solution of CSPs through parallel computing, developing cooperative parallel algorithms and applying auto-tuning techniques to choose the proper parameters for them, which is a field not explored yet.

## Thesis outline

In Part I,

# Part I

## Description of Thesis

# 1

# State of the art

*This chapter presents an overview to the state of the art of Combinatorial Optimization Problems and different approaches to tackle them. We introduce in Section 1.1 the definition of a Combinatorial Optimization Problem and the its link with Constraint Satisfaction Problems (CSP), where we concentrate our main efforts, and we give some examples. The basic techniques used to solve these problems are introduced, like Constraint Propagation (1.2), meta– and hyper–heuristic methods (Sections 1.3 and 1.4). We also present some advanced techniques like hybridization in Section 1.5, parallel computing in Section 1.6, and Solvers cooperation in Section 1.7. Finally we present ıparameter setting techniques in Section 1.8.*

## Contents

This chapter presents an overview to the state of the art of *Combinatorial Optimization Problems* and different approaches to tackle them. In Section 1.1 the definition of a *Constraint Satisfaction Problem* (*CSP*), emphasizing in the concept of *Constraint Satisfaction Problems*, where we concentrate our main efforts. Constraint propagation techniques are deterministic methods to attack these kind of problems (presented in Section 1.2), but in some cases they are incapable to solve them (they are mostly used to reduce the problem's search space or to prove it unsatisfiable). For that reason, the model presented in this thesis is based on *meta-heuristic* methods (Section 1.3). The *Hybridization* approach combines different techniques in the same solution strategy, so the progresses in this field are exposed in Section 1.5.

The evolution of computer architecture is leading us toward massively multi-core computers for tomorrow, composed of thousands of computing units. A parallel model to solve *CSPs* is the core of this work, and its advances, as well as those obtained in the field of *cooperation between solvers*, are presented in Sections 1.6 and 1.7 respectively. Finally, this chapter presents in Section 1.8 an overview of the progresses in the field of *parameter settings*.

## 1.1 Combinatorial Optimization

An *Optimization Problem* consists in finding the best solution among all possible ones, subject or not, to a set of constraints, depending on it is a restricted or an unrestricted problem. The suitable values for the involved variables belong to a set called *domain*. When this domain contains only discrete values, we are facing a *Combinatorial Optimization Problem*, and its goal is to find the best possible solution satisfying a global criterion, named *objective function*. *Resource Allocations* [1], *Task Scheduling* [2], *Master-keying* [3], *Traveling Salesman*, *Knapsack Problem*, among others, are well-known examples of *Combinatorial Optimization Problems* [4].

Sometimes, the main goal is not to find the best solution, but finding one feasible solution. This is the case of *Constraint Satisfaction Problems*. Formally, we present the definition of a *CSP* (sometimes also called *Constraint Network*).

**Definition 1 (Constraint Satisfaction Problem)** *A Constraint Satisfaction Problem (CSP, denoted by $\mathcal{P}$) is a triple $\langle X, D, C \rangle$, where:*

- $X = \{X_1, \ldots, X_n\}$ *is finite a set of variables,*
- $D = \{D_1, \ldots, D_n\}$ *is the set of associated domains. Each domain $D_i$ specifies the set of possible values to the variable $X_i$.*

- $C = \{c_1, \ldots, c_m\}$ *is a set of constraints. Each constraint is defined involving a set of variables, and specifies the possible combinations of values for these variables.*

In *CSPs*, a *configuration* $s \in D_1 \times D_2 \times \cdots \times D_n$ is a combination of values for the variables in $X$. Following we define the concept of solution, which is in other words, a configuration satisfying all the constraints $c_i \in C$.

**Definition 2 (Solution of a CSP)** *Given a CSP* $\mathcal{P} = \langle X, D, C \rangle$ *and a configuration* $S \in D_1 \times D_2 \times \cdots \times D_n$ *we say that it is a solution if and only if:*

$$c_i(S) \text{ is true } \forall c_i \in C$$

*The set of all solutions of* $\mathcal{P}$ *is denoted by* $Sol(\mathcal{P})$

This field, also called *Constraint Programing* is a famous research topic developed by the field of artificial intelligence in the middle of the 70's, and a programming paradigm since the end of the 80's. A *CSP* can be considered as a special case of *Combinatorial Optimization Problems*, where the objective function is to reduce to the minimum the number of violated constraints in the model. A solution is then obtained when the number of violated constraints reach the value zero. We focus our work in solving this particular case of problems.

*CSPs* find a lot of "real-world" applications in the industry. In practice, these problems are intractable for classical constraint programming approaches, like *tree search-based solvers* or *backtracking-based solvers* [5], exploring the whole solution space, wish is huge. For that reason, these kinds of problems are mostly tackled by *meta-heuristic methods* or hybrid approaches, like *Monte Carlo Tree Search* methods, which combine precision (tree search) with randomness (meta-heuristic) showing good results in artificial intelligence for games [6, 7].

## 1.2 Constraint propagation

Constraint propagation techniques are methods used to modify a *Constraint Satisfaction Problem* in order to reduce its variables domains, and turning the problem into one that is equivalent, but usually easier to solve [8]. The main goal is to choose one (or some) constraint(s) and analyzing *local consistency*, which means trying to find values in the variables domain which make constraint unsatisfiable, in order to remove them from the domain. The applied procedure to reduce the variable domains is called *reduction function*,

and it is applied until a new, "smaller" and easier to solve is obtained, and it can not be further reduced: a *fixed point*.

*Chaotic Iterations* is a technique, that comes from numerical analysis and adapted for computer science needs, used for computing limits of iterations of finite sets of functions [9, 10]. In [11, 12] a formalization of constraint propagation is proposed through *chaotic iterations*. In [13], a coordination-based chaotic iteration algorithm for constraint propagation is proposed. It is a scalable, flexible and generic framework for constraint propagation using coordination languages, not requiring special modeling of *CSPs*. We can find an implementation of this algorithm in DICE (Distributed Constraint Environment) [14] using the MANIFOLD coordination language. Coordination services implement existing protocols for constraint propagation, termination detection and splitting of *CSPs*. DICE combines these protocols with support for parallel search and the grouping of closely related components into cooperating solvers.

MANIFOLD is a strongly-typed, block-structured, event-driven language for managing events, dynamically changing interconnections among sets of independent, concurrent and cooperative processes. A MANIFOLD application consists of a number of processes running on a heterogeneous network. Processes in the same application may be written in different programming languages. MANIFOLD has been successfully used in a broad range of applications [15].

In [16] is proposed an implementation of constraint propagation by composition of reductions. It is a general algorithmic approach to tackle strategies that can be dynamically tuned with respect to the current state of constraint propagation, using composition operators. A composition operator models a sub–sequence of an iteration, in which the ordering of application of reduction functions is described by means of combinators for sequential, parallel or fixed–point computation, integrating smoothly the strategies to the model. This general framework provides a good level of abstraction for designing an object-oriented architecture of constraint propagation. Composition can be handled by the *Composite Design Pattern* [17], supporting inheritance between elementary and compound reduction functions. The propagation mechanism uses the *Observer (Listener) Design Pattern* [18], that makes the connection between domain modifications and re–invocation of reduction functions (event-based relations between objects); and the generic algorithm has been implemented using the *Strategy Design Pattern* [19], that allows to parametrize parts of algorithms.

A propagation engine prototype with a *Domain Specific Language* (DSL) was implemented in [20]. It is a solver–independent language able to configure constraint propagations at the modeling stage. The main contributions are a DSL to ease configure constraint propagation engines, and the exploitation of the basic properties of DSL in order to ensure both completeness and correctness of the produced propagation engine, like: i) *Solver*

*independent description*: The DSL does not rely on specific solver requirements (but assuming that solvers provide full access to variable and propagator properties), ii) *Expressivity*: The DSL covers commonly used data structures and characteristics, iii) *Extensibility*: New attributes can be introduced to make group definition more concise. New collections and iterators can provide new propagation schemes, iv) *Unique propagation*: The top-bottom left-right evaluation of the DSL ensures that each arc is only represented once in the propagation engine.

Some characteristics are required to fully benefit from the DSL. Due to their positive impact on efficiency, modern constraint solvers already implement these techniques: i) Propagators are discriminated thanks to their priority (deciding which propagator to run next): lighter propagators (in the complexity sense) are executed before heavier ones. ii) A controller propagator is attached to each group of propagators. iii) Open access to variable and propagator properties: for instance, variable cardinality, propagator arity or propagator priority.

To be more flexible and more accurate, they assume that all arcs from the current *CSP*, are explicitly accessible. This is achieved by explicitly representing all of them and associating them with *watched literals* [21] (controlling the behavior of variable–value pairs to trigger propagation) or *advisors* [22] (Method for supporting incremental propagation in propagator–centered setting). *Advisors* in [22] are used to modify propagator state and to decide whether a propagator must be propagated or "scheduled". They also present a concrete implementation of the DSL based on *Choco* [23] (an open source java constraint programming library) and an extension of the *MiniZinc*, a simple but expressive constraint programming modeling language which is suitable for modeling problems for a range of solvers. It is the most used language for codding *CSP*s [24].

However, we can not solve some *CSPs* only applying constraint propagation techniques. It is necessary to combine them with other methods.

## 1.3 Meta-heuristic methods

*Meta-heuristic Methods* are non problem-specific techniques that efficiently explore the search space in order to find the solution, and can often find them with less computational effort than iterative methods, so an effective way to face the *CSPs*. Their algorithms are approximate and usually non-deterministic.

A *Meta-heuristic Method* is formally defined as an iterative generation process which guides a subordinate heuristic by combining smartly different concepts for *exploring* (also called

*diversification*, is guiding the search process through a much larger portion of the search space with the hope of finding promising solutions that are not yet visited) and *exploiting* (also called *intensification*, is guiding the search process into a limited (but promising) region of the search space with the hope of improving a promising already found solution) the search space (the finite set of candidate solutions or configurations) [25], avoiding getting trapped in lost areas of the search space (local minimums). Sometimes they may make use of domain-specific knowledge in the form of heuristics that are controlled by the upper level strategy. Nowadays more advanced meta-heuristics use search experience to guide the search [26].

They are often nature-inspired and are divided in two groups [27]:

*a) Single Solution Based:* more exploitation oriented, intensifying the search in some specific areas. (We will focus our attention on this first group)

*b) Population Based:* more exploration oriented, identifying areas of the search space where there are (or where there could be) the best solutions.

### 1.3.1 Single Solution Based Meta-heuristic

Methods of the first group are also called *trajectory methods*, and they are based on choosing a solution taking into account some criterion (usually random), and they move from a solution to his *neighbor*, following a trajectory into the search space. They can be seen as an intelligent extension of *Local Search Methods* [27]. Local Search Methods are the most widely used approaches to solve *Combinatorial Optimization Problems* because they often produces high–quality solutions in reasonable time.

*Simulated Annealing* (SA) [28] is one of the first algorithms with an explicit strategy to scape from local minima. Is a method inspired by the annealing technique used by the metallurgists to obtain a "well ordered" solid state of minimal energy. Its main feature is to allow moves resulting in solutions of worse quality than the current solution, in order to scape from local minima, under certain probability, which is decreased during the search process [26]. In [29] is presented a SA algorithm (TTSA) for the Traveling Tournament Problem (TPP) that explores both feasible and infeasible schedules that includes advanced techniques such as strategic oscillation to balance the time spent in the feasible and infeasible regions, by varying the penalty for violations; and reheats (increasing the temperature again) to balance the exploration of the feasible and infeasible regions and to escape local minima.

*Tabu Search* (TS) [30], is among the most used meta-heuristics for *Combinatorial Optimization Problems*. It explicitly maintain a history of the search, as a short term memory keeping

track of the most recently visited solutions, to scape from local minima, to avoid cycles, and to deeply explore the search space. A TB meta-heuristic guides the search on the approach presented in [31] to solve instances of the *Social Golfers* problem.

The idea of *Guided Local Search* (GLS) [32] is to dynamically change the objective function to help the search to gradually scape from local minima, by changing the search landscape. The set of solutions and the neighborhood are fixed, while the objective function is dynamically changed with the aim of making the current local optimum less attractive [26]. In [33] an implementation of a GLS is used to solve the satisfiability (SAT) problem, which is a special case of a *CSP* where the variables take booleans values an the constraints are disjunctions (logical OR) of literals (variables or theirs negations).

The *Variable Neighborhood Search* (VNS) is another meta-heuristic that systematically changes the size of neighborhood during the search process. These neighborhoods can be arbitrarily chosen, but often a sequence $|\mathcal{N}_1| < |\mathcal{N}_2| < \cdots < |\mathcal{N}_{k_{max}}|$ of neighborhoods with increasing cardinality is defined. The choice of neighborhoods of increasing cardinality yields a progressive diversification of the search [34, 26]. In [35] is introduced a *generalized Variable Neighborhood Search* for *Combinatorial Optimization Problems*, and in [36] is presented a model combining integer programming and VNS for *Constrained Nurse Rostering* problems.

One meta-heuristic that can be efficiently implement on parallel processors is *Greedy Randomized Adaptive Search Procedures* (GRASP). GRASP is an iterative randomized sampling technique in which each iteration provides a solution to the target problem at hand through two phases (constructive and search) within each iteration: the first smartly constructs an initial solution via an adaptive randomized greedy function, and the second applies a local search procedure to the constructed solution in to find an improvement [37]. GRASP does not make any smart use of the history of the search process. It only stores the problem instance and the best found solution. That is why GRASP is often outperformed by other meta-heuristics [26]. However, in [38] some extensions like alternative solution construction mechanisms and techniques to speed up the search are presented.

Galinier et al. present in [39] a general approach for solving constraint based problems by local search. In this work, authors present the concept of *penalty functions*, that we pick up in order to write a *CSP* as an *Unrestricted Optimization Problem* (UOP). This formulation was useful in this thesis for modeling the tackled benchmarks. In this formulation, the *objective function* of this new problem must be such that its set of optimal solutions is equal to the solution set of the original (associated) *CSP*.

**Definition 3 (Local penalty function)** *Let a* ***CSP*** $\mathcal{P}\langle X, D, C\rangle$ *and a configuration* $S$

*be. We define the operator* **local penalty function** *as follow:*

$$\omega_i : D(X) \times 2^{D(X)} \rightarrow \mathbb{R}^+ \cup 0 \text{ where:}$$
$$\omega_i(S, c_i) = \begin{cases} 0 & \text{if } c_i(S) \text{ is true} \\ k \in \mathbb{R}^+ & \text{if not} \end{cases}$$

This penalty function defines the cost of a configuration with respect to a given constraint. In consequence, we define the *global penalty function*, to define the cost of a configuration with respect to all constraint on a *CSP*:

**Definition 4 (Global penalty function)** *Let a* ***CSP*** $\mathcal{P}\langle X, D, C\rangle$ *and a configuration* $S$*. We define the operator* **global penalty function** *as follows:*

$$\Omega : D(X) \times 2^{D(X)} \rightarrow \mathbb{R}^+ \cup 0 \text{ where:}$$
$$\Omega(S, C) = \sum_{i=1}^{m} \omega_i(S, c_i)$$

We can now formulate a *Constraint Satisfaction Problem* as an *UOP*:

**Definition 5 (CSP's Associated Unrestricted Optimization Problem)** *Given a* ***CSP*** $\mathcal{P}\langle X, D, C\rangle$ *we define its* **associated Unrestricted Optimization Problem** $\mathcal{P}_{opt}\langle X, D, f\rangle$ *as follows:*

$$\min_X f(X, C)$$
*Where:* $f(X, C) \equiv \Omega(X, C)$ *is the objective function to be minimized over the variable* $X$

It is important to note that a given $S$ is optimum if and only if $f(S, C) = 0$, which means that $S$ satisfies all the constrains in the original *CSP* $\mathcal{P}$.

*Adaptive Search* is also another efficient algorithm based *local search method*, that takes advantage of the structure of the problem in terms of constraints and variables. It uses also the concept of *penalty function* and relies on iterative repair, based on this information, seeking to reduce the *error* (a projected cost of a variable, as a measure of how responsible is the variable in the cost of a configuration) on the worse variable so far. It computes the penalty function of each constraint, then combines for each variable the *errors* of all constraints in which it appears. This allows to chose the variable with the maximal *error* will be chosen as a "culprit" and thus its value will be modified for the next iteration with the best value, that is, the value for which the total error in the next configuration is minimal [40, 41, 42]. In [43] Munera et al. based their solution method in *Adaptive Search* to solve the *Stable Marriage with Incomplete List and Ties* problem [44], a natural variant of the *Stable Marriage Problem* [45].

Michel and Van Hentenryck [46] propose a constraint-based, object-oriented, architecture to reduce the development time of local search algorithms significantly. The architecture consists of two main components: a declarative component which models the application in terms of constraints and functions, and a search component which specifies the meta-heuristic. Its main feature is to allow practitioners to focus on high-level modeling issues and to relieve them from many tedious and error-prone aspects of local search. The architecture is illustrated using COMET, an optimization platform that provides a Java-like programming language to work with constraint and objective functions (a high level constraint programming) [47, 48], that supports the local search architecture with a number of novel concepts, abstractions, and control structures.

### 1.3.2 Population Based Meta-heuristic

Also there exist heuristic methods based on populations. These methods do not work with a single solution, but with a set of solutions named *population*. In this other group we can find the *Evolutionary Algorithms*. This is the general definition to name the algorithms inspired by the "Darwin's principle", that says that only the best adapted individuals will survive. They involve *operators* to handle the population to guide it through the search process. The evolutionary algorithm's operators are another branch of study, because they have to be selected properly according to the specific problem, due to they will play an important roll in the algorithm behavior [49].

Probably the most popular in this group are the *Genetic Algorithms* [50], and theirs operators are based on the simulation of the genetic variation process to achieve individuals (solutions in this case) more adapted; and the *Ant Colony* algorithms [51], that simulate the behavior of an ant swarm to find the shortest path from the food source to the nest.

## 1.4 Hyper-heuristic Methods

*Hyper-heuristics* are automated methodologies for selecting or generating heuristics to solve hard computational problems [52]. This can be achieved with a learning mechanism that evaluates the quality of the algorithm solutions, in order to become general enough to solve new instances of a given problem. *Hyper-heuristics* are related with the *Algorithm Selection Problem*, so they establish a close relationship between a problem instance, the algorithm to solve it and its performance [53].

*Hyper-heuristic frameworks* are also known as Algorithm-Portfolio-based frameworks, and their goal is predicting the running time of algorithms using statistical regression. Then the fastest predicted algorithm is used to solved the problem until a suitable solution is found or a time-out is reached [54]. In [55] is presented a *Simple Neighborhood-based Algorithm Portfolio* written in *Python* (Snappy), a very recent framework. Its aim is to provide a tool that can improve its own performances through on-line learning. Instead of using the traditional off-line training step, a neighborhood search predicts the performance of the algorithms.

HYPERION$^2$ [56] is a Java$^{TM}$ framework for meta– and hyper– heuristics which allows the analysis of the trace taken by an algorithm and its constituent components through the search space, built with the principles of interoperability, generality and efficiency. The main goals of HYPERION$^2$ are:

*a)* Promoting interoperability via component interfaces,

*b)* Allowing rapid prototyping of meta– and hyper– heuristics, with the potential to use the same source code in either case,

*c)* Providing generic templates for a variety of local search and evolutionary computation algorithms,

*d)* Making easier the construction of novel meta- and hyper- heuristics by hybridization (via interface interoperability) or extension (subtype polymorphism),

*e)* *Only pay for what you use* – a design philosophy that attempts to ensure that generality doesn't necessarily imply inefficiency.

*hMod* is inspired by the previous frameworks, and using a new object-oriented architecture, encodes the core of the hyper-heuristic in several modules, referred as algorithm containers. *hMod* directs the programmer to define the heuristic using two separate XML files; one for the heuristic selection process and another one for the acceptance criteria [57].

*Evolving evolutionary algorithms* are specialized hyper-heuristic method which attempt to readjust an evolutionary algorithm to the problem needs. An Evolutionary Algorithm (EA) discover the rules and knowledge, to find the best algorithm to solve a problem. In [58] is used linear genetic programming and multi-expression genetic programming, to optimize the EA solving unimodal mathematical functions and another EA adjusts the sequence of genetic and reproductive operators. A solution consists of a new evolutionary algorithm capable of outperforming genetic algorithms when solving a specific class of unimodal test functions.

## 1.5 Hybridization

The *Hybridization* approach is the one who combine different approaches into the same solution strategy, and recently, it leads to very good results in the constraint satisfaction field. For example, constraint propagation may find a solution to a problem, but they can fail even if the problem is satisfiable, because of its local nature. At each step, the value of a small number of variables are changed, with the overall aim of increasing the number of constraints satisfied by this assignment, and applying other techniques to avoid local solutions, for example adding a stochastic component to choose variables to affect. Integrations of global search (complete search) with local search have been developed, leading to hybrid algorithms.

Hooker J.N. presents in [59] some ideas to illustrate the common structure present in exact and heuristic methods, to encourage the exchange of algorithmic techniques between them. The goal of this approach is to design solution methods ables to smoothly transform its strategy from exhaustive to non-exhaustive search as the problem become more complex.

In [60] a taxonomy of hybrid optimization algorithms is presented in an attempt to provide a mechanism to allow qualitative comparison of hybrid optimization algorithms, combining meta-heuristics with other optimization algorithms from mathematical programming, machine learning and constraint programming.

Monfroy et al. present in [61, 62] a general hybridization framework is proposed to combine complete constraints resolution techniques with meta-heuristic optimization methods in order to reduce the problem through domain reduction functions, ensuring not loosing solutions. Other interesting ideas are TEMPLAR, a framework to generate algorithms changing predefined components using hyper-heuristics methods [63]; and *ParadisEO*, a framework to design parallel and distributed hybrid meta-heuristics showing very good results [64], including a broad range of reusable features to easily design evolutionary algorithms and local search methods.

Another technique has been developed, the called *autonomous search*, based on the supervised or controlled learning. This systems improve their functioning while they solve problems, either modifying their internal components to take advantage of the opportunities in the search space, or to adequately chose the solver to use (*portfolio point of view*) [65].

In [66] is proposed another portfolio-based technique, *time splitting*, to solve optimization problems. Given a problem $P$ and a schedule $Sch = [(\Sigma_1, t_1), \ldots, (\Sigma_n, t_n)]$ of $n$ solvers, the corresponding time-split solver is defined as a particular solver such that:

a) runs $\Sigma_1$ on $P$ for a period of time $t_1$,

b) then, for $i = 1, \ldots, n-1$, runs $\Sigma_{i+1}$ on $P$ for a period of time $t_{i+1}$ exploiting or not the best solution found by the previous solver $\Sigma_i$ $t_i$ units of time.

In [67] is proposed a tool (`xcsp2mzn`) for converting problem instances from the XCSP[i] format [68] to MiniZinc that is a simple but expressive constraint programming modeling language which is suitable for modeling problems for a range of solvers. It is the most used language for codding *CSPs* [24]. The second contribution of this work is the development of `mzn2feat` a tool to extract static and dynamic features from the MiniZinc representation, with the help of the Gecode[ii] [69] interpreter, and allows a better and more accurate selection of the solvers to be used according to the instances to solve. Some results are showed proposing that the performances that can be obtained using these features are competitive with state of the art on *CSP* portfolio techniques.

## 1.6 Parallel computing

Parallel computing is a way to solve problems using some calculus resources at the same time. It is a powerful alternative to solve problems which would require too much time by using the traditional ways, i.e., sequential algorithms [70]. That is why this field is in constant development and it is the topic where we will put most of our effort.

For a couple of years, all processors in modern machines are multi-core. Massively parallel architectures, previously expensive and so far reserved for super–computers, become now a trend available to a broad public through hardware like the Xeon Phi or GPU cards. The power delivered by massively parallel architectures allow us to treat faster these problems [71]. However this architectural evolution is a non-sense if algorithms do not evolve at the same time: the development and the implementation of algorithms should take this into account and tackling the problems with very different methods, changing the sequential reasoning of researchers in Computer Science [72, 73]. We can find in [74] a survey of the different parallel programming models and available tools, emphasizing on their suitability for high-performance computing.

Falcou propose in [75] a programming model: *Parallel Algorithmic Skeletons* (along with a C++ implementation called Quaff to make parallel application development easier. Writing efficient code for parallel machines is less trivial, as it usually involves dealing with low-level

[i] Is a XML-like language for coding *CSPs*. Is not more used than MiniZinc but although it was mainly used as the standard in the *International Constraint Solver Competition* (ended in 2009), the *ICSC* dataset is for sure the biggest dataset of CSP instances existing today

[ii] Is an efficient open source environment for developing constraint-based system and applications.

APIs such as OpenMP, message-passing interfaces (MPI), among others. However, years of experience have shown that using those frameworks is difficult and error-prone. Usually many undesired behaviors (like deadlocks) make parallel software development very slow compared to the classic, sequential approach. In that sense, this model is a high-order pattern to hide all low-level, architecture or framework dependent code from the user, and provides a decent level of organization. QUAFF is a skeleton-based parallel programming library, which has demonstrated its efficiency and expressiveness solving some application from computer vision, that relies on C++ template meta-programming to reduce the overhead traditionally associated with object-oriented implementations of such libraries: the code generation is done at compile time.

Some results have been obtained on this field. The contribution in terms of hardware has been crucial, achieving powerful technologies to perform large–scale calculations. But the development of the techniques and algorithms to solve problems in parallel is also visible, focusing the main efforts in three fundamentals concepts:

*a) Problem subdivision*,

*b) Scalability* and

*c) Inter-process communication*.

In a preliminary review of literature on parallel constraint solving [76], addressing the literature in constraints on exploitation of parallel systems for constraint solving, is starting first by looking at the justification for the multi-core architecture. The direction it is most likely to take and limiting factors on performance such as *Amdahl*'s law, and then reviewing recent literature on parallel constraint programming. For their part, they group the paper into four areas: i) parallelizing the search process, ii) parallel and distributed arc-consistency, iii) multi-agent and cooperative search and iv) combined parallel search and parallel consistency.

The issue of sub-dividing a given problem in some smaller sub-problems is sometimes not easy to address. Even when we can do it, the time needed by each process to solve its own part of the problem is rarely balanced. In [77] are proposed some techniques to tackle this problem, taking into account that sometimes, the more can be sub-divided a problem, the more balanced will be the execution times of the process. In [78] is presented an comparison between Transposition-table Driven Scheduling (TDS) and a parallel implementation of a best-first search strategy (Hash Distributed A*), that uses the standard approach of of *Work Stealing* for partitioning the search space. This technique is based on maintaining a local work queue, (provided by a *root process* through hash-based distribution that assign an unique processor to each work) accessible to other process that "steal" work from if they become unoccupied. The same approach is used in [79] to evaluate *Zobrist Hashing*, an

efficient hash function designed for table games like chess and Go, to mitigate communication overheads.

In [80] is presented a study of the impact of space-partitioning techniques on the performance of parallel local search algorithms to tackle the *k-medoids* clustering problem. Using a parallel local search, this work aims to improve the scalability of the sequential algorithm, which is measured in terms of the quality of the solution within the same time with respect to the sequential algorithm. Two main techniques are presented for domain partitioning: first, *space-filling curves*, used to reduce any N-dimensional representation into a one-dimension space (this technique is also widely used in the nearest-neighbor-finding problem [81]); and second, *k-Means* algorithm, one of the most popular clustering algorithms [82].

In [83] is proposed a mechanism to create sub-*CSPs* (whose union contains all the solutions of the original *CSP*) by splitting the domain of the variables though communication between processes. The contribution of this work is explained in details in section 1.7.

Related to the search process, we can find two main approaches. First, the *single walk* approach, in which all the processes try to follow the same path towards the solution, solving their corresponding part of the problem, with or without cooperation (communication). The other is known as *multi walk*, and it proposes the execution of various independent processes to find the solution. Each process applies its own strategies (portfolio approach) or simply explores different places inside the search space. Although this approach may seem too trivial and not so smart, it is fair to say that it is in fashion due to the good obtained results using it [40].

*Scalability* is the ability of a system to handle the increasing growth of workload. A system which has improved over time its performance after adding work resources, and it is capable of do it proportionally is called *scalable*. The increase has not been only in terms of calculus resources, but also in the amount of sub-problems coming from the sub-division of the original problem. The more we can divide a problem into smaller sub-problems, the faster we can solve it [84]. *Adaptive Search* is a good example of local search method that can scale up to a larger number of cores, e.g., a few hundreds or even thousands [40]. For this algorithm, an implementation of a cooperative multi-walks strategy has been published in [85]. In this framework, the processes are grouped in teams to achieve search intensification, which cooperate with others teams through a head node (process) to achieve search diversification. Using an adaptation of this method, Munera et al. propose a parallel solution strategy able to solve hard instances of *Stable Marriage with Incomplete List and Ties Problem* quickly. In [86] is presented a combination of this method with an *Extremal Optimization* procedure: a nature-inspired general-purpose meta-heuristic [87].

A lot of studies have been published around this topic. A parallel solver for numerical *CSPs* is presented in [88] showing good results scaling on a number of cores. In [89], an estimation

of the speed-up (a performance measure of a parallel algorithm) through statistical analysis of its sequential algorithm is presented. This is a very interesting result because it a way to have a rough idea of the resources needed to solve a given problem in parallel.

Another issue to treat is the interprocess communication. Many times a close collaboration between process is required, in order to achieve the solution. But the first inconvenient is the slowness of the communication process. Some work have achieved to identify what information is viable to share. One example is [90] where an idea to include low-level reasoning components in the SAT problems resolution is proposed. This approach allow us to perform the clause-shearing, controlling the exchange between any pair of process.

In [85] is presented a new paradigm that includes cooperation between processes, in order to improve the independent multi-walk approach. In that case, cooperative search methods add a communication mechanism to the independent walk strategy, to share or exchange information between solver instances during the search process. This proposed framework is oriented towards distributed architectures based on clusters of nodes, with the notion of *teams* running on nodes and controlling several search engines (*explorers*) running on cores, and the idea that all teams are distributed and thus have limited inter–node communication. The communication between teams ensures diversification, while the communication between explorers is needed for intensification. This framework is oriented towards distributed architectures based on clusters of nodes, where teams are mapped to nodes and explorers run on cores. This framework was developed using the *X10 programming language*, which is a novel language for parallel processing developed by IBM Research, giving more flexibility than traditional approaches, e.g., MPI communication package.

In [91] have been presented an implementation of the meta-solver framework which coordinates the cooperative work of arbitrary pluggable constraint solvers. This approach intents to integrate arbitrary, new or pre–existing constraint solvers, to form a system capable of solving complex mixed–domain constraint problems. The existing increased cooperation overhead is reduced through problem-specific cooperative solving strategies.

In [92] is proposed the first *Deterministic Parallel DPLL* (A complete, backtracking-based search algorithm for deciding the satisfiability of propositional logic formulas in conjunctive normal form) engine. The experimental results show that their approach preserves the performance of the parallel portfolio approach while ensuring full reproducibility of the results. Parallel exploration of the search space, defines a controlled environment based on a total ordering of solvers interactions through synchronization barriers. To maximize efficiency, information exchange (conflict-clauses) and check for termination are performed on a regular basis. The frequency of these exchanges greatly influences the performance of the solver. The paper explores the trade off between frequent synchronizing which allows

the fast integration of foreign conflict–clauses at the cost of more synchronizing steps, and infrequent synchronizing at the cost of delayed foreign conflict–clauses integration.

Considering the problem of parallelizing restarted back–track search, in [93] was developed a simple technique for parallelizing restarted search deterministically and it demonstrates experimentally that it can achieve near–linear speed–ups in practice, when the number of processors is constant and the number of restarts grows to infinity. They propose the following: Each parallel search process has its own local copy of a scheduling class which assigns restarts and their respective fail–limits to processors. This scheduling class computes the next *Luby* restart fail–limit and adds it to the processor that has the lowest number of accumulated fails so far, following an *earliest–start–time–first strategy*. Like this, the schedule is filled and each process can infer which is the next fail–limit that it needs to run based on the processor it is running on – without communication. Overhead is negligible in practice since the scheduling itself runs extremely fast compared to CP search, and communication is limited to informing the other processes when a solution has been found.

In [94], were explored the two well–known principles of diversification and intensification in portfolio–based parallel SAT solving. To study their trade–off, they define two roles for the computational units. Some of them classified as *Masters* perform an original search strategy, ensuring diversification. The remaining units, classified as *Slaves* are there to intensify their master's strategy. There are some important questions to be answered: i) what information should be given to a slave in order to intensify a given search effort?, ii) how often, a subordinated unit has to receive such information? and iii) the question of finding the number of subordinated units and their connections with the search efforts? The results lead to an original intensification strategy which outperforms the best parallel SAT solver *ManySAT*, and solves some open SAT instances.

Multi-objective optimization problems involve more than one objective function to be optimized simultaneously. Usually these problems do not have an unique optimal solution because the exist a trade-off between one objective function and the others. For that reason, in a multi-objective optimization problem, the concept of Pareto optimal points is used. A Pareto optimal point is a solution that improving one objective function value, implies the deterioration of at least one of the other objective function. A collection of Pareto optimal points defines a Pareto front. In [95], is proposed a new search method, called *Multi-Objective Embarrassingly Parallel Search* (MO–EPS) to solve multi-objective optimization problems, based on: i) Embarrassingly Parallel Search (EPS): where the initial problem is split into a number of independent sub-problems, by partitioning the domain of decision variables [77, 96]; and ii) Multi-Objective optimization adding cuts (MO–AC): an algorithm that transforms the multi-objective optimization problem into a feasibility one, searches a feasible solution and then the search is continued adding constraints to the problem until either the problem becomes infeasible or the search space gets entirely explored [97].

A component-based constraint solver in parallel is proposed in [98]. In this work, a parallel solver coordinates autonomous instances of a sequential constraint solver, which is used as a software component. The component solvers achieve load balancing of tree search through a time-out mechanism. It is implemented a specific mode of solver cooperation that aims at reducing the turn-around time of constraint solving through parallelization of tree search. The main idea is to try to solve a *CSP* before a time-out. If it can not find a solution, the algorithm defines a set of disjoint sub-problems to be distributed among a set of solvers running in parallel. The goal of the time-out mechanism is to provide an implicit load balancing: when a solver is busy, and there are no subproblems available, another solver produces new sub-problems when its time-out elapses.

## 1.7 Solvers cooperation

The interaction between solvers exchanging some information is called *solver cooperation* and it is very popular in this field due to their good results. Its main goal is to improve some kind of limitations or inefficiency imposed by the use of unique solver. In practice, each solver runs in a computation unit, i.e. thread or processor. The cooperation is performed through inter–process communication, by using different methods: *signals*, asynchronous notifications between processes in order to notify an event occurrence; *semaphore*, an abstract data type for controlling access, by multiple processes, to a common resource; *shared memory*, a memory simultaneously accessible by multiple processes; *message passing*, allowing multiple programs to communicate using messages; among others.

Kishimoto et al. present in [99] a parallelization of the an algorithm A* (Hash Distributed A*) for *optimal sequential planning* [100], exploiting distributed memory computers clusters, to extract significant speedups from the hardware. In classical planning solving, both the memory and the CPU requirements are main causes of performance bottlenecks, so parallel algorithms have the potential to provide required resources to solve changeling instances. In [78], authors study scalable parallel best-first search algorithms, using MPI, a paradigm of *Message Passing Interface* that allows parallelization, not only in distributed memory based architectures, but also in shared memory based architectures and mixed environments (clusters of multi-core machines) [101].

In [102] is presented a paradigm that enables the user to properly separate computation strategies from the search phases in solver cooperations. The cooperation must be supervised by the user, through *cooperation strategy language*, which defines the solver interactions in order to find the desired result.

In [90], an idea to include low-level reasoning components in the SAT problems resolution is proposed, dynamically adjusting the size of shared clauses to reduce the possible blow up in communication. [102] presents a paradigm that enables the user to properly separate strategies combining solver applications in order to find the desired result, from the way the search space is explored.

*Meta–S* is an implementation of a theoretical framework proposed in [91], which allows to tackle problems, through the cooperation of arbitrary domain–specific constraint solvers. *Meta–S* [91] is a practical implementation and extension of a theoretical framework, which allows the user to attack problems requiring the cooperation of arbitrary domain–specific constraint solvers. Through its modular structure and its extensible strategy specification language it also serves as a test–bed for generic and problem–specific (meta–)solving strategies, which are employed to minimize the incurred cooperation overhead. Treating the employed solvers as black boxes, the meta–solver takes constraints from a global pool and propagates them to the individual solvers, which are in return requested to provide newly gained information (i.e. constraints) back to the meta–solver, through variable projections. The major advantage of this approach lies in the ability to integrate arbitrary, new or pre–existing constraint solvers, to form a system that is capable of solving complex mixed–domain constraint problems, at the price of increased cooperation overhead. This overhead can however be reduced through more intelligent and/or problem–specific cooperative solving strategies. HYPERION [56] is an already mentioned framework for meta– and hyper–heuristics built with the principle of interoperability, generality by providing generic templates for a variety of local search and evolutionary computation algorithms and efficiency, allowing rapid prototyping with the possibility of reusing source code.

Arbab and Monfory propose in [83] a technique to guide the search by splitting the domain of variables. A *Master* process builds the network of variables and domain reduction functions, and sends this informations to the worker agents. They workers concentrate their efforts on only one sub-CSP and the *Master* collects solutions. The main advantage is that by changing only the search agent, different kinds of search can be performed. The coordination process is managing using the MANIFOLD coordination language [15].

## 1.8 Parameter setting techniques

Most of these methods to tackle combinatorial problems, involve a number of parameters that govern their behavior, and they need to be well adjusted, and most of the times they depend on the nature of the specific problem, so they require a previous analysis to study their behavior [103]. That is way another branch of the investigation arises: *parameter*

*tuning*. It is also known as a meta optimization problem, because the main goal is to find the best solution (parameter configuration) for a program, which will try to find the best solution for some problem as well. In order to measure the quality of some found parameter setting for a program (solver), one of these criteria are taken into consideration: the speed of the run or the quality of the found solution for the problem that it solves.

The are tow classes to classify these methods:

*a) Off-line tunning*: Also known just as parameter tuning, were parameters are computed before the run.

*b) On-line tunning*: Also known as parameter control, were parameters are adjusted during the run, and

### 1.8.1 Off-line tunning

The technique of parameter tuning or off-line tunning, is used to computed the best parameter configuration for an algorithm before the run (solving a given instance of a problem), to obtain the best performance. Most of algorithms are very sensible to their parameters. This is the case of Evolutionary Algorithms (EA), were some parameters define the behavior of the algorithm. In [104] is presented a study of methods to tune these algorithms.

In [105] is presented *EVOCA*, a tool which allows meta-heuristics designers to obtain good results searching a good parameter configuration with no too much effort, by using the tool during the iterative design process. Holger H. Hoos highlights in [106] the efficacy of the technique named *racing procedure*, that is based on choosing a set of model problems and adjusting the parameters through a certain number of solver runs, discarding configurations that show a behavior substantially worse than the best already obtained so far.

ParamsILS (version 2.3) is a tool for parameter optimization for parametrized algorithms, which uses powerful stochastic local search methods and it has been applied with success in many combinatorial problems in order to find the best parameter configuration [107]. It is an open source program written in *Ruby*, and the public source include some examples and a detailed and complete User Guide with a compact explanation about how to use it with a specific solver [108].

Revac is a method based on information theory to measure parameter relevance, that calibrates the parameters of EAs in a robust way. Instead of estimating the performance of an EA for different parameter values, the method estimates the expected performance when parameter values are chosen from a given probability density distribution $C$. The method iteratively refines the probability distribution $C$ over possible parameter sets, and starting

with a uniform distribution $C_0$ over the initial parameter space $\mathcal{X}$, the method gives a higher and higher probability to regions of $\mathcal{X}$ that increase the expected performance of the target EA [109]. In [110] is presented a case study demonstrating that using the REVAC the "world champion" EA (the winner of the CEC-2005 competition) can be improved with few effort.

Another technique was successfully used to tune automatically parameters for EAs, through a model based on a *case-based reasoning* system. It attempts to imitate the human behavior in solving problems: look in the memory how we have solved a similar problem [111] .

### 1.8.2 On-line tunning

Although parameter tunning shows to be an effective way to adjust parameters to sensibles algorithms, in some problems the optimal parameter settings may be different for various phases of the search process. This is the main motivation to use on-line tuning techniques to find the best parameter setting, also called *Parameter Control Techniques*. Parameter control techniques are further divided into i) *deterministic parameter control*, where the value of a strategy parameter is altered by some deterministic rule, ignoring any feedback; ii) *adaptive parameter control*, which continually update their parameters using feedback from the population or the search, and this feedback is used to determine the direction or magnitude of the parameter changes; and iii) *self-adaptive parameter control*, which assign different parameters to each individual, Here the parameters to be adapted are coded into the chromosomes that undergo mutation and recombination, but these parameters are coded into the chromosomes that undergo mutation and recombination [112].

Differential Evolution (DE) algorithm has been demonstrated to be an efficient, effective and robust optimization method. However, its performance is very sensitive to the parameters setting, and this dependency changes from problem to problem. The selection of proper parameters for a particular optimization problem is a quite complicate subject, especially in the multi-objective optimization field. This is the reason why many researchers are motivated to develop techniques to set the parameters automatically.

Liu et al. propose in [113] an adaptive approach which uses fuzzy logic controllers to guide the search parameters, with the novelty of changing the mutation control parameter and the crossover during the optimization process. A self-adaptive DE (SaDE) algorithm is proposed in [114], where both trial vector generation strategies and their associated control parameter values are gradually adjusted by learning from the way they have generated their previous promising solutions, eliminating this way the time-consuming exhaustive search for the most suitable parameter setting. This algorithm has been generalized to multi-objective realm, with objective-wise learning strategies (OW-MOSaDE) [115].

Drozdik et al. present in [116] a study of various approaches to find out if one can find an inherently better one in terms of performance and whether the parameter control mechanisms can find favorable parameters in problems which can be successfully optimized only with a limited set of parameters. They focused in the most important parameters: 1) the *scaling factor*, which controls the structure of new invidious; and 2) the *crossover probability*.

Meta-GAs [117] is a genetic self-adapting algorithm, adjusting genetic operators of genetic algorithms. In this paper the authors propose an approach of moving towards a Genetic Algorithm that does not require a fixed and predefined parameter setting, because it evolves during the run.

# 2

# Prior works leading to POSL

*In this chapter are presented Prior works leading to* POSL. *In Section 2.1 we explain how to tackle CSPs by modeling it through a continuous optimization problem, as a first attempt aiming for the right direction in order to find the proper approach. In Section 2.2 we present a brief work where we applied the the problem subdivision approach to solve the k-medoids problem in parallel. Finally we present in Section 2.3 a study applying the* ParamILS *tool in order to find the optimum parameter configuration to Adaptive Search solver.*

## Contents

## 2.1 Relaxation model for discrete *CSPs*

Aiming for the right direction in order to find the proper approach, our first attempt to tackle the problem of reducing the search space of a *CSP*, we model it through a continuous optimization problem, and then, applying efficient methods to solve it. This way we do not reach an optimal solution, but an approximation of it. The new variables domain would be the neighborhood of the found approximation.

On a first attempt to tackle the problem of reducing the search space of a CSP, we thought to model it through a continuous problem, and after, trying to apply efficient methods to solve this kind of problems. This way we do not reach an optimal solution, but an approximation of it. The new variables domain would be the neighborhood of the found approximation. In [118] some issues to take into account in order to model combinatorial problems using continuous problems are showed.

To illustrate the fallowed procedure, we will use a widely known problem: *n-queens problem*.

In the *n-queens problem*, you want to place $n$ queens on an $n \times n$ chessboard (square grid). Each queen occupies one square on a grid and no two queens share the same square. Two queens are attacking each other if one of them can travel horizontally, vertically, or diagonally and hit the square the other queen is on. The problem is to place the queens such that no two queens are attacking each other.

The Constraint Satisfaction Problem associated to the $n \times n$ N-Queens problem:

$$\begin{aligned} \text{CSP} = &\ \{\mathbb{X}, \mathbb{D}(\mathbb{X}), \mathbb{C}\} \\ \mathbb{X} = &\ (x_1, x_2, \ldots, x_n) \\ \mathbb{D}(\mathbb{X}) = &\ \{\mathbb{D}(x_i)\}, \mathbb{D}(x_i) = \{1, 2, \ldots, n\}, \forall i = 1 \ldots n \\ \mathbb{C} = &\ \{\mathbb{C}_1, \mathbb{C}_2\} \end{aligned} \tag{2.1}$$

In Model 2.1, the value of $x_i$ represent the position of a queen in the column $i$. This representation ensures that only one queen will be placed in a given column. To ensure that only one queen can be placed in a row, we can write the following constraint:

$$\mathbb{C}_1 \equiv x_i \neq x_j$$
$$\mathbb{C}_1 \equiv x_i - x_j > 0 \text{ or } x_i - x_j < 0$$
$$\mathbb{C}_1 \equiv (x_i - x_j)^2 > 0, \forall i < j$$

Then, we only have to ensure that two queens can not be connected diagonally:

$$\begin{aligned}
&\mathbb{C}_2 \equiv |x_i - x_j| \neq |j - i| \\
&\mathbb{C}_2 \equiv (x_i - x_j)^2 \neq (j - i)^2 \\
&\mathbb{C}_2 \equiv (x_i - x_j)^2 - (j - i)^2 > 0 \text{ or } (x_i - x_j)^2 - (j - i)^2 < 0 \\
&\mathbb{C}_2 \equiv \left((x_i - x_j)^2 - (j - i)^2\right)^2 > 0, \forall i < j
\end{aligned}$$

We describe below the transformation into a non-linear optimization problem penalized.

$$\text{NLOP} = \begin{cases}
\min \quad f(X) = & -P_1 \cdot \sum\limits_{i<j} (x_i - x_j)^2 \\
& -P_2 \sum\limits_{i<j} \left((x_i - x_j)^2 - (j - i)^2\right)^2 - \\
& -P_3 \sum\limits_{i<j} \left((x_j - x_i)^2 - (j - i)^2\right)^2 \\
X = & (x_i),\ i = 1 \ldots n \text{ and } i < j \\
\text{s.t.:} \quad 1 \leq x_i \leq n &
\end{cases} \tag{2.2}$$

The global minimum $\zeta$ exists and $f(\zeta) = 0$. The main key here is to choose $P_i$, $i = 1..3$ big enough. In this way we obtained a non-linear optimization model with box-constraints, and from [119, 120, 121] we have an efficient program to solve this kind of problems (*L-BFGS-B*), even for large instances [122]. This work provide an open source code in C#, and to use it we had to implement a subroutine (method) called **fungrad**, telling to the algorithm how to evaluate the objective function and his gradient.

This approach did not show good results. One possible reason is that, as we can see, a quadratic model at first became in a more complex model with 4-exponent. It is fear to remark also that this problem can have several local optimums ($\bigtriangledown F(X^*) = 0$) different than the global optimum, and this algorithm returns them as solutions. In a continuous point of view two different variables have different values even when they are too close, and this can be the other reason. The previous model can be improved in order to obtain better results. Also a good idea is to choose others benchmark problems as case of study, but we decide to not keep working on that because the investigation in this field will take us further apart from the thesis main goal.

## 2.2 Domain Split

A way to tackle huge combinatorial problems in parallel is to split the search space. In this section, the *problem subdivision* approach was adopted to divide the domain of a given problem, in this particular case, to solve the *k-medoids problem* in parallel. [i]

As was previously said, a way to tackle huge combinatorial problems in parallel is to split the search space. In this section, the *problem subdivision* approach was adopted to divide the domain of a given problem, in this particular case, to solve the *k-medoids problem* in parallel [ii].

In [80] two main techniques are presented for domain partitioning: first, *space-filling curves*, used to reduce any N-dimensional representation into a one-dimension space (this technique is also widely used in the nearest-neighbor-finding problem [81]); and second, *k-Means* algorithm, one of the most popular clustering algorithms [82]. In this work we found that the methods used for domain partitioning do not take into account the number of clients associated to each new sub-domain. This results in an unbalanced distributions of workload phenomenon. That's why our goal was designing some ideas to tackle this problem, perhaps not through an exact manner, but at least more approximated.

First, we propose **Algorithm 1**, which represents the backbone of our idea. This algorithm takes a set of $\mathbb{R}^2$ *points* (representing the locations of the clients) and returns a partition of size $K$. Such a set of points is called a *domain* and the partition a *sub-domain*. At each intermediate step $i$, we have a set (list) of sub-domains. The algorithm takes the most populated one, splits it into two (or four, depending on the strategy) new sub-domains and includes them in the list. The stop condition will depends on the fallowed approach (see

[i] This work falls within the framework of the *Ulysses* project between France and Ireland
[ii] This work falls within the framework of the Ulysses project between France and Ireland

below).

**Algorithm 1:** Domain_Split

**input** : U: Set of client locations

**output**: Q = $\{Q_i\}_{i=1...K}$: $K$ subsets of U

```
1 A ← U
2 Q.Insert(A)
3 repeat
4 |   A ← Q.GetNext() /* It also removes the returned element */
5 |   [a1, a2] ← Split(A)
6 |   Q.Insert(a1, a2)
7 until <some condition>
```

First of all, we make clear some details of the algorithm exposed above:

- `Insert(...)` Inserts a set (or two) into the data structure [iii].
- `GetNext()` Returns the next sub-set to be divided, tacking into account the *split strategy* (see below).
- `Split(...)` Returns two sub-domains as a subdivision of the given domain (parameter).

In the next sub-sections we will answer two main question arising at this point:

*a) How to split the each sub-domain?* (<*Split*> function on line 5) It refers to, given a set of points (locations), how to decide which of them will be included in one sub-domain and which of them in the other.

*b) How much to split each sub-domain?* (<*some condition*> on line 3) It refers to decide when to stop splitting the domain.

### 2.2.1 Domain Splitting. General point of view

In order to split the domain, we can think in some approaches, taking into account the number of available cores and the number of metro-nodes we want to place in the system. In the article of A. Arbelaez and L. Quesada a domain split taking into account the number of available cores for parallel calculus is proposed. In our approach we intend to extend this idea keeping in mind also the number of metro-nodes to allocate. Following, we propose three variants to face the problem:

[iii] The design is not decided yet, because first we need to know some implementations details about the tool already coded by the Ireland team

*a)* **one metro-node per core**: In this variant we can assign one metro-node to each core, and in this case, split the domain in $K$ sub-domains ($K$ is the number of cores). It means that the algorithm will compute the best position for a metro-node in a current sub-domain.

- In this case we only have to replace the line 3 by something like: **for** $i \leftarrow 1$ **to** $N$ **do** ..., where $N$ is the number of metro-nodes.
- The ideal scenario here is when $N = K$ which is not probable at all. So we only should study the case when they are different
- In that case, we need to distribute efficiently the metro-nodes into the domain subdivisions, but here, one possible scenario arise: it can be happen that (depending on the followed domain-split strategy) we were trying to allocate a metro-node into an area with a few clients. This produce a very local point of view of the problem. That's why we propose the following *second variant*.

*b)* **Incomplete partition**: To split a sub-domain if it can generate sub-sub-domains containing at least $C$ clients. It means, for example, if there exist in list a sub-domain with 8 clients, and the number $C$ is fixed in $C = 5$, this sub-domain can not be divided, because it will generate for sure a sub-sub-domain with less that 5 clients. In this case more than one metro-node would be allocated in some of those specific sub-domains, because we will have more metro-nodes than sub-domains in our model.

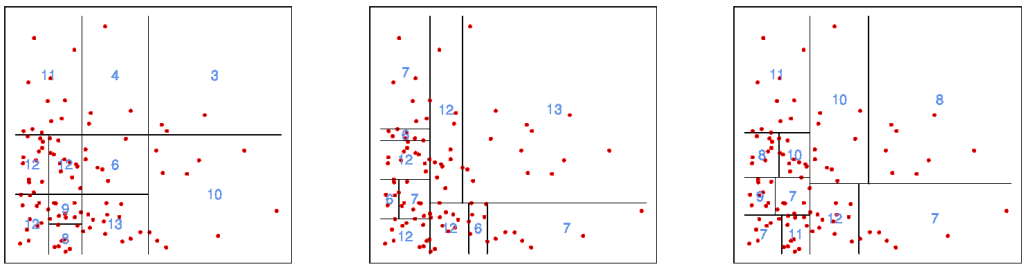
- Of course, using this variant we can find situations described in the first variant. I that case we should proceed consequently.

*c)* **Combination**: This variant is just a combination of the two previous variants. Then, we will be working with two parameters:

- $C \rightarrow$ Lower bound of clients for new sub-domains. It means that a sub-domain can be divided iff the new produced sub-domains will contains more than $C$ clients.
- $M \rightarrow$ Lower bound of metro-nodes to be allocated in a sub-domain. In this case we will split the domain while it can be ensure that at least $M$ metro-nodes will be assigned to each sub-domain.

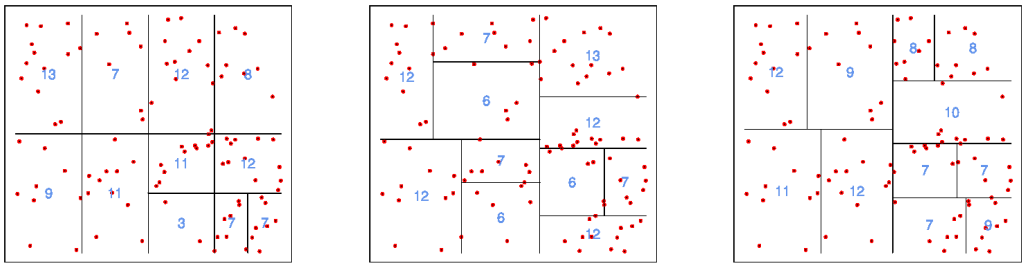
### 2.2.2 Split strategies

In this sub-section we first discuss three ideas to split the domain. They take a sub-domain and produce other two, dividing the space vertically or horizontally, depending on the shape



**(a)** Geometrical Split **(b)** Cardinal Split **(c)** Mean Split

**Figure 2.1:** Split domain of point normally distributed $\mathcal{N}(0, 0.35)$



**(a)** Geometrical Split **(b)** Cardinal Split **(c)** Mean Split

**Figure 2.2:** Split domain of point uniformly distributed

of the current sub–domain. In other section we expose another strategy to follow, in which the subdivision of a sub-domain produces four sub-domains.

In all of them the number of clients in each sub-domain is taken into account, but in different ways. A common feature between them is that, at the moment of splitting a given sub-domain, it will be done in a perpendicular way (either to the x–axis or to the y–axis, depending on the characteristic of the sub-domain to be divided). The reference point to divide the sub-domain is called the *cut point* of the sub-domain.

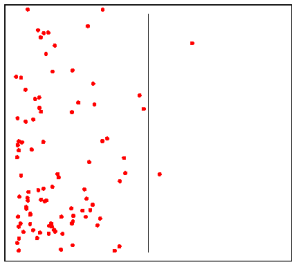


**Figure 2.3:** If we assume $C = 3$ then this set can't be divided

*a)* **Geometrical Split**:

- Split criterion: *Geometrical* → Dividing the region in tow parts with the same area.
- Cut point: The middle point of the x–axis (or y–axis, depending on the shape)
- Result: Two equal regions, but with different amount of clients (see Figure 2.1a). In the next step, the next sub-domain to divide will be, from those already divided, the most populated one (the sub-domain with more clients).
- Problem: Maybe we need to divide a sub-domain because it has a lot of client, but in the other hand it generate a sub-domain with a few clients, as you can see in the Figure 2.3.
- Benefit: Very fast split. If we attack scenarios with point uniformly distributed, the behavior is pretty much desirable, as we can see in the Figure 2.2a.

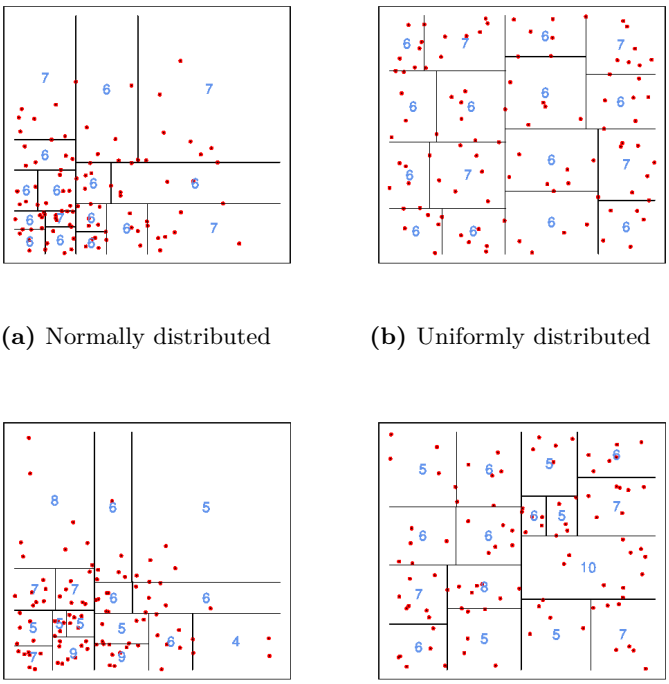
*b)* **Cardinal Split**

- Split criterion: The number of clients, i.e., a sub-domain is divided in such a way that in the resulting sub-sub-domains there will be the same number of clients.
- Cut point: The current sub-domain is ordered and the x–axis (or y–axis, depending on the shape) of the element (location of the client) right in the center is selected to be the *cut point*
- Result: Two regions with the same ($\pm 1$) amount of clients (see Figure 2.1a). In the next step, the next sub-domain to divide will be, from those already divided, the most populated one (the sub-domain with more clients).
- Problem: The subdivision process is a bit more costly: we need to group the clients on both sides of a *perfect pivot*[iv].
- Benefit: It guarantees the same cardinality in both new sub-sub-domains.

*c)* **Mean Split**

- Split criterion: This is a mid-point strategy between the two previous. The goal is to find a *cut point* to group the elements of the current sub-domain, but in a easy way (fast), that's why we don't compute the exact middle point to produce two sub-sub-domains with the same cardinality, as in the previous approach, instead of that we work with his expected value: the arithmetic mean.
- Cut point: We compute the mean of the $x$–axis (or $y$–axis) of the elements (locations) of the current sub-domain, and it will be the *cut point*

[iv] Element of a set with the same number of elements lower and greater than him



**(a)** Normally distributed **(b)** Uniformly distributed

**(c)** Normally distributed **(d)** Uniformly distributed

**Figure 2.4:** Domain divided in $2^4 = 16$ sub-domains: (a) - (b) Cardinal; (c) - (d) Mean.

- Result: Two regions with not exact information about their sizes or their cardinality.

- Problem: *Idem.*

- Benefit: The *cut point* can be obtained by a $O(n)$ number of operations. As we can see in the preliminary results[v] (Figure 2.4c and 2.4d), the behaviour of this technique is near to the *cardinal split* technique, at least for normally and uniformly distributed sets of point.

### 2.2.3 Conclusion

This is a theoretical work that we have not validated, because we decide to tackle the problem using a different approach that we will explain in the next section.

[v] The experiments are coded in R[123].

## 2.3 Tunning methods for local search algorithms

Another performed study was applying ParamILS (version 2.3), a tool to find the optimum parameter settings for parametrized algorithms [107], to *Adaptive Search* solver. It is an open source program (project) in *Ruby*, and the public source code include some examples and a detailed and complete User Guide with a compact explanation about how to use it with a specific solver [108].

The first thing we had to do was building a *wrapper* to be able to use the tool. This wrapper was coded in C++ language.

Tacking advantage of the fact that we have already built a wrapper to our solver, the next step can be tuning more interesting problems and after that, using the tool to tune the solver but not only to find the best parameter configuration for a specific problem, but also the best parameter configuration to solve any of the benchmarking (a general parameter configuration).

In a first study, we decided to work with the parameters detailed in Table 2.1.

**Table 2.1:** Adaptive Search parameters

| Parameter | Type | Description |
|---|---|---|
| -P | PERCENT | probability to select a local min (instead of staying on a plateau) |
| -f | NUMBER | freeze variables of local min for NUMBER swaps |
| -F | NUMBER | freeze variables swapped for NUMBER swaps |
| -l | LIMIT | reset some variables when LIMIT variable are frozen |
| -p | PERCENT | reset PERCENT of variables |

In this document, we explain in details the implementation and experimentation process.

### 2.3.1 ParamILS_2.3

ParamILS is a open source program (project) in *Ruby*. It can be downloaded from

`http://cs.ubc.ca/labs/beta/Projects/ParamILS`.

The `zip` file includes some examples that you can run and see how the tool works. It In addition, it brings a complete User Guide with a compact explanation about how to use it with a specific solver.

### 2.3.2 Using ParamILS

To use the tool ParamILS, we have installed Ruby 1.8.7 in our computer. In this firs step, we are using a laptop with UBUNTU 14.4 but our idea is to make some tests in *Curiosiphi*[vi].

To run the tool, we need to use the following command line:

```
$ ruby param_ils_2_3_run.rb -numRun 0 -scenariofile
      /.../<scenario_file> -validN 100
```

All the information that ParamILS needs to tune the solver has to be written in the file <scenario_file>. We explain its content in the next section.

### 2.3.3 Tuning scenario files

The *tuning scenario file* is a text file with all the information concerning to how to tune the solver with ParamILS. It includes where to find the solver binary, the parameter domains, etc. In our case, the *tuning scenario file* looks like the following:

```
algo = ./QtWrapper_wrapper
execdir = /.../src
deterministic = 0
run_obj = runtime
overall_obj = mean
cutoff_time = 50.0
cutoff_length = max
tunerTimeout = 3600
paramfile = instances/all_intervals-params.txt
outdir = instances/all_intervals-paramils-out
instance_file = instances/.../all_intervals-lower-instances.txt
test_instance_file = instances/.../all_intervals-upper-instances.txt
```

Next, we will explain in details what each line in this file means:

- **algo** An algorithm executable or a call to a wrapper script around an algorithm that conforms with the input/output format of ParamILS. (our wrapper)

[vi] ...

- **execdir** Directory to execute `algo` from: "cd <`execdir`>; <`algo`>"
- **run_obj** A scalar quantifying how "good" a single algorithm execution is, such as its required runtime.
- **overall_obj** While **run_obj** defines the objective function for a single algorithm run, **overall_obj** defines how those single objectives are combined to reach a single scalar value to compare two parameter configurations. Implemented examples include **mean**, **median**, **q90** (the 90% quantile), **adj_mean** (a version of the mean accounting for unsuccessful runs: total runtime divided by number of successful runs), **mean1000** (another version of the mean accounting for unsuccessful runs: (total runtime of successful runs + 1000 x runtime of unsuccessful runs) divided by number of runs – this effectively maximizes the number of successful runs, breaking ties by the runtime of successful runs; it is the criterion used in most of Frank experiments), and **geomean** (geometric mean, primarily used in combination with **run_obj** = `speedup`. The empirical statistic of the cost distribution (across multiple instances and seeds) to be minimized, such as the mean (of the single run objectives). [vii].
- **cutoff_time** The time after which a single algorithm execution will be terminated unsuccessfully. This is an important parameter: if chosen too high, lots of time will be wasted with unsuccessful runs. If chosen too low the optimization is biased to perform well on easy instances only.
- **tunerTimeout** The timeout of the tuner. Validation of the final best found parameter configuration starts after the timeout.
- **paramfile** Specifies the file with the parameters of the algorithms.
- **outdir** Specifies the directory ParamILS should write its results to.
- **instance_file** Specifies the file with a list of training instances.
- **test_instance_file** Specifies the file with a list of test instances.

Another important file that we have to compose properly is the *algorithm parameter file*, just following the instruction from [108] –*[...] each line lists one parameter, in curly parentheses the possible values considered, and in square parentheses the default value [...].* Our *algorithm parameter file* looks like this:

```
P {20, 25, 30, 35, 40, 45, 50, 55, 60} [50]
f {0, 1, 2, 3} [1]
F {0, 1, 2, 3} [0]
```

[vii] We use **mean** but maybe we can experiment with other values

```
l {0, 1, 2, 3} [1]
p {1, 2, 3, 5, 10, 20} [5]
```

In the actual *Adaptive Search* implementation, the solver binary file and the problem instance are the same thing. It means that we only have to use the following command to solve the *All—intervals* problem of size $K$:

```
$ ./all-intervas K
```

So, to use ParamILS we modified a little our code: now our solver takes the size parameter from a text file. That way, the instance file is a text file only containing a number.

In order to use ParamILS to tune *Adaptive Search*, the last must comply with specific input/output rules. One solution could be modified the code of the actual *Adaptive Search* implementation, but we preferred to build a wrapper instead.

### 2.3.4 Building the wrapper

The algorithm executable must follow the input/output criteria presented below:

**Called as:**

```
$ <algo_exectuable> <instance_name> <instance-specific_information>
<cutoff_time> <cutoff_length> <seed> <params>
```

- <algo_exectuable> Solver
- <instance_name> In our case, a text file containing only the problem size
- <instance-specific_information> We don't use it
- <cutoff_time> Cut off time for each run of the solver (see above)
- <cutoff_length> We don't use it
- <seed> Random seed
- <params> Parameters and its values

Exmaple:

```
$ ./QtWrapper _320.txt "" 50.0 214483647 524453158 -p 5 -l 1 -f 1 -P 50 -F 0
```

**Outputs:**

`Result for ParamILS: <solved>, <runtime>, <runlength>, <best_sol>, <seed>`

- <**solved**> `SAT` if the algorithm terminates successfully. `TIMESOUT` if the algorithm times out.
- <**runtime**> Runtime
- <**runlength**> -1 (as Frank Hutter recommended)
- <**best_sol**> -1 (as Frank Hutter recommended)
- <**cutoff_length**> We don't use it
- <**seed**> Used random seed

Exmaple:

```
$ SAT, 2.03435, -1, -1, 524453158
```

To build the wrapper we have just followed a simple algorithm: launch two process "in parallel". In the parent process we translate the input of the wrapper to the input of *Adaptive Search*. The solver is executed, and the runtime is measured. After that we post the output properly. In the child process a *sleep* command is executed for <`runtime`> seconds and after that, if the parent process has not finished yet, it is killed, posting a times out message. See the **Algorithm** 2 for more details.

**Algorithm 2:** Costas Wrapper

**input** : $Pth_{\pi}$: problem instance path, $k$: cut off time, s: random seed, $\theta$: parameters configuration

**output**: $PiLS_{out}$: Output in a ParamILS way

```
1  fork() /* Divide the execution in two processes                    */
2  if <in child process> then
3      t_0 ← clock_TIC()
4      strCall ← build_str(" ./AS_Wrapper %1 -s %2 %3", Pth_π, s, θ)
5      systemCall(strCall)
6      t_e ← clock_TOC()
7      killProcess(<parent process>)
8      t ← t_e- t_0
9      return paramilsOutput(SAT, t, s)
10 else
11     sleep(k)
12     killProcess(<child process>)
13     return paramilsOutput(TIMESOUT, k, s)
14 end
```

### 2.3.5 Using the wrapper

#### 2.3.5.1 Factory call

The first thing we have to do is to implement the class ICallFactory. Here, the string-binary-name for the command call is statically obtained.

We present an example, the class All_IntervalCallFactory

```
// all_interval_call_factory.h
 class All_IntervalCallFactory:  public ICallFactory
{
    public:
       std::string BuildCall();
       std::string BuildDefaultCall();
};
```

```
// all_interval_call_factory.cpp
 #define ALGO_EXECUTABLE "./all-interval"
 #define DEFAULT_CALL "./all-interval _100.txt"

std::string All_IntervalCallFactory::BuildCall()
{
    return ALGO_EXECUTABLE;
}
std::string All_IntervalCallFactory::BuildDefaultCall()
{
    return DEFAULT_CALL;
}
```

All we have to do is to define our new macro **ALGO_EXECUTABLE** (**DEFAULT_CALL** is not being used)

#### 2.3.5.2 Main method

Let's suppose now that we want to run an algorithm called *mySolver* that receives a file as a parameter, called *my_instance_size.txt* (this is mandatory).

So, we have to create (as we've explained before) the class My_SolverCallFactory and to define the macro as follows:

```
#define ALGO_EXECUTABLE "./mySolver"
```

Now, the main method would be exactly like this:

```
int main( int argc,  char* argv[])
{
   shared_ptr<ICallFactory> problem =
      make_shared<My_SolverCallFactory>();
   shared_ptr<TuningData> td =
      (make_shared<TuningData>(argc, argv, problem));

   shared_ptr<ADWrapper> w (make_shared<ADWrapper>());
   string output = w->tune(td);

   cout << output << endl;
    return 0;
}
```

### 2.3.6 Results

#### 2.3.6.1 Tuning *All-intervals*

Study cases:

*a)* The *training instances set* is composed by instances of *All–Intervals* problems of order $N$ with

$$N \in \{100, 110, 120, 130, 140, 150, 160, 170, 180\}$$

*b)* The *test instances set* is composed by instances of *All–Intervals* problems of order $N$ with

$$N \in \{190, 200, 210, 220, 230, 240, 250, 260, 265\}$$

*c)* The cutoff for each run was 50.0 seconds

*d)* The test quality is based on 100 runs

**Experiment 1** **Parameters domains**:

- **P** `{41, 46, 51, 56, 60, 66, 71, 76, 80}`
- **F, f, l** `{0, 1, 2, 3}`
- **p** `{5, 10, 15, 20, 25, 30, 35}`

The results are presented in Table 2.2.

**Experiment 2** **Parameters domains**:

- **P** `{41, 46, 51, 56, 60, 66, 71, 76, 80}`
- **F, f, l** `{0, 1, 2, 3}`
- **p** `{5, 10, 15, 20, 25, 30, 35}`

The results are presented in Table 2.3.

**Experiment 3** **Parameters domains**:

- **P** `{10, 20, 30, 40, 50, 60, 70, 80, 90}`
- **F, f, l** `{0, 1, 2, 3, 4, 5, 6, 7, 8}`
- **p** `{10, 20, 30, 40, 50, 60, 70}`

The results are presented in Table 2.4.

**Testing parameters** To the next experiment, only the results obtained in the Tables 2.2 and 2.4 were took into account, since they were obtained by using longer cut-off times. As it can be observed in those tables, *Adaptive Search* seems to show a good behavior if the parameters **F**, **P** and **l** are in the following sets: **F** ∈ `{ 0, 1}`, **P** ∈ `{ 80, 90}` and **l** ∈ `{ 0, 1}`.

In that sense, an specific configuration was extracted from the results above, and 60 runs of *Adaptive Search* were performed solving *All–Intervals* ($N = 600$) benchmark:

- 30 using the default parameter configuration (`-F 0 -P 66 -f 1 -l 1 -p 25`)
- 30 with an optimal parameter configuration extracted from the Tables 2.2, 2.4 (`-F 0 -P 80 -f 6 -l 1 -p 10`)

Tables 2.5 shows the results by using the default parameter configuration, and Table 2.6 shows the results by using the parameter configuration found by *ParamILS*, and it is clear that the default configuration shows better results than *ParamILS*'s one.

### 2.3.6.2 Tuning *Costas Array*

Study cases:

*a)* The *training instances set* is composed by instances of *Costas Array* problems of order $N$ with $9 \leq N \leq 15$

*b)* The *test instances set* is composed by instances of *Costas Array* problems of order $N$ with $14 \leq N \leq 19$

*c)* The cutoff for each run was 60.0 seconds

*d)* The test quality is based on 100 runs

**Tuning experiments** **Parameters domains**:

- **P** `{10, 20, 30, 40, 50, 60, 70, 80, 90}`
- **F, f, l** `{0, 1, 2, 3, 4, 5, 6, 7, 8}`
- **p** `{5, 10, 20, 30, 40, 50, 60, 70}`

The results are presented in Table 2.7.

**Testing parameters** The Table 2.7 shows, how *Adaptive Search* seems to be not sensitive to parameters **F** and **p**, i.e. they don't change during the tuning process. On the other hand, the tuner seems to find some optimum values for the other parameters: **P** $\in$ `{ 80, 90}`, **f** $\in$ `{ 4, 5}` and **l** = 2.

In that case also, an specific configuration was extracted from the results showed in Table 2.7, and 60 runs of *Adaptive Search* were performed solving *Costas Array* ($N = 20$) benchmark:

- 30 using the default parameter configuration (`-F 0 -P 50 -f 1 -l 0 -p 5`)
- 30 with an optimal parameter configuration extracted from the Table 2.7 (`-F 3 -P 90 -f 5 -l 2 -p 30`)

Table 2.8 shows the results by using the default parameter configuration, and Table 2.9 shows the results by using the parameter configuration found by *ParamILS*. One more time, "in the mean", the default configuration wins.

## 2.3.7 Tuning comparison

### 2.3.7.1 Experiment 1: Around Default parameters

**Parameters domains**:

- **P** `{43, 45, 47, 50, 53, 55, 57}`
- **F**, **f**, **l** `{0, 1, 2}`
- **p** `{5, 7, 10}`

The results are presented in Table 2.10.

### 2.3.7.2 Experiment 2: Around ParamILS parameters

**Parameters domains**:

- **P** `{75, 77, 80, 83, 85, 87, 90, 93, 95}`
- **f** `{4, 5, 6}`
- **F** `{2, 3, 4}`
- **l** `{1, 2, 3}`

- **p** `{20, 25, 30, 35, 40}`

The results are presented in Table 2.11.

### 2.3.8 Conclusion

The conclusion of this study is that the tunning process by hand in this case was more effective than using *ParamILS*. The results show that the found parameters are less effectives than the default parameters in both cases.

**Table 2.2:** Results with `tunerTimeout` = 20000 seconds

| Initial configuration | | | | | Final best configuration | | | | | Training quality | Number of runs | Test quality |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| F | P | f | l | p | F | P | f | l | p | | | |
| 0 | 66 | 1 | 1 | 25 | 0 | 80 | 2 | 1 | 35 | 0.79666 | 1780 | 8.274 |
| 2 | 56 | 2 | 2 | 20 | 1 | 80 | 1 | 1 | 10 | 0.795 | 1637 | 5.508 |
| 0 | 41 | 0 | 0 | 5 | 1 | 80 | 3 | 0 | 15 | 0.789 | 1547 | 5.8478 |
| 3 | 80 | 3 | 3 | 35 | 1 | 80 | 2 | 0 | 10 | 0.880686 | 1258 | 6.15398 |

**Table 2.3:** Results with `tunerTimeout` = 3600 seconds

| Initial configuration | | | | | Final best configuration | | | | | Training quality | Number of runs | Test quality |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| F | P | f | l | p | F | P | f | l | p | | | |
| 0 | 66 | 1 | 1 | 25 | 0 | 80 | 0 | 1 | 25 | 0.815 | 384 | 5.8191 |
| 0 | 66 | 1 | 1 | 25 | 1 | 80 | 1 | 1 | 35 | 0.737 | 452 | 6.267 |
| 0 | 66 | 1 | 1 | 25 | 1 | 56 | 0 | 1 | 35 | 1.03 | 371 | 9.056 |
| 0 | 66 | 1 | 1 | 25 | 0 | 76 | 0 | 1 | 20 | 0.814 | 385 | 4.915 |
| 0 | 66 | 1 | 1 | 25 | 0 | 80 | 3 | 1 | 20 | 0.76 | 469 | 5.417 |
| 2 | 56 | 2 | 2 | 20 | 0 | 41 | 0 | 1 | 10 | 0.919 | 239 | 18.364 |
| 2 | 56 | 2 | 2 | 20 | 0 | 56 | 1 | 1 | 20 | 0.819 | 407 | 5.409 |
| 2 | 56 | 2 | 2 | 20 | 1 | 80 | 1 | 1 | 35 | 0.772 | 457 | 5.43 |
| 2 | 56 | 2 | 2 | 20 | 1 | 80 | 0 | 1 | 10 | 0.858 | 504 | 5.566 |
| 2 | 56 | 2 | 2 | 20 | 0 | 80 | 1 | 1 | 10 | 0.7845 | 562 | 18.944 |
| 0 | 41 | 0 | 0 | 5 | 0 | 41 | 1 | 0 | 10 | 0.9749 | 367 | 5.97813 |
| 0 | 41 | 0 | 0 | 5 | 0 | 41 | 1 | 0 | 10 | 0.885 | 450 | 5.706 |
| 0 | 41 | 0 | 0 | 5 | 0 | 41 | 1 | 0 | 10 | 0.906 | 335 | 18.707 |
| 0 | 41 | 0 | 0 | 5 | 0 | 41 | 1 | 0 | 10 | 0.995 | 335 | 19.558 |
| 0 | 41 | 0 | 0 | 5 | 0 | 41 | 0 | 0 | 5 | 0.855 | 404 | 5.686 |
| 3 | 80 | 3 | 3 | 35 | 0 | 66 | 3 | 1 | 25 | 0.9118 | 230 | 26.585 |
| 3 | 80 | 3 | 3 | 35 | 0 | 80 | 1 | 1 | 10 | 0.732 | 310 | 7.875 |
| 3 | 80 | 3 | 3 | 35 | 0 | 80 | 0 | 1 | 20 | 0.816 | 303 | 7.2896 |
| 3 | 80 | 3 | 3 | 35 | 1 | 80 | 3 | 1 | 35 | 0.821 | 327 | 6.812 |
| 3 | 80 | 3 | 3 | 35 | 0 | 80 | 0 | 1 | 30 | 0.9203 | 443 | 5.401 |

**Table 2.4:** Results with `tunerTimeout` = 18000 seconds

| Initial configuration | | | | | Final best configuration | | | | | Training quality | Number of runs | Test quality |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| F | P | f | l | p | F | P | f | l | p | | | |
| 0 | 10 | 0 | 0 | 10 | 0 | 40 | 7 | 0 | 50 | 0.883188 | 936 | 6.3191 |
| 0 | 10 | 0 | 0 | 10 | 0 | 80 | 2 | 1 | 40 | 0.774659 | 1584 | 5.45674 |
| 0 | 10 | 0 | 0 | 10 | 0 | 40 | 2 | 0 | 10 | 0.96885 | 1104 | 6.82643 |
| 4 | 60 | 4 | 4 | 40 | 0 | 60 | 8 | 1 | 40 | 0.90358 | 1566 | 5.48127 |
| 4 | 50 | 4 | 4 | 40 | 0 | 80 | 5 | 1 | 20 | 0.78536 | 1662 | 11.5649 |
| 3 | 50 | 4 | 2 | 30 | 0 | 90 | 6 | 1 | 70 | 0.79440 | 1395 | 5.08108 |
| 0 | 90 | 0 | 0 | 10 | 1 | 90 | 6 | 1 | 10 | 0.859569 | 1379 | 5.4286 |
| 0 | 90 | 0 | 0 | 10 | 1 | 90 | 6 | 1 | 30 | 0.80738 | 1117 | 5.47126 |
| 8 | 90 | 8 | 8 | 60 | 0 | 80 | 5 | 1 | 10 | 0.834934 | 1384 | 5.5377 |
| 5 | 30 | 2 | 3 | 60 | 0 | 90 | 1 | 0 | 20 | 0.862013 | 1707 | 5.21837 |
| 3 | 20 | 2 | 4 | 60 | 0 | 80 | 6 | 1 | 10 | 0.805604 | 1630 | 5.4467 |
| 6 | 70 | 1 | 3 | 50 | 0 | 80 | 5 | 1 | 10 | 0.792600 | 1344 | 5.46558 |
| 6 | 40 | 1 | 3 | 30 | 1 | 80 | 7 | 0 | 20 | 0.822703 | 1977 | 5.41185 |

**Table 2.5:** Default configuration runtimes (secs)

| | | | | |
|---|---|---|---|---|
| 37.210 | 411.300 | 112.510 | 171.000 | 73.770 |
| 327.880 | 214.910 | 124.910 | 482.740 | 530.440 |
| 212.660 | 99.370 | 287.400 | 533.540 | **18.410** |
| 197.290 | 1016.950 | 110.230 | 566.480 | **1362.010** |
| 94.860 | 819.700 | 434.460 | 620.600 | 95.920 |
| 80.580 | 333.370 | 121.590 | 489.700 | 248.370 |
| **mean: 341.005333** | | | | |

**Table 2.6:** ParamILS configuration runtimes (secs)

| | | | | |
|---|---|---|---|---|
| 154.460 | 264.530 | 169.840 | 26.990 | 108.790 |
| 550.210 | 104.900 | 31.100 | **9.870** | 1242.900 |
| 678.760 | 475.570 | 201.200 | 622.410 | 297.960 |
| 526.930 | 375.620 | 293.380 | 598.850 | 350.270 |
| 540.290 | 252.940 | 673.630 | 423.030 | 589.210 |
| 32.080 | 254.640 | **2034.020** | 571.100 | 207.090 |
| **mean: 422.085667** | | | | |

**Table 2.7:** Results with `tunerTimeout` = 1800 seconds

| Initial configuration | | | | | Final best configuration | | | | | Training quality | Number of runs | Test quality |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| F | P | f | l | p | F | P | f | l | p | | | |
| 0 | 10 | 0 | 0 | 5 | 2 | 90 | 2 | 2 | 5 | 0.0493699 | 957 | 5.8461 |
| 0 | 10 | 0 | 0 | 5 | 0 | 90 | 5 | 2 | 5 | 0.0509388 | 1783 | 6.52742 |
| 0 | 10 | 0 | 0 | 5 | 0 | 90 | 5 | 2 | 5 | 0.049901 | 1759 | 5.21828 |
| 3 | 40 | 4 | 4 | 30 | 3 | 90 | 5 | 2 | 30 | 0.053974 | 856 | 6.3539 |
| 4 | 50 | 3 | 5 | 20 | 4 | 90 | 5 | 2 | 20 | 0.0500355 | 2000 | 5.4047 |
| 4 | 60 | 5 | 3 | 50 | 4 | 60 | 5 | 3 | 50 | 0.0520575 | 2000 | 6.09106 |
| 8 | 90 | 8 | 8 | 70 | 8 | 80 | 4 | 2 | 70 | 0.052685 | 550 | 3.85682 |
| 8 | 90 | 8 | 8 | 70 | 8 | 80 | 4 | 2 | 70 | 0.054104 | 536 | 4.17855 |
| 8 | 90 | 8 | 8 | 70 | 8 | 80 | 4 | 2 | 70 | 0.0497819 | 1284 | 3.90945 |
| 3 | 10 | 1 | 6 | 60 | 3 | 90 | 5 | 2 | 60 | 0.054934 | 2000 | 6.81675 |
| 5 | 70 | 6 | 1 | 10 | 5 | 90 | 4 | 2 | 10 | 0.0499895 | 2000 | 4.07365 |
| 1 | 30 | 5 | 7 | 5 | 1 | 90 | 4 | 2 | 5 | 0.0525747 | 1237 | 2.70091 |
| 7 | 80 | 2 | 0 | 70 | 7 | 90 | 5 | 2 | 70 | 0.0502264 | 212 | 5.2637 |

**Table 2.8:** Default configuration runtimes (secs)

| | | | | |
|---|---|---|---|---|
| 452.980 | 91.420 | 31.510 | **827.860** | 96.670 |
| 635.030 | 295.830 | 272.360 | 151.040 | 170.660 |
| 183.550 | 161.340 | 91.240 | 426.470 | 62.020 |
| 138.090 | 236.030 | **2.850** | 187.240 | 21.510 |
| 165.370 | 90.440 | 195.580 | 15.390 | 229.720 |
| 170.840 | 174.210 | 30.520 | 6.570 | 115.880 |
| **mean: 191.007** | | | | |

**Table 2.9:** ParamILS configuration runtimes (secs)

| | | | | |
|---|---|---|---|---|
| 546.260 | 263.230 | 17.200 | 29.220 | 495.940 |
| 237.340 | 187.760 | **7.810** | 43.120 | 94.370 |
| 59.930 | 128.690 | 247.810 | 265.010 | 231.260 |
| 209.640 | 465.340 | 21.840 | 8.740 | **1264.610** |
| 57.700 | 122.890 | 450.610 | 229.580 | 174.540 |
| 414.080 | 402.250 | 91.150 | 677.190 | 58.640 |
| **mean: 250.125** | | | | |

**Table 2.10:** Results with `tunerTimeout` = 18000 seconds

| Initial configuration | | | | | Final best configuration | | | | | Training quality | Number of runs | Test quality |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| F | P | f | l | p | F | P | f | l | p | | | |
| 2 | 43 | 0 | 0 | 7 | 0 | 45 | 1 | 0 | 5 | 0.0438025 | 952 | 3.13061 |
| 1 | 55 | 2 | 2 | 10 | 1 | 53 | 2 | 0 | 5 | 0.0434366 | 1120 | 6.8108005 |
| 1 | 55 | 2 | 2 | 10 | 1 | 53 | 2 | 0 | 5 | 0.0435660 | 2000 | 4.6961601 |

**Table 2.11:** Results with `tunerTimeout` = 18000 seconds

| Initial configuration | | | | | Final best configuration | | | | | Training quality | Number of runs | Test quality |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| F | P | f | l | p | F | P | f | l | p | | | |
| 2 | 85 | 6 | 1 | 35 | 2 | 85 | 6 | 1 | 35 | 0.0447855 | 2000 | 5.1182902 |
| 4 | 75 | 4 | 3 | 25 | 4 | 75 | 4 | 3 | 25 | 0.0458100 | 2000 | 3.4968102 |
| 3 | 95 | 5 | 2 | 40 | 3 | 95 | 5 | 2 | 40 | 0.0470930 | 2000 | 4.6591102 |

# 3

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# Part II

Appended Papers